Dimesons and dibaryons with heavy quarks using lattice QCD

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EHS 2024 @ Swansea

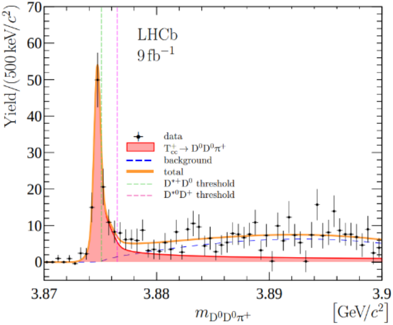
1. T_{cc} tetraquark
2. T_{bc} tetraquark
3. Heavy dibaryons

Only heavy 'flavored' hadrons

T_{cc} : PRL 129 (2022) 032002, PRD 109 (2024) 094509
T_{bc} : PRL 132 (2024) 201902, PRD XXXXXX
Heavy dibaryons : PRL 130 (2023) 111901, PRL 123 (2019) 162003, PRD 106 (2022) 054511

T_{cc} tetraquark

Doubly heavy tetraquarks: T_{cc}^+



LHCb: 2109.01038, 2109.01056

$$\delta m \equiv m_{T_{cc}^+} - (m_{D^{*+}} + m_{D^0})$$

$$\delta m_{\text{pole}} = -360 \pm 40^{+4}_{-0} \text{ keV}/c^2 ,$$
$$\Gamma_{\text{pole}} = 48 \pm 2^{+0}_{-14} \text{ keV} .$$

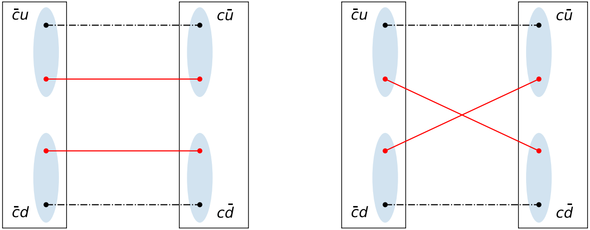
- The doubly charmed tetraquark T_{cc}^+, $I = 0$ and favours $J^P = 1^+$. Nature Phys., Nature Comm. 2022
  Striking similarities with the longest known heavy exotic, X(3872).
- No features observed in $D^0 D^+ \pi^+$: possibly not $I = 1$.
- Many more exotic tetraquark candidates discovered recently, T_{cs}, $T_{c\bar{s}}$, $X(6900)$.
  Prospects also for T_{bc} in the near future. See talk by Ivan Polyakov at Hadron 2023
- Doubly heavy tetraquarks: theory proposals date back to 1980s.
  c.f. Ader&Richard PRD25(1982)2370

Hadron spectroscopy using lattice QCD

- Compute matrices of two point correlation functions

$$C_{ji}(t) = \langle 0|\bar{O}_j(t) O_i(0)|0\rangle = \sum_n \frac{Z_i^n Z_j^{n*}}{2E_n} e^{-E_n(t)}$$

- For doubly charmed four quark systems near DD^* threshold.
  we use 'only' O of type $(\bar{q}\Gamma_1 c)_{1_c}(\bar{q}'\Gamma_2 c)_{1_c}$.
- Wick contractions to compute: $[c,\ q,\ q']$



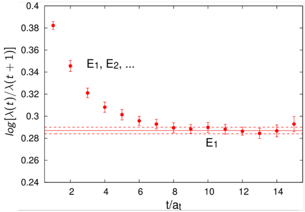
- Lattice QCD ensembles : CLS Consortium
  $m_\pi \sim$ 280 MeV, $m_K \sim$ 467 MeV, $a \sim 0.086$ fm
  Spatial volumes: $L \sim 2$ fm and $L \sim 2.7$ fm
  Charm quark masses ($m_c^?$): $m_D \sim$ 1762 MeV and 1927 MeV

Mass/Energy extraction

- Euclidean two point current-current correlation matrices

$$C_{ji}(t_f - t_i) = \langle 0|\mathcal{O}_j(t_f)\bar{\mathcal{O}}_i(t_i)|0\rangle = \sum_n \frac{Z_i^{n*}Z_j^n}{2m_n} e^{-E_n(t_f-t_i)}$$

where $\mathcal{O}_j(t_f)$ and $\bar{\mathcal{O}}_i(t_i)$ are the desired interpolating operators and $Z_j^n = \langle 0|\mathcal{O}_j|n\rangle$.

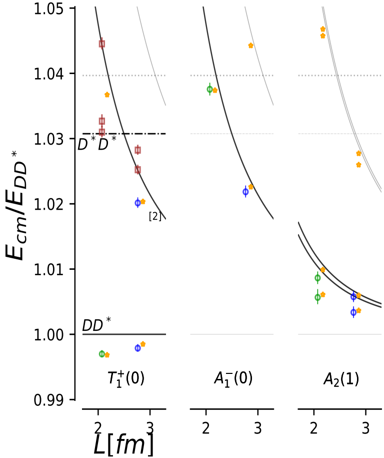


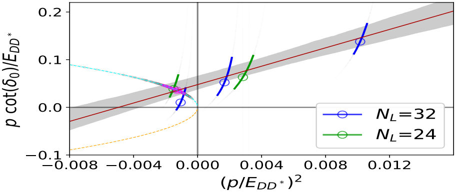
- Solving generalized eigenvalue problem

$$C(t)v^{(n)}(t) = \lambda^{(n)}(t)C(t_0)v^{(n)}(t)$$

- Effective mass defined as $\log[\frac{\lambda(t)}{\lambda(t+1)}]$
- The ground state : from the exponential fall off at large times.
  Non-linear fitting techniques.

