One Born-Oppenheimer Effective Theory for all Exotics

Exotic Hadron Spectroscopy
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Exotic Hadron

- Exotics : more complex structures

- XYZ mesons: Exotic states with at-least 2-heavy quarks

✓ States that don't fit traditional $Q\bar{Q}$ spectrum.

✓ Exotic quantum numbers:

- $J^{PC} = 0^{--}, 0^{+-}, 1^{-+}, 2^{+-}$ etc. are exotic

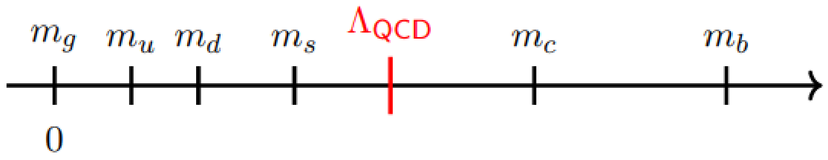
- Charged: Ex. Z_c and Z_b states: minimal 4-quarks:

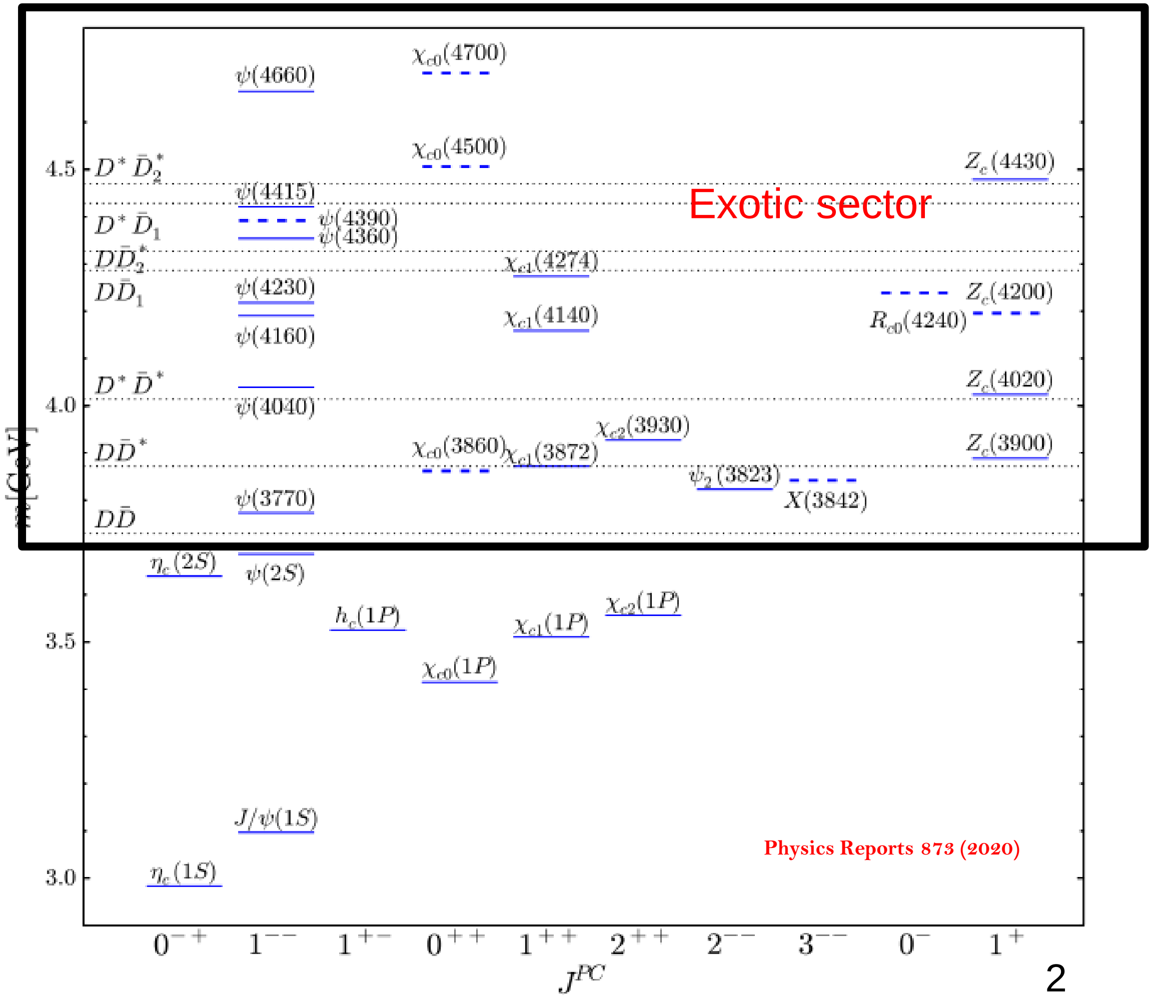
$Z_c(4430)^{\pm}$ $Z_b(10650)^{\pm}$

For review see Brambilla et al. Phys. Reports. 873 (2020)

- Dozens of XYZ mesons discovered since 2003.

$m_c \approx 1.5\,\mathrm{GeV}$ $m_b \approx 5\,\mathrm{GeV}$





Exotic Hadron

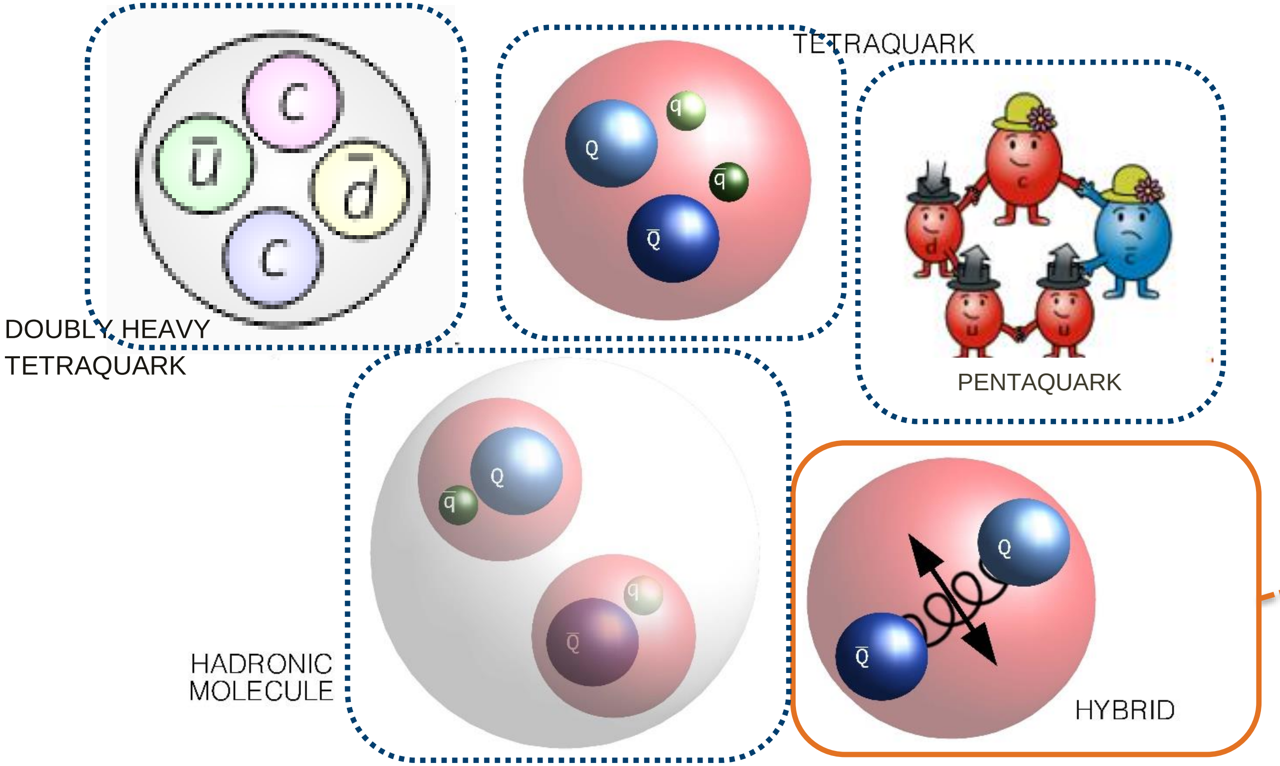


Figure from https://www.fz-juelich.de/en/ias/ias-4/research/exotic-hadrons/exotics_pad.jpg

Figure from Nat Rev Phys 1, 480-494 (2019)   Figure from Montesinos Meson 2023 talk

QUESTION:

Coherent comprehensive framework based on QCD for all X Y Z hadrons ???

Hybrids ($Q\bar{Q}g$): Isospin scalar exotic state.

Use EFT + Lattice
Multiple lattice results on static energies

Brambilla, Lai, AM, Vairo Phys. Rev. D 107, 054034 (2023)

Berwein, Brambilla, Castellà , Vairo Phys. Rev. D. 92, (2015), 114019

Braaten, Langmack, Smith Phys. Rev. D. 90, 014044 (2014)

Soto, Oncala Phys. Rev. D. (2017)

Non-zero isospin states. Use EFT + Lattice.
However, some lattice results on the static energies are available

Brambilla, AM, Vairo arXiv 2406.xxxxx   Soto & Castella Phys. Rev. D. 102, (2020), 014012

BOEFT: Exotic Hadron

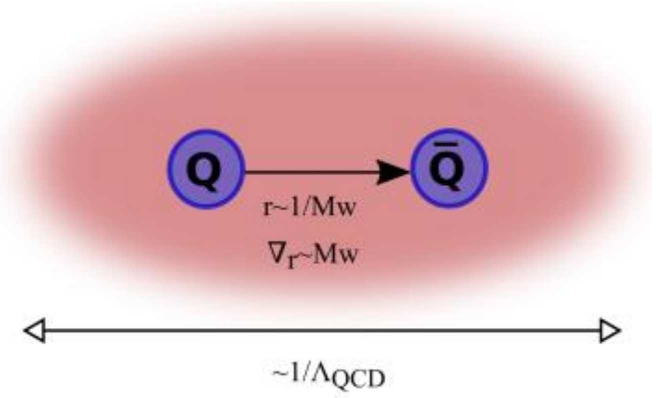
- Exotic hadron $(Q\bar{Q}X, QQX, \ldots\ldots)$, X: any combination of light quark and gluons for color singlet.

- Hierarchy of scales in hybrids:

$$m \gg mv \gtrsim \Lambda_{\mathrm{QCD}} \gg mv^2$$

- ❖ Mass of heavy quark: $\boldsymbol{m}$
- ❖ Energy scale for light d.o.f: $\boldsymbol{\Lambda_{QCD}}$
- ❖ Relative separation between heavy quarks: $\mathrm{r} \sim 1/mv$
- ❖ Hybrids are extended objects: $< r > \gtrsim 0.7\ fm$
- ❖ Heavy Quark dynamics scale: mv^2

Extended objects: $< r > \gtrsim 0.7\ fm$



- Time-scale for dynamics of $Q\bar{Q}$: $\sim \dfrac{1}{mv^2} \gg \dfrac{1}{\Lambda_{QCD}}$

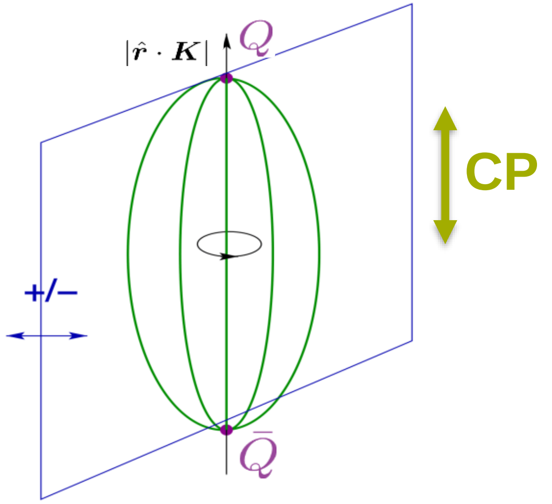
Born-Oppenheimer (BO) Approximation

Juge, Kutti, Morningstar, Phys. Rev. Lett. 90, 161601 (2003)

Braaten, Langmack, Smith Phys. Rev. D. 90, 014044 (2014)

R. Oncala, J. Soto, Phys. Rev. D96 (2017) Berwein, Brambilla, Castellà , Vairo Phys. Rev. D. 92, (2015) Brambilla, Krein, Castellà , Vairo Phys. Rev. D. 97, (2018)

BOEFT: Quantum #'s



- **BOEFT potentials ($V_\Gamma(r)$):** LDF (light quarks, gluons) **static energies.**
  Potential between Q & $\bar{Q}$

- **Static limit (m→ ∞):** heavy quarks are fixed in position.

