Thresholds effects on quarkonium spectrum in an EFT formalism

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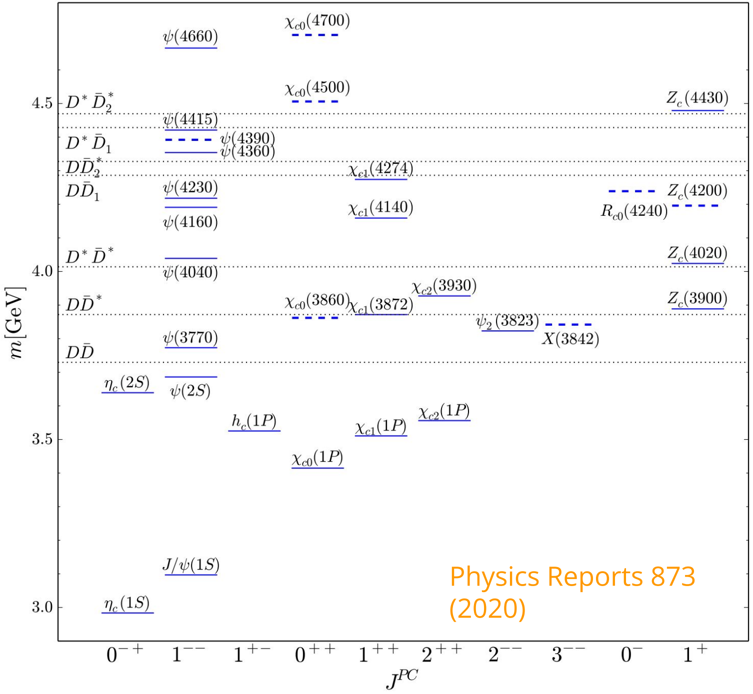
Outline

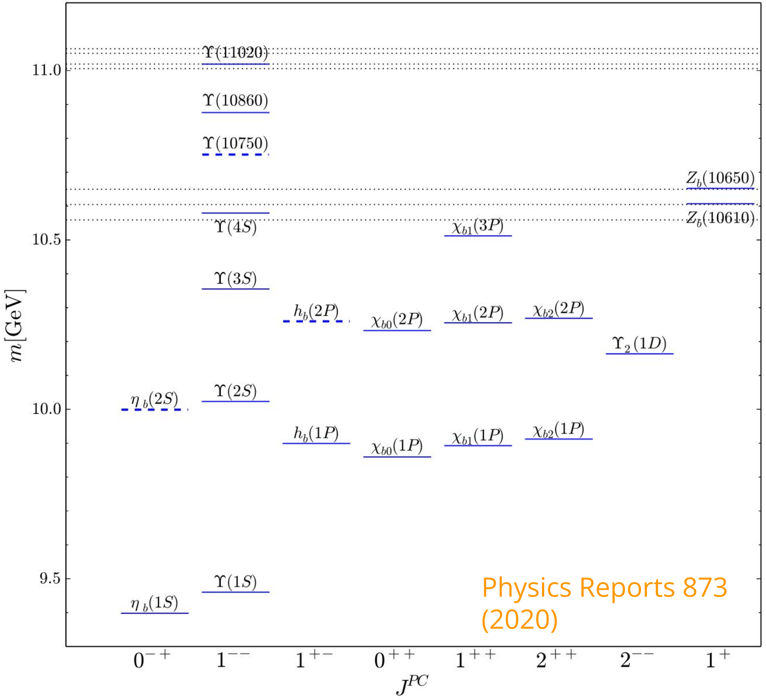
- Experimental Picture
- The BOEFT approach from a lattice perspective
- BOEFT Lagrangian and spectra alignment
- Threshold corrections: coupled system approach
- Threshold corrections: self-energy approach
- Comparison among the methods
- Conclusions

Experimental Picture

Spectrum: charmonium sector



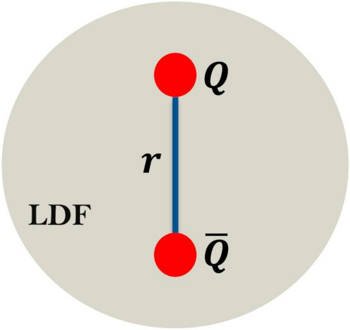
Spectrum: bottomonium sector



How do the threshold states mix with the quarkonium ones and modify its spectrum?