DD^* scattering in $l = 0, 1$ @ $m_c^{(h)}$ with an ERE





MP, Prelovsek PRL 2022, Prelovsek Talk@EHS 2023

$m_\pi \sim 280$ MeV

- $t^{-1} = p\cot\delta_0 - ip$
  Real (virtual) bound pole at $p = +(-)i|p|$.
- Fit quality: $\chi^2/d.o.f. = 3.7/5$.
- Binding energy: $\delta m_{T_{cc}} = -9.9(^{+3.6}_{-7.2})$ MeV.
- First evaluation of the DD^* amplitude in T_{cc} channel.

+/g refers to positive parity, -/u refers to negative parity.

More recent lattice studies:

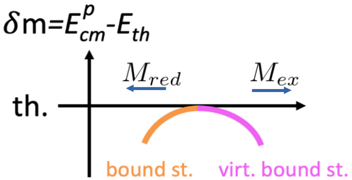
Chen *et al* 2206.06185 [PLB], Lyu *et al* 2302.04505 [PRL], Green @ Lattice 2023, Whyte, Wilson, Thomas 2405.15741

Results and inferences with ERE approach

- A shallow virtual bound state pole in s-wave related to T_{cc}.

	m_D [MeV]	$\delta m_{T_{cc}}$ [MeV]	T_{cc}
lat. ($m_\pi \simeq 280$ MeV, $m_c^{(h)}$)	1927(1)	$-9.9^{+3.6}_{-7.2}$	virtual bound st.
lat. ($m_\pi \simeq 280$ MeV, $m_c^{(l)}$)	1762(1)	$-15.0(^{+4.6}_{-9.3})$	virtual bound st.
exp.	1864.85(5)	$-0.36(4)$	bound st.

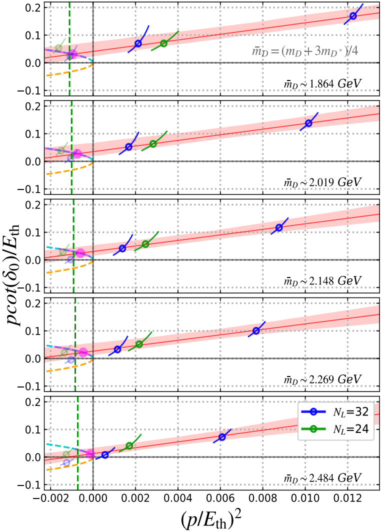
- Observations in line with the expected behaviour of a near-threshold molecular bound state pole in simple Quantum Mechanical potentials.

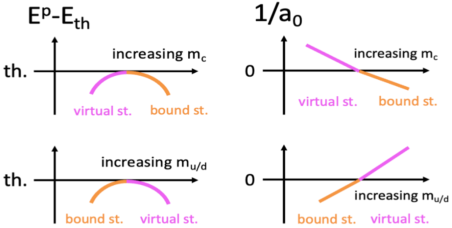


MP, Prelovsek PRL 2022

- $M_{red}(\propto m_c)$ is the reduced mass of the DD^* system.
- The mass of the particle exchanged during the interaction $M_{ex}(\propto m_{u/d})$.

m_c dependence of the T_{cc} pole [ERE]





Collins, Nefediev, MP, Prelovsek 2402:14715

$m_\pi \sim 280$ MeV

- Virtual bound poles at all values of m_c.
- m_c dependence: Purely attractive, roughly m_Q independent DD^* potential.
- Critical mass: $\bar{m}_D^{\rm crit}(\rm ERE) = 2.71(^{+34}_{-26})$ GeV
- ERE: Questionable [OPE interactions and lhc]

Pion exchange interactions/left-hand cut: ERE and QC

- A two fold problem: (Unphysical pion masses used in lattice)

  $m_\pi > m_{D^*} - m_D \quad \Rightarrow \quad D^* \to D\pi$ is kinematicaly forbidden.

  $2 \to 2$ Generalized LQC: does not subthreshold lhc effects.

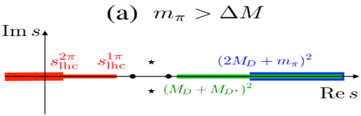
  Raposo&Hansen 2311.18793, Dawid *et al* 2303.04394, Hansen *et al* 2401.06609

  ERE convergence fails at the nearest singularity.

  Left-hand cut in the DD^* system close below the DD^* threshold.

  Du, Baru, Hanhart *et al* 2303.09441[PRL]

- Unphysical pion masses ($m_\pi > \Delta M = M_{D^*} - M_D$, stable D^* meson):



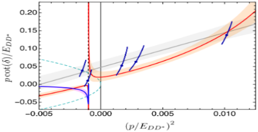


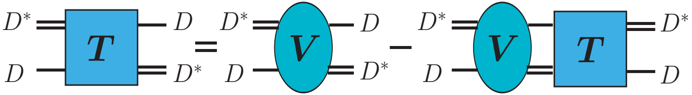
Figure taken from Du, Baru, Hanhart *et al* 2303.09441[PRL]

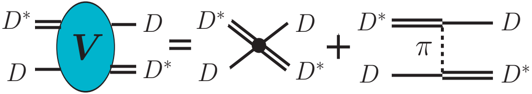
Long range pion exchange interactions: the origin of left-hand singularity and cut.

