Dispersive analysis of the σ resonance in $\pi\pi$ scattering, from lattice QCD

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hadspec

EXOHAD
EXOTIC HADRONS TOPICAL COLLABORATION

Spectroscopy in lattice QCD

"hadron spectroscopy explores the possible bound combinations of quarks and gluons allowed by the interactions of QCD"

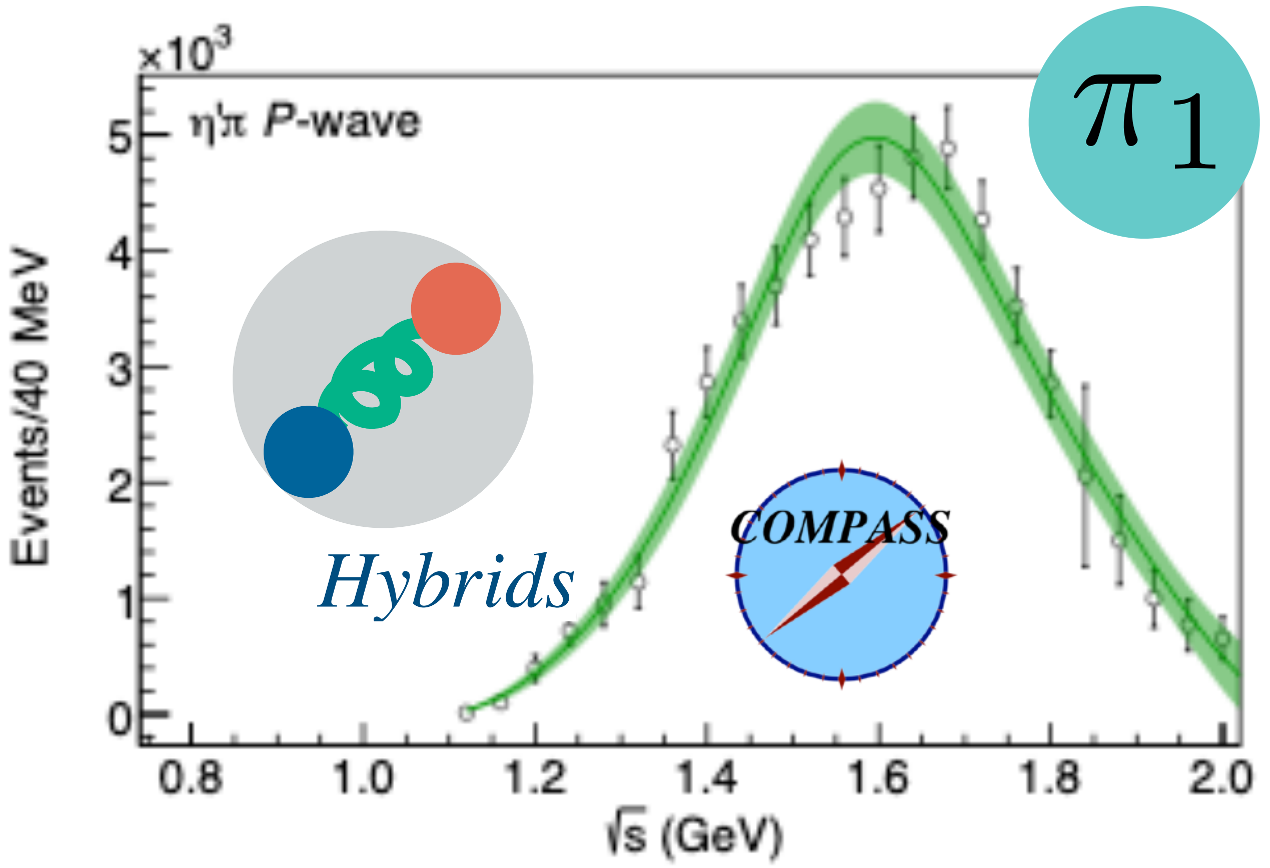
How do quark and gluons combine inside unstable hadrons?

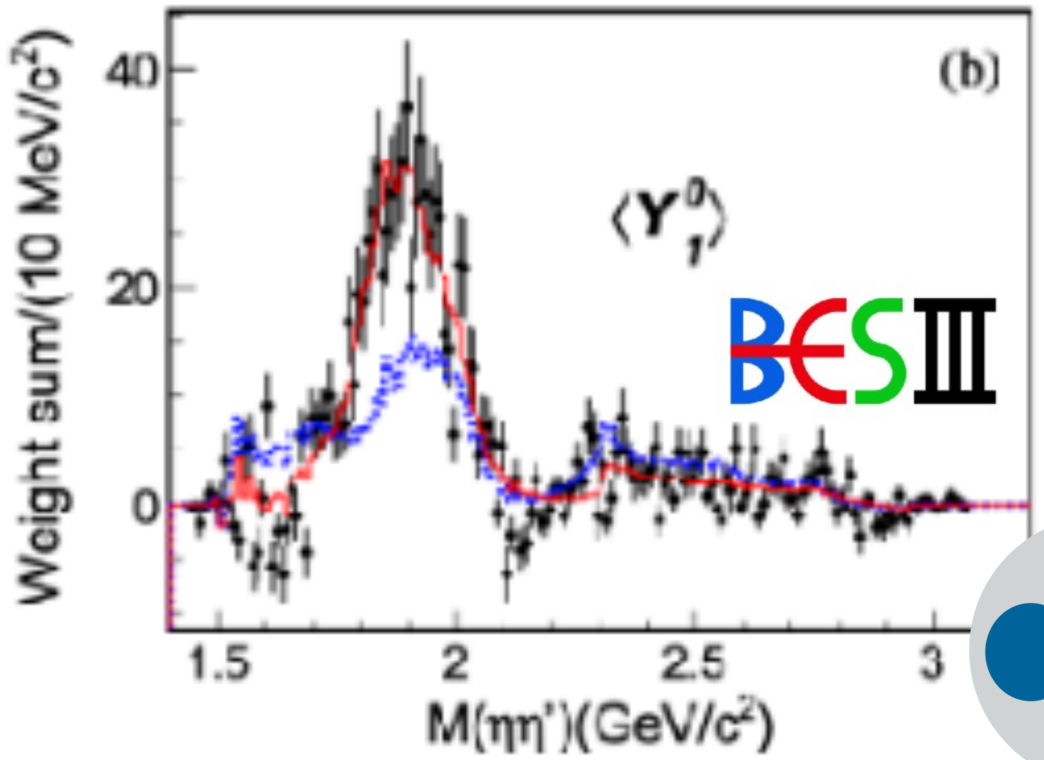
We need a combination of lattice QCD and experiment to answer that question

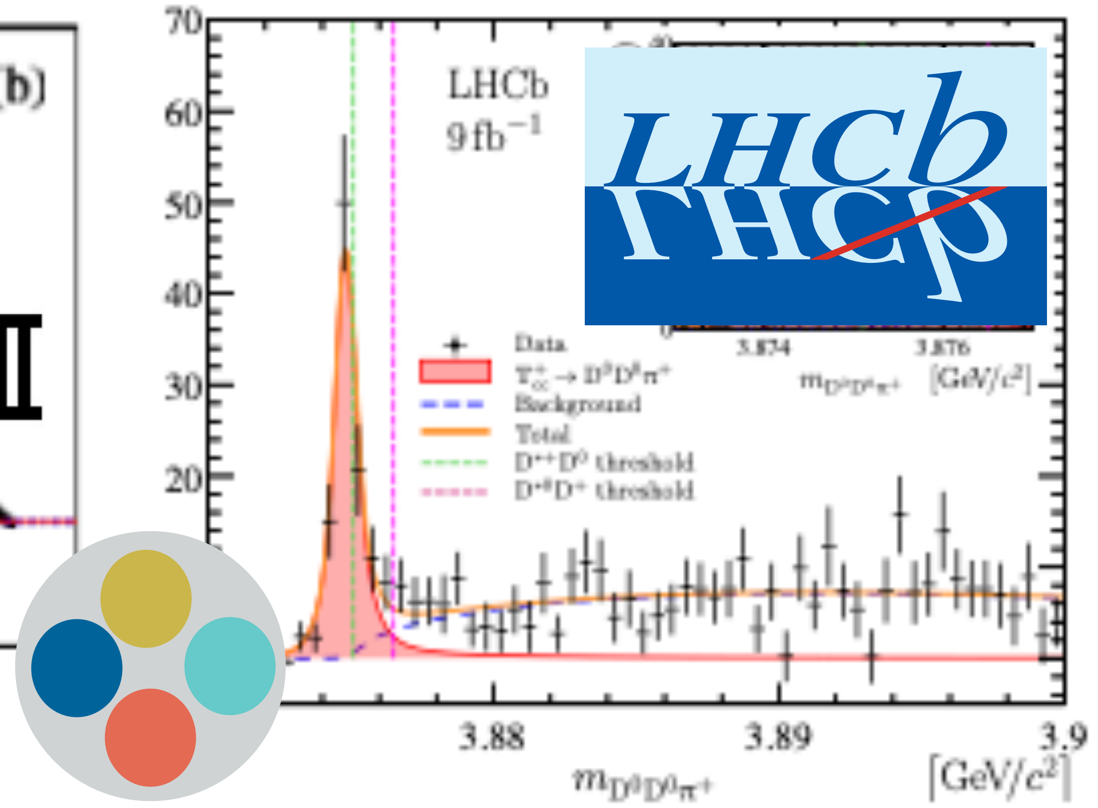
Guide experimental searches (π_1,η_1)

Confirm existence (tetraquarks, pentaquarks, glueballs)

Intermediate particle

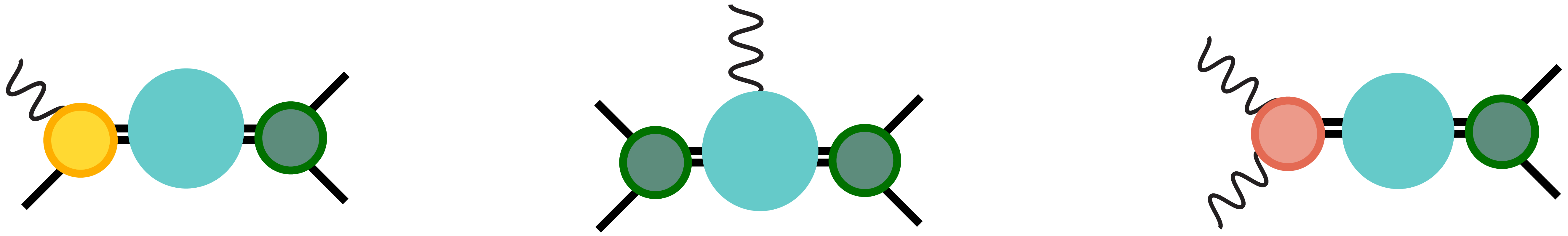


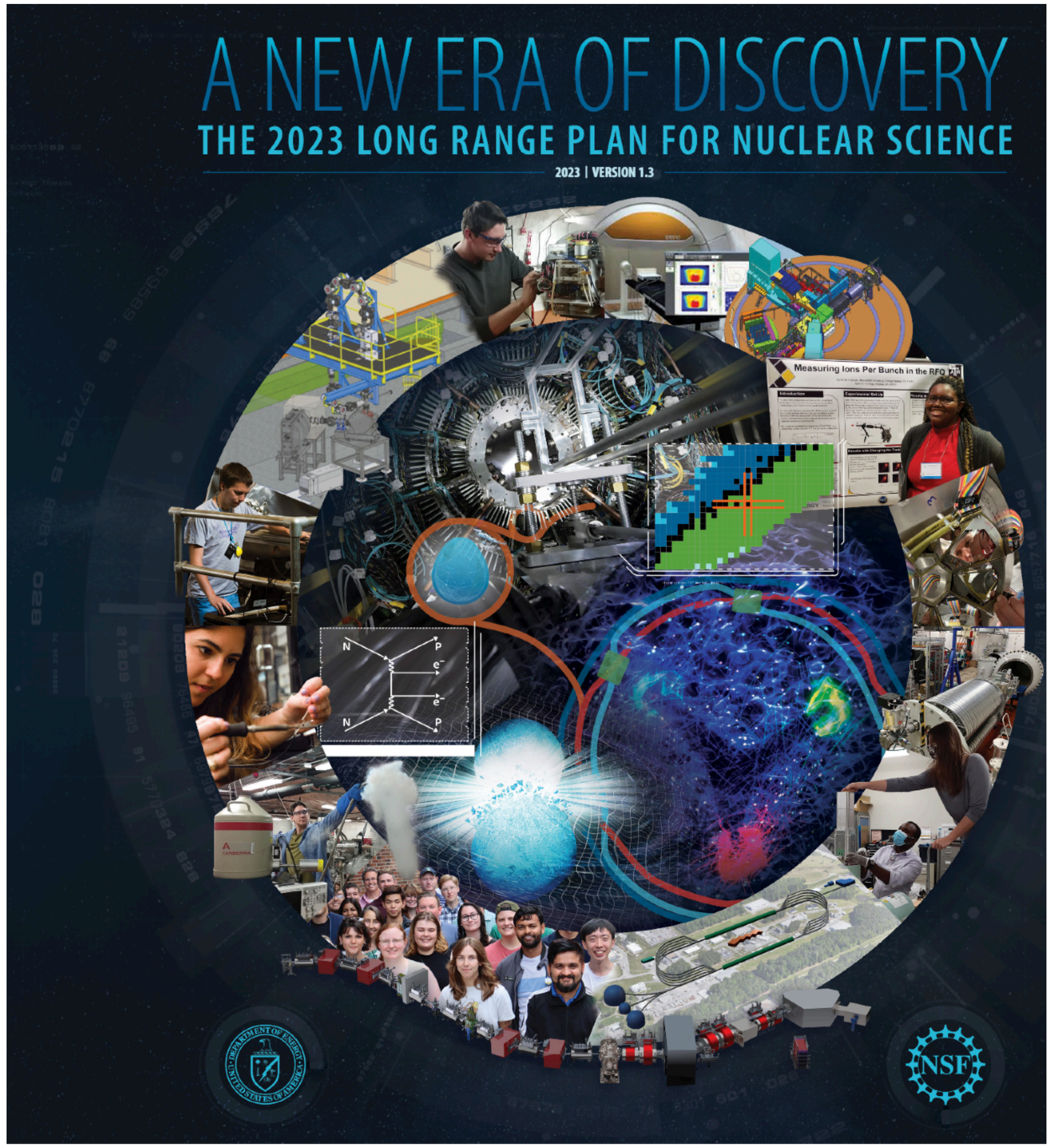




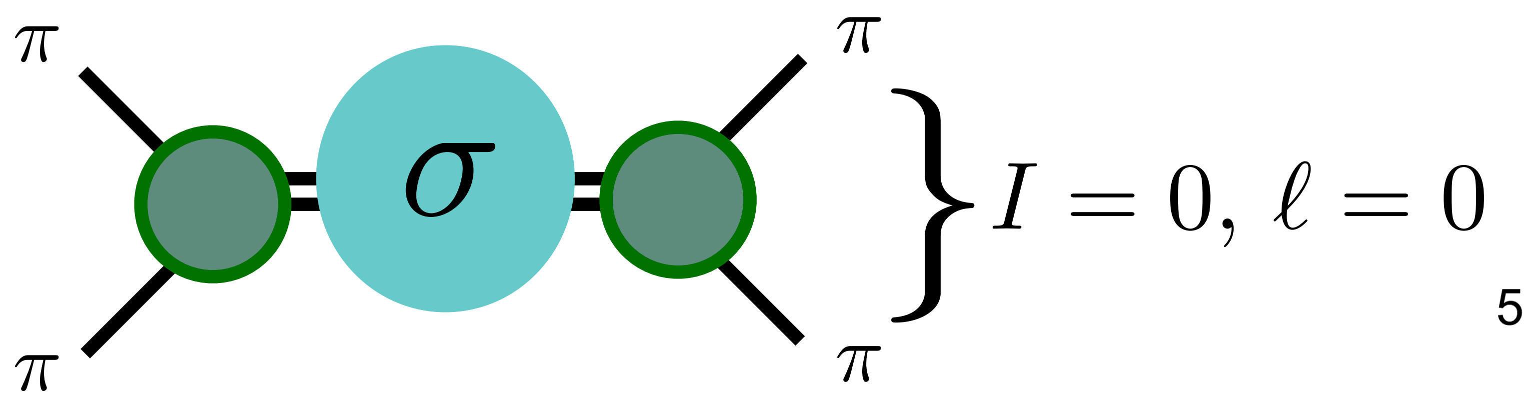
Tetraquarks

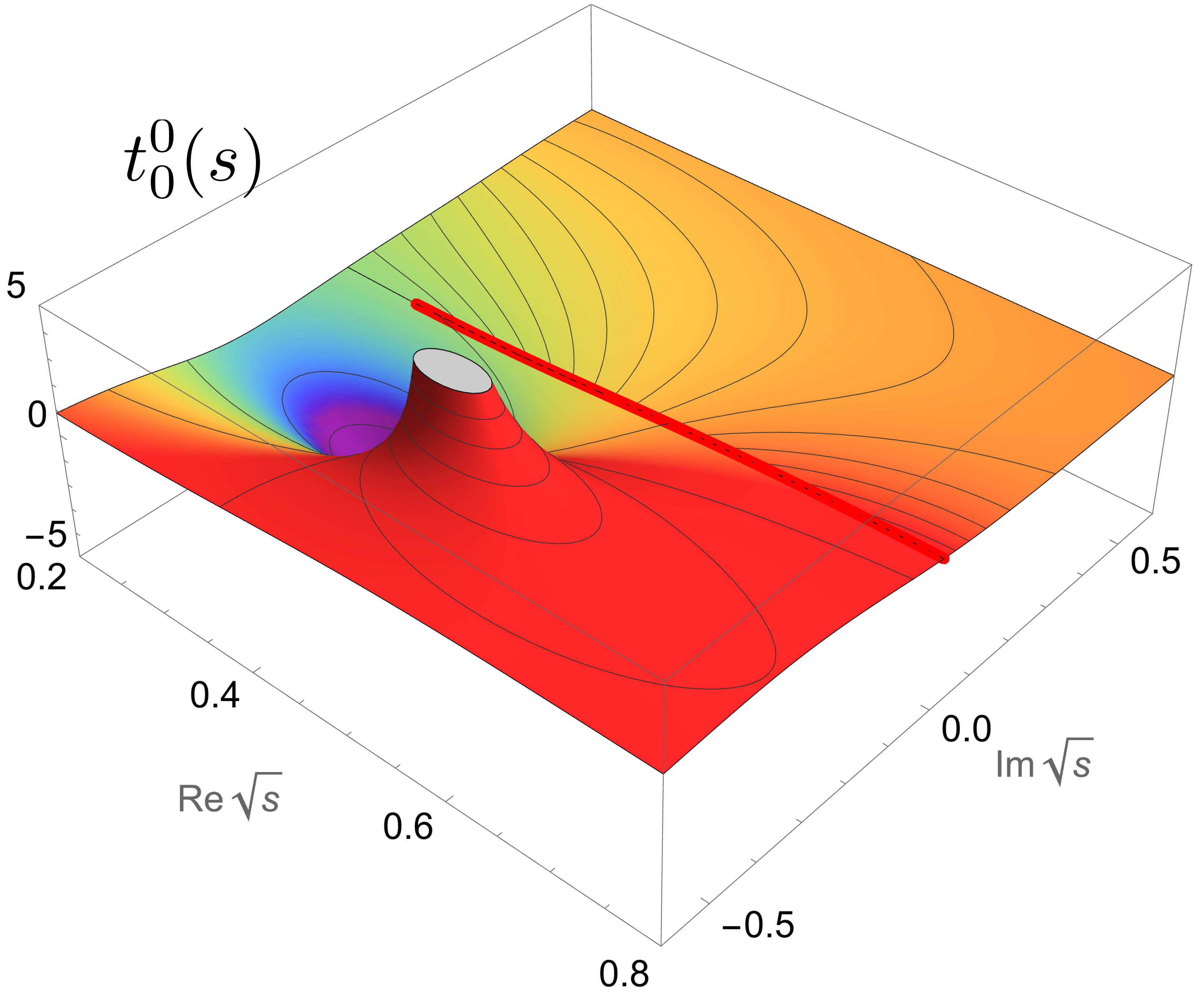
Understand their nature (*observations are not enough*!)





Light Scalars: the σ



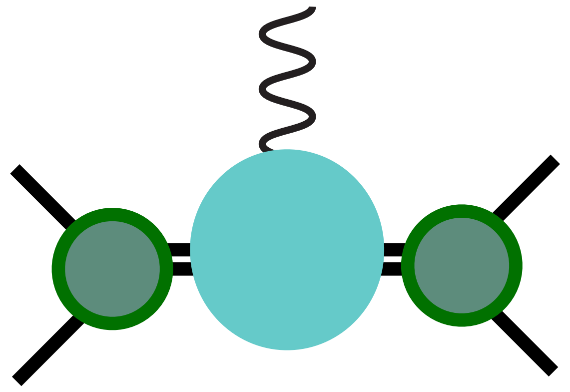


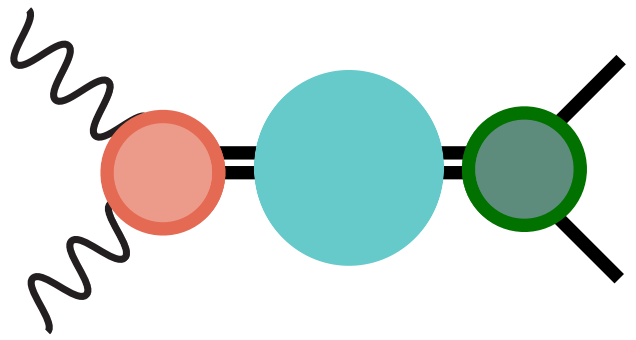
Lightest resonance in QCD

Extremely broad → extremely short-lived

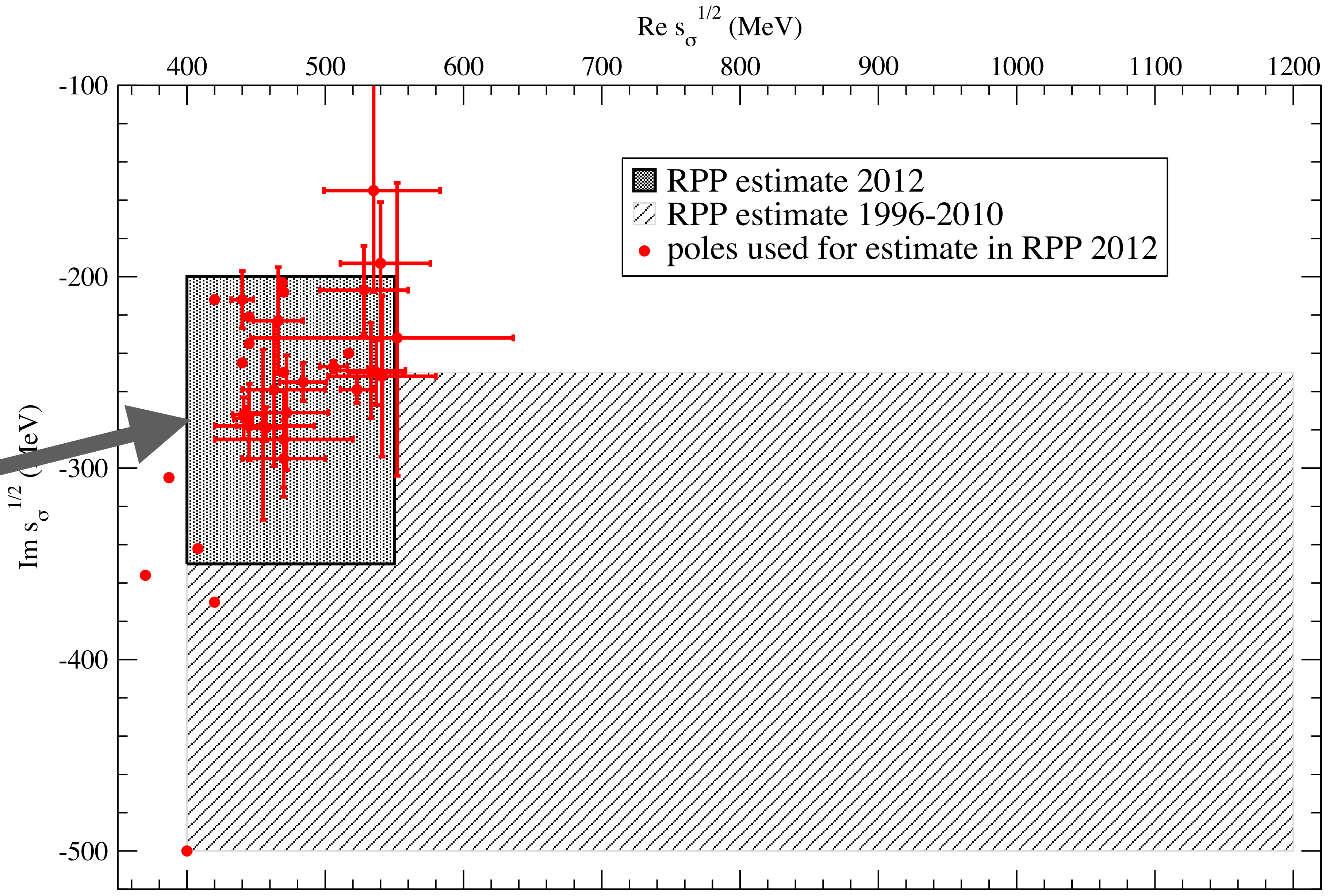
Correlated with chiral symmetry-breaking phenomena (Adler zero)

Not well-understood → new observables ??





Very challenging experimental extraction



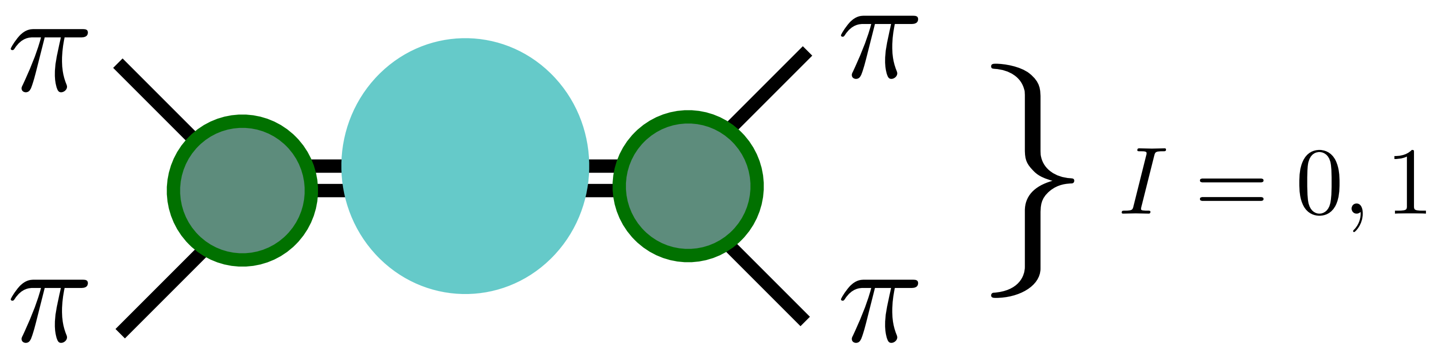
Spectroscopy in lattice QCD

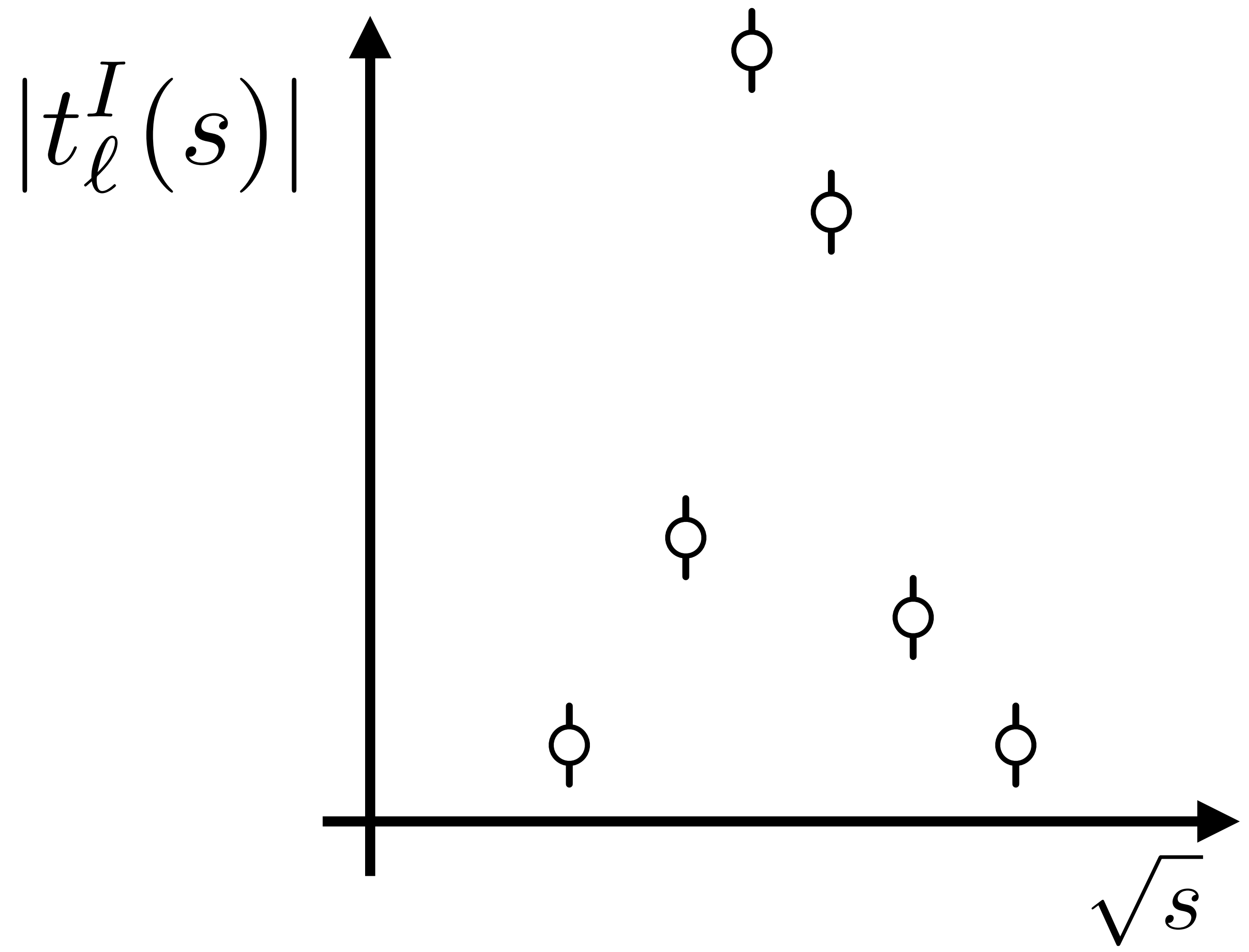
Extracting resonances from 2-body data 101

Assume we have scattering data for well-defined angular momentum

Assume the resonance is narrow and isolated

$$t_\ell^I(s) = \frac{1}{\rho(s)} \frac{\sqrt{s}\Gamma}{m_{\rm BW}^2 - s - i\sqrt{s}\Gamma}$$





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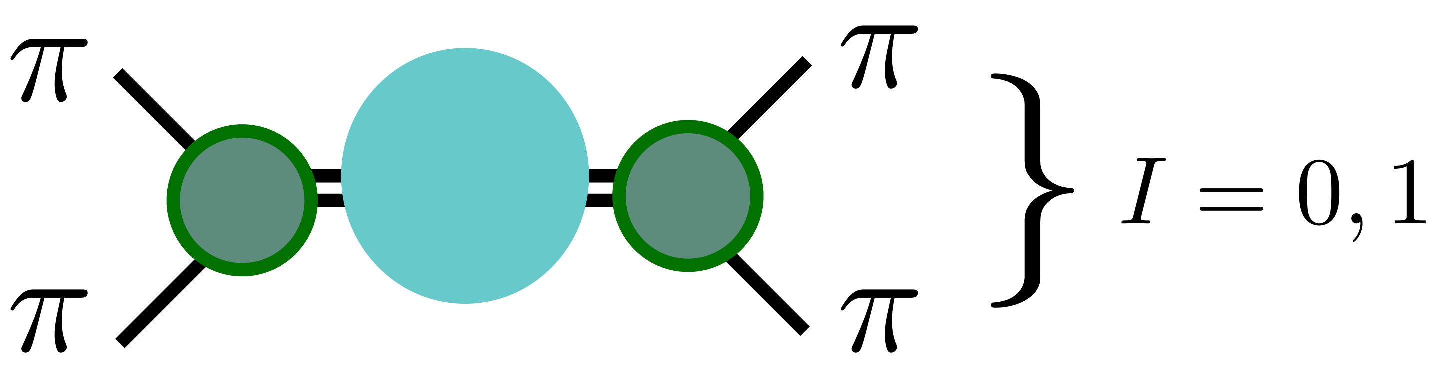
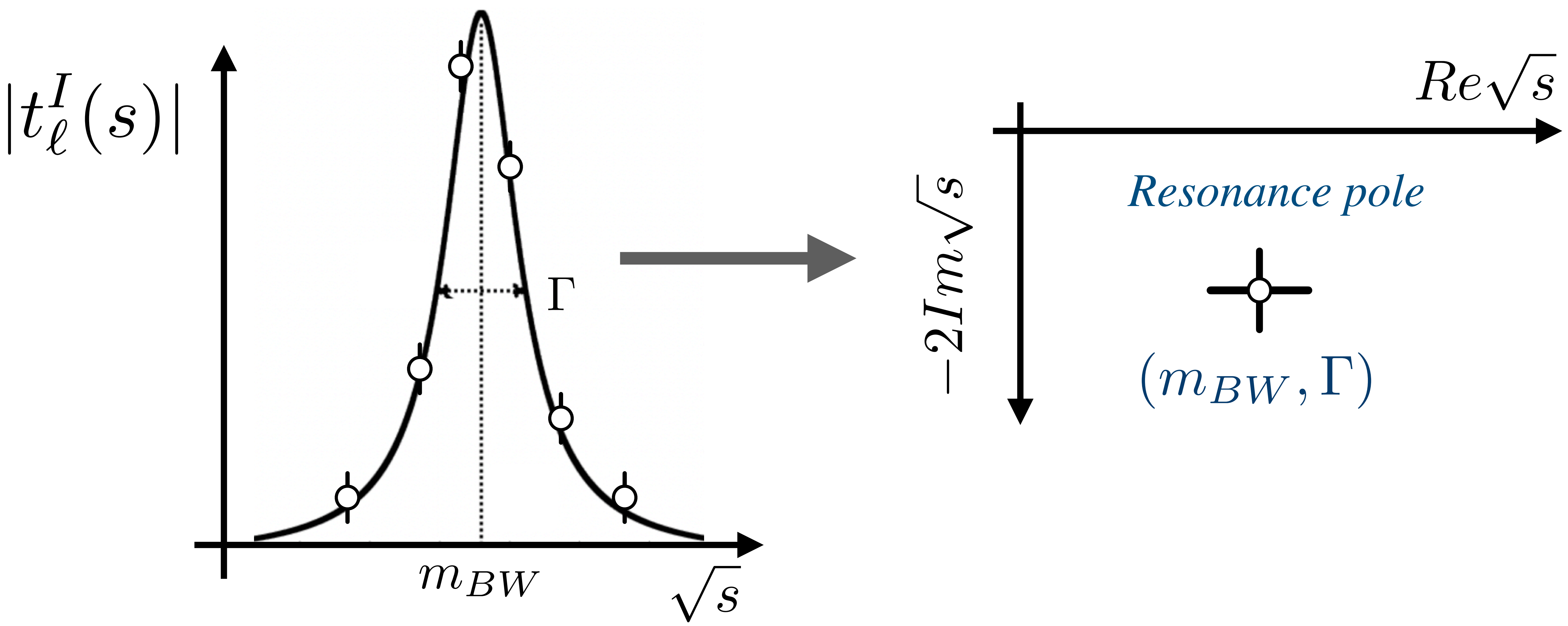


$$t_\ell^I(s) = \frac{1}{\rho(s)} \frac{\sqrt{s}\Gamma}{m_{\rm BW}^2 - s - i\sqrt{s}\Gamma}$$

Pole at $\sqrt{s_p} \sim (m_{BW} - i\Gamma/2)$



More general form for the amplitude

$$t_\ell^I(s) = \frac{K(s)}{1 - i\rho(s)K(s)} = \frac{1}{\rho(s)} e^{i\delta_\ell^I(s)} \sin\delta_\ell^I(s)$$