The BOEFT approach from a lattice perspective

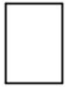
A Pictorial Representation

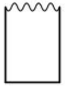


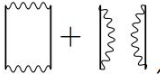
Quarkonium

Σ_g^+

Wilson Loops

$W_{\Psi\Psi}(\boldsymbol{r}) \equiv$ 

$W_{\Psi M_{(s)}}(\boldsymbol{r}) \equiv \sqrt{n_f}$ 

$W_{M_{(s)}M_{(s)}}(\boldsymbol{r}) \equiv -n_f$ 

Meson-Antimeson threshold

S-wave S-wave threshold

S-wave P-wave threshold

$K_{\bar q}^P \otimes K_q^P$	K^{PC}	Static energies $D_{\infty h}$
$(1/2)^- \otimes (1/2)^+$	0^{-+}	$\{\Sigma_u^-\}$
	1^{--}	$\{\Sigma_g^+, \Pi_g\}$
$(1/2)^- \otimes (1/2)^-$	0^{++}	$\{\Sigma_g^+\}$
	1^{+-}	$\{\Sigma_u^-, \Pi_u\}$
$(1/2)^- \otimes (3/2)^-$	1^{+-}	$\{\Sigma_u^-, \Pi_u\}$
	2^{++}	$\{\Sigma_g^+, \Pi_g, \Delta_g\}$

For two systems to interact they need to be in the same $D_{\infty h}$ reps.

The cylindrical group $D_{\infty h}$ reps: Λ_η^σ (LDF)

$|\boldsymbol{r} \cdot \boldsymbol{K}_{light}| \equiv \Lambda = 0, 1, 2, \ldots$ corresponding to $\Sigma, \Pi, \Delta, \ldots$

CP eigenvalue: $\eta = +1\ (g), -1\ (u)$

Reflection symm. about a plane containing $Q\bar{Q}$: $\sigma = \pm 1$

Static quarks correlation matrix

$$C(\boldsymbol{r}, t) = \begin{pmatrix} W_{\Psi\Psi}(\boldsymbol{r}) & W_{\Psi M}(\boldsymbol{r}) & W_{\Psi M_s}(\boldsymbol{r}) \\ W_{M\Psi}(\boldsymbol{r}) & W_{MM}(\boldsymbol{r}) & W_{MM_s}(\boldsymbol{r}) \\ W_{M_s\Psi}(\boldsymbol{r}) & W_{M_sM}(\boldsymbol{r}) & W_{M_sM_s}(\boldsymbol{r}) \end{pmatrix}, \quad \boldsymbol{r} = \boldsymbol{r}_1 - \boldsymbol{r}_2$$

The BOEFT approach from a lattice perspective

A parametrization for the static Hamiltonian

from *Phys.Lett.B* 854 (2024)

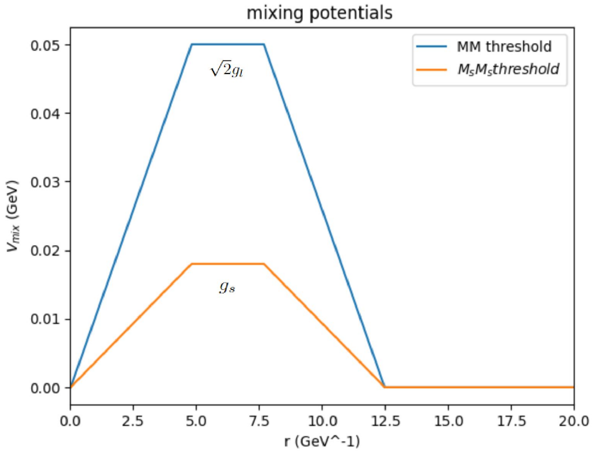
$$H(r,t) = \begin{pmatrix} \hat{V}(r) & \sqrt{2}g_l & g_s \\ \sqrt{2}g_l & E_1 & 0 \\ g_s & 0 & E_2 \end{pmatrix}, \quad \hat{V}(r) = \hat{V}_0 + \frac{\gamma}{r} + \sigma r$$