Fits with a potential that incorporates the one pion exchange:

Virtual bound states $\Rightarrow$ Virtual resonances

Solving Lippmann-Schwinger Equation for the DD^* amplitude





- The potential: a sum of short range and long range interactions

$$V(\boldsymbol{p},\boldsymbol{p}') = V_{\rm CT}(p,p') + V_\pi^S(p,p') \quad \text{with} \quad V_{\rm CT}(p,p') = 2c_0 + 2c_2(p^2 + p'^2) + \mathcal{O}(p^4, p'^4)$$

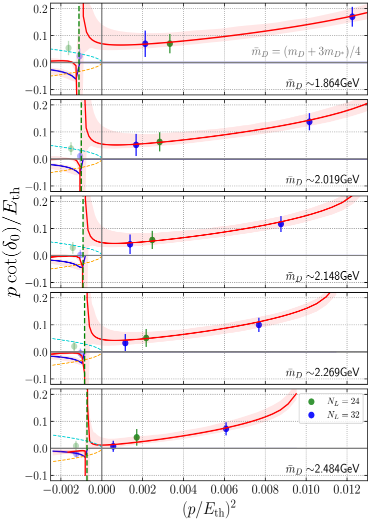
- The scattering amplitude $\qquad T^{-1} \propto p\cot\delta_0 - ip$

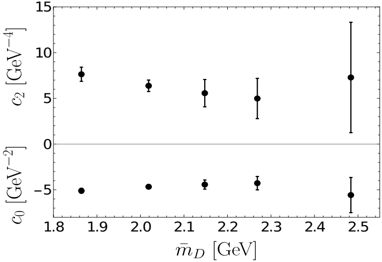
- The pion decay constant f_π and $DD^*\pi$ coupling g_c
  at $m_\pi \sim 280$ MeV following the 1-loop χPT.

Du, Baru, Hanhart *et al* 2303.09441[PRL]

Collins, Nefediev, MP, Prelovsek 2402:14715

m_c dependence of the T_{cc} pole [EFT]



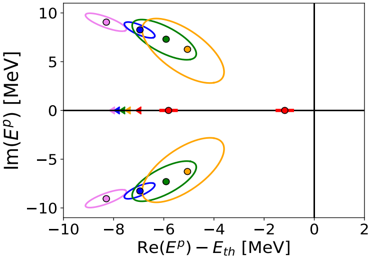


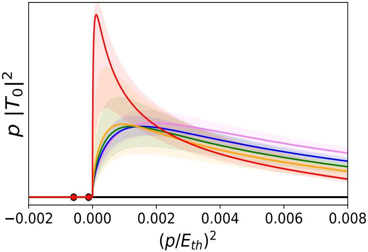
- Resonance poles below threshold at all values of m_c except the heaviest.
- At the heaviest m_c: virtual bound poles
- Weak m_c dependence in $V(\boldsymbol{p}, \boldsymbol{p}')$.

Collins, Nefediev, MP, Prelovsek 2402:14715

$m_\pi \sim 280$ MeV

Pole positions and scattering rate [EFT]

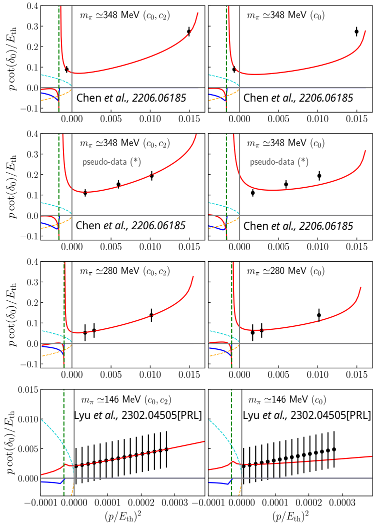




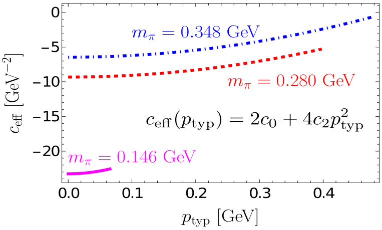
- Subthreshold resonance pole pair moving towards the real axis with increasing m_c.
- Collide on the real axis below threshold and turn back-to-back.
  At the heaviest m_c: virtual bound poles [in Red]
- With increasing m_c, subthreshold resonance poles evolves to become
  a pair of virtual bound poles.
- Enhancement in the DD^* scattering rate $(p|T_0|^2)$.

Collins, Nefediev, MP, Prelovsek 2402:14715

m_π dependence of the T_{cc} pole [EFT]

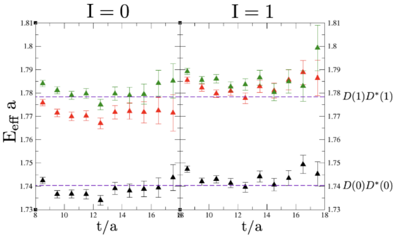


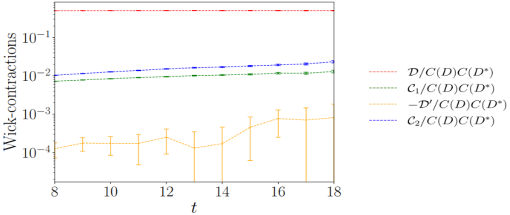
Collins, Nefediev, MP, Prelovsek 2402:14715

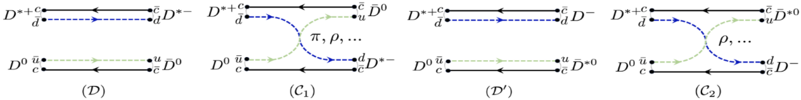


- Qualitative study of m_π dependence using $V_{\rm CT}(p,p') = 2c_0 + 2c_2(p^2 + p'^2)$
- Two parameter fit (c_0, c_2) [left] and a single parameter fit (c_0, with $c_2 = 0$) [right].
- Resonance poles at m_π ∼348 and ∼280 MeV. Shallow virtual bound poles at m_π =146 MeV.
- Stronger attraction for lighter m_π. [$c_{\rm eff}$] stronger binding in T_{cc} for lighter pions.
- m_π =146 MeV: HALQCD procedure.