Elastic case

Spectroscopy in lattice QCD

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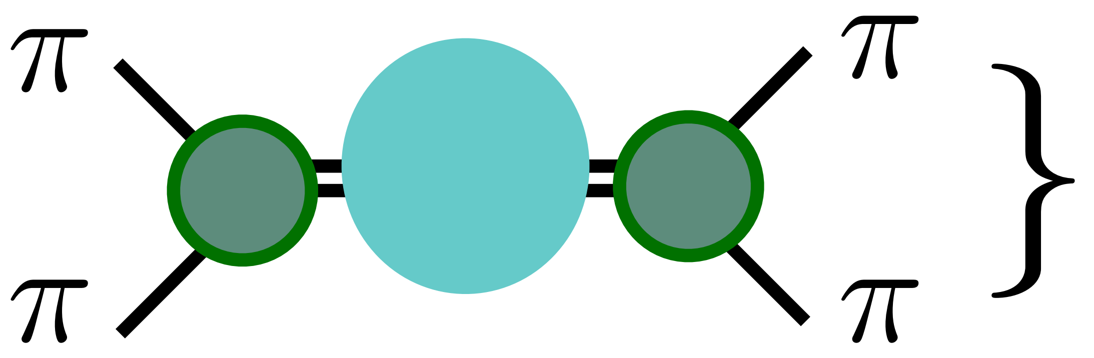

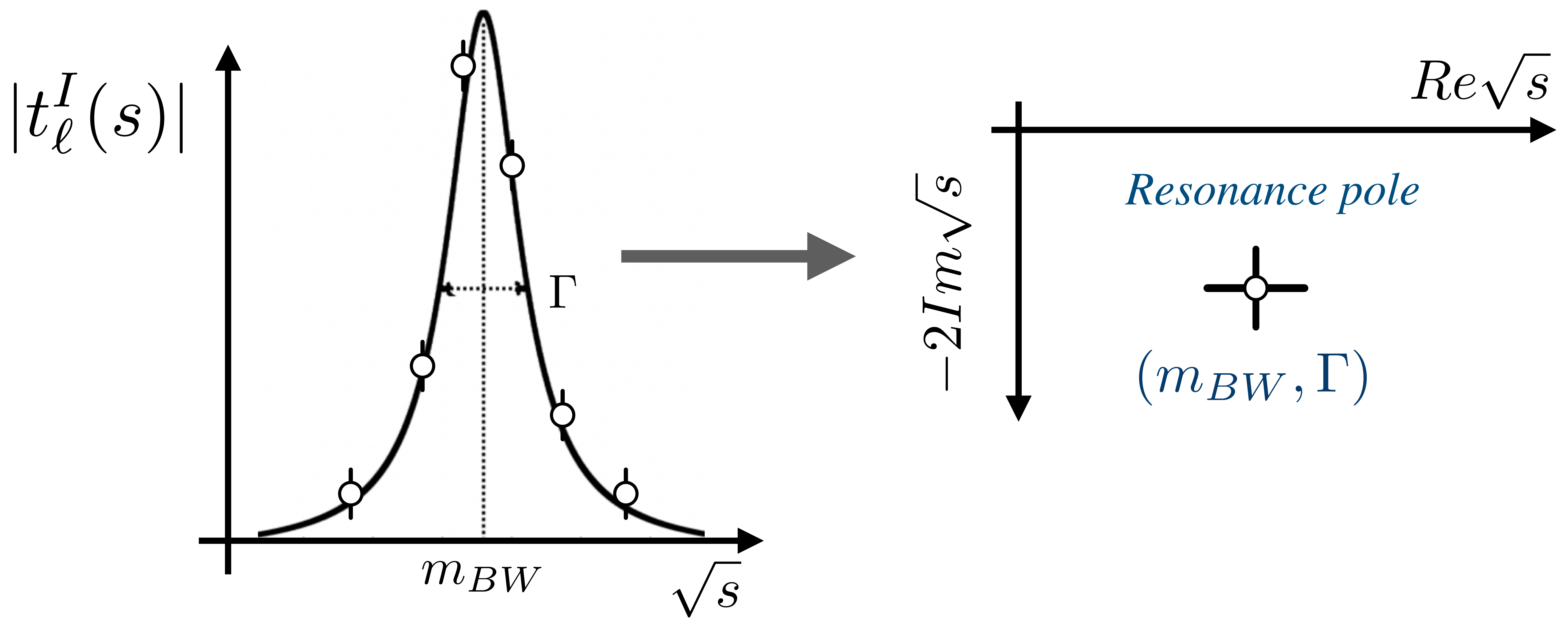

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In lattice QCD, our basic equation is the Lagrangian

$$\mathcal{L}_{\mathrm{QCD}} = \sum_f \bar{\psi}_f \left(i\not{D} - m_f\right)\psi_f - \frac{1}{2g_s^2} \mathrm{Tr}\, G_{\mu\nu}G^{\mu\nu}$$

Quark masses are a parameter for us → m_π is a "choice"

Our basic observables are correlation functions

$$\langle O_f(t) O_i^\dagger(0)\rangle = \frac{1}{Z_T} \int \mathcal{D}[\Phi] \mathrm{e}^{-S_E[\Phi]} O_f[\Phi] O_i^\dagger[\Phi]$$

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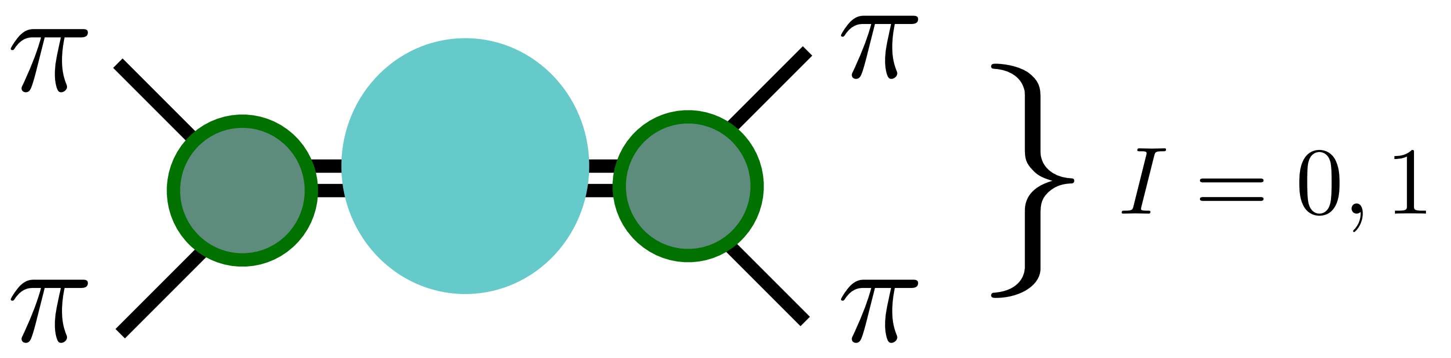

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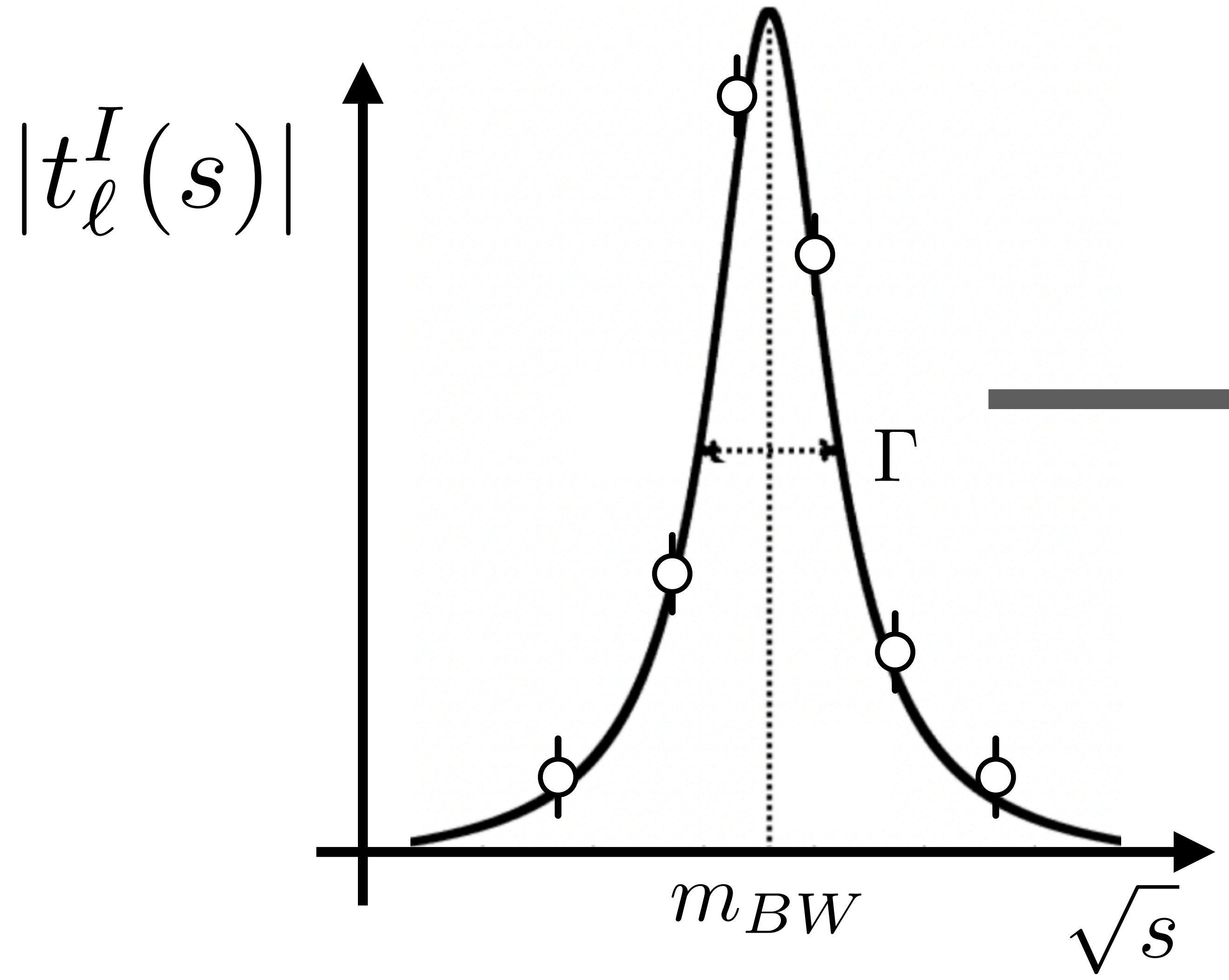

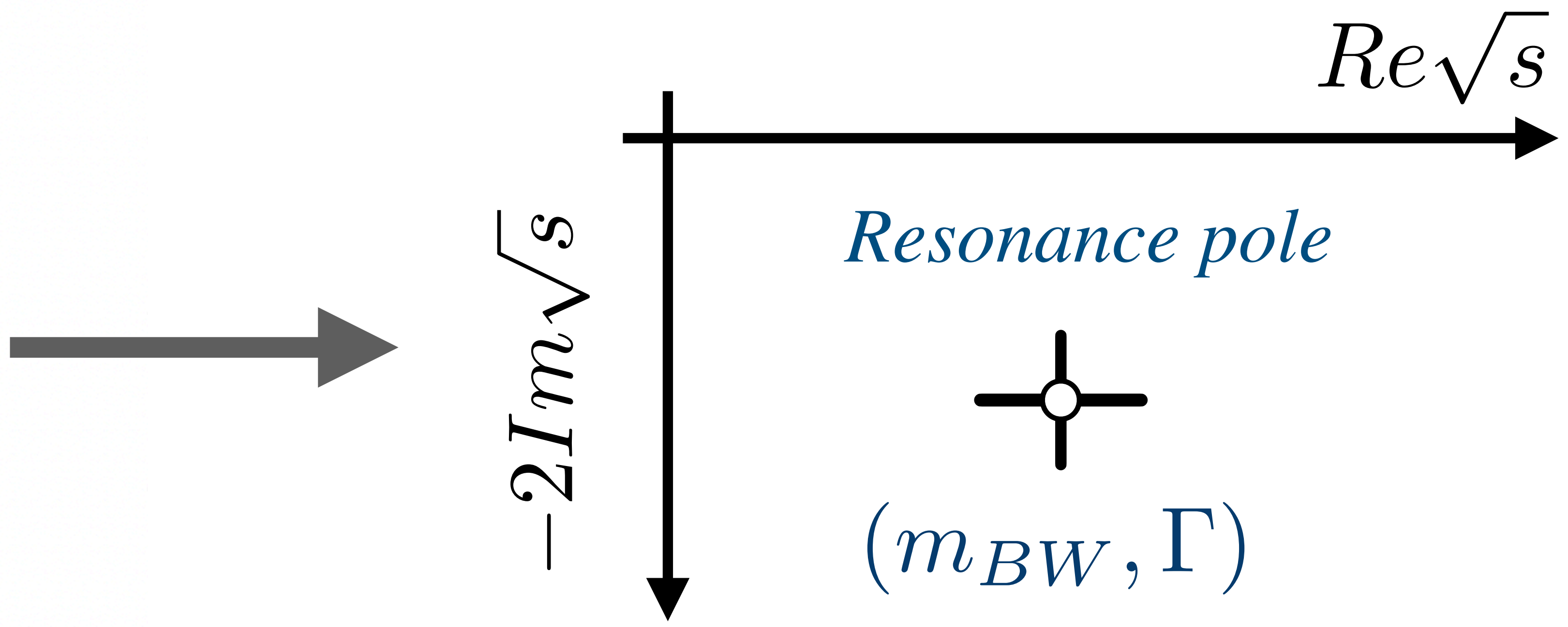

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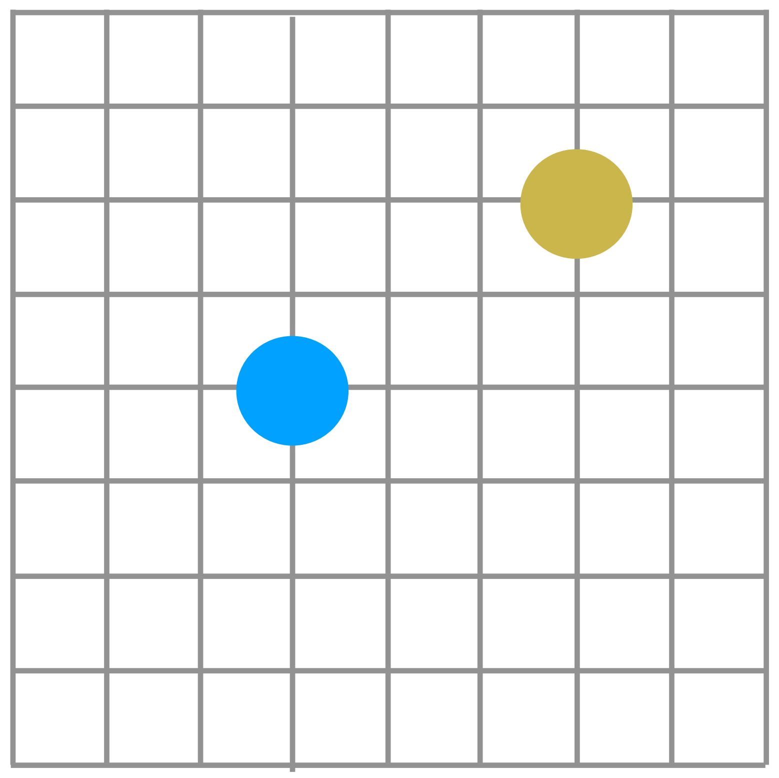
How do we go from here to there??

Spectroscopy in lattice QCD



We start by formulating our theory in a discretized box

$$\int \mathcal{D}[\phi] = \prod_x \int d\phi_x$$

Imagine our quark living on the sites

We perform a time rotation $it \to t \quad iS \to S_E$

$$\int \mathcal{D}[\phi] e^{-iS[\phi]} = \prod_x \int d\phi_x e^{-S_E[\phi_x]}$$

$$0 < e^{-S_E[\phi_x]} < 1$$

Probability-like function

Numerical, Montecarlo sampling of our gluon fields

$$\langle O \rangle = \lim_{N \to \infty} \frac{1}{N} \sum_{n=1}^{N} O\left[U_n\right]$$

N is the number of samples

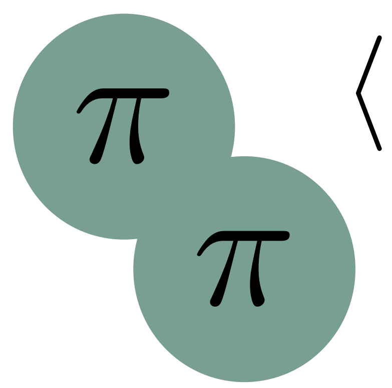
$$\langle O_f(t) O_i^\dagger(0) \rangle$$

Our observables come with a central value and error associated to the number of samples ("measurements")

Spectroscopy in lattice QCD

Quantum mechanical time evolution

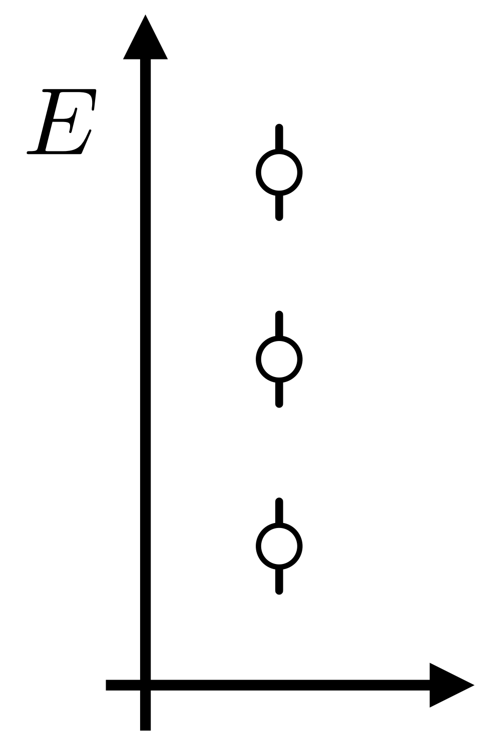
Energy levels are ALWAYS quantized on a box, what we determine from lattice QCD is the values of these energies



$$\langle O_f(t) O_i^\dagger(0)\rangle \sim \sum_n \underline{e^{-E_n t}} \langle 0|O_f(0)|n\rangle \langle n|O_i^\dagger(0)|0\rangle$$

Time is imaginary

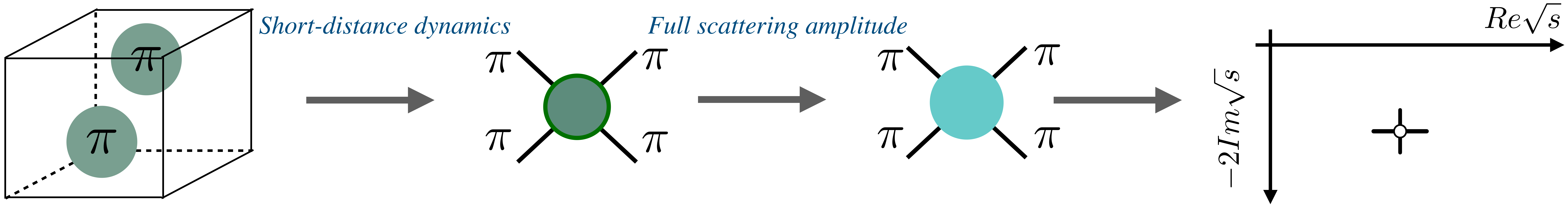
Extract E_n



We determine the strength of the reaction from the difference between non-interacting and interacting energies

Attraction reduces energies, repulsion increases it

Lüscher, Nucl. Phys. B 354 (1991)



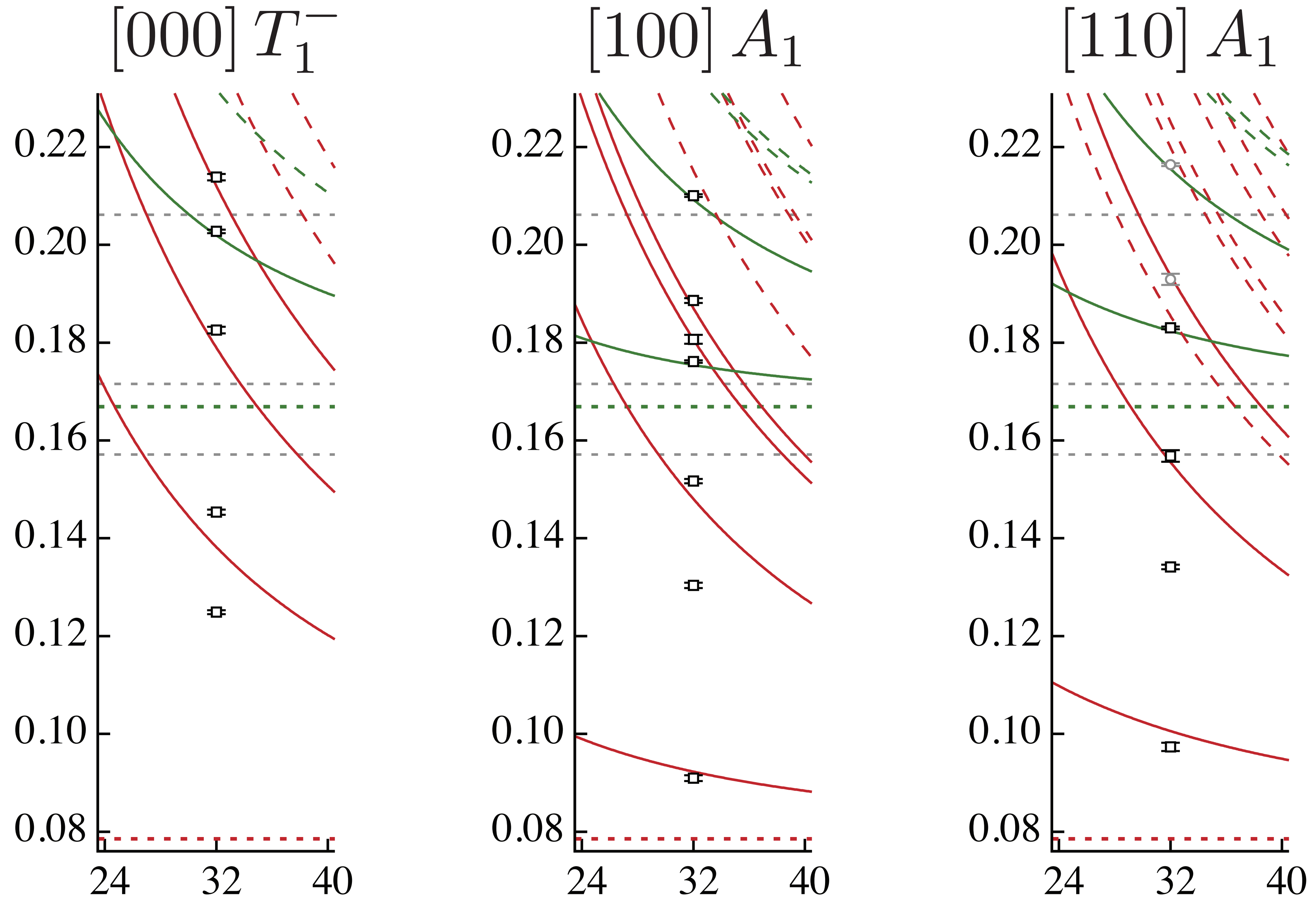
General $\det\left[\underline{F^{-1}(E_n, L)} + K(s_n)\right] = 0 \qquad t_\ell^I(s) = \frac{K(s)}{1 - i\rho(s)K(s)}$

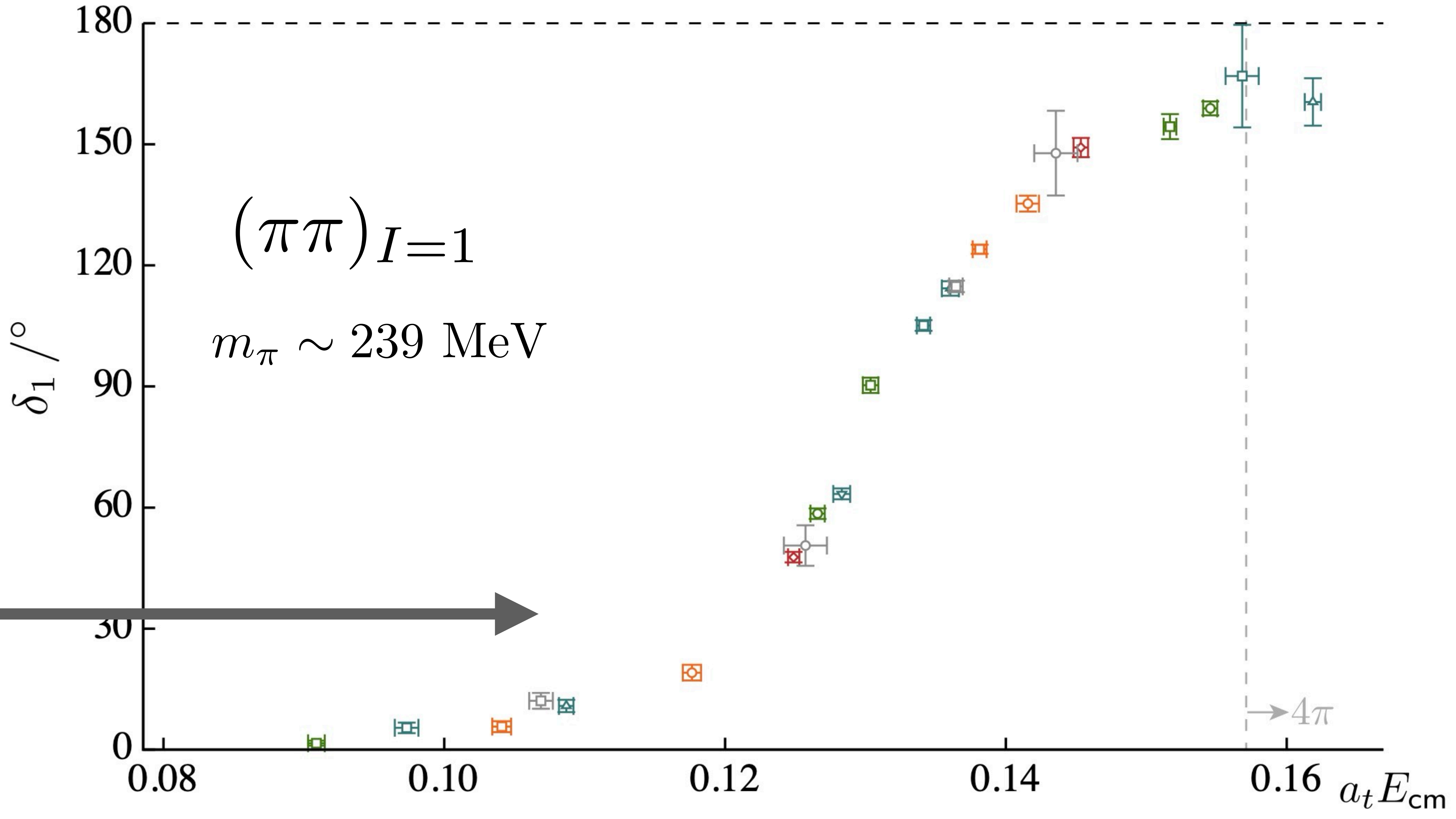
Known kinematic function

Elastic analysis

Every energy corresponds to one "data" point

$$\det\left[F^{-1}\left(E_n, L\right) + K(s_n)\right] = 0$$





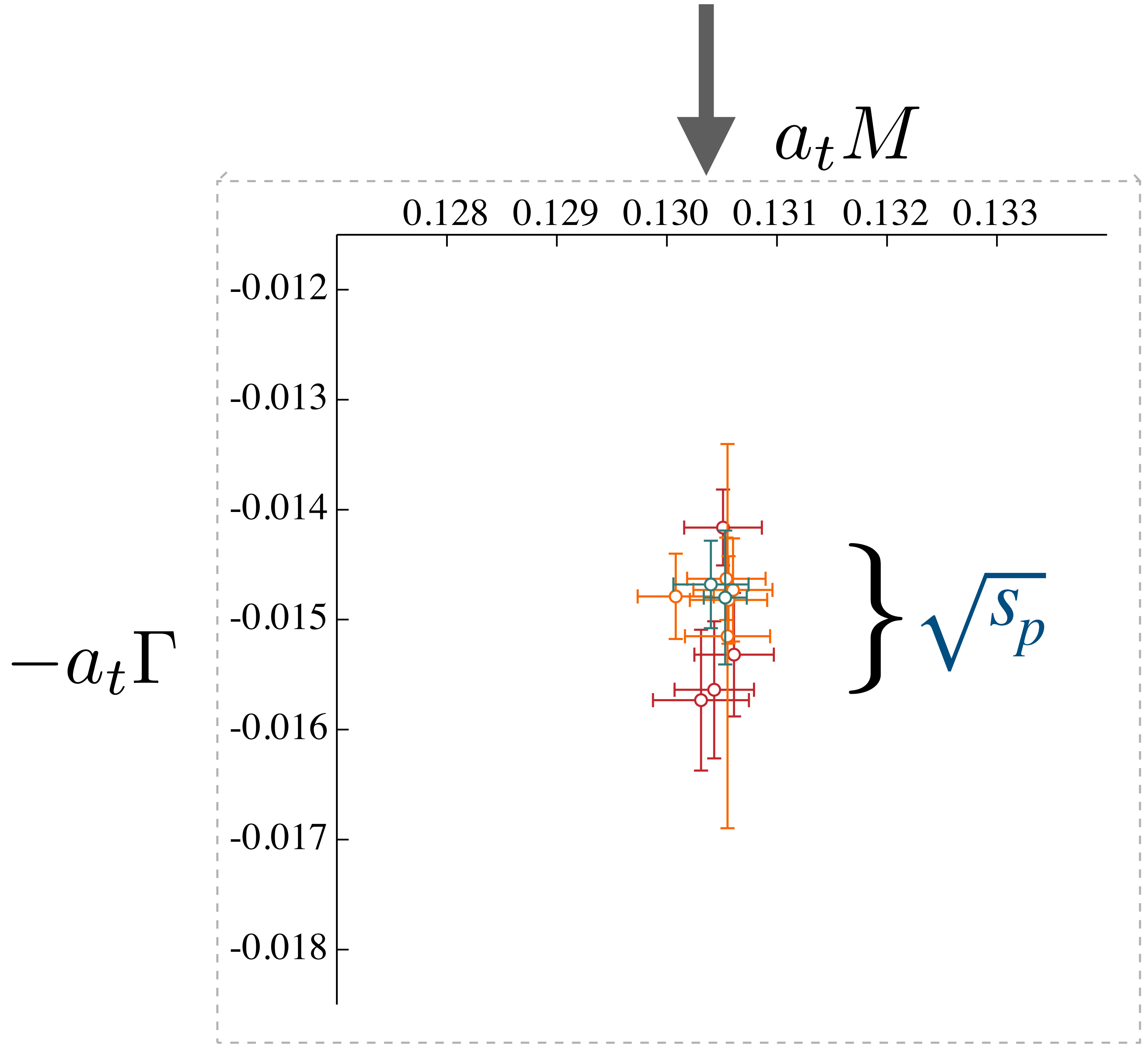
Elastic analysis

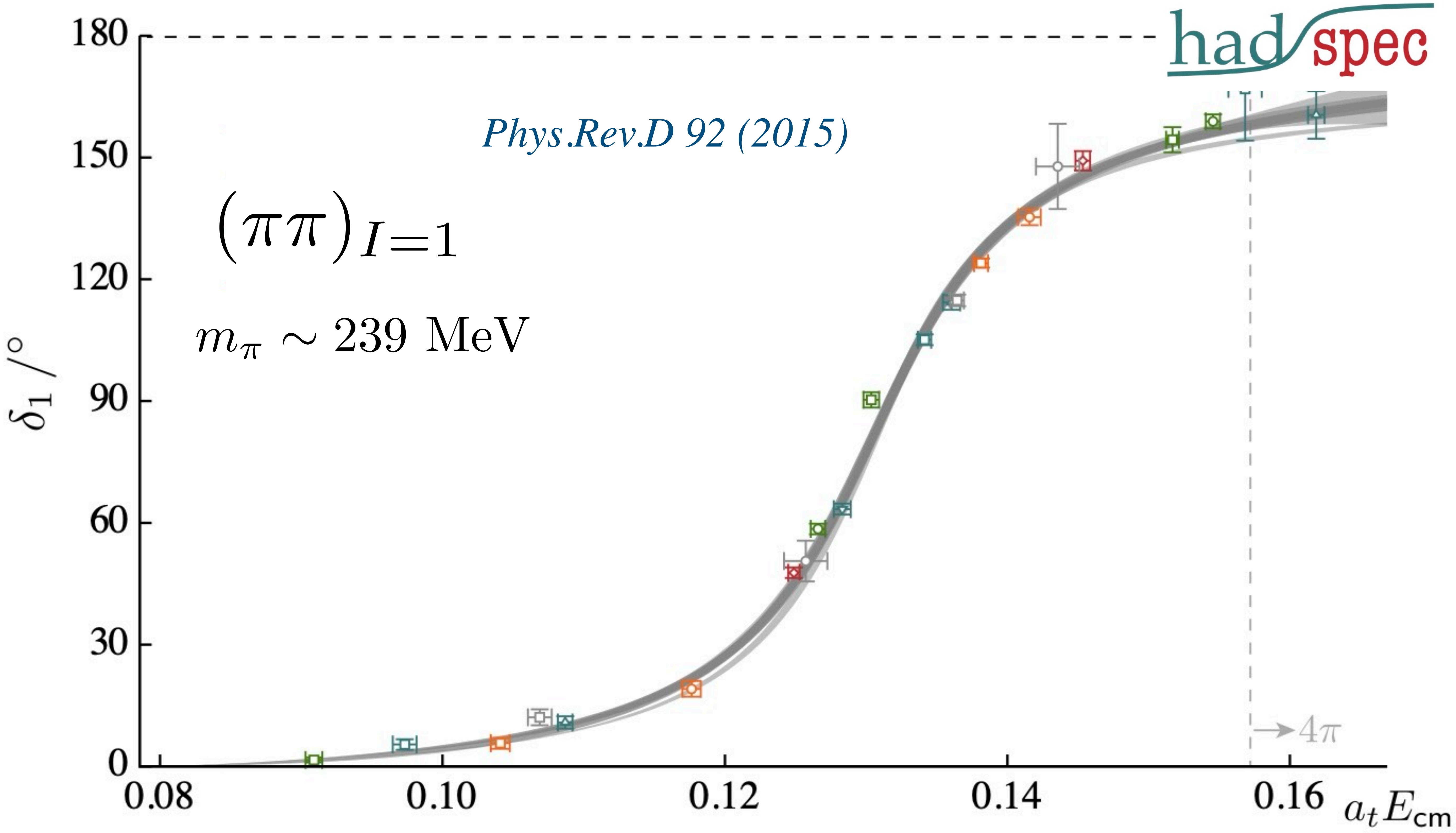
This amplitude can be easily fitted

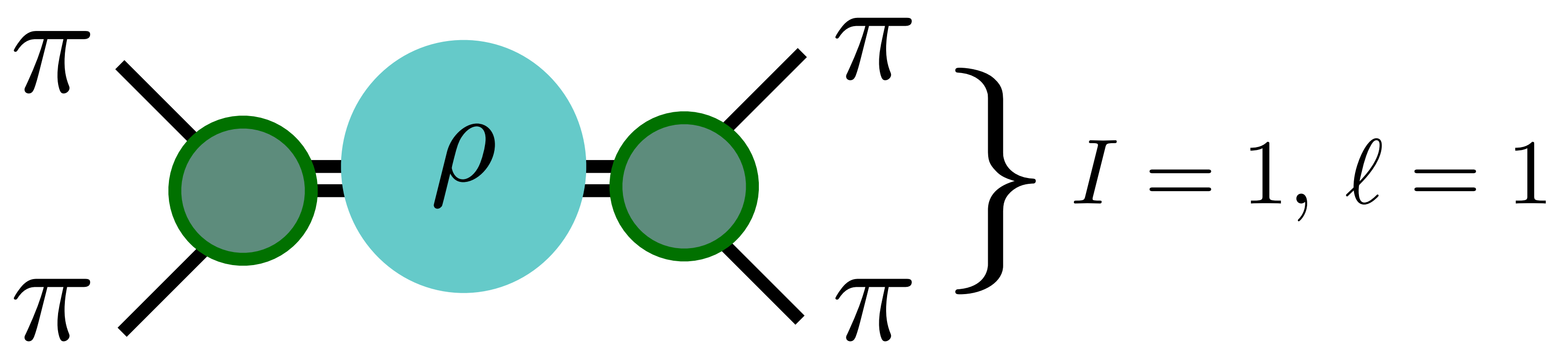
$$t_\ell^I(s) = \frac{1}{\rho(s)} \frac{\sqrt{s}\Gamma}{m_{\rm BW}^2 - s - i\sqrt{s}\Gamma}$$

Pole at $\sqrt{s_p} \sim (m_{BW} - i\Gamma/2)$

We can fit other parameterizations





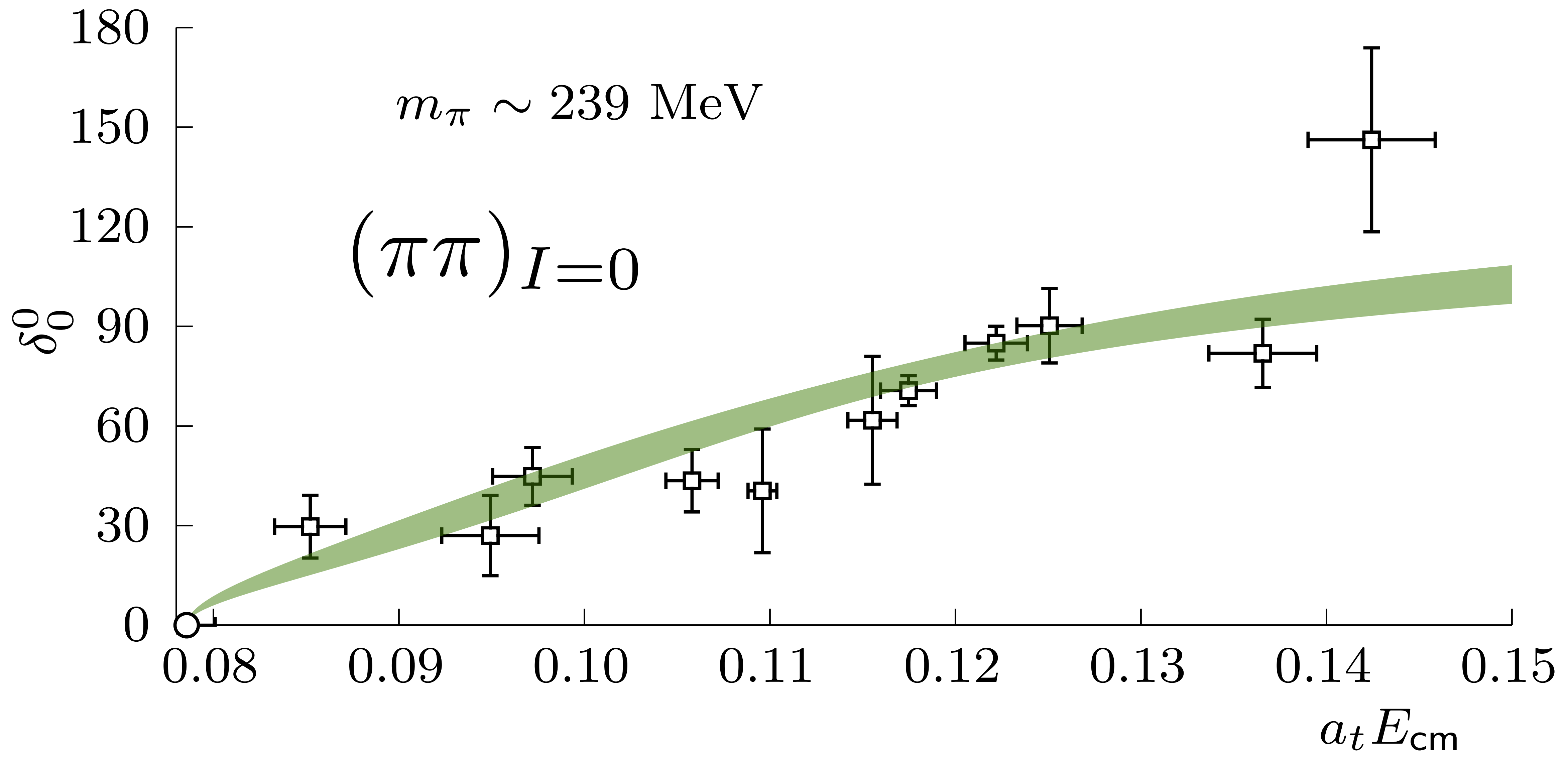


Great accuracy and precision!!