  Cylindrical symmetry ($D_{\infty h}$) due to preferred quark-antiquark axis

- **BO-quantum number (r ≠ 0)**: $D_{\infty h}$ representations (diatomic molecules):

  ✓ Absolute value of component of angular momentum of light d.o.f

  $$|\boldsymbol{r}\cdot \boldsymbol{K}_{\mathbf{light}}| \equiv \boldsymbol{\Lambda} = \mathbf{0}, \mathbf{1}, \mathbf{2}, \dots \dots . . (\mathbf{or}\ \boldsymbol{\Sigma}, \boldsymbol{\Pi}, \boldsymbol{\Delta}, \boldsymbol{\Phi}, \dots . .)$$

  ✓ Product of charge conjugation and parity ($\boldsymbol{CP}$):

  $$\boldsymbol{\eta} = +\mathbf{1}\ (\mathbf{g}), -\mathbf{1}\ (\boldsymbol{u})$$

  ✓ $\boldsymbol{\sigma}$: Eigenvalue of reflection about a plane containing static sources.

  $$\sigma = P\,(-1)^{K_{\text{light}}} = \pm 1$$

  Born, Oppenheimer, Annalen der Physik 389 (1927)

  Landau, Lifshitz & Pitaevskii, QM book

  $$\Gamma \equiv \Lambda^{\sigma}_{\eta}$$

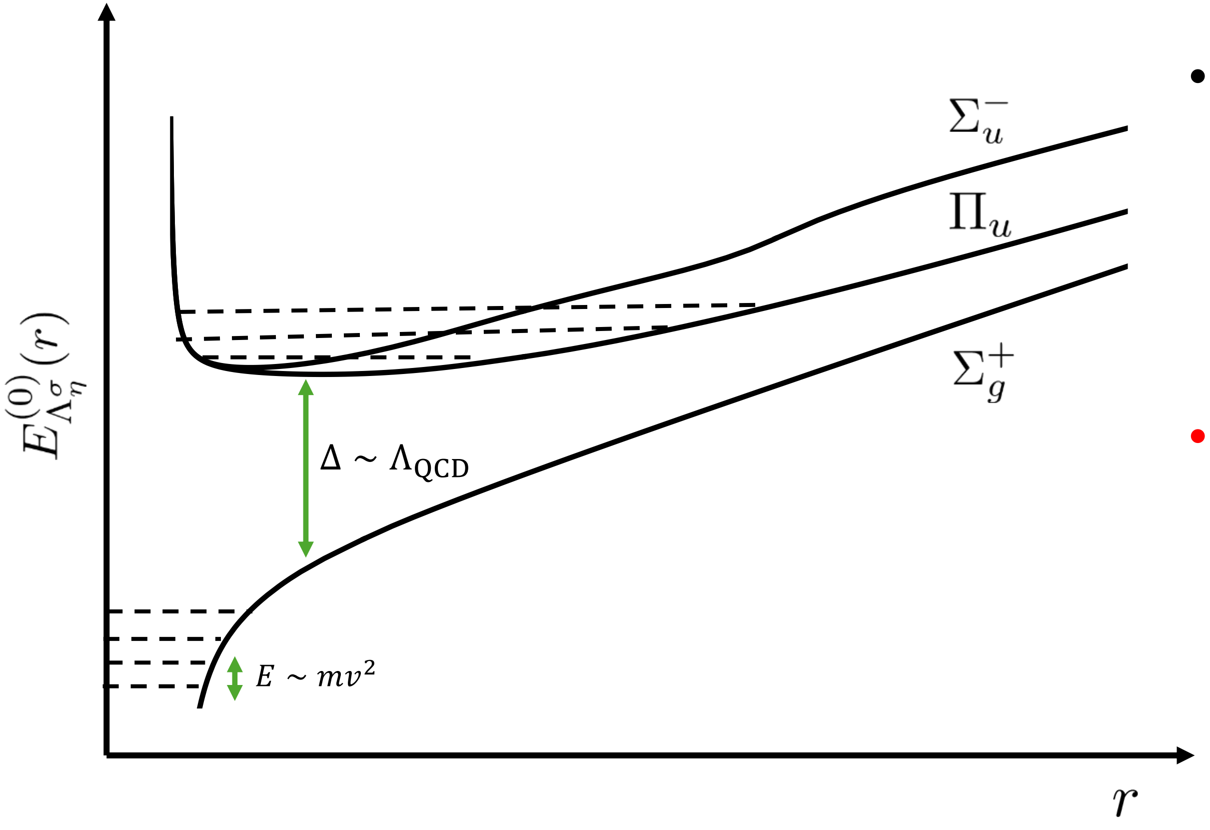
- **BO−quantum number (r → 0)**: **Spherical symmetry restored**: Labelled by gluon quantum #'s $\Gamma \equiv K^{PC}$.

BOEFT

Brambilla, AM, Vairo arXiv 2406.xxxxx

- BOEFT Lagrangian: $$L_{\rm BOEFT} = L_{Q\bar{Q}} + L_{Q\bar{Q}g} + L_{Q\bar{Q}q\bar{q}} + L_{\rm mixing} + \cdots$$

Castellà , Soto Phys. Rev. D. 102, (2020)



- Gap of order $\Lambda_{\rm QCD}$ allows us to focus individually on low-lying states corresponding to quarkonium, hybrid, tetraquark etc.

- $L_{\rm mixing}$: Mixing between different states with similar masses and same quantum-numbers.

Ex: Hybrid-quarkonium mixing, Tetraquark-hybrid & Tetraquark-quarkonium mixing etc.

BOEFT

Brambilla, AM, Vairo
arXiv 2406.xxxxx

- BOEFT Lagrangian:

$$L_{\mathrm{BOEFT}} = \int d^3\boldsymbol{R} \int d^3\boldsymbol{r} \sum_{\kappa\lambda\lambda'} \mathrm{Tr}\Big\{ \Psi^{\dagger}_{\kappa\lambda}(\mathbf{r}, \mathbf{R}, t) \Big[i\partial_t \, \delta_{\lambda\lambda'} - V_{\kappa\lambda\lambda'}(r) + P^{i\dagger}_{\kappa\lambda}(\theta,\phi) \frac{\boldsymbol{\nabla}^2_r}{m_Q} P^{i}_{\kappa\lambda'}(\theta,\phi) \Big] \Psi_{\kappa\lambda'}(\mathbf{r}, \mathbf{R}, t) \Big\}$$

Light d.o.f / BO-quantum #: $\kappa = \{ \boldsymbol{K}^{PC} (\Lambda^{\sigma}_{\eta}) , f \}$ $\quad \lambda = \pm\Lambda$

Projection vectors for fixing $D_{\infty h}$ representations: $P^{i}_{K\lambda}(\theta, \varphi) = D^{\lambda *}_{K-i}(0, \theta, \varphi)$

BOEFT potential: $V_{\kappa\lambda\lambda'}(r) = \boxed{V^{(0)}_{\kappa\lambda}(r)} \delta_{\lambda\lambda'} + \boxed{\frac{V^{(1)}_{\kappa\lambda\lambda'}(r)}{m_Q}} + \ldots ,$

Static Energy

Spin-dependent potentials

Explicit spin-dependent potentials have been determined in case of Hybrids

Brambilla, Lai, Segovia, Castellà, Vairo Phys. Rev. D. 101, (2020)

Brambilla, Lai, Segovia, Castellà, Vairo Phys. Rev. D. 99, (2019)

Soto, Valls, arXiv 2302.01765

Castellà , Soto Phys. Rev. D. 102, (2020)

BOEFT

Brambilla, AM, Vairo
arXiv 2406.xxxxx

Wave-function for Exotic State:

$$|X_N\rangle = \sum_{\lambda} \int d^3r |\boldsymbol{r}\rangle \otimes |k,\lambda\rangle \phi^{(N)}_{\kappa\lambda}(\boldsymbol{r})$$

$|\boldsymbol{r}\rangle$: Heavy quark pair state separated by position r

$|k,\lambda\rangle$: Light quark or gluon state: Parametrically depends on r

Total orbital momentum for Exotic State:

$$\boldsymbol{L} = \boldsymbol{L}_Q + \boldsymbol{K}$$

$\boldsymbol{K}$: angular-momentum of light d.o.f

$\boldsymbol{L_Q}$: orbital-angular momentum of QQ or $Q\bar{Q}$ pair.

Angular wave-function:

$$|l,m;k,\lambda\rangle = \int \frac{d\Omega}{\sqrt{2\pi}} |\theta,\phi\rangle\, |k,\lambda\rangle\, D^{\lambda}_{lm}(\psi,\theta,\varphi)$$

BOEFT

Brambilla, AM, Vairo
arXiv 2406.xxxxx

- Adiabatic Radial Schrödinger equation: Mixing different static energies at short-distances:

$$\sum_{\lambda}\left[-\frac{1}{m_Q r^2}\partial_r\, r^2\partial_r + \frac{1}{m_Q r^2}\boxed{M_{\lambda'\lambda}} + V^{(0)}_{\kappa\lambda'\lambda}\right]\psi^{(N)}_{\kappa\lambda}(r) = \mathcal{E}_N\, \psi^{(N)}_{\kappa\lambda'}(r)\,,$$

Mixing term from angular momentum piece:
Coupling different static energies Λ^{σ}_{η} at short-distance

- General expression of $M_{\lambda'\lambda}$ (matrix in $\lambda' - \lambda$ basis) :

$$\lambda, \lambda' = \pm\Lambda$$

$$\begin{aligned} M_{\lambda'\lambda} &= \langle l, m; k, \lambda' | \boldsymbol{L}_Q^2 | l, m; k, \lambda\rangle \\ &= \left(l(l+1) - 2\lambda^2 + k(k+1)\right)\delta^{\lambda'\lambda} - \sqrt{k(k+1)-\lambda(\lambda+1)}\sqrt{l(l+1)-\lambda(\lambda+1)}\,\delta^{\lambda'\lambda+1} \\ &\quad - \sqrt{k(k+1)-\lambda(\lambda-1)}\sqrt{l(l+1)-\lambda(\lambda-1)}\,\delta^{\lambda'\lambda-1} \end{aligned}$$

BOEFT

Brambilla, AM, Vairo arXiv 2406.xxxxx

Coupled Equations for lowest Hybrids and Tetraquarks ($QQ\bar{q}\bar{q}$ or $Q\bar{Q}q\bar{q}$):

Gluon or light quark spin K=1

$$\left[-\frac{1}{m_Q r^2}\partial_r r^2 \partial_r + \frac{1}{m_Q r^2}\begin{pmatrix} l(l+1)+2 & -2\sqrt{l(l+1)} \\ -2\sqrt{l(l+1)} & l(l+1)\end{pmatrix} + \begin{pmatrix} V_\Sigma & 0 \\ 0 & V_\Pi \end{pmatrix}\right]\begin{pmatrix}\psi^{(N)}_{\Sigma,\sigma_p} \\ \psi^{(N)}_{\Pi,\sigma_P}\end{pmatrix} = \mathcal{E}_N \begin{pmatrix}\psi^{(N)}_{\Sigma,\sigma_P} \\ \psi^{(N)}_{\Pi,\sigma_P}\end{pmatrix}$$

$$\left[-\frac{1}{m_Q r^2}\partial_r\, r^2\, \partial_r + \frac{l(l+1)}{m_Q r^2} + V_\Pi\right]\psi^{(N)}_{\Pi,-\sigma_P} = \mathcal{E}_N\, \psi^{(N)}_{\Pi,-\sigma_P}$$

Gluon or light quark spin K=2

$$\left[-\frac{1}{m_Q r^2}\partial_r r^2 \partial_r + \frac{1}{m_Q r^2}\begin{pmatrix} l(l+1)+6 & -2\sqrt{3l(l+1)} & 0 \\ -2\sqrt{3l(l+1)} & l(l+1)+4 & -2\sqrt{(l-1)(l+2)} \\ 0 & -2\sqrt{(l-1)(l+2)} & (l-1)(l+2)\end{pmatrix} + \begin{pmatrix} V_\Sigma & 0 & 0 \\ 0 & V_\Pi & 0 \\ 0 & 0 & V_\Delta \end{pmatrix}\right]\begin{pmatrix}\psi^{(N)}_{\Sigma,\sigma_P} \\ \psi^{(N)}_{\Pi,\sigma_P} \\ \psi^{(N)}_{\Delta,\sigma_p}\end{pmatrix} = \mathcal{E}_N \begin{pmatrix}\psi^{(N)}_{\Sigma,\sigma_P} \\ \psi^{(N)}_{\Pi,\sigma_P} \\ \psi^{(N)}_{\Delta,\sigma_p}\end{pmatrix}$$

$$\left[-\frac{1}{m_Q r^2}\partial_r r^2 \partial_r + \frac{1}{m_Q r^2}\begin{pmatrix} l(l+1)+4 & -2\sqrt{(l-1)(l+2)} \\ -2\sqrt{(l-1)(l+2)} & (l-1)(l+2)\end{pmatrix} + \begin{pmatrix} V_\Pi & 0 \\ 0 & V_\Delta \end{pmatrix}\right]\begin{pmatrix}\psi^{(N)}_{\Pi,-\sigma_P} \\ \psi^{(N)}_{\Delta,-\sigma_p}\end{pmatrix} = \mathcal{E}_N \begin{pmatrix}\psi^{(N)}_{\Pi,-\sigma_P} \\ \psi^{(N)}_{\Delta,-\sigma_p}\end{pmatrix}$$

BOEFT

Brambilla, AM, Vairo
arXiv 2406.xxxxx

Coupled Equations for Doubly Heavy Baryons and Pentaquarks (QQq̄qq or QQ̄qqq):

Light quark spin K=1/2

$$\left[-\frac{1}{m_Q r^2}\,\partial_r\, r^2\,\partial_r + \frac{(l-1/2)(l+1/2)}{m_Q r^2} + V_{K_\eta}\right]\psi^{(N)}_{K_\eta,\sigma_P} = \mathcal{E}_N\,\psi^{(N)}_{K_\eta,\sigma_P}$$

Light quark spin K=3/2

$$\left[-\frac{1}{m_Q r^2}\partial_r r^2 \partial_r + \frac{1}{m_Q r^2}\begin{pmatrix} l(l-1)-\frac{9}{4} & -\sqrt{3l(l+1)-\frac{9}{4}} \\ -\sqrt{3l(l+1)-\frac{9}{4}} & l(l+1)-\frac{3}{4}\end{pmatrix} + \begin{pmatrix} V_{(1/2)_u} & 0 \\ 0 & V_{(3/2)_u}\end{pmatrix}\right]\begin{pmatrix}\psi^{(N)}_{1/2,\sigma_P} \\ \psi^{(N)}_{3/2,\sigma_P}\end{pmatrix} = \mathcal{E}_n \begin{pmatrix}\psi^{(N)}_{1/2,\sigma_P} \\ \psi^{(N)}_{3/2,\sigma_P}\end{pmatrix}$$

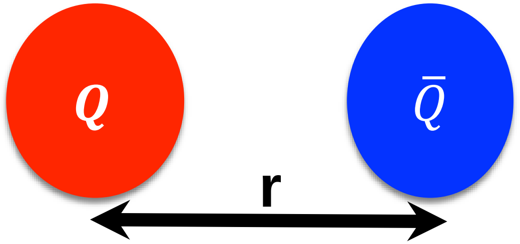
$$\left[-\frac{1}{m_Q r^2}\partial_r r^2 \partial_r + \frac{1}{m_Q r^2}\begin{pmatrix} l(l+3)+\frac{17}{4} & -\sqrt{3l(l+1)-\frac{9}{4}} \\ -\sqrt{3l(l+1)-\frac{9}{4}} & l(l+1)-\frac{3}{4}\end{pmatrix} + \begin{pmatrix} V_{(1/2)_u} & 0 \\ 0 & V_{(3/2)_u}(r)\end{pmatrix}\right]\begin{pmatrix}\psi^{(N)}_{1/2,-\sigma_P} \\ \psi^{(N)}_{3/2,-\sigma_P}\end{pmatrix} = \mathcal{E}_n \begin{pmatrix}\psi^{(N)}_{1/2,-\sigma_P} \\ \psi^{(N)}_{3/2,-\sigma_P}\end{pmatrix}$$

Exotic

Brambilla, AM, Vairo arXiv 2406.xxxxx

- Exotic hadron ($Q\bar{Q}X, QQX, \ldots\ldots$), X is light d.o.f.