Isovector T_{cc}: Weak repulsion







$$C_I^{DD^*}(t) = \mathcal{D} - \mathcal{C}_1 + (-)^{I+1}(\mathcal{D}' - \mathcal{C}_2)$$

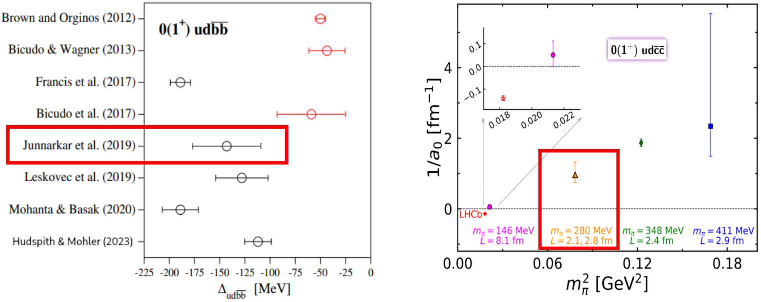
- $\mathcal{C}_2$ determining the isospin effects. Weakly repulsive interactions.

Ortiz-Pacheco, MP, *et al.*, 2312.13441

- Consistent with observations reported in Chen *et al.*, 2206.06185 [PLB]

T_{bc} tetraquark

Doubly heavy tetraquarks using lattice QCD, T_{bb} and T_{cc}: $I(J^P) = 0(1^+)$

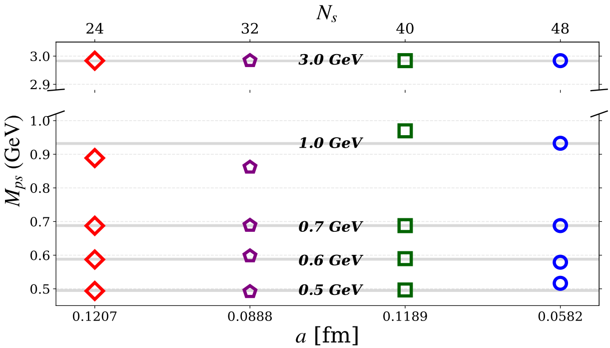


- Deeper binding in doubly bottom tetraquarks $\mathcal{O}(100MeV)$. Fig: Hudspith&Mohler 2023
  Red box: ILGTI work on QQ tetraquarks: Junnarkar, Mathur, MP PRD 2019

- Shallow bound state in doubly charm tetraquarks $\mathcal{O}(100keV)$. Fig: Lyu *et al.*PRL 2023
  Red box: T_{cc} (RQCD) [PRL 2022] and its quark mass dependence [2402.14715].
  Whyte, Wilson and Thomas arXiv:2405.15741

- No concrete conclusions in the bottom-charm tetraquark sector.
  A summary of different lattice investigations → see review by Pedro Bicudo, 2212.07793

Lattice setup for bottom hadrons



- MILC dynamical ensembles with $N_f = 2 + 1 + 1$ HISQ fields.
- Valence quark fields with masses ranging from light to charm: overlap action
- Bottom quark evolution using a NRQCD Hamiltonian.
  tuned using kinetic mass of 1S bottomonium spin averaged $\overline{M}^{\bar{b}b}$ Mathur *et al* Lattice 2016

Correlation functions and Interpolators: $I(J^P) = 0(1^+)$

- Focus on the T_{1g} finite volume irrep in the rest frame.

- Two point correlations computed as

$$\mathcal{C}_{ij}(t) = \sum_{\mathbf{x}} \left\langle \mathcal{O}_i(\mathbf{x},t)\mathcal{O}_j^\dagger(0) \right\rangle = \sum_n \frac{Z_i^n Z_j^{n\dagger}}{2E^n} e^{-E^n t},$$

with wall smearing for quark fields at source.

- Focus only on the ground state energy splitting. Relevant low lying two meson thresholds

$$\begin{aligned} DB^*\ [included]: &\qquad E_{et}^{phys} \sim\ 7.190GeV \\ BD^*\ [included]: &\qquad E_{it1}^{phys} \sim\ 7.290GeV \\ D^*B^*\ [excluded]: &\qquad E_{it2}^{phys} \sim\ 7.334GeV \end{aligned}$$

- Local 2 two-meson-like interpolators and one diquark-antidiquark-like interpolator

$$\begin{aligned} \mathcal{O}_1(x) &= [\bar{u}\gamma_i b][\bar{d}\gamma_5 c](x) - [\bar{d}\gamma_i b][\bar{u}\gamma_5 c](x), \\ \mathcal{O}_2(x) &= [\bar{u}\gamma_5 b][\bar{d}\gamma_i c](x) - [\bar{d}\gamma_5 b][\bar{u}\gamma_i c](x), \\ \mathcal{O}_3(x) &= [(\bar{u}^T\Gamma_5\bar{d} - \bar{d}^T\Gamma_5\bar{u})(b\Gamma_i c)](x). \end{aligned}$$

MP *et al* PRL 2024

Spectrum extraction: $I(J^P) = 0(1^+)$

- $\mathcal{C}_{ij}(t)$ are solved for the generalized eigenvalue problem [GEVP]

$$\mathcal{C}(t)v^n(t) = \lambda^n(t)\mathcal{C}(t_0)v^n(t)$$

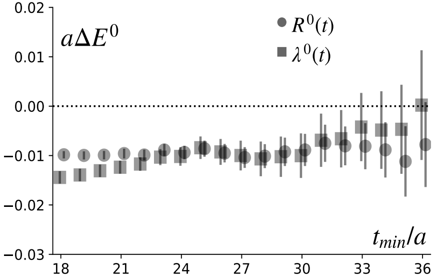
- Fits to the eigenvalue correlators $[\lambda^n]$ and the ratio of eigenvalue correlators with a non-interacting correlator $[R^n(t) = \frac{\lambda^n(t)}{\mathcal{C}_{m_1}(t)\mathcal{C}_{m_2}(t)}]$.

MP *et al* Lattice 2021

- Fits to the ground state in the finest ensembl with $M_{ps} \sim 0.7$ GeV in terms of energy splittings from $M_{B^*} + M_D$.

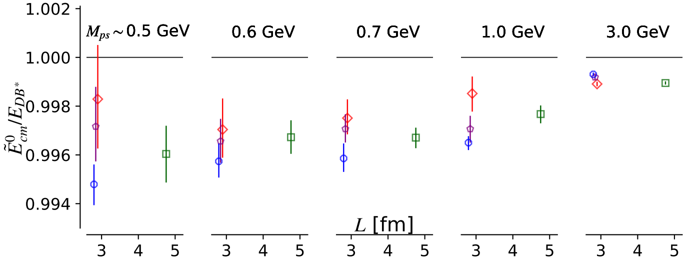
$$\Delta E^0 = E^0 - M_{B^*} - M_D$$

- t_{min} dependence of energy estimates from fits to $R^0(t)$ and $\lambda^0(t) \rightarrow$



MP *et al* PRL 2024

The ground state spectrum: $I(J^P) = 0(1^+)$

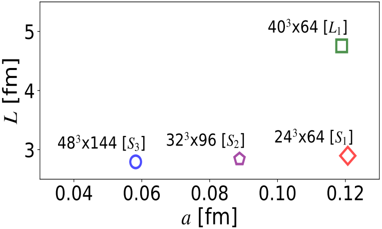


- Energy spectrum determined based on fits to $R^0(t)$.
  Automatic accounting for NRQCD additive correction.

- Energy reconstructed using $\tilde{E}^0 = \Delta E^0 + \overline{M}^{lat}_{B^*} + M^{lat}_D$
  where $\overline{M}^{lat}_{B^*} = M^{lat}_{B^*} - 0.5\overline{M}^{\bar{b}b,\ lat} + 0.5\overline{M}^{\bar{b}b,\ phys}$

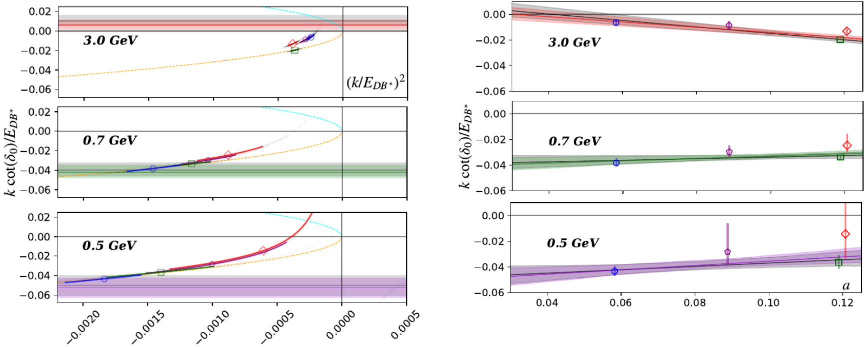
- Consistent negative energy shifts.
  Decreasing magnitude with increasing $m_{u/d}$ or M_{ps}

- Non-trivial lattice spacing dependence.



MP *et al* PRL 2024

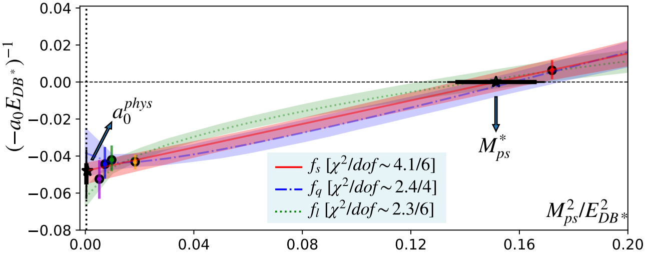
Finite volume analysis and continuum extrapolation: $I(J^P) = 0(1^+)$



- Elastic DB^* scattering: finite volume analysis *à la* Lüscher. Briceño PRD 2014
  Only ground states used and only scattering length in an ERE. $[k cot\delta_0 \sim -1/a_0]$
- A linear lattice spacing dependence assumed for the fitted amplitude.
- Determined DB^* scattering length in the continuum limit for all M_{ps}.
  Results indicate attractive interaction between D and B^* mesons at all M_{ps}.

MP *et al* PRL 2024

M_{ps} dependence of $DB^{(*)}$ scattering length



- Light quark mass ($m_{u/d}$ or M_{ps}) dependence.

  $f_l(M_{ps}) = \alpha_c + \alpha_l M_{ps}, \quad f_s(M_{ps}) = \beta_c + \beta_s M^2_{ps}, \quad$ and $\quad f_q(M_{ps}) = \theta_c + \theta_l M_{ps} + \theta_s M^2_{ps}.$

  indicates a real bound state at physical pion mass.