$$\sigma = 0.199\ GeV^2 \qquad \gamma = -0.434 \qquad \hat{V}_0 = -1.144\ GeV$$

$$\sqrt{2}g_l = 0.050\ GeV \qquad g_s = 0.018\ GeV$$

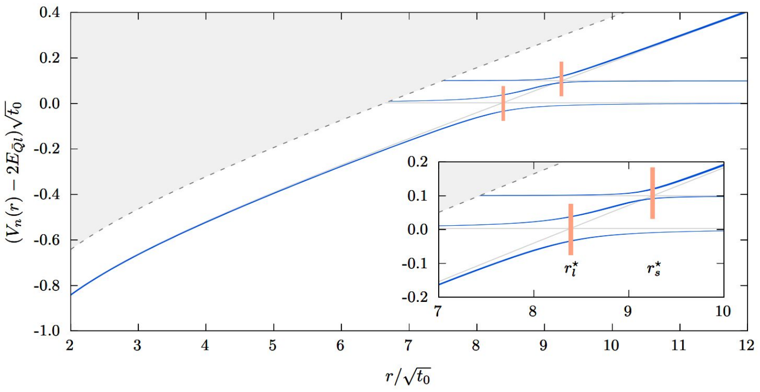
$$E_1 = 0.005\ GeV \qquad E_2 = 0.138\ GeV$$

Our mixing potential beyond the fit region



Adiabatic levels of the static system

from *Phys.Lett.B* 854 (2024)



$$\sqrt{t_0} \sim 0.144\ fm$$

$$m_\pi \sim 200 - 340\ GeV$$

$$m_K \sim 440 - 480\ GeV$$

$$Fit\ region_{exc.\ levels} \sim 0.9 - 1.5\ fm$$

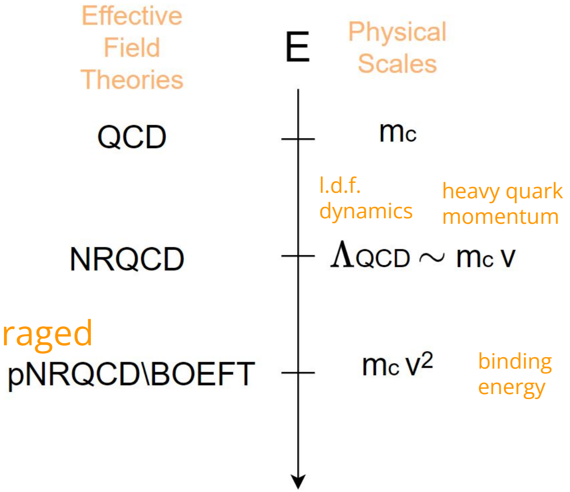
$$String\ breaking\ region \sim 1.2 - 1.3\ fm$$

BOEFT Lagrangian and spectra alignment

The Lagrangian (leading order)

$$\mathcal{L}_{BOEFT} = \int d^3\boldsymbol{r} \left\{ \Psi^\dagger(\boldsymbol{r},t) \left(i\partial_t + \frac{\nabla_r^2}{m_Q} - V_\Psi \right) \Psi(\boldsymbol{r},t) - \sum_{M,M_s} V_{\Psi M_\Sigma} \left(M^\dagger_{\Sigma_g^+}(\boldsymbol{r},t)\, \Psi(\boldsymbol{r},t) + h.c. \right) + \sum_{M,M_s} M^\dagger_{\Sigma_g^+}(\boldsymbol{r},t) \left(i\partial_t + \boldsymbol{P}^\dagger_{\Sigma_g^+} \frac{\nabla_r^2}{m_Q} \boldsymbol{P}_{\Sigma_g^+} - V_{M_{\Sigma_g^+}} \right) M_{\Sigma_g^+}(\boldsymbol{r},t) \right\}$$

Spin independent lagrangian: quarkonium and threshold states are spin averaged



Fixing the spectrum and the thresholds

Compatible with the value of Phys. Rev. D 98

$$\left\langle (\Upsilon)_{1s} \middle| \left(-\frac{\nabla_r^2}{m_b} + V_\Psi \right) \middle| (\Upsilon)_{1s} \right\rangle \stackrel{!}{=} m^b_{1s} - 2m_B^{spin\ avg.} + \frac{a_1}{m_b} + \mathcal{O}(v^4) \Longrightarrow a_1 = 0.025\ GeV^2$$