$Q\bar{Q}X$



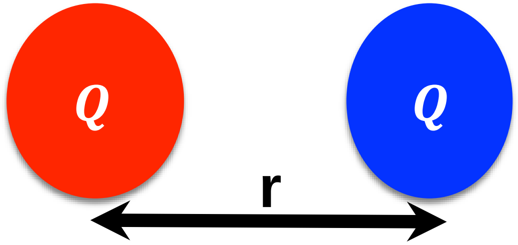
color: $3 \otimes \bar{3} = 1 \oplus 8$

X_8= gluon → **Hybrid**

$X_8 = q\bar{q}$ → **Tetraquark / Molecule**

$X_8 = qqq$ → **Pentaquark / Molecule** and so on

QQX



color: $3 \otimes \bar{3} = \bar{3} \oplus 6$

X= q → **Double heavy baryon**

$X = \bar{q}\bar{q}$ → **Tetraquark**

$X = q\bar{q}q$ → **Pentaquark** and so on

BOEFT potentials ($V_\Gamma(r)$): LDF (light quarks, gluons) **static energies.**

Potential between Q & $\bar{Q}$

BOEFT can address all these states with inputs from Lattice QCD

BOEFT: Static Energy

- Total angular momentum of **$Q\bar{Q}$X** or **QQX** :

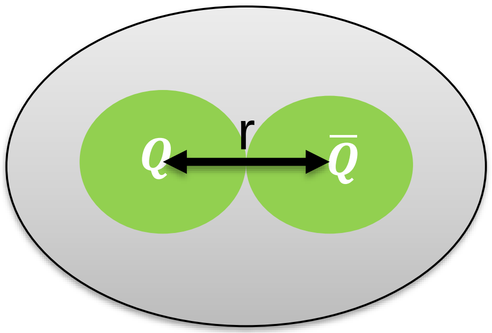
:

$$\mathbf{J} = \mathbf{L}_{\mathbf{Q\bar{Q}}} + \mathbf{K} + \mathbf{S}_{\mathbf{Q\bar{Q}}}$$

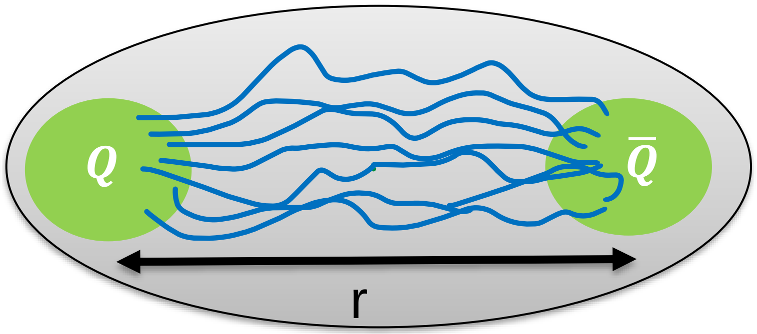
NRQCD operator (gauge invariant) for exotic hadron $Q\bar{Q}X$:

$$\mathcal{O}_K(t,\, \mathbf{r},\, \mathbf{0}) = \chi^\dagger(\mathbf{t},\, \mathbf{r}/\mathbf{2})\phi(\mathbf{t},\, \mathbf{r}/\mathbf{2}, \mathbf{0})\mathbf{H}_\mathbf{K}(\mathbf{t},\, \mathbf{0})\phi(\mathbf{t},\, \mathbf{0}, -\mathbf{r}/\mathbf{2})\psi(\mathbf{t},\, -\mathbf{r}/\mathbf{2})$$

$H_K(t,\, \mathbf{0})$: Gluon or light-quark operator characterizing ***X*** corresponding to quantum # **K**, isospin, color etc..



Short-distance ($r \to 0$)

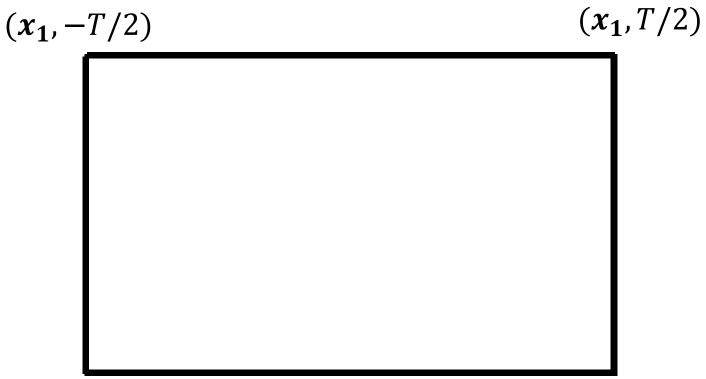


Large-distance ($r \to \infty$)

Berwein, Brambilla, Castellà , Vairo Phys. Rev. D. 92, (2015) Castellà , Soto Phys. Rev. D. 102, (2020) Brambilla, AM, Vairo arXiv 2406.xxxx

BOEFT: Potentials

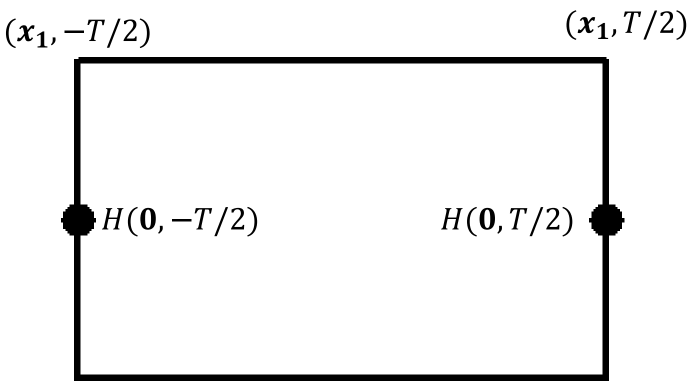
Brambilla, AM, Vairo
arXiv 2406.xxxxx



(a)

$$W_\square \equiv \langle 1 \rangle_\square$$

Wilson loop for quarkonium



(b)

$$\langle H(\mathbf{0}, T/2)\, H(\mathbf{0}, -T/2)\rangle_\square$$

Wilson loop for exotics

(BO-quantum #:)

$$\kappa = \{\boldsymbol{K}^{PC}(\Lambda_\eta^\sigma), f\}$$

$$V_s(r) = -\frac{4\alpha_s}{3r},\; V_o(r) = \frac{\alpha_s}{6r}$$
$$V_T(r) = -\frac{2\alpha_s}{3r},\; V_\Sigma(r) = \frac{\alpha_s}{3r}$$

Short-distance behavior of BO-Potentials:

$Q\bar{Q}$: $$V^{(0)}_{\Sigma_g^+}(r) = V_s(r) + b_{\Sigma_g^+} r^2 + \cdots$$

$Q\bar{Q}X$: $$V^{(0)}_{\Lambda_\eta^\sigma}(r) = V_o(r) + \Lambda_{H_\kappa} + b_{\Lambda_\eta^\sigma} r^2 + \cdots$$

QQX: $$V^{(0)}_{\Lambda_\eta^\sigma}(r) = V_l(r) + \Lambda_{H_\kappa,l} + b_{\kappa\lambda,l}\, r^2 + \cdots$$

$$(l = T, \Sigma)$$

Long-distance behavior of BO-Potentials:

- String behavior (**only pure SU(3) gauge theory**)

$$E_N(r) = \sqrt{\sigma^2 r^2 + 2\pi\sigma\,(N - 1/12)}$$

K. Juge, J. Kuti, C. Morningstar, Phys. Rev. Lett. 90 (2003)

- Mixing with pair of heavy-light states based on BO-quantum numbers or $\boldsymbol{\Lambda_\eta^\sigma}$ representations

Static Energies: Quenched

K. Juge, J. Kuti, C. Morningstar, Phys. Rev. Lett. 90 (2003)

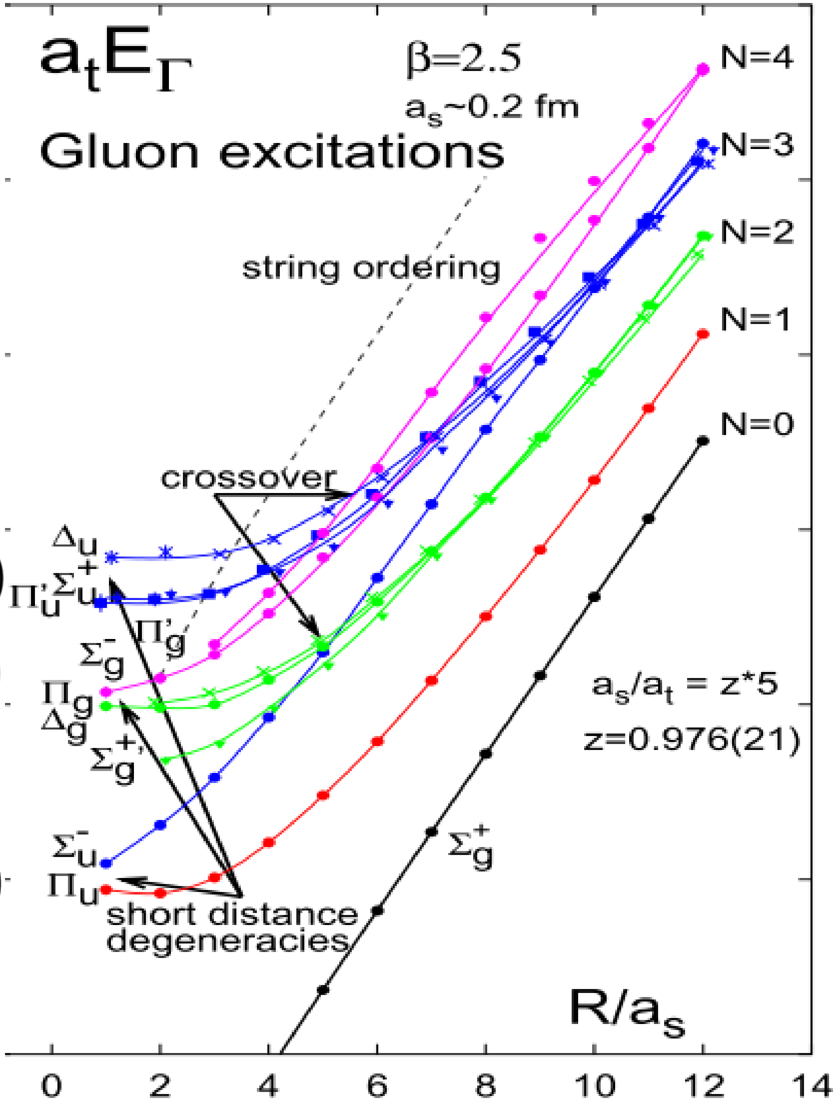
Sharifian, Cardoso, Bicudo, Phys. Rev. D. 107 (2023)

$2^{+-}\ (\Sigma_u^+, \Pi_u', \Delta_u)$

$2^{--}\ (\Delta_g, \Sigma_g^-, \Pi_g')$

$1^{--}\ (\Sigma_g^{+\prime}, \Pi_g)$

$1^{+-}\ (\Pi_u, \Sigma_u^-)$



$N = 3\ (\Sigma_u^-, \Sigma_u^+, \Pi_u', \Delta_u, \cdots)$

$N = 2\ (\Sigma_g^{+\prime}, \Pi_g, \Delta_g)$

$N = 1\ (\Pi_u)$

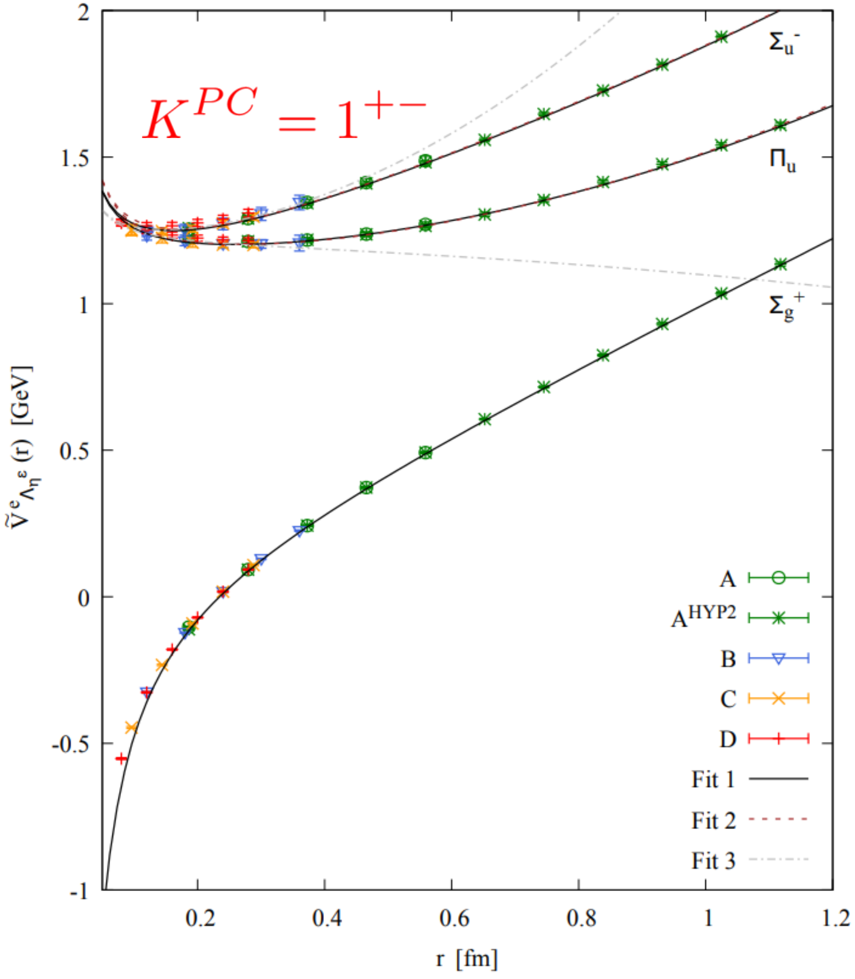
$N = 0\ (\Sigma_g^+)$

0 ← r → ∞

Hybrids

BOEFT: Hybrids

- Coupled Schrödinger Eq:



$$\left[-\frac{1}{m_Q r^2}\,\partial_r r^2 \partial_r + \frac{1}{m_Q r^2}\begin{pmatrix} l(l+1)+2 & 2\sqrt{l(l+1)} \\ 2\sqrt{l(l+1)} & l(l+1)\end{pmatrix} + \begin{pmatrix} E_{\Sigma_u^-} & 0 \\ 0 & E_{\Pi_u}\end{pmatrix}\right]\begin{pmatrix}\psi^{(m)}_{\Sigma} \\ \psi^{(m)}_{-\Pi}\end{pmatrix} = E_m^{Q\bar{Q}g}\begin{pmatrix}\psi^{(m)}_{\Sigma} \\ \psi^{(m)}_{-\Pi}\end{pmatrix}$$

$$\left[-\frac{1}{m_Q r^2}\,\partial_r\, r^2\, \partial_r + \frac{l(l+1)}{m_Q r^2} + E_{\Pi_u}\right]\psi^{(m)}_{+\Pi} = E_m^{Q\bar{Q}g}\,\psi^{(m)}_{+\Pi}$$