- DB^* scattering length and binding energy in the continuum limit

$$a_0^{phys} = 0.57(^{+4}_{-5})(17) \text{ fm} \quad \text{and} \quad \delta m_{T_{bc}} = -43(^{+6}_{-7})(^{+14}_{-24}) \text{ MeV}$$

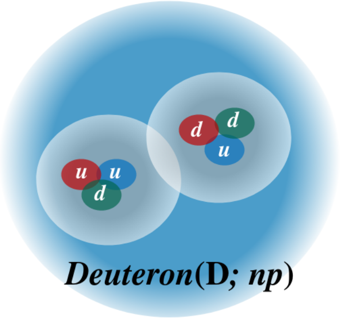
- DB scattering length and binding energy in the continuum limit

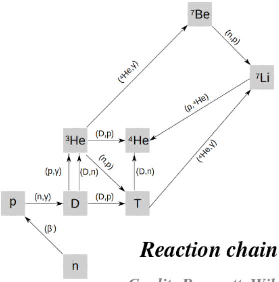
$$a_0^{phys} = 0.61(^{+3}_{-4})(18) \text{ fm} \quad \text{and} \quad \delta m_{T_{bc}} = -39(^{+4}_{-6})(^{+8}_{-18}) \text{ MeV}$$

MP *et al* PRL 2024, Radhakrishnan, MP, Mathur, PRD 2024

Heavy dibaryons

Deuteron: the longest known dibaryon



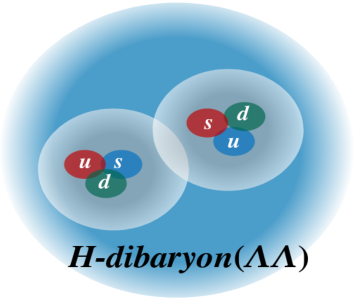


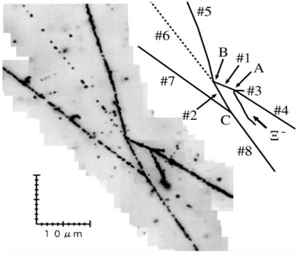
Reaction chain in BBN

Credit: Pamputt, Wikimedia Commons

- Nucleus of Deuterium discovered in 1932. Urey, Brickwedde & Murphy
- A very fine-tuned binding energy $\Delta E = M_D - M_p - M_n = 2.2$ MeV.
- Big bang Nucleosynthesis (BBN) has a deuteron bottleneck:
  Determines the abundances of light nuclei.
- How will the binding energy vary with quark masses?
  Could there be dineutrons or diprotons with heavier light quark masses.

The scalar dihyperon

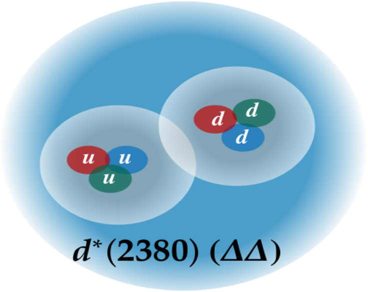


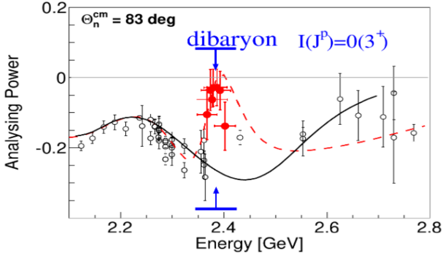


***NAGARA event*:**
Takahashi PRL 87, 212502 (2001)

- Bound *uuddss* flavor-singlet dihyperon with $J^P = 0^+$:
  Perhaps a stable Dihyperon — Jaffe PRL 38 195 (1977)
- NAGARA event: Strongest constraint on binding energy.
  $$B_H < B_{\Lambda\Lambda}^{Nagara} = 6.91 \pm 0.16 \text{ MeV}$$
  Takahashi *et al.*, PRL 87, 212502 (2001)
- ALICE @ LHC: constraints on ΛΛ interactions from femtoscopic measurements
  ALICE 1905.07209 PLB

d^* resonance





***d^* dibaryon*:** Clement 1610.05591, WASA@COSY/SAID 1402.6844 PRL

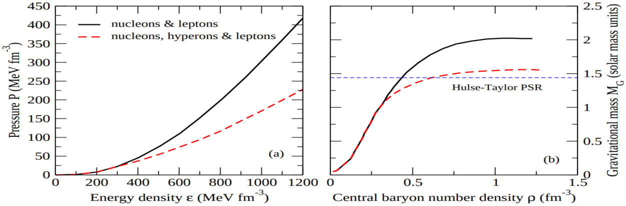
- Prediction for an isoscalar $\Delta\Delta$ configuration with $J^P = 3^+$. Assumed SU(6) symmetry. Dyson and Xuong PRL 13 815 (1964)
- Resonance feature at 2.38 GeV with $\Gamma \sim 70$ MeV and $I(J^P) = 0(3^+)$. Pole in the coupled $^3D_3 - ^3G_3$ partial waves. Adlarson *et al.*, 1402.6844 PRL
- Whether isosymmetric partner of d^* with maximal isospin exists? Other possible nonstrange dibaryon candidates, if any.

Baryon-baryon interactions: Other prospects

- Hyperon formation $\Leftarrow$ Large nuclear densities in astrophysical objects

Bazavov *et al*, 1404.6511 PRL, 1404.4043 PLB

Chatterjee and Vidaña 1510.06306 EPJA, Vidaña *et al* 1706.09701 PLB



- A handful of experimental efforts using large nuclei reactions.
  Inputs on LECs to EFTs $\Rightarrow$ nuclear many body calculations.