Mass relations

All the physical values are spin averaged

$$\left\langle (\Upsilon)_{1s} \middle| \left(-\frac{\nabla_r^2}{m_b} + \frac{\gamma}{r} + \sigma r + 2m_b \right) \middle| (\Upsilon)_{1s} \right\rangle = m^b_{1s} + \mathcal{O}(v^4)$$

Natural value of the mass: $m_b = m_B$

$$m_B^{spin\ avg} = m_B^{static} + \frac{a_1}{m_b} + \mathcal{O}\left(\frac{1}{m_b^2}\right)$$

BOEFT Lagrangian and spectra alignment

Charmonium spectrum

$m_c = m_D$

nL	charmonium $E_{PDG}^{spin\ avg.}(MeV)$	$E_{th}(MeV)$
$1s$	3068	3120
$2s$	3674	3704
$3s$		4134
$4s$		4501
$5s$		4831
$1p$	3525	3531
$2p$		3982
$3p$		4362
$4p$		4700
$5p$		5011
$1d$		3817
$2d$		4212
$3d$		4561
$4d$		4880
$5d$		5177

Threshold values

Thr.	$E_{PDG}^{spin\ avg.}(GeV)$	$E_{M\bar{M}}^{th.}$(GeV)
$D\bar{D}$	3.946	3.938
$D_s\bar{D}_s$	4.152	4.070
$B\bar{B}$	10.628	10.628
$B_s\bar{B}_s$	10.806	10.760

Experimental values taken from R.L. Workman *et al.* (Particle Data Group)

Bottomonium spectrum

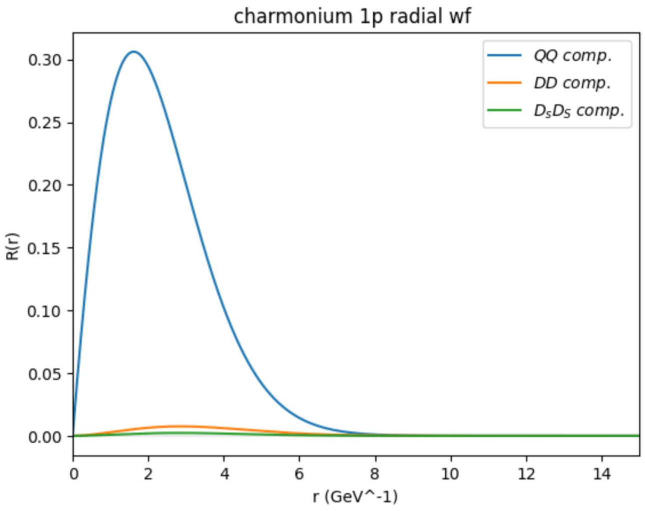
$m_b = m_B$

nL	bottomonium $E_{PDG}^{spin\ avg.}(MeV)$	$E_{th}(MeV)$
$1s$	9445	9445
$2s$	10017	9990
$3s$		10335
$4s$		10616
$5s$		10865
$1p$	9900	9884
$2p$	10260	10240
$3p$		10529
$4p$		10782
$5p$		11012
$1d$		10127
$2d$		10427
$3d$		10687
$4d$		10923
$5d$		11141

Threshold corrections: coupled system approach

Bound state (below threshold) e. o. m.

$$\begin{pmatrix} -\frac{\nabla^2}{m_Q}+V_\Psi & V_{\Psi M_\Sigma} & V_{\Psi M_{s\Sigma}} \\ V_{\Psi M_\Sigma} & \vec{P}_\Sigma^*(-\frac{\nabla^2}{m_Q})\vec{P}_\Sigma + V_M & 0 \\ V_{\Psi M_{s\Sigma}} & 0 & \vec{P}_\Sigma^*(-\frac{\nabla^2}{m_Q})\vec{P}_\Sigma + V_{M_s} \end{pmatrix} \begin{pmatrix} \psi_{Q\bar{Q}}(\boldsymbol{r}) \\ \psi_{M\bar{M}}(\boldsymbol{r}) \\ \psi_{M_s\bar{M}_s}(\boldsymbol{r}) \end{pmatrix} = E \begin{pmatrix} \psi_{Q\bar{Q}}(\boldsymbol{r}) \\ \psi_{M\bar{M}}(\boldsymbol{r}) \\ \psi_{M_s\bar{M}_s}(\boldsymbol{r}) \end{pmatrix}$$