Light Scalars: the σ

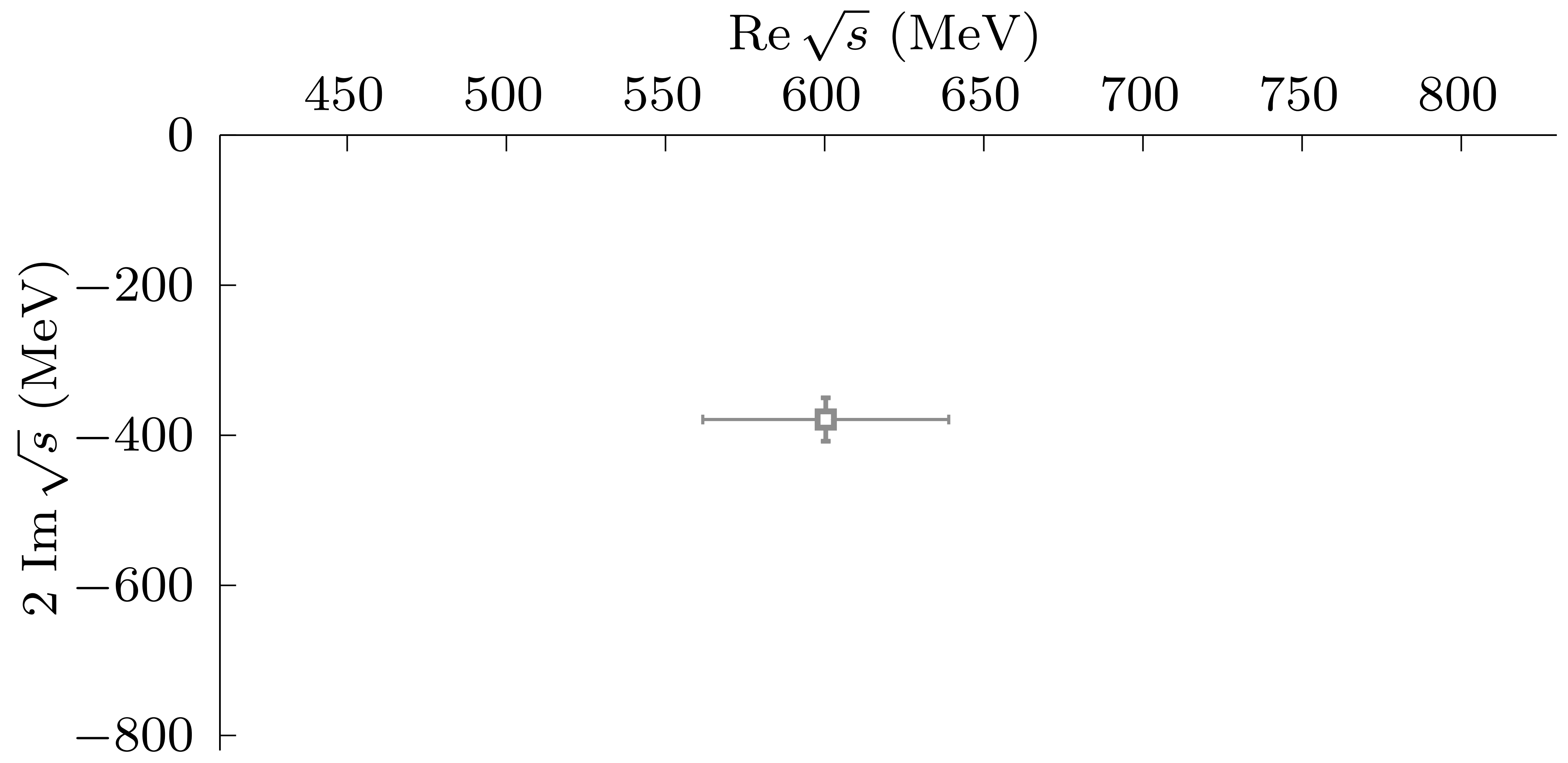
Data is precise !!

$t_0^0(s)\ (S0)$



σ pole positions

$$t_\ell^I(s) = \frac{1}{\rho(s)} e^{i\delta_\ell^I(s)} \sin \delta_\ell^I(s)$$



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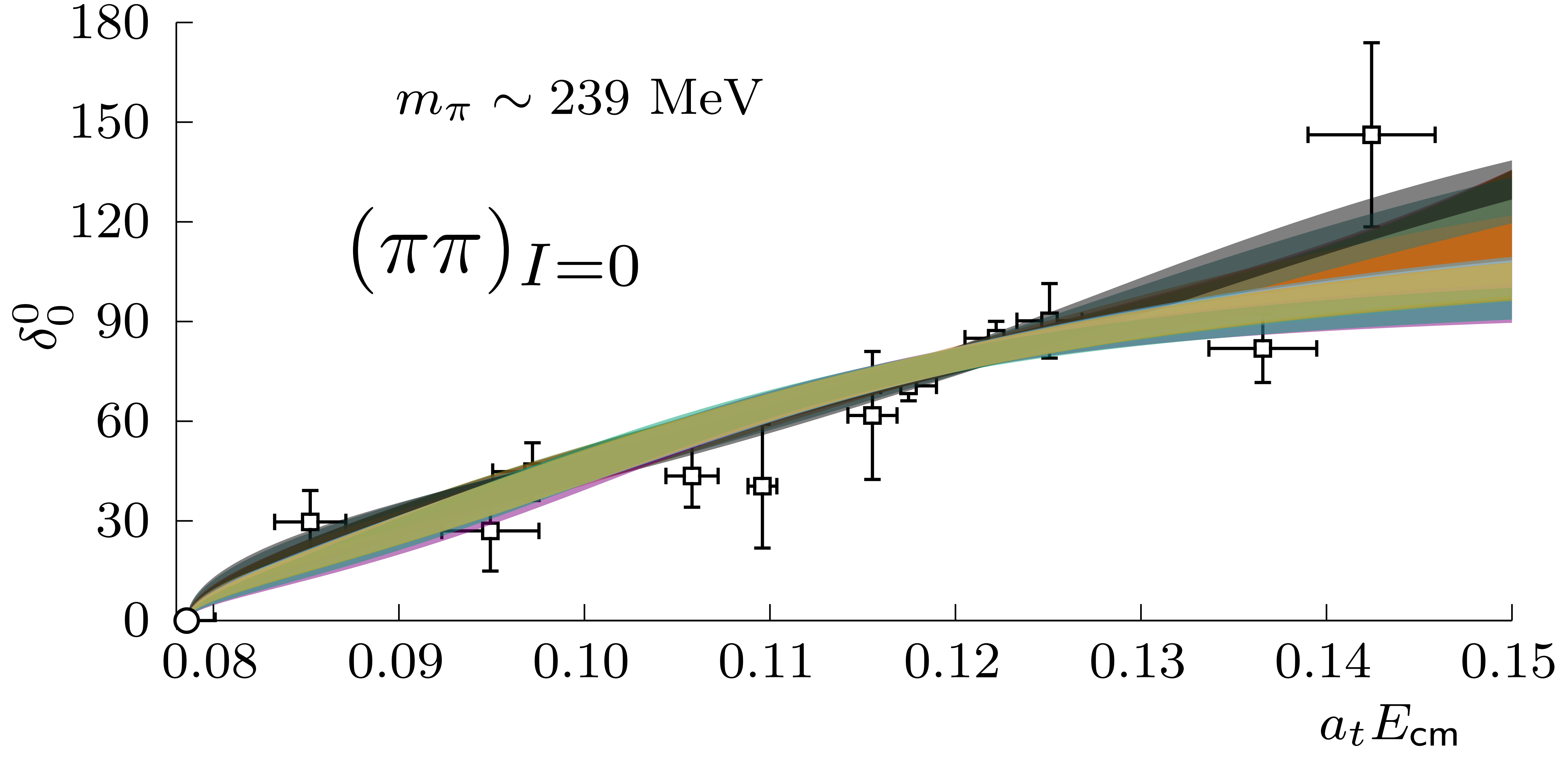
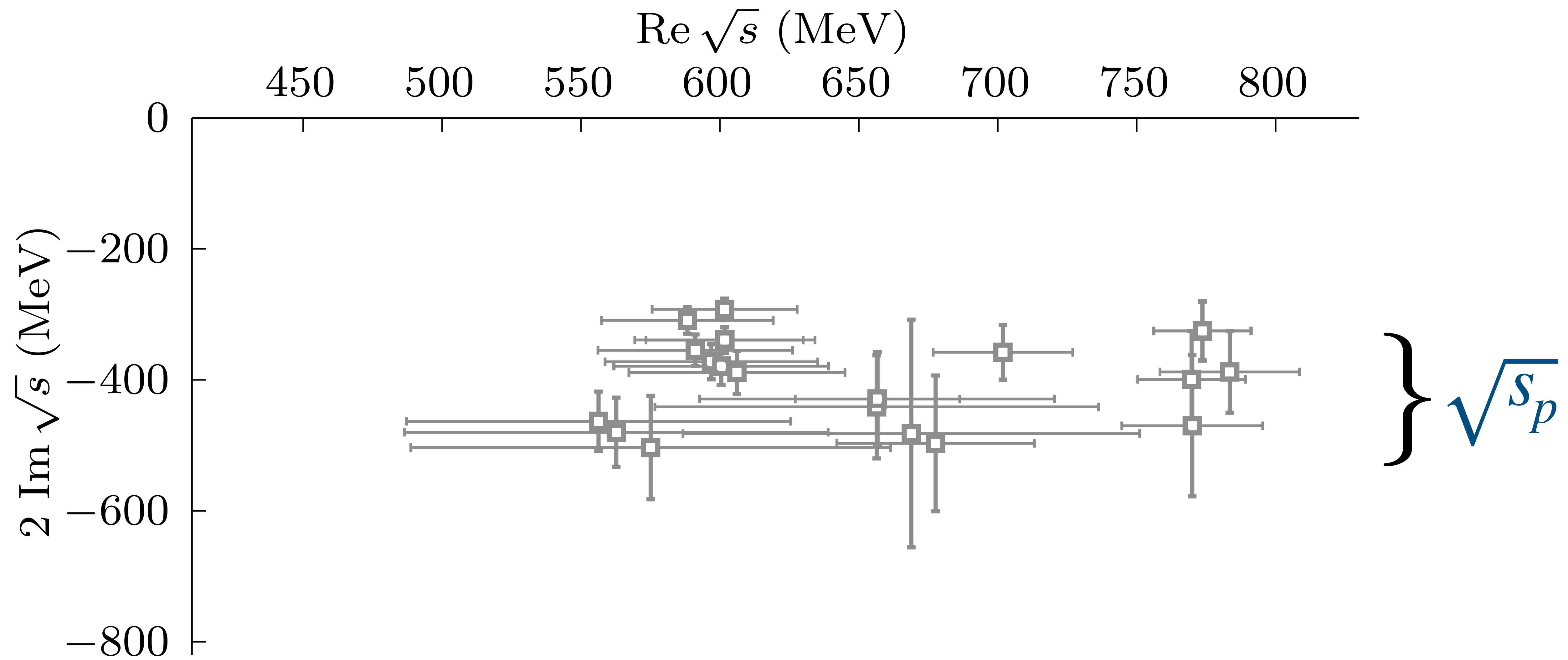


σ **pole positions**

$$t_\ell^I(s) = \frac{1}{\rho(s)} e^{i\delta_\ell^I(s)} \sin \delta_\ell^I(s)$$

VERY large model (systematic) spread!!

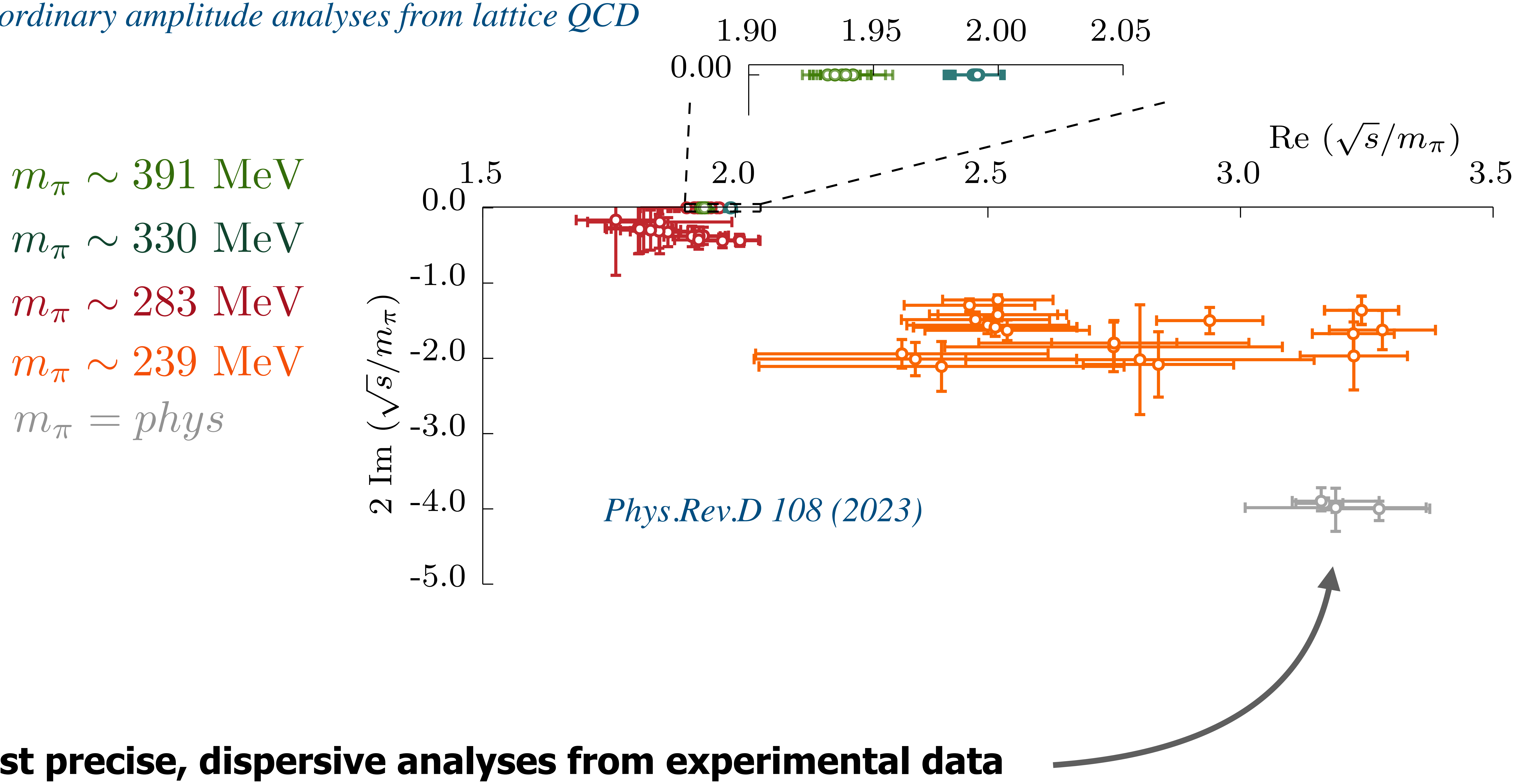
We can repeat this for varying pion mass



Light Scalars: the σ

Total error becomes really large when the state is a resonance

Color poles come from ordinary amplitude analyses from lattice QCD



Compare to the most precise, dispersive analyses from experimental data

S-matrix

Basic principles that scattering amplitudes must preserve (more general than QCD)!

- [x] **Probability is conserved →Unitarity**

  Amplitudes can be described by the K-matrix formalism

$$t_\ell^I(s) = \frac{K(s)}{1 - i\rho(s)K(s)}$$

- [ ] **Particle-antiparticle relation → Crossing symmetry**

  All $I = 0, 1, 2$ amplitudes T^I are part of a global amplitude T

- [ ] **Causality → Analyticity**

  We can write dispersion relations for this amplitude T

Most analyses only apply the first one, but we ultimately need all three

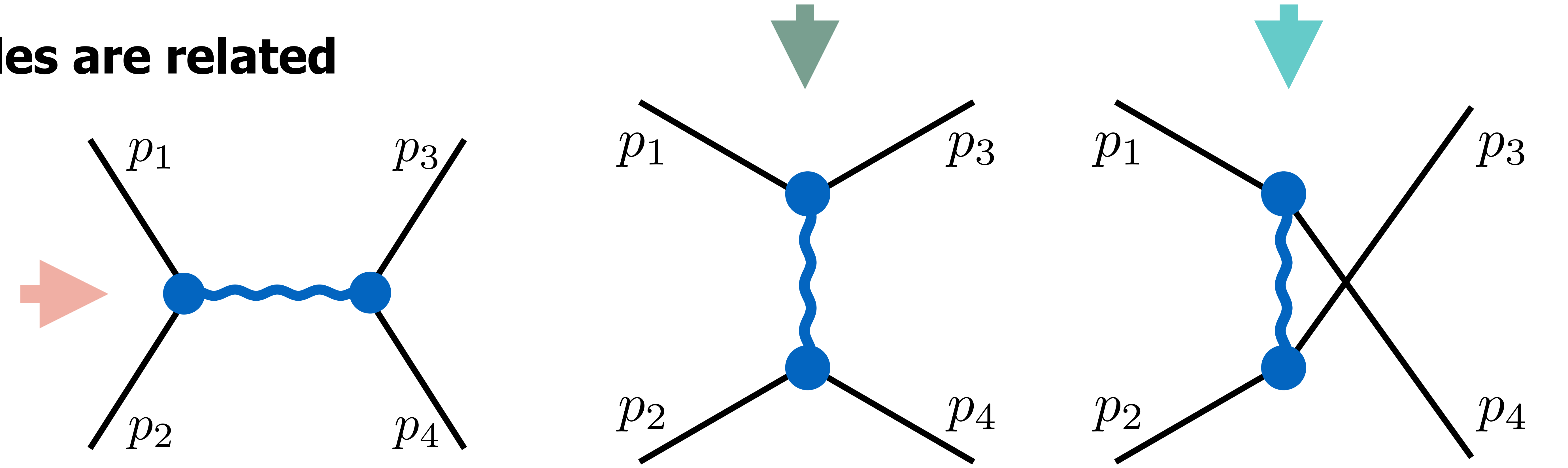
Crossing

Particles and anti-particles are related

s-channel

t-channel

u-channel



$$s = (p_1 + p_2)^2$$
$$t = (p_1 - p_3)^2$$
$$u = (p_1 - p_4)^2$$

Example, for $\pi\pi$, we can relate all amplitudes through a single function T

The three different isospins are related through the same function, evaluated over different kinematical ranges

$$(\pi\pi)_{I=0} \to \; T^{I=0}(s,t,u) = 3T(s,t,u) + T(t,s,u) + T(u,t,s)$$
$$(\pi\pi)_{I=1} \to \; T^{I=1}(s,t,u) = T(t,s,u) - T(u,t,s)$$
$$(\pi\pi)_{I=2} \to \; T^{I=2}(s,t,u) = T(t,s,u) + T(u,t,s)$$

Our isospin-defined amplitude is defined through its partial-wave expansion

We obtain the partial waves $t^I_\ell(s)$ from lattice QCD

$$T^I(s,t,u) = 32\pi \sum_\ell (2\ell+1) t^I_\ell(s) P_\ell(\cos\theta_s)$$

We can obtain T from fitting all different isospins, or vice-versa

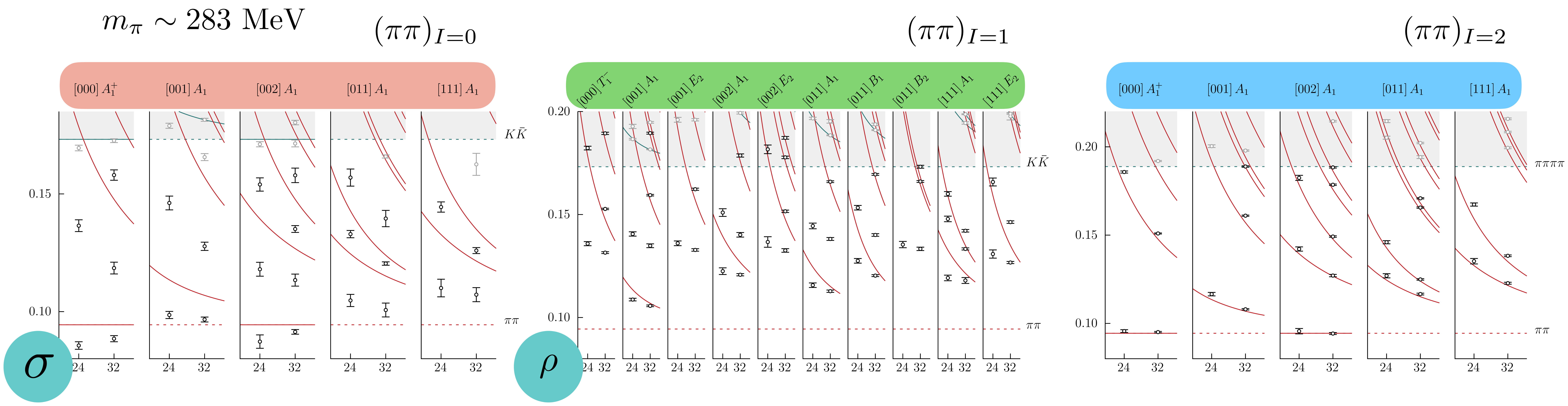
$$T(s,t,u) = \frac{1}{3}(T^{I=0}(s,t,u) - T^{I=2}(s,t,u))$$

$$T(t,s,u) = \frac{1}{2}\left(T^{I=1}(s,t,u) + T^{I=2}(s,t,u)\right)$$

$$T(u,t,s) = \frac{1}{2}\left(T^{I=2}(s,t,u) - T^{I=1}(s,t,u)\right)$$

Crossing

Lattice QCD gives us information on well-defined isospin partial waves



We combine partial waves to create amplitudes

$$(\pi\pi)_{I=0} \rightarrow T^{I=0}(s,t,u) = 3T(s,t,u) + T(t,s,u) + T(u,t,s)$$

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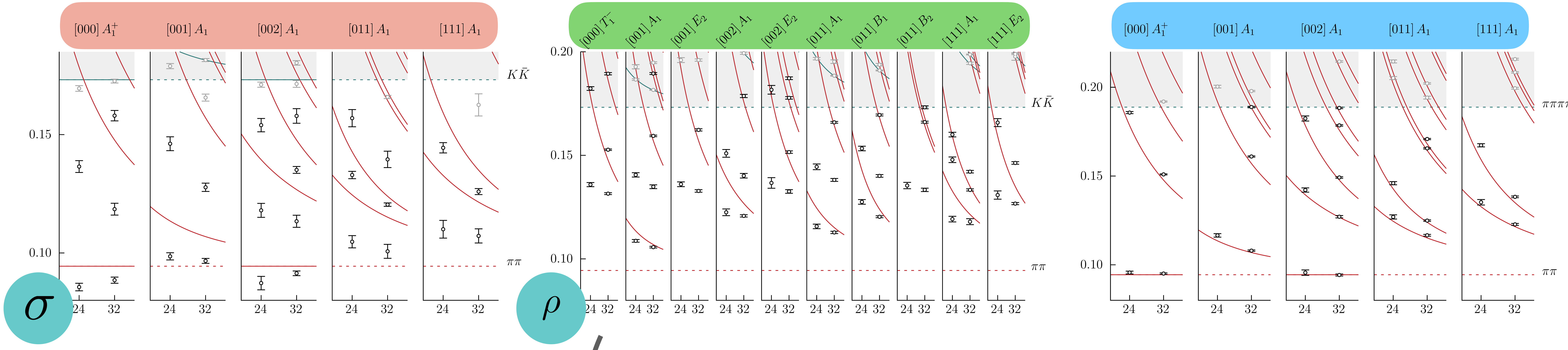
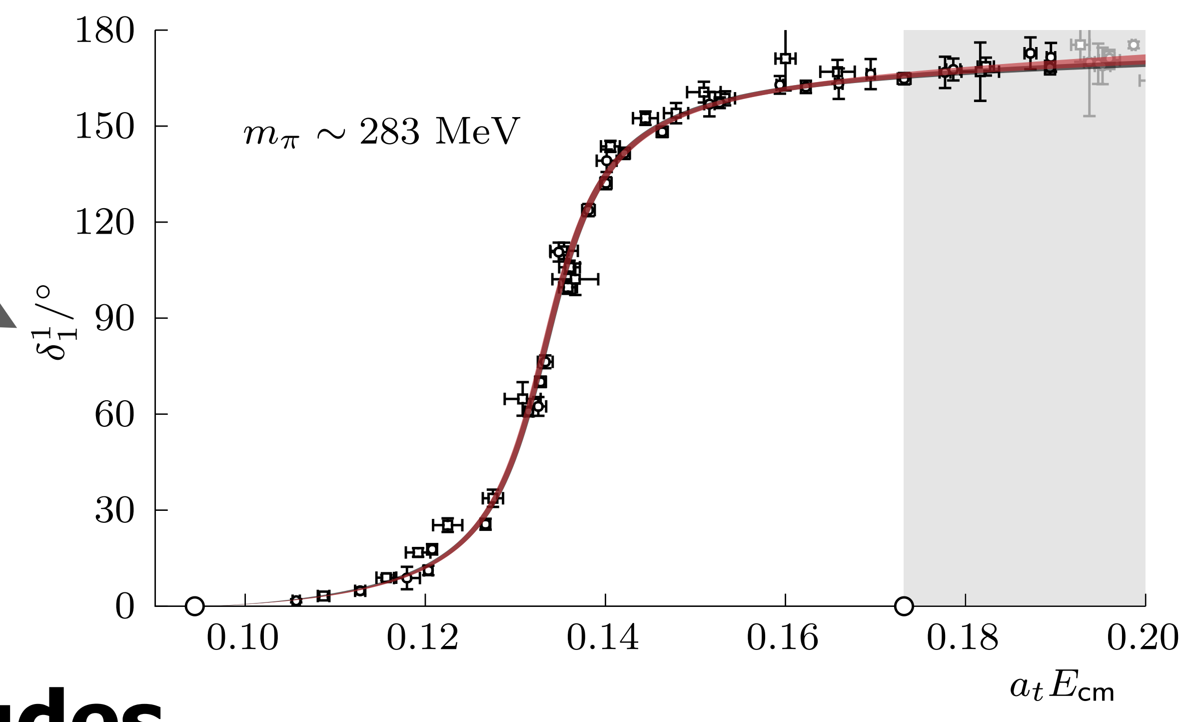
$m_\pi \sim 283$ MeV

$(\pi\pi)_{I=0}$ $(\pi\pi)_{I=1}$ $(\pi\pi)_{I=2}$





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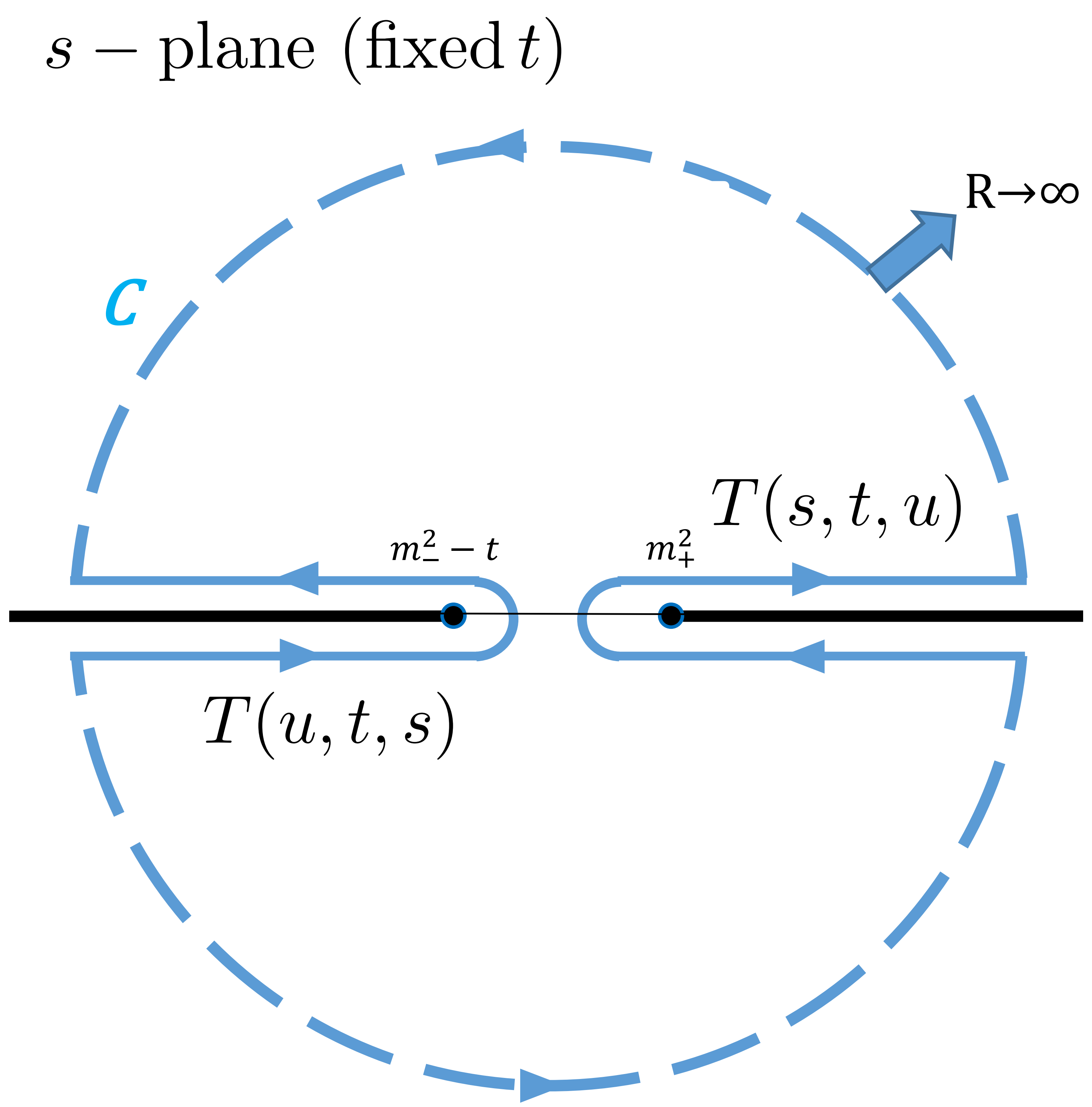
Analyticity: Dispersion relations

Our amplitude has two branch-cuts (phase space square root)

Use Cauchy's theorem over contour C, circular parts go to zero

$$T(s,t,u) = \frac{1}{2\pi i}\int_C \frac{T(s',t,u'(s',t))}{s'-s}ds'$$

+ maybe subtractions



How is this useful?? → "hooks" are given by (Schwarz reflection principle)

$$T(s+i\epsilon,t,u-i\epsilon) - T(s-i\epsilon,t,u+i\epsilon) = 2i\,\text{Im}\,T(s,t,u) \longrightarrow$$ **data**

Project the integral into partial waves to get your dispersion relations (ex. Roy eqs.):

Roy Phys.Lett.B 36 (1971)

$$t_\ell^I(s) \to \tilde{t}_\ell^I(s) = \tau_\ell^I(s) + \sum_{I',\ell'}\int_{4m_\pi^2}^{\infty} ds' K_{\ell\ell'}^{II'}(s',s)\,\text{Im}\,t_{\ell'}^{I'}(s')$$

Initial fit to data — *Final dispersive output*

We compare now this dispersive equation (output) with the initial fit to data for the amplitude $t_\ell^I(s)$ (input)

Lat step: model selection

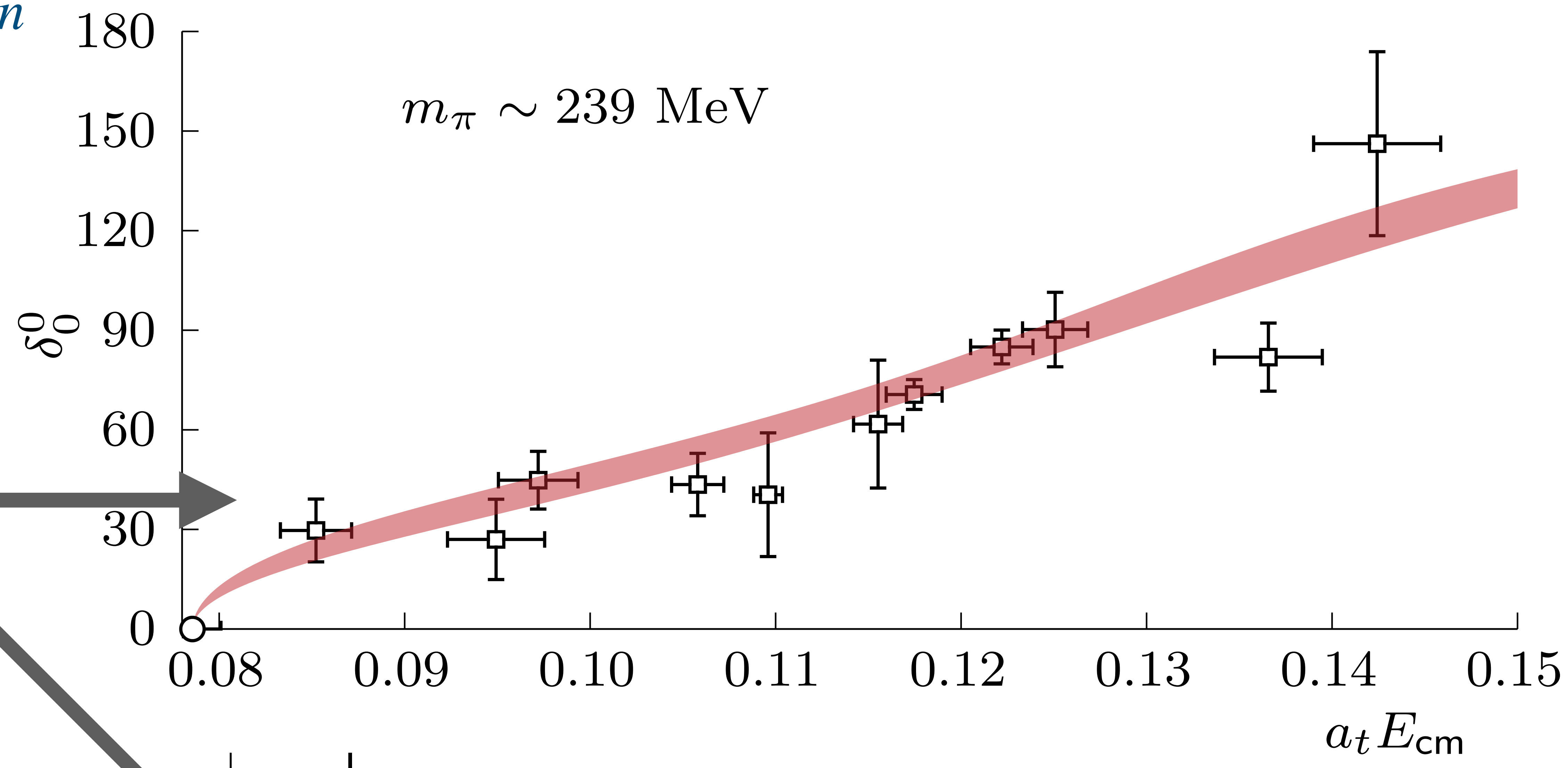
Within the many models ($t_\ell^I(s)$) fitted to lattice data, only a few will be compatible with the dispersive output $\tilde{t}_\ell^I(s)$