$\lambda = 0, \pm 1$

Hybrid Spectrum:

Multiplet	J^{PC}	$M_{c\bar{c}g}$	$M_{b\bar{b}g}$
H_1	$\{1^{--}, (0,1,2)^{-+}\}$	4155	10786
H_1'		4507	10976
H_1''		4812	11172
H_2	$\{1^{++}, (0,1,2)^{+-}\}$	4286	10846
H_2'		4667	11060
H_2''		5035	11270
H_3	$\{0^{++}, 1^{+-}\}$	4590	11065
H_3'		5054	11352
H_3''		5473	11616
H_4	$\{2^{++}, (1,2,3)^{+-}\}$	4367	10897
H_5	$\{2^{--}, (1,2,3)^{-+}\}$	4476	10948

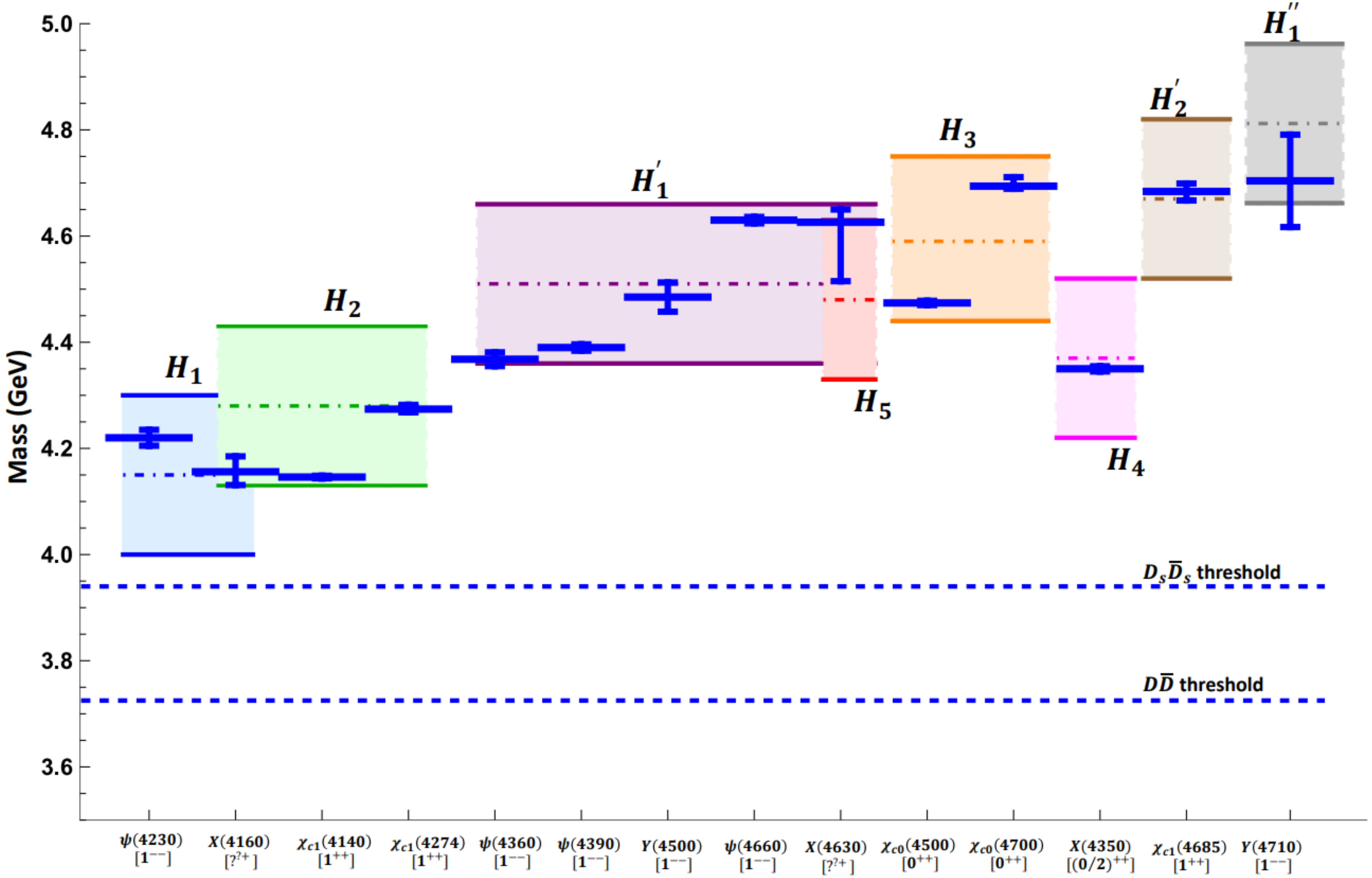
Λ- doubling: opposite parity states non-degenerate.

Brambilla, Lai, AM, Vairo Phys. Rev. D 107, 054034 (2023)

Berwein, Brambilla, Castellà , Vairo Phys. Rev. D. 92, (2015)

BOEFT: Hybrids

- **Charmonium hybrids**: comparison with experimental results:



	l	$J^{PC}\{s=0, s=1\}$	$E_n^{(0)}$
H_1	1	$\{1^{--}, (0,1,2)^{-+}\}$	Σ_u^-, Π_u
H_2	1	$\{1^{++}, (0,1,2)^{+-}\}$	Π_u
H_3	0	$\{0^{++}, 1^{+-}\}$	Σ_u^-
H_4	2	$\{2^{++}, (1,2,3)^{+-}\}$	Σ_u^-, Π_u
H_5	2	$\{2^{--}, (1,2,3)^{-+}\}$	Π_u

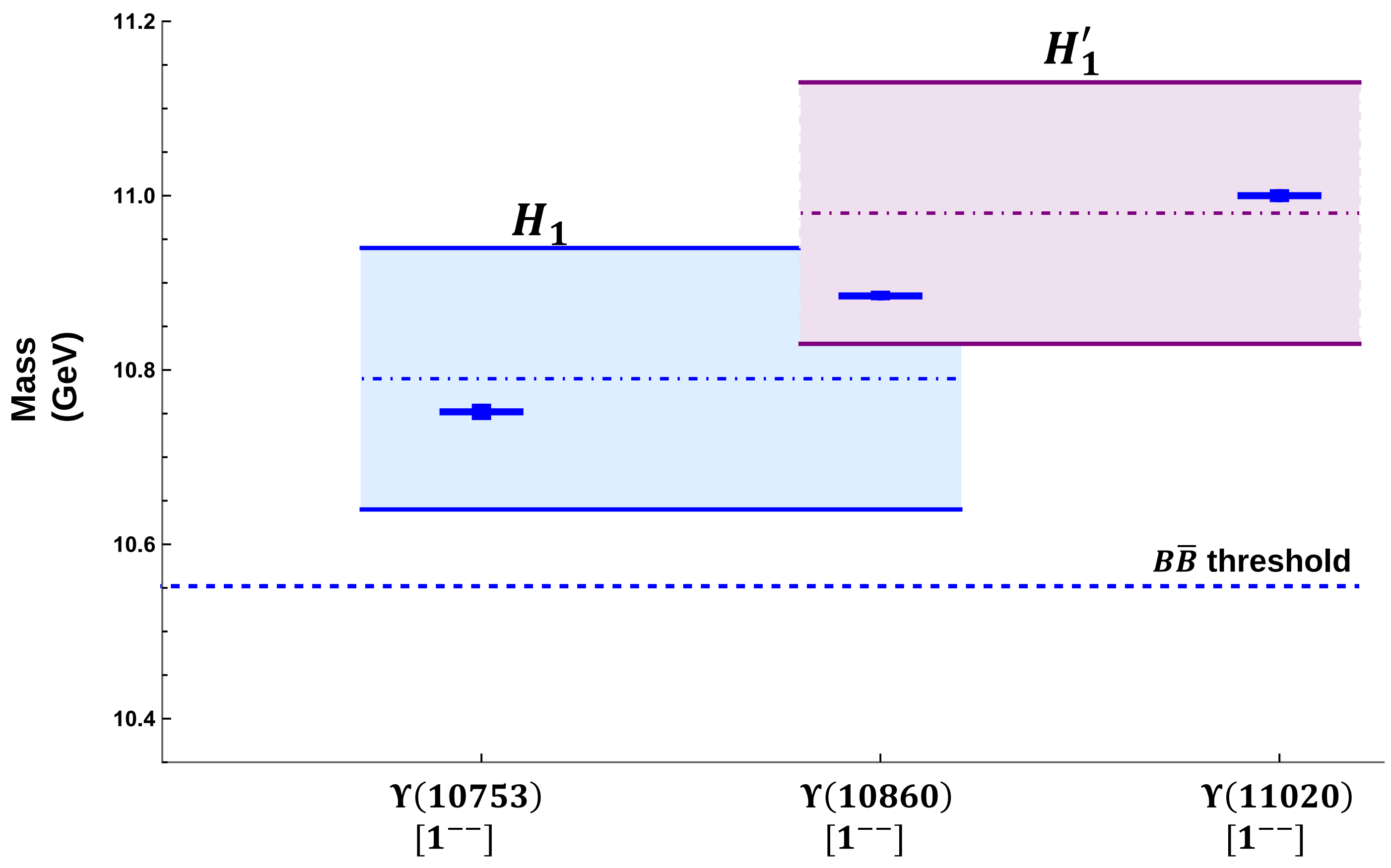
PDG 2022

Brambilla, Lai, AM, Vairo

Phys. Rev. D 107, 054034 (2023)

BOEFT: Hybrids

- **Bottomonium hybrids**: comparison with experimental results:



PDG 2022

	l	$J^{PC}\{s=0, s=1\}$	$E_n^{(0)}$
H_1	1	$\{1^{--}, (0,1,2)^{-+}\}$	Σ_u^-, Π_u
H_2	1	$\{1^{++}, (0,1,2)^{+-}\}$	Π_u
H_3	0	$\{0^{++}, 1^{+-}\}$	Σ_u^-
H_4	2	$\{2^{++}, (1,2,3)^{+-}\}$	Σ_u^-, Π_u
H_5	2	$\{2^{--}, (1,2,3)^{-+}\}$	Π_u

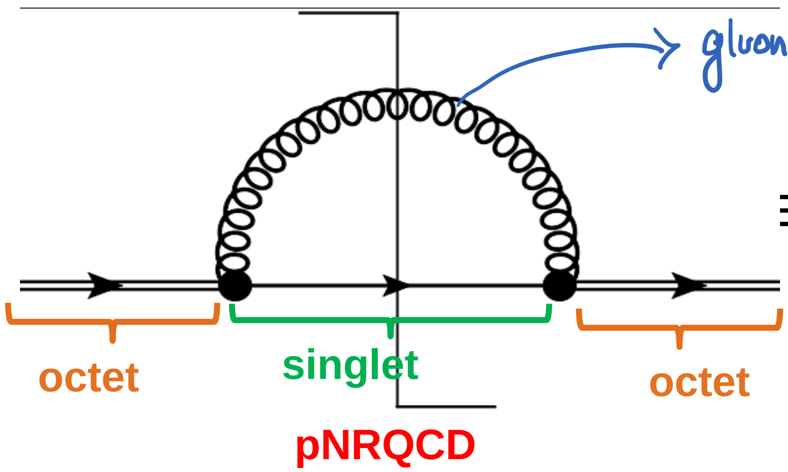
Brambilla, Lai, AM, Vairo arXiv:2212.09187

Hybrid Decays

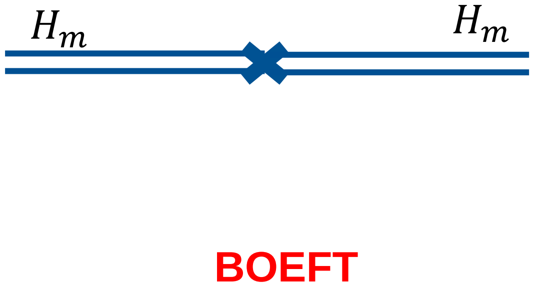
Brambilla, Lai, AM, Vairo Phys. Rev. D 107, 054034 (2023)

- BOEFT can describe decays of hybrids to quarkonium.
- Semi-inclusive process: $H_m \to Q_n + X$; Q_n: low-lying quarkonium (states below threshold) & X: light hadrons.
  - ✓ ΔE: Large energy difference ⇒ $\Delta E \equiv E_{H_m} - E_{Q_n} \gtrsim 1$ GeV.
  - ✓ Constituent gluon of the hybrid is a spectator.
  - ✓ Hierarchy of scales: $\Delta E \gg \Lambda_{QCD} \gg mv^2$ (Perturbative computation)

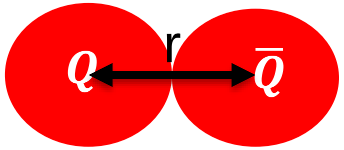
matching **pNRQCD** and **BOEFT:**



$\equiv \mathrm{Im}$



Virtual gluon resolves color structure of $Q\bar{Q}$ pair ($\mathbf{r} \to \mathbf{0}$) in quarkonium and hybrid in short-distance limit



Quarkonium ---> Singlet

Hybrid ---> Octet

- Decays are computed from local imaginary terms in the hybrid potential (BOEFT potential).

Optical theorem: $\sum_n \Gamma(H_m \to Q_n) = -2\,\mathrm{Im}\,\langle H_m|V|H_m\rangle$

DISCLAIMER!!!
Decay to open-flavor threshold states not accounted here.

Hybrid Decays

Brambilla, Lai, AM, Vairo Phys. Rev. D 107, 054034 (2023)

- Spin-conserving decay due to $\boldsymbol{r} \cdot \boldsymbol{E}$ term :

$|S_H = 1 > \dashrightarrow |S_Q = 1 >$
$|S_H = 0 > \dashrightarrow |S_Q = 0 >$

$$\Gamma(H_m \to Q_n) = \frac{4\alpha_s\,(\Delta E)\,T_F}{3N_c}\, T^{ij}\,(T^{ij})^\dagger\,\Delta E^3$$

DISCLAIMER!!!
Decay to open-flavor threshold states not accounted here.

$$T^{ij} \equiv \langle H_m | r^j | Q_n \rangle = \int d^3\mathbf{r}\, \Psi^{i\dagger}_{(m)}(\mathbf{r})\, r^j\, \Phi^{Q\bar{Q}}_{(n)}(\mathbf{r})$$

$\Psi^i_{(m)}$: Hybrid wf

Φ^Q_n : Quarkonium wf

R. Oncala, J. Soto, Phys. Rev. D96, 014004 (2017).

J. Castellà, E. Passemar, Phys. Rev. D104, 034019 (2021)

$$\langle H_m | \boldsymbol{r} | Q_n \rangle = \sqrt{T^{ij}(T^{ij})^\dagger}$$

- Spin-flipping decay due to $\boldsymbol{S}.\boldsymbol{B}$ term:

$|\chi_H\rangle$: Hybrid spin wf

$|\chi_Q\rangle$: Quarkonium spin wf

$|S_H = 1 > \dashrightarrow |S_Q = 0 >$
$|S_H = 0 > \dashrightarrow |S_Q = 1 >$

$$T^{ij} \equiv \langle H_m | \left(S_1^j - S_2^j\right) | Q_n \rangle = \left[\int d^3\mathbf{r}\, \Psi^{i\dagger}_{(m)}(\mathbf{r})\, \Phi^Q_{(n)}(\mathbf{r})\right] \langle \chi_H | \left(S_1^j - S_2^j\right) | \chi_Q \rangle$$

Depends on overlap of quarkonium and hybrid wavefunctions.

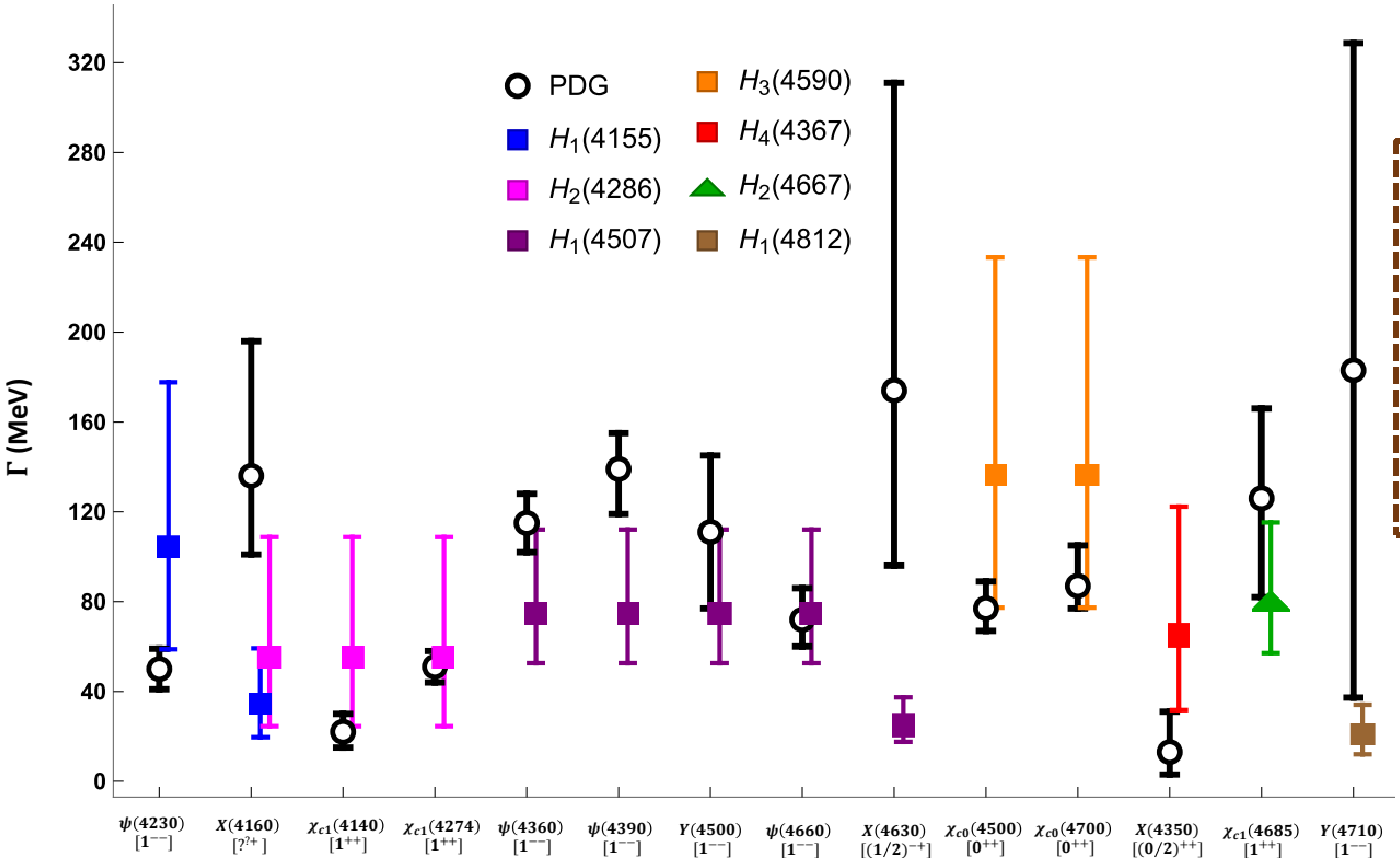
Hybrid-to-Quarkonium transition decay rate
= **spin-conserving** + **spin-flipping** decay rates.

Our estimate of decay rate are lower-bounds for the **total width of hybrids**

Results

Brambilla, Lai, AM, Vairo Phys. Rev. D 107, 054034 (2023)

- Comparison: charm exotic states with corresponding charmonium hybrid state:



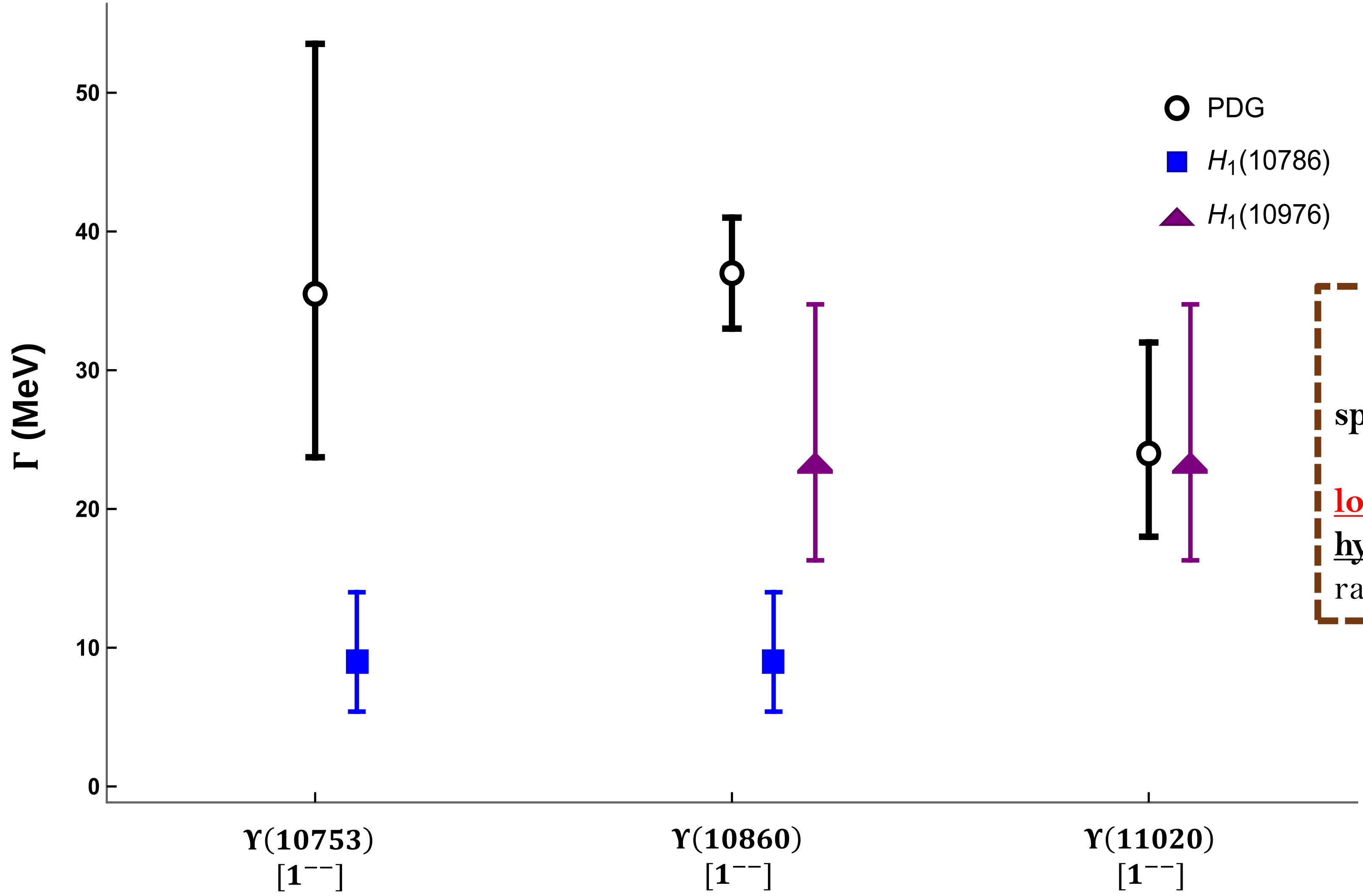
$H_m \rightarrow Q_n + X$

spin-conserving + spin-flipping decays
=
lower bound on the total decay widths of hybrids
which is compared with inclusive rate of physical states in PDG.

Results

Brambilla, Lai, AM, Vairo Phys. Rev. D 107, 054034 (2023)

- Comparison: bottom exotic states with corresponding bottomonium hybrid state:



$$H_m \rightarrow Q_n + X$$

spin-conserving + spin-flipping decays
=
lower bound on the total decay widths of hybrids which is compared with inclusive rate of physical states in PDG.

Hybrid: Mixing with heavy-light

- **Hybrid decays to meson-pair threshold states:** $\Delta E \lesssim \Lambda_{QCD}$

Conventional Wisdom: Hybrid decays to two S-wave mesons forbidden! $H_m \nrightarrow D^{(*)}\bar{D}^{(*)}$

Kou & Pene, Phys Lett B 631 (2005) Page, Phys Lett B 407 (1997) Farina, Tecocoatzi, Giachino, Santopinto & Swanson, Phys Rev D 102 (2020)

Born Oppenheimer quantum numbers for hybrids and ground state meson pair **does allow for decay to two s-wave mesons.**

Bruschini 2306.17120

	l	$J^{PC}\{s=0, s=1\}$	$E_n^{(0)}$
H_1	1	$\{1^{--}, (0,1,2)^{-+}\}$	Σ_u^-, Π_u
H_2	1	$\{1^{++}, (0,1,2)^{+-}\}$	Π_u
H_3	0	$\{0^{++}, 1^{+-}\}$	Σ_u^-
H_4	2	$\{2^{++}, (1,2,3)^{+-}\}$	Σ_u^-, Π_u
H_5	2	$\{2^{--}, (1,2,3)^{-+}\}$	Π_u

Most quarkonium hybrids can decay into pair of s-wave mesons !!!

forbidden for decay into pair of s-wave mesons

Recent lattice computation for $c\bar{c}$ hybrid 1^{-+} **decay** to

$D_1\bar{D}$: 258(133) MeV

$D^*\bar{D}$: 88(18) MeV

$D^*\bar{D}^*$: 150(118) MeV

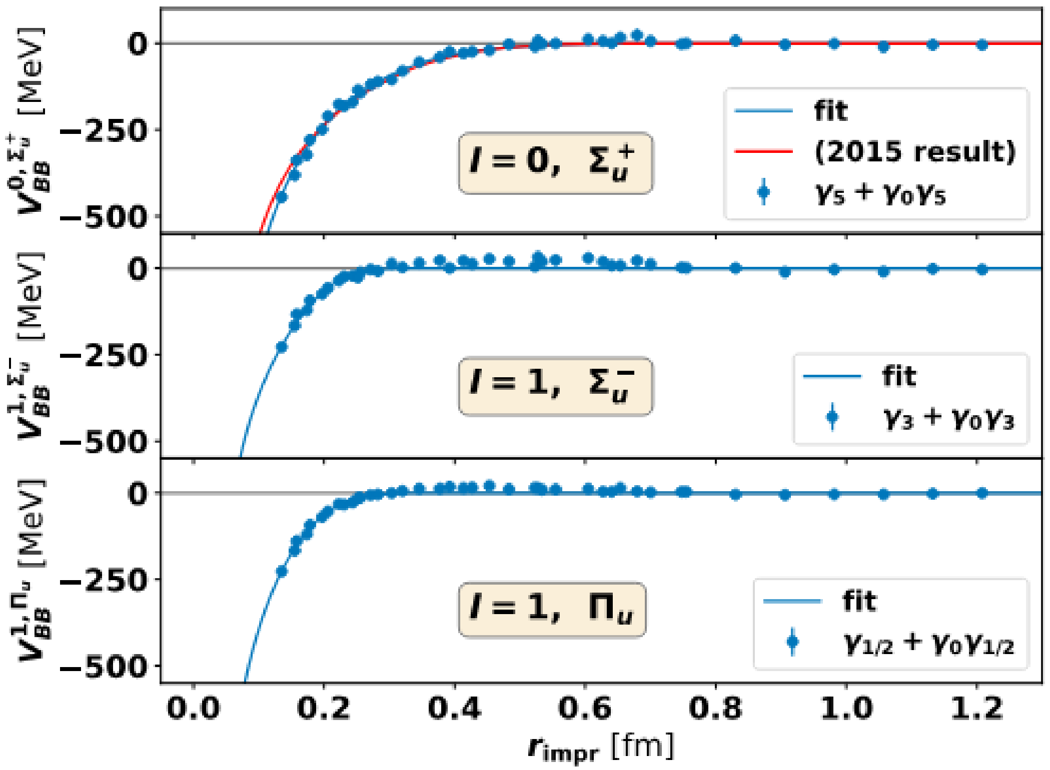
Shi et al 2306.12884

See R. Bruschini talk on Tuesday on Branching Ratios !!!

Tetraquarks
&
Pentaquarks

Static Energies: Tetraquark

$QQ\bar{q}\bar{q}$

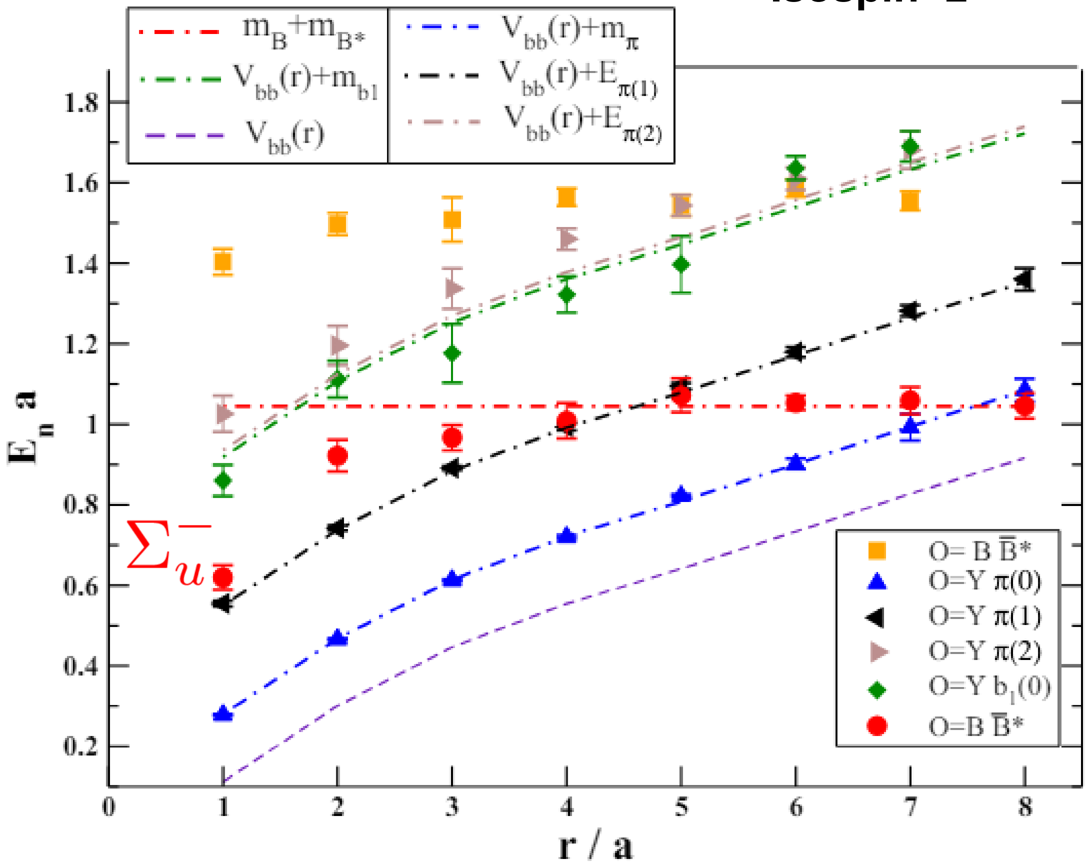


Mueller et al, PoS LATTICE2023, 64 (2024)

Bicudo, Cichy, Peters, Wagner, Phys. Rev. D. 93, (2016)

$Q\bar{Q}q\bar{q}$

Isospin=1

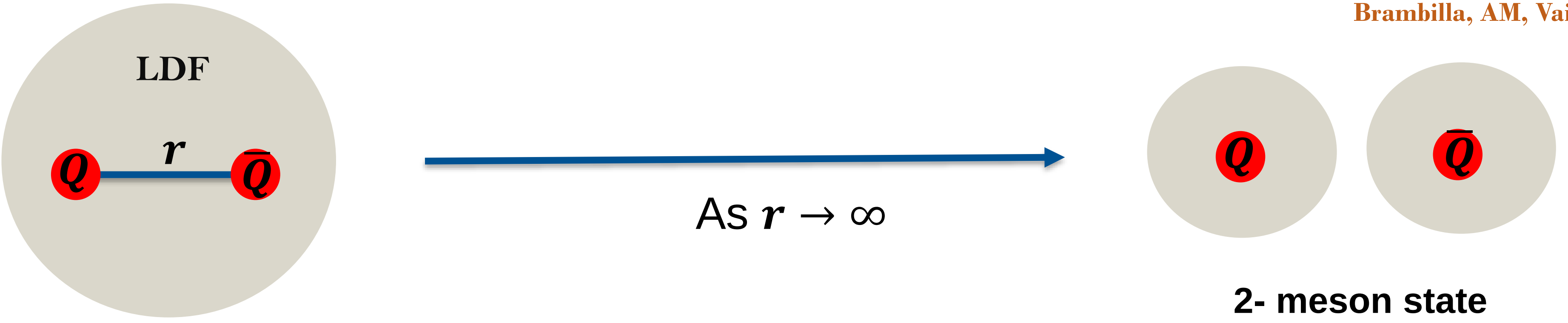


Prelovsek, Bahtiyar, Petkovic, Phys. Lett. B. 805, (2020)

Meson pair operators considered on lattice for these potentials !!

Static Energies: Tetraquark

Brambilla, AM, Vairo arXiv 2406.xxxx



Consider $Q\bar{Q}q\bar{q}$ system:

BO-quantum # Λ_η^σ:

$Q\bar{Q}$ (color)	Light Spin K^{PC}	Λ_η^σ $(D_{\infty h})$
Octet	0^{-+}	Σ_u^-
	1^{--}	$\{\Sigma_g^+, \Pi_g\}$

BO-quantum # Λ_η^σ for meson-antimeson

$K_{\bar{q}}^P \otimes K_q^P$	K^{PC}	Static energies $D_{\infty h}$
$(1/2)^- \otimes (1/2)^+$	0^{-+}	$\{\Sigma_u^-\}$
	1^{--}	$\{\Sigma_g^+, \Pi_g\}$

s-wave+s-wave
Ex. $D\bar{D}$ threshold

Meson-antimeson have same BO-quantum # Λ_η^σ

$Q\bar{Q}q\bar{q}$: Operator Overlap

Brambilla, AM, Vairo arXiv 2406.xxxx

Foster & Michael, UKQCD, 1997

NRQCD operator (gauge invariant) for exotic hadron: $\boldsymbol{Q\bar{Q}}$ pair in **octet** color

$$\mathcal{O}_K(t,\, \mathbf{r},\, \mathbf{0}) = \chi^\dagger(\mathbf{t},\, \mathbf{r}/\mathbf{2})\phi(\mathbf{t},\, \mathbf{r}/\mathbf{2}, \mathbf{0})\mathbf{H_K}(\mathbf{t},\, \mathbf{0})\phi(\mathbf{t},\, \mathbf{0},\, -\mathbf{r}/\mathbf{2})\psi(\mathbf{t},\, -\mathbf{r}/\mathbf{2})$$

$$\mathbf{H}_{\boldsymbol{K}}(t,\boldsymbol{x}) = \left[\bar{q}(t,\boldsymbol{x})\tilde{\boldsymbol{\Gamma}}T^a q(t,\boldsymbol{x})\right] T^a$$

$\tilde{\boldsymbol{\Gamma}}$: Dirac matrices based on quantum #'s

Quarkonium + Pions

Quarkonium state:

$$|\mathcal{Q}\rangle = \mathcal{N}\int d^3\boldsymbol{r}\, \Psi^{(n)}(\boldsymbol{r})\, \psi_b^\dagger(t,-\boldsymbol{r}/2)\, \phi_{bc}(t;-\boldsymbol{r}/2,\boldsymbol{r}/2)\, \chi_c(t,\boldsymbol{r}/2)\, |\Omega\rangle$$

Overlap of our operator on quarkonium + pion:

$$\langle\mathcal{Q}|\mathcal{O}_K^{Q\bar{Q}}(t,\boldsymbol{r})\,|\mathcal{Q}\rangle = 0$$

Meson-antimeson

Meson-antimeson state:

$$\begin{aligned}|M\bar{M}\rangle = \Bigg[&\mathcal{N}\int d^3\boldsymbol{x}\,\Psi_J(\boldsymbol{x})\\ &\times\int d^3\boldsymbol{y}\,\varphi_{J_1}(\boldsymbol{y}+\boldsymbol{x}/2)\,\psi_c^\dagger(t,-\boldsymbol{x}/2)\,\phi_{cd}(t;-\boldsymbol{x}/2,\boldsymbol{y}+\boldsymbol{x}/2)\,[P_+\Gamma_1 q_d(t,\boldsymbol{y}+\boldsymbol{x}/2)]\\ &\times\int d^3\boldsymbol{z}\,\varphi_{J_2}(\boldsymbol{z}-\boldsymbol{x}/2)\,[\bar{q}_b(t,\boldsymbol{z}-\boldsymbol{x}/2)\,\Gamma_2 P_-]\,\phi_{be}(t;\boldsymbol{z}-\boldsymbol{x}/2,\boldsymbol{x}/2)\,\chi_e(t,\boldsymbol{x}/2)\Bigg]|\mathrm{vac}\rangle\end{aligned}$$

Overlap of our operator on meson-antimeson:

$$\langle M\bar{M}|\mathcal{O}_K^{Q\bar{Q}}(t,\boldsymbol{r})\,|M\bar{M}\rangle \neq 0$$

Above operator is good operator for lattice computation for $Q\bar{Q}q\bar{q}$ potentials !!!

BOEFT: $QQ\bar{q}\bar{q}$ multiplets

Brambilla, AM, Vairo arXiv 2407.xxxx

doubly heavy core

spin: $1/2 \otimes 1/2 = 0 \oplus 1$

Q_1 Q_2 → Q_1Q_2

color: $3 \otimes 3 = 6 \oplus 3^*$

flavor: $\{Q_1Q_2\}$ $[Q_1Q_2]$

J^P: 1^+ 0^+

light antiquarks

q q'

{qq'}, 1^+ [qq'], 0^+

Defines the Born-Oppenheimer static potentials $\Sigma_g^+, \{\Sigma_g^-, \Pi_g\}$

doubly heavy tetraquarks

QQ color state	Light spin K^{PC}	Static energies	Isospin I	l	J^P $S_Q=0$	J^P $S_Q=1$
anti-triplet $\bar{3}$	0^+	$\{\Sigma_g^+\}$	0	0	—	1^+
				1	1^-	—
	1^+	$\{\Sigma_g^-, \Pi_g\}$	1	0	0^-	—
				1	1^-	$(0,1,2)^+$

J^P for T_{cc}^+

Limited lattice inputs available on Born-Oppenheimer static potentials $\Sigma_g^+, \{\Sigma_g^-, \Pi_g\}$

Bicudo, Cichy, Peters, & Wagner PRD 93, 034501 (2016)

BOEFT: $Q\bar{Q}q\bar{q}$ multiplets

Brambilla, AM, Vairo arXiv 2402.xxxx

$Q\bar{Q}$ color state	Light spin K^{PC}	Static energies	l	J^{PC} $\{S_Q=0, S_Q=1\}$	Multiplets
Octet	0^{-+}	$\{\Sigma_u^-\}$	0	$\{0^{++}, 1^{+-}\}$	T_1^0
			1	$\{1^{--}, (0,1,2)^{-+}\}$	T_2^0
			2	$\{2^{++}, (1,2,3)^{+-}\}$	T_3^0
	1^{--}	$\{\Sigma_g^{+\prime}, \Pi_g\}$	1	$\{1^{+-}, (0,1,2)^{++}\}$	T_1^1
		$\{\Sigma_g^{+\prime}\}$	0	$\{0^{-+}, 1^{--}\}$	T_2^1
		$\{\Pi_g\}$	1	$\{1^{-+}, (0,1,2)^{--}\}$	T_3^1
		$\{\Sigma_g^{+\prime}, \Pi_g\}$	2	$\{2^{-+}, (1,2,3)^{--}\}$	T_4^1

J^{PC} for neutral partner of Z_c, Z_b states. Probably mixing between both channels required ?

J^{PC} for $X(3872)$

Lattice inputs only available on Born-Oppenheimer static potentials Σ_u^- for r/a >1

Prelovsek, Bahtiyar, & Petkovic
Phys. Lett. B 805 (2020)

BOEFT: Pentaquark multiplets

Brambilla, AM, Vairo arXiv 2407.xxxx

$Q\bar{Q}qqq$

$Q\bar{Q}$ color state	Light spin $\boldsymbol{K}^P$	Static energies	l	J^P $\{S_Q = 0, S_Q = 1\}$
Octet	$(1/2)^+$	$(1/2)_g$	$1/2$	$\{1/2^-, (1/2, 3/2)^-\}$
	$(3/2)^+$	$(3/2)_g$	$3/2$	$\{3/2^-, (1/2, 3/2, 5/2)^-\}$

No lattice inputs available on Born-Oppenheimer static potentials for pentaquarks

$QQqq\bar{q}$

QQ color state	Light spin $\boldsymbol{K}^P$	heavy spin	
		$S_Q = 0$	$S_Q = 1$
sextet	$(1/2)^-$	$\{(1/2)^-\}$	$\{(1/2, 3/2)^+, (1/2, 3/2, 5/2)^+\}$
	$(3/2)^-$	$\{(3/2)^-\}$	$\{(1/2, 3/2)^+\}, \{(1/2, 3/2, 5/2)^+\}, \{(3/2, 5/2, 7/2)^+\}$
antitriplet	$(1/2)^-$	$\{(1/2)^+, (3/2)^+\}$	$\{(1/2, 3/2)^-\}$
	$(3/2)^-$	$\{(1/2)^+\}, \{(3/2)^+\}, \{(5/2)^+\}$	$\{(1/2, 3/2, 5/2)^-\}$

Coupled Schrödinger equation for these pentaquark states derived in Brambilla, AM, Vairo arXiv 2407.xxxxx.

Summary/Outlook

- Born-Oppenheimer EFT: Tool based on QCD and Born-Oppenheimer approximation to study Exotic states.

- BOEFT: model-independent & systematic framework with inputs required from lattice QCD.

- Hybrids ($Q\bar{Q}g$): Candidates based on mass and decays to quarkonium:

  Charm sector:

  - $X(4160)$: could be **charm hybrid** $H_1[2^{-+}](4155)$.
  - $X(4630)$: could be **charm hybrid** $H_1[(1/2^{-+})](4507)$.
  - $\psi(4390)$: could be **charm hybrid** $H_1[1^{--}](4507)$.
  - $\chi_{c1}(4685)$: could be **charm hybrid** $H_2[(1^{++})](4667)$.

  Bottom sector:

  - $\Upsilon(10753)$: could be **bottom hybrid** $H_1[(1^{--})](10786)$.

- Inputs needed from lattice QCD: adjoint meson or baryon spectrum, triplet & sextet meson or baryon spectrum, computation of tetraquark & pentaquark static energies.

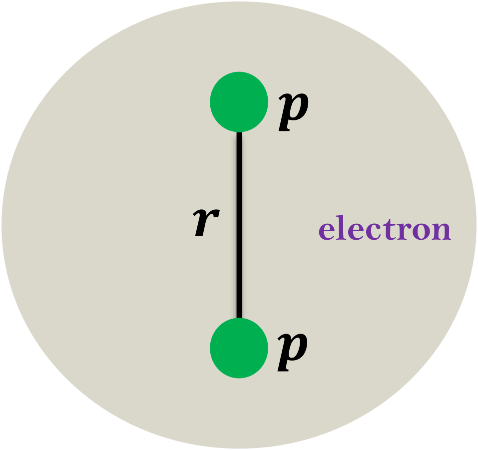
- Phenomenological studies of X(3872), Z_c & Z_b and T_cc are underway. Stay Tuned !!

Backup Slides

Born-Oppenheimer Philosophy

- Sharp difference between time or energy scales of heavy & light degrees of freedom.

Ex. H_2^+ molecule: 2 protons & 1 electron. $m_p \sim 1\,\mathrm{GeV} \gg m_e \sim 0.5\,\mathrm{MeV}$



Protons (nuclei) move very slowly compared to electrons and can be considered **static** (fixed) when considering the motion of the electrons

Electrons instantaneously adjust as **r** changes

1. Solve electron Schrödinger eq. for fixed **r**

$$H_{\mathrm{el}}\,(\mathbf{r})\,|\psi^i_{\mathrm{el}};\mathbf{r}\rangle = E^i_{\mathrm{el}}\,(r)\,|\psi^i_{\mathrm{el}};\mathbf{r}\rangle$$

$\boldsymbol{E^i_{\mathrm{el}}(r)}$: Electronic static energy

2. Solve nuclei (proton) Schrödinger eq. with $\boldsymbol{E^i_{\mathrm{el}}(r)}$ as **potential**.

QCD states with 2-heavy quarks (XYZ mesons): analogous of molecules in atomic systems !!!

Heavy quarks ↔ nuclei **Gluons & light quarks ↔ electrons**

Lattice input required for gluon and light-quark energies

Brambilla, AM, Vairo arXiv 2406.xxxx

Light quark operators characterized by K^{PC} essential for determinig BO-potentials $V_{\Lambda_\eta^\sigma}(r)$

Gluonic operators $\mathbf{H}_{\mathbf{K}^{\mathbf{PC}}}$ in lattice characterizing Hybrids $\mathbf{Q\bar{Q}g}$

$$\mathbf{H}_{1+-}(t,\boldsymbol{x}) = \boldsymbol{B}(t,\boldsymbol{x})$$
$$\mathbf{H}_{1--}(t,\boldsymbol{x}) = \boldsymbol{E}(t,\boldsymbol{x})$$

Light quark operators $\mathbf{H}_{\mathbf{K}^{\mathbf{PC}}}$ relevant for lattice computation of static energies for tetraquarks $\mathbf{Q\bar{Q}q\bar{q}}$

$$\mathbf{H}_{0++}(t,\boldsymbol{x}) = [\bar{q}(t,\boldsymbol{x})T^a q(t,\boldsymbol{x})]\, T^a$$
$$\mathbf{H}_{0-+}(t,\boldsymbol{x}) = \left[\bar{q}(t,\boldsymbol{x})\gamma^5 T^a q(t,\boldsymbol{x})\right] T^a$$
$$\mathbf{H}_{1++}(t,\boldsymbol{x}) = \left[\bar{q}(t,\boldsymbol{x})\boldsymbol{\gamma}\gamma^5 T^a q(t,\boldsymbol{x})\right] T^a$$
$$\mathbf{H}_{1--}(t,\boldsymbol{x}) = [\bar{q}(t,\boldsymbol{x})\boldsymbol{\gamma} T^a q(t,\boldsymbol{x})]\, T^a$$
$$\mathbf{H}_{1+-}(t,\boldsymbol{x}) = [\bar{q}(t,\boldsymbol{x})\,(\boldsymbol{\gamma}\times\boldsymbol{\gamma})\,T^a q(t,\boldsymbol{x})]\, T^a$$

Light quark operators $\mathbf{H}_{\mathbf{K}^{\mathbf{PC}}}$ relevant for lattice computation of static energies for pentaquarks $\mathbf{Q\bar{Q}qqq}$

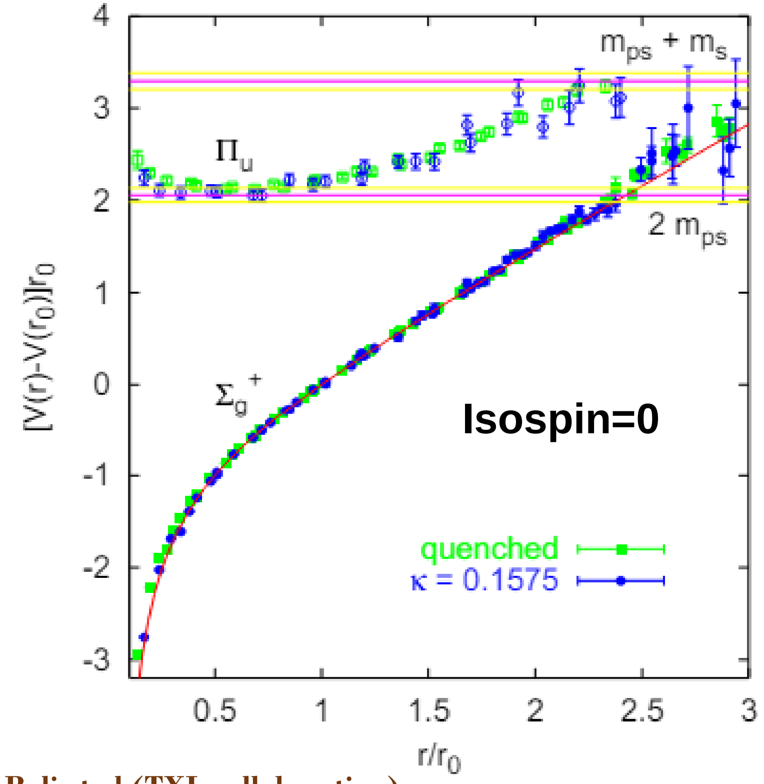
Castellà , Soto Phys. Rev. D. 102, (2020)

$$\begin{aligned}
H^{\alpha}_{I_3,(1/2)^+}(t,\boldsymbol{x}) = \\
\Big[&\left(\delta_{\alpha\beta_1}\sigma^2_{\beta_2\beta_3} + \delta_{\alpha\beta_2}\sigma^2_{\beta_1\beta_3} + \delta_{\alpha\beta_3}\sigma^2_{\beta_1\beta_2}\right)\left(\delta_{I_3 f_1}\tau^2_{f_2 f_3} + \delta_{I_3 f_2}\tau^2_{f_1 f_3} + \delta_{I_3 f_3}\tau^2_{f_1 f_2}\right)(T_2)^a_{l_1,l_2,l_3} \\
&+ \left(\delta_{\alpha\beta_1}\sigma^2_{\beta_2\beta_3} + \delta_{\alpha\beta_2}\sigma^2_{\beta_3\beta_1} + \delta_{\alpha\beta_3}\sigma^2_{\beta_2\beta_1}\right)\left(\delta_{I_3 f_1}\tau^2_{f_2 f_3} + \delta_{I_3 f_2}\tau^2_{f_3 f_1} + \delta_{I_3 f_3}\tau^2_{f_2 f_1}\right)(T_3)^a_{l_1,l_2,l_3} \\
&+ \left(\delta_{\alpha\beta_1}\sigma^2_{\beta_3\beta_2} + \delta_{\alpha\beta_2}\sigma^2_{\beta_3\beta_1} + \delta_{\alpha\beta_3}\sigma^2_{\beta_1\beta_2}\right)\left(\delta_{I_3 f_1}\tau^2_{f_3 f_2} + \delta_{I_3 f_2}\tau^2_{f_3 f_1} + \delta_{I_3 f_3}\tau^2_{f_1 f_2}\right)(T_1)^a_{l_1,l_2,l_3} \Big] \\
&(P_+ q_{l_1 f_1}(t,\boldsymbol{x}))^{\beta_1}\,(P_+ q_{l_2 f_2}(t,\boldsymbol{x}))^{\beta_2}\,(P_+ q_{l_3 f_3}(t,\boldsymbol{x}))^{\beta_3}\, T^a \,. \qquad (53)
\end{aligned}$$

Similar operator list can be written for Doubly heavy tetraquark $\mathbf{QQ\bar{q}\bar{q}}$ and Pentaquark states $\mathbf{QQqq\bar{q}}$. List of operators will be addressed in Brambilla, AM, Vairo arXiv 2406.xxxxx

Static Energies: Un-Quenched

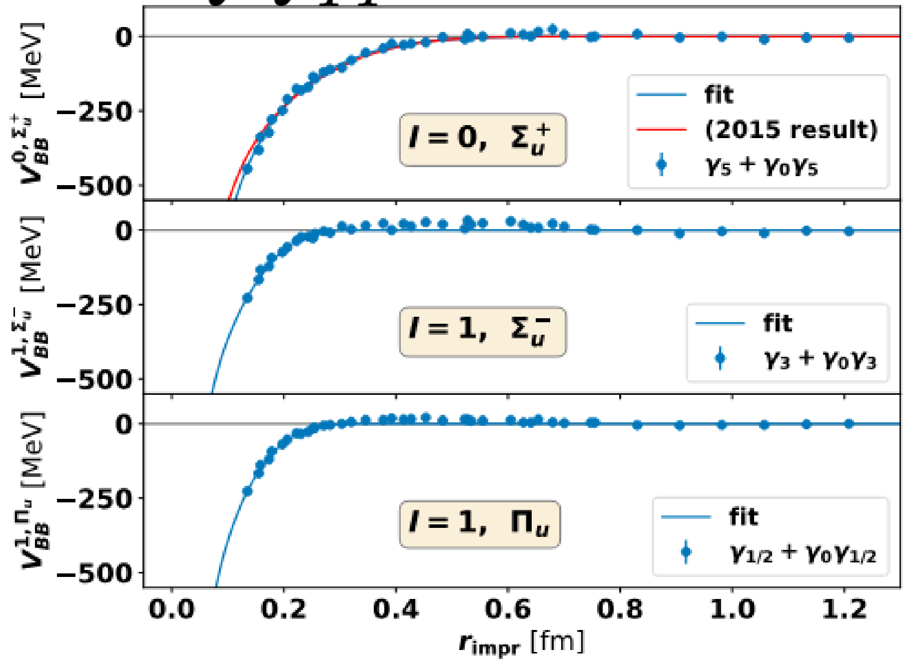
$Q\bar{Q}g$



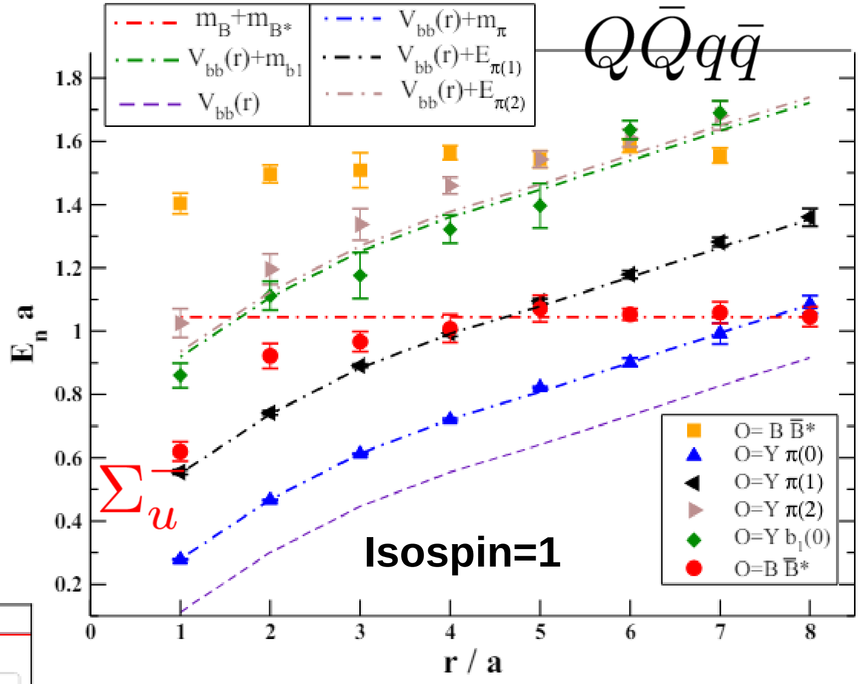
Bali et al (TXL collaboration)