Wavefunctions

$$\psi_{Q\bar{Q}}^{nlm_l}(\boldsymbol{r}) = \phi_{nl}(r)Y_{lm_l}(\hat{\boldsymbol{r}})$$

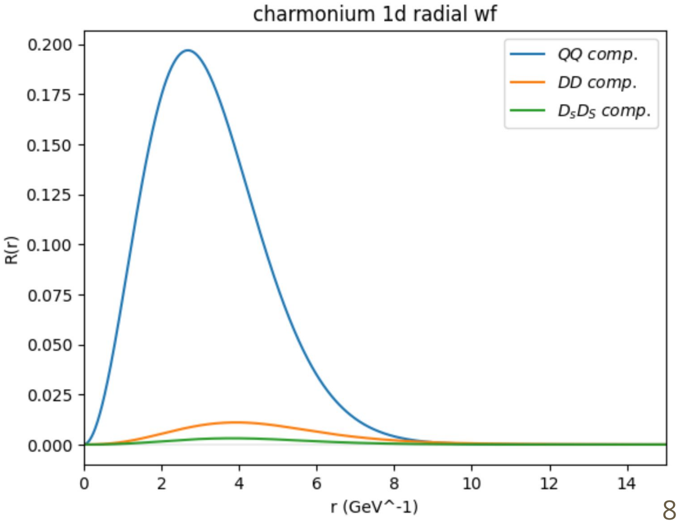
$$\psi_{M\bar{M}}^{nLm_L}(\boldsymbol{r}) = \phi_{nL}(r)Y_{Lm_L}(\hat{\boldsymbol{r}})$$

$$\psi_{M_s\bar{M}_s}^{nL_sm_{L_s}}(\boldsymbol{r}) = \phi_{nL_s}(r)Y_{L_sm_{L_s}}(\hat{\boldsymbol{r}})$$

Selection rules

$$l \overset{!}{=} L \overset{!}{=} L_s$$

$$m_l \overset{!}{=} m_L \overset{!}{=} m_{L_s}$$



Threshold corrections: coupled system approach

Threshold corrections: $\Delta E^{nl}_{M\bar{M}} = E^{nl\ coupl.}_{M\bar{M}} - E^{nl}$

Charmonium

nL	$\Delta E_{D\bar{D}}(MeV)$	$\Delta E_{D_s\bar{D}_s}(MeV)$	$\Delta E_{tot}(MeV)$
$1s$			
$2s$	-3		-4
$1p$	-1		-1
$2p$		-1	
$1d$	-3		-3

- threshold corrections of few MeVs
- bigger corrections for states nearer to the threshold

Bottomonium

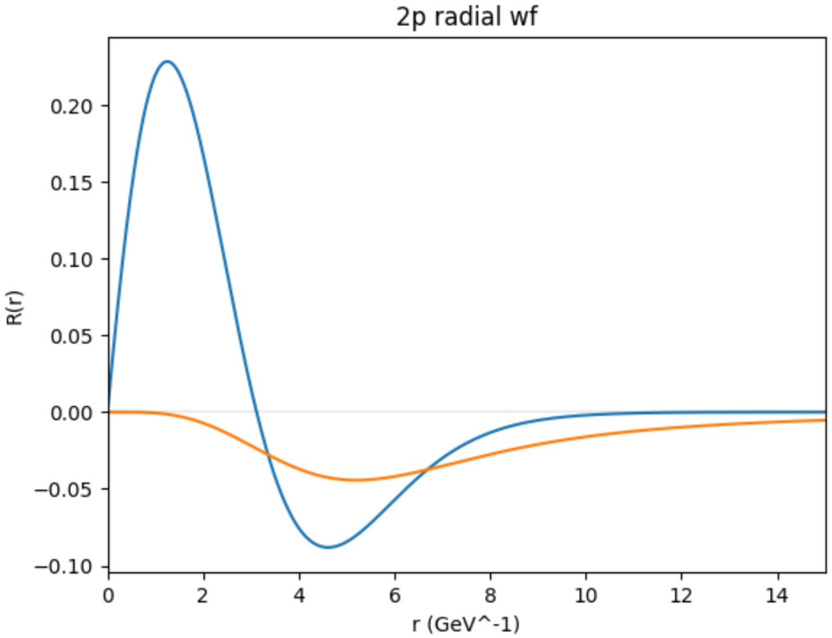
nL	$\Delta E_{B\bar{B}}(MeV)$	$\Delta E_{B_s\bar{B}_s}(MeV)$	$\Delta E_{tot}(MeV)$
$1s$			
$2s$			
$3s$	-3		-3
$4s$	-16		-16
$1p$			
$2p$	-2		-2
$3p$	-7		-7
$1d$	-1		-1
$2d$	-4		-4
$3d$		-1	

Threshold corrections: coupled system approach

A 2p charmonium state just below the DD threshold via mixing coupling fine-tuning?

original lattice coupling $\sqrt{2}g_l = 0.050\ GeV$

fine-tuned coupling $\sqrt{2}g_l = 0.098\ GeV$



Mass

$$D\bar{D}^{spin\ avg.} = 3.938\ GeV$$

$$M_{\chi_c\ 2p}^{fine-tun.} = 3.937\ GeV$$

Composition

$$P_{c\bar{c}} = 61\% \qquad P_{D\bar{D}} = 39\%$$

Radius

$$\left\langle (\chi_c)_{2p} \middle| \left(\frac{1}{r}\right) \middle| (\chi_c)_{2p} \right\rangle = 1.24\ fm^{-1}$$

Threshold corrections: self-energy approach

Exact charmonium propagator

$$\int dt e^{iEt}\langle 0|T\left\{\boldsymbol{\Psi}(t,\boldsymbol{r})\boldsymbol{\Psi}^\dagger(t,\boldsymbol{r}')\right\}|0\rangle_{exact} = \sum_{nl} i\,\frac{\psi_{nl}(\boldsymbol{r})\psi^*_{nl}(\boldsymbol{r}')}{E-E_{nl}-\Sigma_{nl}(E)}$$

similar methods in *Phys.Rev.D* 106 (2022)

Quarkonium bound states

$$|q\bar{q}\rangle_{nl} = \int d^3\boldsymbol{r}\;\phi_{nl}(r)Y_{lm_l}(\hat{\boldsymbol{r}})\;\boldsymbol{\Psi}^\dagger(\boldsymbol{r})\;|0\rangle$$

Meson-Antimeson states

$$|M\bar{M}\rangle_{\Sigma_g^+} = (-i)^K\int d^3\boldsymbol{r}\sum_{l=|L-K|}^{L+K}\left(4\pi i^{-l}j_l(kr)\right)C^{l0}_{L0K0}Y_{L,m_L}M^\dagger_{\Sigma_g^+}|0\rangle$$

Form factor and selection rules

$$f^{l'}_{nl}(k) = C^{l'0}_{l0K0}\int dr\; r^2\phi_{nl}(r)V_{\Psi M_\Sigma}(r)j_{l'}(kr)$$

$$l \overset{!}{=} L$$

$$m_l \overset{!}{=} m_L$$

Diagrams resummation

Poles shift and decay rates

$$Re\;\Sigma^{M\bar{M}}_{nl}(E) = \frac{2m_Q}{\pi}\int dk\;k^2\,\frac{\left(\sum_{l'=l-1}^{l+1} f^{l'}_{nl}(k)\right)^2}{m_Q(E-V_M)-k^2}$$

$$Im\;\Sigma^{M\bar{M}}_{nl}(E) = \Gamma^{M\bar{M}}_{nl}(E) = 2m_Q\;\sqrt{m_Q\,(E-V_M)}\left(\sum_{l'=l-1}^{l+1} f^{l'}_{nl}(k)\right)^2$$

Threshold corrections: self-energy approach

Threshold corrections: $\Sigma_{nl}(E_{nl})$

nL	$\Delta E_{D\bar{D}}(MeV)$	$\Delta E_{D_s\bar{D}_s}(MeV)$
$1s$	-1	
$2s$	-4	
$1p$	-2	
$2p$		-1
$1d$	-4	

nL	$\Delta E_{B\bar{B}}(MeV)$	$\Delta E_{B_s\bar{B}_s}(MeV)$
$1s$		
$2s$	-2	
$3s$	-5	-1
$4s$	-23	-1
$1p$	-1	
$2p$	-5	-1
$3p$	-14	-1
$1d$	-3	
$2d$	-4	-1
$3d$		-2

Experimental data

$$\Gamma(\psi(3770) \to D\bar{D}) \sim O(20)\ MeV$$

S-wave D-wave quarkonium mixing not considered in our calculations

$$\Gamma(\Upsilon(4s) \to B\bar{B} \sim O(20)\ MeV$$

This state is below the BB spin avg. threshold

$$\Gamma(\Upsilon(10860) \to B\bar{B}_{spin\ avg.}) \sim O(20)\ MeV$$

Experimental values taken from R.L. Workman *et al.* (Particle Data Group)

Decay rates: $\Gamma_{nl}^{M\bar{M}}(E_{nl})$

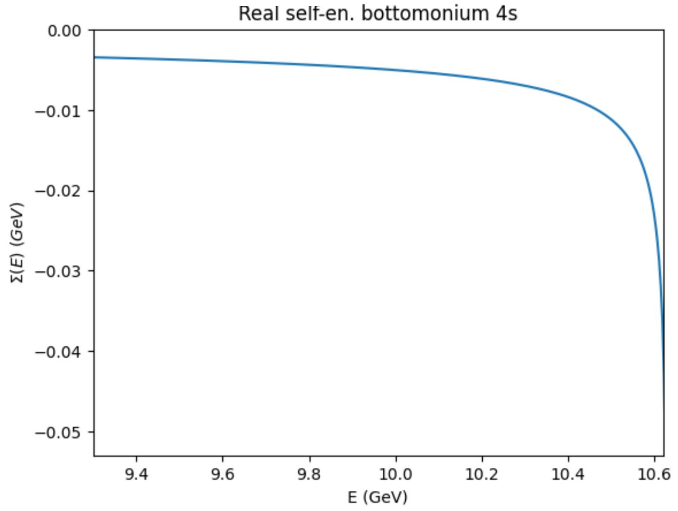
nL	$\Gamma_{D\bar{D}}(MeV)$	$\Gamma_{D_s\bar{D}_s}(MeV)$
$3s$	5	7
$4s$	3	
$5s$	5	
$2p$	8	
$3p$	12	1
$4p$	16	
$5p$	15	2
$2d$	27	
$3d$	24	1
$4d$	18	1
$5d$	15	2

nL	$\Gamma_{B\bar{B}}(MeV)$	$\Gamma_{B_s\bar{B}_s}(MeV)$
$5s$		2
$4p$		1
$5p$	34	
$3d$	1	
$4d$	42	3
$5d$	29	1

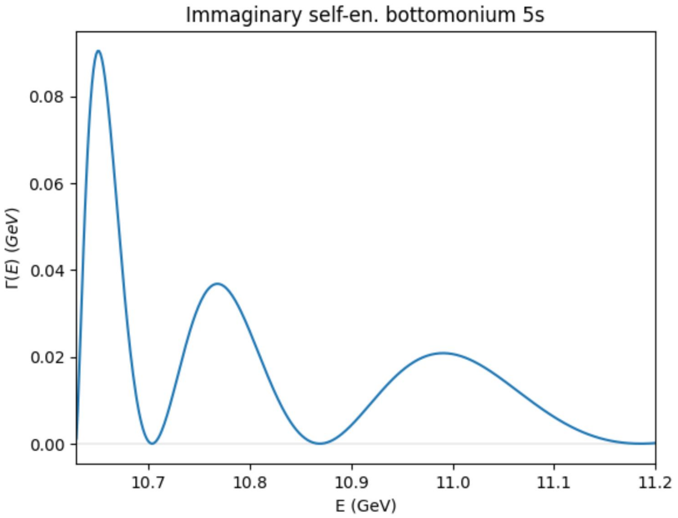
Threshold corrections: self-energy approach

