Meson-meson scattering at large N_c

Jorge Baeza-Ballesteros

In collaboration with P. Hernández and F. Romero-López
Based on arXiv/2202.02291 and ongoing work

IFIC, University of Valencia-CSIC

Exotic Hadron Spectroscopy 2024 - 4th July 2024

Overview

1. The large N_c limit of QCD
2. Chiral Perturbation Theory
3. Scattering in the lattice
4. $\pi\pi$ scattering at threshold
5. Meson-meson scattering at large N_c
6. Summary and outlook

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The large N_{c} limit of QCD

QCD simplifies at the
large N_{c} or 't Hooft limit
$N_{\mathrm{c}} \to \infty \quad N_{\mathrm{f}} = \text{const}$

- Keeps **non-perturbative features** (asymptotic freedom, confinement...)
- Has **predictive power** in the non-perturbative regime
- Lattice allows to study **subleading N_{c} effects**

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Long-term goal: Understand subleading N_c effects in the lattice:

- Pion mass and decay constant [Hernández et al. 2019]
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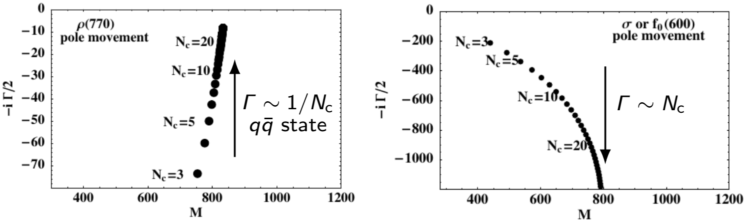
- **Meson-meson scattering** [JBB et al. 2022 and ongoing] ⟶ **This talk**

Large N_c in phenomenology

Large N_c + Unitarized ChPT ⟶ N_c scaling of resonances [Peláez 2004]



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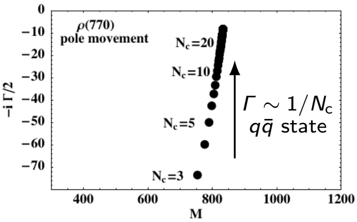

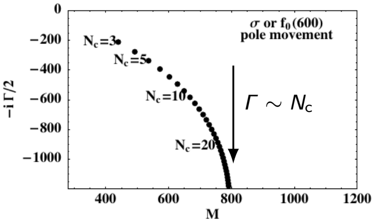

Model dependent + neglects **subleading N_c**

Tetraquarks at large N_c

Recent controversy about the existence of tetraquarks at large N_c

- [Coleman 1985]: Tetraquarks do not exist at large N_c
- [Weinberg 2013]: Tetraquarks can exist at large N_c, with $\Gamma \sim 1/N_c$ (as ordinary resonances)
- [Knetch, Peris 2013]: $\Gamma \sim 1/N_c$ or $\Gamma \sim 1/N_c^2$ depending on the flavor structure
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Lattice QCD can allow us to directly answer this question

Meson-meson scattering at large N_c

This talk: N_c scaling of meson-meson scattering

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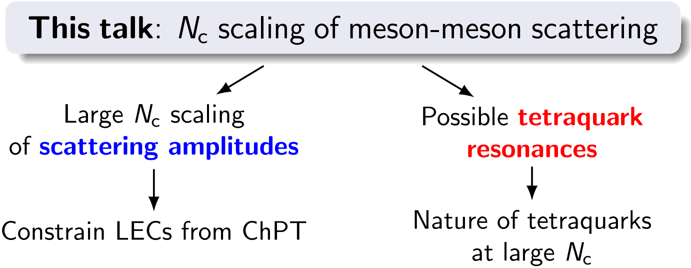
Large N_c scaling
of **scattering amplitudes**

↓

Constrain LECs from ChPT

Meson-meson scattering at large N_c



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Large N_c scaling of **scattering amplitudes**

Constrain LECs from ChPT

Possible **tetraquark resonances**

Nature of tetraquarks at large N_c

Meson-meson scattering at large N_c

$\boldsymbol{N_f = 4}$ $(m_u = m_d = m_s = m_c)$

Used to study $K \to \pi\pi$

[Donini et al. 2020]