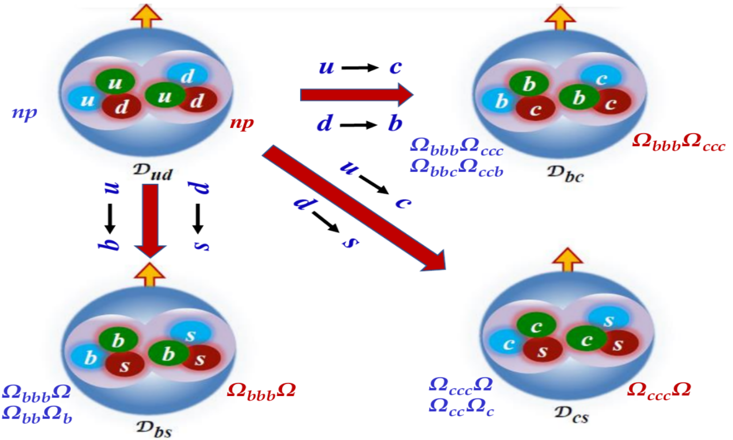
Epelbaum 2005, INT-NFPNP 2022, $0\nu\beta\beta$ PSWR 2022

- Heavy dibaryons: Relatively free of the light quark chiral dynamics.

- Heavy dibaryons: no near three or four particle thresholds.
  Simple model studies ($\Omega\Omega$ scattering): widely different inferences.

Richard *et al* 2005.06894 PRL, Liu *et al* 2107.04957 CPL, Huang *et al* 2011.00513 EPJC

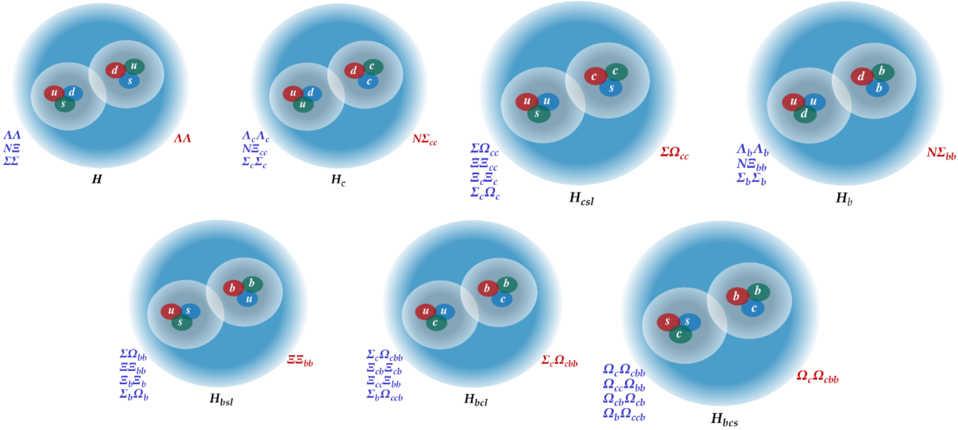
Deuteron-like Heavy dibaryons



Elastic thresholds in red text

Junnarkar and Mathur 1906.06054 PRL

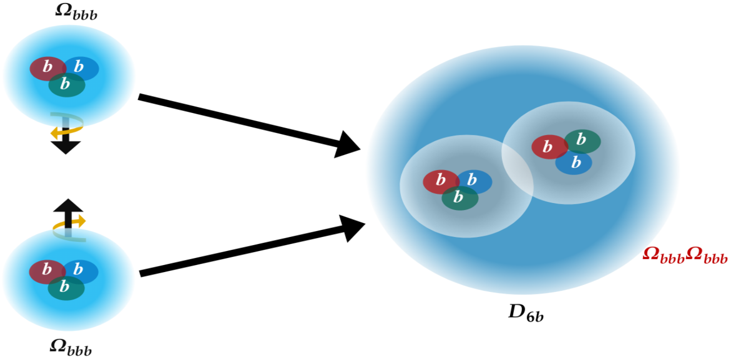
Triply flavored heavy dibaryons



Elastic thresholds in red text

Junnarkar and Mathur 2206.02942 PRD

Single flavored heavy dibaryons ($\mathcal{D}_{6q}$)



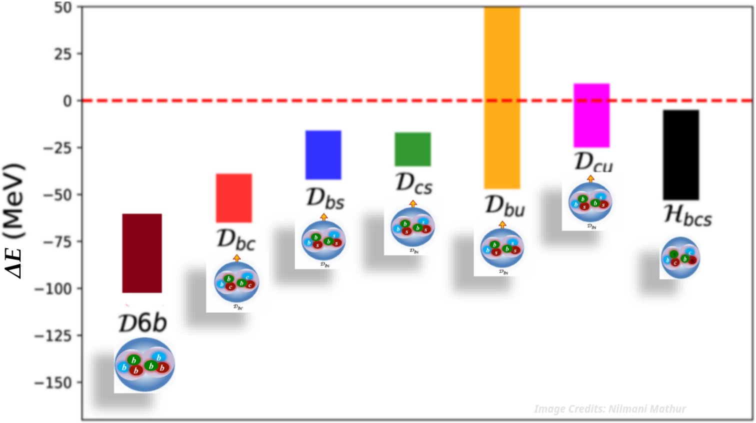
Heavy spin 0 single flavored partner of $d^*(2380)$?? Dyson and Xuong PRL 13 815 (1964)

Leading m_l dependence could arise from pair produced 2π exchanges.

Calculations at m_Q: Relatively cheap calculations with clean signals.