Some illustrative plots

$Re\ \Sigma_{4s}^{B\bar{B}}(E)$



$Im\ \Sigma_{5s}^{B\bar{B}}(E)$



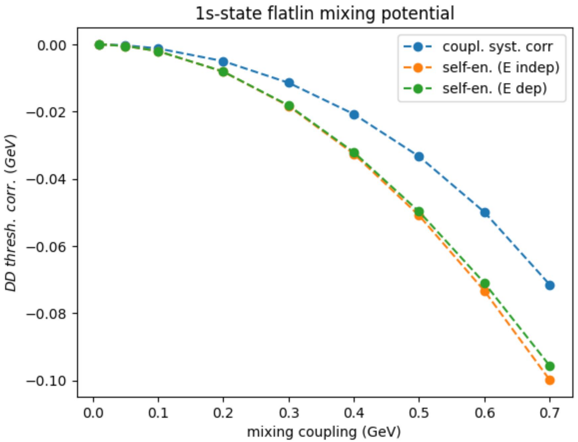
Weak dependence from the energy of the state far from threshold

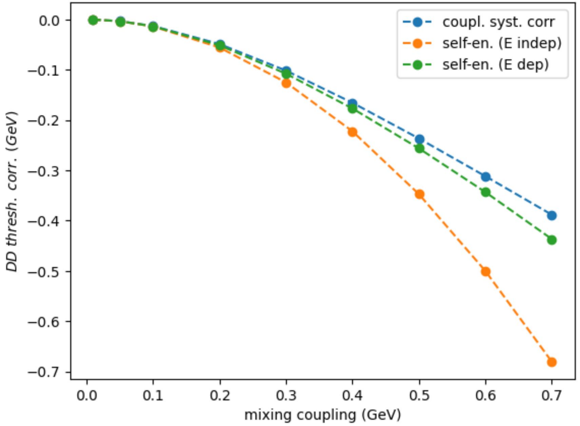
Strong dependence by the energy of the state even far from threshold

Order of magnitude of the decay rates within the uncertainties is compatible with the experimental data

Comparison among the methods

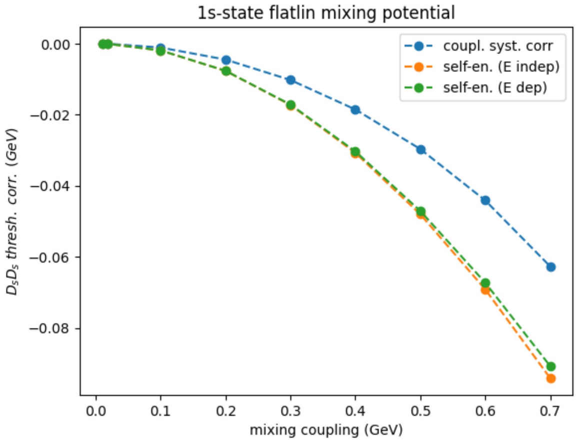
DD threshold

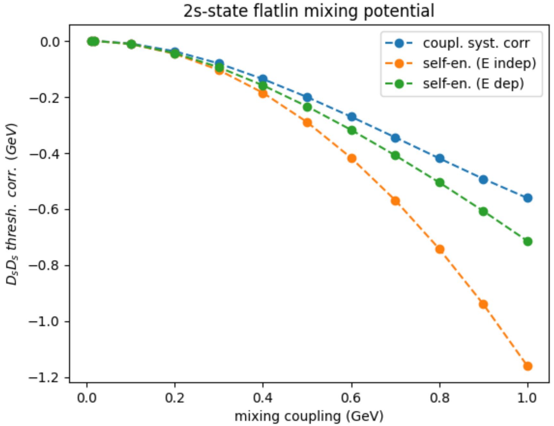




- the different methods agree in the weak-coupling limit
- in the strong-coupling limit a fully non-perturbative treatment (coupled system) is needed

DsDs threshold





Conclusions

- LQCD inputs needed for the BOEFT approach
- Different approaches to study the problem: coupled system & self-energy
- Possibility to fine-tune the mixing coupling to get just below threshold states for coupled system
- States close to the threshold receive bigger threshold corrections w.r.t. further ones
- Same predictions from the different methods (coupled system, self-energy) in the weak-coupling limit
- Decay rates key observable to test the model