Born-Oppenheimer Approximation for Exotic Hidden-Heavy Hadrons

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The Ohio State University

Exotic Hadrons Topical Collaboration

Outline

The Dilemma of Exotic Hadrons

- For a long time, it was believed that every hadron is either:
  - a quark-antiquark meson;
  - a 3-quark baryon.

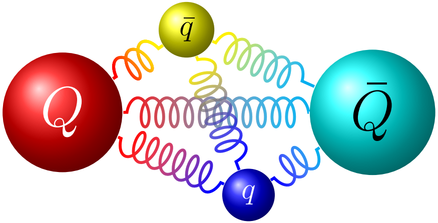
- Dozens of exotic hadrons with additional constituents have been discovered in the last 20 years.

- Among them are several hidden-heavy tetraquarks:
  - 44 $c\bar{c}$ tetraquarks;
  - 5 $b\bar{b}$ tetraquarks.

- No theoretical scheme has yet unveiled their general pattern.

The Born-Oppenheimer Approximation for QCD

Juge, Kuti & Morningstar (hep-ph/9902336)

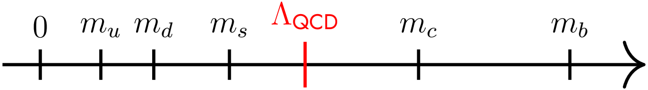


Light-QCD fields

Adjust instantaneously to the motion of the heavy (anti)quarks.

Heavy quark and antiquark

Move in potentials given by QCD with static color sources.



The Born-Oppenheimer Hamiltonian

Expansion in powers of $1/m_Q$

$$H_{\text{BO}}(\vec{r},\vec{p}) = H_{\text{static}}(\vec{r}) + \frac{p^2}{m_Q} + \ldots$$

Leading order ($m_Q \to \infty$): the static limit

$$H_{\text{static}}(\vec{r}) = \sum_n |\zeta_n(\vec{r})\rangle\, V_n(r)\, \langle\zeta_n(\vec{r})|$$

- n Born-Oppenheimer quantum numbers
- $V_n(r)$ energy levels of light QCD with static $Q, \bar{Q}$ at distance r
- $|\zeta_n(\vec{r})\rangle$ eigenstates of light QCD with static $Q, \bar{Q}$ at $+\vec{r}/2, -\vec{r}/2$

Symmetries of the Born-Oppenheimer Approximation

The static $Q\bar{Q}$ break

- rotations,
- parity,
- charge-conjugation,

down to

- cylindrical symmetries,
- combined CP symmetry.

The quantum numbers are not

J angular momentum,

P parity,

C charge-conjugation,

but rather

λ angular momentum projection,

η (g or u) $CP = +$ or $-$.

Heavy-quark spin symmetry

Static energy levels are independent of the heavy-quark spins.

Connection with Lattice QCD

Correlation matrix in light QCD with static $Q, \bar{Q}$ at $+\vec{r}/2, -\vec{r}/2$

$$C_{ij}(r,\tau,\tau_0) = \langle 0|\mathcal{O}_i(\vec{r},\tau)U(\tau,\tau_0)\mathcal{O}_j^\dagger(\vec{r},\tau_0)|0\rangle$$

The correlation matrix $\mathbf{C}$ can be calculated using lattice QCD.

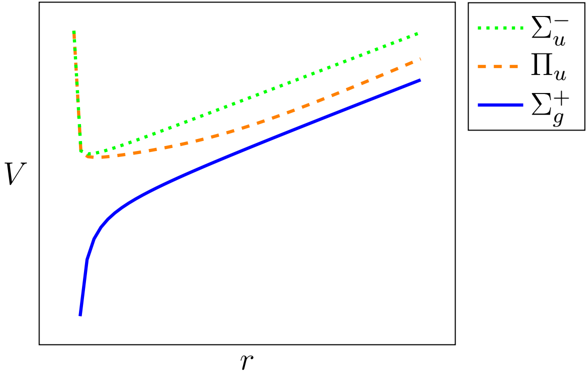
QCD	quantity that is determined	B-O Hamiltonian
$\mathbf{C}$ eigenvalues at large τ	adiabatic potentials	$V_n(r)$
$\mathbf{C}$ eigenvectors at large τ	nonadiabatic couplings	$\vert\zeta_n(\vec{r})\rangle$