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Degenerate ~~mesons~~ pions

$M_\pi = M_K = M_D = M_\eta$

7 scattering channels

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$$15 \otimes 15 = \underset{\pi^+\pi^+}{\overset{\text{even } J}{\mathbf{84\,(SS)}}} \oplus \overset{\text{odd } J}{\mathbf{45\,(SA)}} \oplus \overset{\text{odd } J}{\mathbf{45\,(AS)}} \oplus \underset{D_s^+\pi^+ - D^+K^+}{\overset{\text{even } J}{\mathbf{20\,(AA)}}} \oplus 15 \oplus 15 \oplus 1$$

$$C_{SS} = D - C + (p_1 \leftrightarrow p_2)$$
$$C_{AA} = D + C + (p_1 \leftrightarrow p_2)$$
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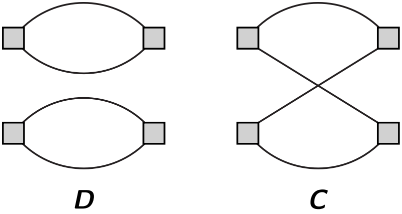

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Large N_c counting

$$\mathcal{M}^{SS,AA} = \mp \frac{1}{N_c}\left(a + b\frac{N_f}{N_c} \pm c\frac{1}{N_c}\right) + \ldots$$

$a, b, c \sim \mathcal{O}(1)$ constants

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Chiral Perturbation Theory (ChPT)

ChPT describes QCD in terms of pseudo-Goldstone bosons (**pions**)

$$\phi = \begin{pmatrix} \pi^0 + \frac{\eta_0}{\sqrt{3}} + \frac{\eta_c}{\sqrt{6}} & \sqrt{2}\pi^+ & \sqrt{2}K^+ & \sqrt{2}D^0 \\ \sqrt{2}\pi^- & -\pi^0 + \frac{\eta_0}{\sqrt{3}} + \frac{\eta_c}{\sqrt{6}} & \sqrt{2}K^0 & \sqrt{2}D^+ \\ \sqrt{2}K^- & \sqrt{2}\bar{K}^0 & -\frac{2\eta_0}{\sqrt{3}} + \frac{\eta_c}{\sqrt{6}} & \sqrt{2}D_s^+ \\ \sqrt{2}\bar{D}^0 & \sqrt{2}D^- & \sqrt{2}D_s^- & -\frac{3\eta_c}{\sqrt{6}} \end{pmatrix} \qquad (N_f = 4)$$

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Most general lagrangian with QCD symmetries

$$U = \mathrm{e}^{i\phi/F}$$

$$\mathcal{L}_2 = \frac{F^2}{4}\mathrm{Tr}[\partial_\mu U \partial^\mu U^\dagger] + \frac{F^2 B_0}{2}\mathrm{Tr}[\chi U^\dagger + \chi^\dagger U] \qquad \text{(2 LECs)}$$

$$F^2 \sim \mathcal{O}(N_\mathrm{c})$$

$$B_0, M_\pi \sim \mathcal{O}(1)$$

$$\mathcal{L}_4 = \sum_{i=0}^{12} L_i O_i \qquad L_i \sim \mathcal{O}(N_\mathrm{c}) \text{ or } \mathcal{O}(1) \qquad \text{(13 LECs)}$$

ChPT at large N_c

At large N_c, the η' needs to be included

$$M_{\eta'}^2 = M_\pi^2 + \frac{2N_f\chi_{top}}{F_\pi^2} \xrightarrow[\text{large } N_c]{F_\pi^2 \sim \mathcal{O}(N_c)} M_\pi^2 + \ldots$$

[Witten-Veneciano]

Large N_c or U(N_f) ChPT [Kaiser, Leutwyler 2000]:

- Include η' in pion matrix

$$\phi|_{U(N_f)} = \phi|_{SU(N_f)} + \eta' \mathbb{1}$$

- Leutwyler counting scheme

$$\mathcal{O}(m_q) \sim \mathcal{O}(M_\pi^2) \sim \mathcal{O}(k^2) \sim \mathcal{O}(N_c^{-1})$$

$\pi\pi$ scattering in ChPT

$\pi\pi$ scattering at LO in ChPT [Weinberg 1979] $k\cot\delta_0 = \frac{1}{a_0} + \ldots$

$$\boxed{M_\pi a_0^{SS} = -\frac{M_\pi^2}{16\pi F_\pi^2}} \propto -\frac{1}{N_c} \qquad \boxed{M_\pi a_0^{AA} = +\frac{M_\pi^2}{16\pi F_\pi^2}} \propto +\frac{1}{N_c}$$