Compare the real parts given by the fitted amplitude and the dispersion relation

Select those amplitudes that deviate only "within uncertainties"

Use the dispersion relation, using the selected amplitudes as input, to calculate observables

$\{t_0^0(s)\}_1$



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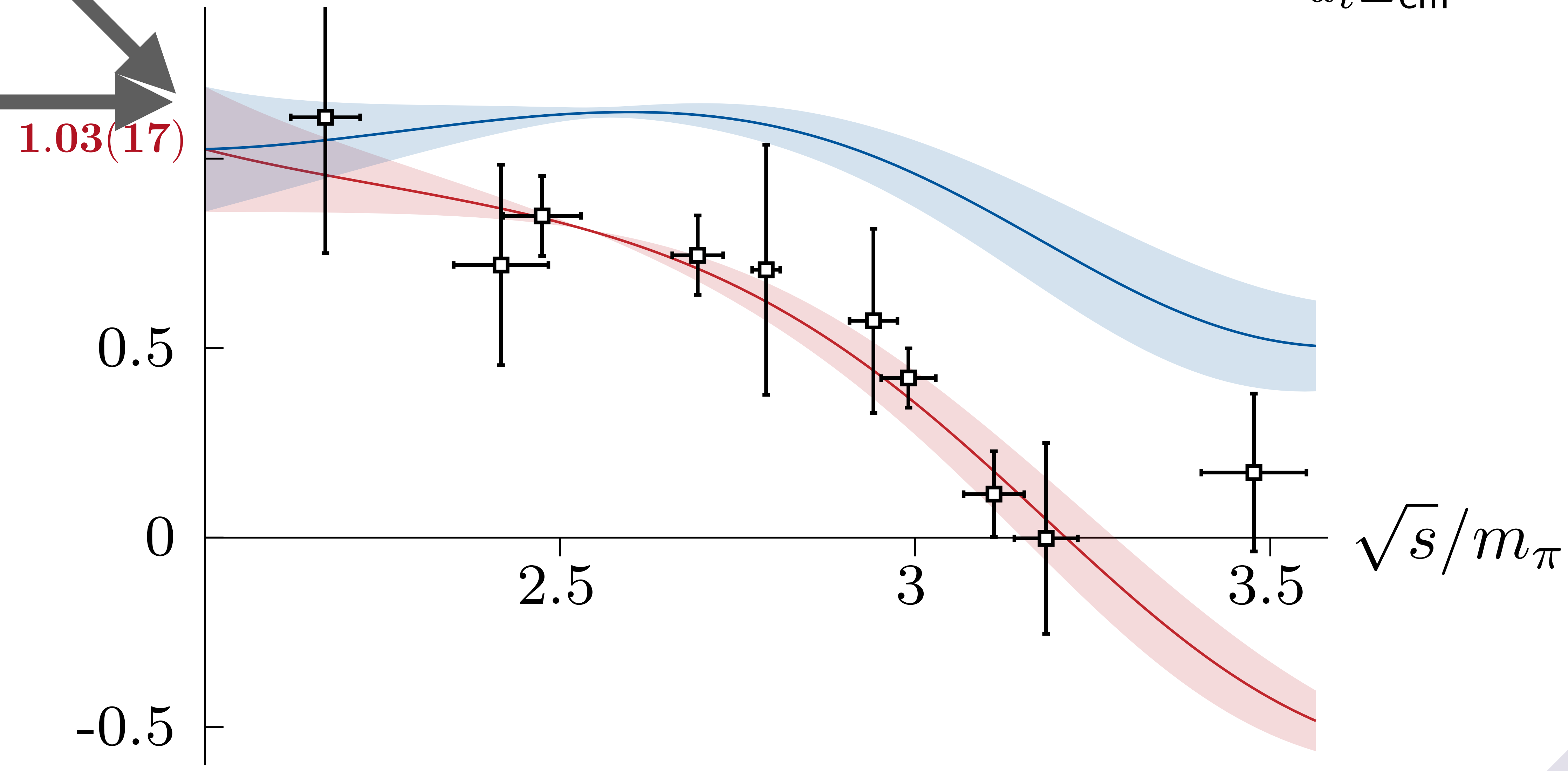

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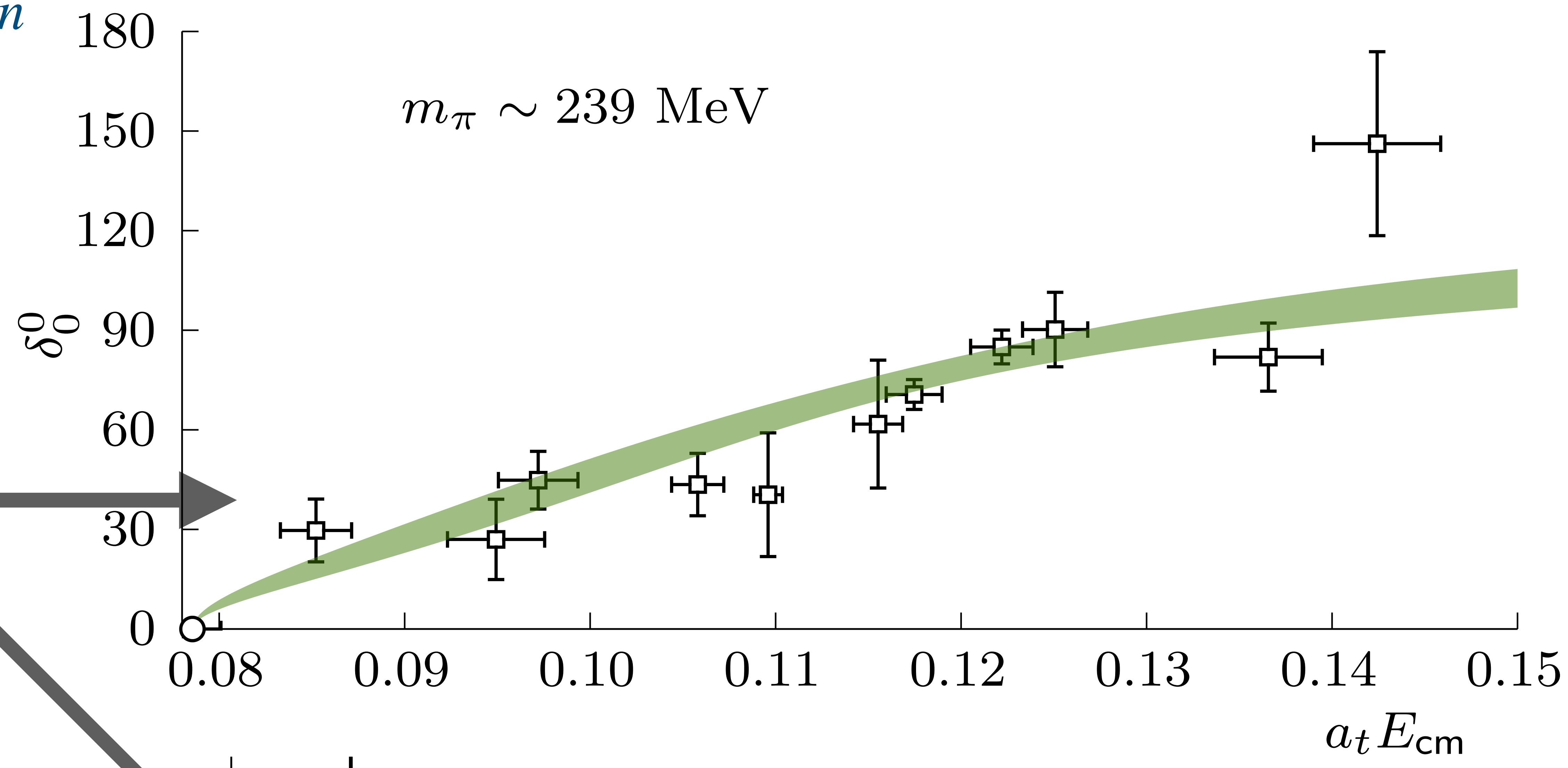
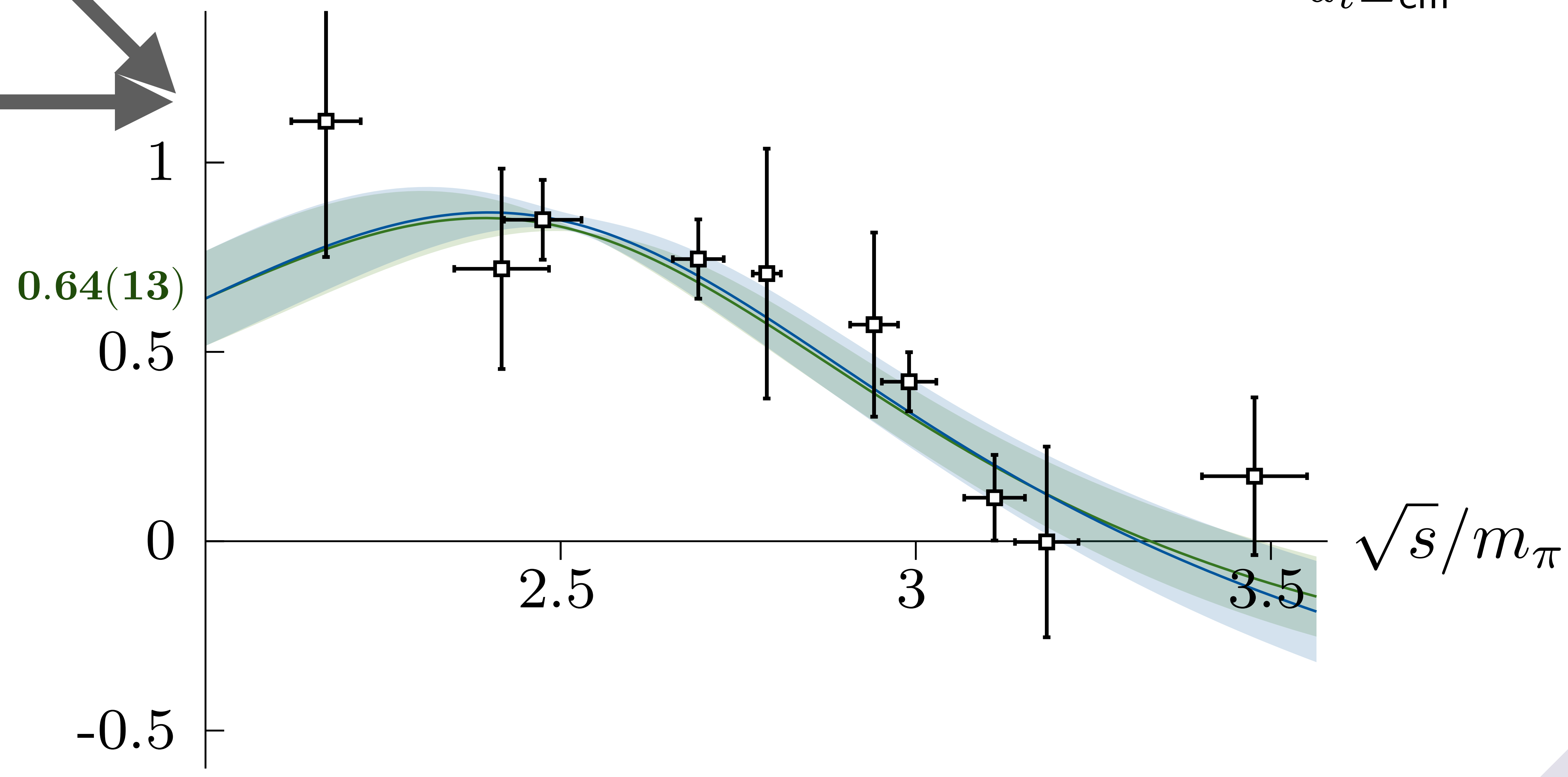


$$\left\{t_0^0(s)\right\}_2$$

$$\tilde{t}_\ell^I(s) = \tau_\ell^I(s) + \sum_{I',\ell'} \int_{4m_\pi^2}^{\infty} ds' K_{\ell\ell'}^{II'}(s', s)\, \mathrm{Im}\, t_{\ell'}^{I'}(s')$$



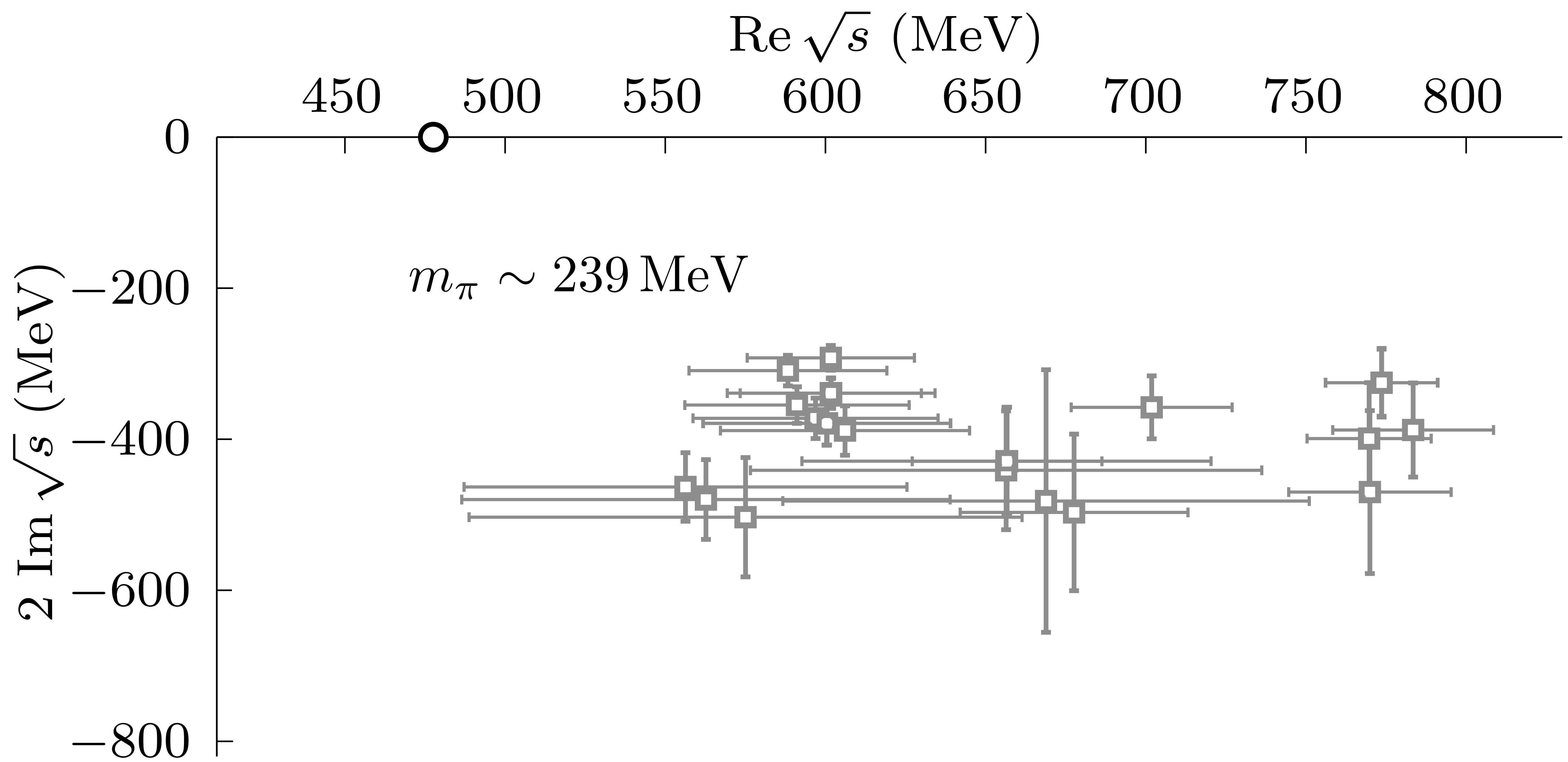
Outside the physical region

Once these dispersive constrains are imposed, the systematic error is drastically reduced

Compare the systematic spread of ordinary parameterizations with the dispersive extractions

Ordinary analysis

Dispersive analysis



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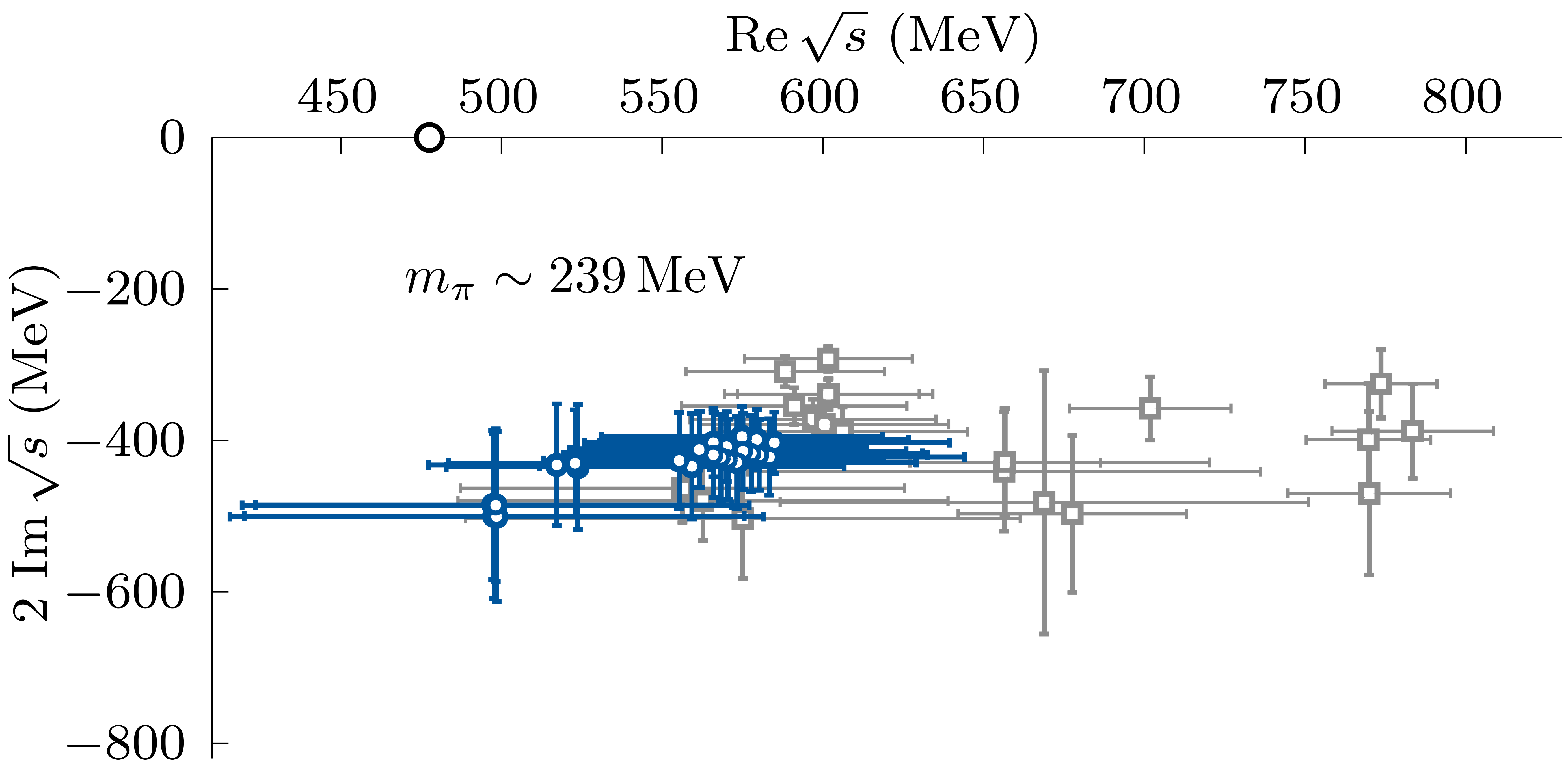
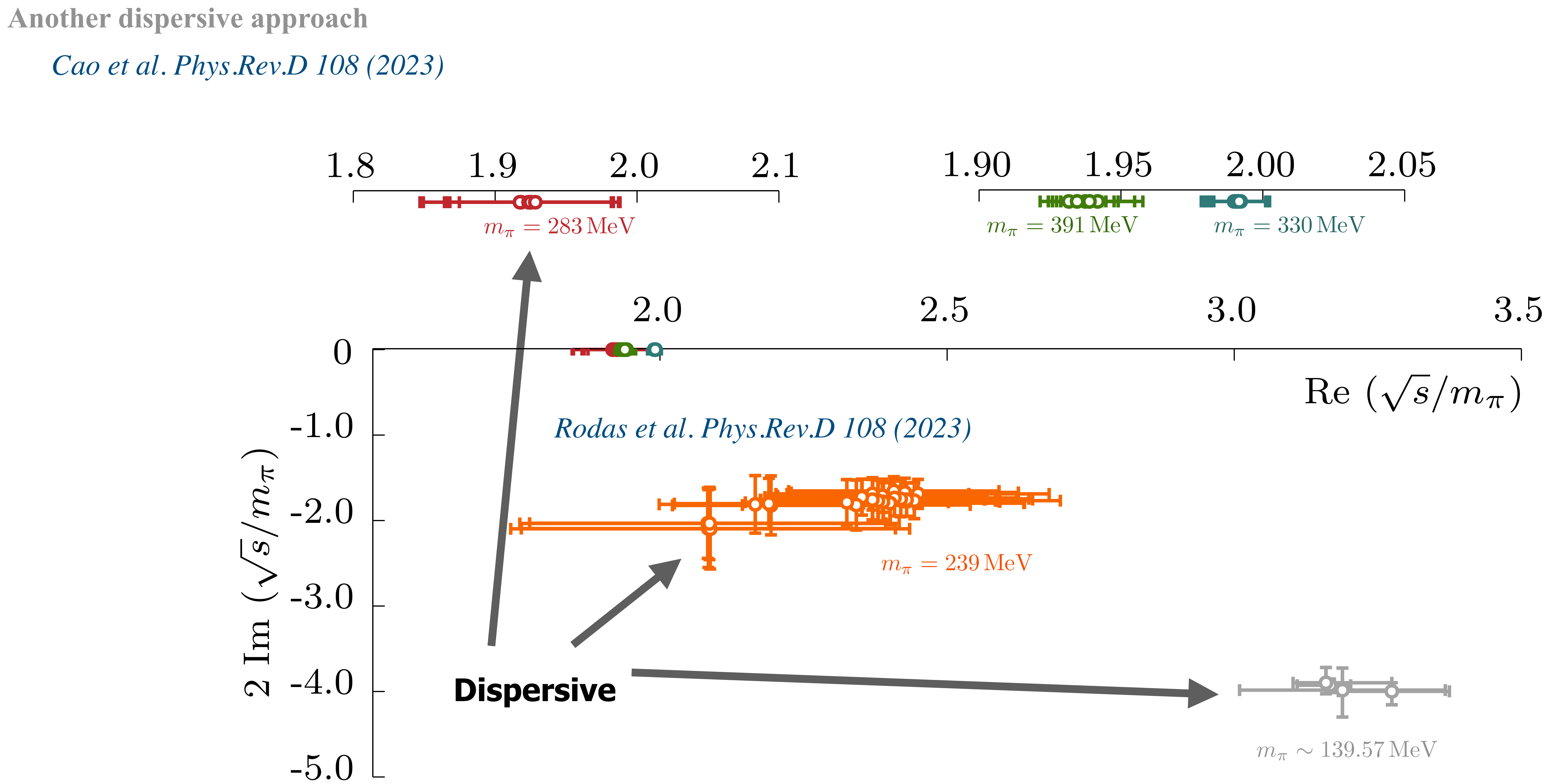


Outside the physical region

Various recent, dispersive determinations

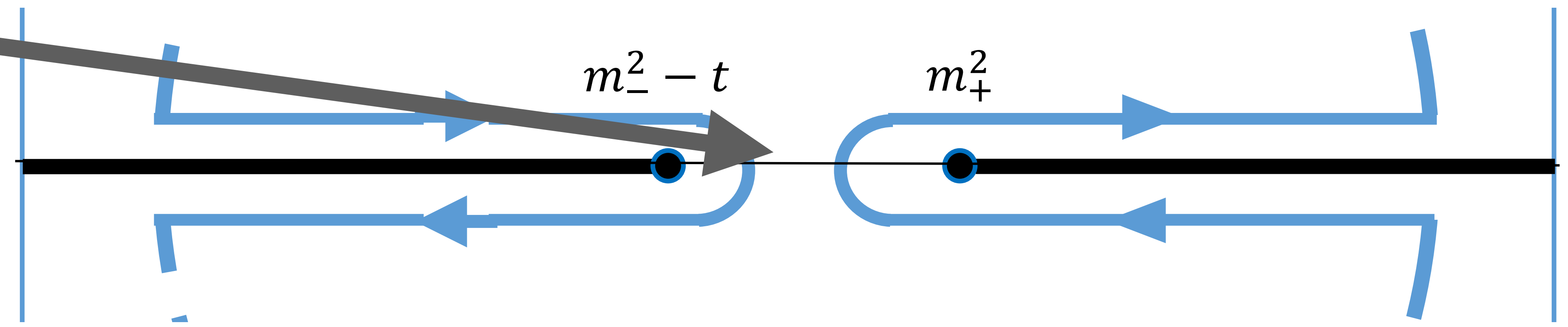


Adler Zeroes

Adler zeroes (amplitude zeroes) are a fundamental result of chiral symmetry

If $m_\pi \simeq 0$ **then** $T(s,t,u) \underset{s\sim 0}{\rightarrow} 0$

Adler, Phys.Rev. 137 (1965)



It is customarily accepted that these zeroes are still there even after the breaking of the symmetry

These zeroes appear on the S-waves and are considered directly linked to ChPT

ChPT predicts Adler zeroes for all pseudo-scalar scattering amplitudes at LO

$$s_{A,I=0} = m_\pi^2/2 \qquad\qquad s_{A,I=2} = 2m_\pi^2$$

In lattice QCD our pion mass is not zero, but the community still uses ChPT in most analyses

No robust analysis of these zeroes has been performed for varying pion masses

Adler Zeroes

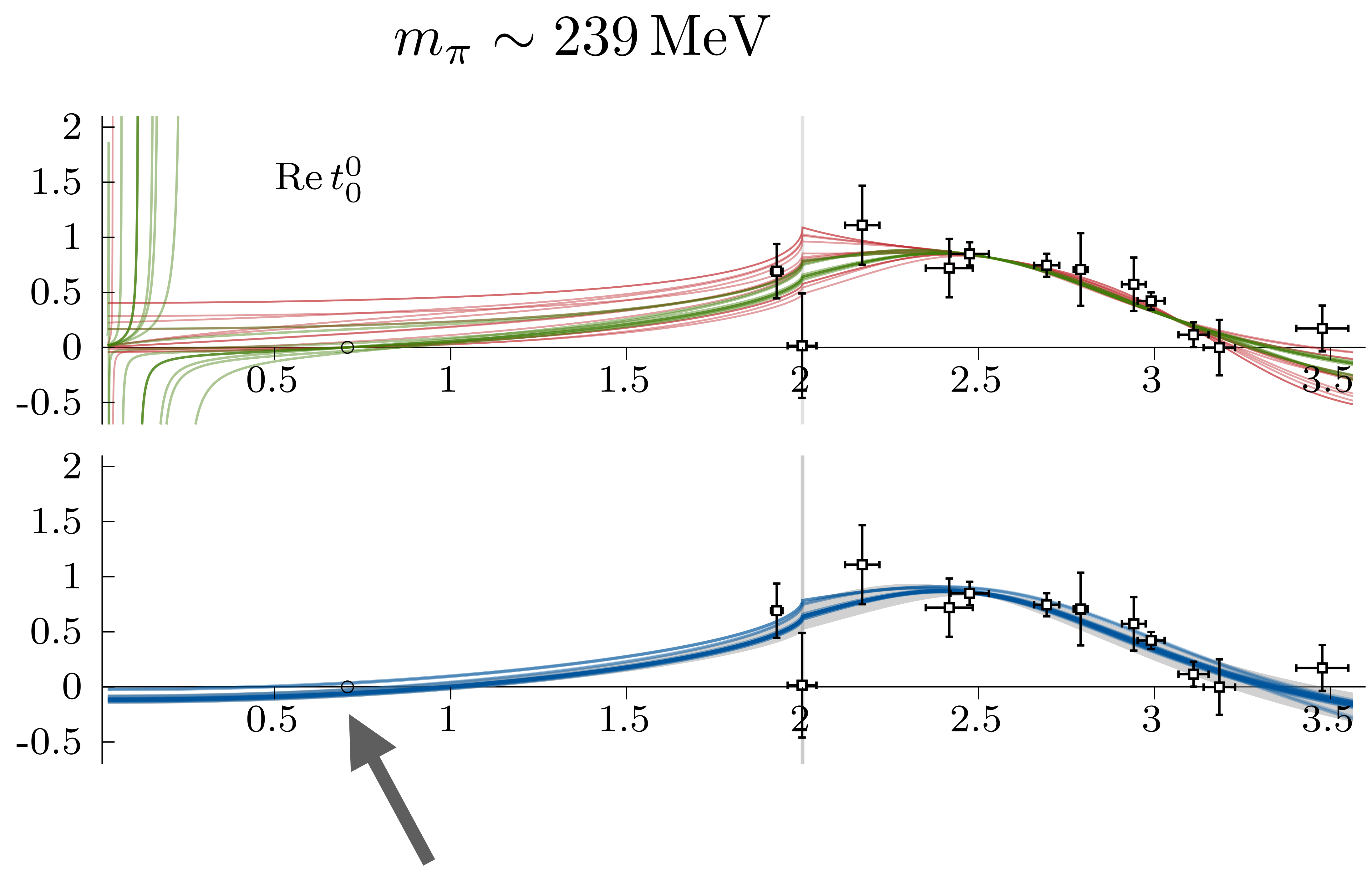
Very "stable" for $I = 2\ \pi\pi$

Even "bad" DRs produce Adler zeroes for I=2, close to the LO prediction $s_{A,I=2} = 2m_\pi^2$

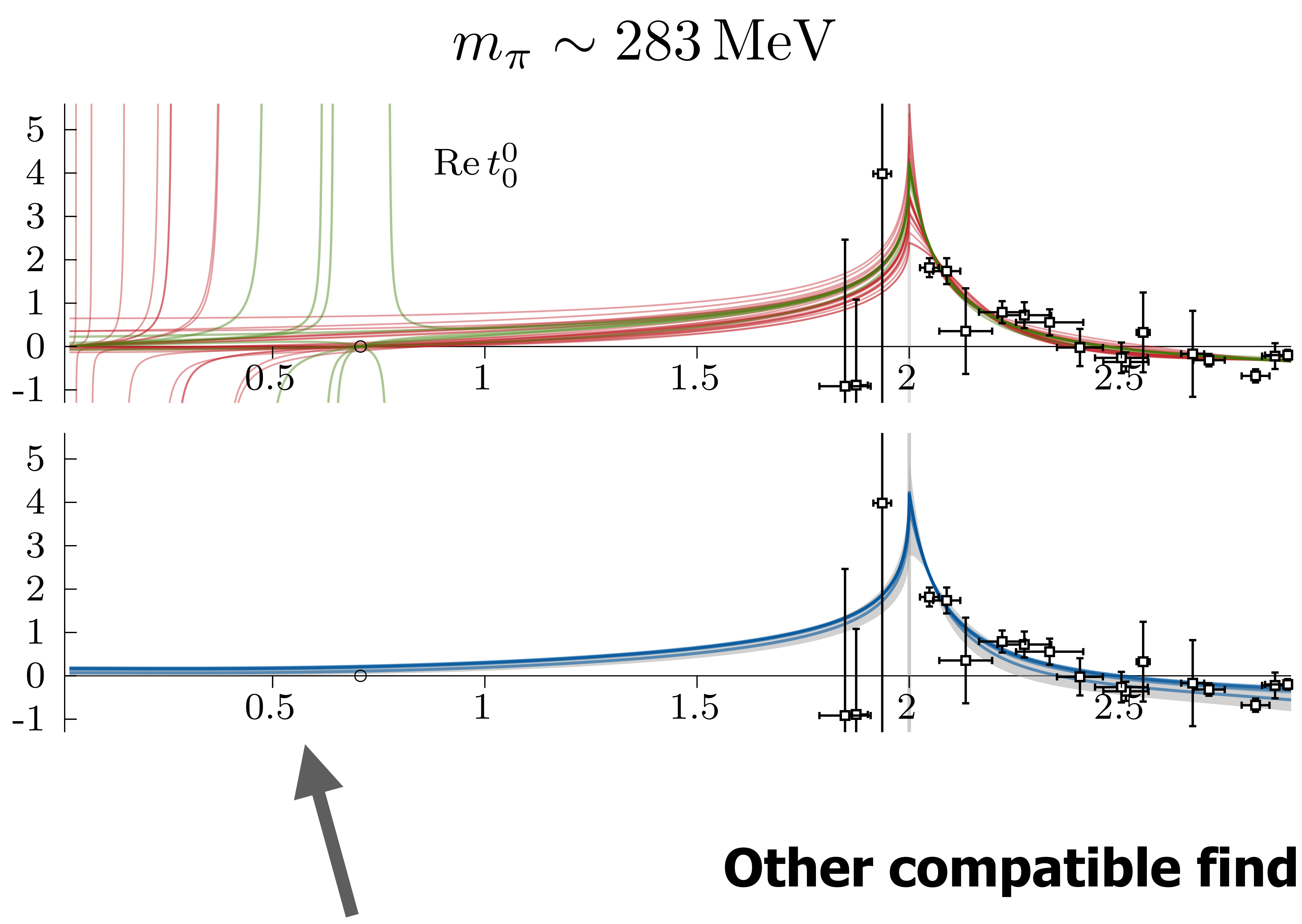
Adler Zeroes

All good DRs produce an $I = 0$ $\pi\pi$ Adler zero for the lighter mass

No good DR produces an $I = 0$ $\pi\pi$ Adler zero for the heavier mass



Adler zero



NO Adler zero

Other compatible findings

Phys.Rev.D 108 (2023)

Phys.Rev.D 109 (2024)

Summary and outlook

First-principles extraction of a broad resonance directly from QCD

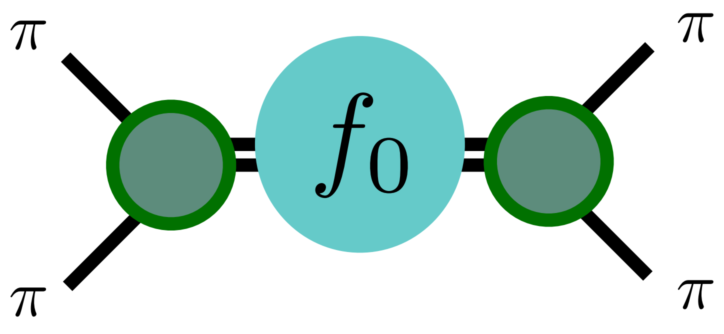
The lighter the π, the more relevant this approach is

(Much) Better constraints over scattering lengths

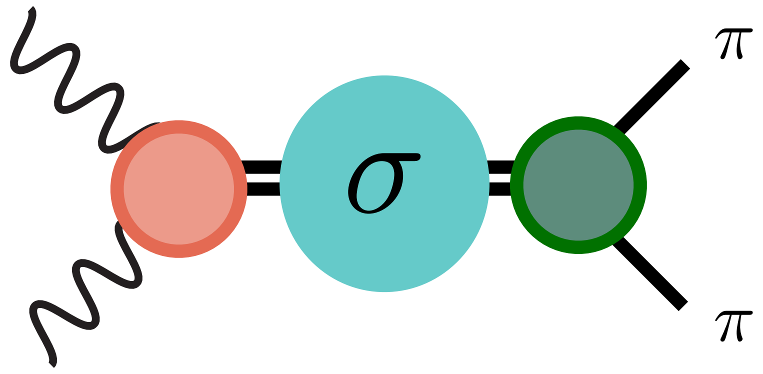
Future

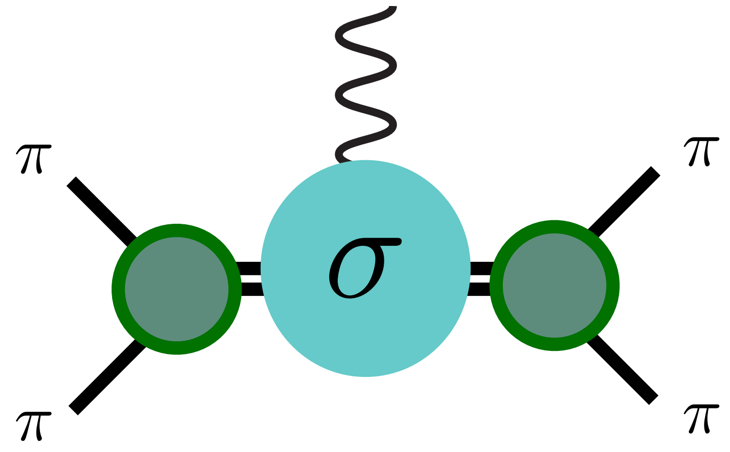
Study low-energy observables in more detail (Adler zeroes move away from real axis, for large m_π)

Extract the $f_0(980)$??