Phys. Rev. D. 62, (2000)

$QQ\bar{q}\bar{q}$



$Q\bar{Q}q\bar{q}$



Prelovsek, Bahtiyar, Petkovic, Phys. Lett. B. 805, (2020)

Mueller et al, PoS LATTICE2023, 64 (2024)

Bicudo, Cichy, Peters, Wagner, Phys. Rev. D. 93, (2016)

Bali, Neff, Duessel, Lippert, Schilling
(SESAM collaboration)
Phys. Rev. D. 71, (2005)

Static Energies: Avoided crossing

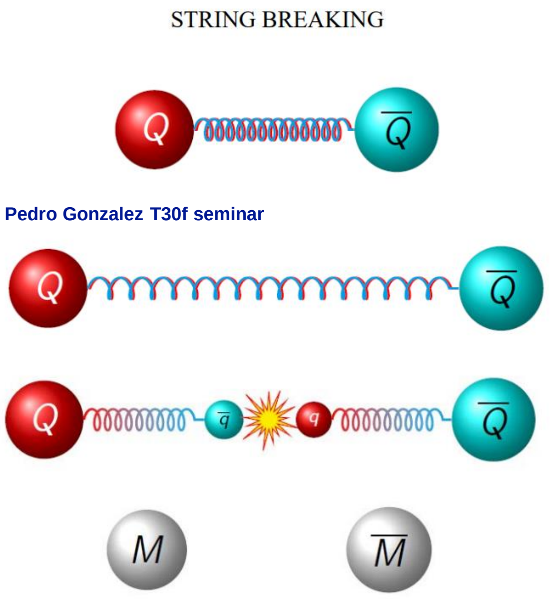
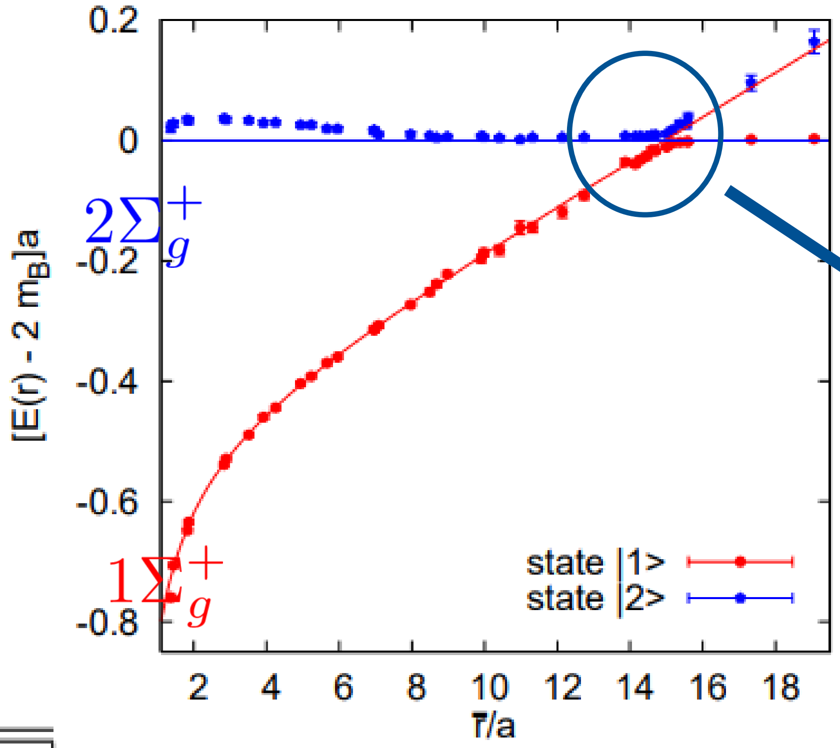
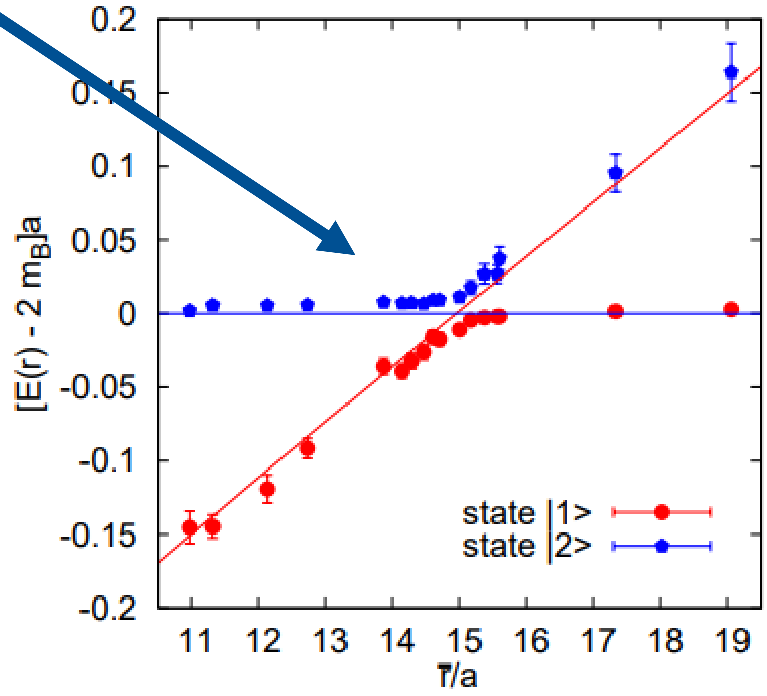


Figure from Pedro Gonzalez T30f seminar





$V_\Psi(r) = -\frac{4}{3}\frac{\alpha_s}{r} + \sigma r$

$m_M + m_{\bar{M}}$

$m_\pi \approx 650$ MeV

String breaking radius $\approx$ 1.25 fm

a $\approx$ 0.083 fm

Meson-antimeson threshold

$K_{\bar{q}}^P \otimes K_q^P$	K^{PC}	Static energies $D_{\infty h}$
$(1/2)^- \otimes (1/2)^+$	0^{-+}	$\{\Sigma_u^-\}$
	1^{--}	$\{\Sigma_g^+, \Pi_g\}$
$(1/2)^- \otimes (1/2)^-$	0^{++}	$\{\Sigma_g^+\}$
	1^{+-}	$\{\Sigma_u^-, \Pi_u\}$
$(1/2)^- \otimes (3/2)^-$	1^{+-}	$\{\Sigma_u^-, \Pi_u\}$
	2^{++}	$\{\Sigma_g^+, \Pi_g, \Delta_g\}$

s-wave+s-wave

Ex. $D\bar{D}$ threshold

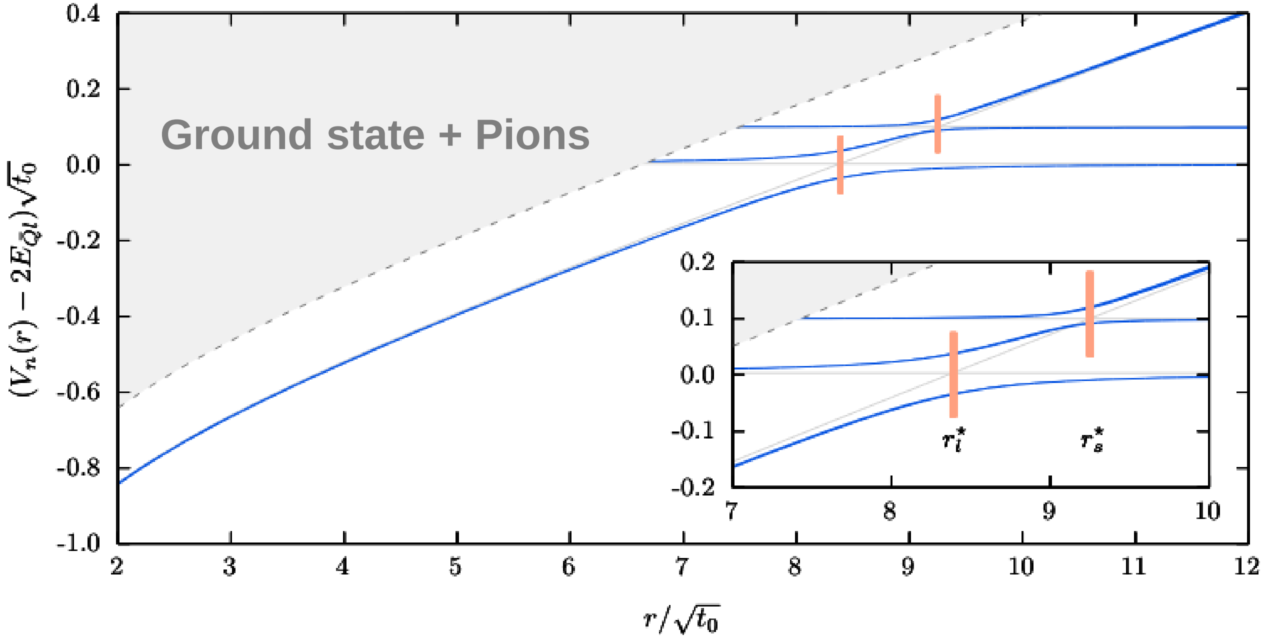
BO-quantum # Σ_g^+ mix: avoided crossing between $Q\bar{Q}$ & $M\bar{M}$

Static Energies: Avoided crossing

More recent computation of string breaking:

Bulava et al, Phys. Lett. B. 793 (2019)

Bulava et al, arXiv 2403.00754



Model Hamiltonian for determining parameters:

$$H(r) = \begin{pmatrix} \hat{V}(r) & \sqrt{2}g_l & g_s \\ \sqrt{2}g_l & \hat{E}_1 & 0 \\ g_s & 0 & \hat{E}_2 \end{pmatrix}, \hat{V}(r) = \hat{V}_0 + \sigma r + \gamma/r$$

$m_\pi \approx 200 - 340$ MeV $\quad m_K \approx 440 - 480$ MeV

String breaking radius ≈ 1.22 fm **a** ≈ 0.063 fm

Hybrid static energies: **(Σ_u^-, Π_u): avoided crossing** with **s-wave + p-wave threshold**

No lattice results available on this till now

Tetraquark / pentaquark static energies: Is there any meaning to avoided crossing ?

Hybrid: Mixing with heavy-light

Brambilla, AM, Vairo

arXiv 2406.xxxxx

- **Unique result from BOEFT:**

Hybrid decays to two S-wave mesons **allowed**: $$H_m \longrightarrow D^{(*)}\bar{D}^{(*)}$$

BOEFT: Mixing allowed if Λ_η^σ (BO-quantum numbers) are same.

Hybrid

Light spin K^{PC}	Static energies $D_{\infty h}$	l	J^{PC} $\{S_Q=0, S_Q=1\}$	Multiplets
1^{+-}	$\{\Sigma_u^-, \Pi_u\}$	1	$\{1^{--}, (0,1,2)^{-+}\}$	H_1
	$\{\Pi_u\}$	1	$\{1^{++}, (0,1,2)^{+-}\}$	H_2
	$\{\Sigma_u^-\}$	0	$\{0^{++}, 1^{+-}\}$	H_3
	$\{\Sigma_u^-, \Pi_u\}$	2	$\{2^{++}, (1,2,3)^{+-}\}$	H_4
	$\{\Pi_u\}$	2	$\{2^{--}, (1,2,3)^{-+}\}$	H_5

BO-quantum # Λ_η^σ for threshold

$K_{\bar{q}}^P \otimes K_q^P$	K^{PC}	Static energies $D_{\infty h}$
$(1/2)^- \otimes (1/2)^+$	0^{-+}	$\{\Sigma_u^-\}$
	1^{--}	$\{\Sigma_g^+, \Pi_g\}$
$(1/2)^- \otimes (1/2)^-$	0^{++}	$\{\Sigma_g^+\}$
	1^{+-}	$\{\Sigma_u^-, \Pi_u\}$
$(1/2)^- \otimes (3/2)^-$	1^{+-}	$\{\Sigma_u^-, \Pi_u\}$
	2^{++}	$\{\Sigma_g^+, \Pi_g, \Delta_g\}$

s-wave+s-wave
Ex. $\mathbf{D\bar{D}}$ threshold

s-wave+p-wave
Ex. $\mathbf{D_1\bar{D}}$ threshold

See R. Bruschini talk on Tuesday on Branching Ratios !!!

Bruschini 2306.17120 Bruschini & Gonzalez, 1912.07337 J. Castella 2401.13393