Mathur, MP and Chakraborty 2205.02862 PRL

Heavy dibaryons results summary

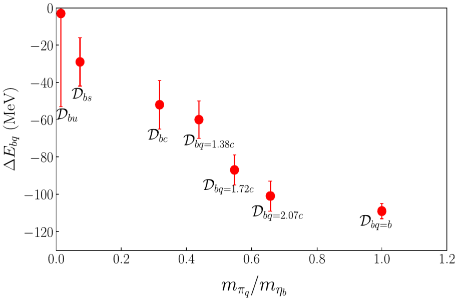


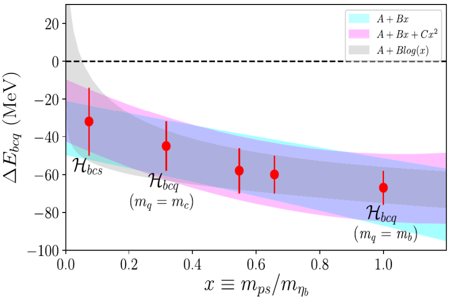
$\Delta E = E - M_{H_1} - M_{H_2}$

Junnarkar and Mathur 1906.06054 PRL ($\mathcal{D}_{bc}$, $\mathcal{D}_{bs}$, $\mathcal{D}_{cs}$, $\mathcal{D}_{bu}$, $\mathcal{D}_{cu}$),
Mathur, MP, Chakraborty 2205.02862 PRL ($\mathcal{D}_{6b}$),

Junnarkar and Mathur 2206.02942 PRD ($\mathcal{H}_{bcs}$)

Light quark mass dependence



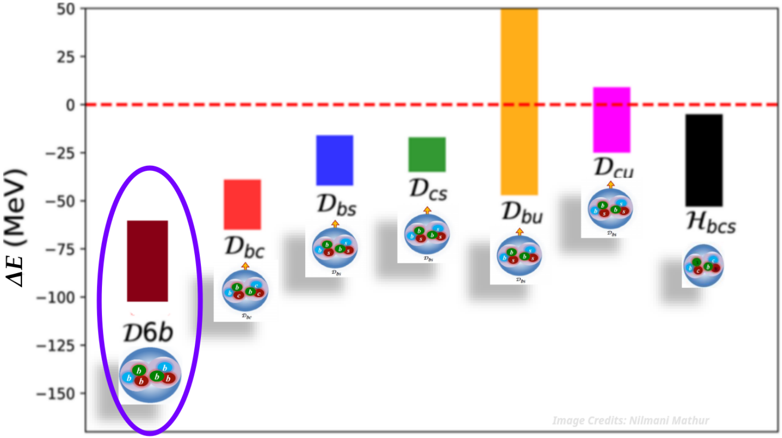


Junnarkar and Mathur 1906.06054 PRL, Junnarkar and Mathur 2206.02942 PRD

Heavier the quark masses, stronger the binding.
Different pattern of binding compared to T_{QQ}

MP, Prelovsek 2202.10110 PRL, Collins, MP, *et al.*, 2402.14715 PRD

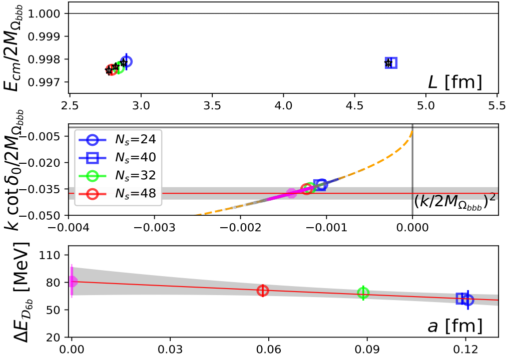
Baryon-baryon interactions in heavy sector



Mathur, MP, Chakraborty 2205.02862 PRL

Not limited to just a finite volume spectrum extraction.
Involved scattering analysis with a zero-range approximation.

Amplitude analysis and binding energy estimate



- Fits with "$-1/a_0^{[0]} - a/a_0^{[1]}$" is found to be the best with $\chi^2/d.o.f. = 0.7/2$

$$a_0^{[0]} = 0.18(^{+0.02}_{-0.02})\ \text{fm},$$

$$a_0^{[1]} = -0.18(^{+0.18}_{-0.11})\ \text{fm}^2$$

- Constraint $k.cot\delta(k) = -\sqrt{-k^2}$ gives us a bound state pole with $\Delta E^{cont}_{\mathcal{D}_{6b}} = -81(^{+14}_{-16})(14)$ MeV.

Using $M^{lphys}_{\Omega_{bbb}} = 14366(7)(9)$ MeV, we compute the mass of this bound state as

$$M^{phys}_{\mathcal{D}_{6b}} = 2M^{lphys}_{\Omega_{bbb}} + \Delta E^{cont}_{\mathcal{D}_{6b}} = 28651(^{+16}_{-17})(15)\ \text{MeV}$$

Summary

- T_{cc} on the lattice, long range pion exchange interactions and left-hand cuts
- Analysis using an Effective Range Expansion and an Effective Field Theory.
  Either parametrizations of DD^* interactions indicate
  a possibly bound system for heavier m_c and lighter m_π.
- The binding of T_{cc}^+ observed in experiments:
  Possibly a delicate interplay between m_c and m_π.

- Attractive interactions between D and $B^{(*)}$ mesons.
  Potential real bound states in axialvector and scalar channels.

- Baryon-baryon interactions in the charm and heavy sector:
  Results for $\mathcal{D}_{6Q}$, $\mathcal{D}_{Qq}$, $\mathcal{H}_{Q_1Q_2q}$
- Expected bound systems for heavier m_c and heavier m_π.
  Quark mass dependence different from that for the tetraquarks.

- T_{cc}: Sara Collins, Luka Leskovec, Alexey Nefediev, Emmanuel Ortiz-Pacheco, Sasa Prelovsek, and Ivan Vujmilovic

- T_{bc}: Nilmani Mathur, Archana Radhakrishnan

- Heavy dibaryons: Debsubhra Chakraborty, Parikshit Junnarkar, and Nilmani Mathur