Study new observables ??





Spare slides

Permutations

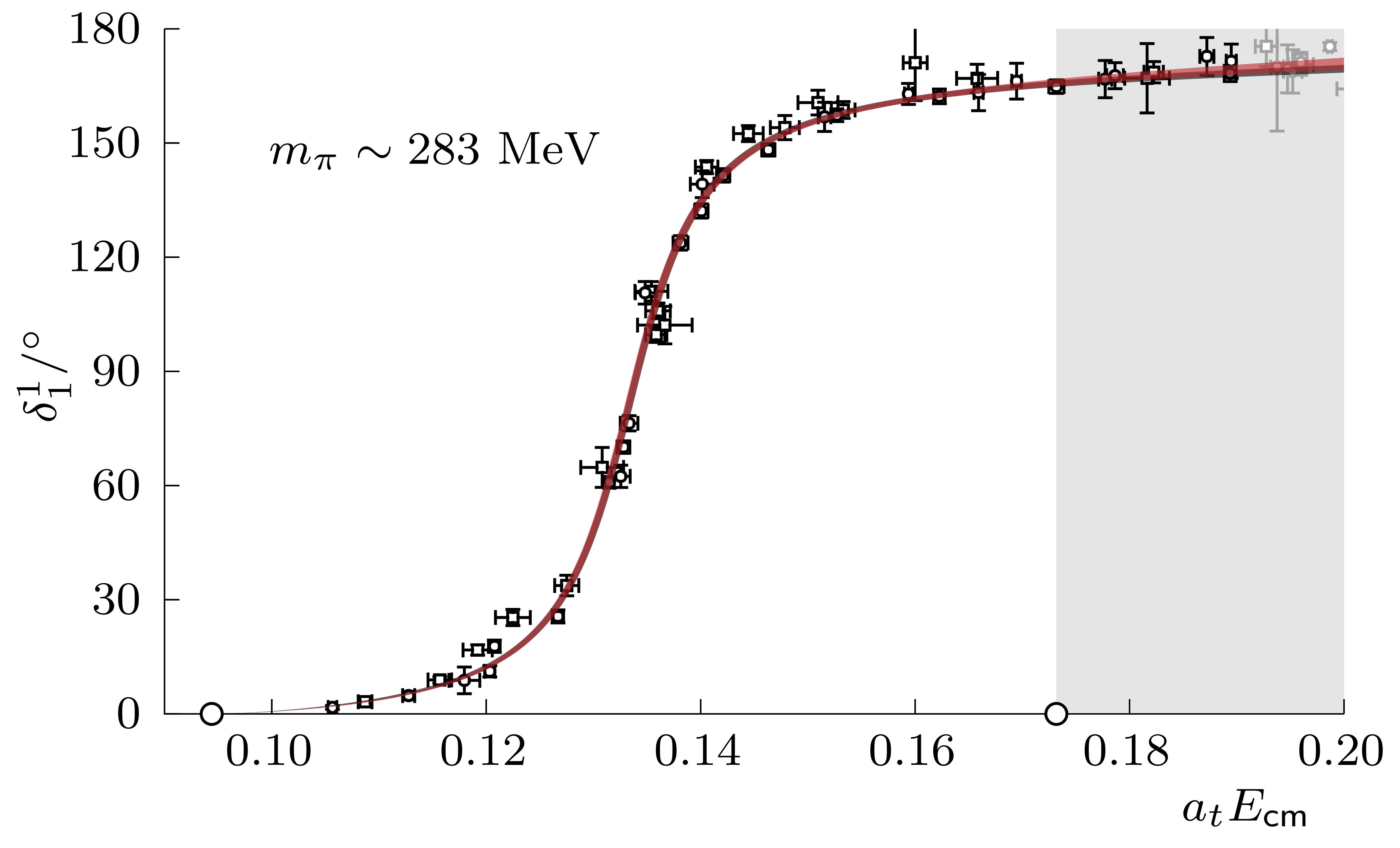
$$\sum_{I',\ell'} \int_{4m_\pi^2}^{\infty} ds'\, K_{\ell\ell'}^{II'}(s',s)\, \mathrm{Im}\, t_{\ell'}^{I'}(s')$$

For ℓ_{max} partial waves

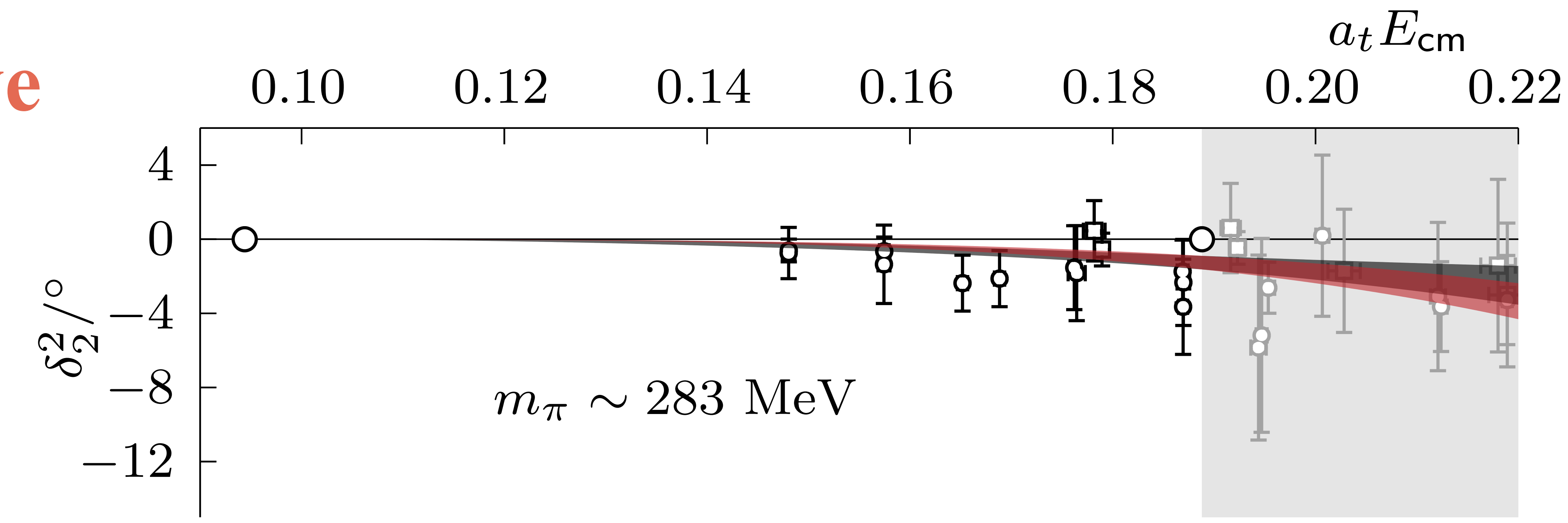
$$N_I \ell_{max} N_{params} \sim 10^5$$

We can fix most

$I = 1$ P-wave



$I = 2$ D-wave

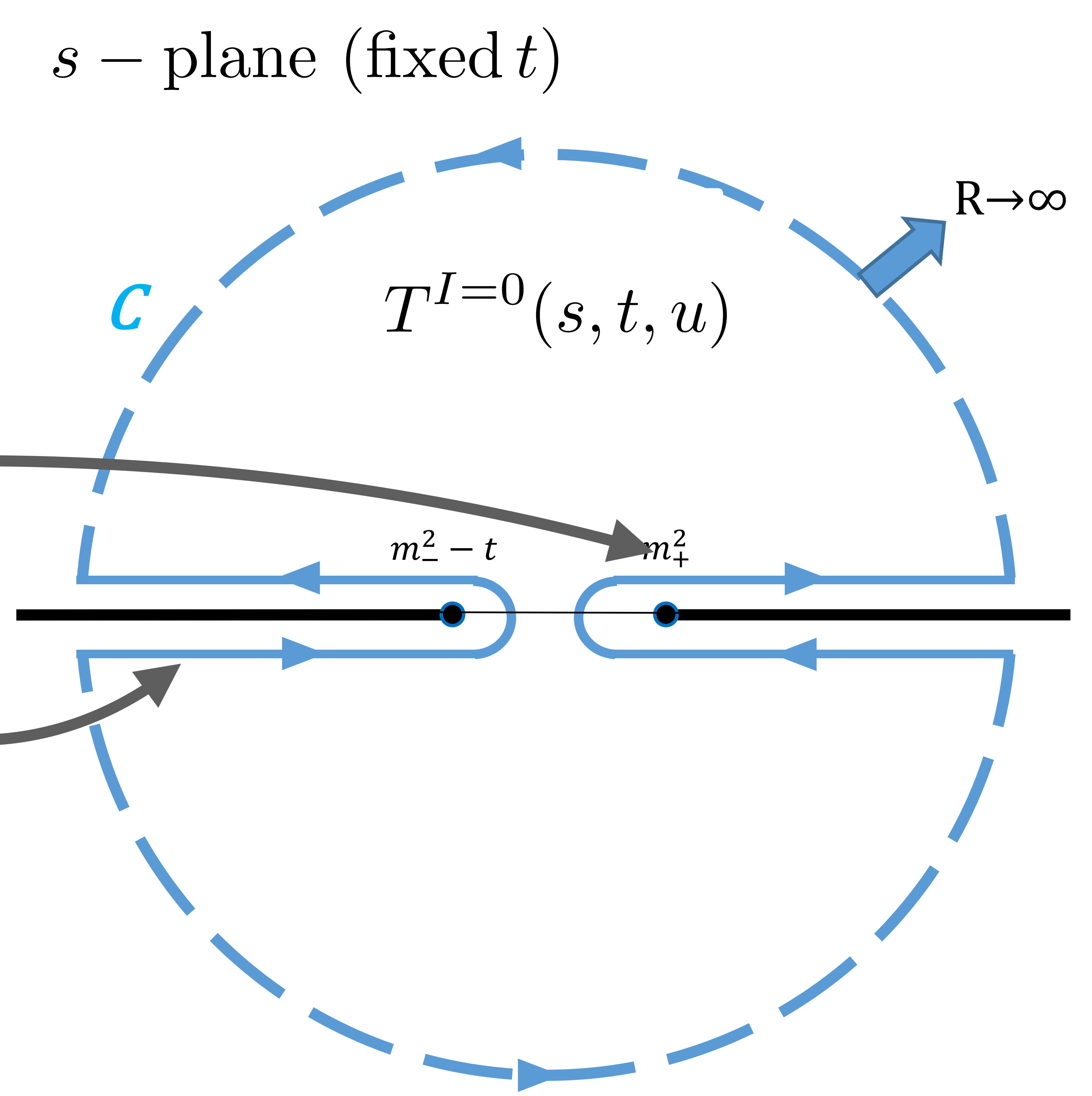


Crossing

Determines the analytic structure of the amplitudes

$T(s,t,u)$ **has a unitarity cut for** $s \geq 4m_\pi^2$

$$T^{I=0}(s,t,u) = 3T(s,t,u) + T(t,s,u) + T(u,t,s)$$



Cauchy theorem over contour C

$$T(s,t,u) = \frac{1}{2\pi i}\int_C \frac{T(s',t,u'(s',t))}{s'-s}ds'$$

How is this useful?? → "hooks" are given by $\mathrm{Im}\, T(s,t,u) \rightarrow$ **direct+crossing data**

Project the integral to get your dispersion relations (ex. Roy eqs.):

$$t_\ell^I(s) \rightarrow \tilde{t}_\ell^I(s) = \tau_\ell^I(s) + \sum_{I',\ell'}\int_{4m_\pi^2}^{\infty} ds' K_{\ell\ell'}^{II'}(s',s)\,\mathrm{Im}\, t_{\ell'}^{I'}(s')$$

Roy Phys.Lett.B 36 (1971)

$$\tau_0^0(s)/m_\pi = \tfrac{1}{3}\left(a_0^0 + 5a_0^2\right) + \tfrac{1}{3}\left(2a_0^0 - 5a_0^2\right)\frac{s}{4m_\pi^2}$$

Outside the physical region

Both sides are good now, we can now apply Cauchy's theorem+crossing

$$T(s,t,u) = \frac{1}{2\pi i}\int_C \frac{T(s',t,u'(s',t))}{s'-s} ds'$$

$$T^{I=0}(s,t,u) = 3T(s,t,u) + T(t,s,u) + T(u,t,s)$$
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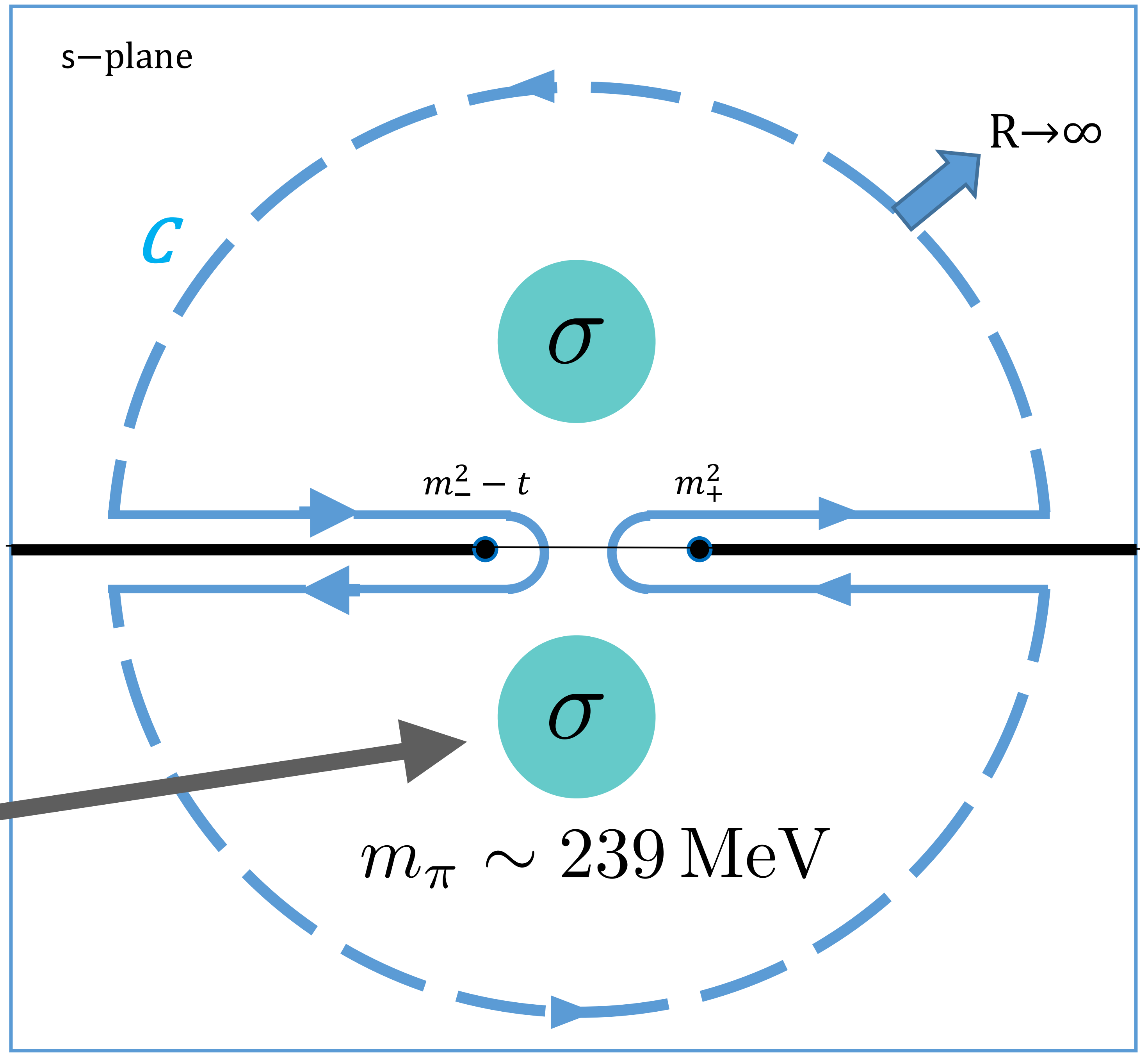
$$T^{I=2}(s,t,u) = T(t,s,u) + T(u,t,s)$$



Now, what happens here??

Tests: good vs bad

We select those models that respect the DRs

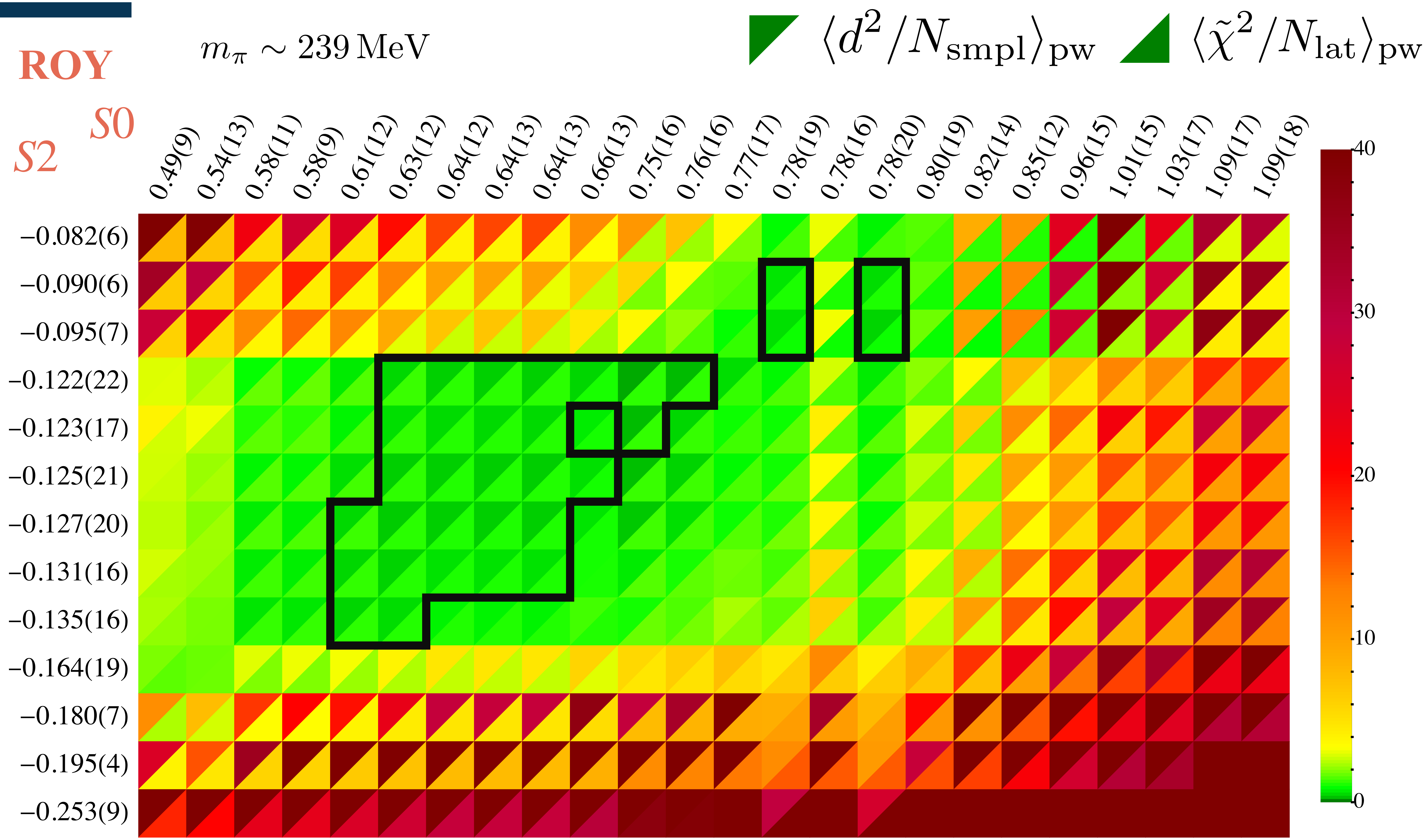
Scattering plane

$N_I \ell_{\max} N_{\text{params}} \sim 300 - 400$

$m_\pi \sim 239\,\text{MeV}$

Black **ROY**

$d^2/N_{\text{smpl}} < 1, \quad \tilde{\chi}^2/N_{\text{lat}} < 2$



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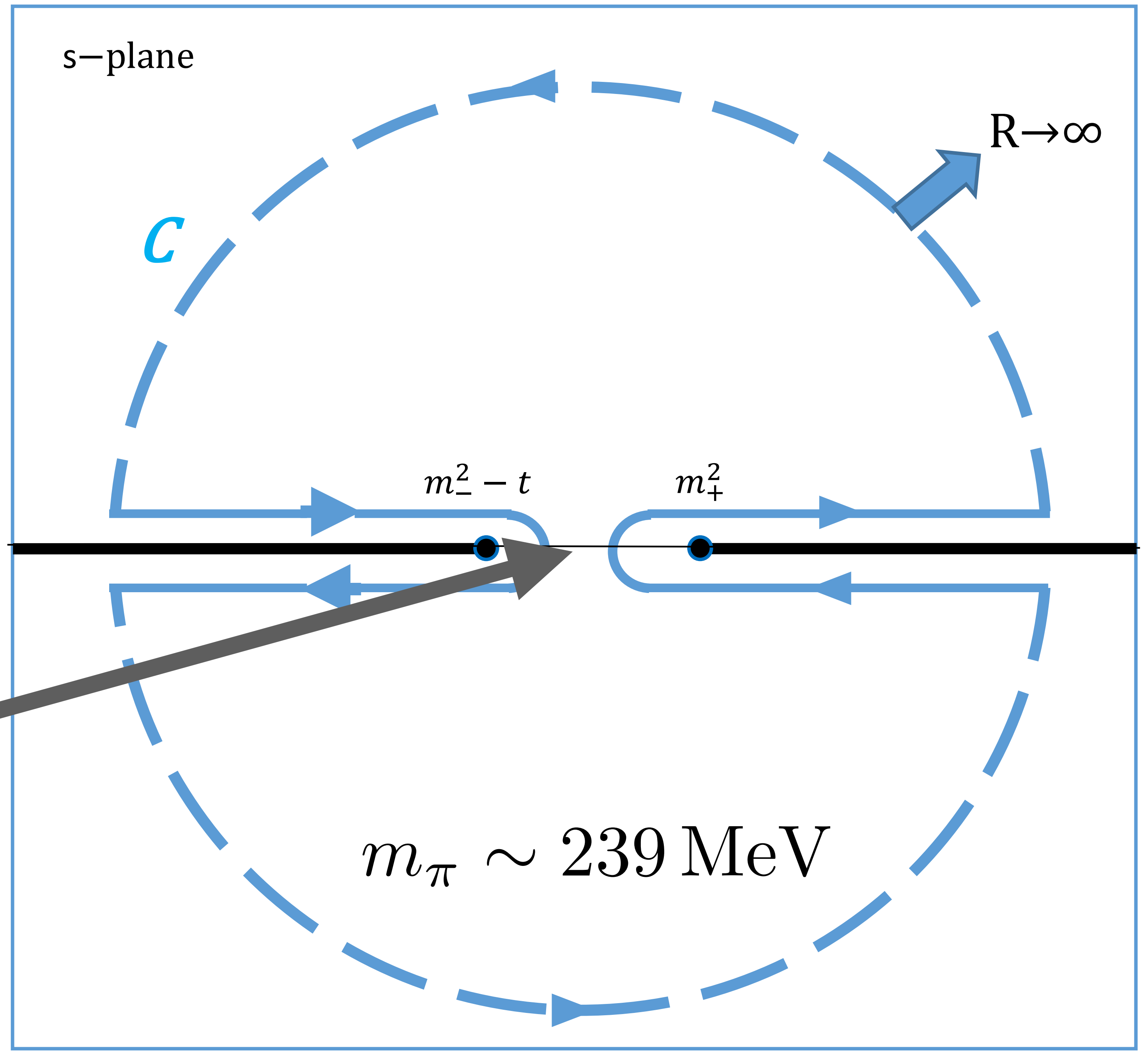

Now, what happens here??

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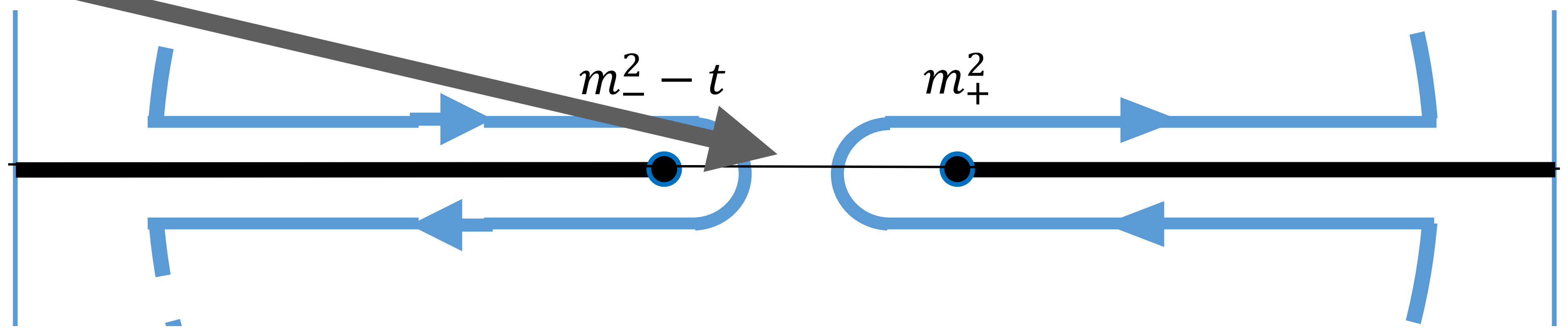
Adler, Phys.Rev. 137 (1965)



These zeroes appear on the S-waves and are considered directly linked to ChPT

ChPT predicts Adler zeroes for all pseudo-scalar scattering amplitudes at LO

$$s_{A,I=0} = m_\pi^2/2 \qquad\qquad s_{A,I=2} = 2m_\pi^2$$

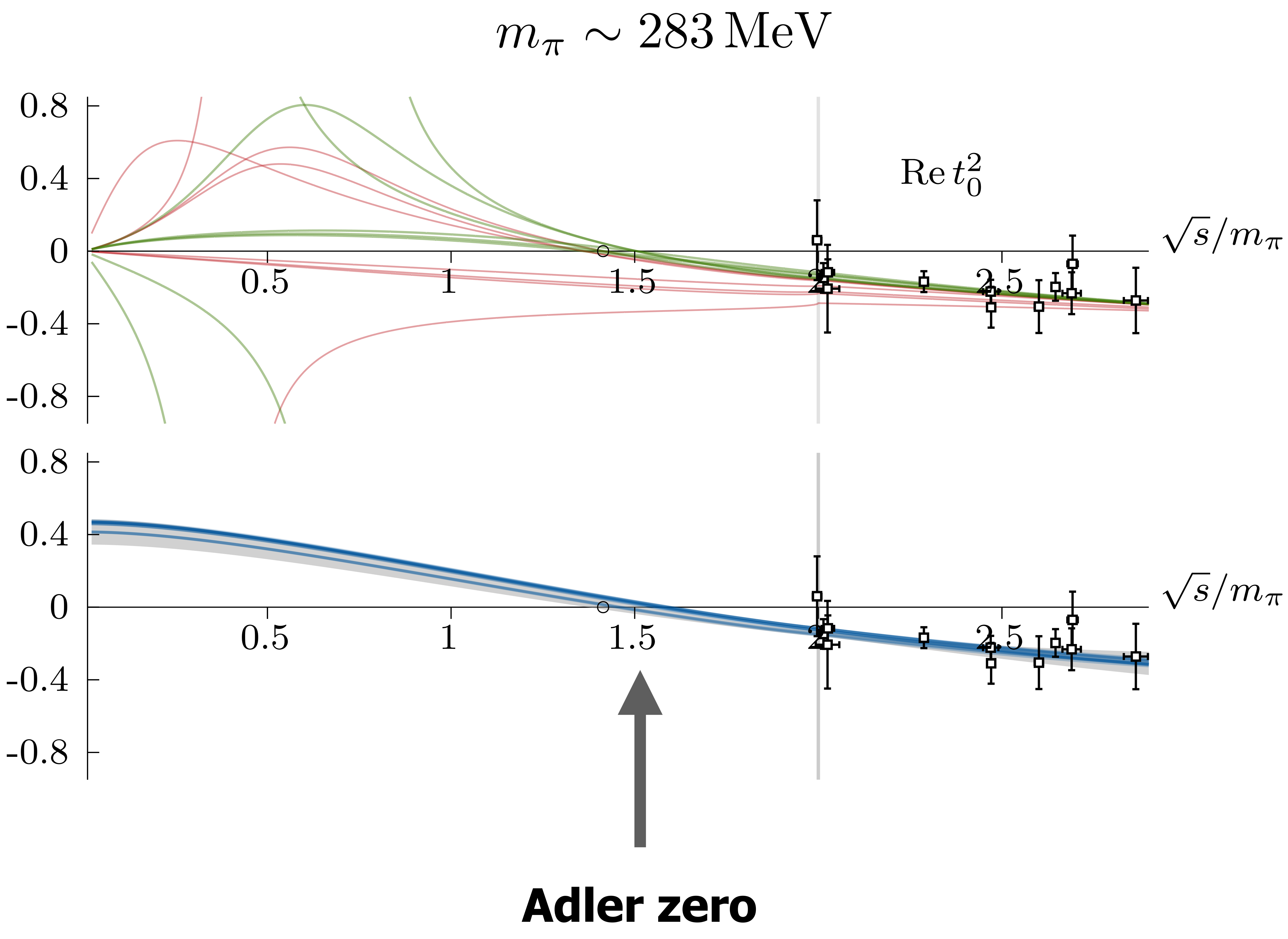
In lattice QCD our $m_\pi \nsim 0$**, but we still use ChPT in most analyses**

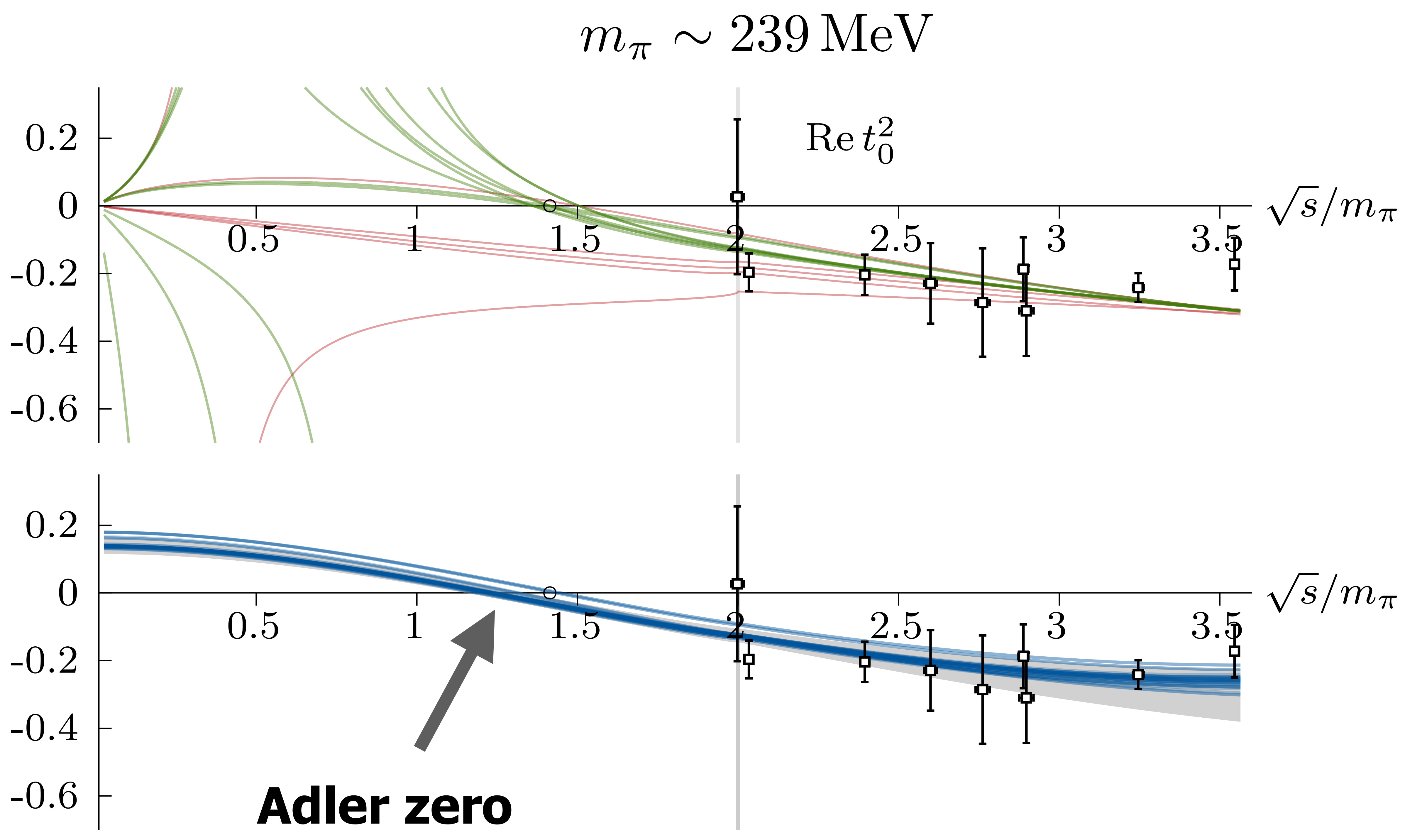
What can our DRs say about that?

Adler Zeroes

Very "stable" for $I = 2\ \pi\pi$

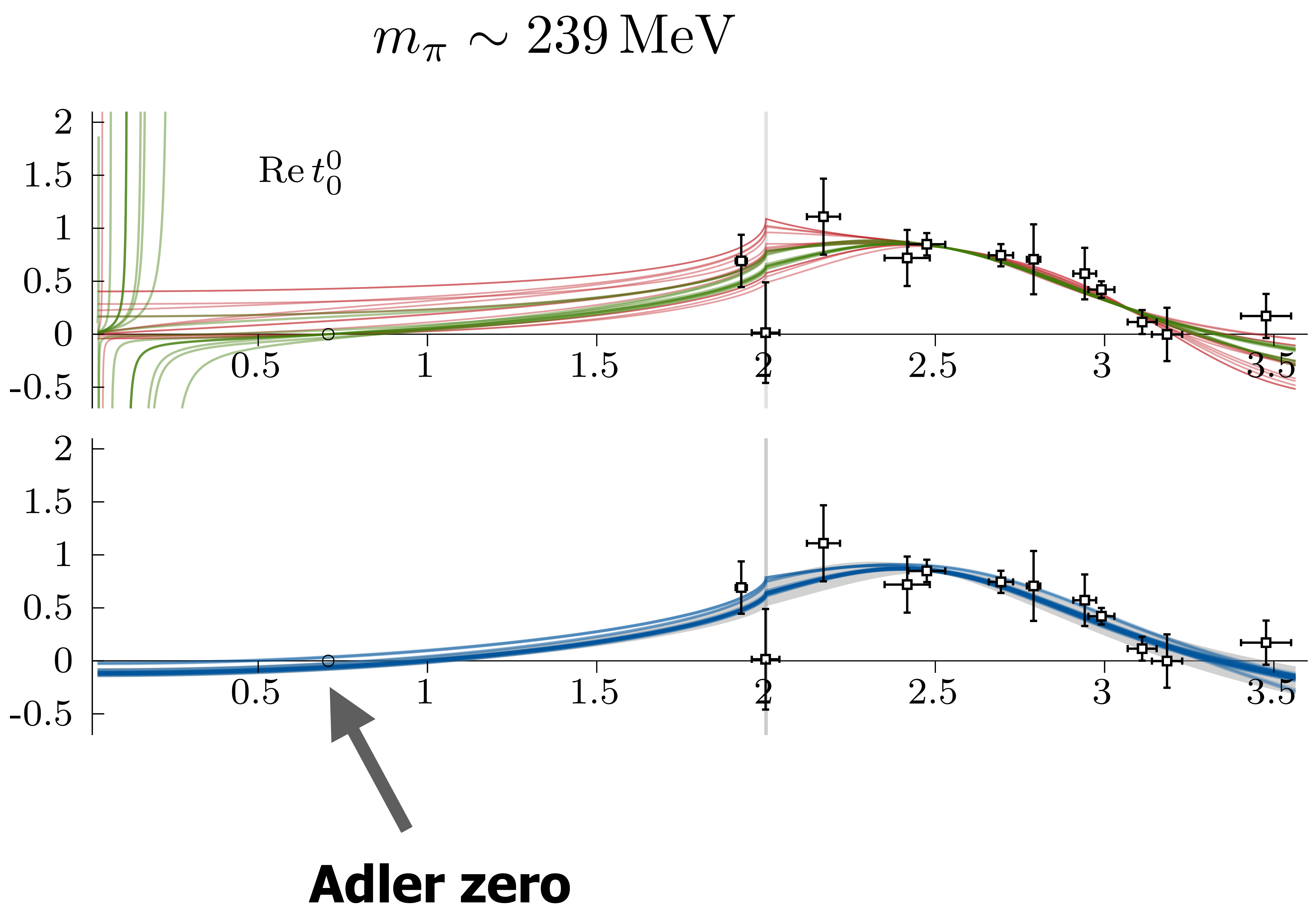
Even "bad" DRs produce Adler zeroes for I=2, close to the LO prediction $s_{A,I=2} = 2m_\pi^2$





Adler Zeroes

All good DRs produce an $I = 0$ $\pi\pi$ Adler zero for the lighter mass



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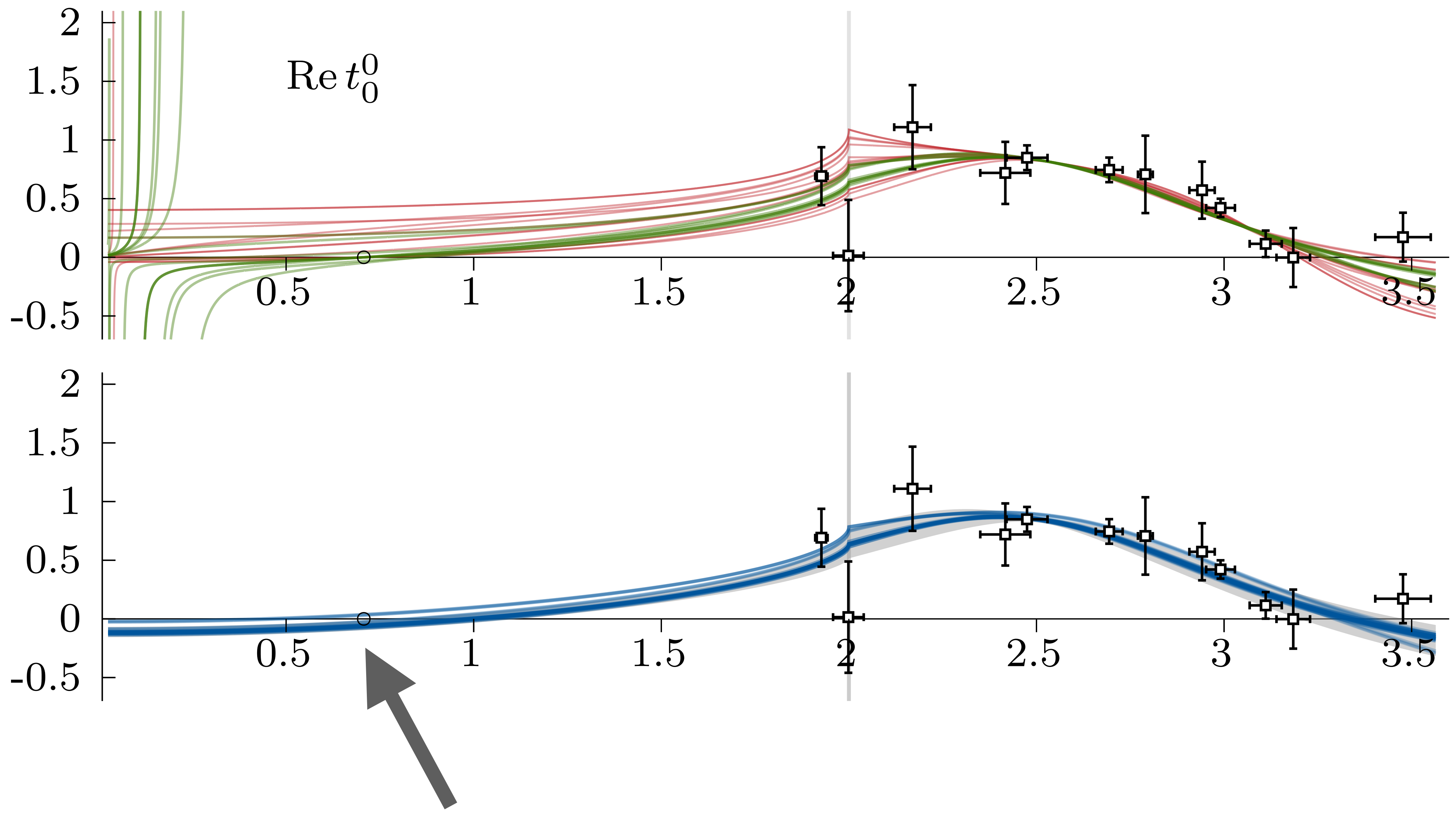
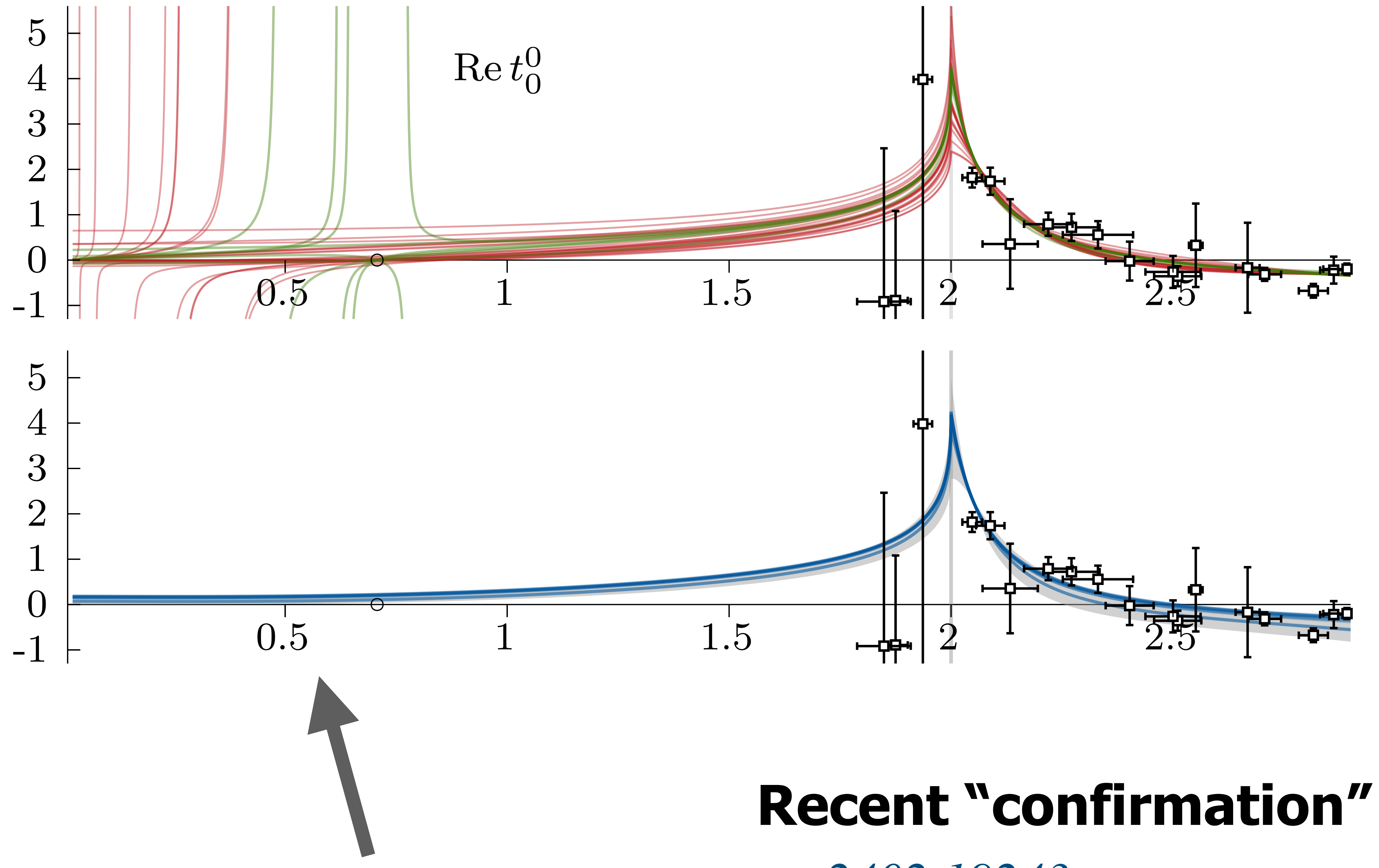
$m_\pi \sim 239\,\mathrm{MeV}$



$m_\pi \sim 283\,\mathrm{MeV}$

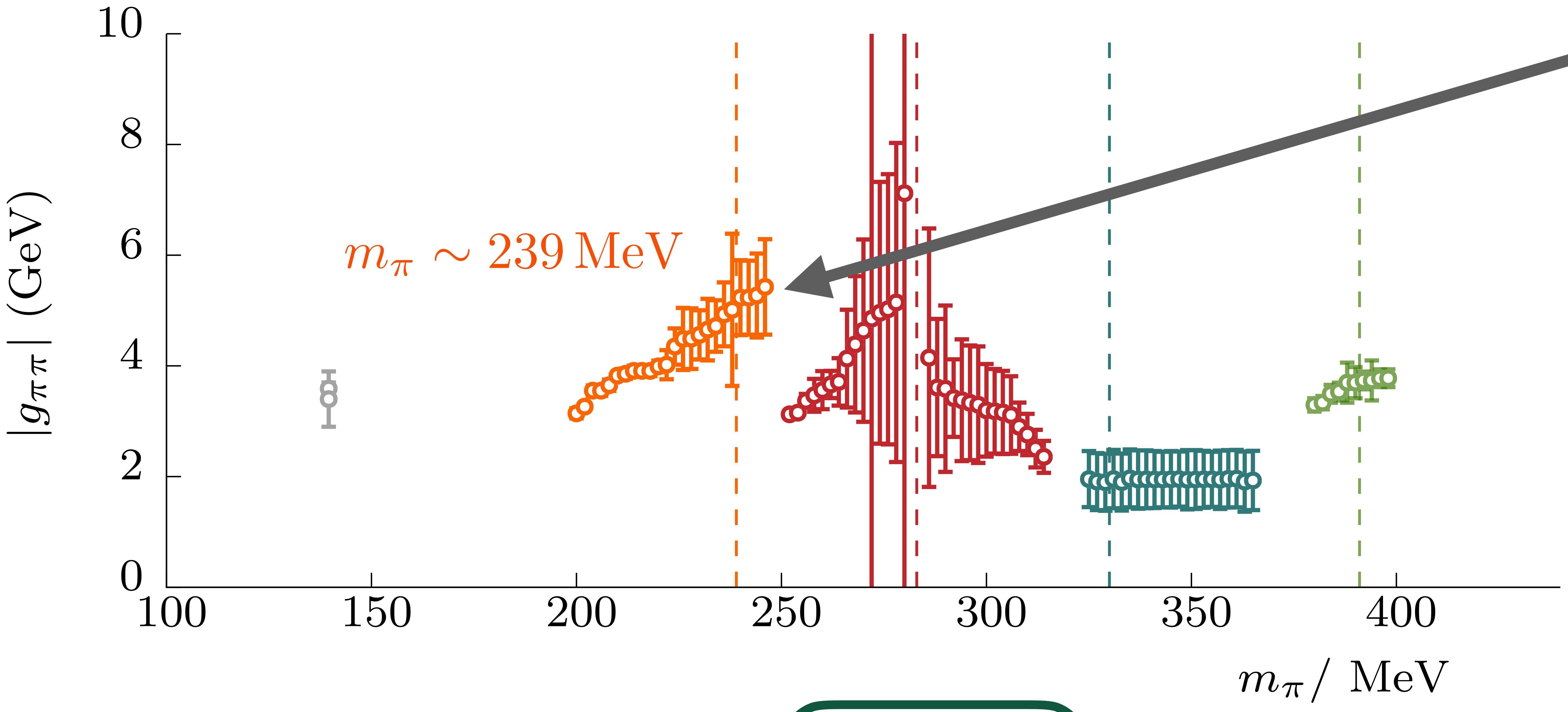


Recent "confirmation"

2402.19243

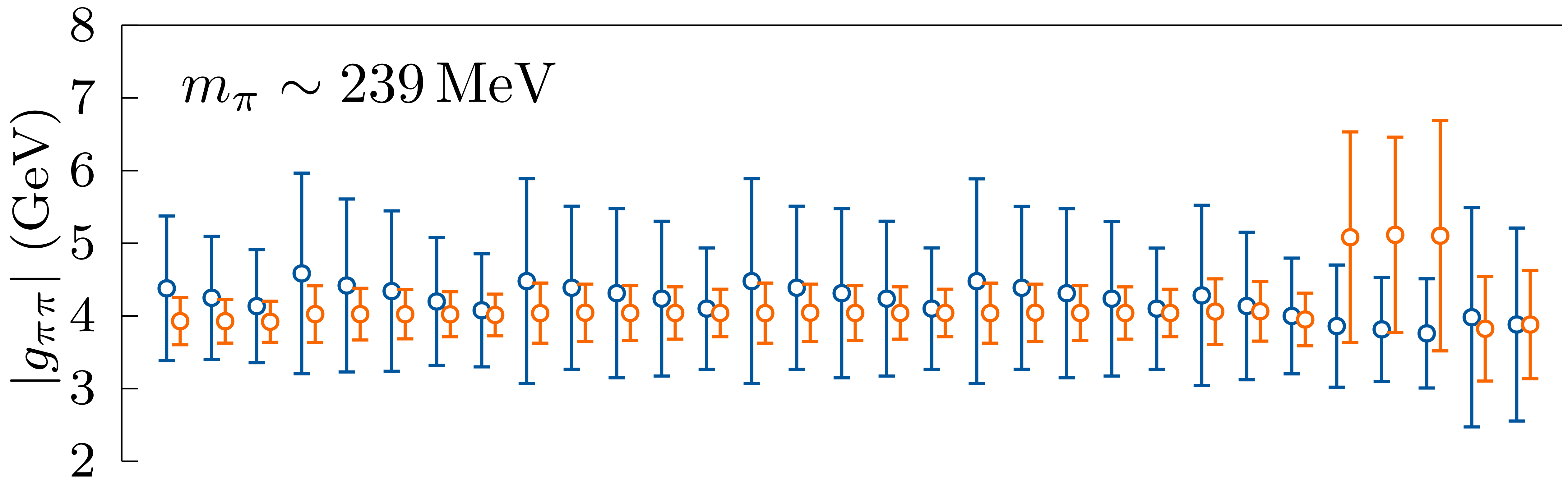
Couplings

Models: Large systematic dependence



2304.03762

DRs: No systematics



Make $Fit \to In$ $DR \to Out$ compatible ⟷ Unitarity

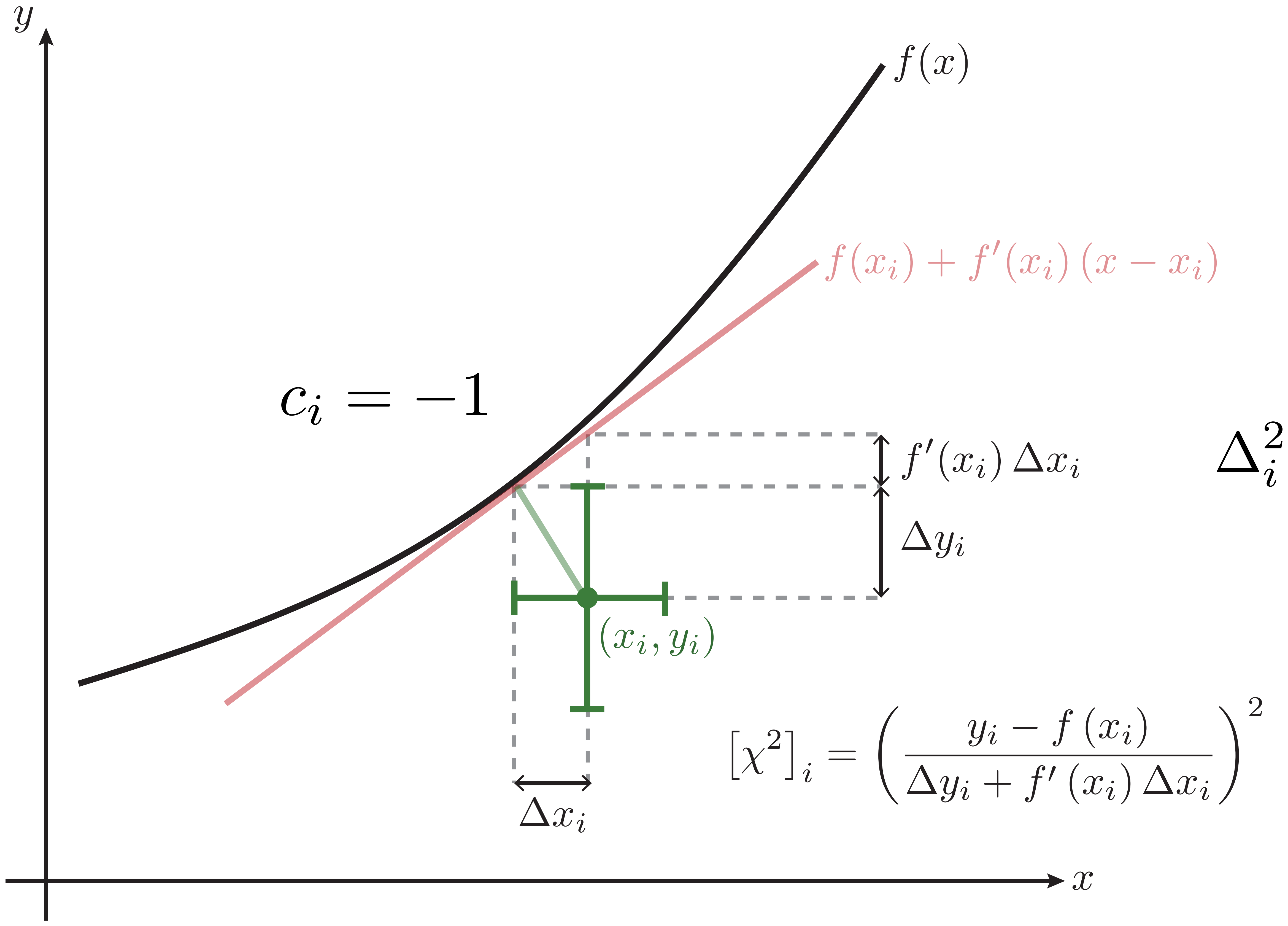
$$\left[d^2\right]_\ell^I \equiv \sum_{i=1}^{N_{\rm smpl}} \left(\frac{\mathrm{Re}\,\tilde{t}_\ell^I(s_i) - \mathrm{Re}\,t_\ell^I(s_i)}{\Delta\left[\mathrm{Re}\,\tilde{t}_\ell^I(s_i) - \mathrm{Re}\,t_\ell^I(s_i)\right]} \right)^2$$

Make $DR \to Out$ and data compatible ⟷ Lattice QCD data description

$$\left[\tilde{\chi}^2\right]_\ell^I \equiv \sum_{i,j=1}^{N_{\rm lat}} \left(\frac{\mathfrak{f}_i - \mathrm{Re}\,\tilde{t}_\ell^I(s_i)}{\Delta_i} \right) \mathrm{corr}\left(\mathfrak{f}_i, \mathfrak{f}_j\right)^{-1} \left(\frac{\mathfrak{f}_j - \mathrm{Re}\,\tilde{t}_\ell^I(s_j)}{\Delta_j} \right)$$

Make $DR \to Out$ **and data compatible** ⟷ **Lattice QCD data description**

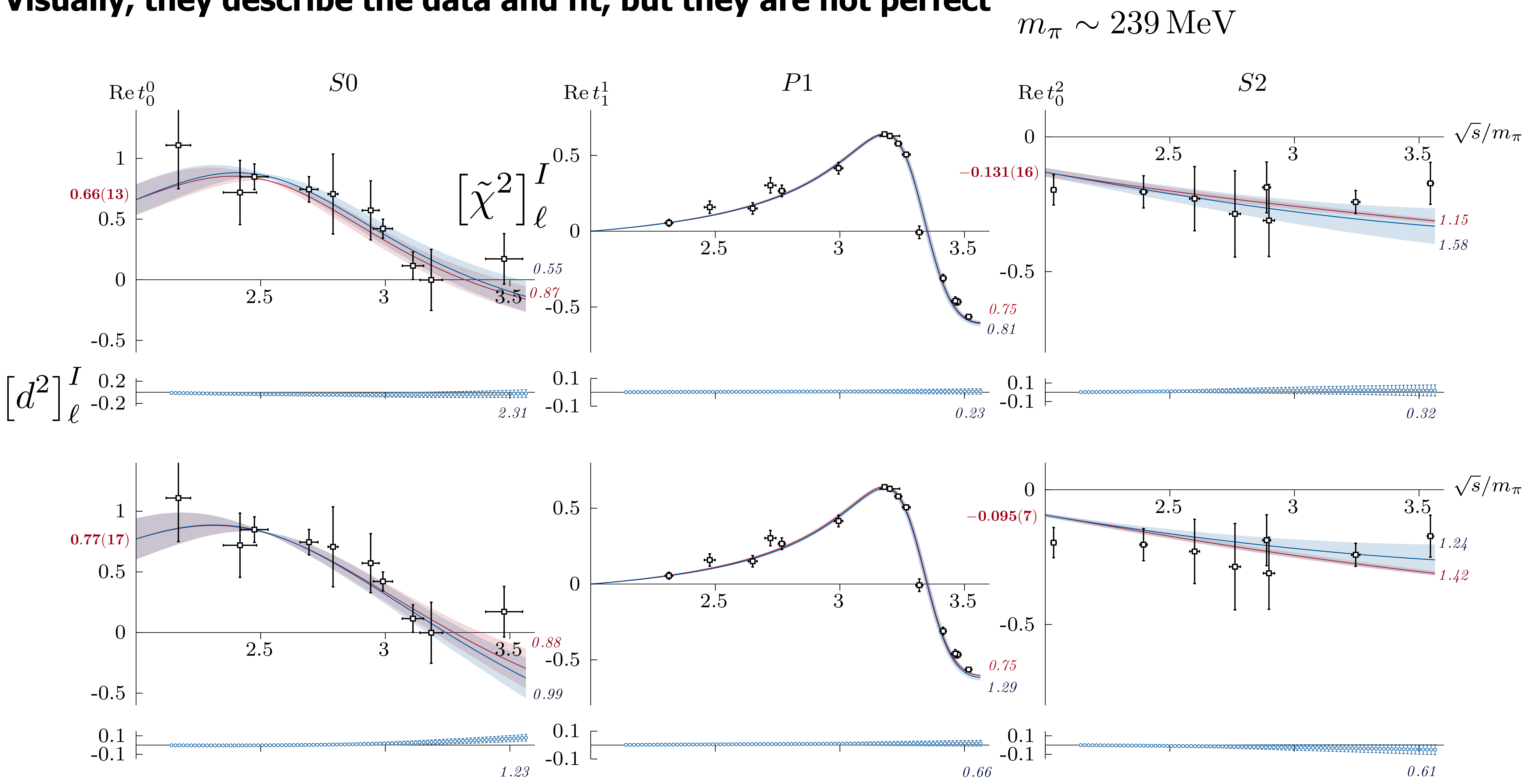
$$\left[\tilde{\chi}^2\right]^I_\ell \equiv \sum_{i,j=1}^{N_{\rm lat}} \left(\frac{\mathfrak{f}_i - \mathrm{Re}\,\tilde{t}^I_\ell(s_i)}{\Delta_i}\right) \mathrm{corr}\left(\mathfrak{f}_i, \mathfrak{f}_j\right)^{-1} \left(\frac{\mathfrak{f}_j - \mathrm{Re}\,\tilde{t}^I_\ell(s_j)}{\Delta_j}\right)$$



$$\Delta_i^2 = \begin{pmatrix} \Delta\mathfrak{f}_i & \frac{d\tilde{f}^I_\ell(s_i)}{dE_i}\Delta E_i \end{pmatrix} \begin{pmatrix} 1 & -c_i \\ -c_i & 1 \end{pmatrix} \begin{pmatrix} \Delta\tilde{f}_i \\ \frac{d\tilde{f}^I_\ell(s_i)}{dE_i}\Delta E_i \end{pmatrix}$$

Ok but not great

Visually, they describe the data and fit, but they are not perfect



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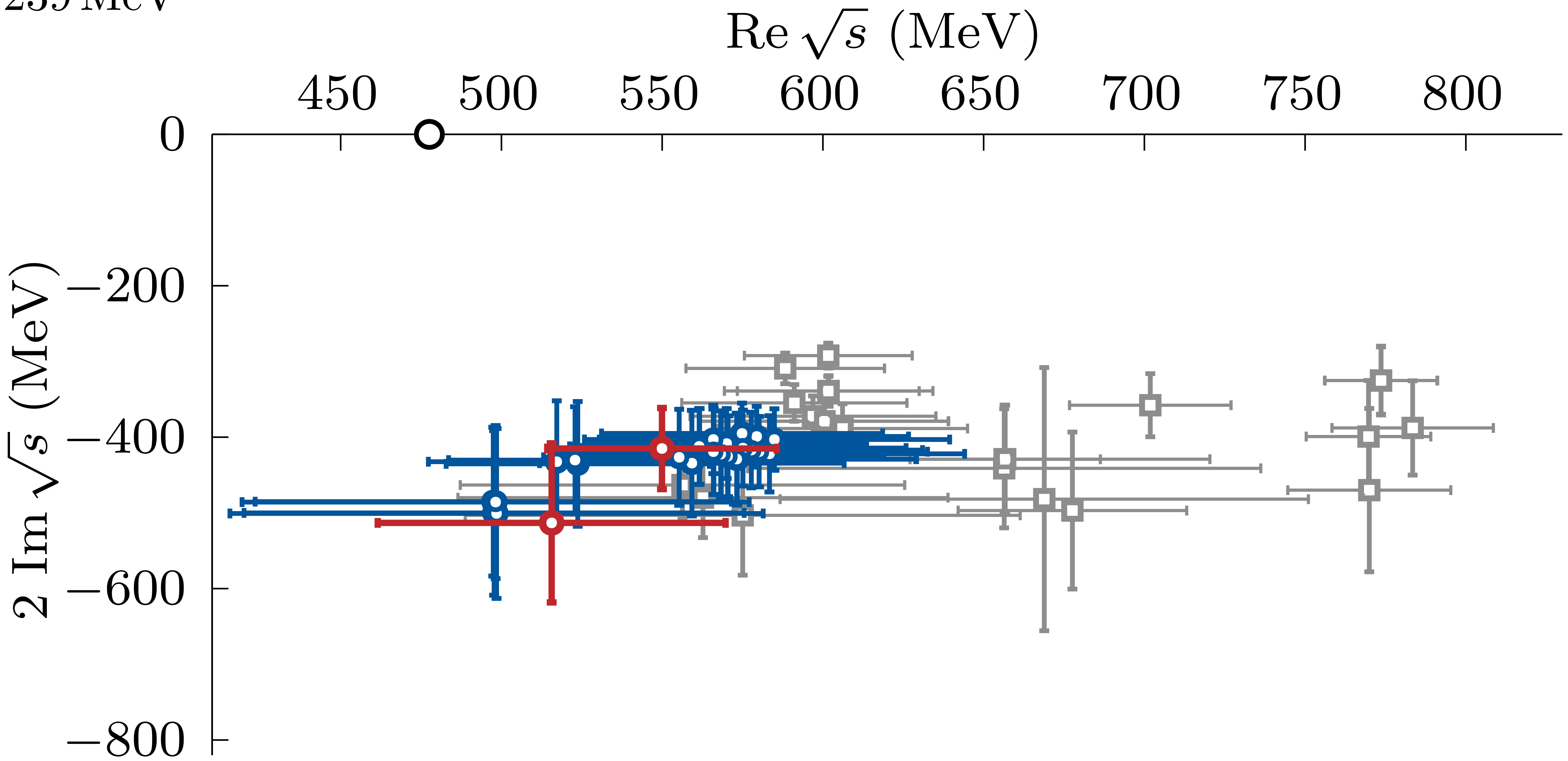
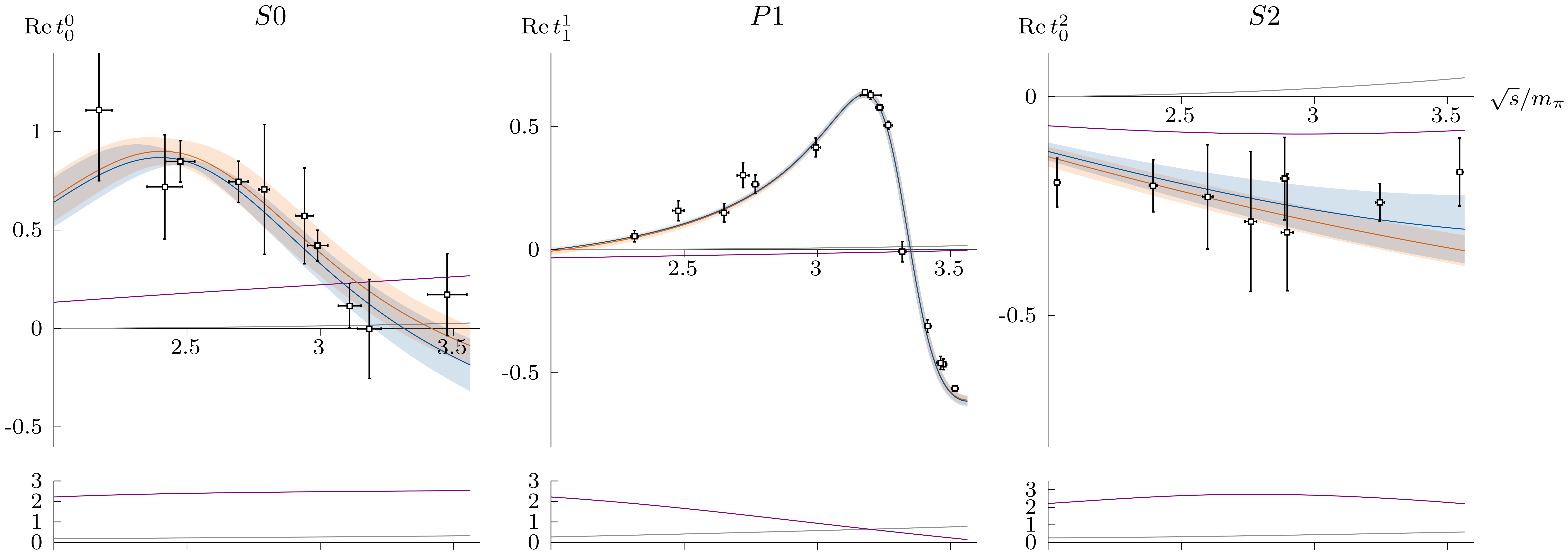


GKPY vs ROY

GKPY: Minimally subtracted → one less subtraction than ROY

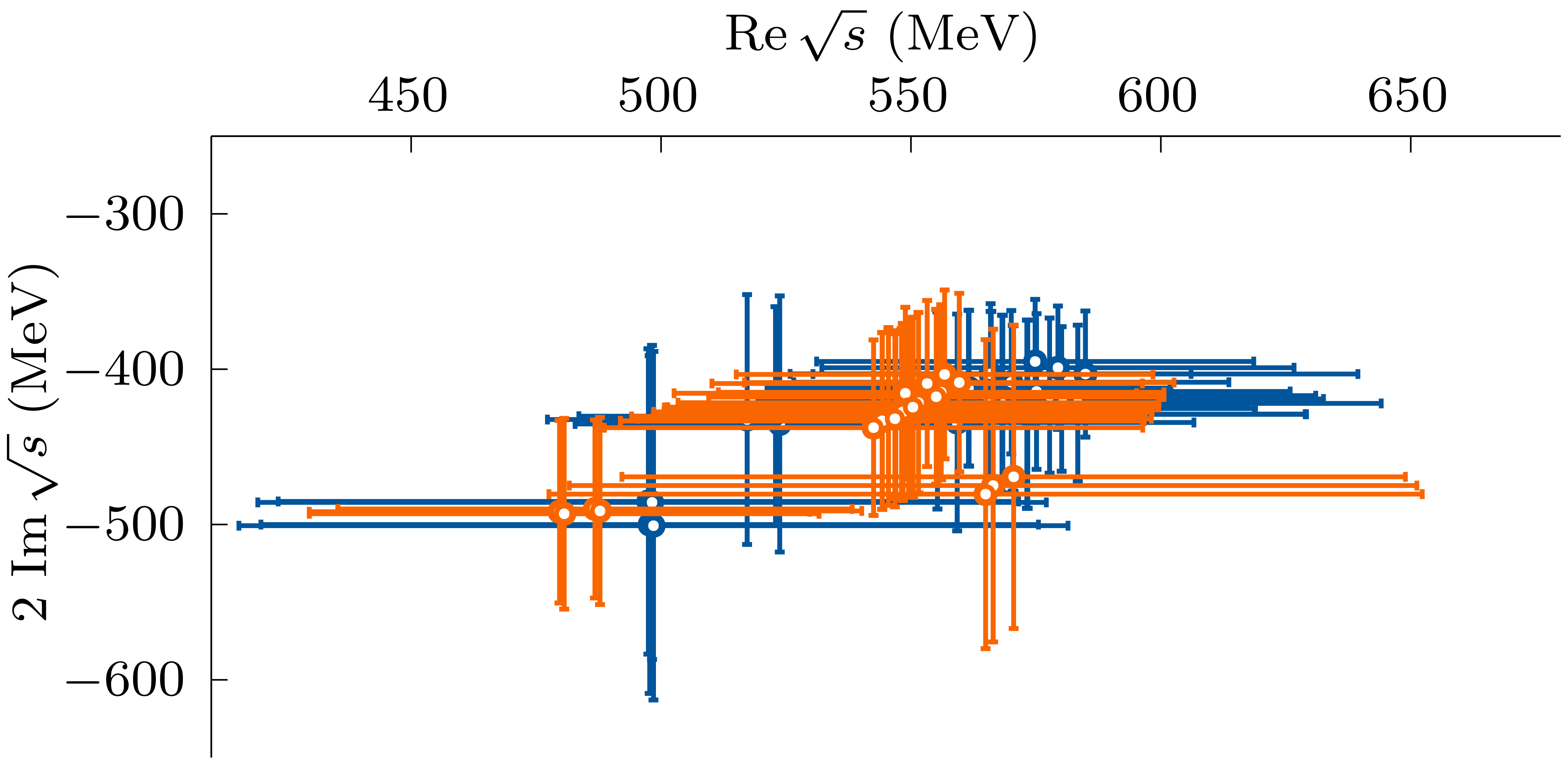
For our analysis, Regge contribution too large for d^2

$m_\pi \sim 239\,\mathrm{MeV}$

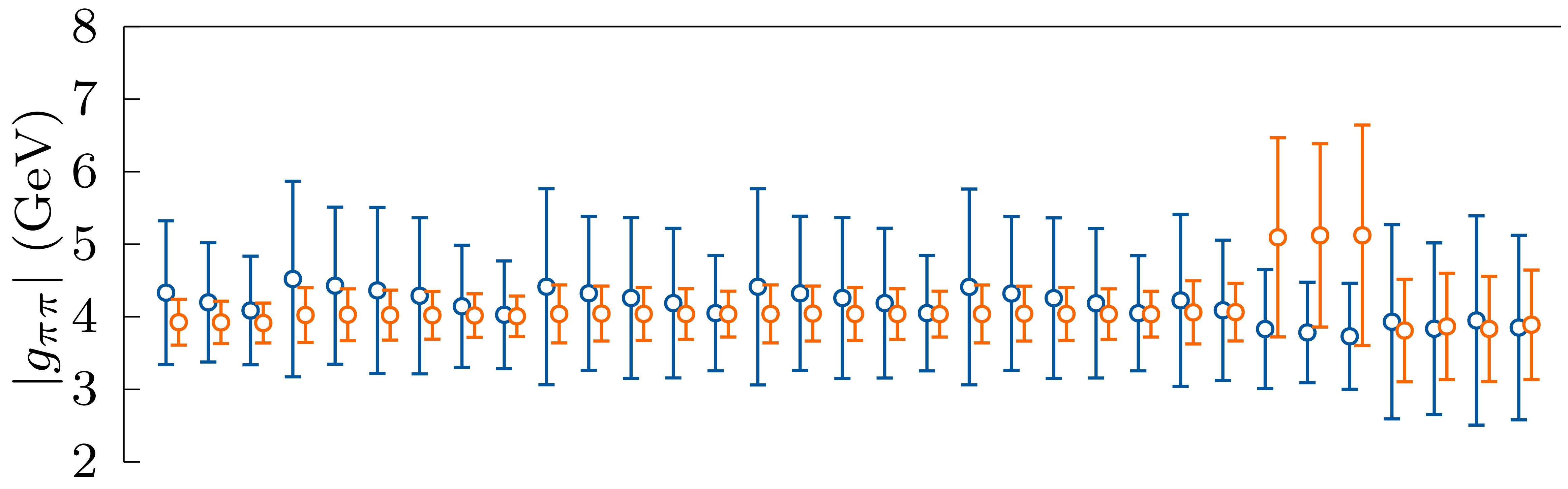


GKPY vs ROY

However, pole extraction is more accurate in most cases, particularly for the coupling



GKPY produces less than half the uncertainty in most cases



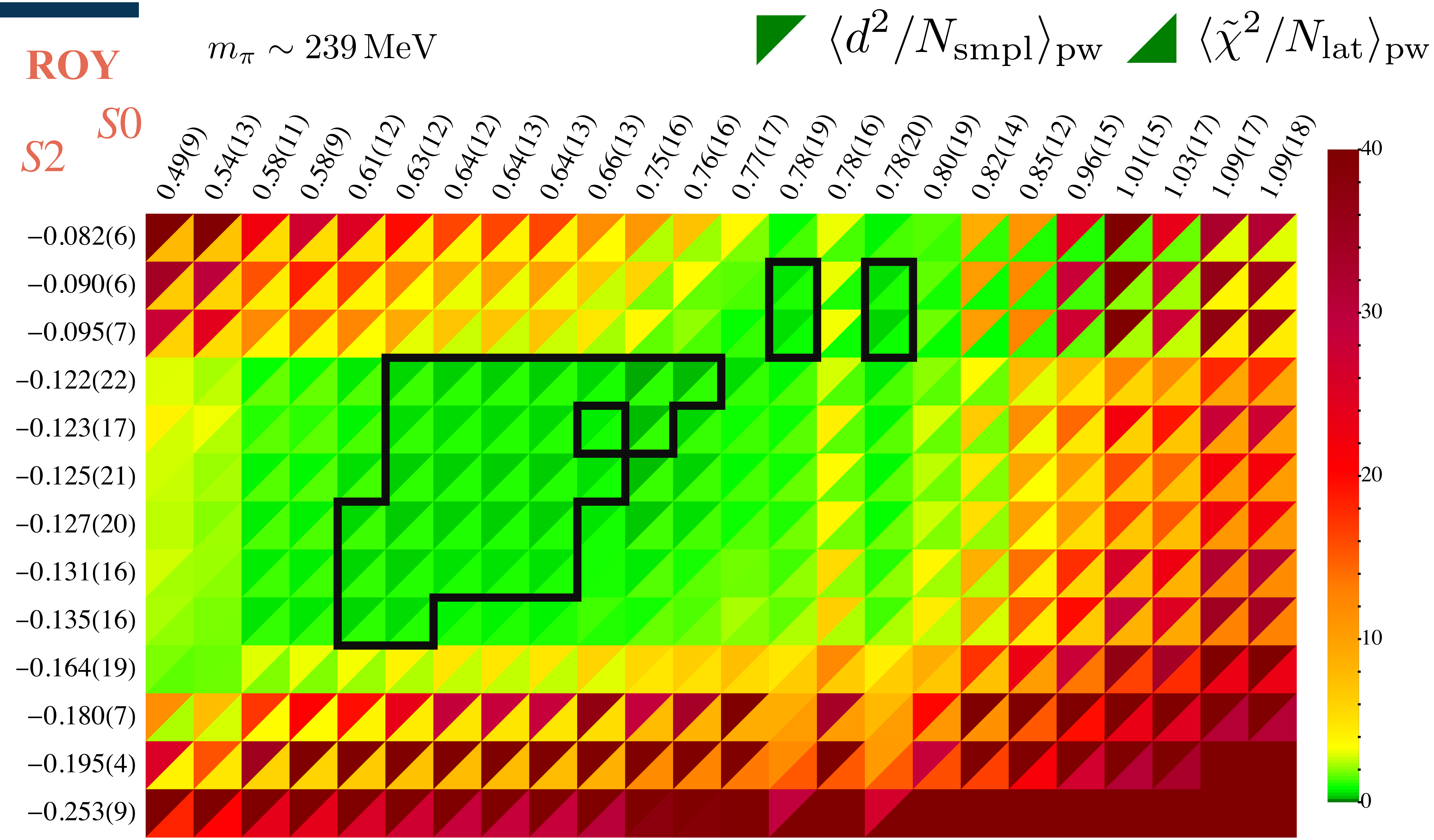
Scattering plane

$N_I \ell_{\max} N_{\text{params}} \sim 300 - 400$

$m_\pi \sim 239\,\text{MeV}$

Black $d^2/N_{\text{smpl}} < 1, \quad \tilde{\chi}^2/N_{\text{lat}} < 2$

ROY



Scattering plane

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$m_\pi \sim 239\,\text{MeV}$

$\langle d^2/N_{\text{smpl}} \rangle_{\text{pw}}$ $\langle \tilde{\chi}^2/N_{\text{lat}} \rangle_{\text{pw}}$

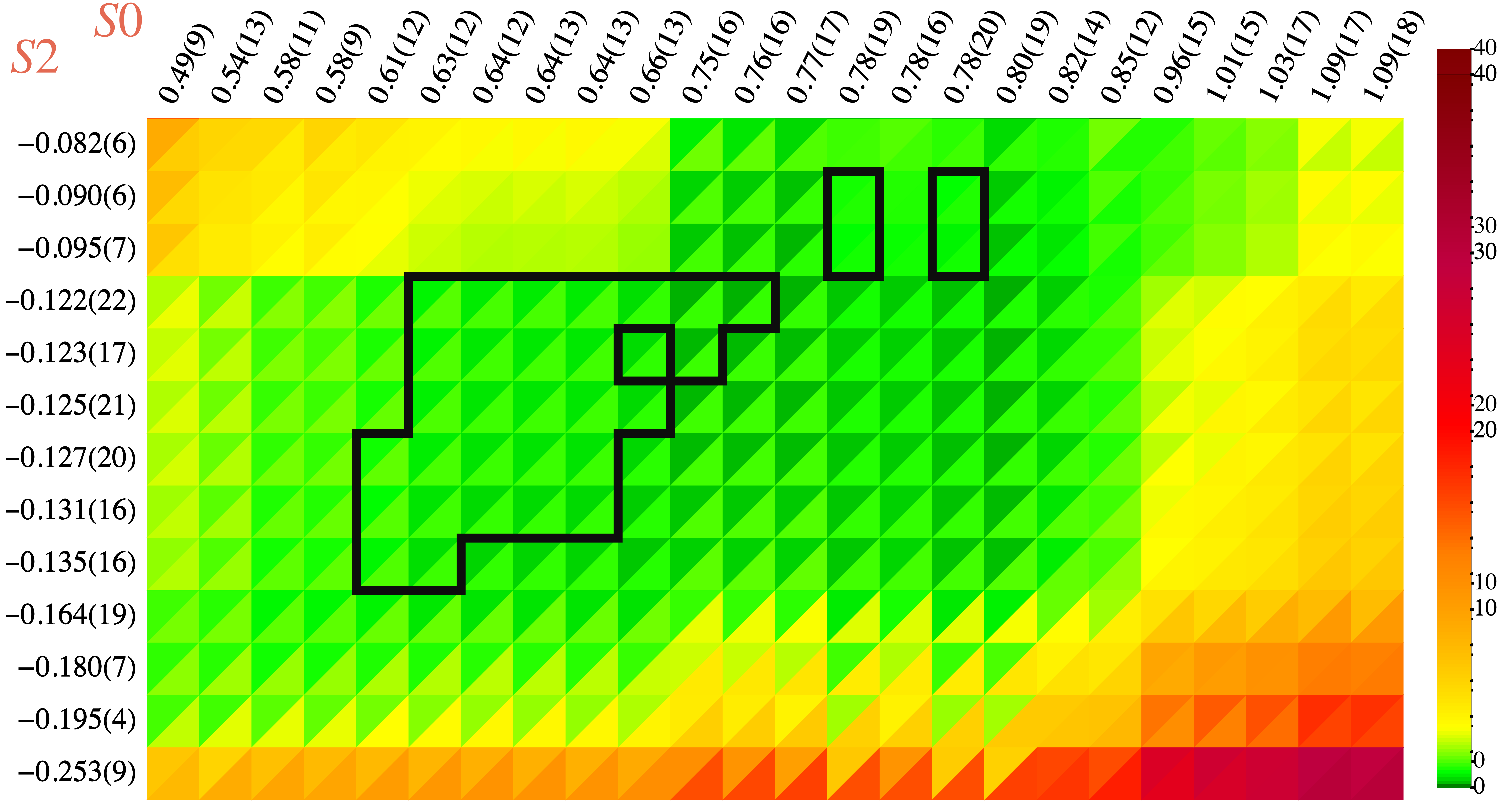
Black

GKPY

$d^2/N_{\text{smpl}} < 1, \quad \tilde{\chi}^2/N_{\text{lat}} < 2$

S0

S2



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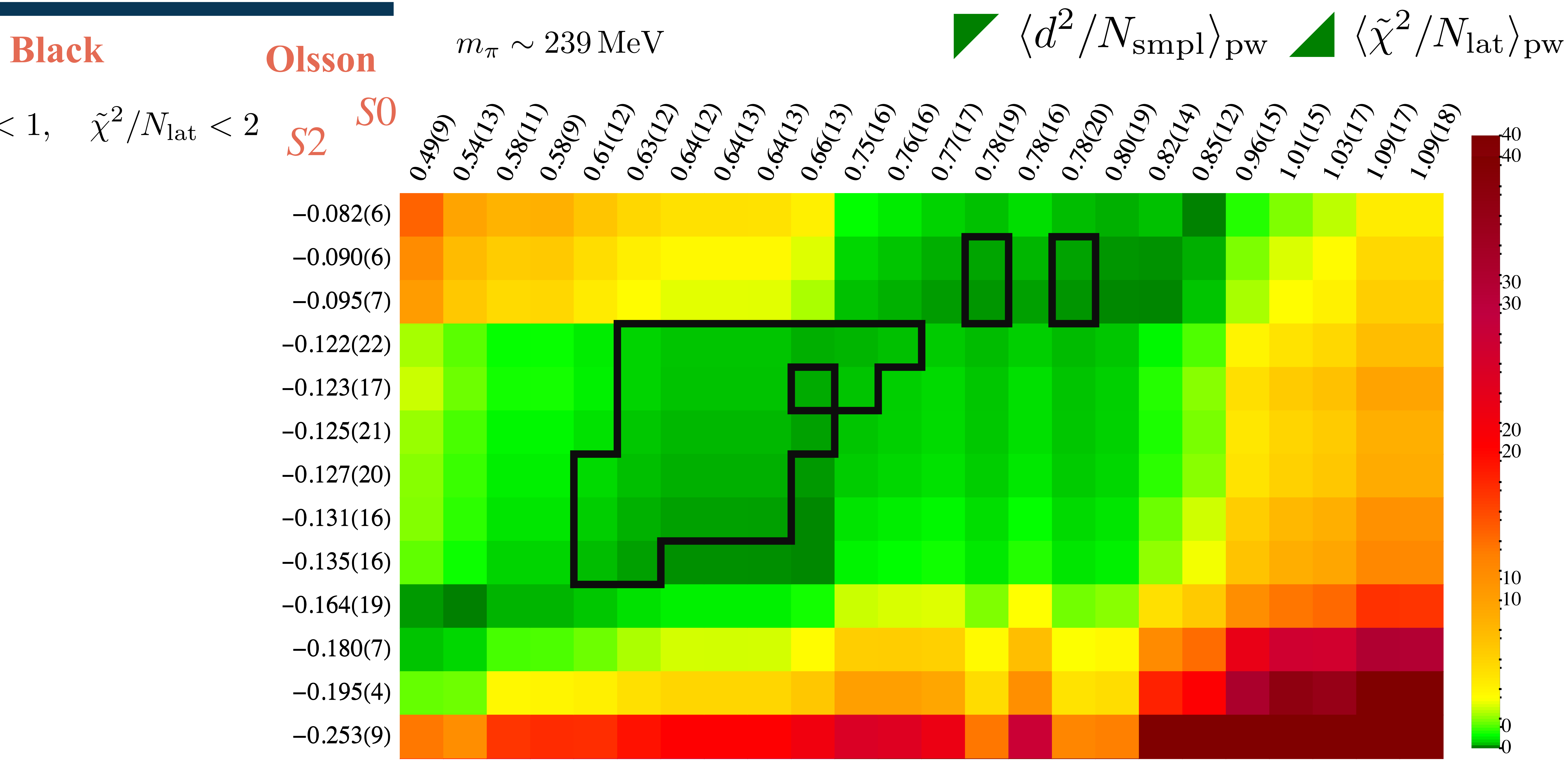


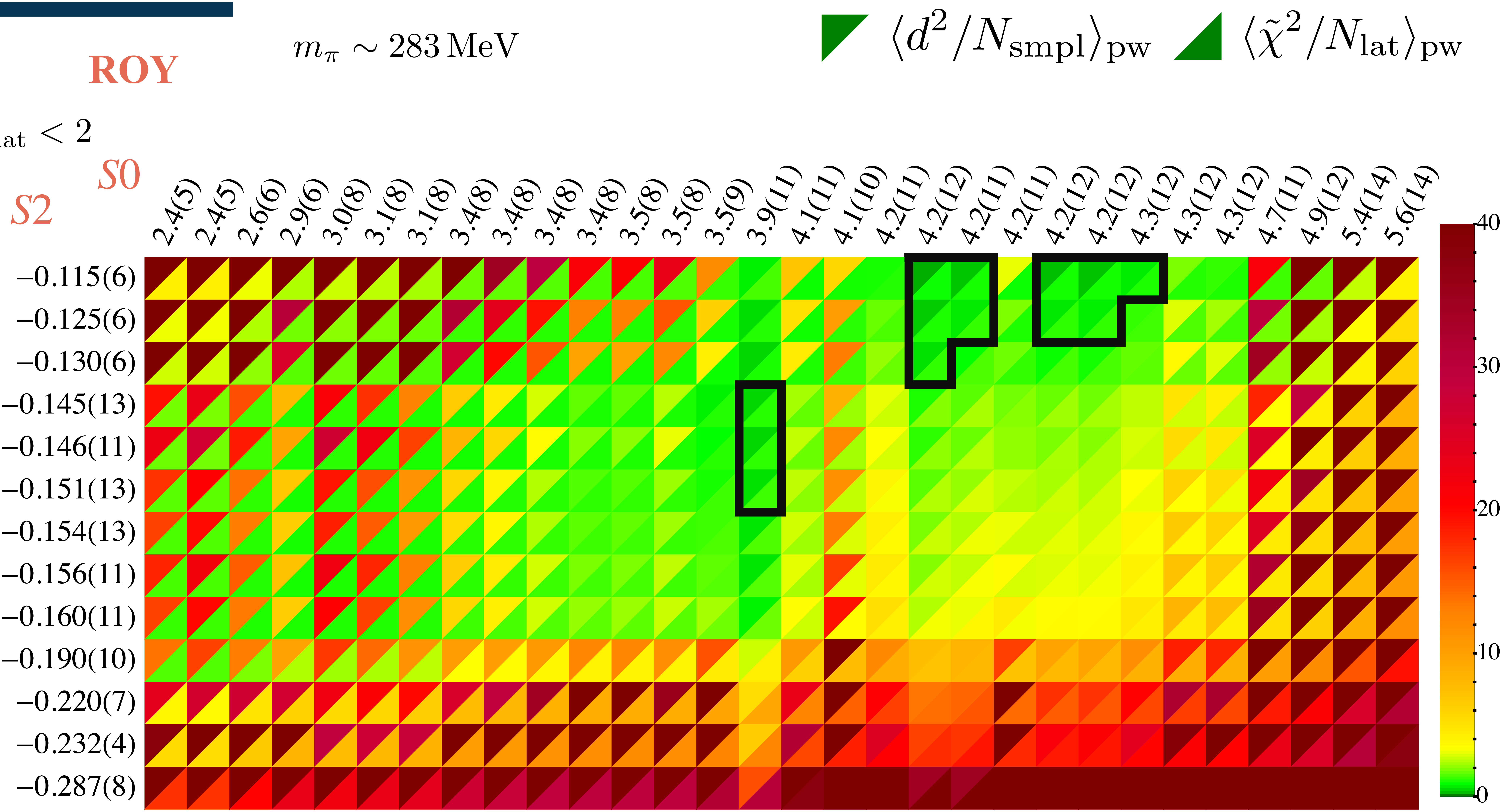
Scattering plane

$N_I \ell_{\max} N_{\text{params}} \sim 300 - 400$

$m_\pi \sim 283\,\text{MeV}$

Black ROY

$d^2/N_{\text{smpl}} < 1, \quad \tilde{\chi}^2/N_{\text{lat}} < 2$



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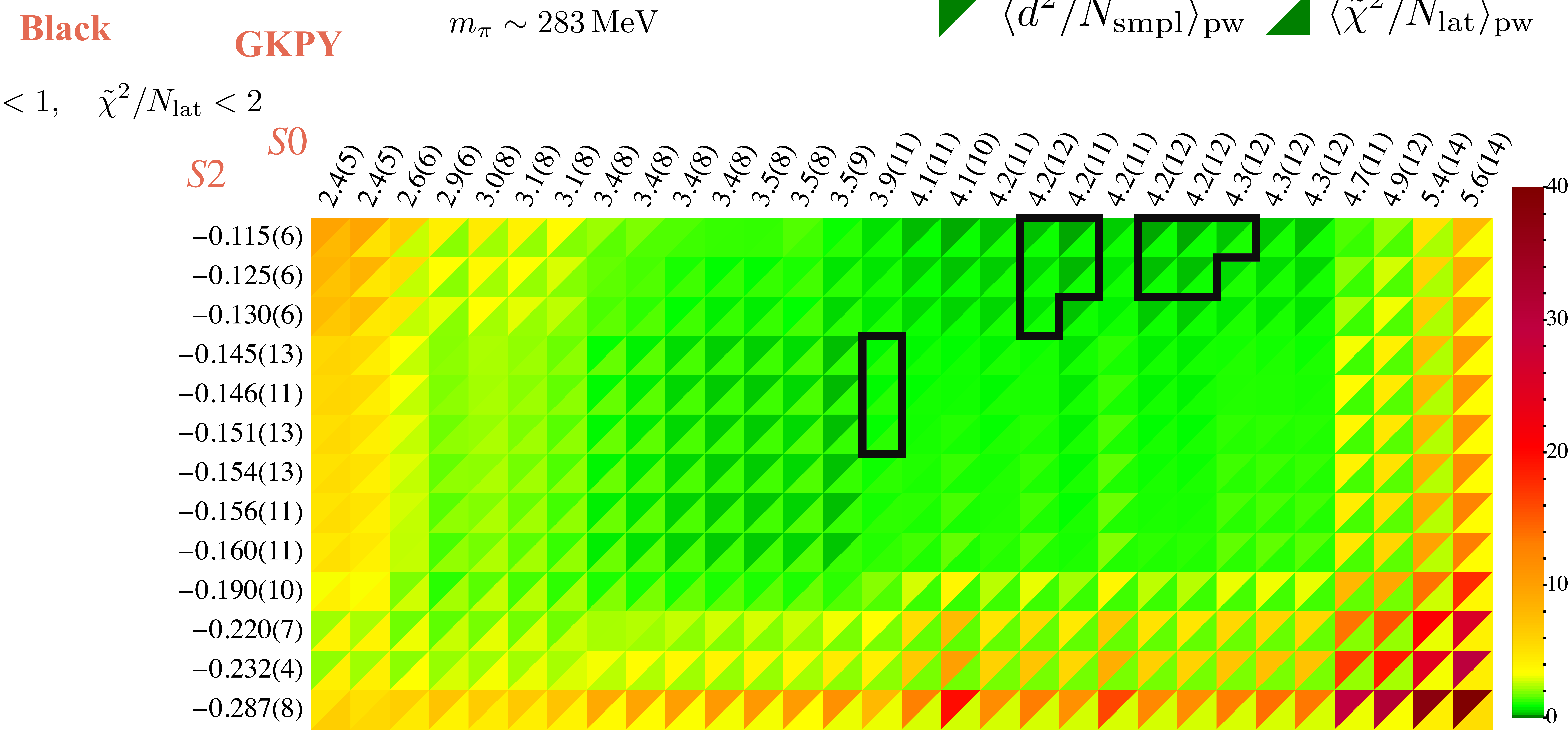

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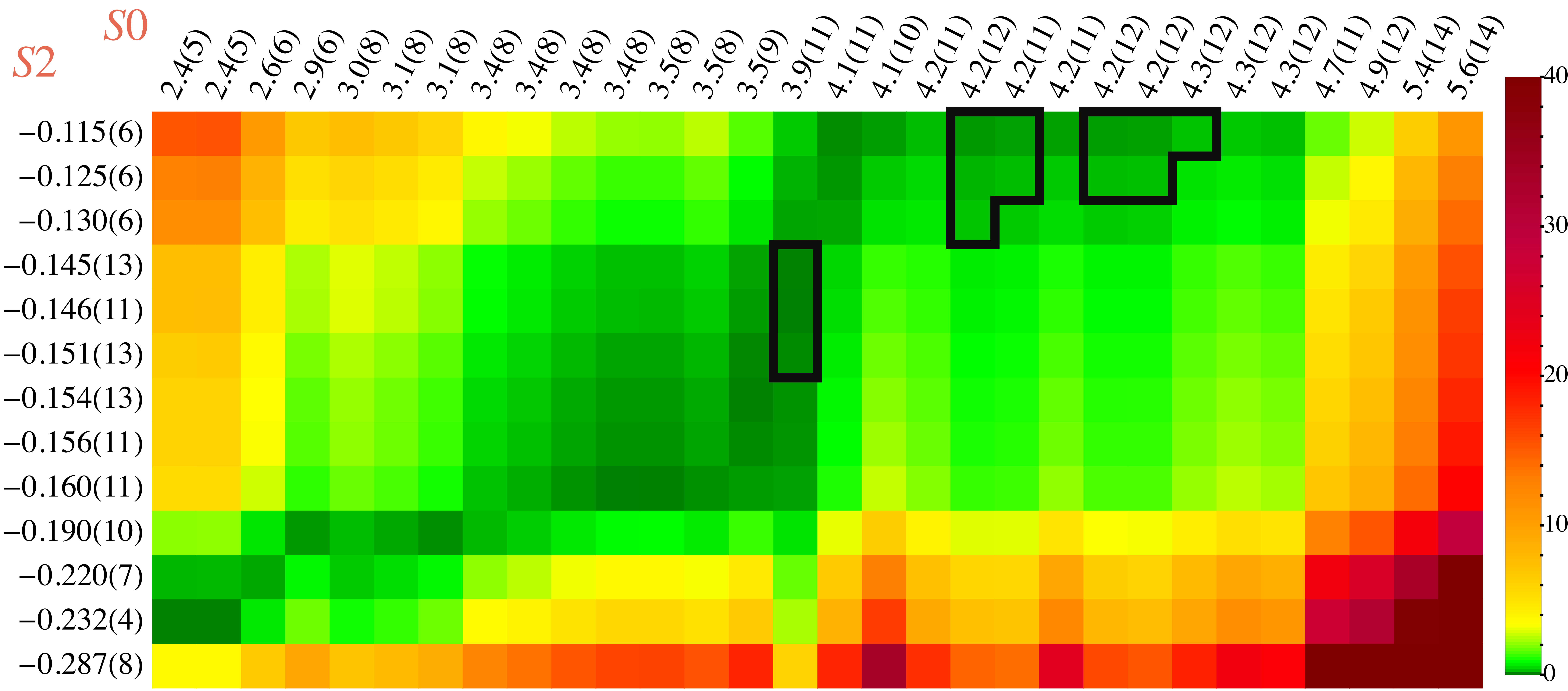
$\langle d^2/N_{\text{smpl}}\rangle_{\text{pw}}$ $\langle \tilde{\chi}^2/N_{\text{lat}}\rangle_{\text{pw}}$

Black Olsson

$d^2/N_{\text{smpl}} < 1, \quad \tilde{\chi}^2/N_{\text{lat}} < 2$



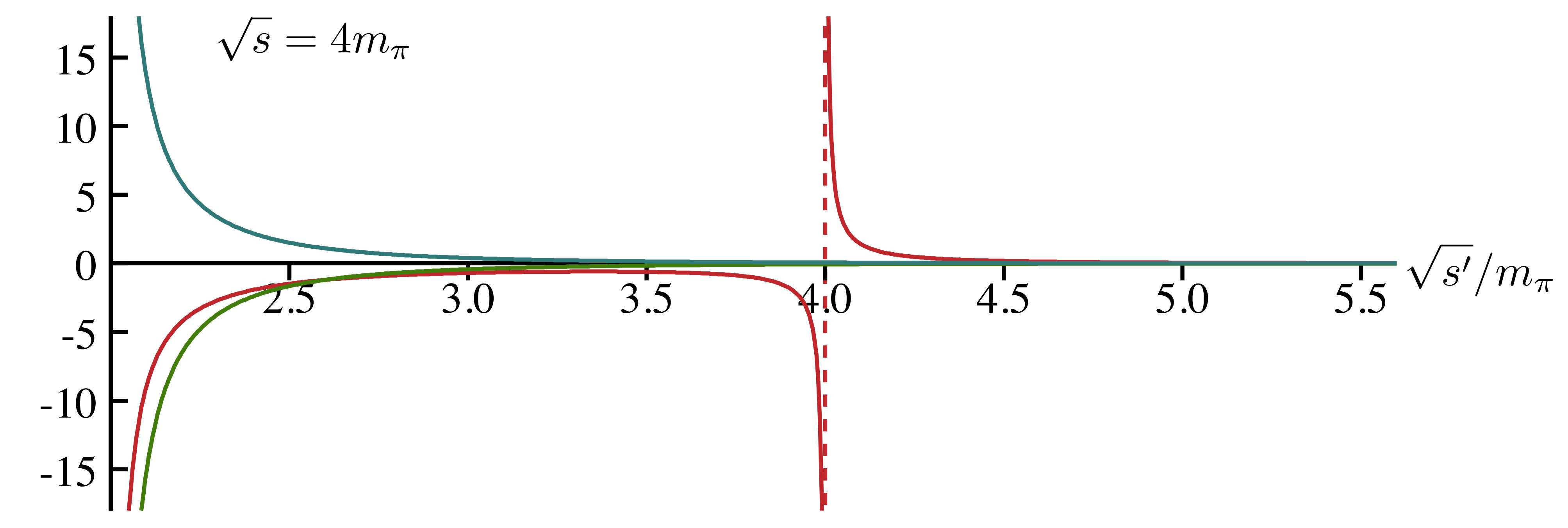
The good

Fit → In

DR → Out

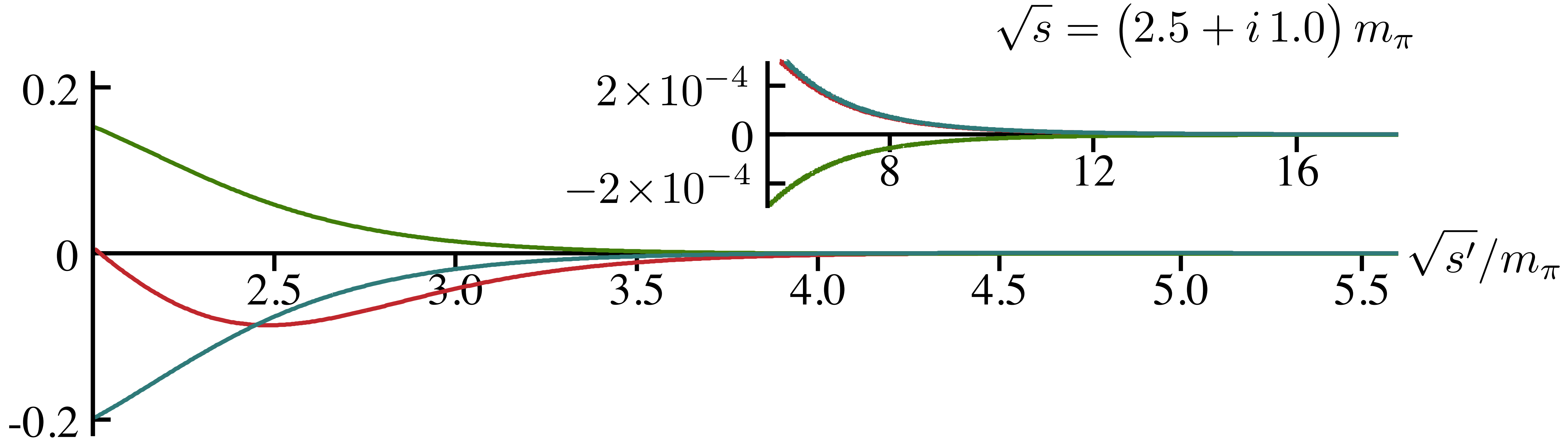
$$\tilde{t}^I_\ell(s) = \tau^I_\ell(s) + \sum_{I',\ell'} \int_{4m_\pi^2}^{\infty} ds' \, K^{II'}_{\ell\ell'}(s',s)\, \mathrm{Im}\, t^{I'}_{\ell'}(s')$$

Real s



Smeared over a large energy region

Complex s



An ϵ on the real axis → ϵ' in the complex plane

The bad

Not happening

$$\sum_{I',\ell'} \int_{4m_\pi^2}^{\infty} ds' \, K_{\ell\ell'}^{II'}(s',s) \, \mathrm{Im}\, t_{\ell'}^{I'}(s')$$

Partial waves

Extrapolated

$$\int_{4m_\pi^2}^{\infty} = \int_{4m_\pi^2}^{s_{max}} + \int_{s_{max}}^{\infty} = \int_{4m_\pi^2}^{s_{fit}} + \int_{s_{fit}}^{s_{max}} + \int_{s_{max}}^{\infty}$$

Regge

- **Regge must be extrapolated from phys. m_π**
- **Regge is wrong below $a_t m_\pi \sim 0.22 - 0.25$**

The Regge

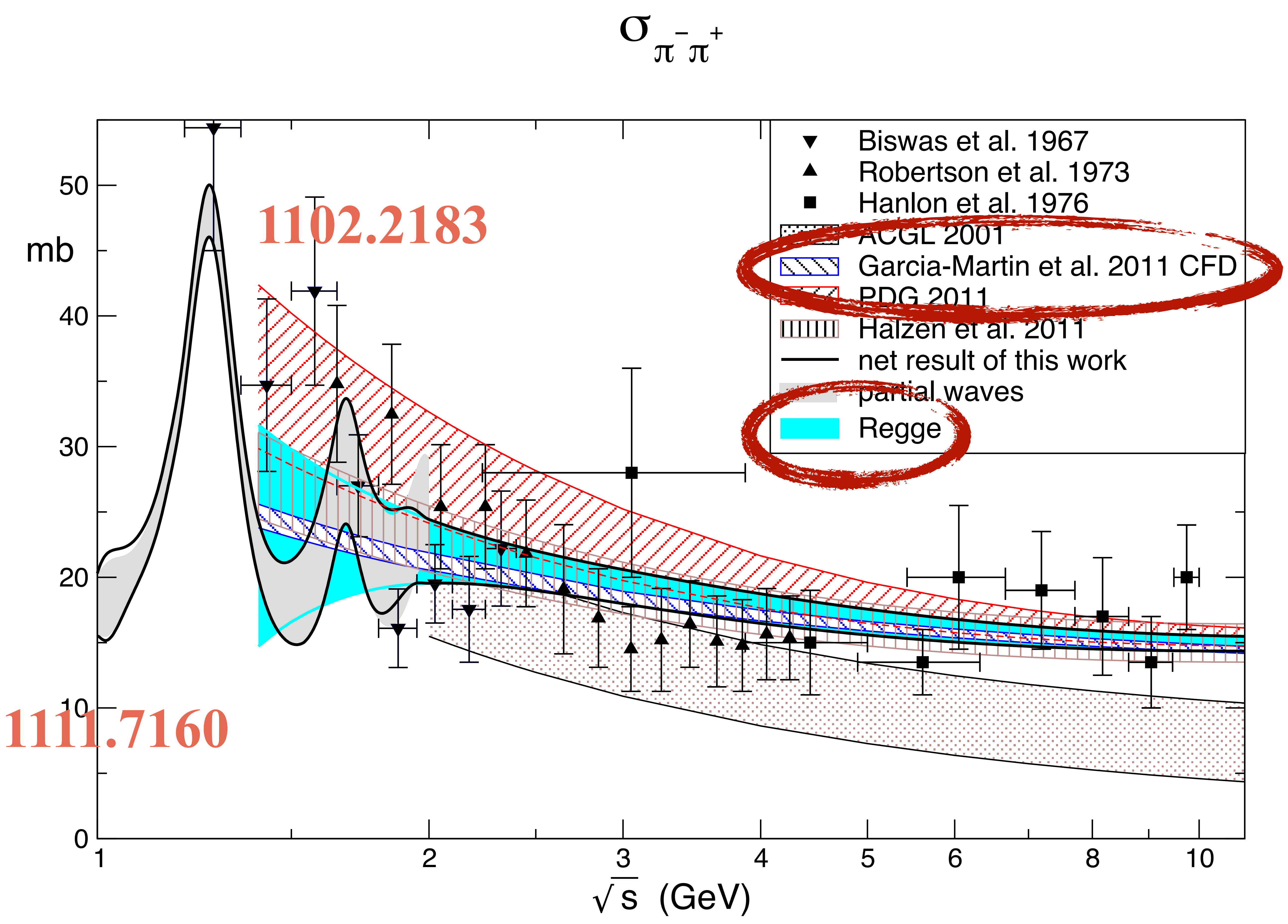
Regge must be extrapolated from phys. m_π

$\mathbb{P} \rightarrow$ gluon exchanges $\rightarrow$ constant over m_q

$\rho, f_2 \rightarrow$ resonances, not constant $\rightarrow \lambda \sim \Gamma/M$

Our $F_{Regge} = \frac{F_{Regge1} + F_{Regge2}}{2}$

Big uncertainty $\Delta F_{Regge} = 0.3\, F_{Regge}$



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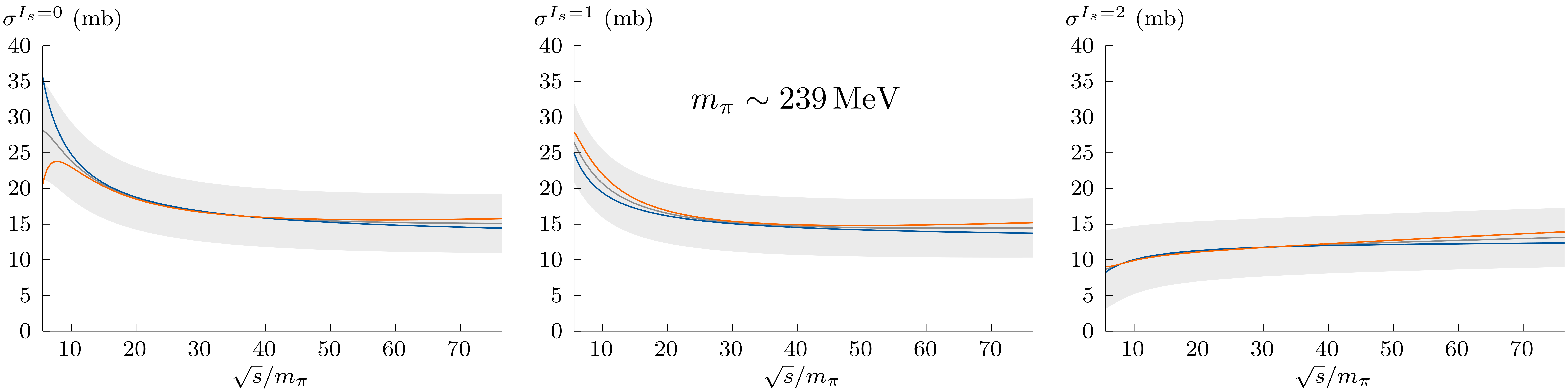