Truncation to N channels

N eigenvalues and eigenvectors $\rightarrow$ truncated B-O approximation

Born-Oppenheimer Potentials for Hidden-Heavy Hadrons

Juge, Kuti & Morningstar (hep-lat/0207004)
Capitani, Philipsen, Reisinger, Riehl & Wagner (1811.11046); Schlosser & Wagner (2111.00741)
Bicudo, Cardoso & Sharifian (2105.12159); Sharifian, Cardoso & Bicudo (2303.15152)



Π_u, Σ_u^-: hybrid potentials

- $r \to 0$: 1^{+-} gluelump
- $r \to \infty$: $N = 1, 3$ string

Σ_g^+: quarkonium potential

- $r \to 0$: 0^{++} vacuum
- $r \to \infty$: $N = 0$ string

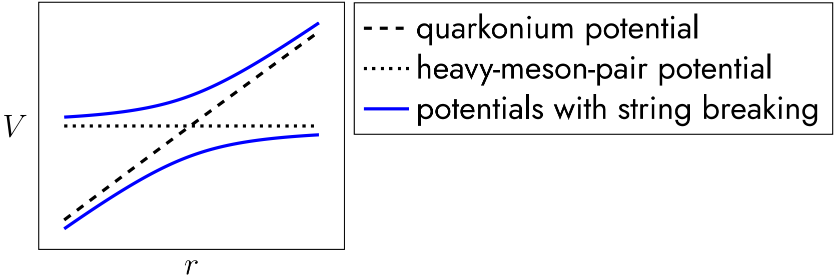
Outline

Hadron-Pair Potentials and Mixing

Bali, Neff, Düssel, Lippert & Schilling (hep-lat/0505012)
Bulava, Hörz, Knechtli, Koch, Moir, Morningstar & Peardon (1902.04006)
Bulava, Knechtli, Koch, Morningstar & Peardon (2403.00754)

Born-Oppenheimer potentials for heavy-hadron pairs

- constant at large distances (threshold)
- mix with potentials for hidden-heavy hadrons → decays!



The Coupled-Channel Schrödinger Equation

Adiabatic representation

- $-\frac{1}{m}\left(\vec{\nabla} + \vec{\mathbf{\Pi}}(\vec{r})\right)^2 \mathbf{\Psi}(\vec{r}) + \mathbf{V}_{\text{diag}}(r)\mathbf{\Psi}(\vec{r}) = E\mathbf{\Psi}(\vec{r})$
- mixed $Q\bar{Q}$ and $B^{(*)}\bar{B}^{(*)}$ channels coupled by nonadiabatic couplings

unitary transformation

Diabatic representation

- $-\frac{1}{m}\nabla^2\mathbf{\Psi}(\vec{r}) + \mathbf{V}(\vec{r})\mathbf{\Psi}(\vec{r}) = E\mathbf{\Psi}(\vec{r})$
- pure $Q\bar{Q}$ and pure $B^{(*)}\bar{B}^{(*)}$ channels coupled by mixing potentials

Mixing Potentials and Selection Rules

$g_{\lambda,\eta}$ transition rates between Born-Oppenheimer potentials

$G_{L,\eta}$ mixing potentials appearing in the coupled-channel Schrödinger equation

$$G_{L,\eta}\big(j^{\pi}, L_Q \to (j_1^{\pi_1}, j_2^{\pi_2})j', L_Q'\big) = (-1)^{L_Q+L_Q'} \times \sum_{\lambda} \left\langle \begin{matrix} j & L \\ \lambda & -\lambda \end{matrix} \middle| \begin{matrix} L_Q \\ 0 \end{matrix} \right\rangle \left\langle \begin{matrix} j' & L \\ \lambda & -\lambda \end{matrix} \middle| \begin{matrix} L_Q' \\ 0 \end{matrix} \right\rangle g_{\lambda,\eta}\big(j^{\pi} \to (j_1^{\pi_1}, j_2^{\pi_2})j'\big)$$

Model-independent selection rules

- conservation of Born-Oppenheimer quantum numbers λ and η
- conservation of angular-momentum vector $\vec{L} = \vec{J}_{\text{light}} + \vec{L}_Q$