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$\pi\pi$ **scattering at NNLO in large N_c ChPT** [JBB at al. 2022]

$$M_\pi a_0^{SS,AA} = \mp\frac{M_\pi^2}{16\pi F_\pi^2} + f_{SS,AA}(M_\pi, F_\pi, L_{SS,AA}, K_{SS,AA})$$

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Large N_c $\longrightarrow$

$$\begin{aligned} L_{SS} &= N_c L^{(0)} + N_f L_c^{(1)} - L_a^{(1)} + \mathcal{O}(N_c^{-1}) \\ L_{AA} &= \underbrace{N_c L^{(0)} + N_f L_c^{(1)}}_{\text{Same sign}} \underbrace{+ L_a^{(1)}}_{\text{Opposite sign}} + \mathcal{O}(N_c^{-1}) \end{aligned}$$

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Meson-meson scattering in the lattice

Particle scattering cannot be directly studied in the lattice

Scattering

Real-time process

Infinite volume

Asymptotic states

Lattice QCD

Euclidean time

Finite volume

Stationary states

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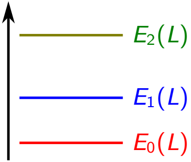

Finite-volume spectrum

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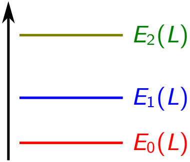
Infinite-volume scattering amplitudes

Quantization condition (QC)



Finite-volume spectrum

Two-particle energy spectrum

Use a **set of operators**, $O_i(t)$, with the correct quantum numbers

$$C_{ij}(t) = \langle O_i(t) O_j(0)^\dagger \rangle \qquad O_i \sim \pi(\boldsymbol{k}_1)\pi(\boldsymbol{k}_2)$$

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Solve **generalized eigenvalue problem**

$$C^{-1/2}(t_0)C(t)C^{-1/2}(t_0)v_n = \lambda_n(t) v_n \longrightarrow \lambda_n(t) \xrightarrow{T \gg t \gg t_0} A_n e^{-E_n t}$$

Fit for **different fit ranges** and extract the energies from **plateaux**

Two-particle quantization condition

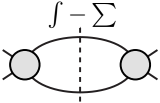
Two-particle QC (matrix equation):

[Lüscher 1986, Rummukainen and Gotlieb 1995, He et al. 2005]:

$$\det[\mathcal{K}_2^{-1} + F(P, L)] = 0$$

$\rho(E)\cot\delta(E) \leftarrow \mathcal{K}_2^{-1}$

$F(P, L) \rightarrow$



$\sim \frac{1}{L^n}$

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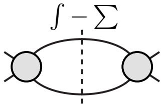

$$\sim \frac{1}{L^n}$$

Reduces to **algebraic equation** assuming lowest partial wave:

Single-channel, *s*-wave ⟶

(Similar for *p*-wave)

$$k\cot\delta_0 = \frac{2}{\gamma L \pi^{1/2}} \mathcal{Z}_{00}^{\boldsymbol{P}}\left(\frac{kL}{2\pi}\right)$$

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Our lattice ensembles

Goal: N_c scaling of $\pi\pi$ scattering and match to ChPT

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Ensembles with $N_f = 4$ dynamical quarks for $N_c = 3 - 6$ generated using **HiRep** [Del Debbio et al., 2010]

Summary of ensembles [Hernández et al., 2019]

$a = 0.075$ fm → $[N_c = 3 - 6] \times [4$ or 5 values of $M_\pi] = 17$ ensembles
$a = 0.065$ fm → $[N_c = 3] \times [2$ values of $M_\pi] = 2$ ensembles
$a = 0.059$ fm → $[N_c = 3] \times [2$ values of $M_\pi] = 2$ ensembles

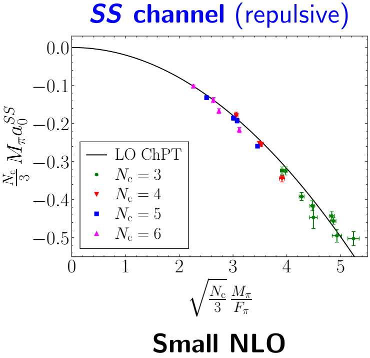
$$M_\pi = 350 - 590\,\mathrm{MeV}$$

↓

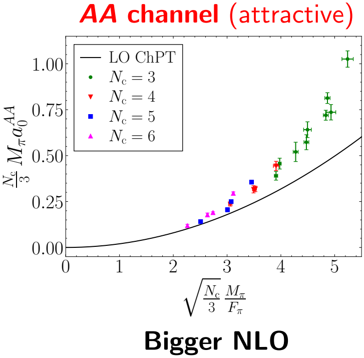
Significant discretization effects in the ***AA* channel**

$\pi\pi$ scattering lengths

We compare scattering lengths to LO ChPT: $M_\pi a_0^{SS,AA} = \mp \frac{M_\pi^2}{16\pi^2 F_\pi^2}$


SS channel (repulsive)

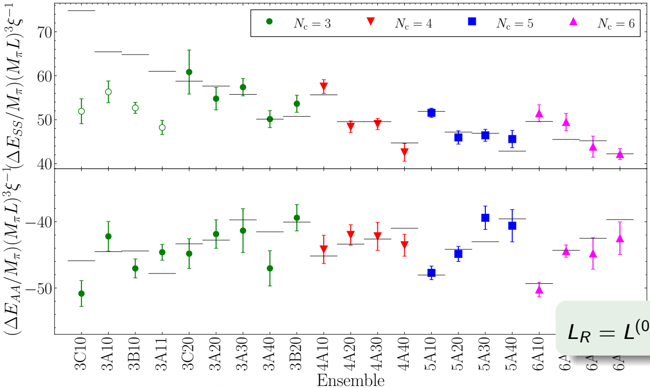
Small NLO


AA channel (attractive)

Bigger NLO

Simultaneous chiral and N_c fit

Simultaneous chiral and N_c fit of both channels to U() ChPT,



***SS* channel**

***AA* channel**

$$L_R = L^{(0)} N_c + L_R^{(1)} + \dots$$

Decreasing $\xi = M_\pi^2/(4\pi F_\pi)^2$

$$\frac{L_{SS,AA}}{N_c} \times 10^3 = -0.02(8) - 0.01(5)\frac{N_f}{N_c} \mp 1.76(20)\frac{1}{N_c}$$

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Recently found **exotic states at LHCb** [LHCb 2020, 2022]:

$$J = 0: \begin{array}{l} T^0_{cs0}(2900) \text{ in } D^+K^- \\ T^{++}_{c\bar{s}0}(2900) \text{ and } T^0_{c\bar{s}0}(2900) \text{ in } D^\pm_s\pi^+ \end{array} \longrightarrow \textbf{\textit{AA} channel}$$

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$$J = 1: \; T^0_{cs1}(2900) \text{ in } D^+K^- \longrightarrow 84 \oplus \mathbf{45}(\boldsymbol{SA}) \oplus \mathbf{45}(\boldsymbol{AS}) \oplus 20 \oplus \ldots$$

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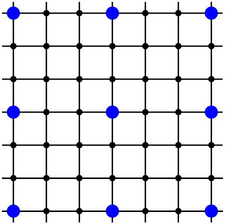
Goal: N_c scaling of meson-meson scattering + tetraquark

Lattice computations

$N_c = 3, 4, 5, 6$ ensembles with $a \sim 0.075$ fm and $M_\pi \sim 590$ MeV

Operator set: $\pi\pi + \rho\rho\,(M_\rho/M_\pi \approx 1.7 - 2)$ + local tetraquark

- Local tetraquark operators ⟶ **Point sources in a sparse lattice $\tilde{\Lambda}$**
  [NPLQCD 2019]



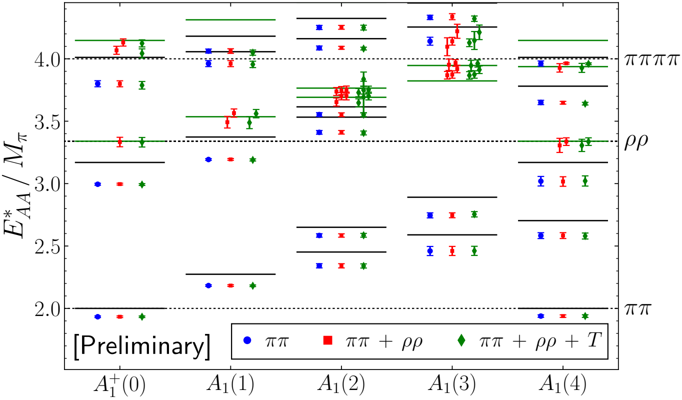
$$T(\boldsymbol{P}) \propto \sum_{\boldsymbol{x}\in\tilde{\Lambda}} e^{-i\boldsymbol{P}\boldsymbol{x}}\, T(\boldsymbol{x})$$

$$T(x) \sim \bar{d}\Gamma_1 u\, \bar{s}\Gamma_2 c - \bar{s}\Gamma_1 u\, \bar{d}\Gamma_2 c$$

Quantum numbers of AA channel

Finite-volume energies: *AA* channel

We study the **effect of different operators** for $N_c = 3$:



└→ Cubic-group irreps

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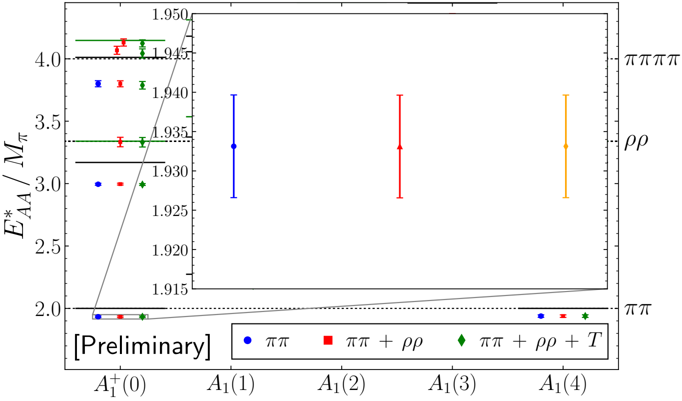

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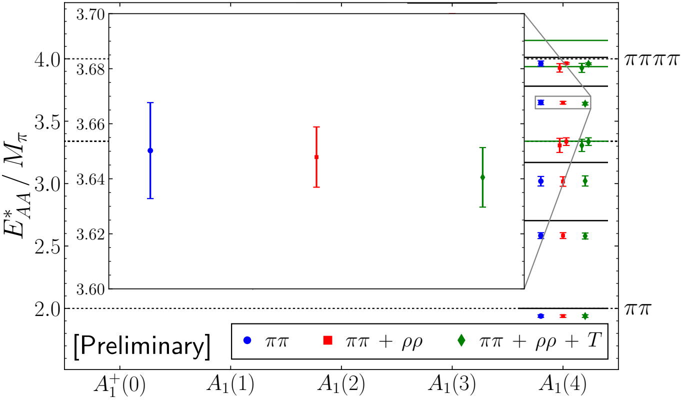
Finite-volume energies: *AA* channel

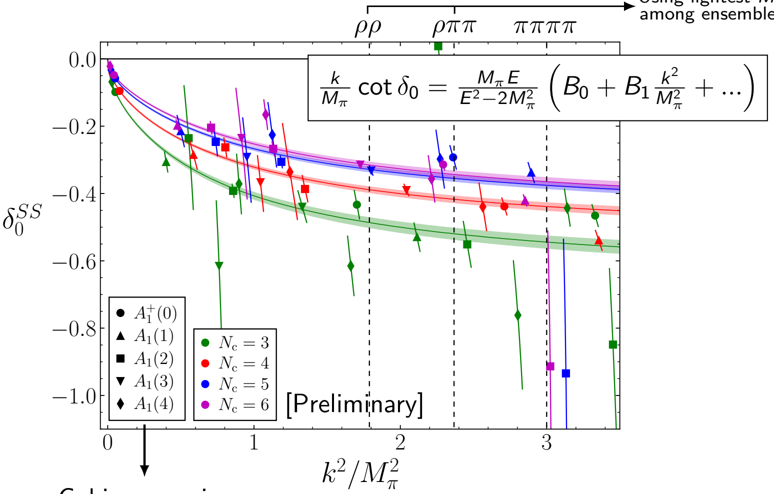
We study the **effect of different operators** for $N_c = 3$:



└→ Cubic-group irreps

Scattering phase shift: *SS* channel

We fit the $\pi\pi$ states to a modified threshold expansion



Using lightest M_ρ among ensembles

$$\frac{k}{M_\pi} \cot\delta_0 = \frac{M_\pi E}{E^2 - 2M_\pi^2}\left(B_0 + B_1 \frac{k^2}{M_\pi^2} + \ldots\right)$$

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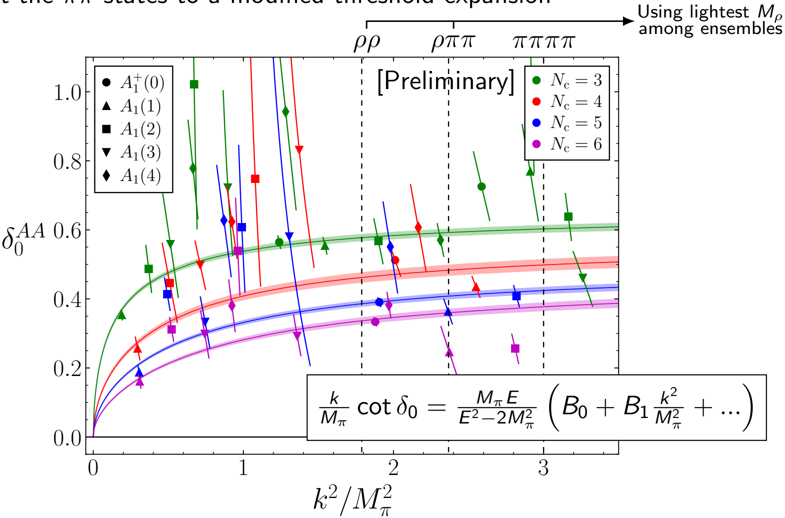

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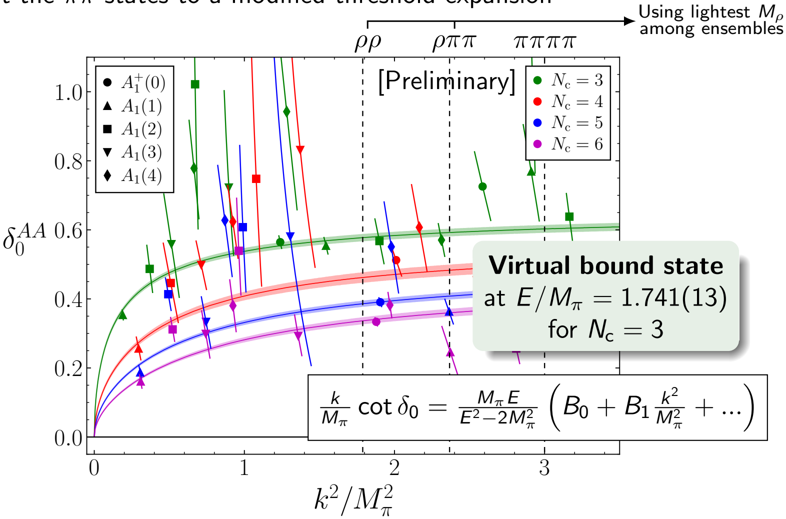

Virtual bound state
at $E/M_\pi = 1.741(13)$
for $N_c = 3$

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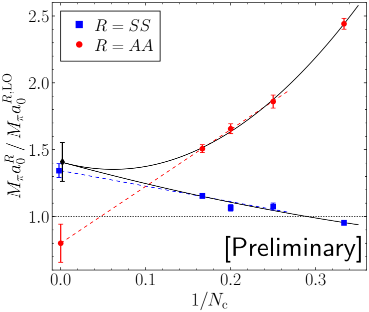
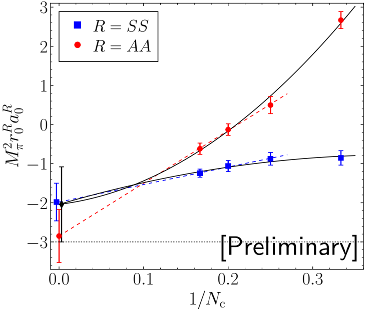
Large N_c scaling of scattering parameters

We study the large N_c scaling of scattering observables

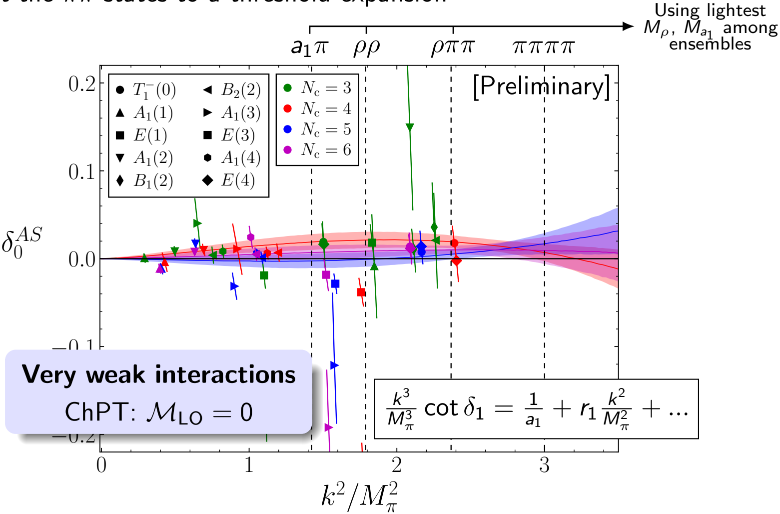




Next step: Constrain LECs from large N_c ChPT

Scattering phase shift: *AS* channel

We fit the $\pi\pi$ states to a threshold expansion



Very weak interactions

ChPT: $\mathcal{M}_{\text{LO}} = 0$

$$\frac{k^3}{M_\pi^3}\cot\delta_1 = \frac{1}{a_1} + r_1\frac{k^2}{M_\pi^2} + \ldots$$

Overview

1. The large N_c limit of QCD
2. Chiral Perturbation Theory
3. Scattering in the lattice
4. $\pi\pi$ scattering at threshold
5. Meson-meson scattering at large N_c
6. Summary and outlook

Summary and outlook

The **large N_c limit** can provide crucial insights on QCD and lattice QCD allows to study **subleading N_c effects**

- We are currently studying the large N_c scaling of scattering observables
- We have successfully studied $\pi\pi$ interactions near threshold and matched to large N_c ChPT, finding **enhanced subleading N_c effects**
- We are able to characterize subleading N_c corrections at higher energies, and find a **virtual bound state** for $N_c = 3$

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Next steps: $\rho\rho$ interactions, constraining LECs

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Thank you for your attention!

$\pi\pi$ scattering in ChPT

$\pi\pi$ scatering amplitudes for N_f flavours are known to NNLO [Weinberg 1979, Gasser, Leutwyler 1985, Bijnens, Lu 2011]

NLO

$$M_\pi a_0^{SS} = -\frac{M_\pi^2}{16\pi F_\pi^2}\left[1 - \frac{16M_\pi^2}{F_\pi^2}L_{SS} + \frac{M_\pi^2}{8F_\pi^2\pi^2 N_\mathrm{f}^2} - \frac{M_\pi^2}{8F_\pi^2\pi^2 N_\mathrm{f}} + \frac{M_\pi^2}{8F_\pi^2\pi^2 N_\mathrm{f}^2}\log\frac{M_\pi^2}{\mu^2} - \frac{M_\pi^2}{8F_\pi^2\pi^2 N_\mathrm{f}}\log\frac{M_\pi^2}{\mu^2} + \frac{M_\pi^2}{8F_\pi^2\pi^2}\log\frac{M_\pi^2}{\mu^2}\right]$$

$$L_R = L^{(0)} N_\mathrm{c} + L_R^{(1)} + \ldots$$

$$M_\pi a_0^{AA} = \frac{M_\pi^2}{16\pi F_\pi^2}\left[1 - \frac{16M_\pi^2}{F_\pi^2}L_{AA} - \frac{M_\pi^2}{8F_\pi^2\pi^2 N_\mathrm{f}^2} - \frac{M_\pi^2}{8F_\pi^2\pi^2 N_\mathrm{f}} - \frac{M_\pi^2}{8F_\pi^2\pi^2 N_\mathrm{f}^2}\log\frac{M_\pi^2}{\mu^2} - \frac{M_\pi^2}{8F_\pi^2\pi^2 N_\mathrm{f}}\log\frac{M_\pi^2}{\mu^2} - \frac{M_\pi^2}{8F_\pi^2\pi^2}\log\frac{M_\pi^2}{\mu^2}\right]$$

Explicit N_f scaling is not the expected at large N_c

Large N_c: $a_0^R \propto \mp\frac{1}{N_\mathrm{c}}\left(\tilde{a} + \tilde{b}\frac{N_\mathrm{f}}{N_\mathrm{c}} \mp \tilde{c}\frac{1}{N_\mathrm{c}}\right) + \mathcal{O}(N_\mathrm{c}^{-3})$

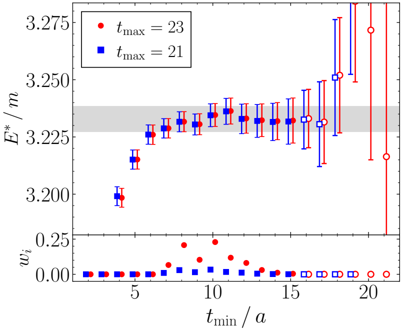
$\tilde{a}, \tilde{b}, \tilde{c} \sim \mathcal{O}(1)$ constants

Two-particle energy spectrum

Average plateaux using **Akaike Information Criterion** [Jay, Neil 2020]

$$w_i \propto \exp\left[-\frac{1}{2}\left(\chi^2 - 2N + 2N_{\text{par}}\right)\right]$$

- Reduces human bias
- Allows to automatically find plateaux for accurate data

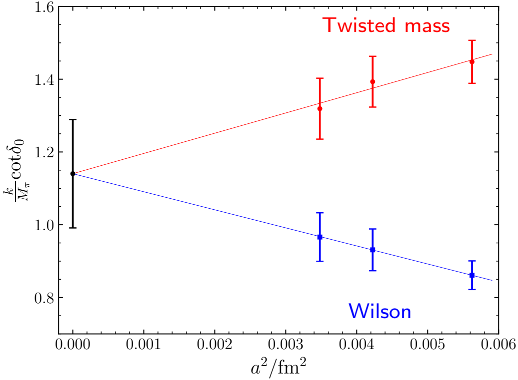


AA-channel: Continuum extrapolation for $N_c = 3$

Continuum extrapolation of $k \cot \delta_0$ for $N_c = 3$ in 3 steps:

1. Extrapolation to $k/M_\pi = -0.08$ using Effective Range Expansion and $M_\pi^2 r_0 a_0 \in [-5, -1]$
2. Interpolation to $\xi = 0.14$
3. Constrained continuum extrapolation

LO ChPT:
$M_\pi^2 r_0 a_0 = -3$



- ⋆ Large $\mathcal{O}(a^2)$ effects for both regularizations
- ⋆ Use TM fermions
- ⋆ Wilson-ChPT inspired parametrization

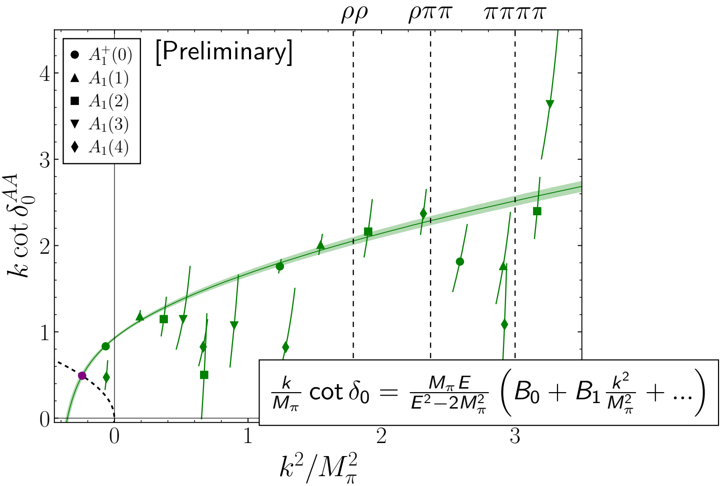
$$\Delta\mathcal{M}_{AA} = 32\pi^2 a^2 W \xi$$

$[W \sim \mathcal{O}(N_c^0)]$

- ⋆ $W = 42(29)$ fm^{-2}

Virtual bound state for $N_c = 3$

We find a **virtual bound state** for $N_c = 3$



$$\frac{k}{M_\pi} \cot \delta_0 = \frac{M_\pi E}{E^2 - 2M_\pi^2} \left(B_0 + B_1 \frac{k^2}{M_\pi^2} + \ldots \right)$$