Our $F_{Regge} = \dfrac{F_{Regge1} + F_{Regge2}}{2}$

Big uncertainty $\Delta F_{Regge} = 0.3\, F_{Regge}$

Pions on the lattice

Connected diagrams

Actual lattices ($32^3 \times 256$)

$$\left[D^{-1}[U]\right]_{00,xt}$$

Size

$$4 \times 3 \times 4 \times 3 \times L^3 \times T$$

Around 10 GB per flavor

Disconnected diagrams

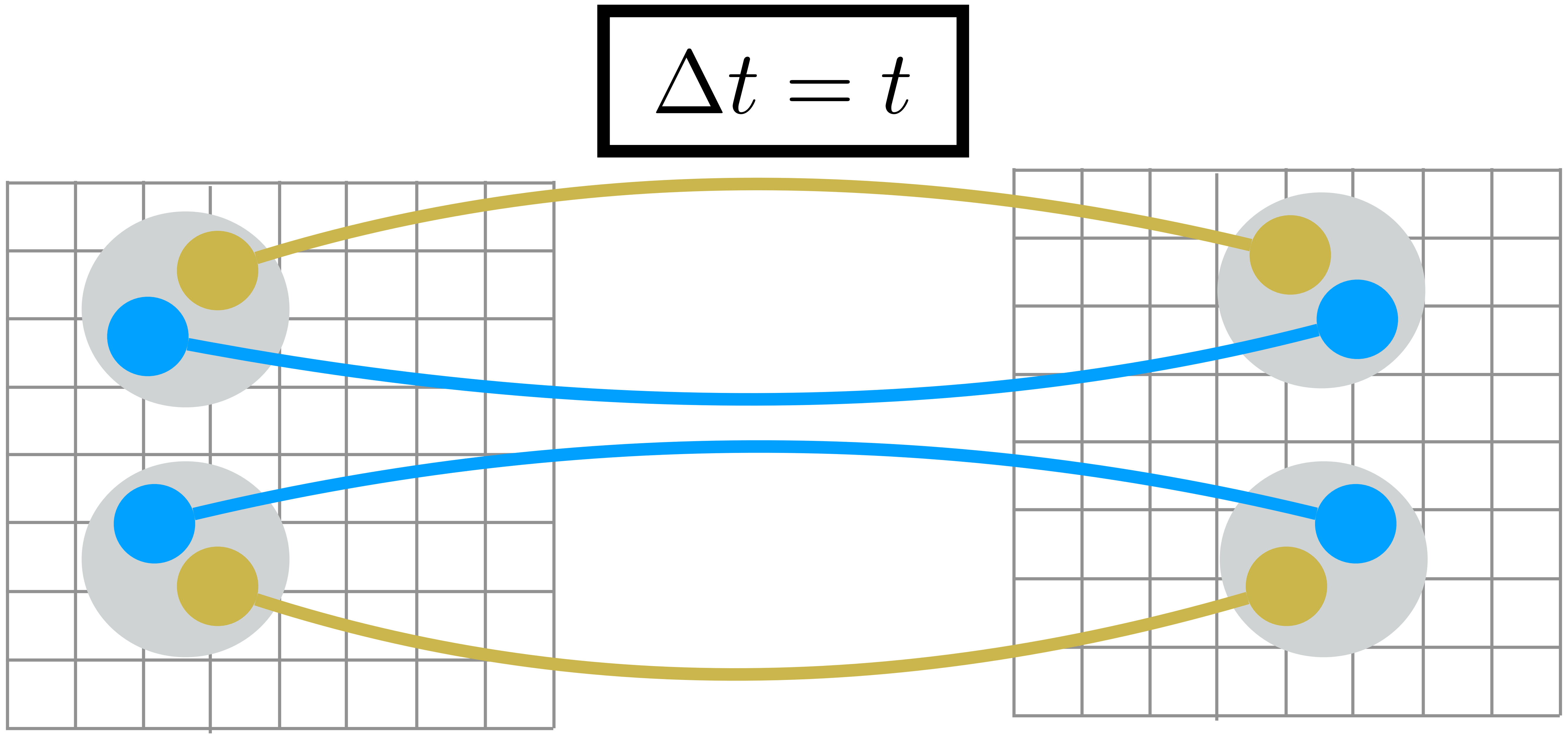
$$\left[D^{-1}[U]\right]_{x_i t_i, x_f t_f}$$

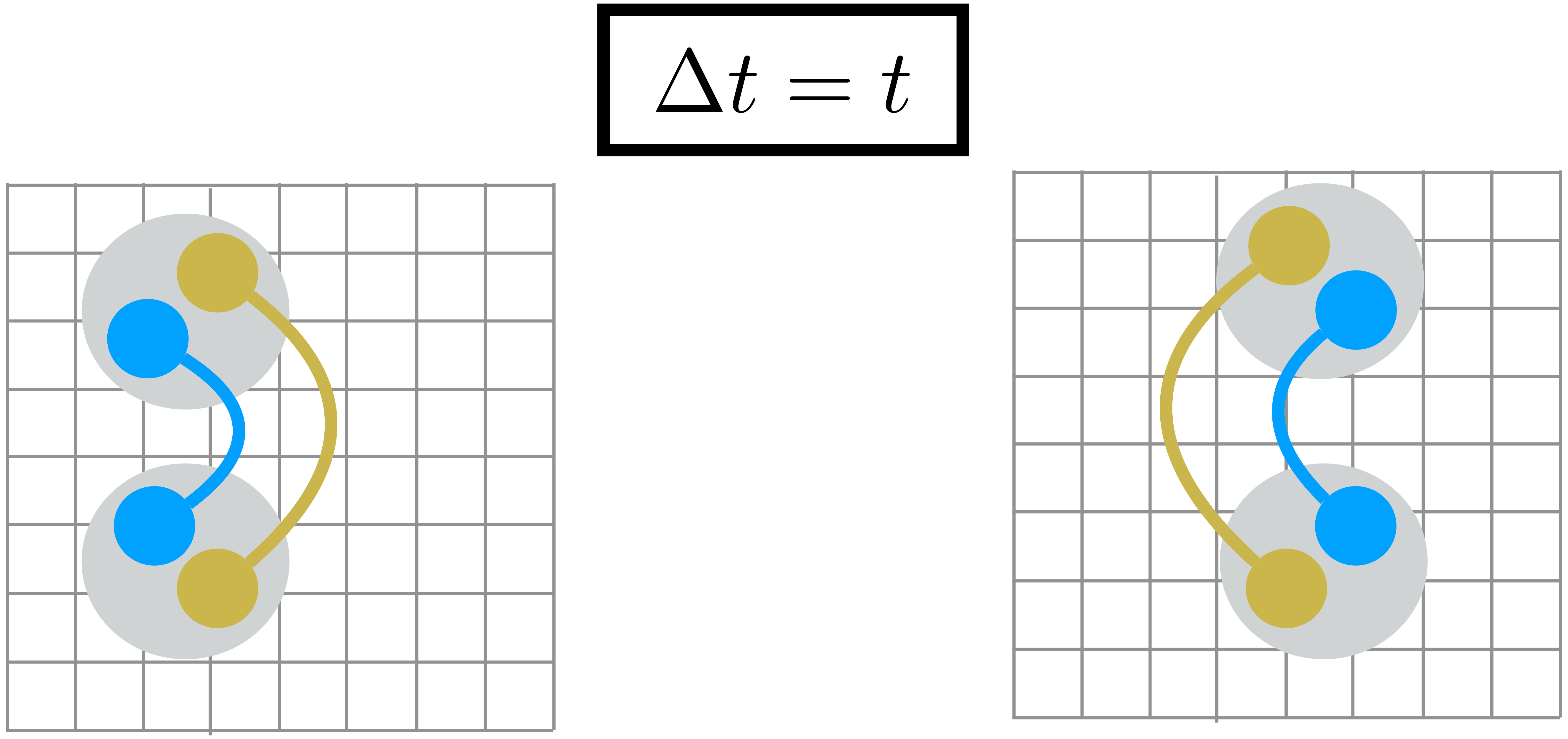
Size

$$(4 \times 3 \times L^3 \times T)^2$$

Around 80 PB per flavor

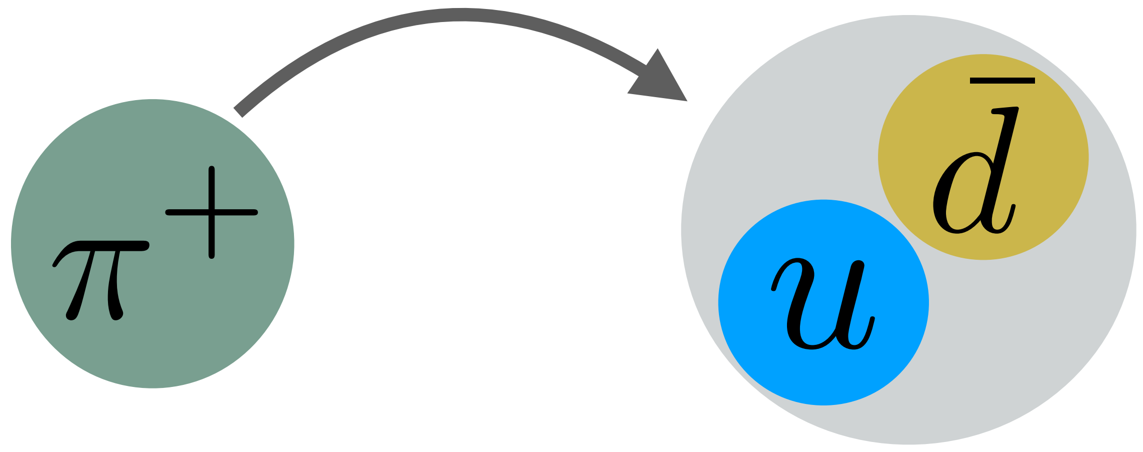
We use clever techniques to "solve" this



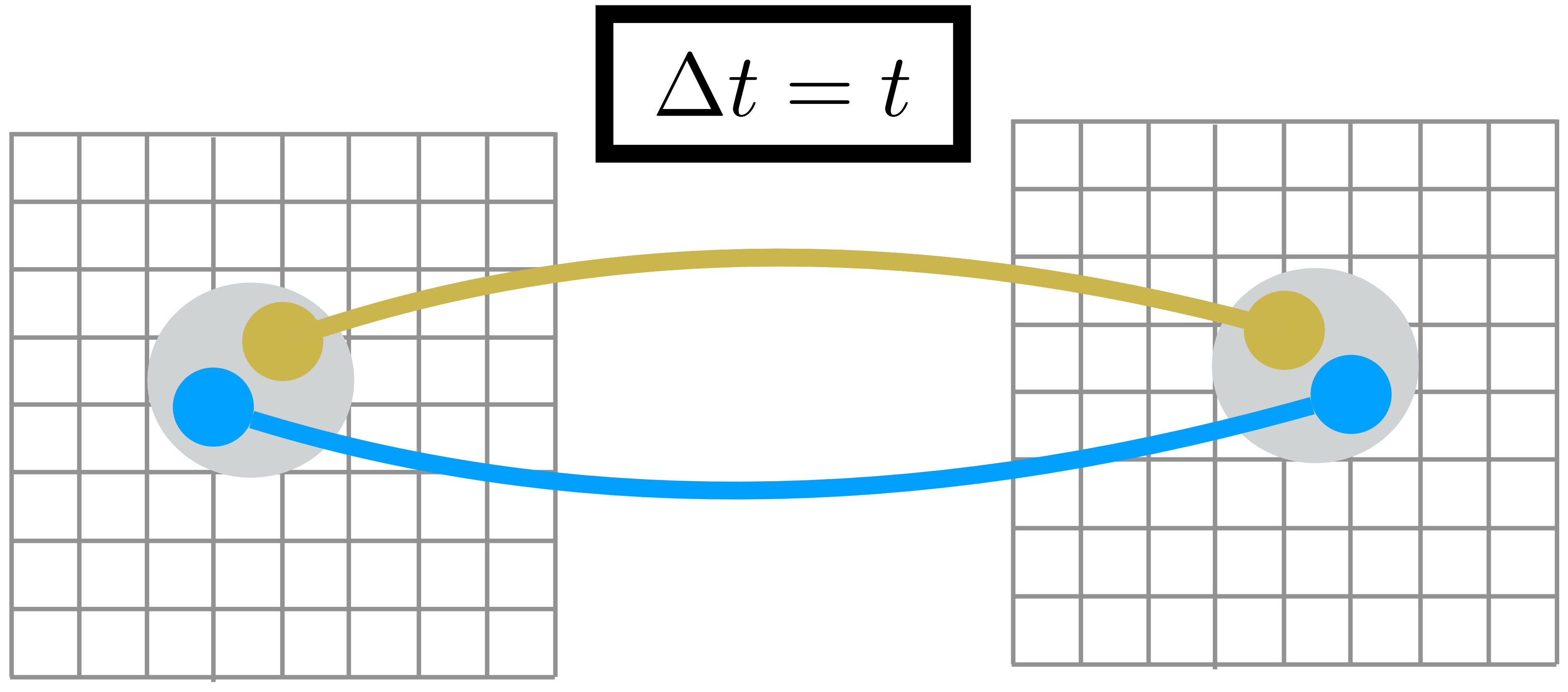


Pions on the lattice

Easiest pion construction in terms of quarks



$$|\pi^+\rangle = |\bar{d}\gamma_5 u\rangle$$



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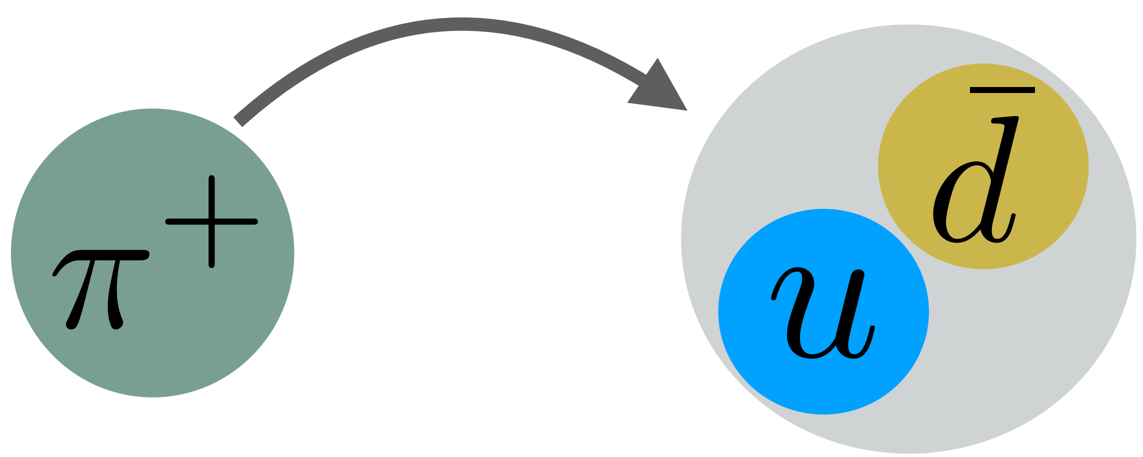

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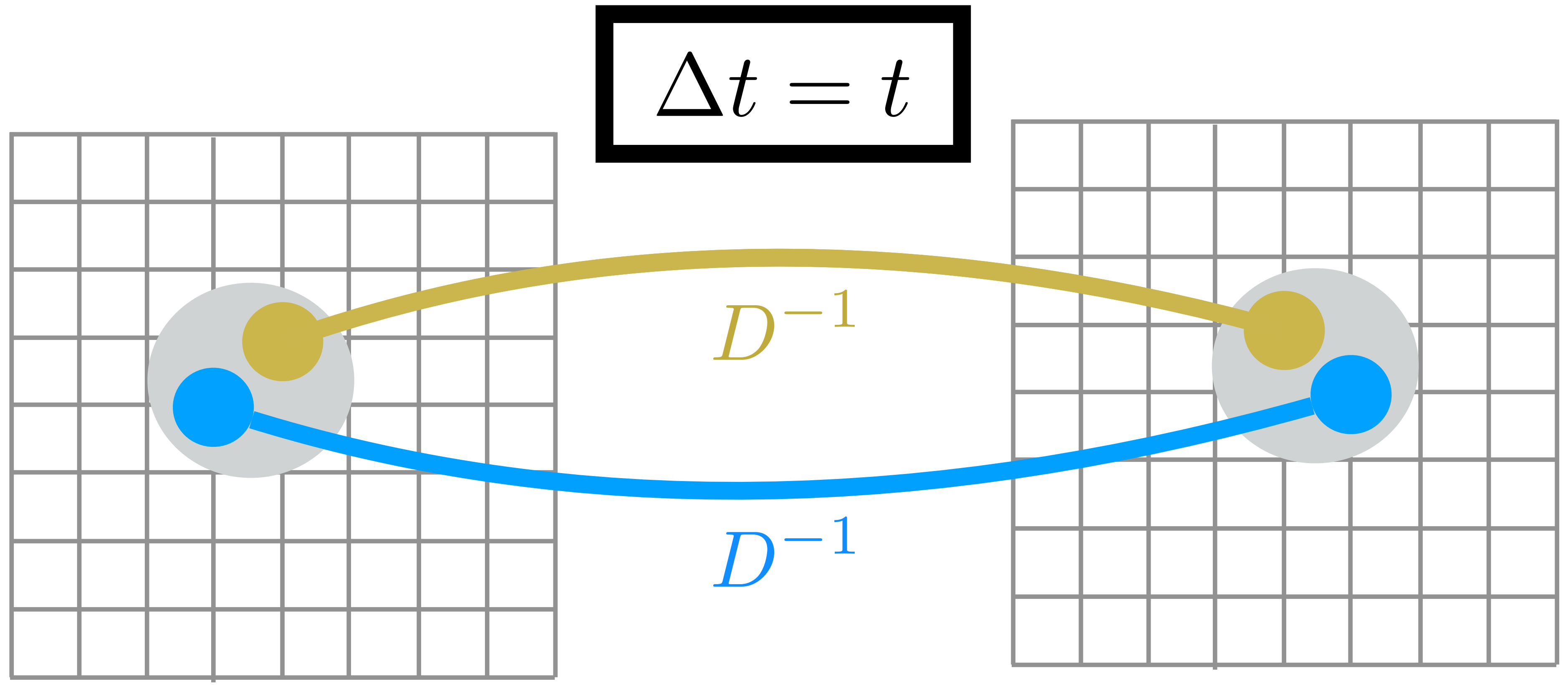
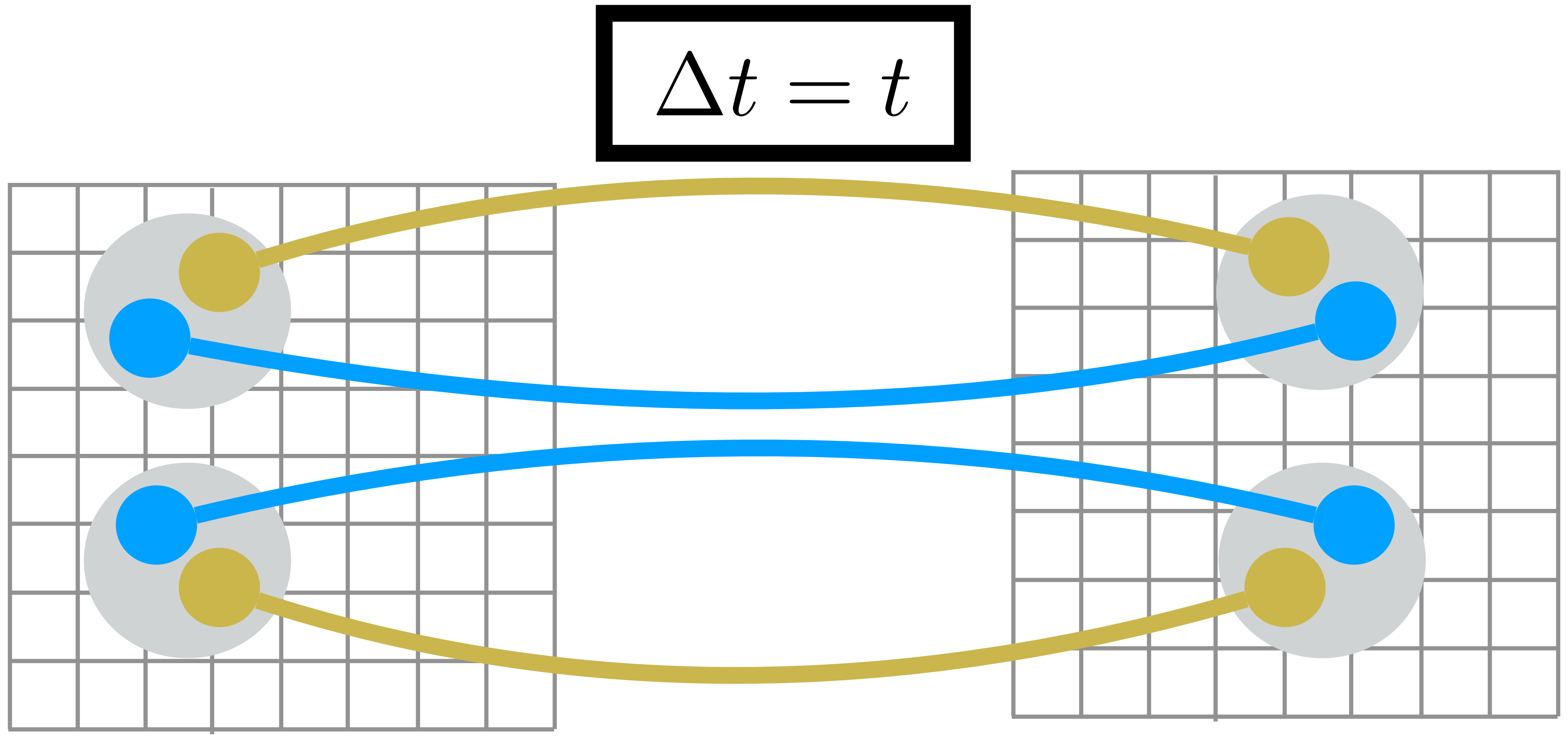


What is the evolution? → contractions

$$\left\langle 0 \middle| \left(\bar{\psi}\gamma_5\psi\right)_{x,t} \left(\bar{\psi}\gamma_5\psi\right)_{0,0} \middle| 0 \right\rangle = -\,\mathrm{tr}\left(\left[D^{-1}[U]\right]_{00,xt}\gamma_5\left[D^{-1}[U]\right]_{xt,00}\gamma_5\right)$$

Point to all propagators

Pions on the lattice



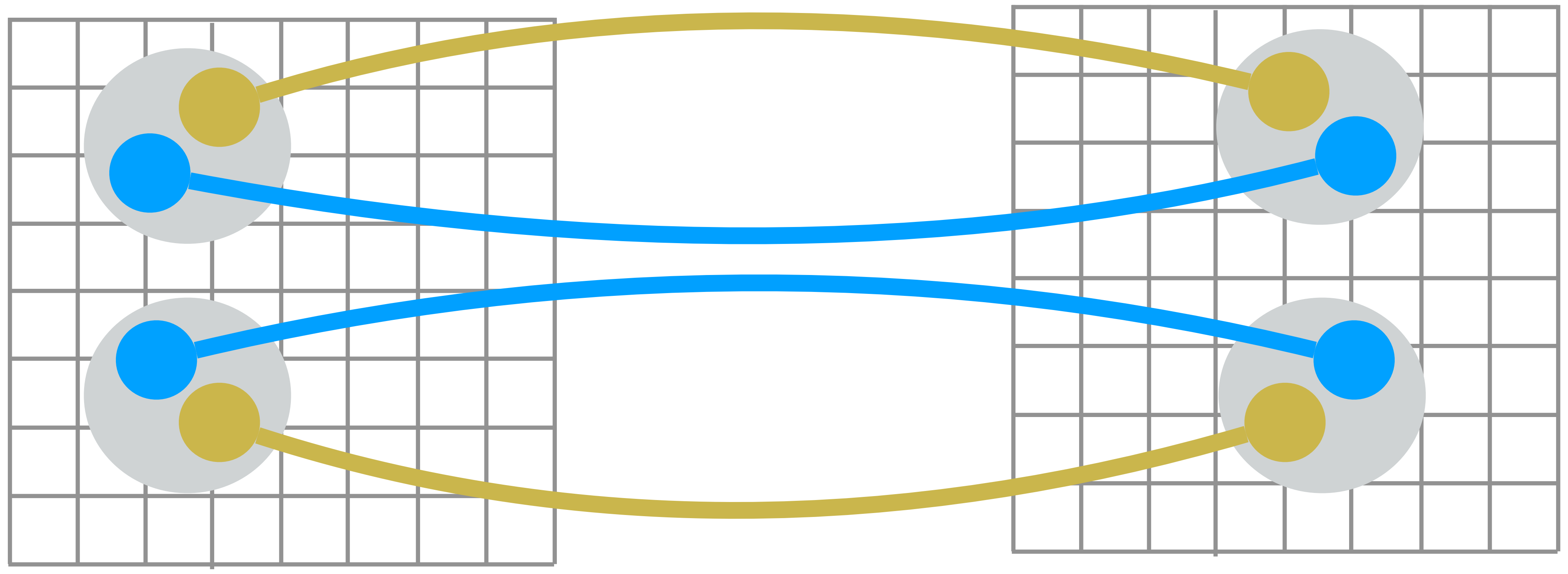
Lets study the temporal evolution of a pair of particles

$$C(t) \equiv \langle 0 | \mathcal{O}(t)\, \mathcal{O}^\dagger(0) | 0 \rangle$$

Basis

$$= \sum_n \langle 0 | \mathcal{O}(t) | n \rangle \langle n | \mathcal{O}^\dagger(0) | 0 \rangle$$

Pions on the lattice



Lets study the temporal evolution of a pair of particles

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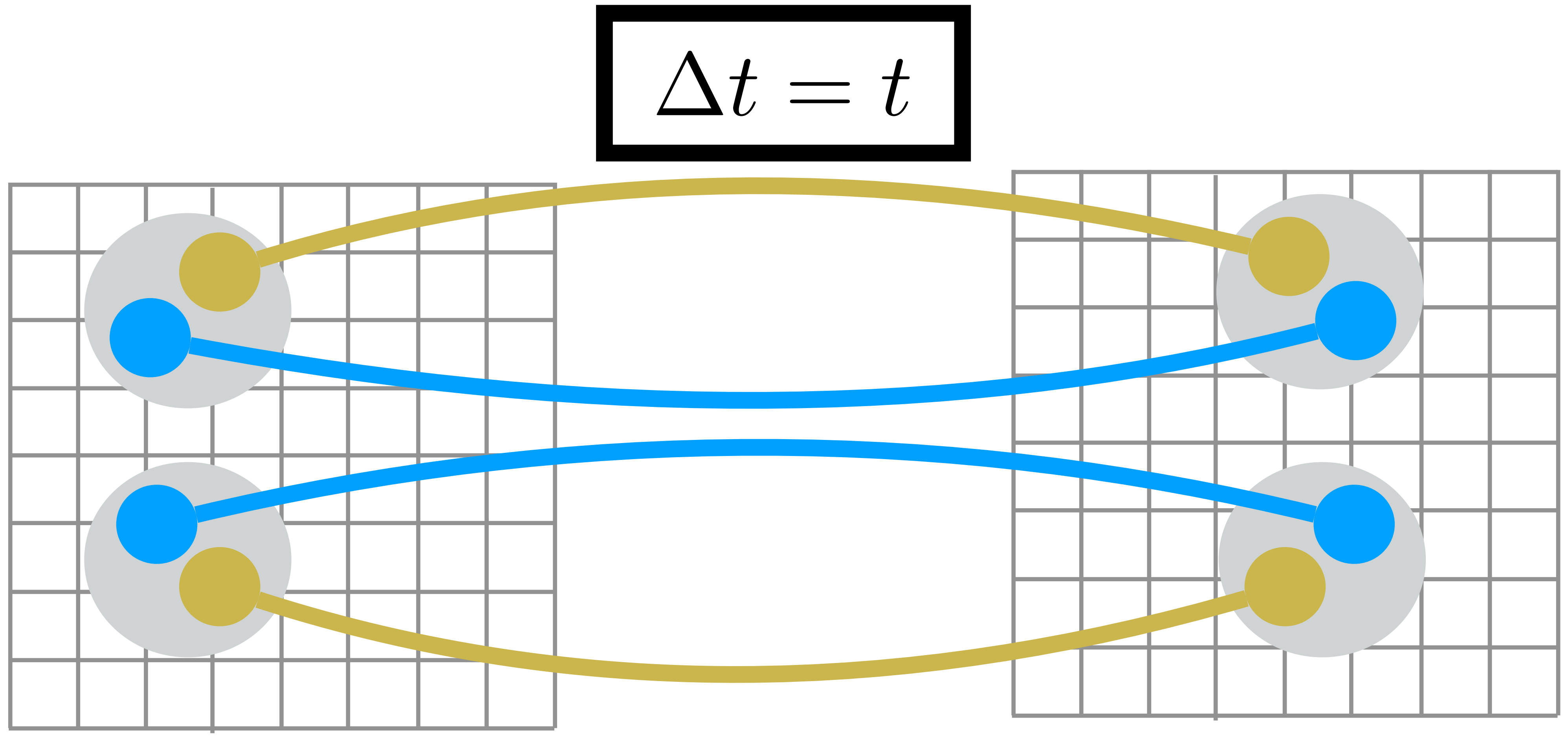
$$e^{-iHt}$$

Pions on the lattice



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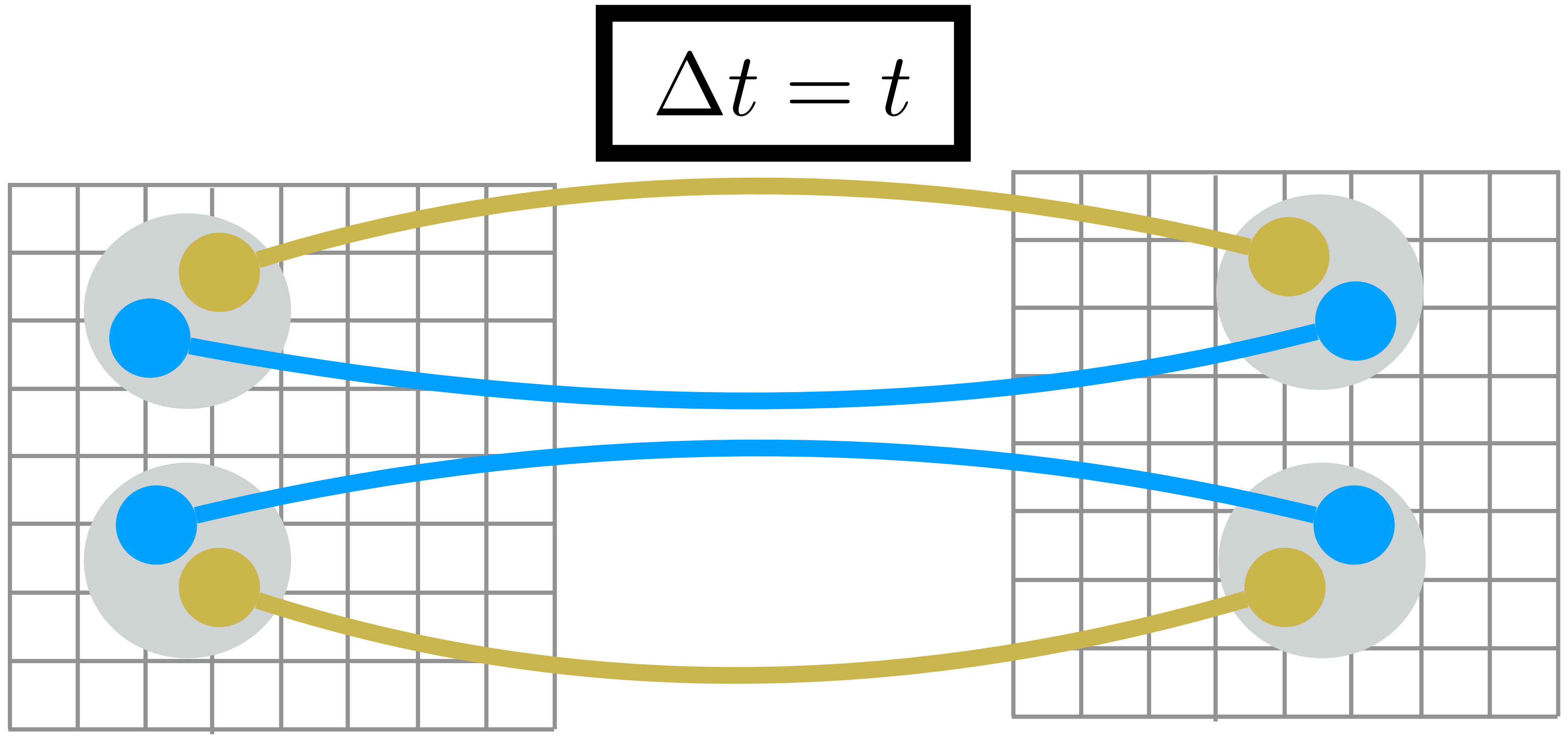
Euclidean time

$$= \sum_n c_n e^{-E_n t}$$

Pions on the lattice



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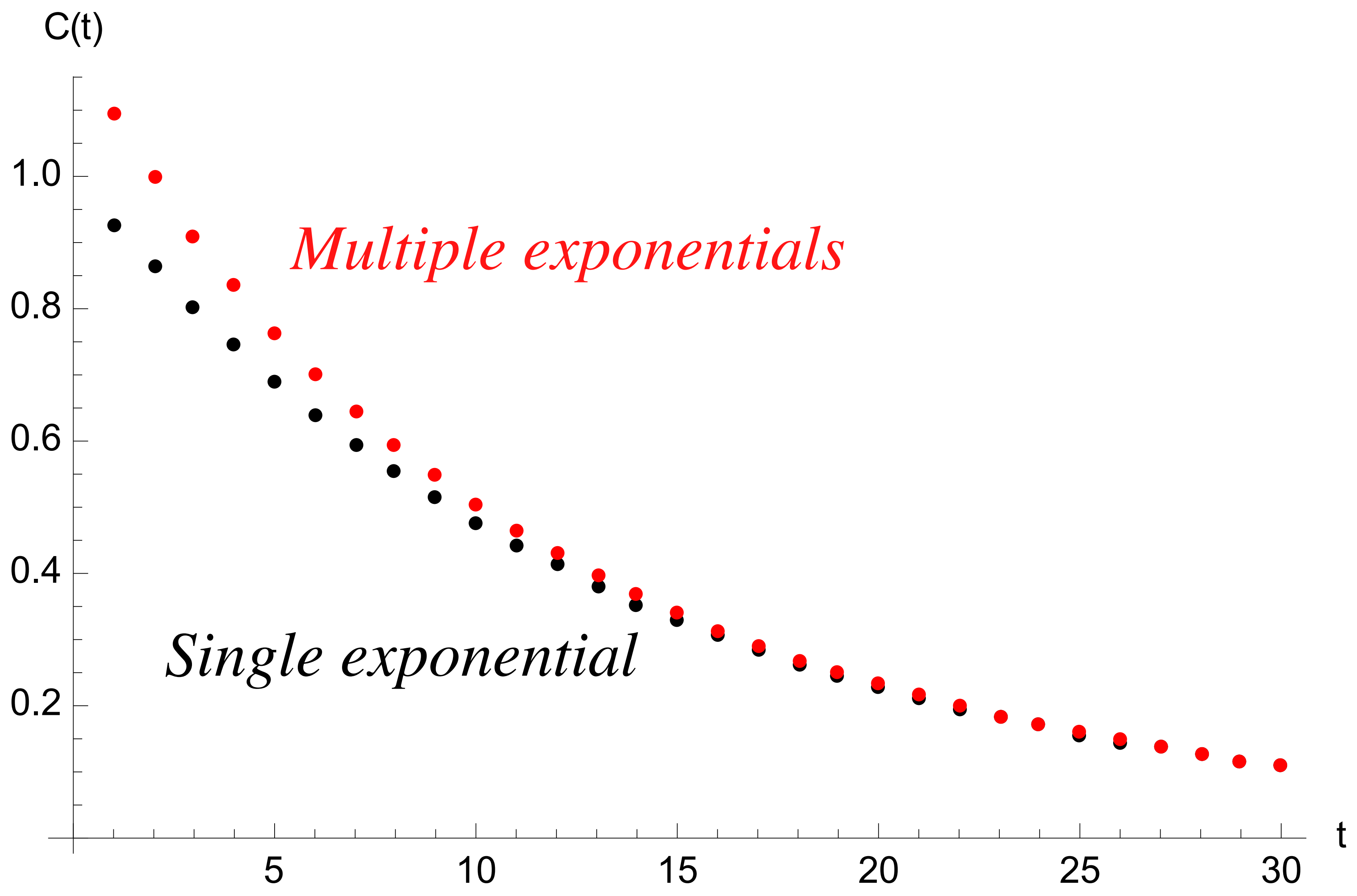

We determine these energies from fitting the temporal evolution of the system

$$m_{\text{eff}} = log\left[\frac{C(t)}{C(t+1)}\right]$$