Heavy Spins and Relative Partial Decay Rates

$G_{L,\eta}$ mixing potentials without heavy spins

$V^J_{S_Q,L,\eta}$ mixing potentials including heavy spins

$$V^J_{S_Q,L,\eta}\Big(j^\pi, L_Q \to \big[\big(\tfrac{1}{2}^+, j_1^{\pi_1}\big)J_1, \big(\tfrac{1}{2}^-, j_2^{\pi_2}\big)J_2\big]S, L'_Q\Big) =$$

$$N(-1)^{2j_1+S_Q+L'_Q+J}\sqrt{\tilde{J}_1\tilde{J}_2\tilde{S}\tilde{S}_Q\tilde{L}}$$

$$\times \sum_{j'}(-1)^{j'}\sqrt{\tilde{j}'}\begin{Bmatrix} S_Q & j' & S \\ L'_Q & J & L \end{Bmatrix}\begin{Bmatrix} \frac{1}{2} & \frac{1}{2} & S_Q \\ j_1 & j_2 & j' \\ J_1 & J_2 & S \end{Bmatrix}$$

$$\times\, G_{L,\eta}\big(j^\pi, L_Q \to (j_1^{\pi_1}, j_2^{\pi_2})j', L'_Q\big)$$

Model-independent relative partial decay rates

Simple predictions in terms of Wigner 6j and 9j symbols.

Outline

A New Understanding of Quarkoium-Hybrid Decays

RB (2306.17120)

Multiplet	J^{PC}		Potential
H_1	1^{--}	$(0,1,2)^{-+}$	$\Pi_u \& \Sigma_u^-$
H_2	1^{++}	$(0,1,2)^{+-}$	Π_u
H_3	0^{++}	1^{+-}	Σ_u^-
H_4	2^{++}	$(1,2,3)^{+-}$	$\Pi_u \& \Sigma_u^-$
H_5	2^{--}	$(1,2,3)^{-+}$	Π_u

- Π_u does not couple to $B\bar{B}$, $B^*\bar{B}$, $B\bar{B}^*$, $B^*\bar{B}^*$
- Σ_u^- does couple to $B\bar{B}$, $B^*\bar{B}$, $B\bar{B}^*$, $B^*\bar{B}^*$

A New Understanding of Quarkoium-Hybrid Decays

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H_1	1^{--}	$(0,1,2)^{-+}$	$\Pi_u \& \Sigma_u^-$	allowed
H_2	1^{++}	$(0,1,2)^{+-}$	Π_u	forbidden
H_3	0^{++}	1^{+-}	Σ_u^-	allowed
H_4	2^{++}	$(1,2,3)^{+-}$	$\Pi_u \& \Sigma_u^-$	allowed
H_5	2^{--}	$(1,2,3)^{-+}$	Π_u	forbidden

- Π_u does not couple to $B\bar{B}$, $B^*\bar{B}$, $B\bar{B}^*$, $B^*\bar{B}^*$ $\rightarrow$ decays are forbidden.
- Σ_u^- does couple to $B\bar{B}$, $B^*\bar{B}$, $B\bar{B}^*$, $B^*\bar{B}^*$ $\rightarrow$ decays are allowed!

Quarkonium Hybrids vs. Conventional Quarkonia

E. Braaten & RB (2403.12868)

Branching ratios for a hidden-heavy meson with $J^{PC} = 1^{--}$

- if conventional quarkonium $\longrightarrow$ $B\bar{B} : B^*\bar{B} : B\bar{B}^* : B^*\bar{B}^* = 1 : 2 : 2 : 7$
- if quarkonium hybrid $\longrightarrow$ $B\bar{B} : B^*\bar{B} : B\bar{B}^* : B^*\bar{B}^* = 1 : 0 : 0 : 3$

Smoking gun for a quarkonium hybrid

- Conventional quarkonia that decay into $B^*\bar{B}$ typically decay also into $B\bar{B}$.
- Quarkonium hybrids that decay into $B^*\bar{B}$ are suppressed in the $B\bar{B}$ channel!
- Quarkonium hybrids that decay into $B\bar{B}$ are suppressed in the $B^*\bar{B}$ channel!

Summary

- The spectrum and decays of exotic hidden-heavy hadrons can be studied *ab initio* using the diabatic Born-Oppenheimer approximation for QCD.

- The Born-Oppenheimer approximation gives model-independent results for:
  - selection rules for decays into heavy-hadron pairs;
  - relative partial decay rates into heavy-hadron pairs.

- Quarkonium hybrids can decay into $B^{(*)}\bar{B}^{(*)}$, contrarily to the conventional wisdom for the last 40 years from constituent models.

Hidden-Heavy Tetraquark Multiplets

Brambilla, Mohapatra & Vairo (in preparation)

1^{--} adjoint meson

Potential	J^{PC}	
$\Sigma_g^{+\prime}/\Pi_g$	1^{+-}	$(0,1,2)^{++}$
$\Sigma_g^{+\prime}$	0^{-+}	1^{--}
Π_g	1^{-+}	$(0,1,2)^{--}$
$\Sigma_g^{+\prime}/\Pi_g$	2^{-+}	$(1,2,3)^{--}$
Π_g	2^{+-}	$(1,2,3)^{++}$

0^{-+} adjoint meson

Potential	J^{PC}	
$\Sigma_u^{-\prime}$	0^{++}	1^{+-}
$\Sigma_u^{-\prime}$	1^{--}	$(0,1,2)^{-+}$
$\Sigma_u^{-\prime}$	2^{++}	$(1,2,3)^{+-}$

Quarkonium Spectrum with Heavy-Meson-Pair Thresholds

RB & P. González (2107.05459); RB (2303.17533)

Simplified potential matrix for mixing $Q\bar{Q}$ and $B^{(*)}\bar{B}^{(*)}$ with $J^{PC} = 1^{++}$

$$\begin{matrix} Q\bar{Q}(P\text{-wave}) \rightarrow \\ B^*\bar{B}(S\text{-wave}) \rightarrow \\ B^*\bar{B}(D\text{-wave}) \rightarrow \\ B^*\bar{B}^*(D\text{-wave}) \rightarrow \end{matrix} \begin{pmatrix} V_{Q\bar{Q}}(r) + 2m_Q & \frac{1}{\sqrt{3}}g(r) & \frac{1}{\sqrt{6}}g(r) & \frac{1}{\sqrt{2}}g(r) \\ \frac{1}{\sqrt{3}}g(r) & M_B + M_{B^*} & 0 & 0 \\ \frac{1}{\sqrt{6}}g(r) & 0 & M_B + M_{B^*} & 0 \\ \frac{1}{\sqrt{2}}g(r) & 0 & 0 & 2M_{B^*} \end{pmatrix}$$

Below threshold

- conventional quarkonia
- heavy-meson molecules

Above threshold

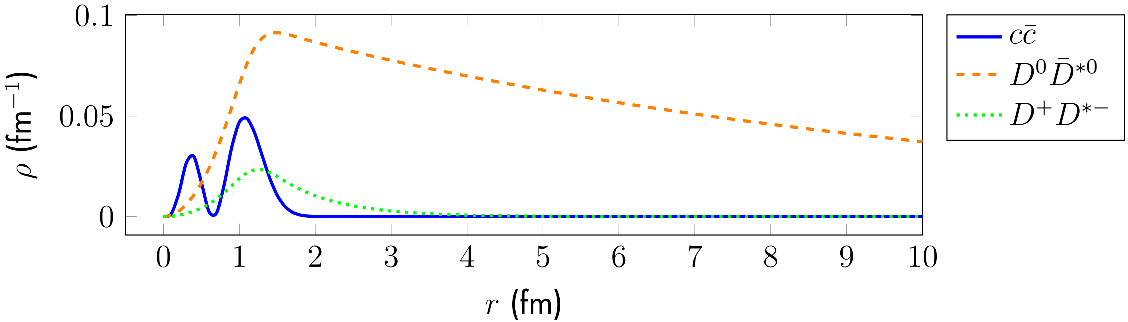
- resonances
- decays into $B^{(*)}\bar{B}^{(*)}$

$X(3872)$ as a Charmonium/Molecule Admixture

RB & P. González (2111.07653)

This oversimplified model assumes a fine tuning of the $c\bar{c}/D^*\bar{D}$ mixing potential.

Calculated radial probability density for $X(3872)$:



Missing Conventional States

RB & P. González (2207.02740)

The conventional charmonium state $\chi_{c1}(2P)$ could be shadowed by $X(3872)$!

Calculated $D^*\bar{D}$ cross section $(J^{PC} = 1^{++})$ times momentum squared: