Scalar and tensor charmonium resonances from lattice QCD

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hadspec
hadspec.org

Exotic Hadron Spectroscopy
Swansea University
2-4 July 2024

based on work:
PRL: arXiv: 2309.14070 (7 pages)
PRD: arXiv: 2309.14071 (55 pages)

UNIVERSITY OF CAMBRIDGE

THE ROYAL SOCIETY

Lattice QCD provides a rigorous approach to hadron spectroscopy

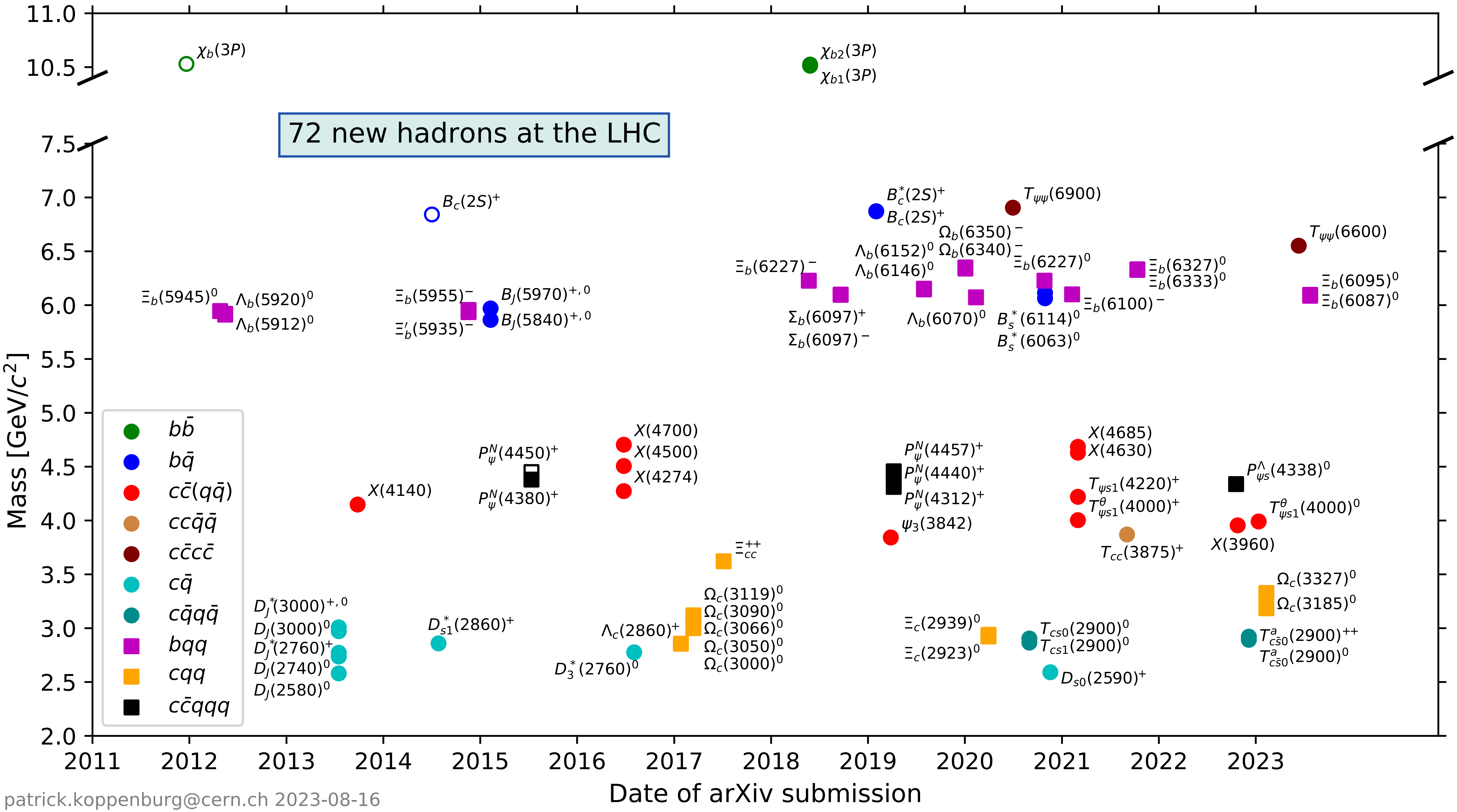
- as **rigorous** as possible
- **all** necessary **QCD** diagrams are computed
- **excited states** appear as **unstable resonances** in a scattering amplitude using the Lüscher method

tremendous progress in recent years
but not yet ready for precision comparisons for most scattering processes

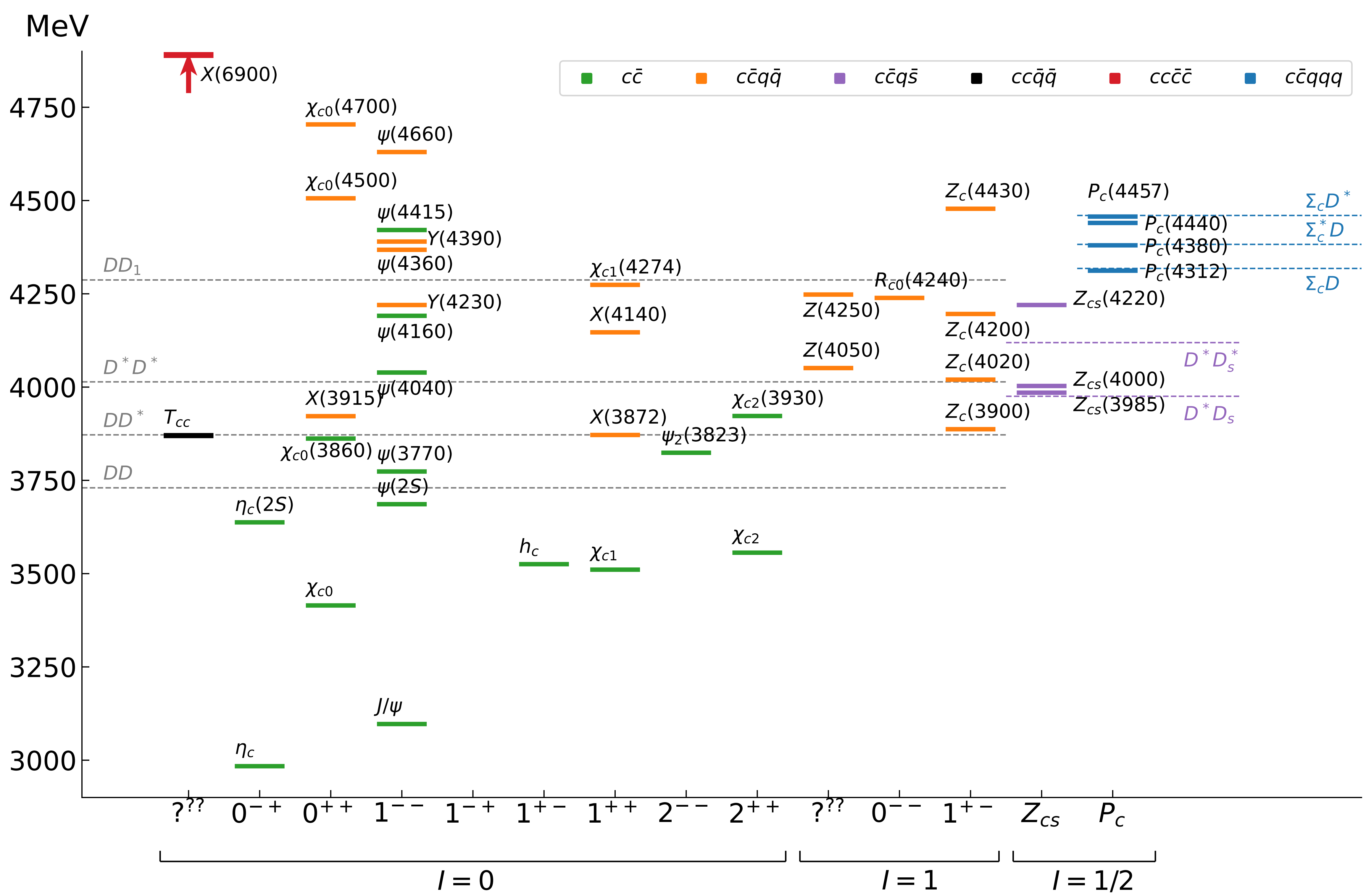
- physical pions are very light
- most interesting states can decay to **many** pions
- control of light-quark mass is a useful tool
- “small” effects not considered in general:

  finite lattice spacing, isospin breaking, EM interactions

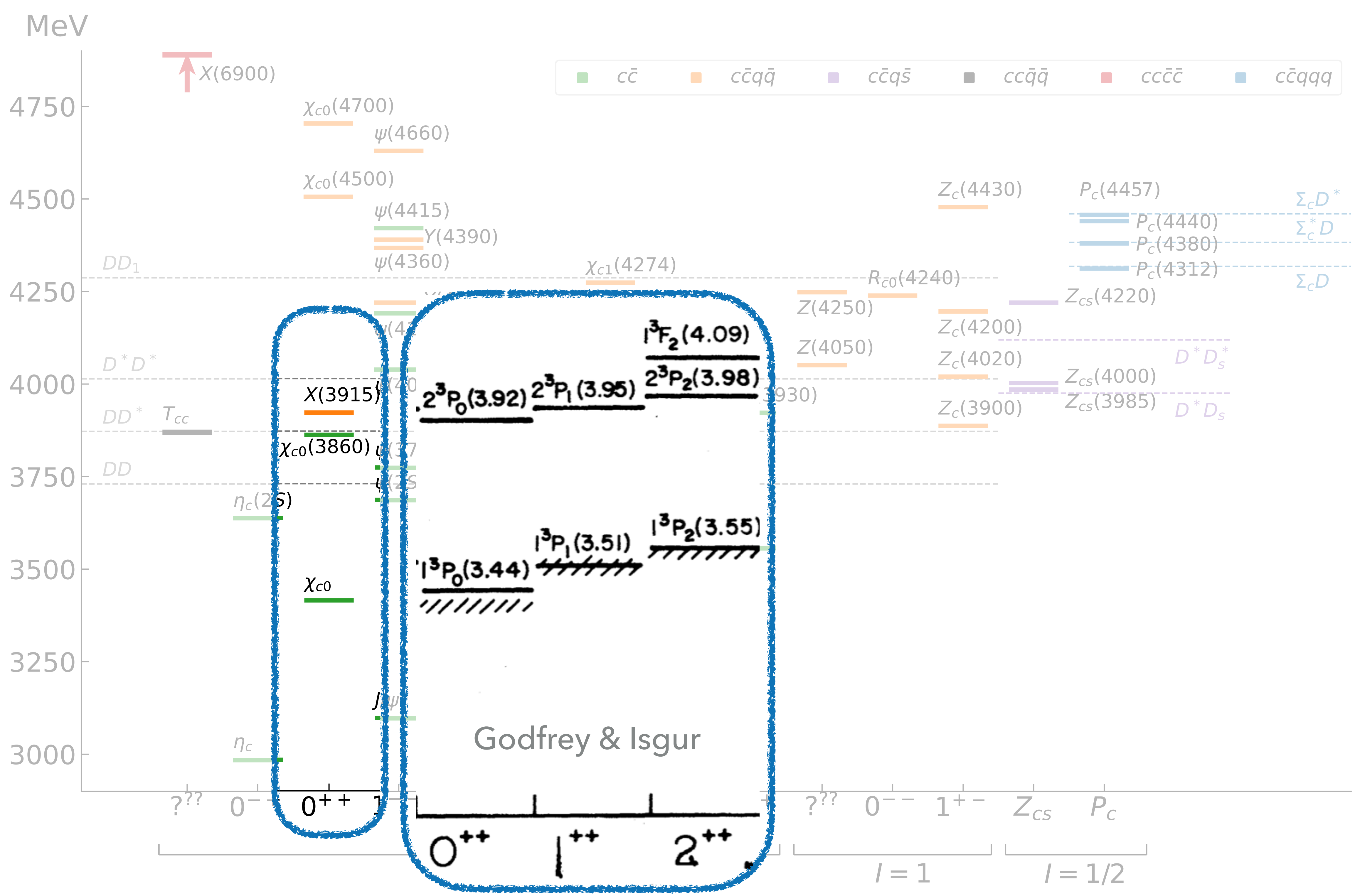
goal: what does **QCD** say about the excited hadron spectrum?



72 new hadrons at the LHC
Mass [GeV/c^2]
Date of arXiv submission
2011 2012 2013 2014 2015 2016 2017 2018 2019 2020 2021 2022 2023
2.0 2.5 3.0 3.5 4.0 4.5 5.0 5.5 6.0 6.5 7.0 7.5 10.5 11.0
$b\bar{b}$
$b\bar{q}$
$c\bar{c}(q\bar{q})$
$cc\bar{q}\bar{q}$
$c\bar{c}c\bar{c}$
$c\bar{q}$
$c\bar{q}q\bar{q}$
bqq
cqq
$c\bar{c}qqq$
$\chi_b(3P)$
$\chi_{b2}(3P)$
$\chi_{b1}(3P)$
$B_c(2S)^+$
$B_c^*(2S)^+$
$B_c(2S)^+$
$T_{\psi\psi}(6900)$
$T_{\psi\psi}(6600)$
$\Xi_b(5945)^0$
$\Lambda_b(5920)^0$
$\Lambda_b(5912)^0$
$\Xi_b(5955)^-$
$\Xi_b'(5935)^-$
$B_J(5970)^{+,0}$
$B_J(5840)^{+,0}$
$\Xi_b(6227)^-$
$\Sigma_b(6097)^+$
$\Sigma_b(6097)^-$
$\Lambda_b(6152)^0$
$\Lambda_b(6146)^0$
$\Omega_b(6350)^-$
$\Omega_b(6340)^-$
$\Lambda_b(6070)^0$
$\Xi_b(6227)^0$
$B_s^*(6114)^0$
$B_s^*(6063)^0$
$\Xi_b(6100)^-$
$\Xi_b(6327)^0$
$\Xi_b(6333)^0$
$\Xi_b(6095)^0$
$\Xi_b(6087)^0$
$X(4140)$
$P_\psi^N(4450)^+$
$P_\psi^N(4380)^+$
$X(4700)$
$X(4500)$
$X(4274)$
$P_\psi^N(4457)^+$
$P_\psi^N(4440)^+$
$P_\psi^N(4312)^+$
$\psi_3(3842)$
$X(4685)$
$X(4630)$
$T_{\psi s1}(4220)^+$
$T_{\psi s1}^\theta(4000)^+$
$T_{cc}(3875)^+$
$P_{\psi s}^\Lambda(4338)^0$
$T_{\psi s1}^\theta(4000)^0$
$X(3960)$
Ξ_{cc}^{++}
$D_J^*(3000)^{+,0}$
$D_J(3000)^0$
$D_J^*(2760)^+$
$D_J(2740)^0$
$D_J(2580)^0$
$D_{s1}^*(2860)^+$
$\Lambda_c(2860)^+$
$D_3^*(2760)^0$
$\Omega_c(3119)^0$
$\Omega_c(3090)^0$
$\Omega_c(3066)^0$
$\Omega_c(3050)^0$
$\Omega_c(3000)^0$
$\Xi_c(2939)^0$
$\Xi_c(2923)^0$
$T_{cs0}(2900)^0$
$T_{cs1}(2900)^0$
$D_{s0}(2590)^+$
$\Omega_c(3327)^0$
$\Omega_c(3185)^0$
$T_{c\bar{s}0}^a(2900)^{++}$
$T_{c\bar{s}0}^a(2900)^0$
patrick.koppenburg@cern.ch 2023-08-16



JPAC arXiv:2112.13436



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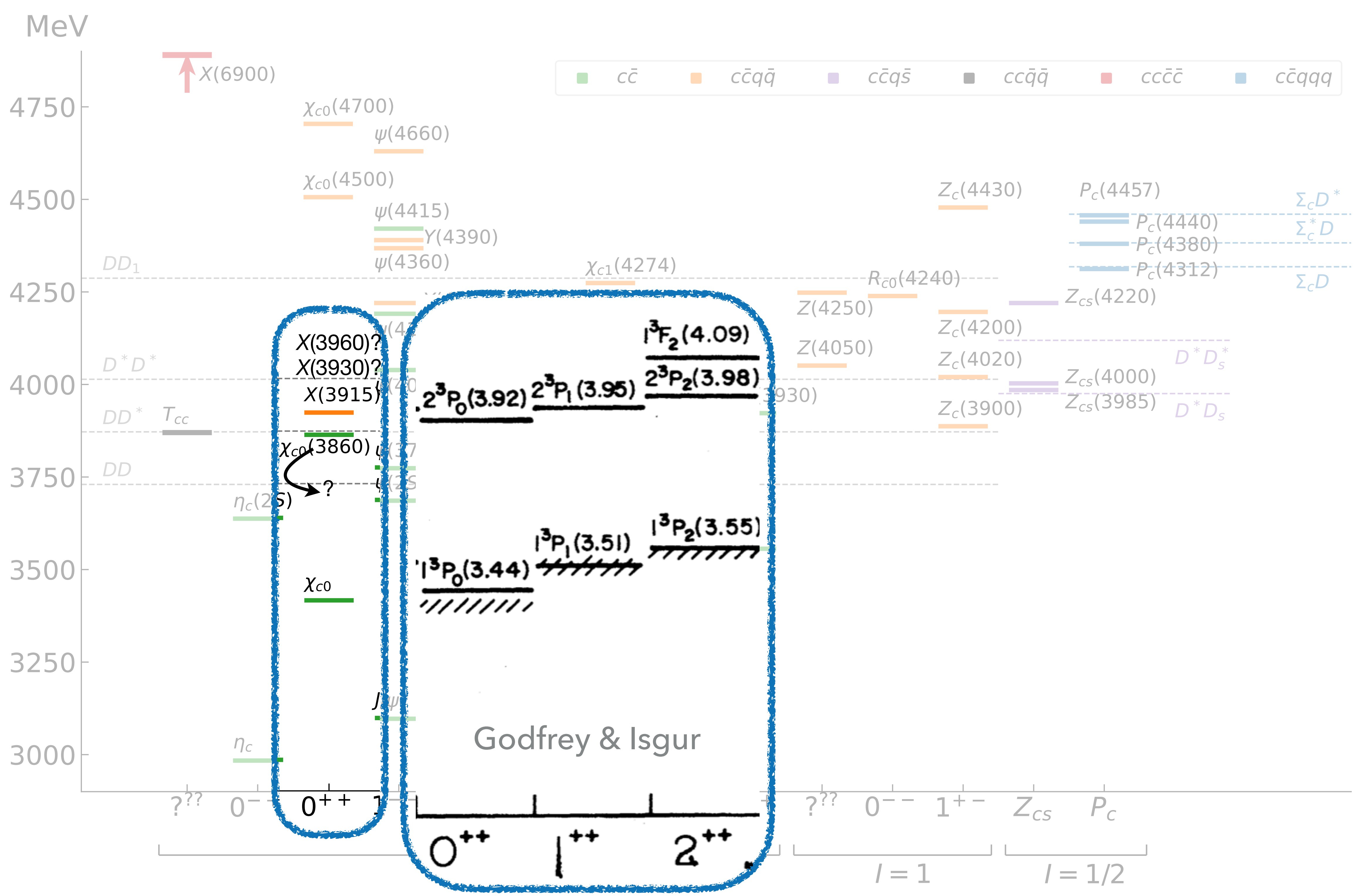

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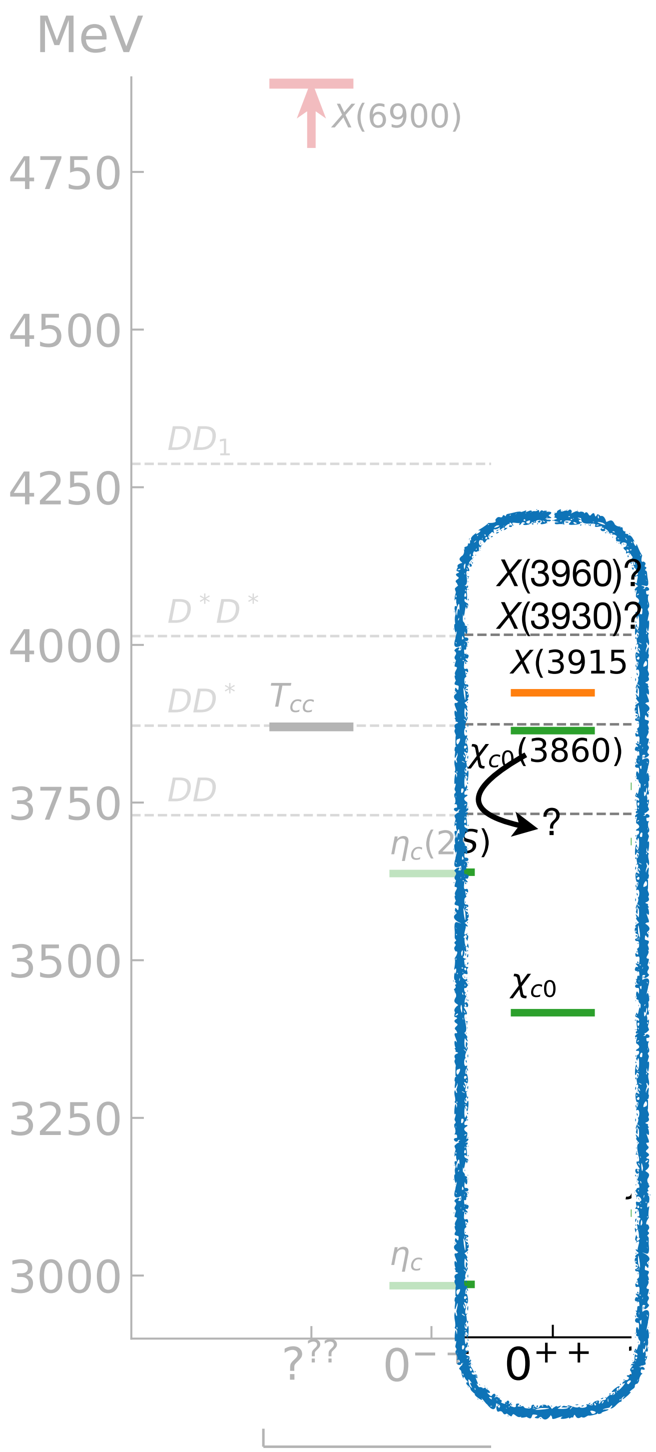
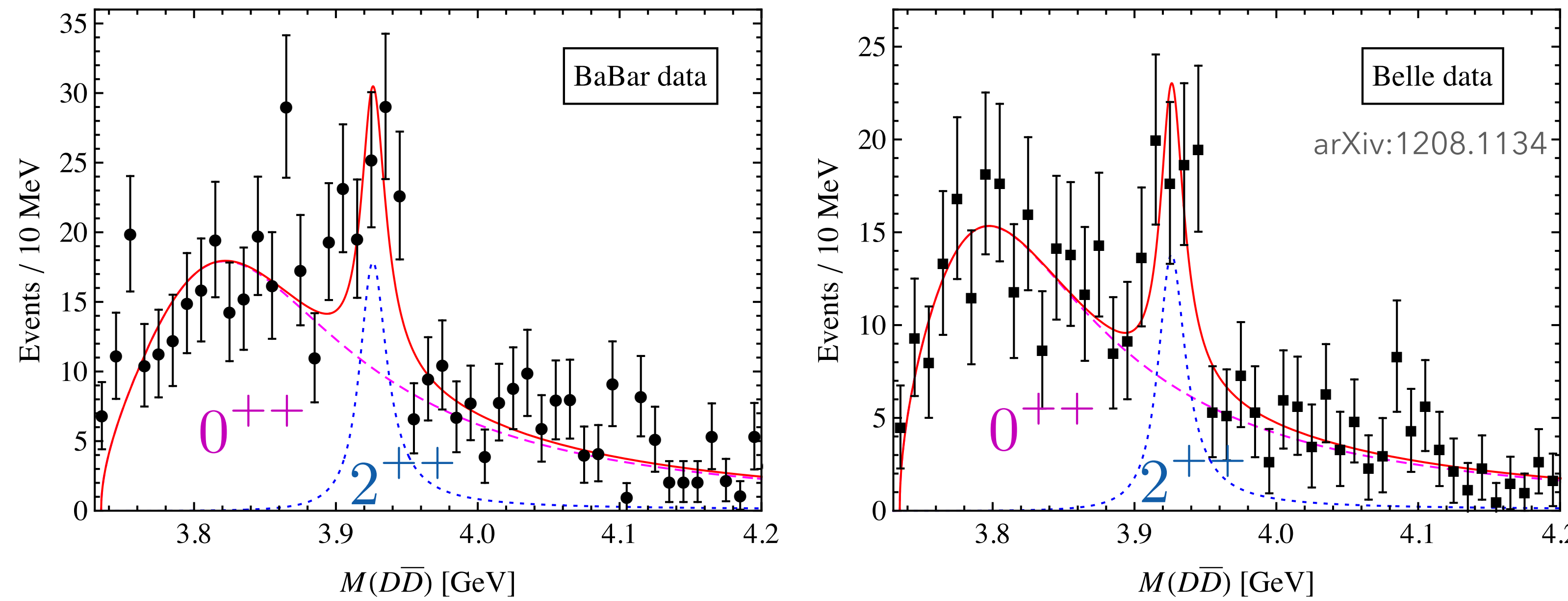
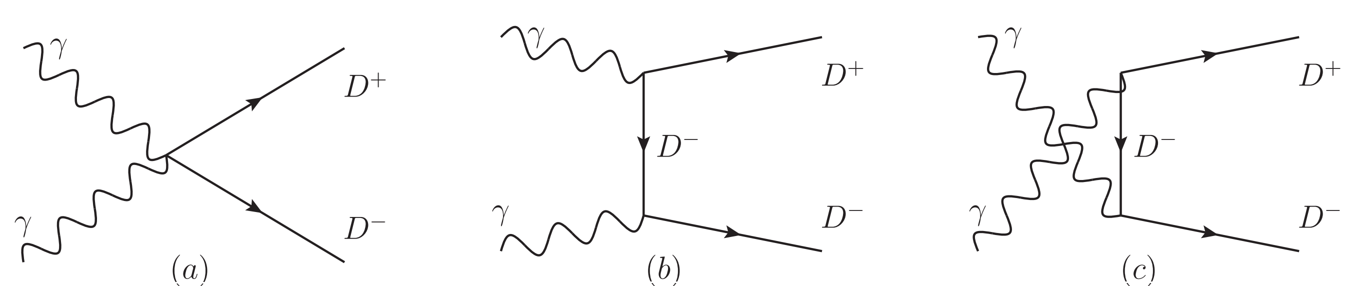


- BaBar, Belle - resonance around 3860 MeV $\gamma\gamma \to D\bar{D}$



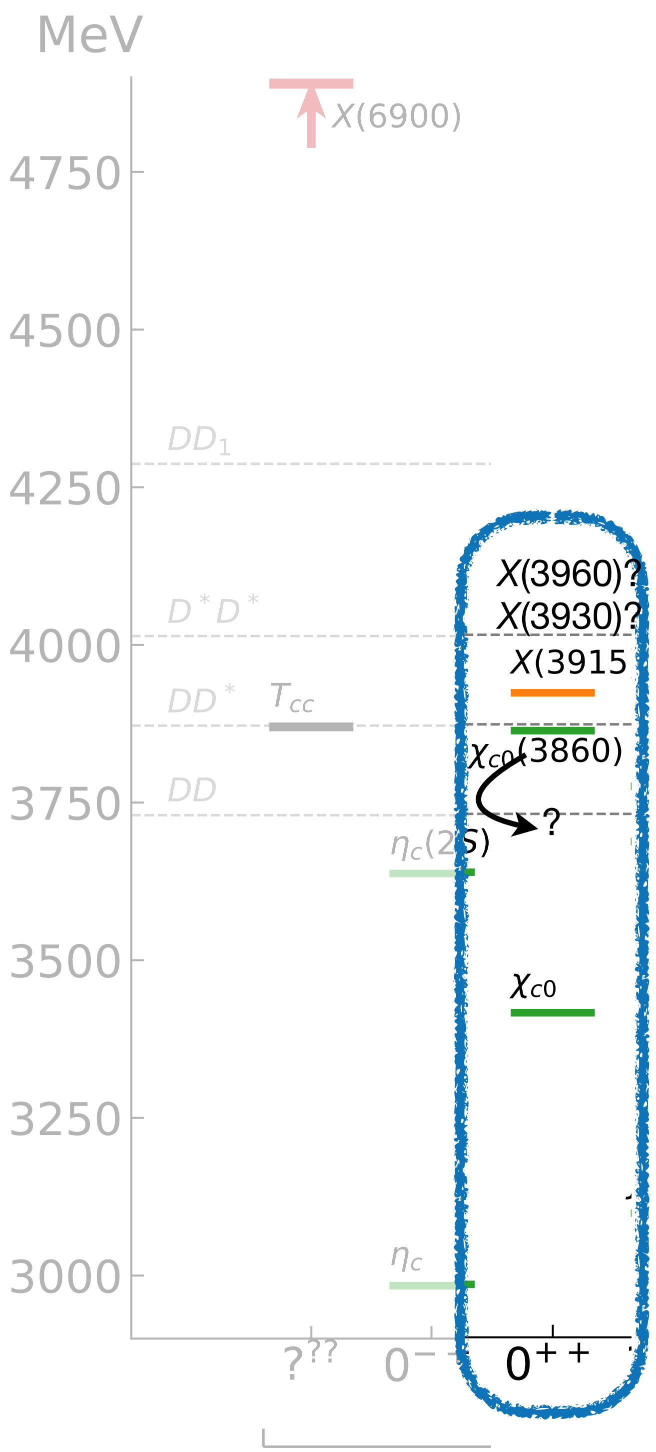
- Guo & Meissner (2012)
m = 3840 MeV, Γ = 220 MeV

- Wang et al (2021), Daneika et al (2022): Complications from Born exchanges lead to a lower state around 3700 MeV

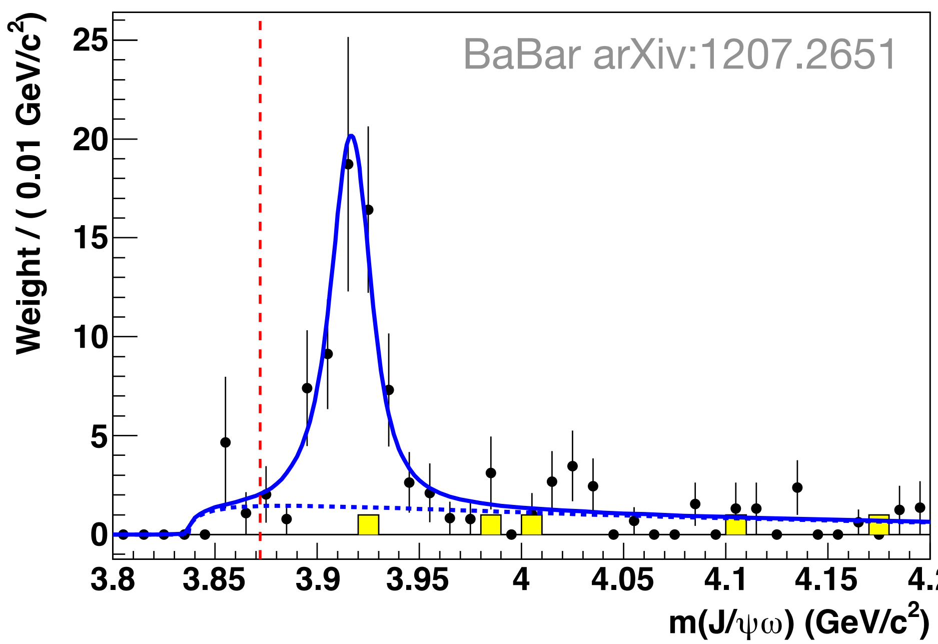


arXiv:2010.15431

no state around 3840-3860 MeV (?)

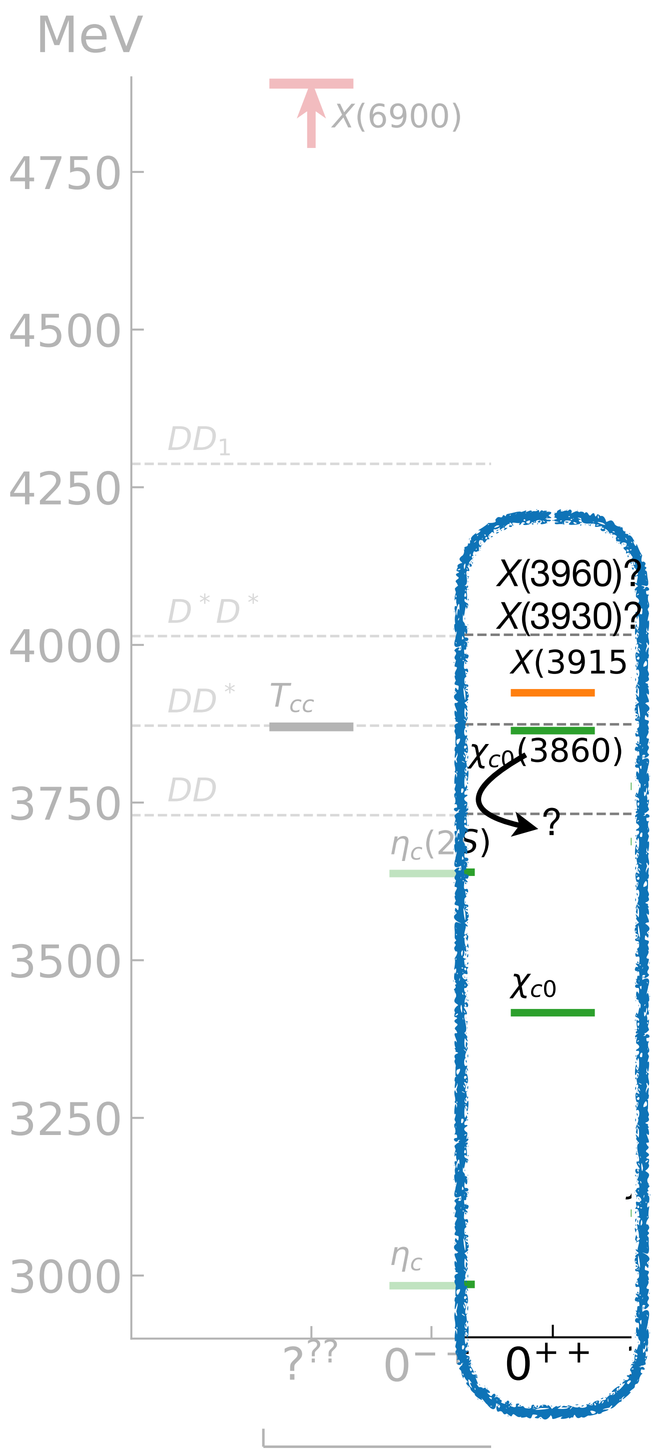


- BaBar, Belle - resonance around 3915 MeV in $J/\psi\omega$

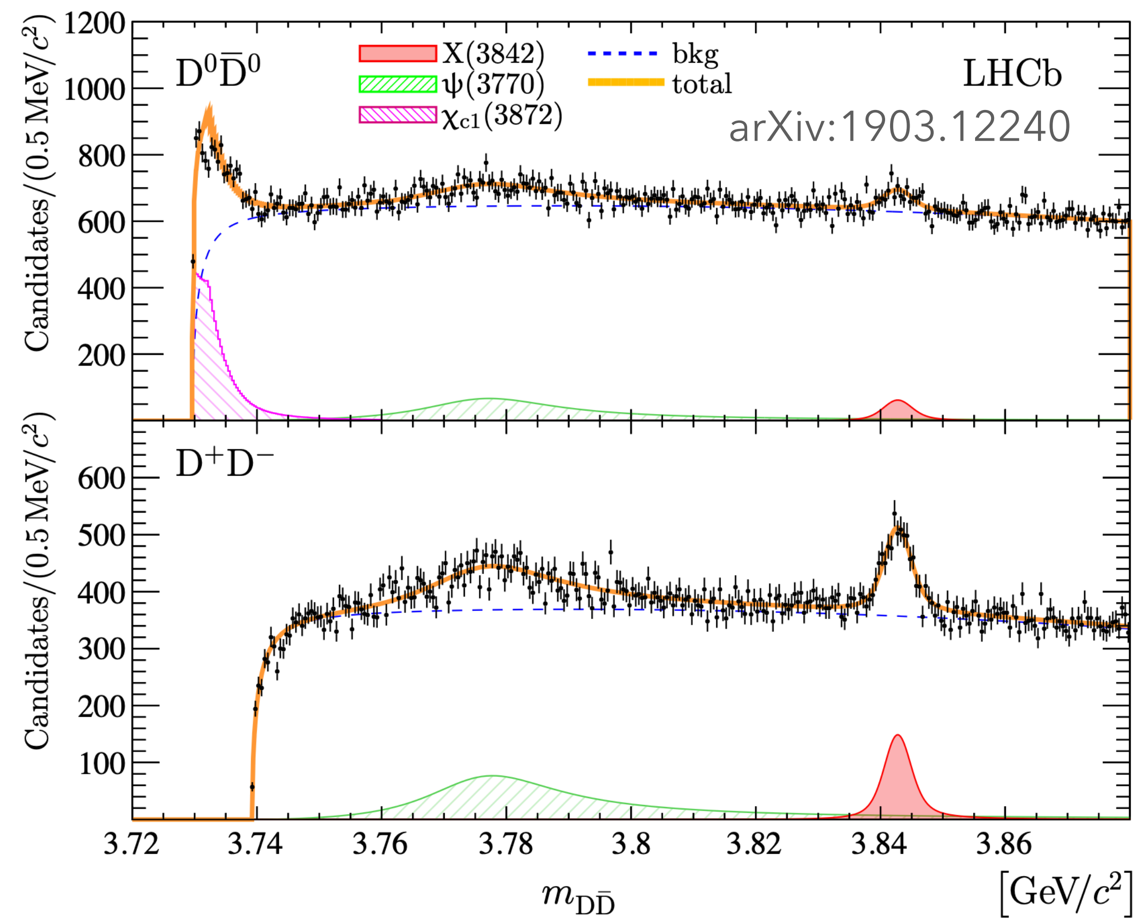


$$m = (3919.4 \pm 2.2 \pm 1.6)\ \text{MeV}$$
$$\Gamma = (13 \pm 6 \pm 3)\ \text{MeV}$$
$$J^P = 0^+$$

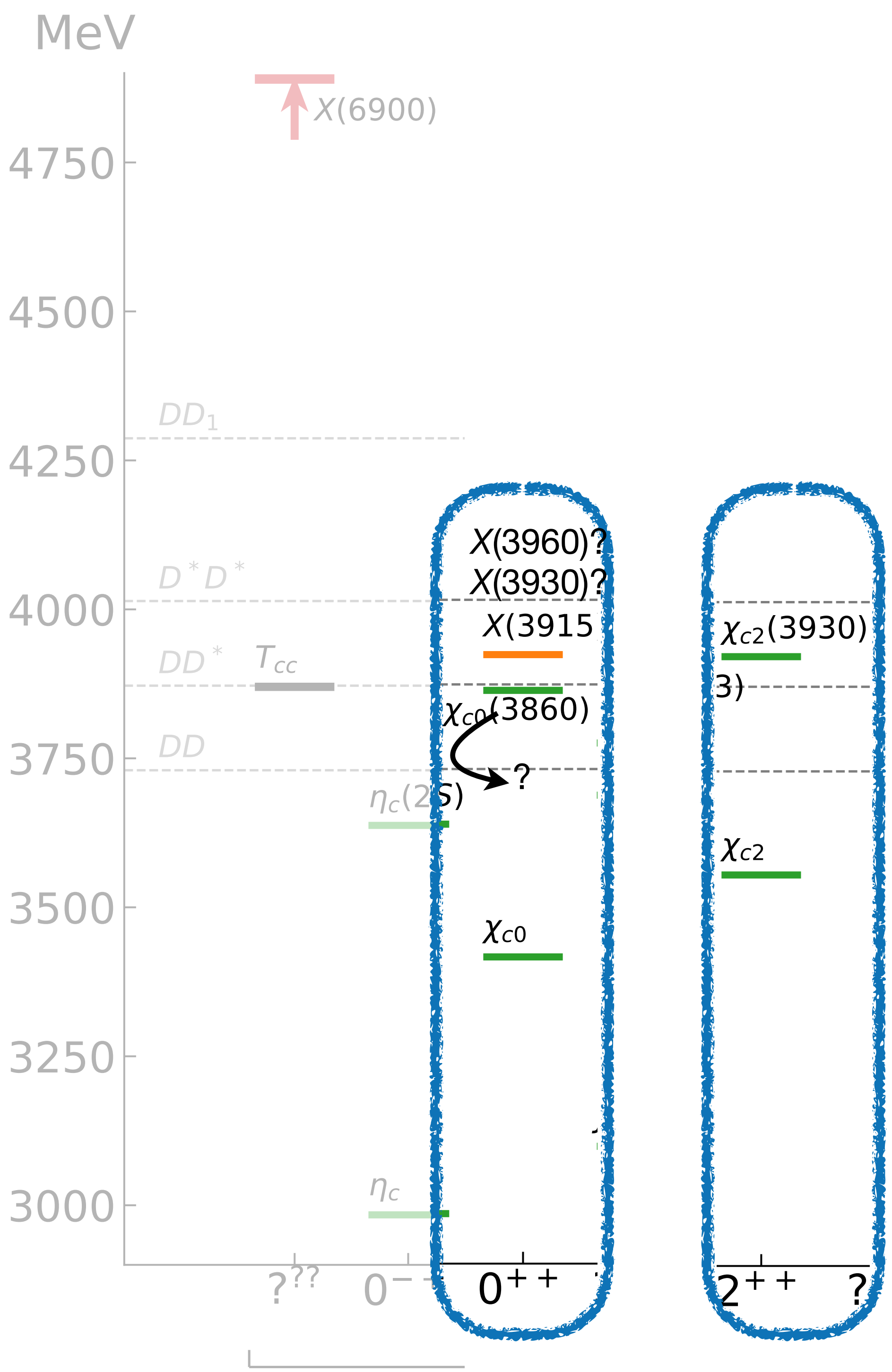
(several other studies of this)



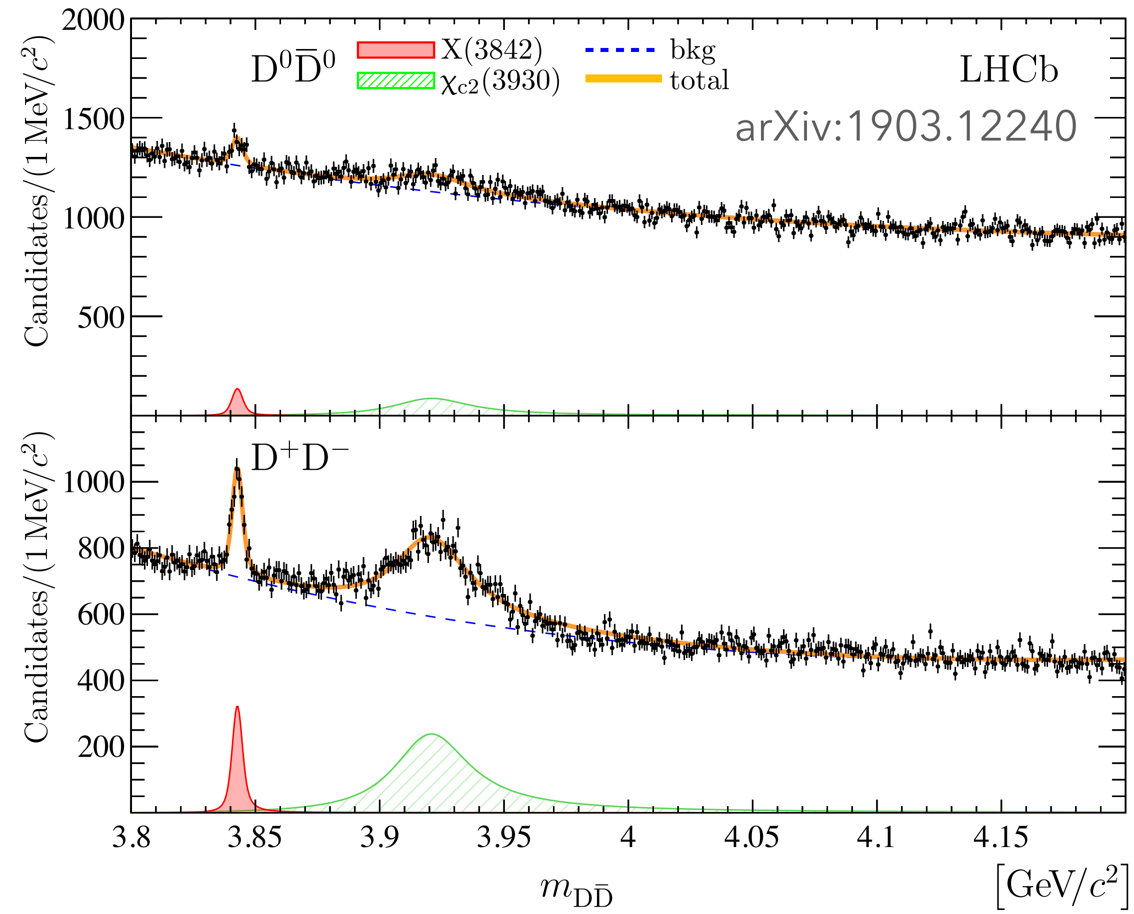
DDbar at LHCb
hadronic production process



Peak at DDbar threshold attributed to "feed-down" from X(3872) decays



Same study from LHCb,
higher energies

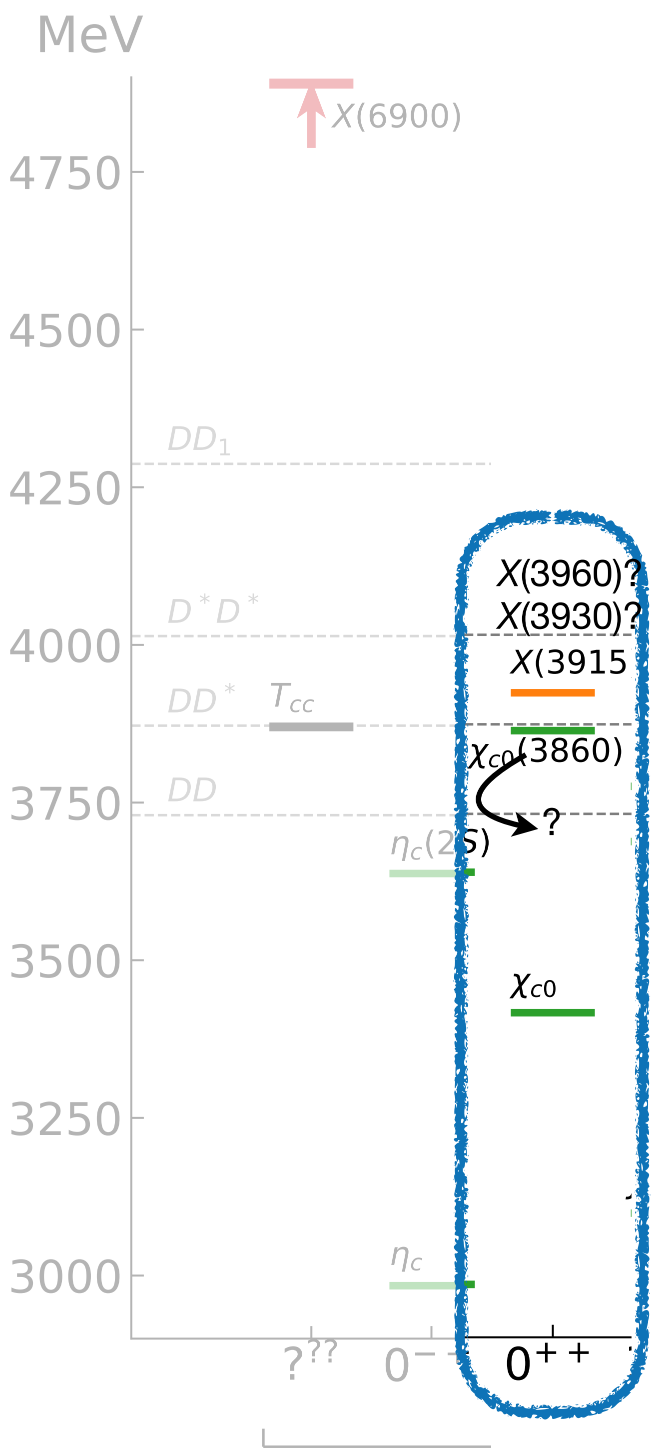


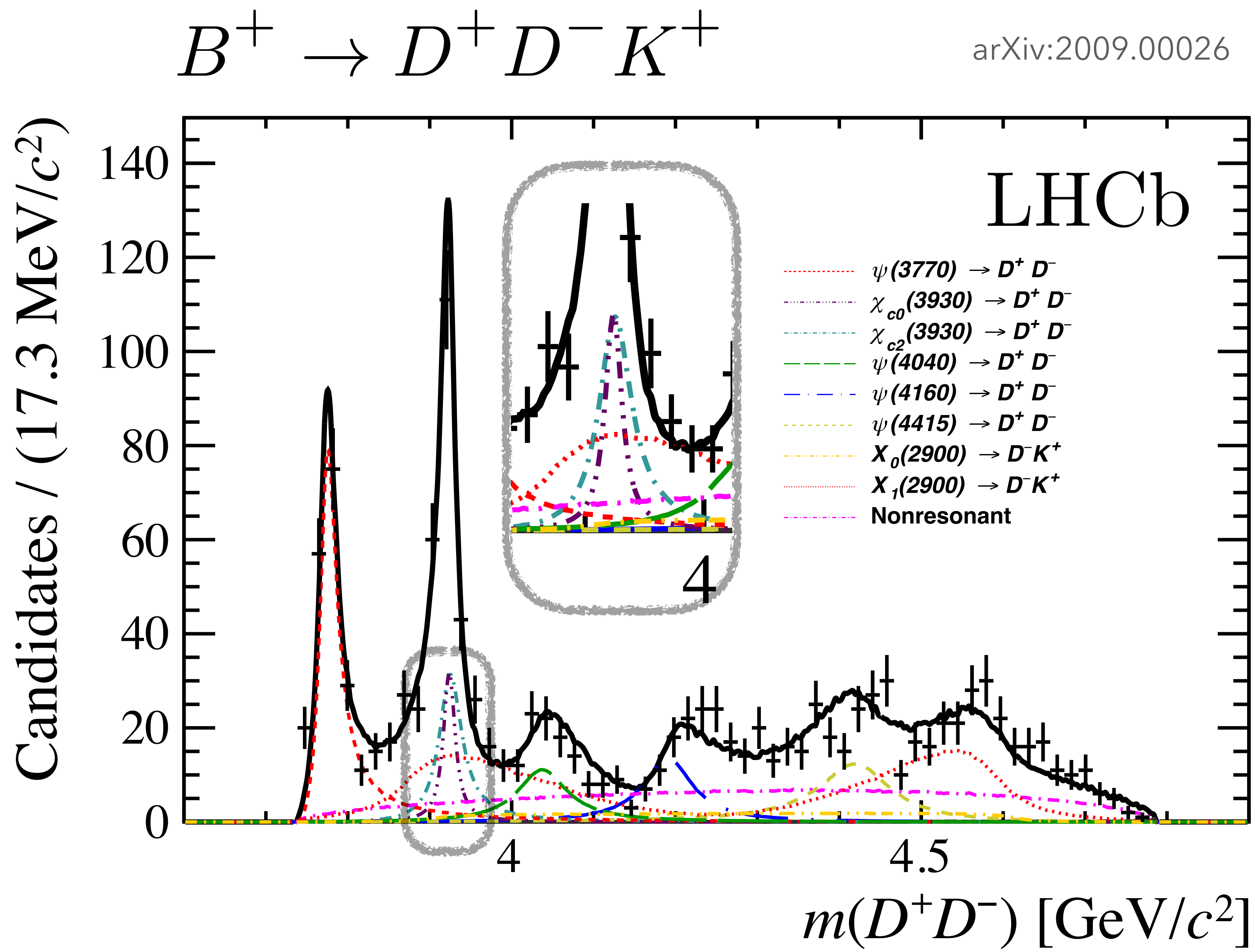
$\chi_{c2}(3930)$

$m \approx 3922$ MeV

$\Gamma \approx 37$ MeV

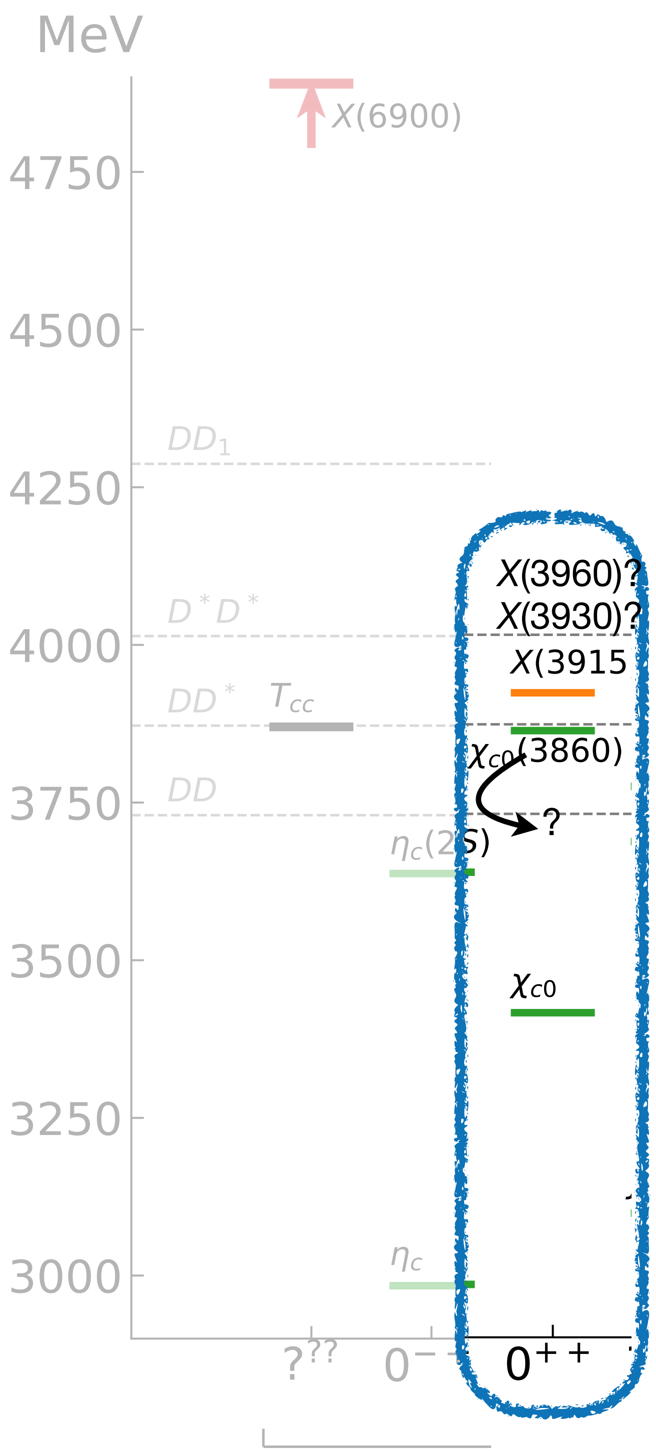
not obviously inconsistent with
earlier Belle & BaBar results





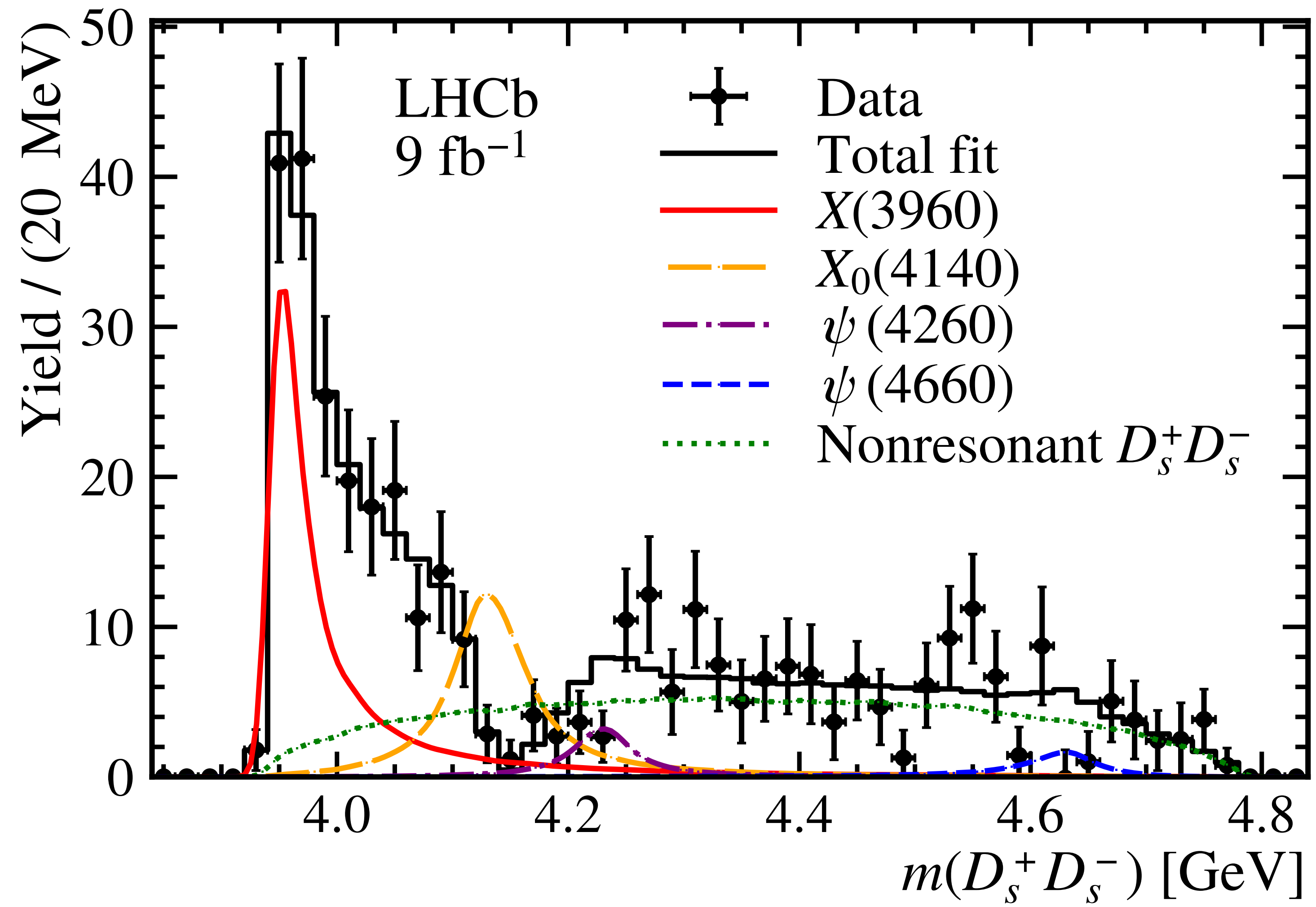
overlapping 0++ and 2++ resonances around 3925 MeV

no need for a low 0++ resonance



$$B^+ \to D_s^+ D_s^- K^+$$

arXiv:2210.15153
LHCb

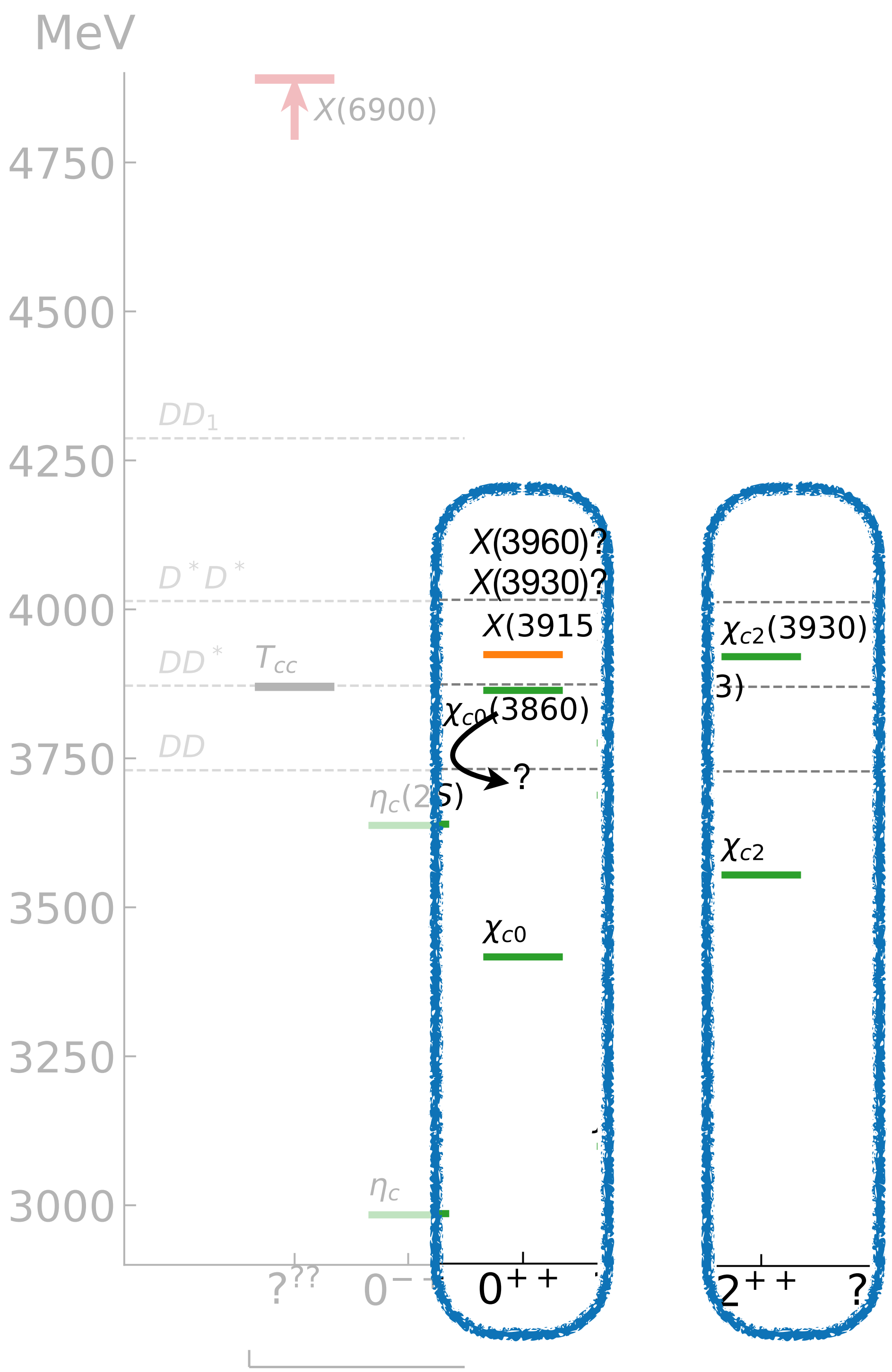


enhancement in DsDs at threshold "X(3960)"

$m \approx 3956$ MeV

$\Gamma \approx 43$ MeV

$J^{PC} = 0^{++}$



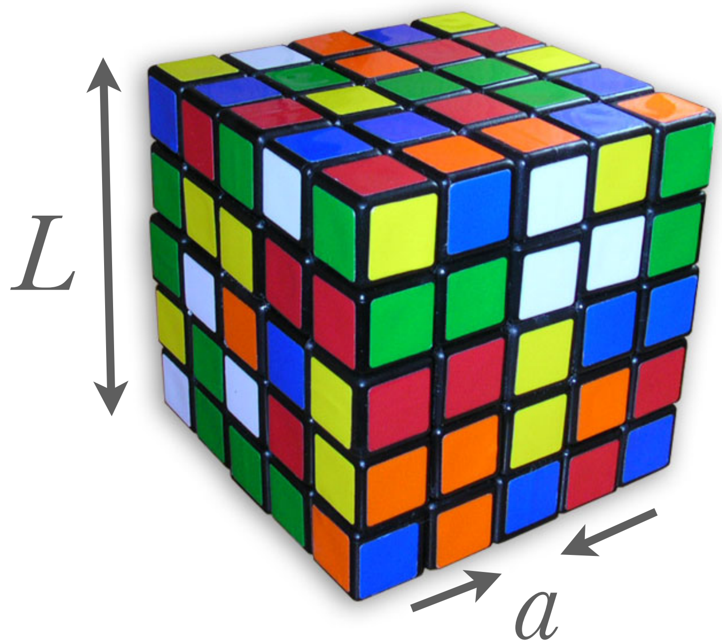
are all of these bumps resonances?

how are these experimental enhancements related to each other?

how many states are there in 0++ and 2++?

can we understand how the quark-model-like states and meson-meson like states contribute to the observed features?

first principles calculations are needed to start to understand this



"HadSpec" lattices

anisotropic (3.5 finer spacing in time)
Wilson-Clover

L/a_s=16, 20, 24
m_π = 391 MeV

rest and moving frames

N_f = 2+1 flavours
all light+strange annihilations included
no charm annihilation

operators used:

$\bar{\psi}\Gamma \overleftrightarrow{D} \ldots \overleftrightarrow{D}\ \psi$ local qq-like constructions

$$\sum_{\vec{p}_1+\vec{p}_2\in\vec{p}} C(\vec{p}_1,\vec{p}_2;\vec{p})\Omega_\pi(\vec{p}_1)\ \Omega_\pi(\vec{p}_2)$$

2&3-hadron constructions

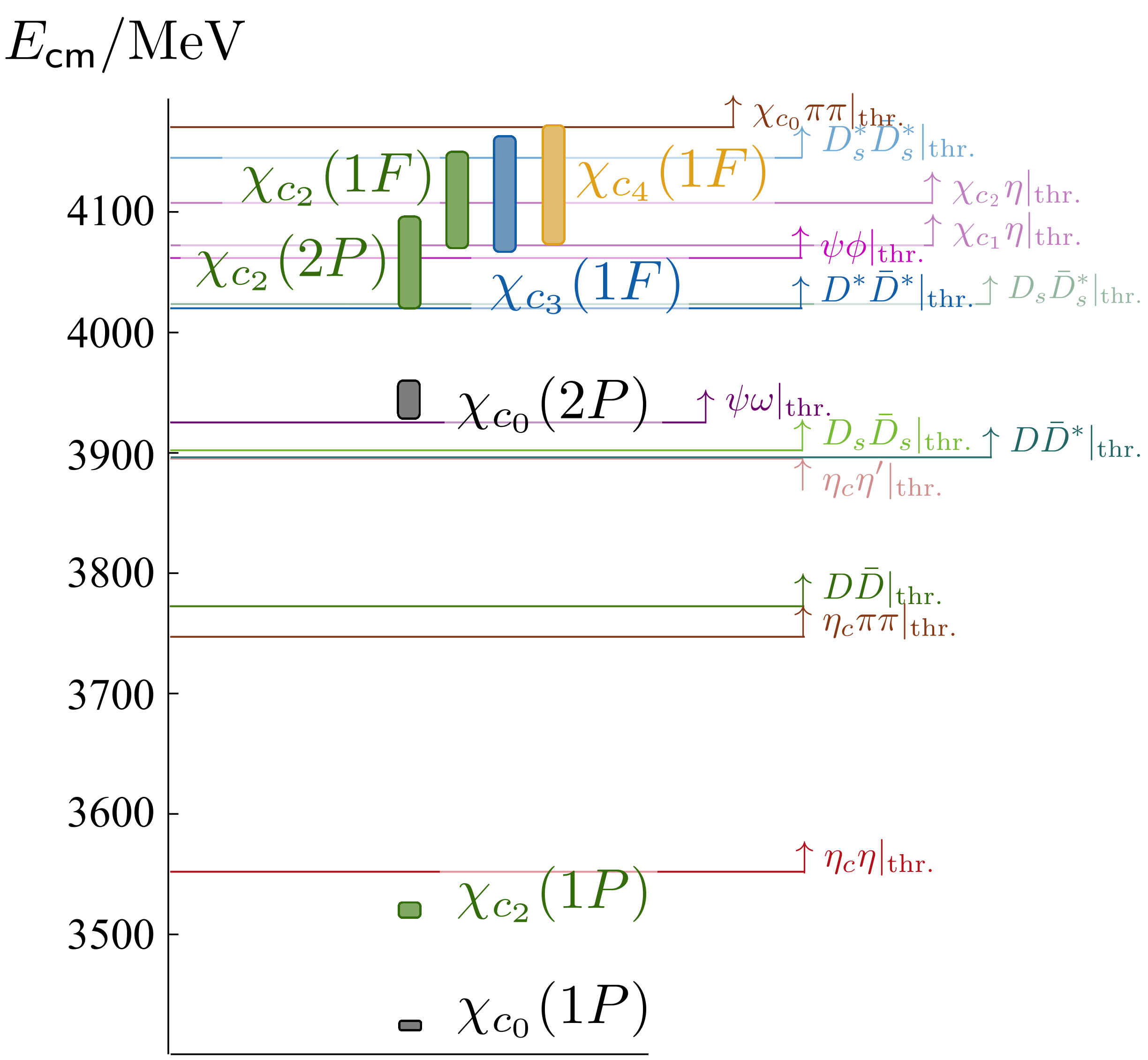
$$\Omega_\pi^\dagger = \sum_i v_i \mathcal{O}_i^\dagger$$

uses the eigenvector from the variational method performed in e.g. pion quantum numbers

using *distillation* (Peardon *et al* 2009)
many channels, many wick contractions

- compute a large correlation matrix
- solve generalised eigenvalue problem to extract energies

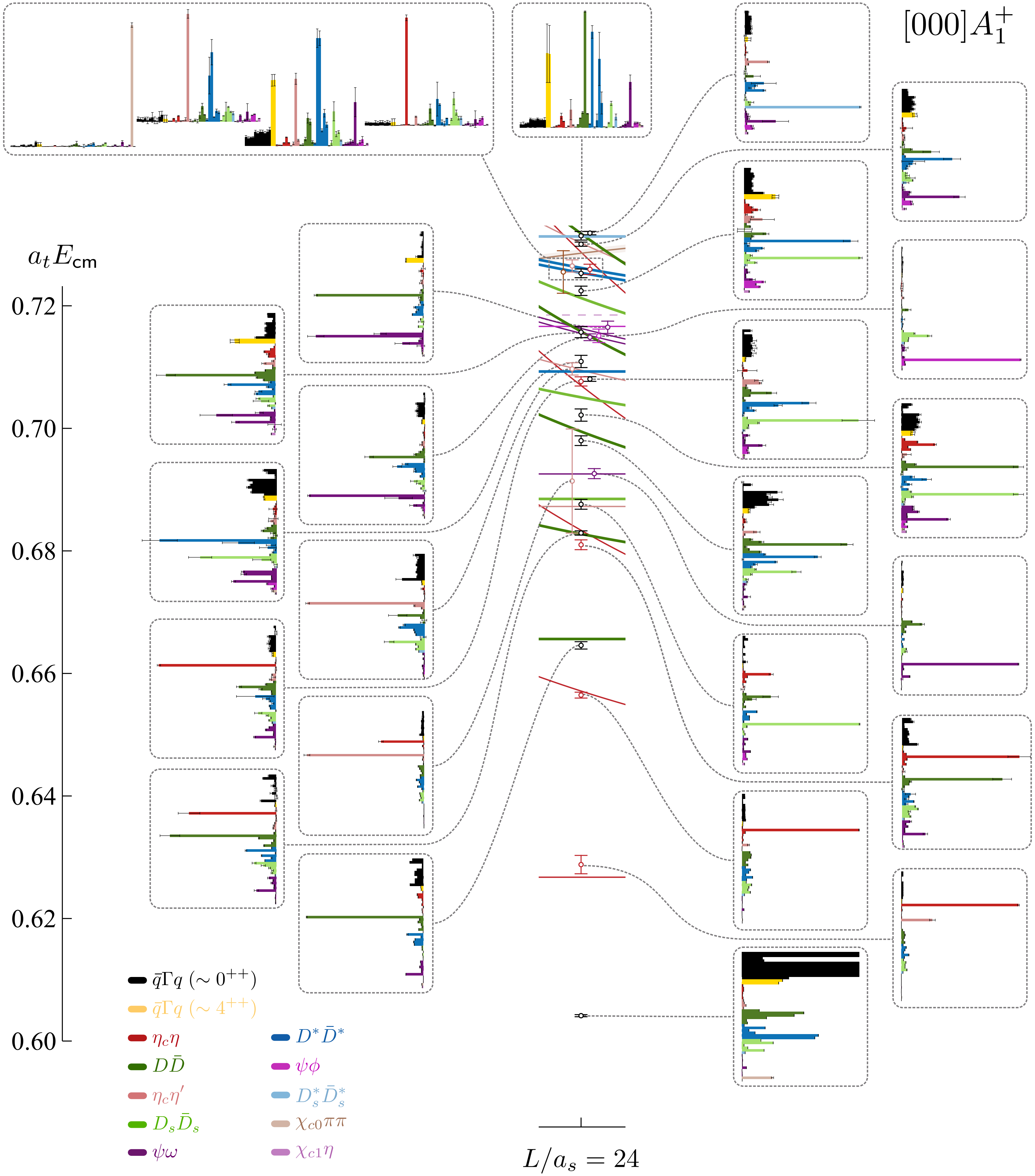
χ_{c0} & χ_{c2}



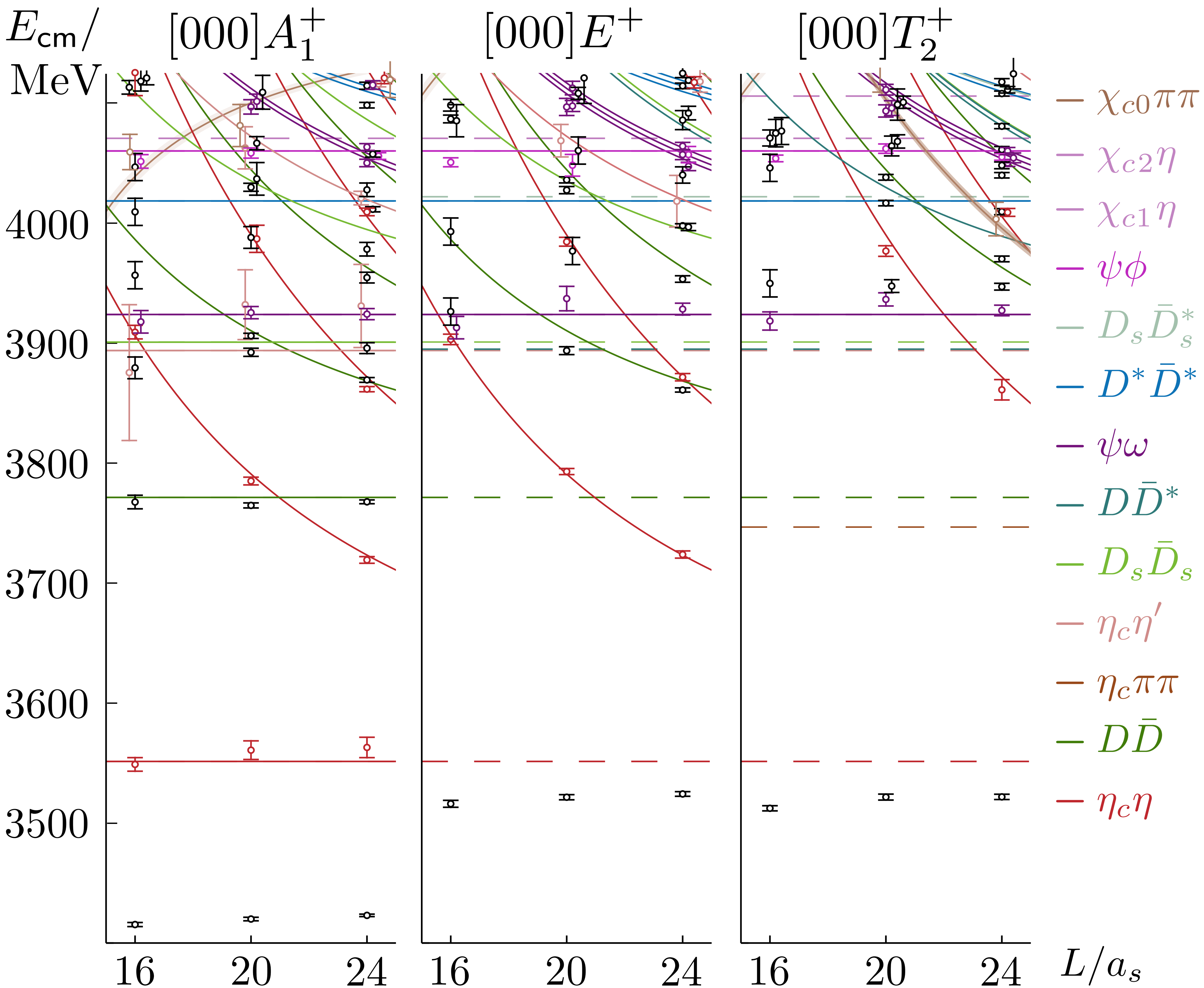
- spectra from qqbar operators only, Liu et al JHEP 1207 (2012) 126

- indicates energy regions where resonance effects are likely

- now: add all necessary meson-meson operators



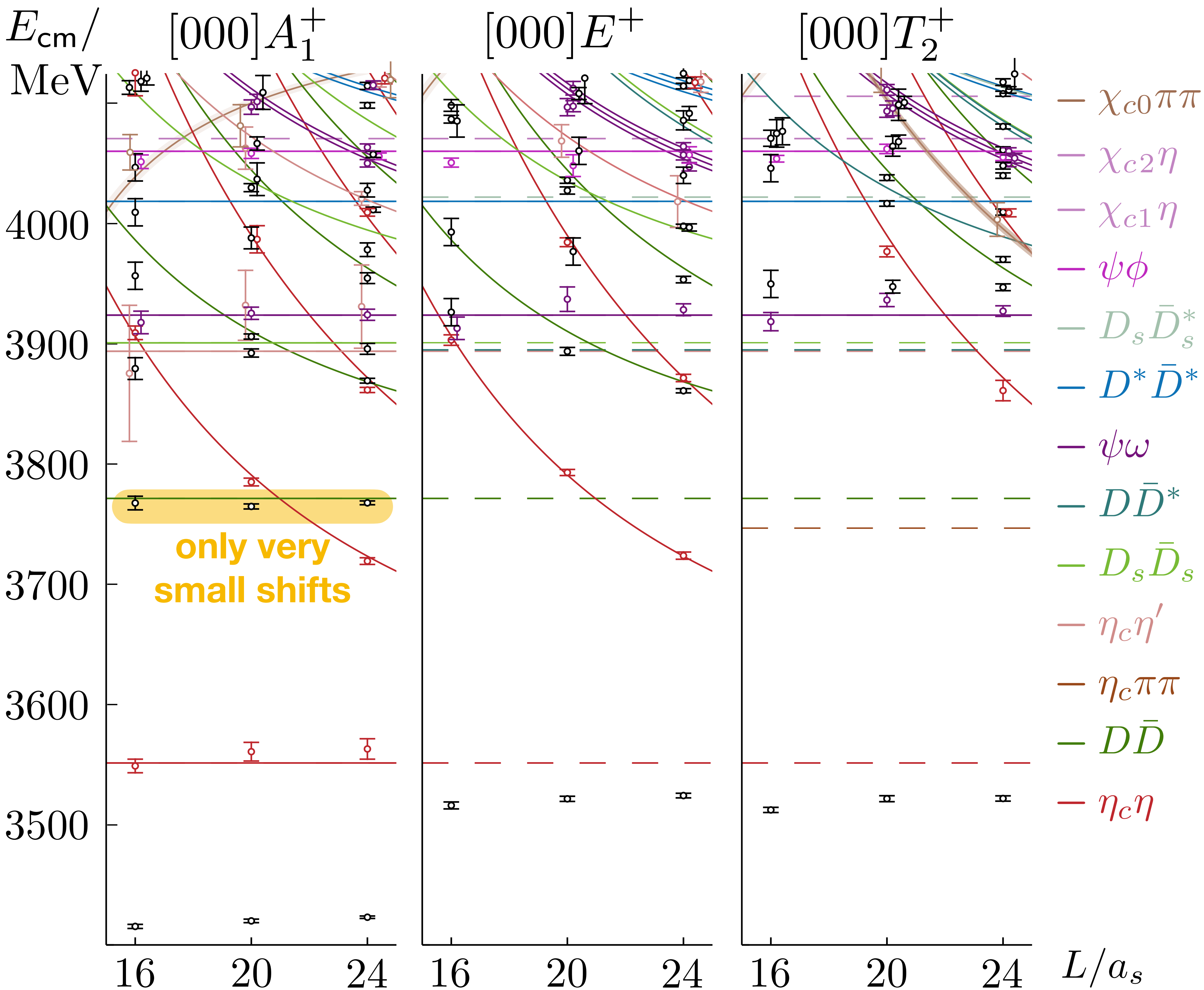
$[000]A_1^+$
$a_t E_{\mathrm{cm}}$
0.72
0.70
0.68
0.66
0.64
0.62
0.60
$\bar{q}\Gamma q\ (\sim 0^{++})$
$\bar{q}\Gamma q\ (\sim 4^{++})$
$\eta_c\eta$
$D\bar{D}$
$\eta_c\eta'$
$D_s\bar{D}_s$
$\psi\omega$
$D^*\bar{D}^*$
$\psi\phi$
$D_s^*\bar{D}_s^*$
$\chi_{c0}\pi\pi$
$\chi_{c1}\eta$
$L/a_s = 24$


E_{cm}/ MeV
$[000]A_1^+$
$[000]E^+$
$[000]T_2^+$
4000
3900
3800
3700
3600
3500
16
20
24
L/a_s
$\chi_{c0}\pi\pi$
$\chi_{c2}\eta$
$\chi_{c1}\eta$
$\psi\phi$
$D_s\bar{D}_s^*$
$D^*\bar{D}^*$
$\psi\omega$
$D\bar{D}^*$
$D_s\bar{D}_s$
$\eta_c\eta'$
$\eta_c\pi\pi$
$D\bar{D}$
$\eta_c\eta$

"0++"

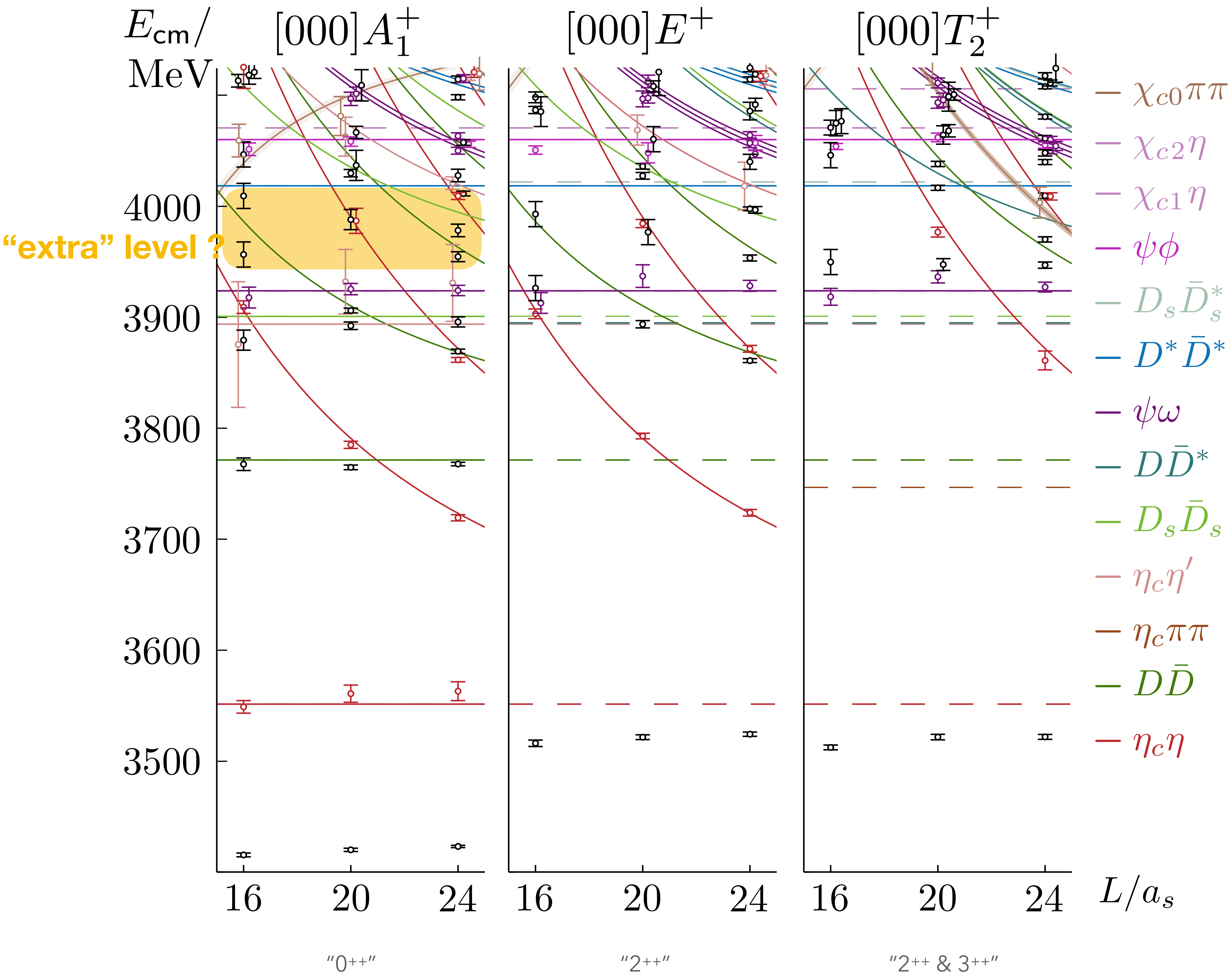
"2++"

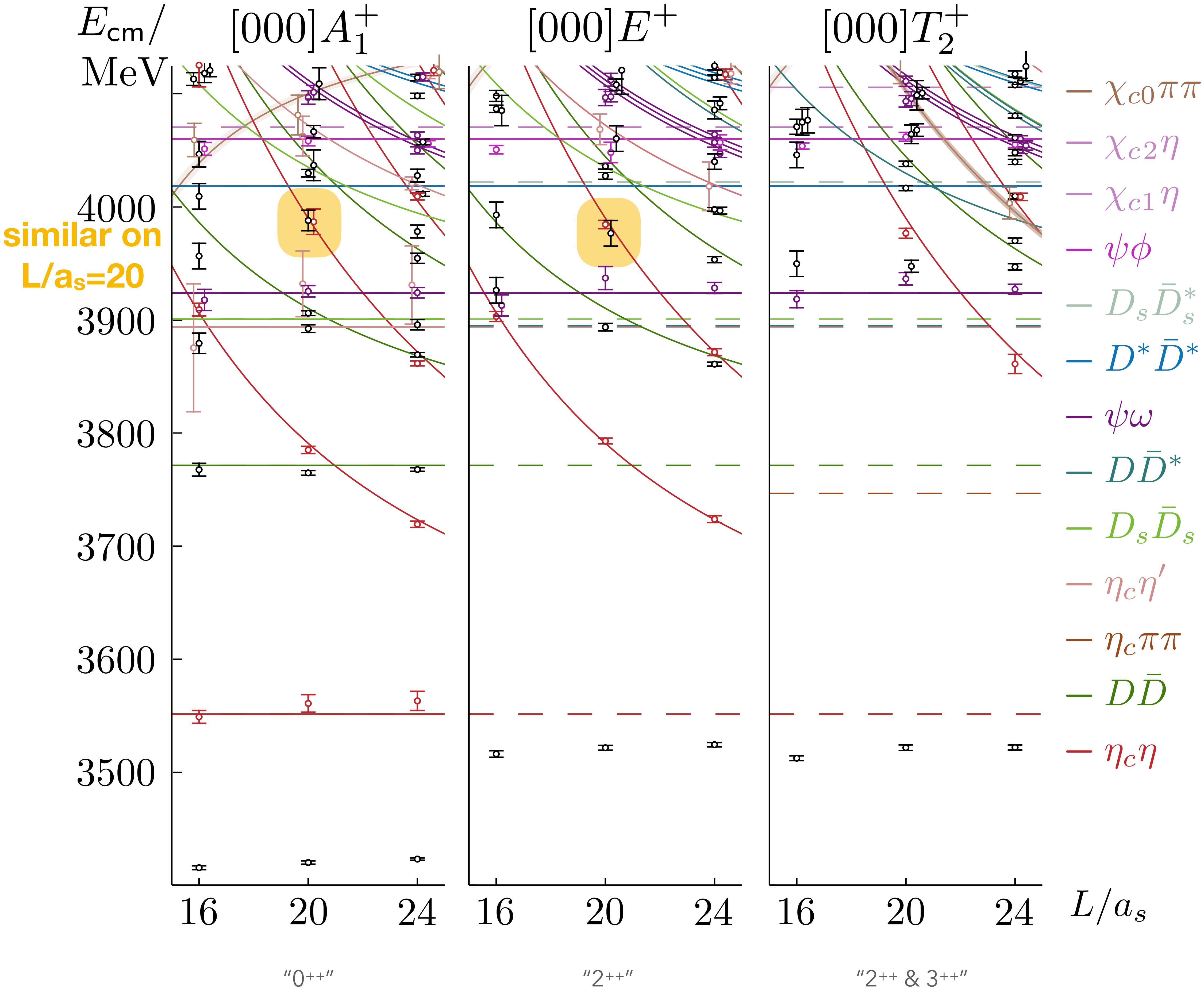
"2++ & 3++"


E_{cm}/ MeV
$[000]A_1^+$
$[000]E^+$
$[000]T_2^+$
4000
3900
3800
3700
3600
3500
16
20
24
L/a_s
only very small shifts
$\chi_{c0}\pi\pi$
$\chi_{c2}\eta$
$\chi_{c1}\eta$
$\psi\phi$
$D_s\bar{D}_s^*$
$D^*\bar{D}^*$
$\psi\omega$
$D\bar{D}^*$
$D_s\bar{D}_s$
$\eta_c\eta'$
$\eta_c\pi\pi$
$D\bar{D}$
$\eta_c\eta$

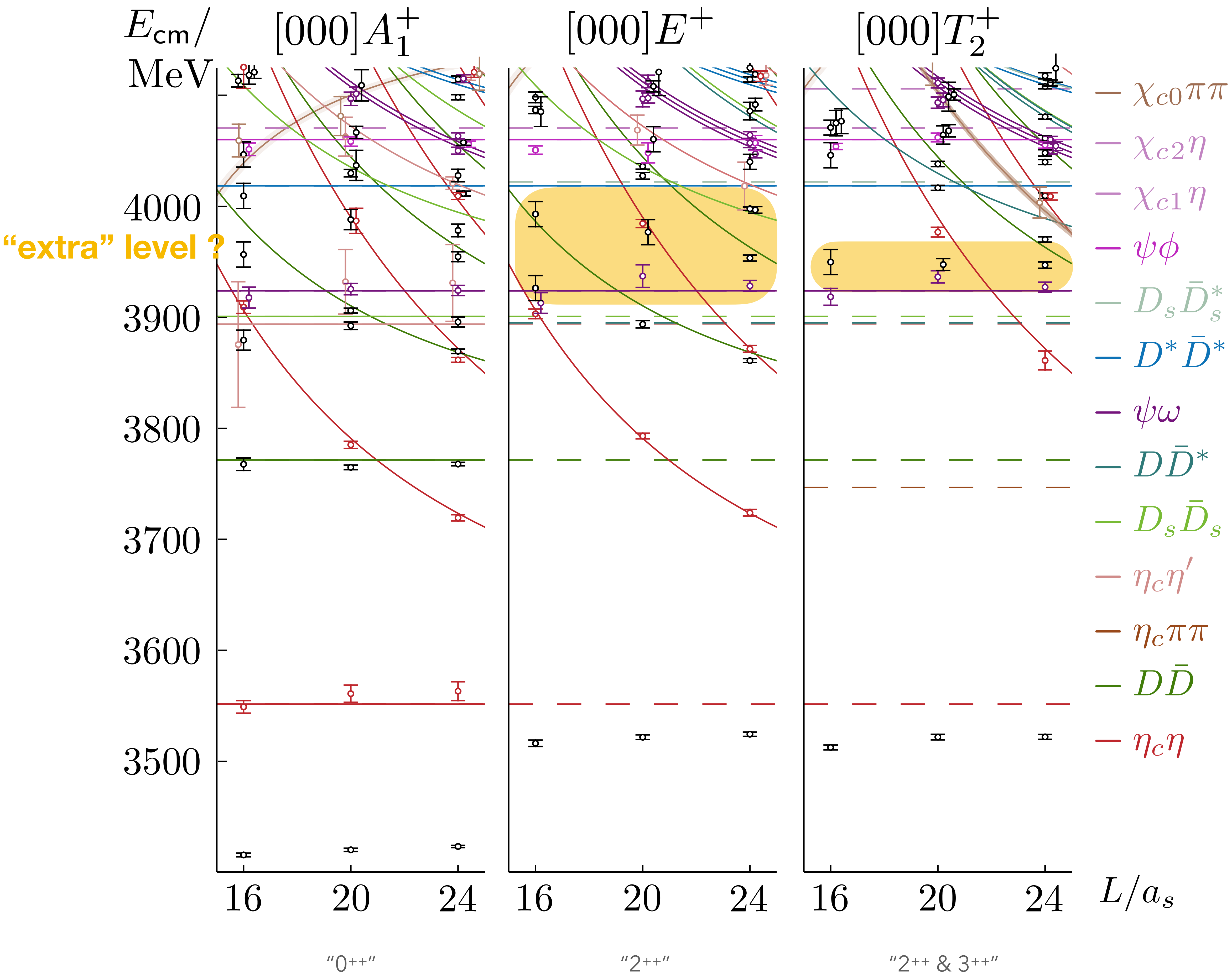
"0++" "2++" "2++ & 3++"

S+D, zero total momentum





E_{cm}/ MeV
$[000]A_1^+$
$[000]E^+$
$[000]T_2^+$
4000
3900
3800
3700
3600
3500
similar on L/a$_s$=20
16
20
24
L/a_s
$\chi_{c0}\pi\pi$
$\chi_{c2}\eta$
$\chi_{c1}\eta$
$\psi\phi$
$D_s\bar{D}_s^*$
$D^*\bar{D}^*$
$\psi\omega$
$D\bar{D}^*$
$D_s\bar{D}_s$
$\eta_c\eta'$
$\eta_c\pi\pi$
$D\bar{D}$
$\eta_c\eta$
"0++"
"2++"
"2++ & 3++"



E_{cm}/ MeV
$[000]A_1^+$
$[000]E^+$
$[000]T_2^+$
"extra" level ?
4000
3900
3800
3700
3600
3500
16
20
24
L/a_s
$\chi_{c0}\pi\pi$
$\chi_{c2}\eta$
$\chi_{c1}\eta$
$\psi\phi$
$D_s\bar{D}_s^*$
$D^*\bar{D}^*$
$\psi\omega$
$D\bar{D}^*$
$D_s\bar{D}_s$
$\eta_c\eta'$
$\eta_c\pi\pi$
$D\bar{D}$
$\eta_c\eta$
"0++"
"2++"
"2++ & 3++"

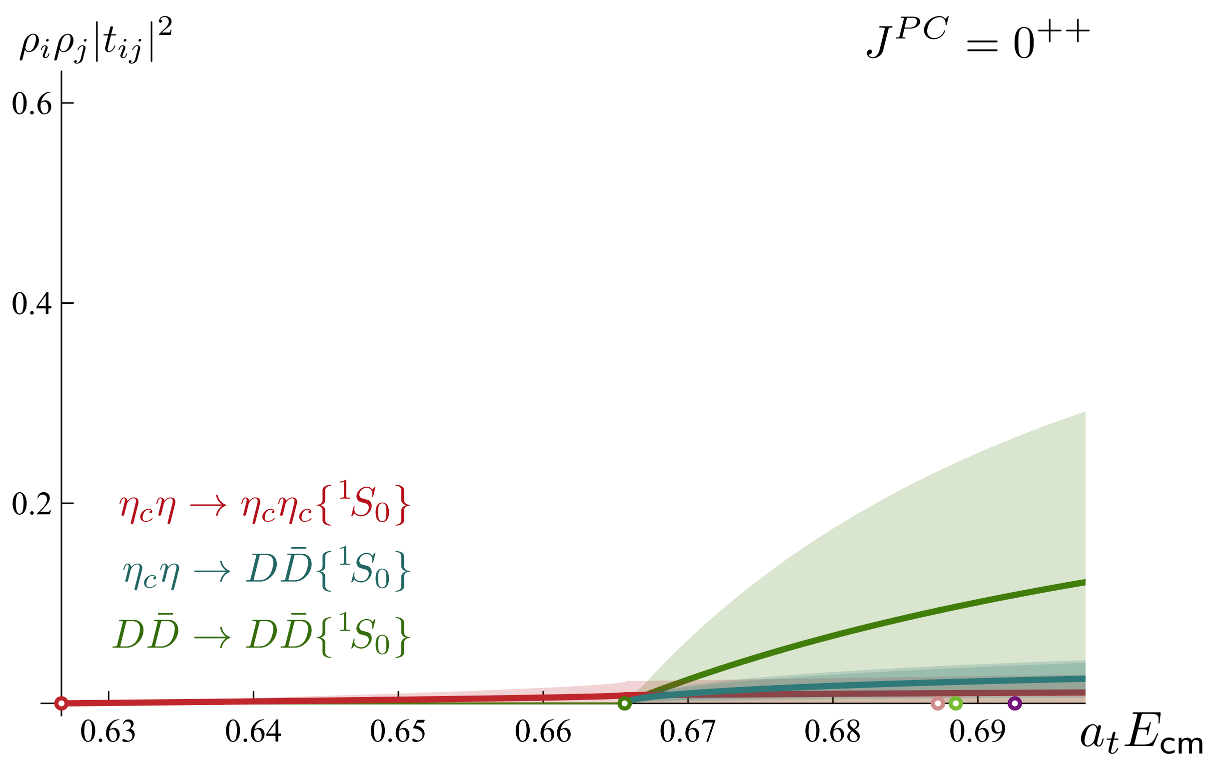
$$\boldsymbol{S} = \mathbf{1} + 2i\boldsymbol{\rho}^{\frac{1}{2}} \cdot \boldsymbol{t} \cdot \boldsymbol{\rho}^{\frac{1}{2}}$$

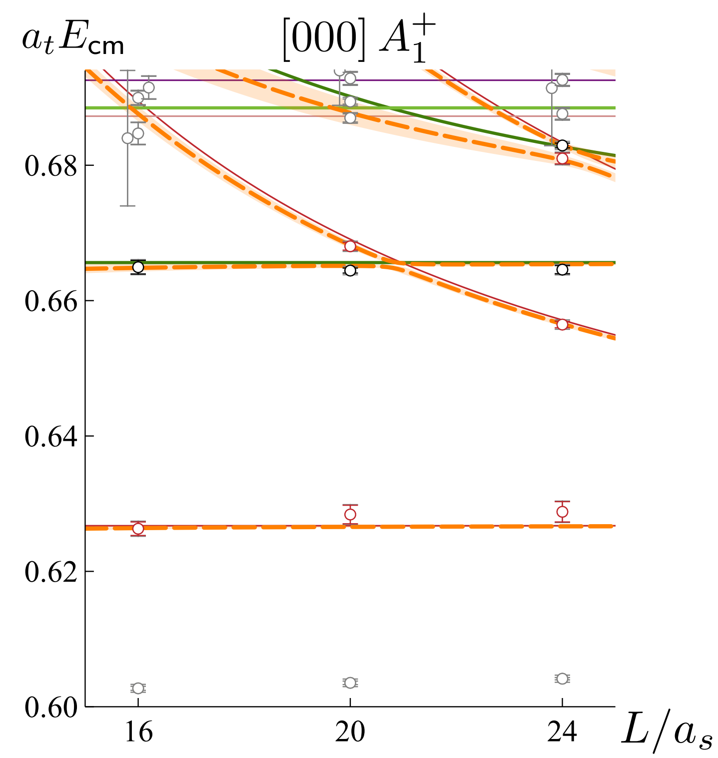
$$\boldsymbol{t}^{-1} = \boldsymbol{K}^{-1} + \boldsymbol{I}$$

$$\mathrm{Im} I_{ij} = -\rho_i = 2k_i/\sqrt{s}$$

$$\det[\mathbf{1} + i\boldsymbol{\rho} \cdot \boldsymbol{t}\,(\mathbf{1} + i\boldsymbol{\mathcal{M}}(L))] = 0$$

$$\boldsymbol{K} = \begin{bmatrix} \gamma_{\eta_c\eta\to\eta_c\eta} & \gamma_{\eta_c\eta\to D\bar{D}} \\ \gamma_{\eta_c\eta\to D\bar{D}} & \gamma_{D\bar{D}\to D\bar{D}} \end{bmatrix}$$



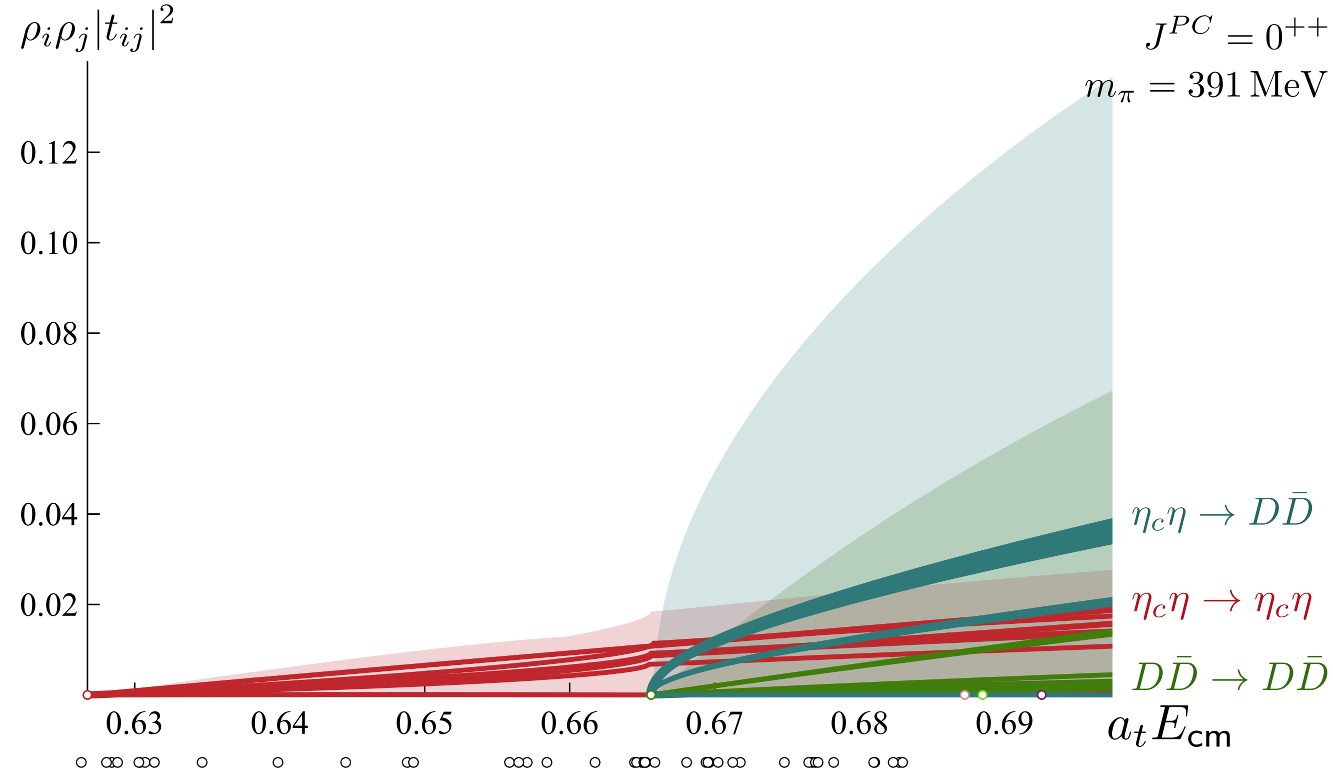


using rest-frame only

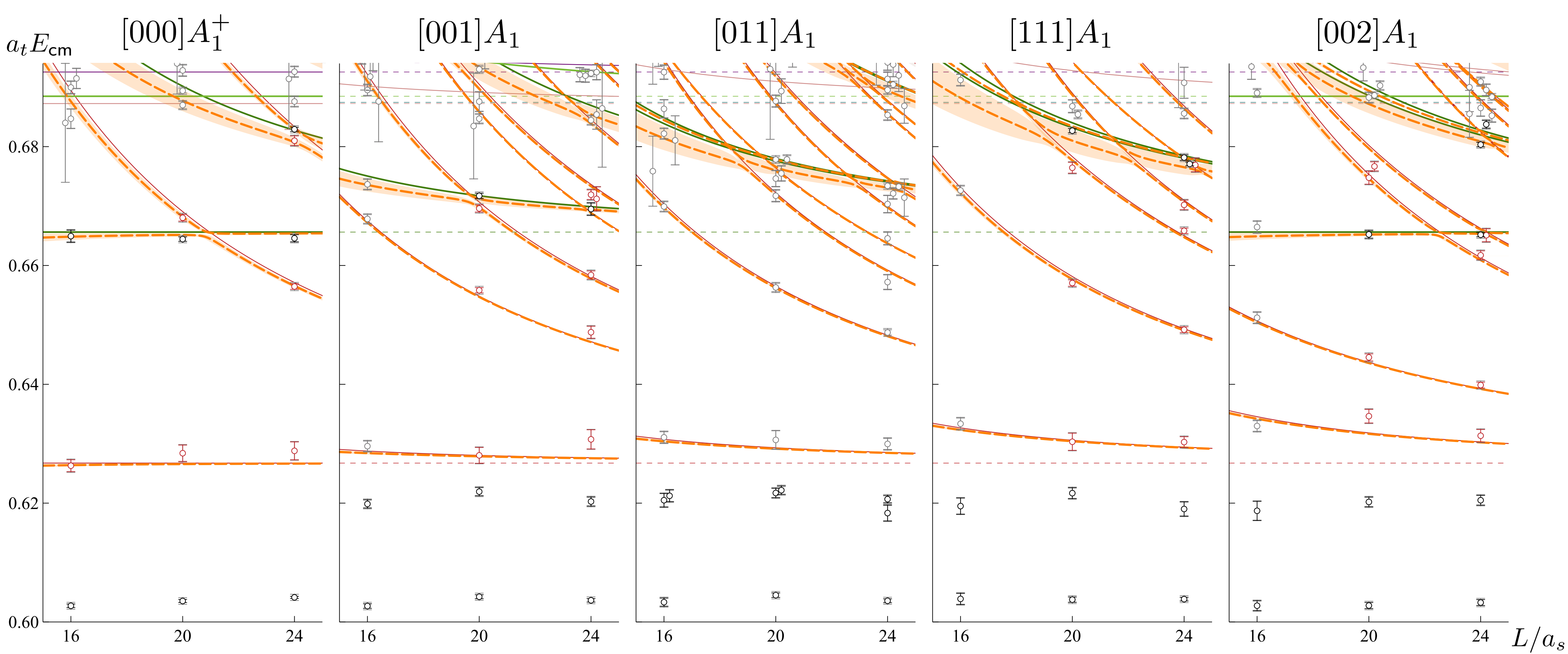
$$\begin{aligned} \gamma_{\eta_c\eta\to\eta_c\eta} &= (0.34 \pm 0.23 \pm 0.09) \\ \gamma_{\eta_c\eta\to D\bar{D}} &= (0.58 \pm 0.29 \pm 0.05) \\ \gamma_{D\bar{D}\to D\bar{D}} &= (1.39 \pm 1.19 \pm 0.24) \end{aligned} \quad \begin{bmatrix} 1.00 & 0.77 & -0.24 \\ & 1.00 & -0.22 \\ & & 1.00 \end{bmatrix}$$

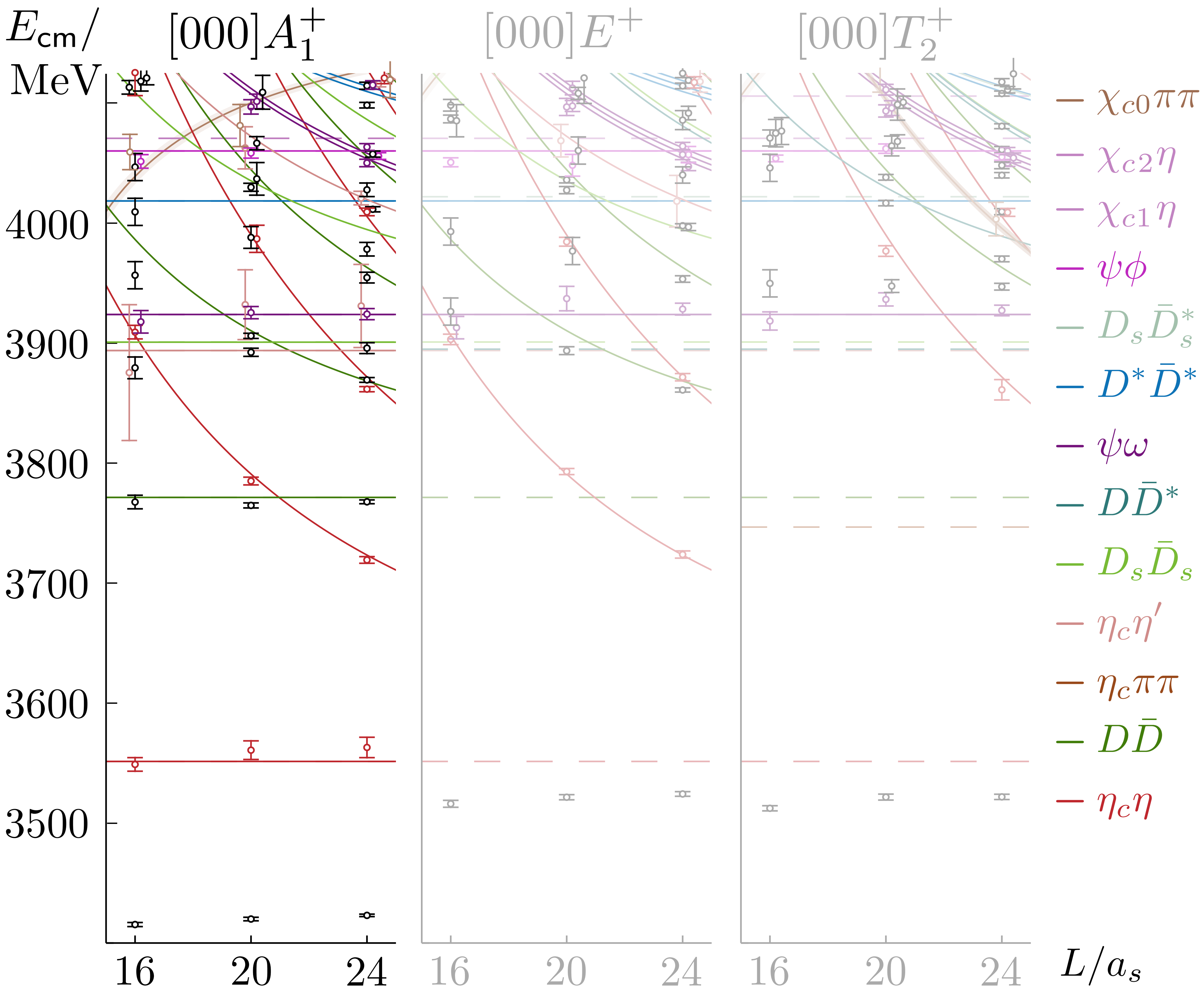
$$\chi^2/N_{\mathrm{dof}} = \frac{5.65}{10-3} = 0.81$$

(smaller range)



using zero and non-zero total momentum





$E_{\mathsf{cm}}/$ MeV
$[000]A_1^+$
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3900
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L/a_s
$\chi_{c0}\pi\pi$
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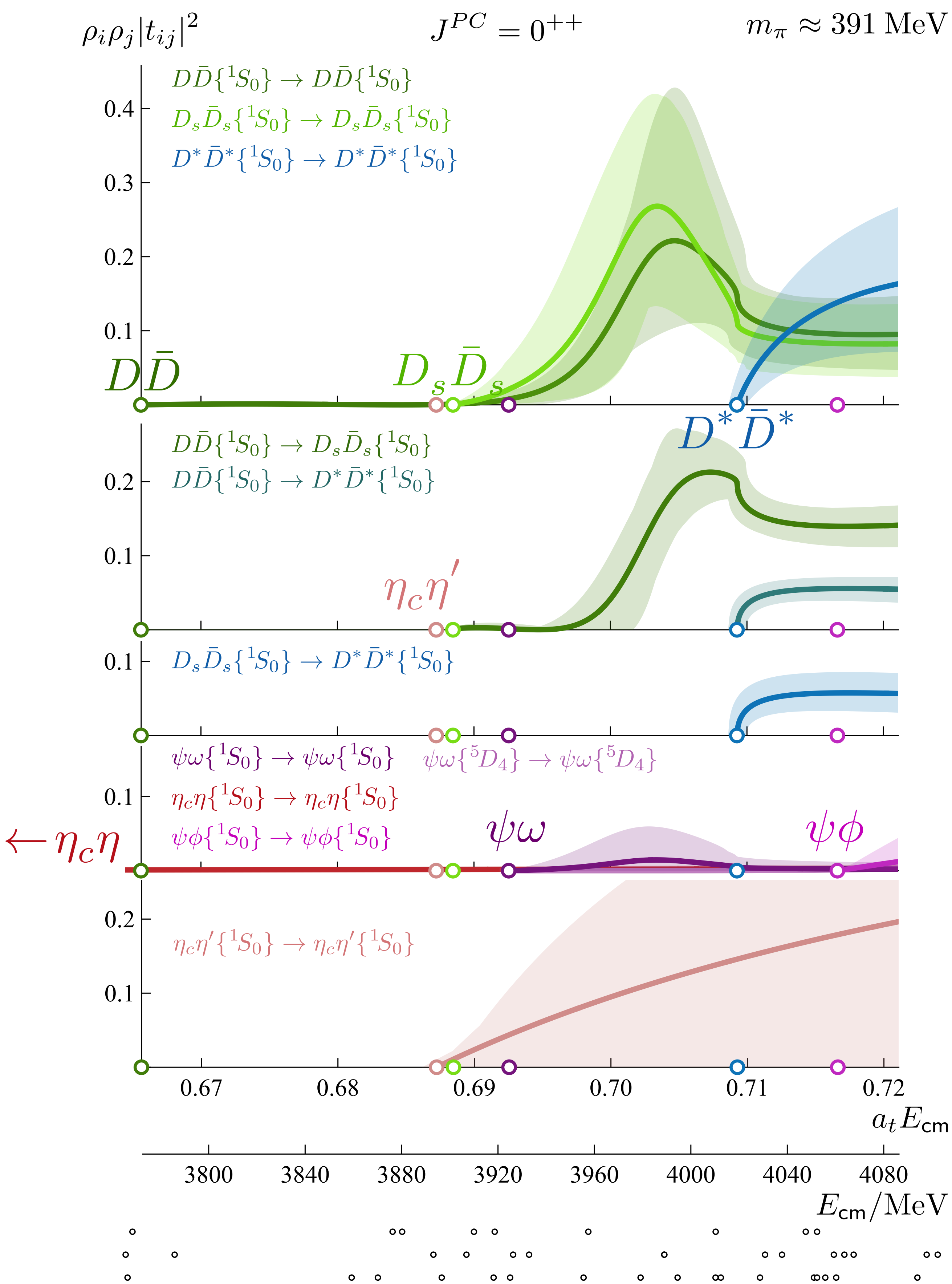
$D_s\bar{D}_s^*$
$D^*\bar{D}^*$
$\psi\omega$
$D\bar{D}^*$
$D_s\bar{D}_s$
$\eta_c\eta'$
$\eta_c\pi\pi$
$D\bar{D}$
$\eta_c\eta$

"0++"

"2++"

"2++ & 3++"



three channels open close together: $\eta_c\eta'$, $D_s\bar{D}_s$, $\psi\omega$

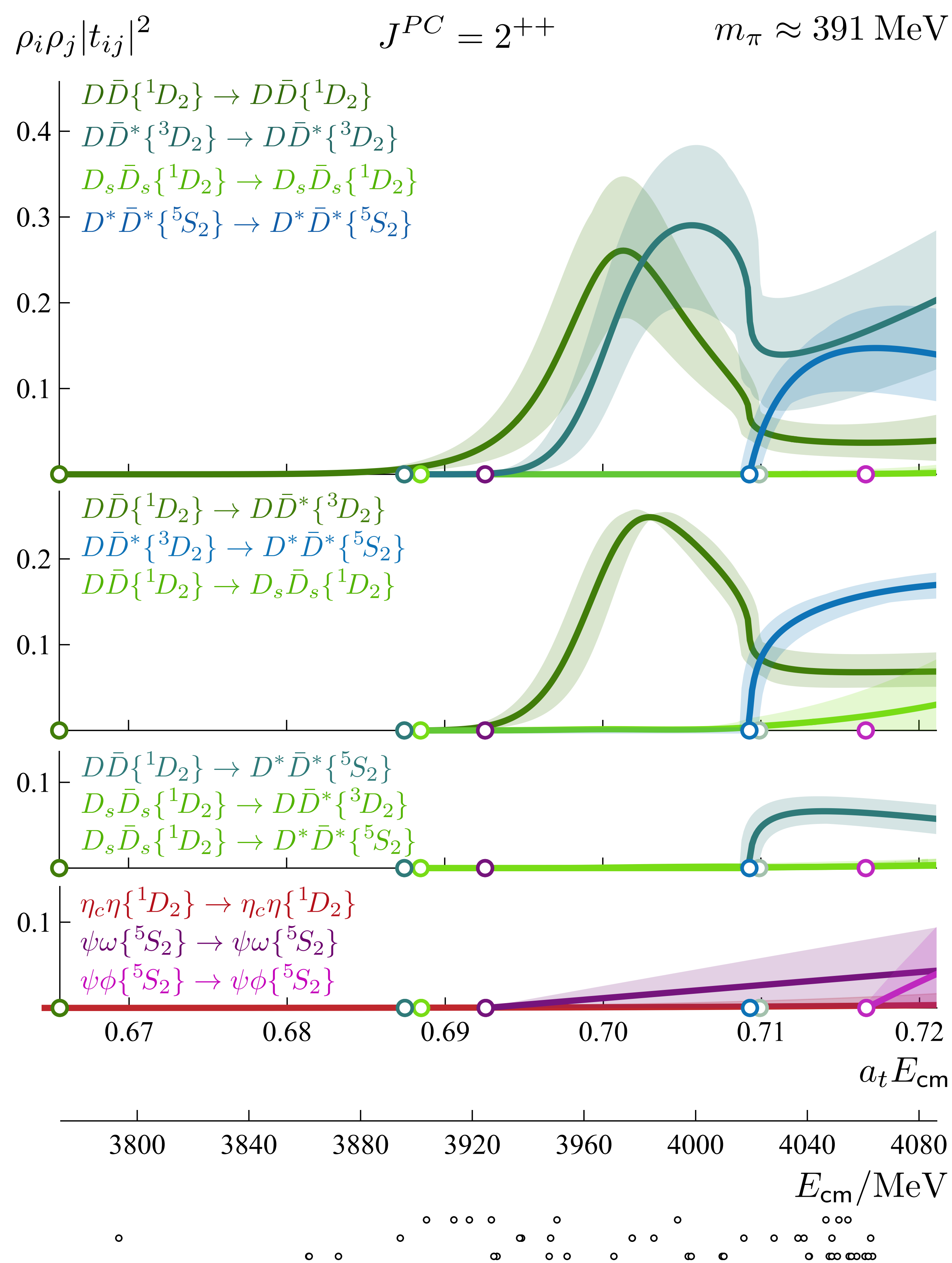
operator overlaps suggest $D^*\bar{D}^*$ is important

$\psi\phi$ has been seen to be important in some places

consider 7-channel system

$$K_{ij} = \frac{g_i g_j}{m^2 - s} + \gamma_{ij}$$

K-matrix pole terms become necessary to obtain a good quality of fit



7-channels, mixture of *S* and *D*

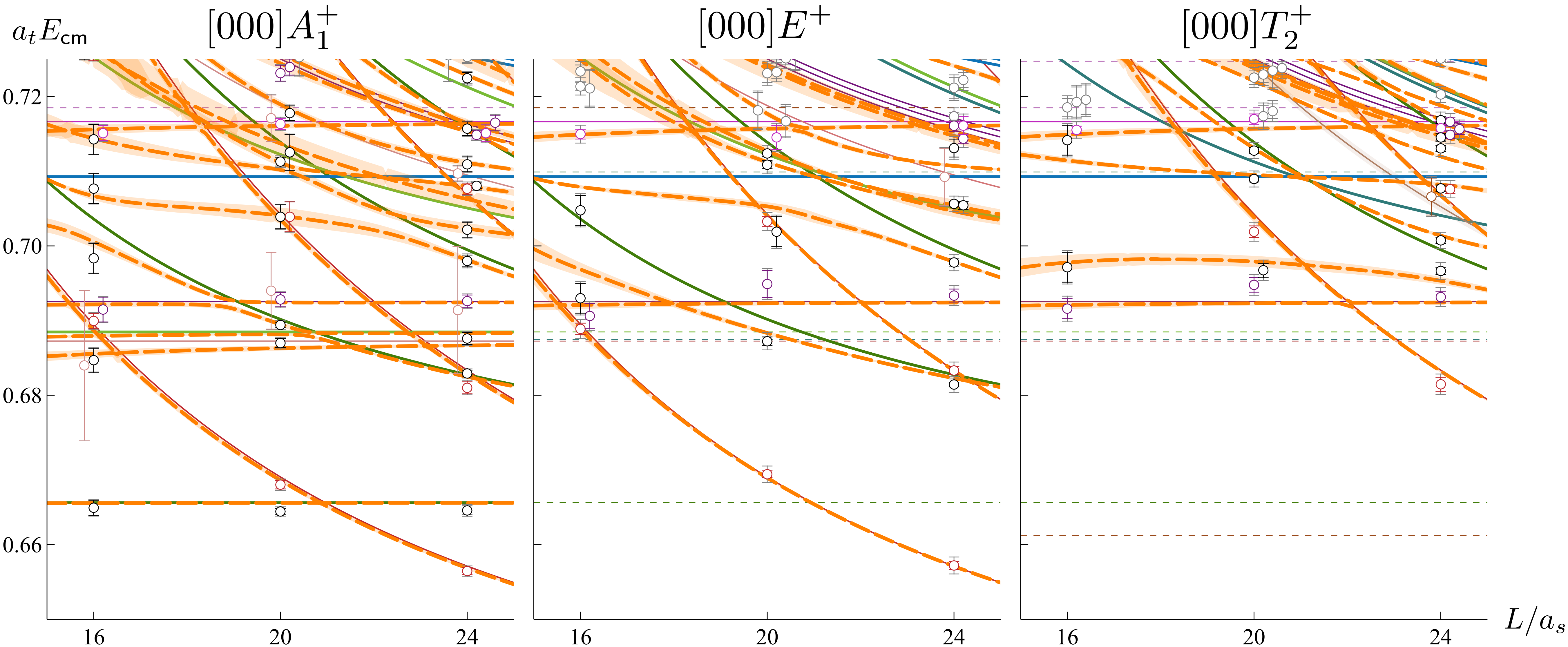
$D\bar{D}, D_s\bar{D}_s\{^1D_2\}$ $D\bar{D}^*\{^3D_2\}$ $D^*\bar{D}^*\{^5S_2\}$

$\eta_c\eta\{^1D_2\}$ $\psi\omega, \psi\phi\{^5S_2\}$

peaks at a similar energy

very small DsDs amplitudes -
some phase space suppression

DD* is large -
similar phase space to DsDs


$a_t E_{\mathsf{cm}}$
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20
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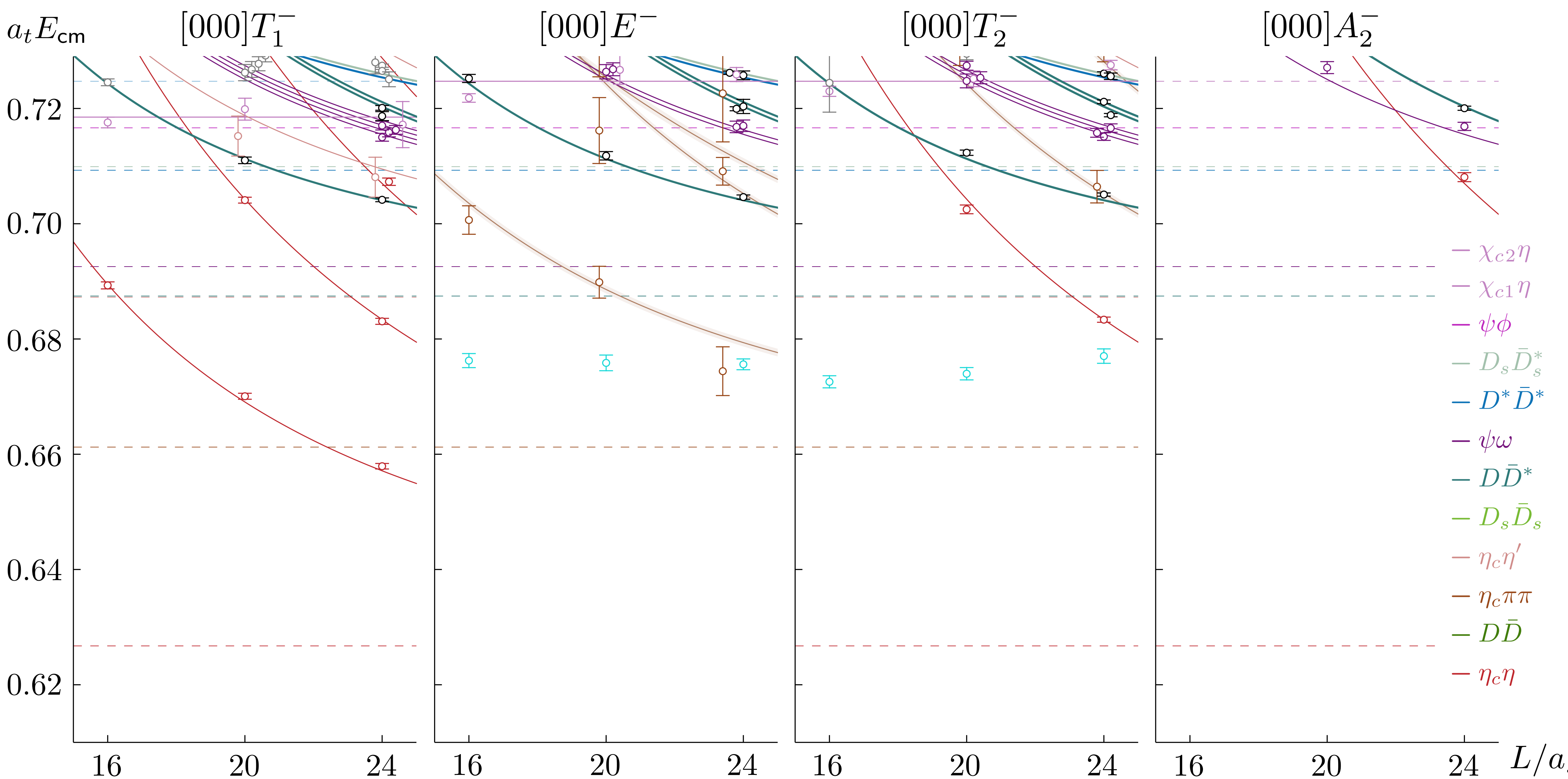
L/a_s

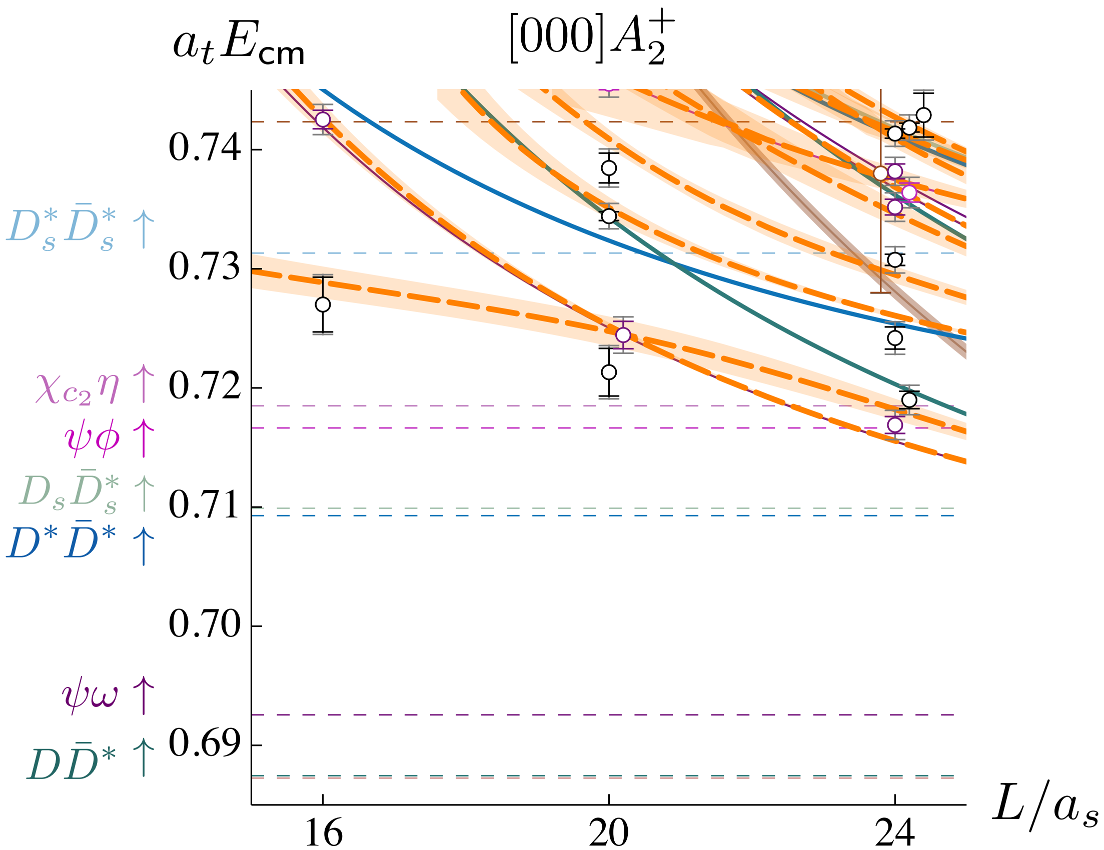


we also computed lattice irreps
with non-zero total momentum

P=- partial waves can then contribute

very little going on

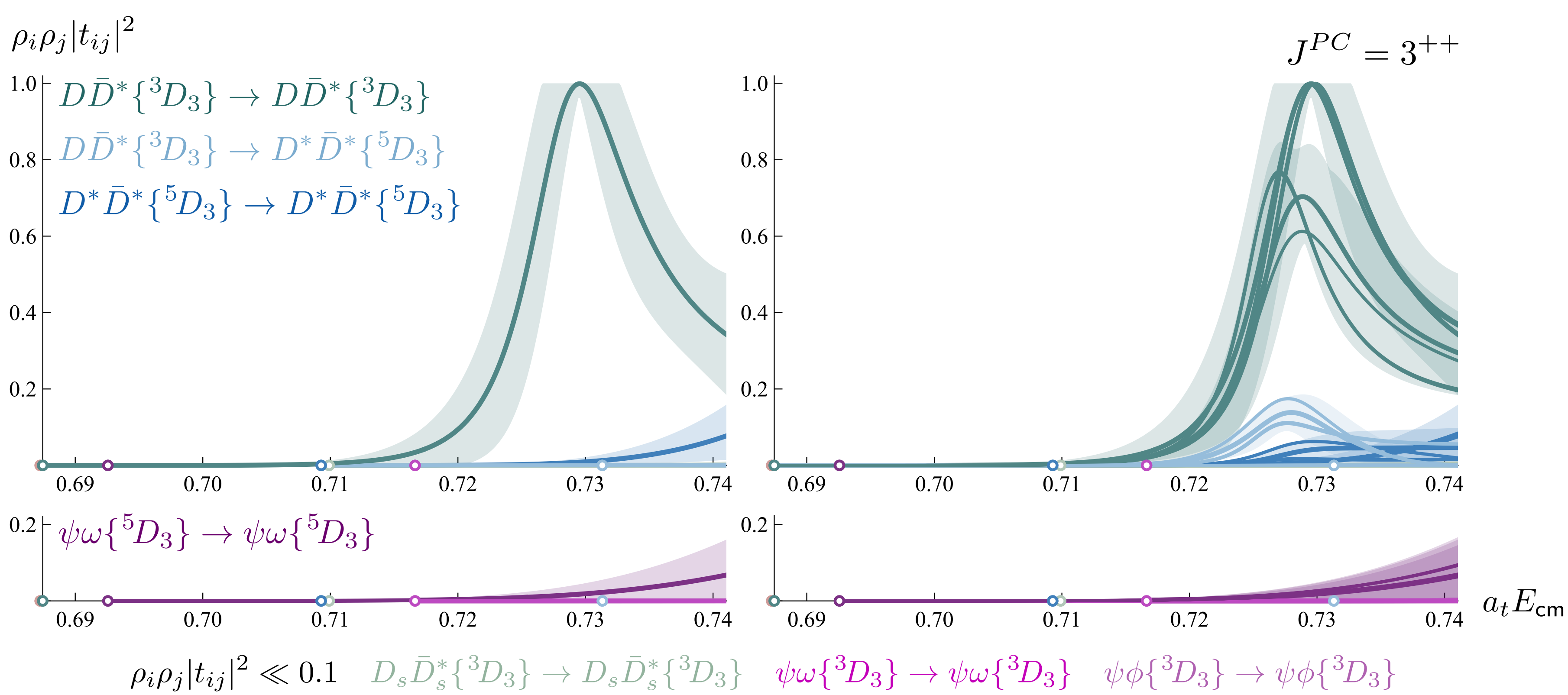
an η_{c2} 2^{-+} state arises below DD*



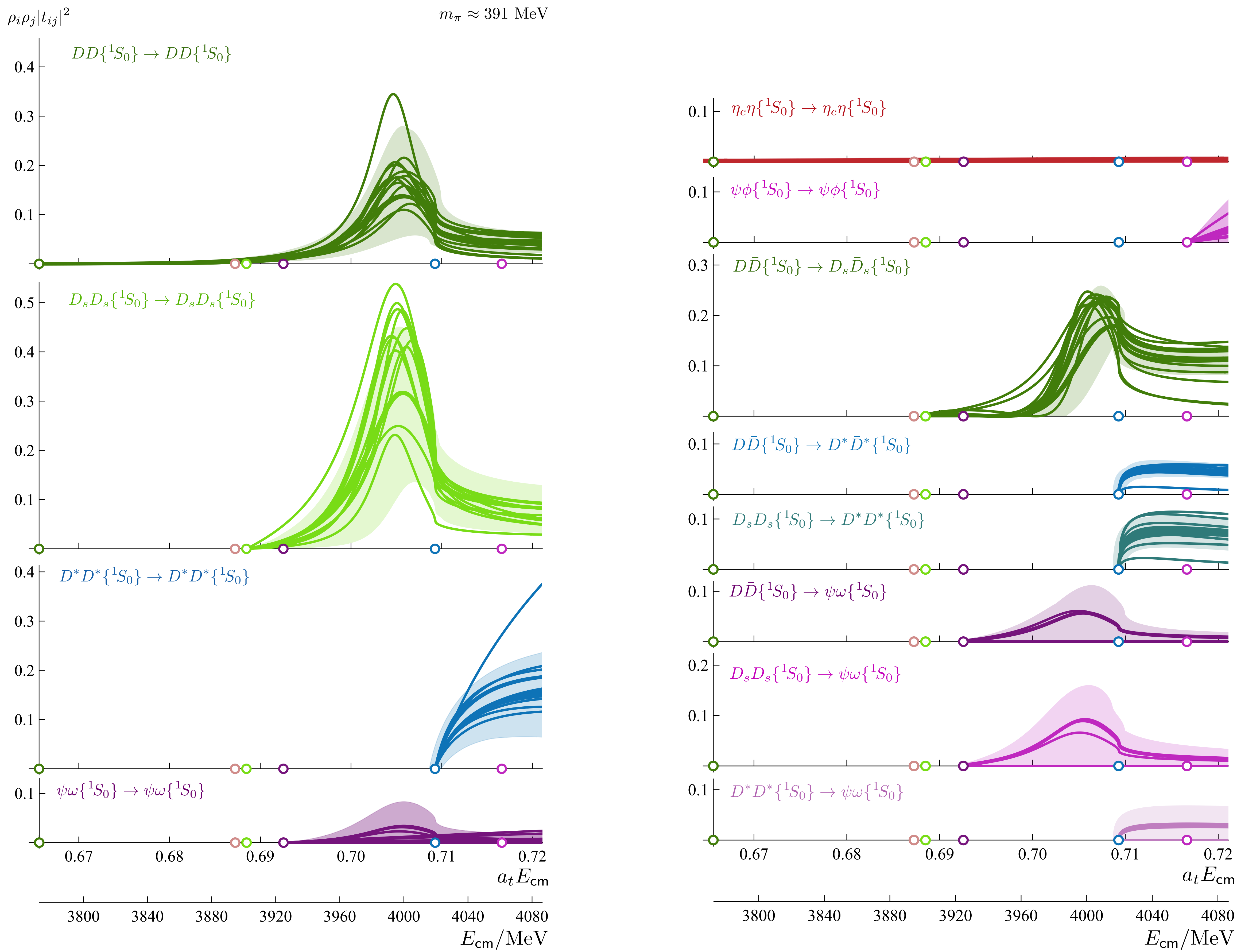
extra level and resonance higher up

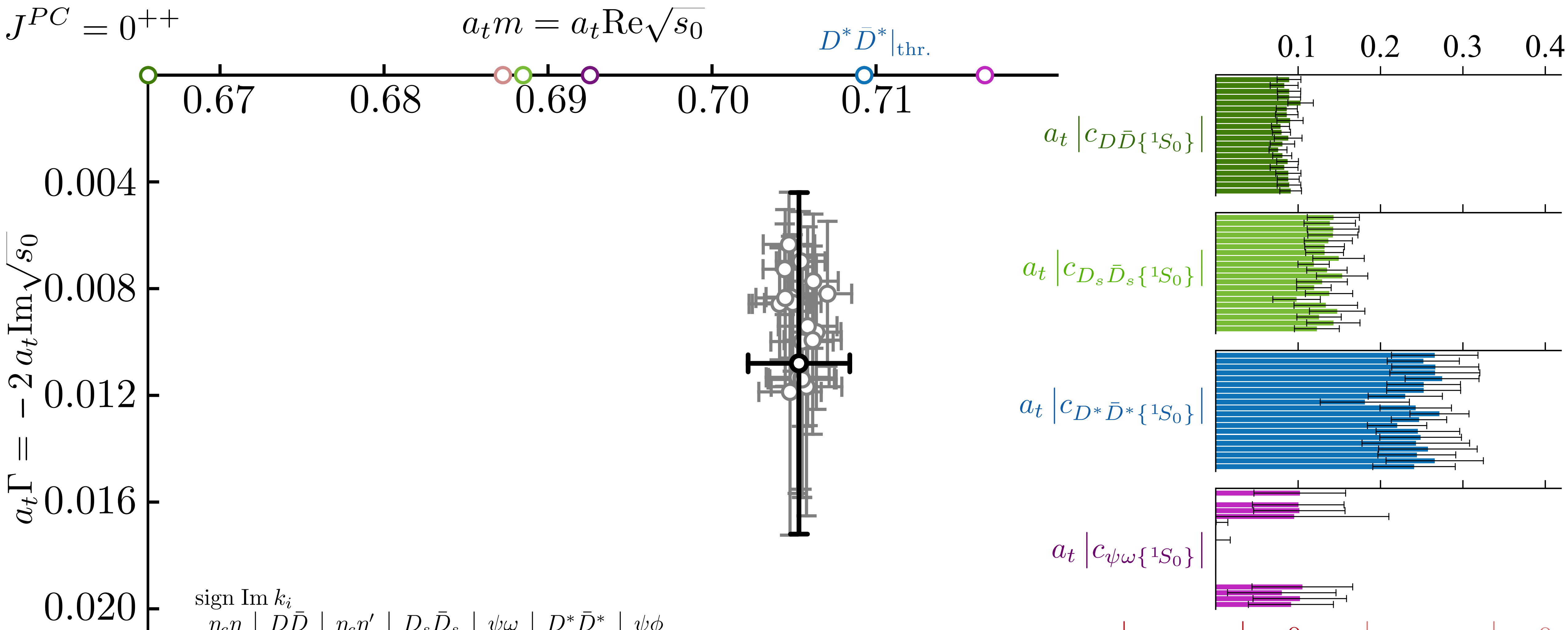
two classes of amplitudes were found:

- zero D*D* coupling
- finite D*D* coupling
- all had significant DD* coupling
- amps very small below 4050 MeV ($a_t E_{cm}$=0.715)



amplitude variations - scalar

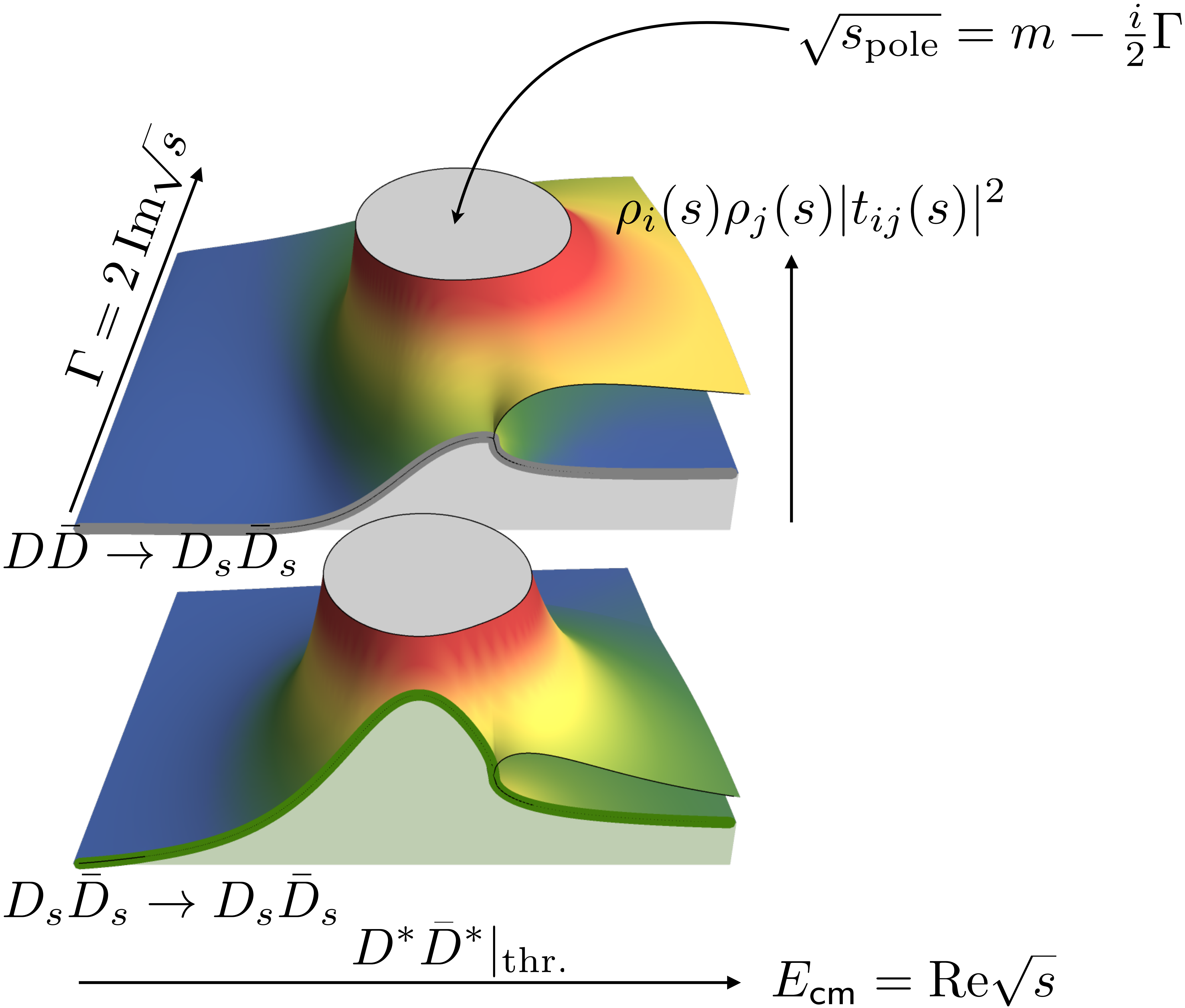


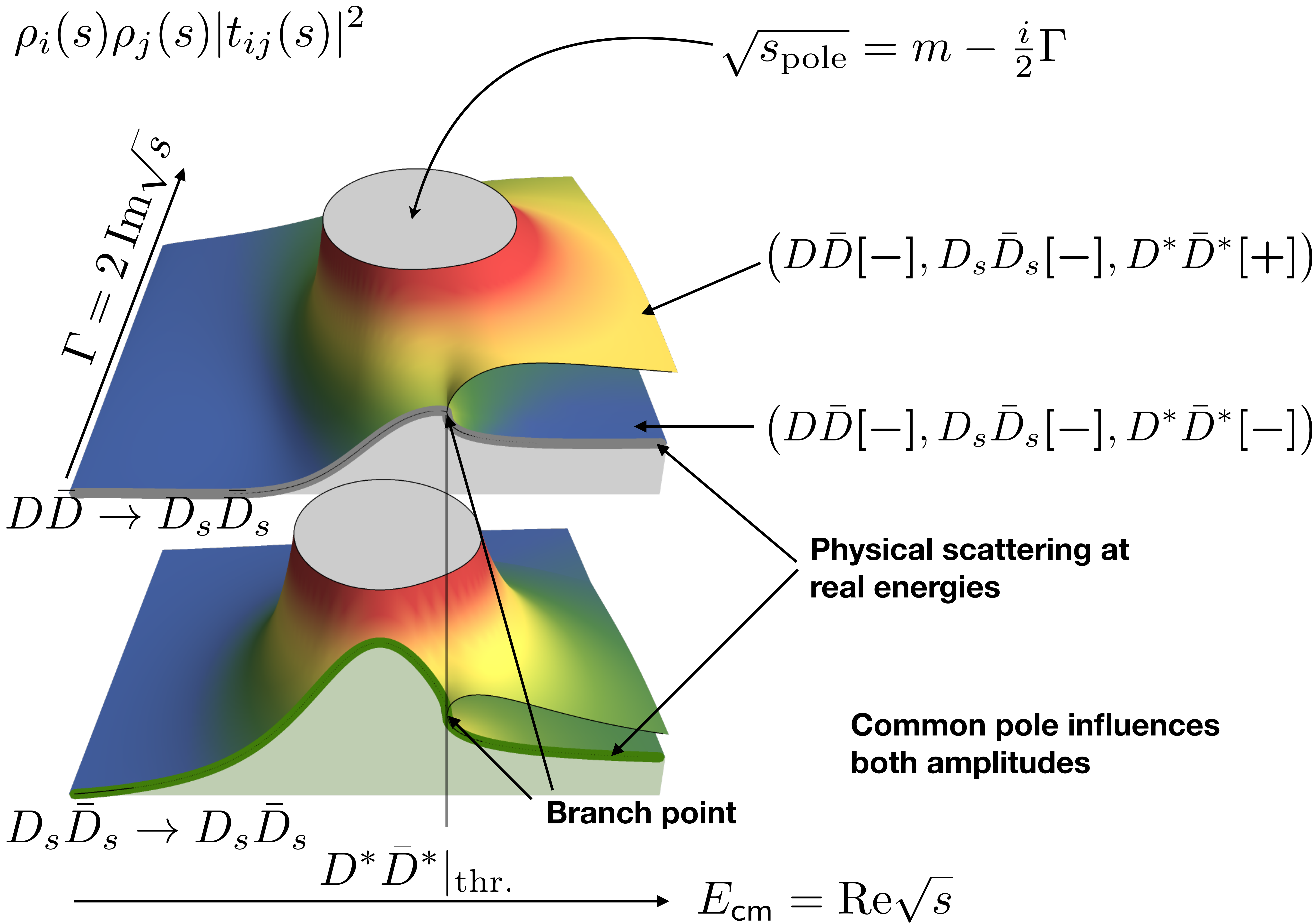


$\eta_c\eta$	$D\bar{D}$	$\eta_c\eta'$	$D_s\bar{D}_s$	$\psi\omega$	$D^*\bar{D}^*$	$\psi\phi$
−	−	−	−	−	+	+

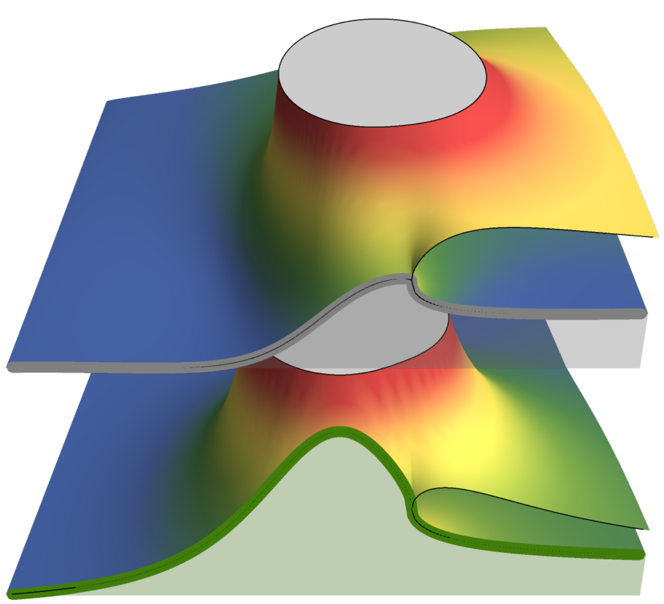
$a_t\,|c_{\eta_c\eta\{^1S_0\}}| \sim 0$ $a_t\,|c_{\eta_c\eta'\{^1S_0\}}| \sim 0$

$a_t\,|c_{\psi\phi\{^1S_0\}}| \sim 0$


$\sqrt{s_{\mathrm{pole}}} = m - \frac{i}{2}\Gamma$
$\Gamma = 2\,\mathrm{Im}\sqrt{s}$
$\rho_i(s)\rho_j(s)|t_{ij}(s)|^2$
$D\bar{D} \to D_s\bar{D}_s$
$D_s\bar{D}_s \to D_s\bar{D}_s$
$D^*\bar{D}^*|_{\mathrm{thr.}}$
$E_{\mathrm{cm}} = \mathrm{Re}\sqrt{s}$


$\rho_i(s)\rho_j(s)|t_{ij}(s)|^2$
$\sqrt{s_{\rm pole}} = m - \frac{i}{2}\Gamma$
$\Gamma = 2\,{\rm Im}\sqrt{s}$
$(D\bar{D}[-], D_s\bar{D}_s[-], D^*\bar{D}^*[+])$
$(D\bar{D}[-], D_s\bar{D}_s[-], D^*\bar{D}^*[-])$
$D\bar{D} \to D_s\bar{D}_s$
Physical scattering at real energies
Common pole influences both amplitudes
$D_s\bar{D}_s \to D_s\bar{D}_s$
Branch point
$D^*\bar{D}^*|_{\rm thr.}$
$E_{\rm cm} = {\rm Re}\sqrt{s}$

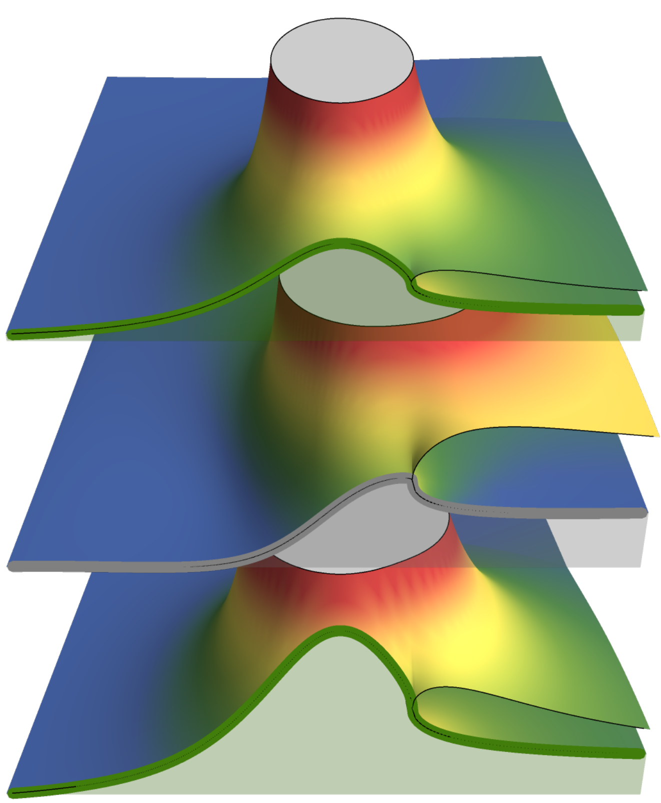
$$\rho_i(s)\rho_j(s)|t_{ij}(s)|^2$$



$D\bar{D} \to D_s\bar{D}_s$

$D_s\bar{D}_s \to D_s\bar{D}_s$

$$\rho_i(s)\rho_j(s)|t_{ij}(s)|^2$$

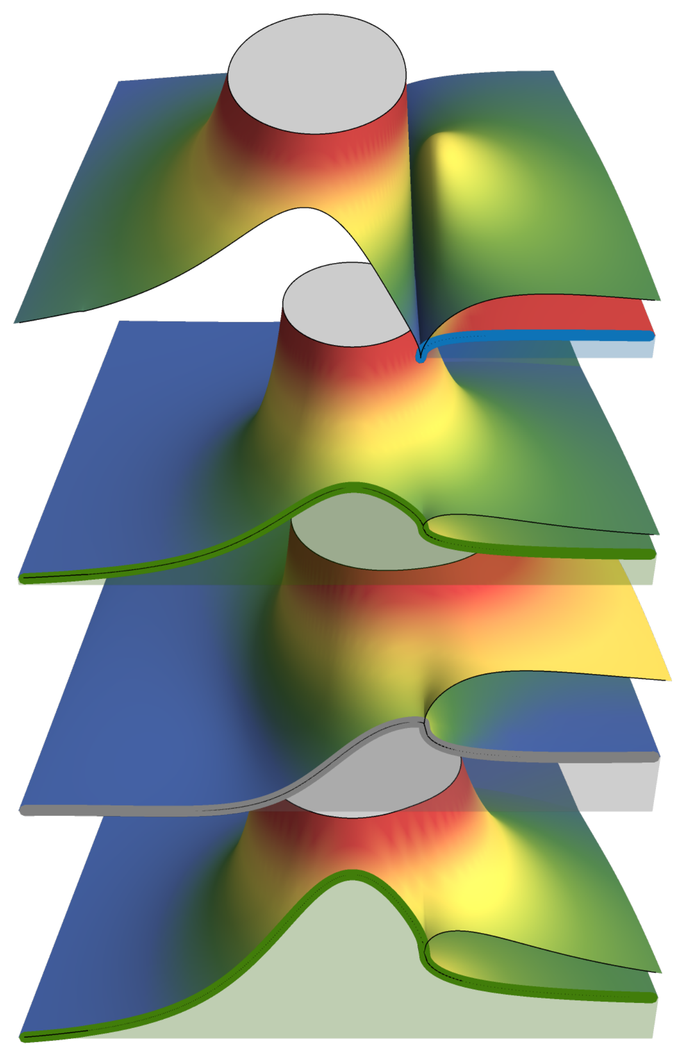


$D\bar{D} \to D\bar{D}$

$D\bar{D} \to D_s\bar{D}_s$

$D_s\bar{D}_s \to D_s\bar{D}_s$

$\rho_i(s)\rho_j(s)|t_{ij}(s)|^2$



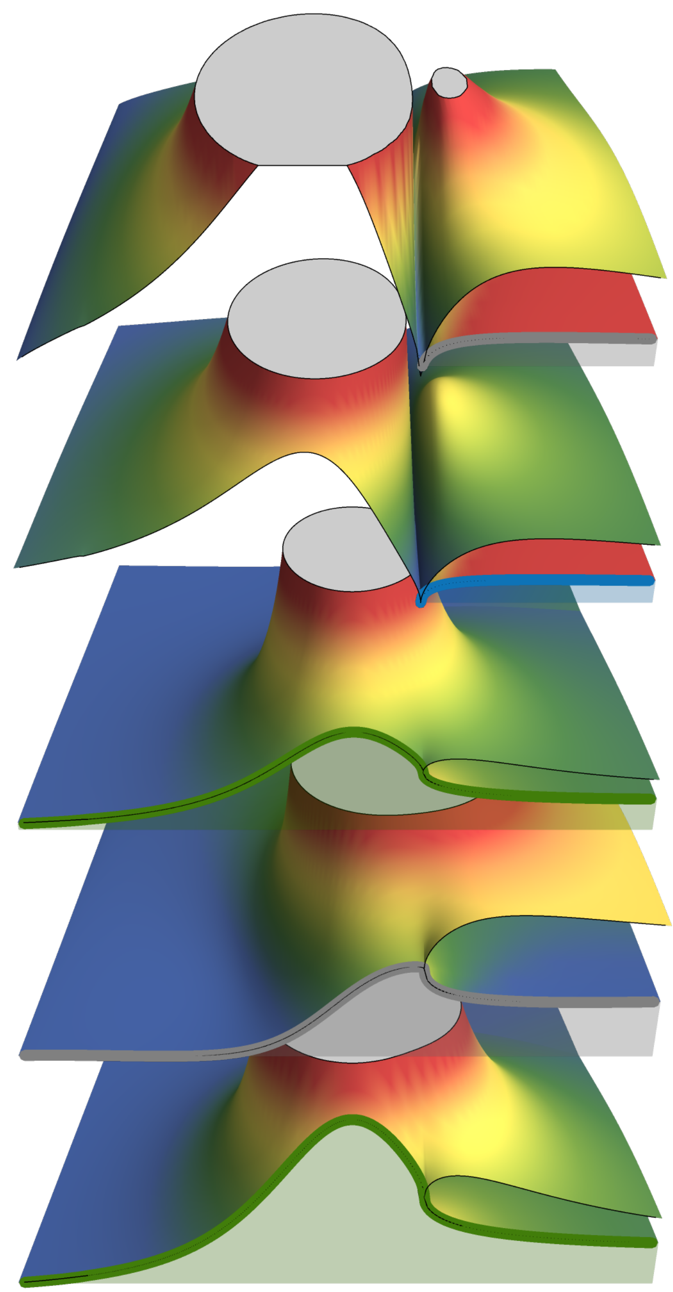
$D\bar{D} \to D^*\bar{D}^*$

$D\bar{D} \to D\bar{D}$

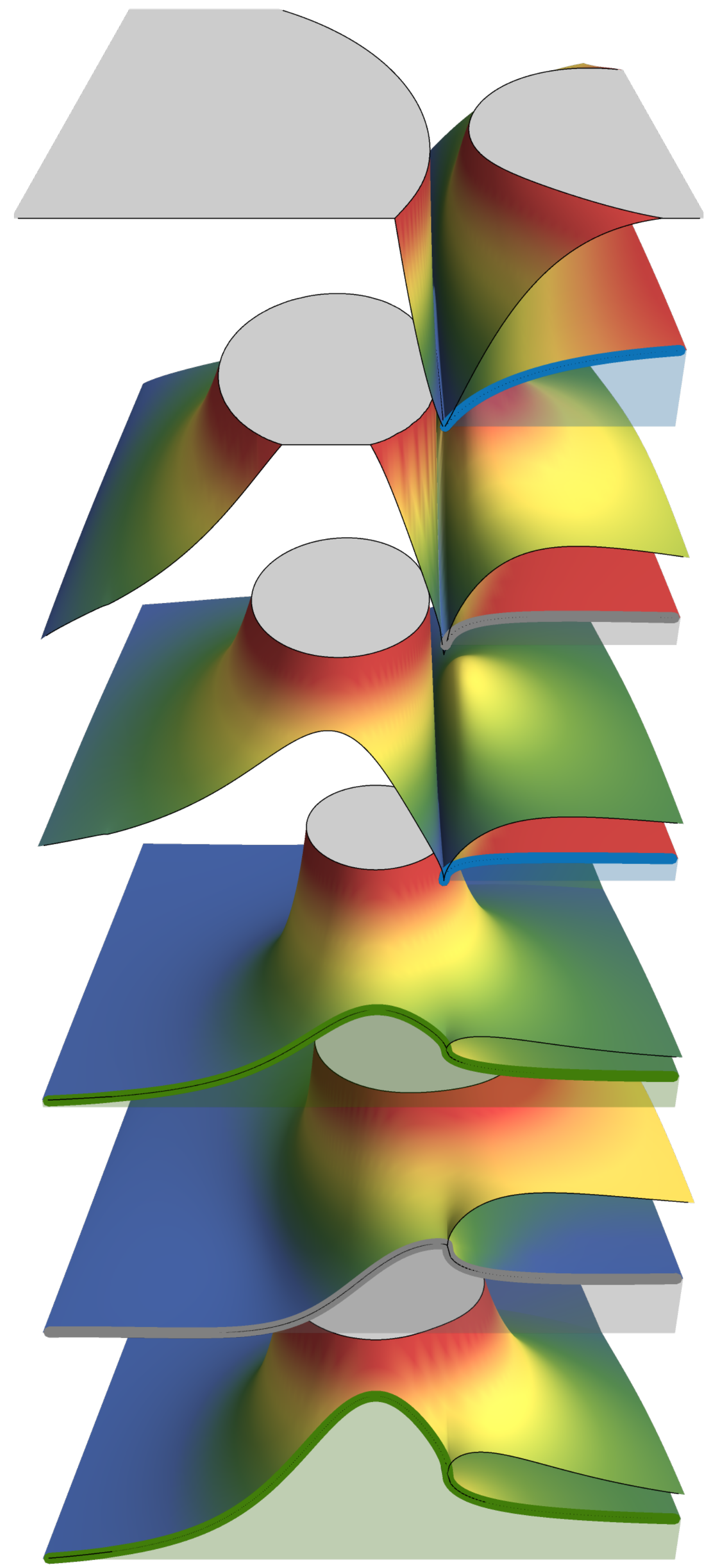
$D\bar{D} \to D_s\bar{D}_s$

$D_s\bar{D}_s \to D_s\bar{D}_s$

$$\rho_i(s)\rho_j(s)|t_{ij}(s)|^2$$



$$\rho_i(s)\rho_j(s)|t_{ij}(s)|^2$$



$D^*\bar{D}^* \to D^*\bar{D}^*$

$D_s\bar{D}_s \to D^*\bar{D}^*$

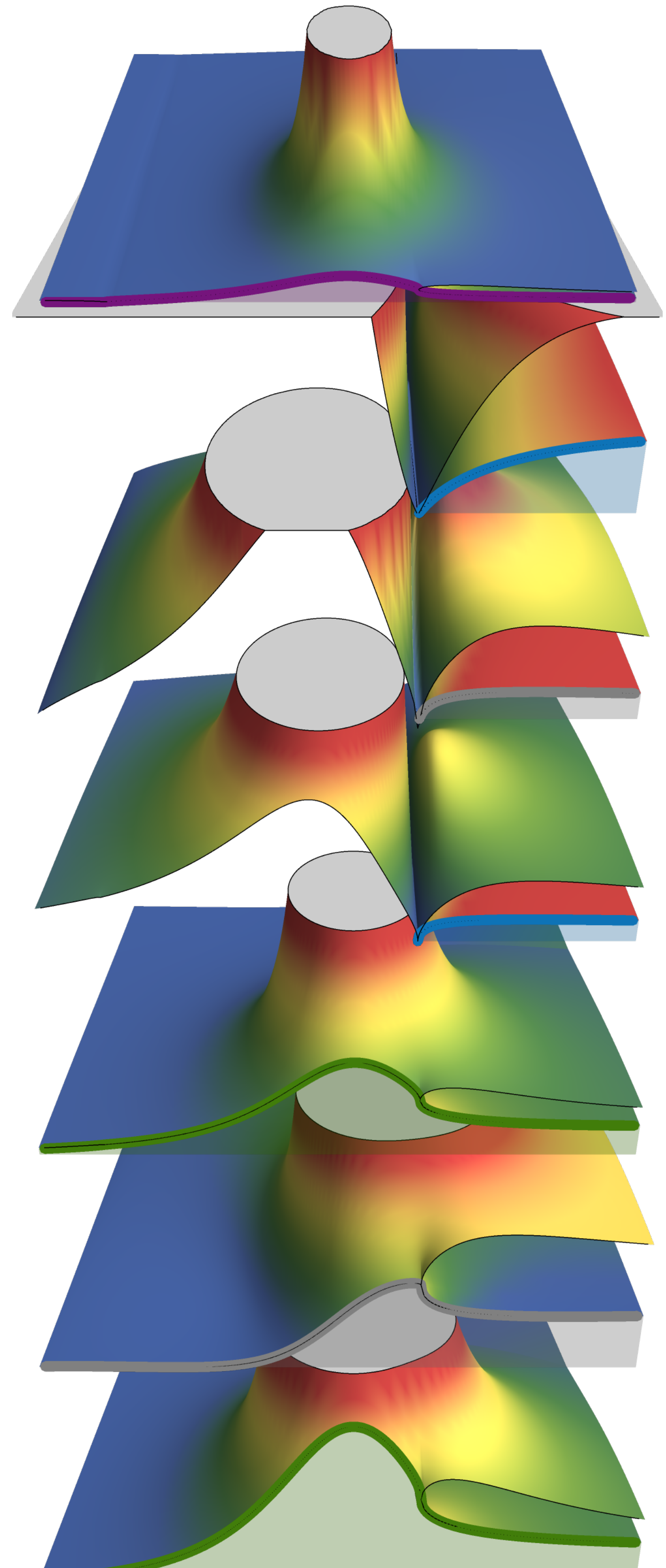
$D\bar{D} \to D^*\bar{D}^*$

$D\bar{D} \to D\bar{D}$

$D\bar{D} \to D_s\bar{D}_s$

$D_s\bar{D}_s \to D_s\bar{D}_s$

$$\rho_i(s)\rho_j(s)|t_{ij}(s)|^2$$



$D\bar{D} \to J/\psi\omega$

$D^*\bar{D}^* \to D^*\bar{D}^*$

$D_s\bar{D}_s \to D^*\bar{D}^*$

$D\bar{D} \to D^*\bar{D}^*$

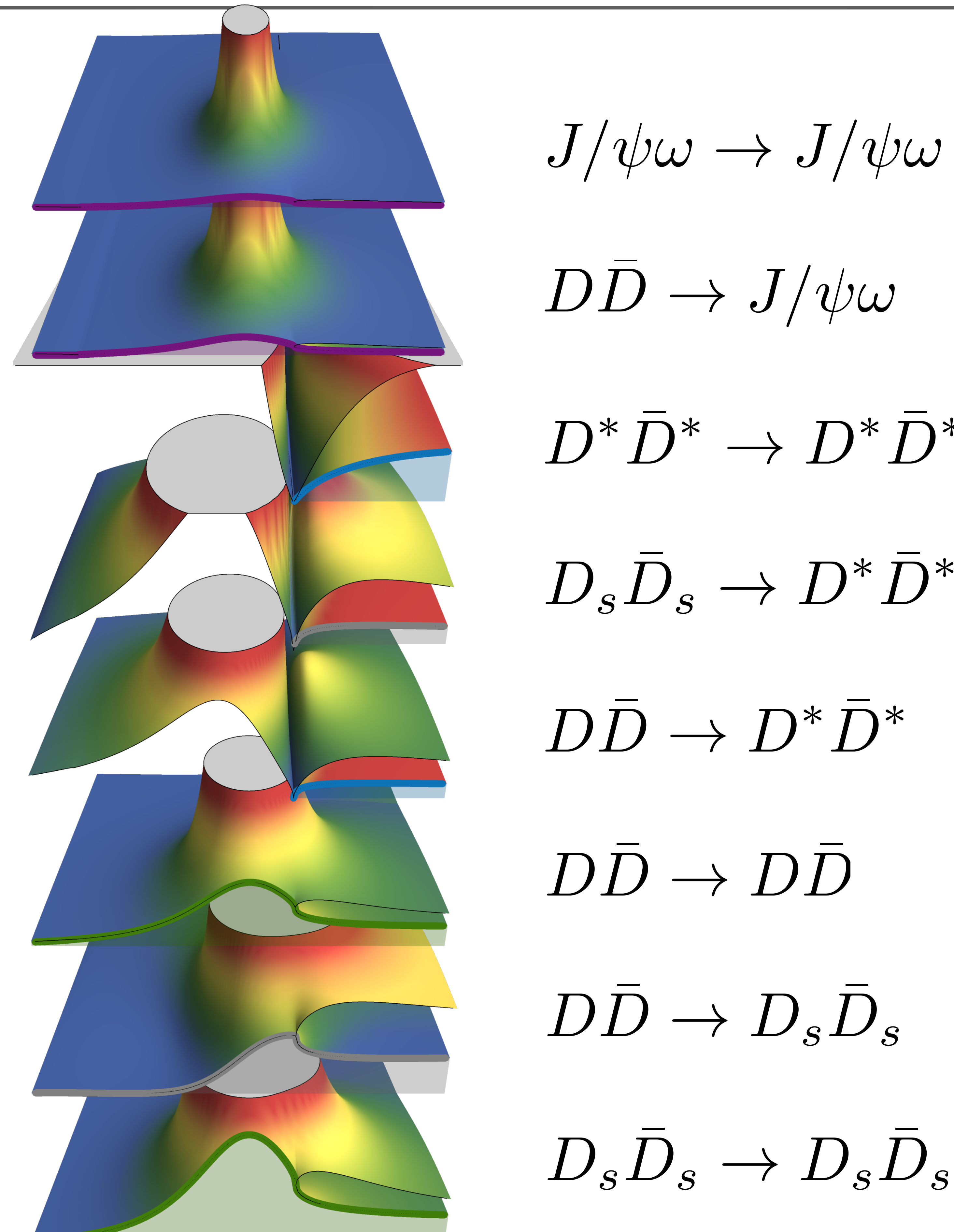
$D\bar{D} \to D\bar{D}$

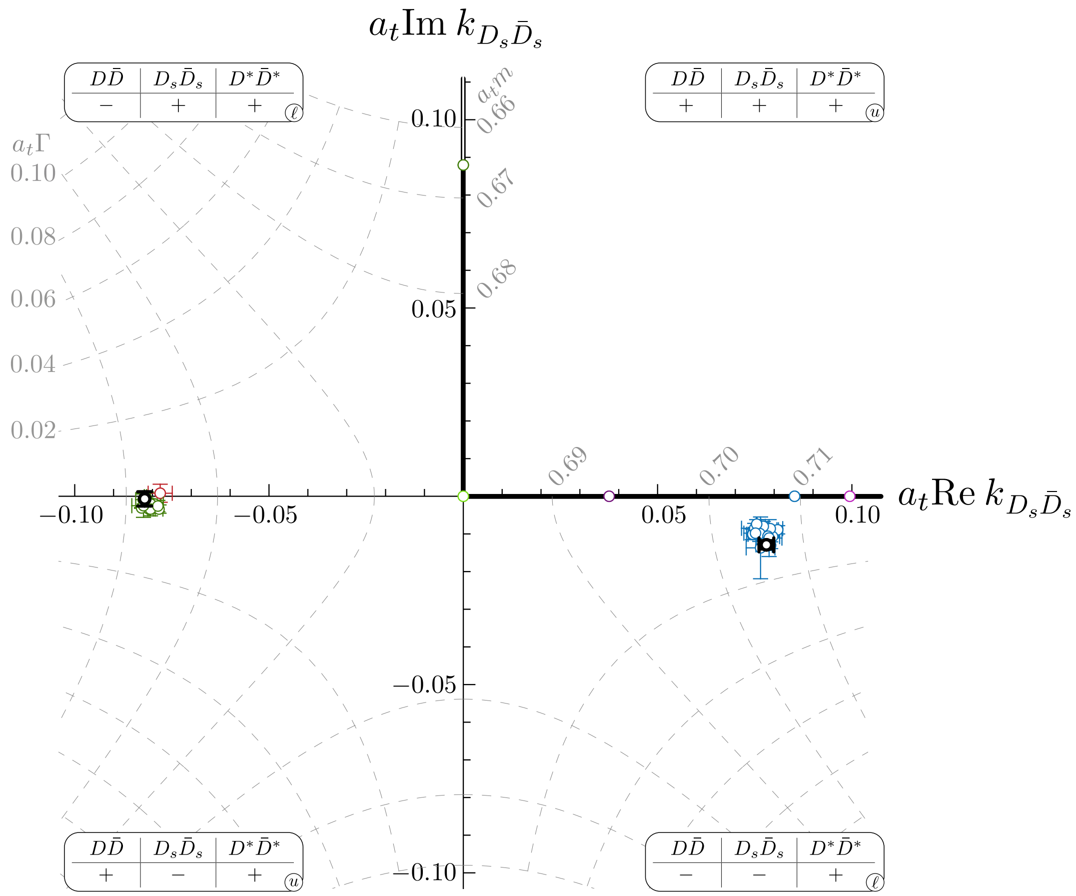
$D\bar{D} \to D_s\bar{D}_s$

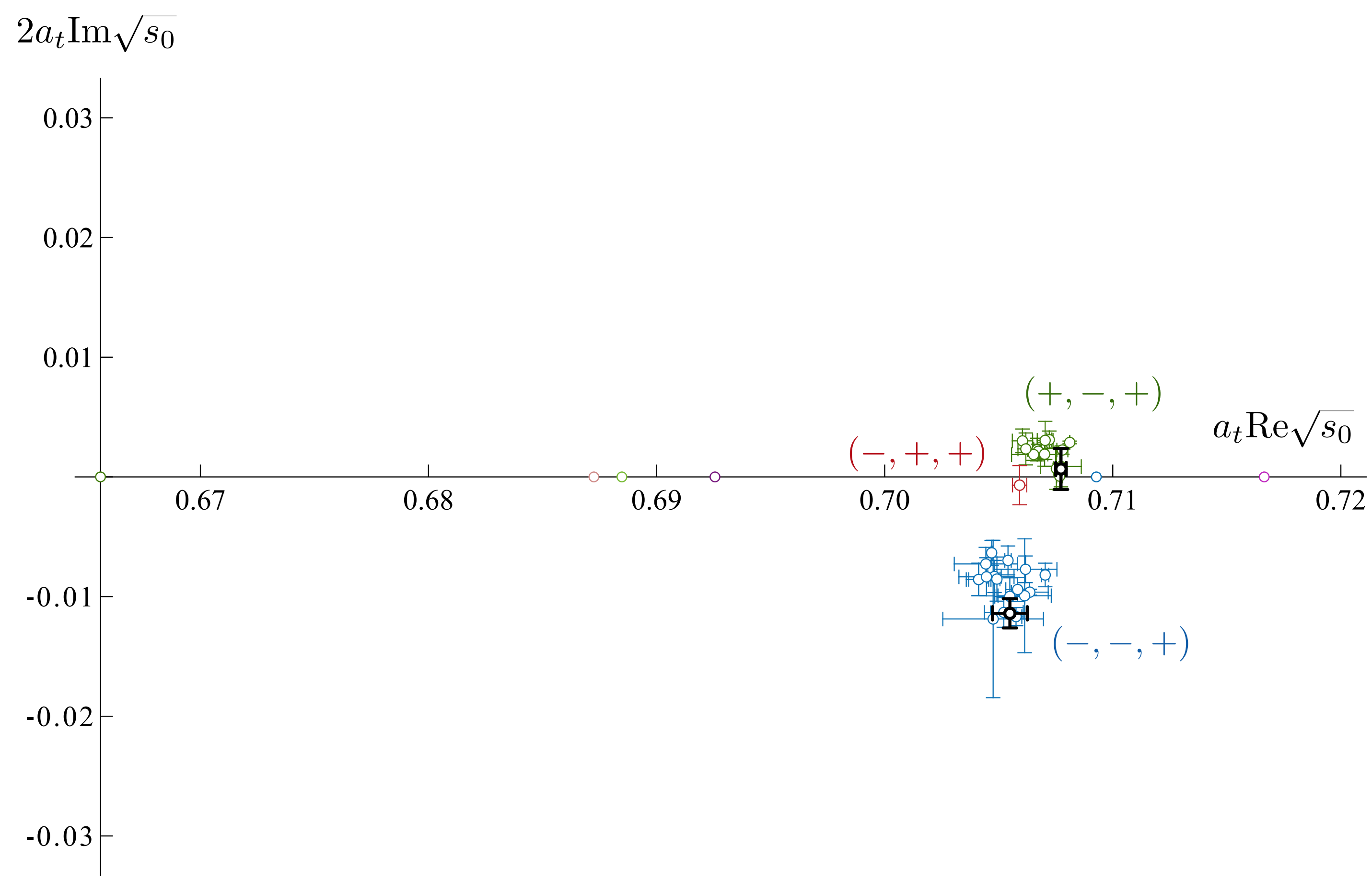
$D_s\bar{D}_s \to D_s\bar{D}_s$

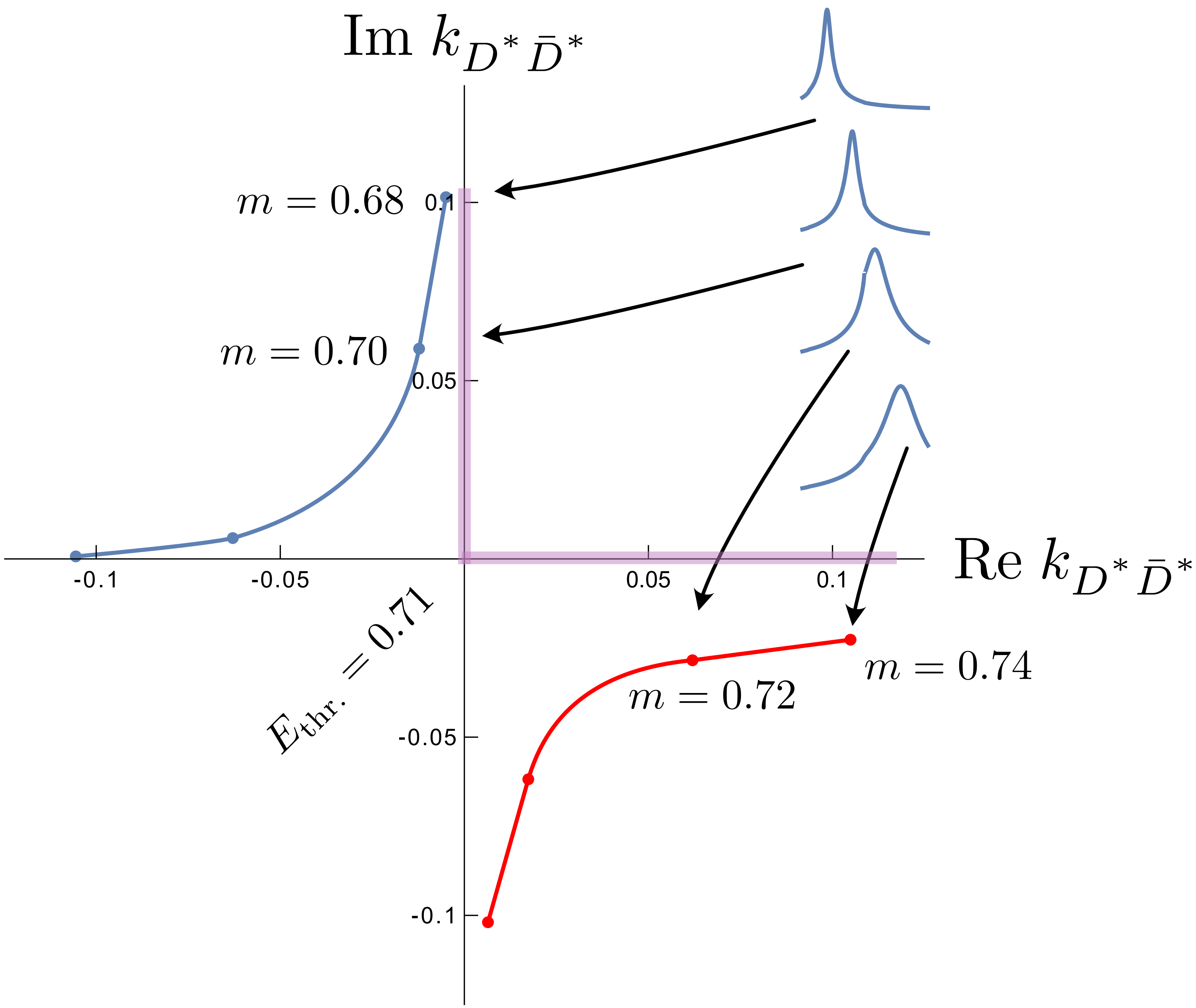
$$\rho_i(s)\rho_j(s)|t_{ij}(s)|^2$$

one resonance pole
— many different amplitudes

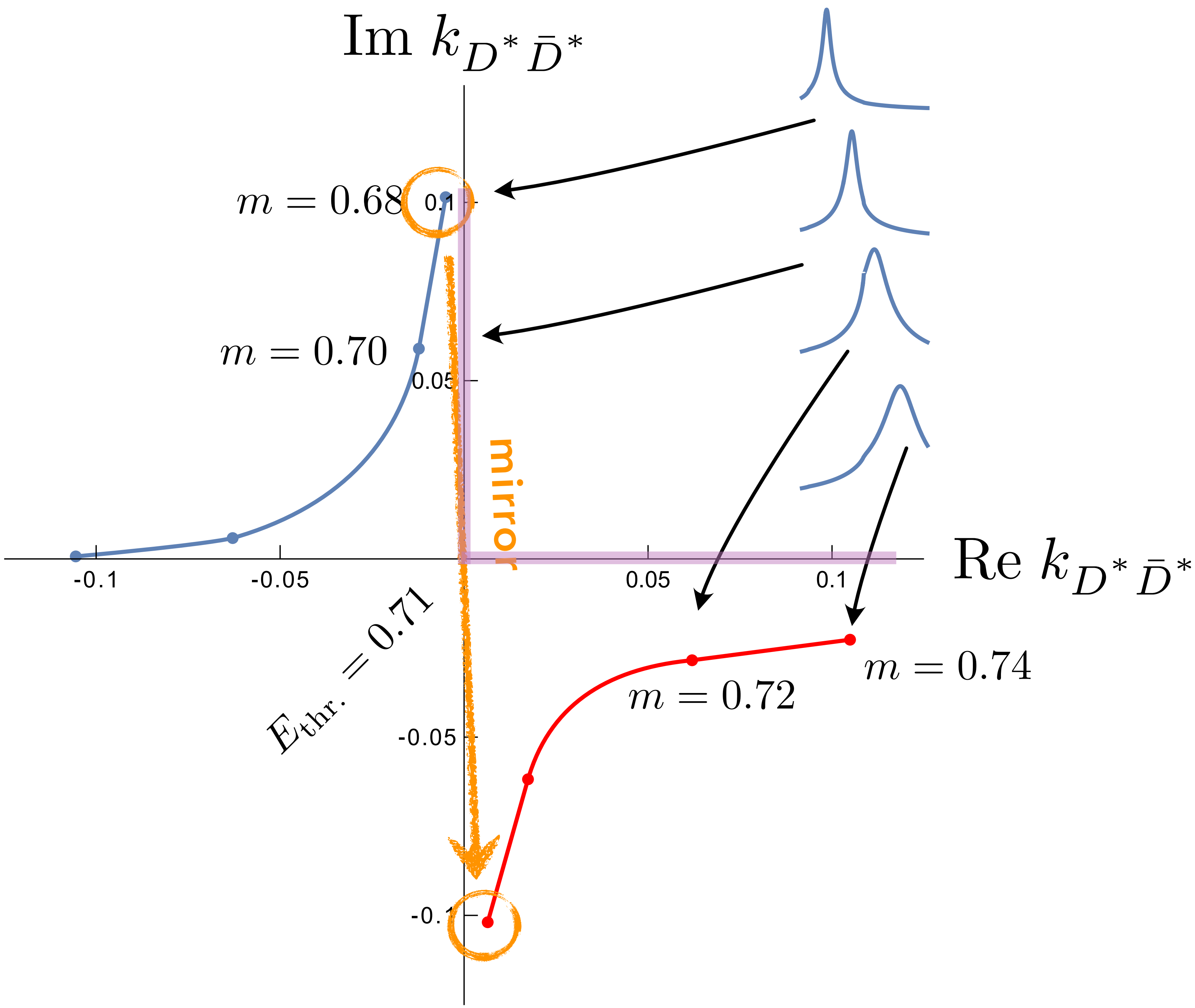



$a_t \mathrm{Im}\, k_{D_s\bar{D}_s}$
$a_t \mathrm{Re}\, k_{D_s\bar{D}_s}$
$a_t\Gamma$
$a_t m$
$D\bar{D}$ | $D_s\bar{D}_s$ | $D^*\bar{D}^*$

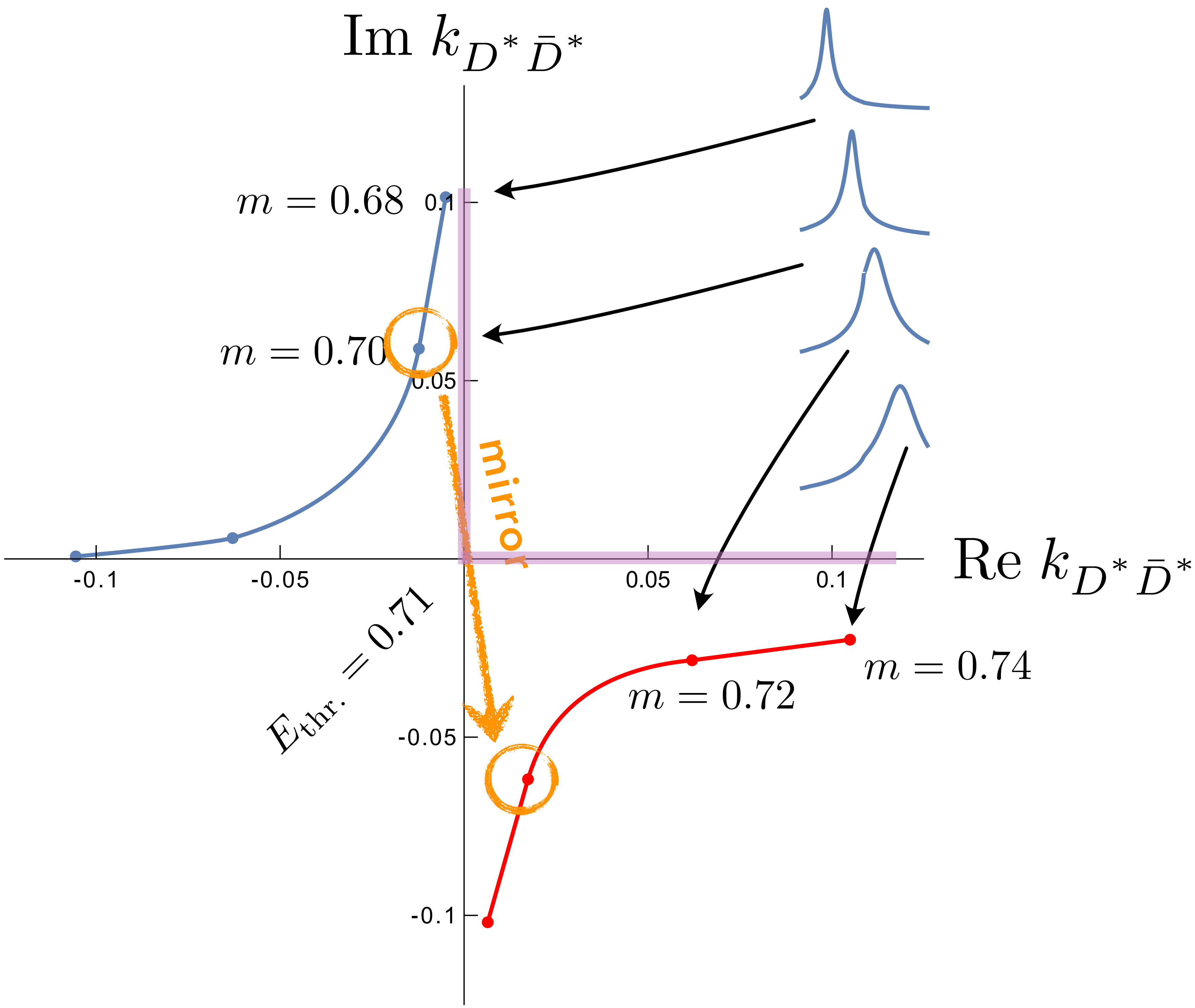

$2a_t\mathrm{Im}\sqrt{s_0}$
$a_t\mathrm{Re}\sqrt{s_0}$
$(+,-,+)$
$(-,+,+)$
$(-,-,+)$



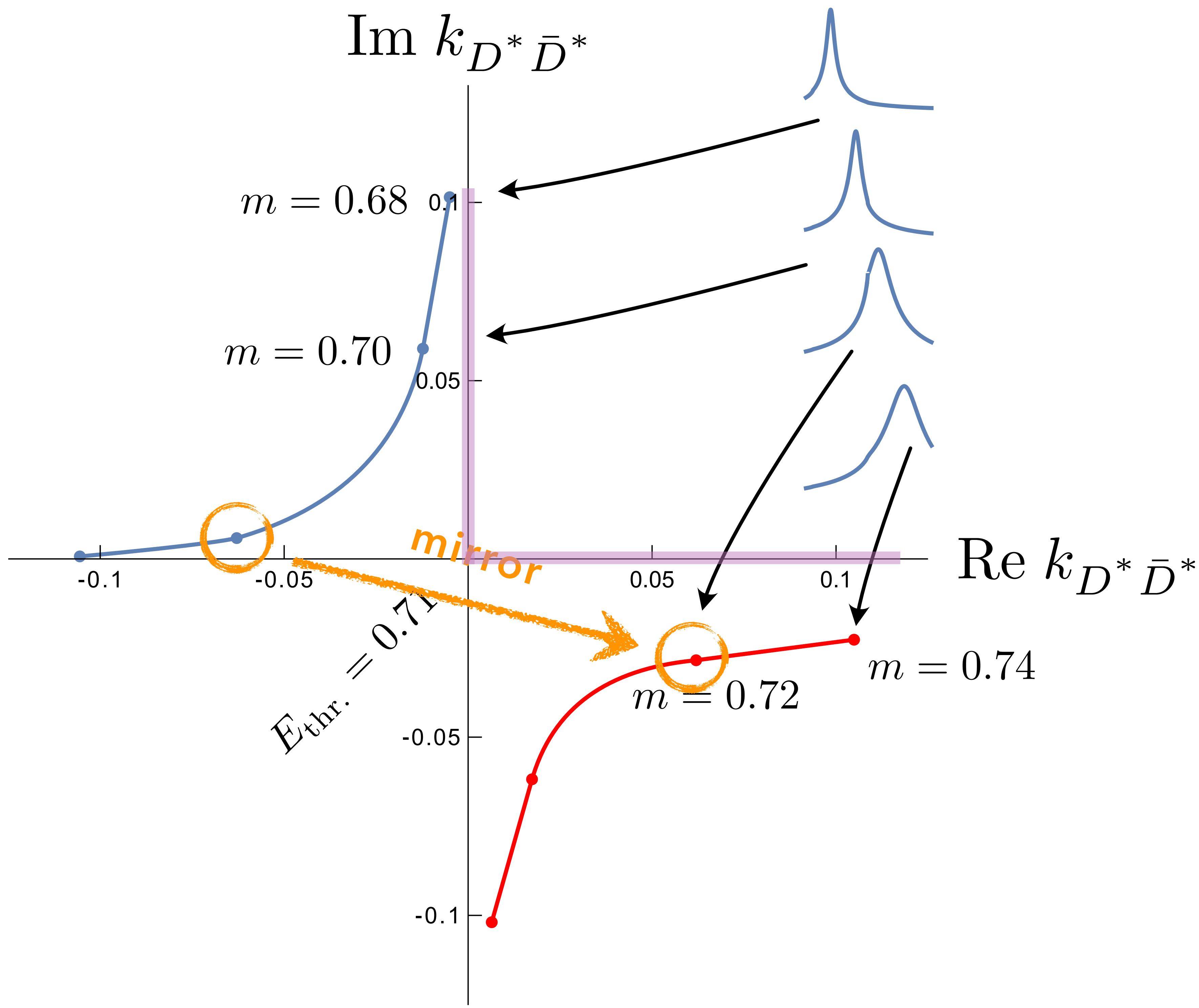
$$t_{ij} = \frac{g_i g_j}{m_0^2 - s - i g_{D\bar{D}}^2 \rho_{D\bar{D}} - i g_{D^*\bar{D}^*}^2 \rho_{D^*\bar{D}^*}}$$



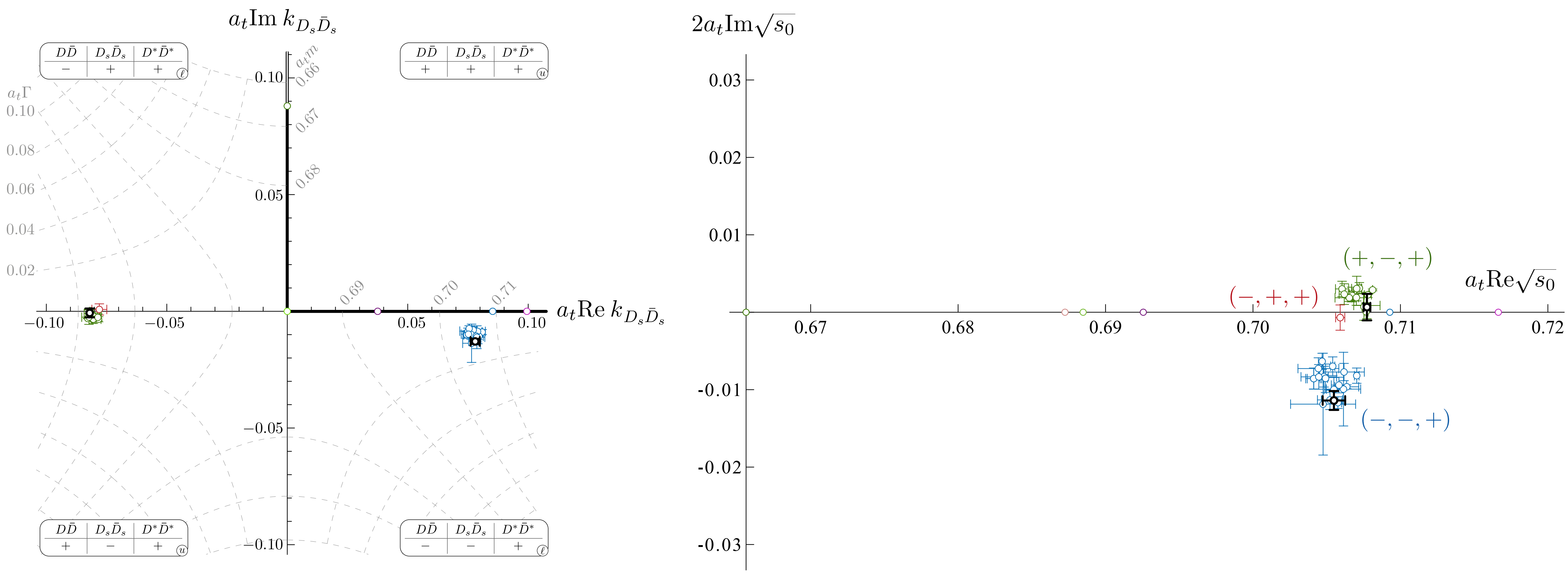
$$t_{ij} = \frac{g_i g_j}{m_0^2 - s - i g_{D\bar{D}}^2 \rho_{D\bar{D}} - i g_{D^*\bar{D}^*}^2 \rho_{D^*\bar{D}^*}}$$



$$t_{ij} = \frac{g_i g_j}{m_0^2 - s - i g^2_{D\bar{D}} \rho_{D\bar{D}} - i g^2_{D^*\bar{D}^*} \rho_{D^*\bar{D}^*}}$$

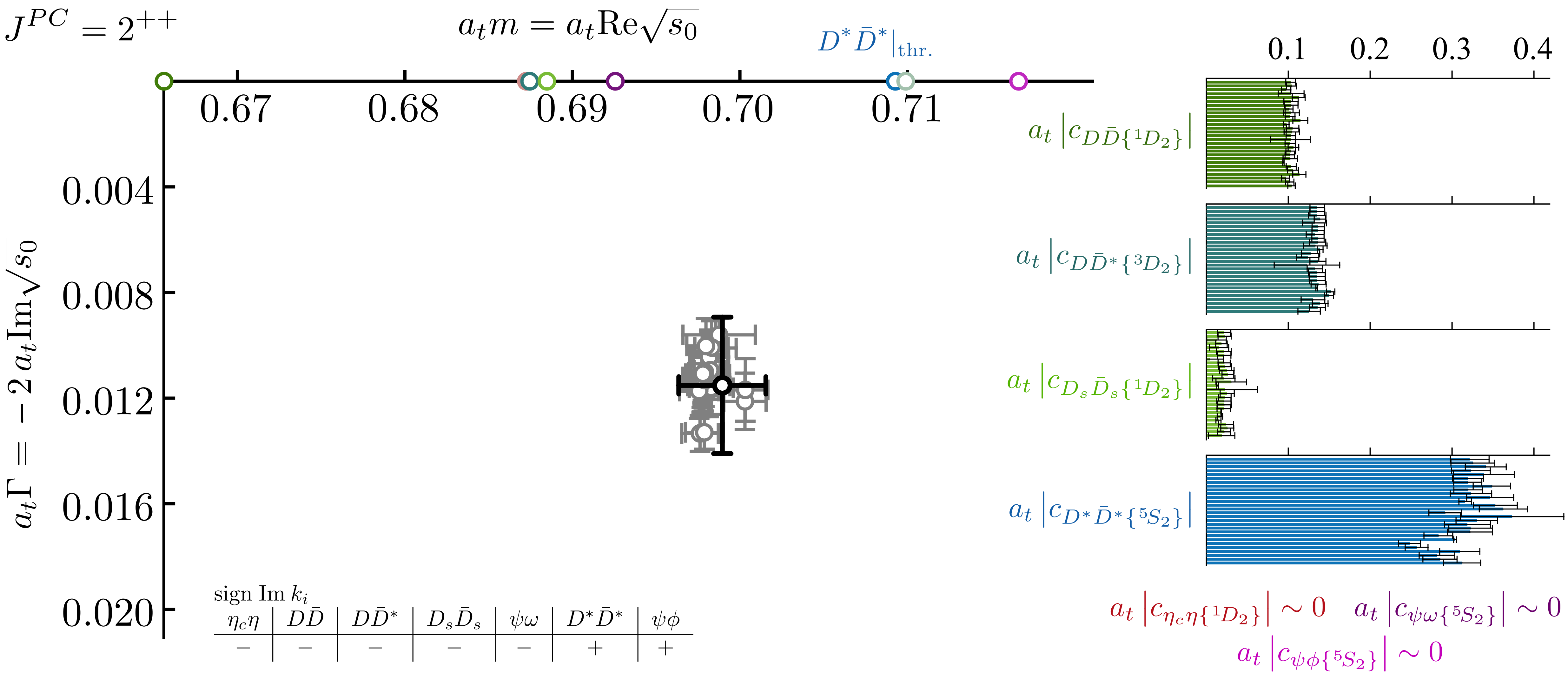


$$t_{ij} = \frac{g_i g_j}{m_0^2 - s - i g_{D\bar{D}}^2 \rho_{D\bar{D}} - i g_{D^*\bar{D}^*}^2 \rho_{D^*\bar{D}^*}}$$



the "green" cluster of poles are just mirror poles

- amplitude is **dominated by a single resonance pole** in this energy region

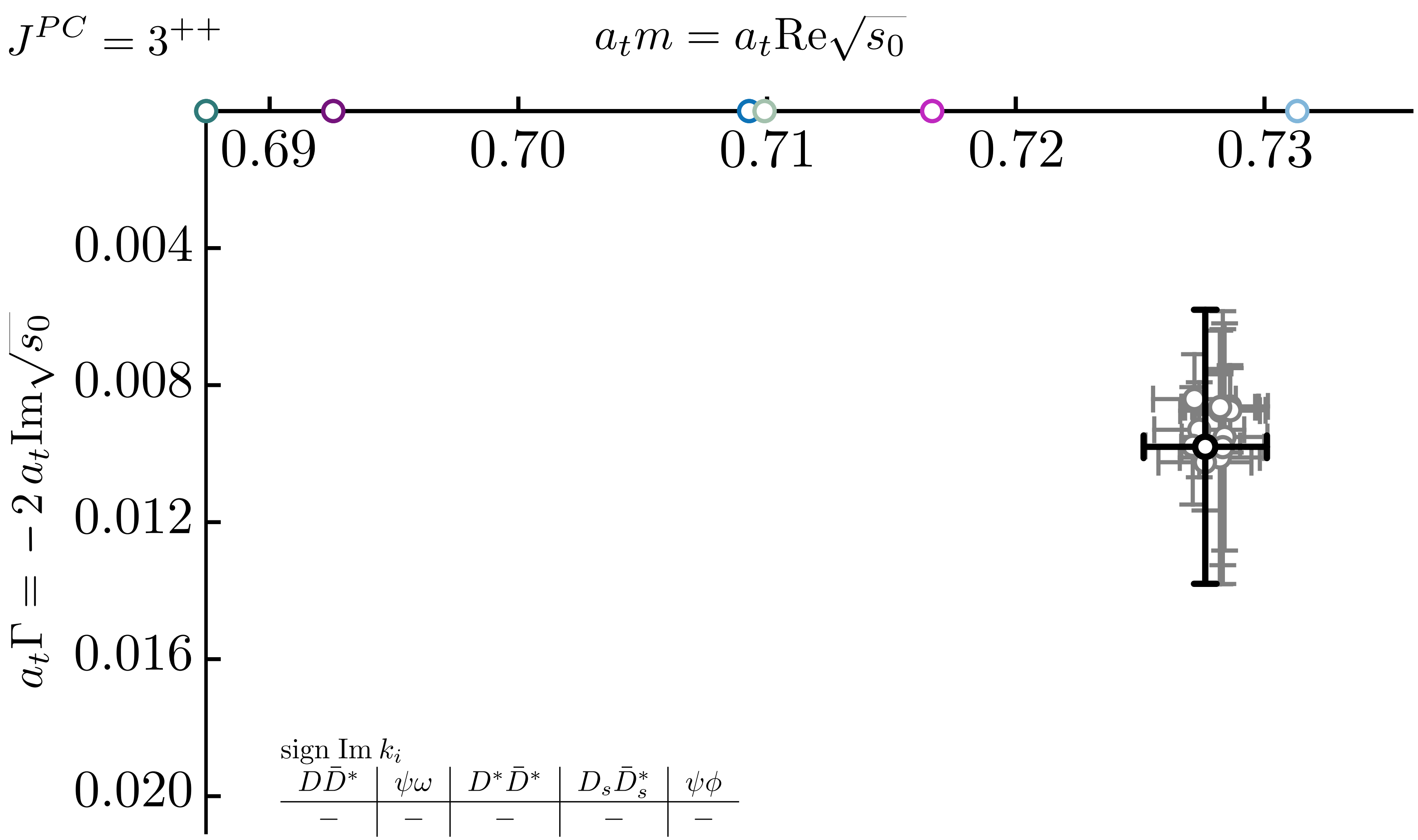


additional poles were found

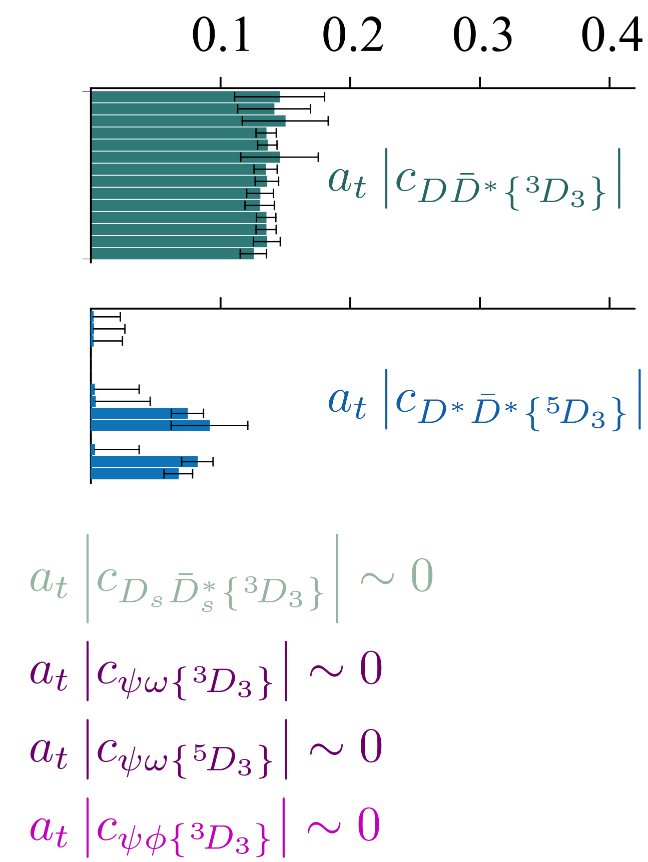
- don't appear to be important

"coupling-ratio" phenomena seen in K-matrix pole parameters

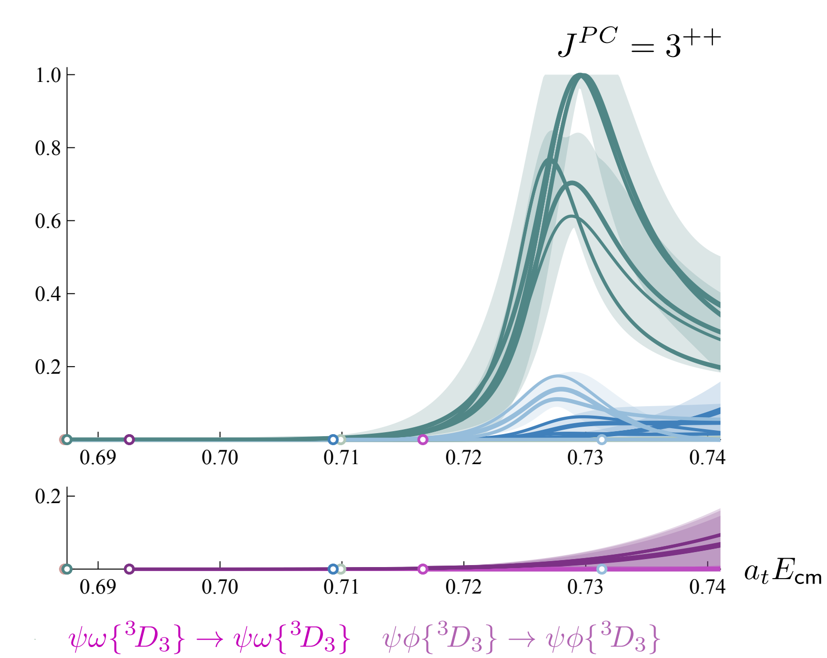
- possible to rescale K-matrix g_i factors and obtain similar amplitudes
- t-matrix couplings are found to be well-determined



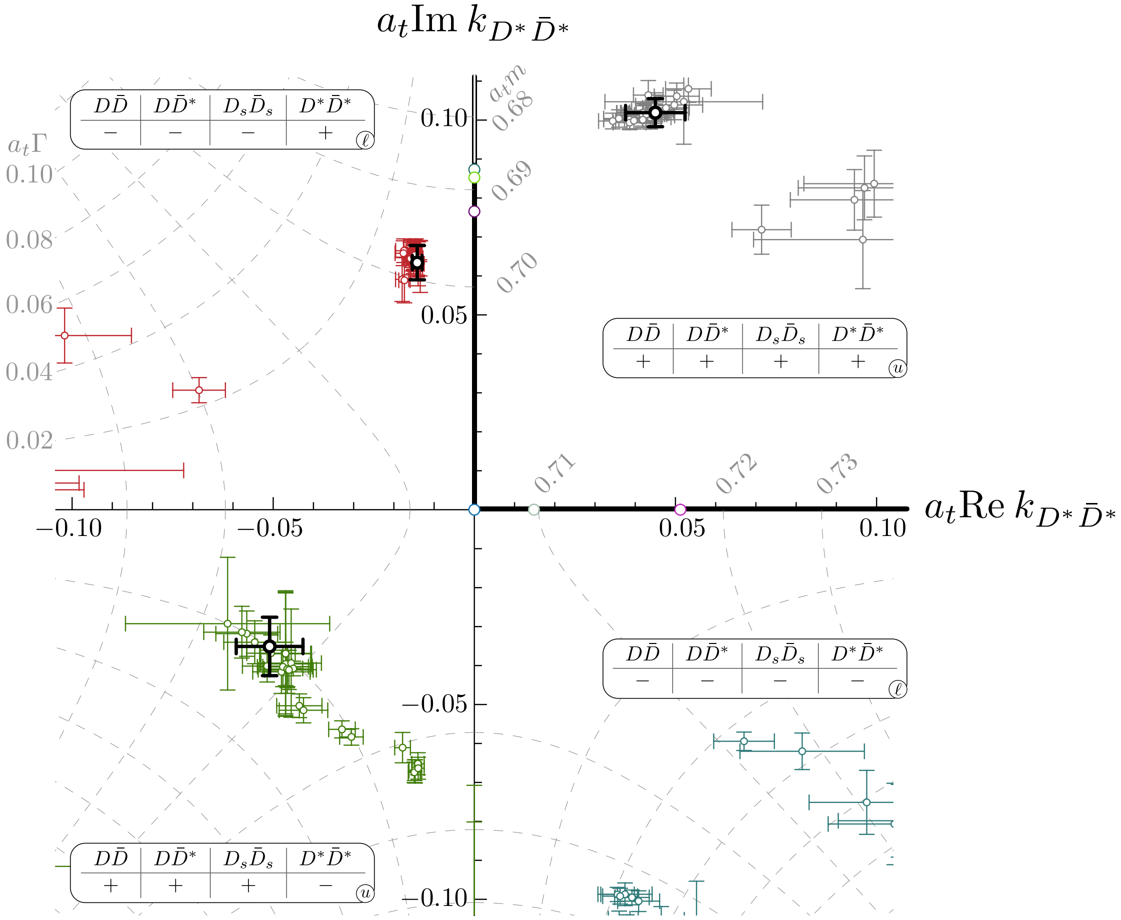
$J^{PC} = 3^{++}$
$a_t m = a_t \mathrm{Re}\sqrt{s_0}$
0.69
0.70
0.71
0.72
0.73
$a_t\Gamma = -2\, a_t \mathrm{Im}\sqrt{s_0}$
0.004
0.008
0.012
0.016
0.020
sign Im k_i
$D\bar{D}^*$ | $\psi\omega$ | $D^*\bar{D}^*$ | $D_s\bar{D}_s^*$ | $\psi\phi$
– | – | – | – | –



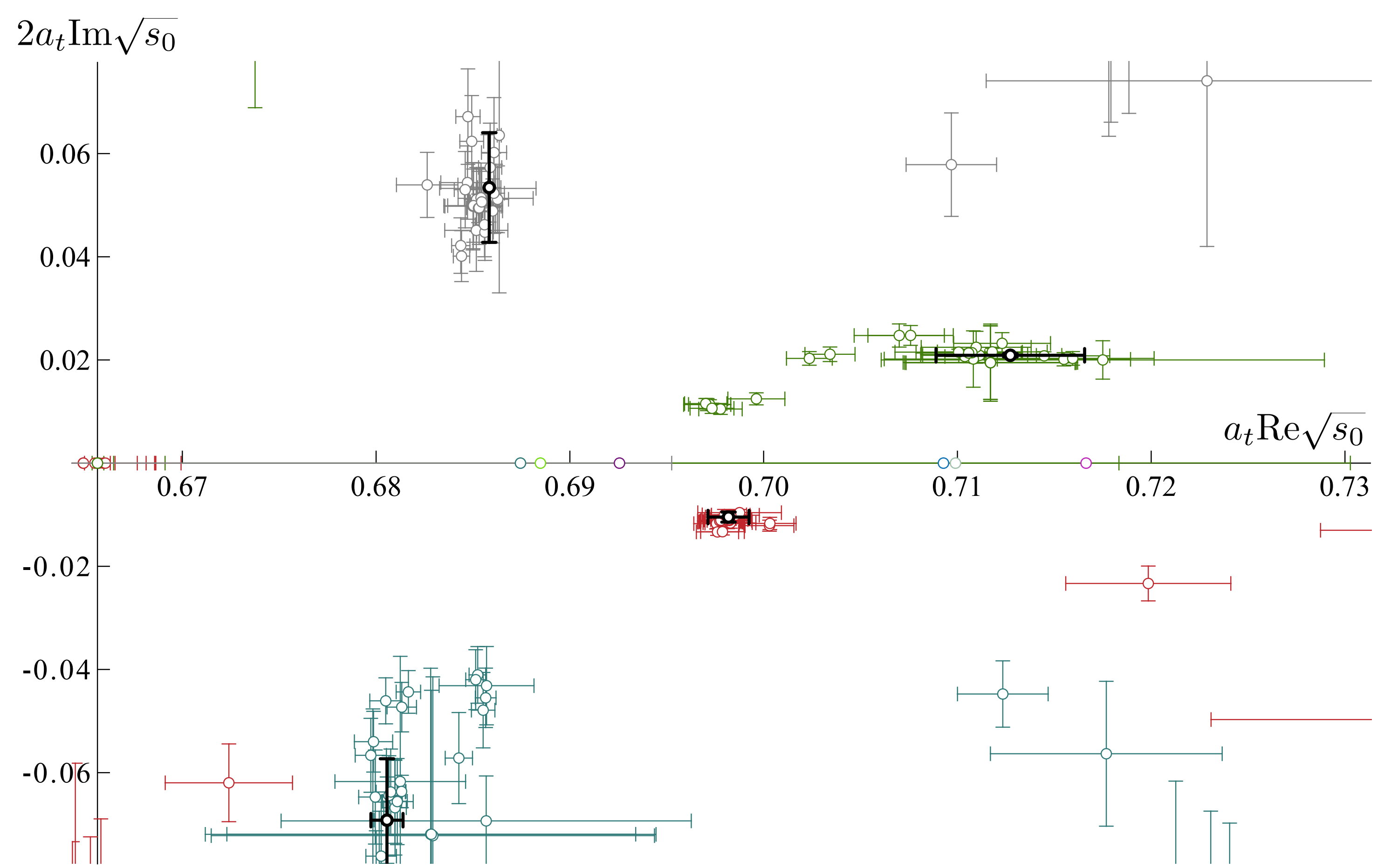
0.1
0.2
0.3
0.4
$a_t \left|c_{D\bar{D}^*\{^3D_3\}}\right|$
$a_t \left|c_{D^*\bar{D}^*\{^5D_3\}}\right|$
$a_t \left|c_{D_s\bar{D}_s^*\{^3D_3\}}\right| \sim 0$
$a_t \left|c_{\psi\omega\{^3D_3\}}\right| \sim 0$
$a_t \left|c_{\psi\omega\{^5D_3\}}\right| \sim 0$
$a_t \left|c_{\psi\phi\{^3D_3\}}\right| \sim 0$



$J^{PC} = 3^{++}$
1.0
0.8
0.6
0.4
0.2
0.69
0.70
0.71
0.72
0.73
0.74
$a_t E_{\mathrm{cm}}$
$\psi\omega\{^3D_3\} \to \psi\omega\{^3D_3\}$
$\psi\phi\{^3D_3\} \to \psi\phi\{^3D_3\}$

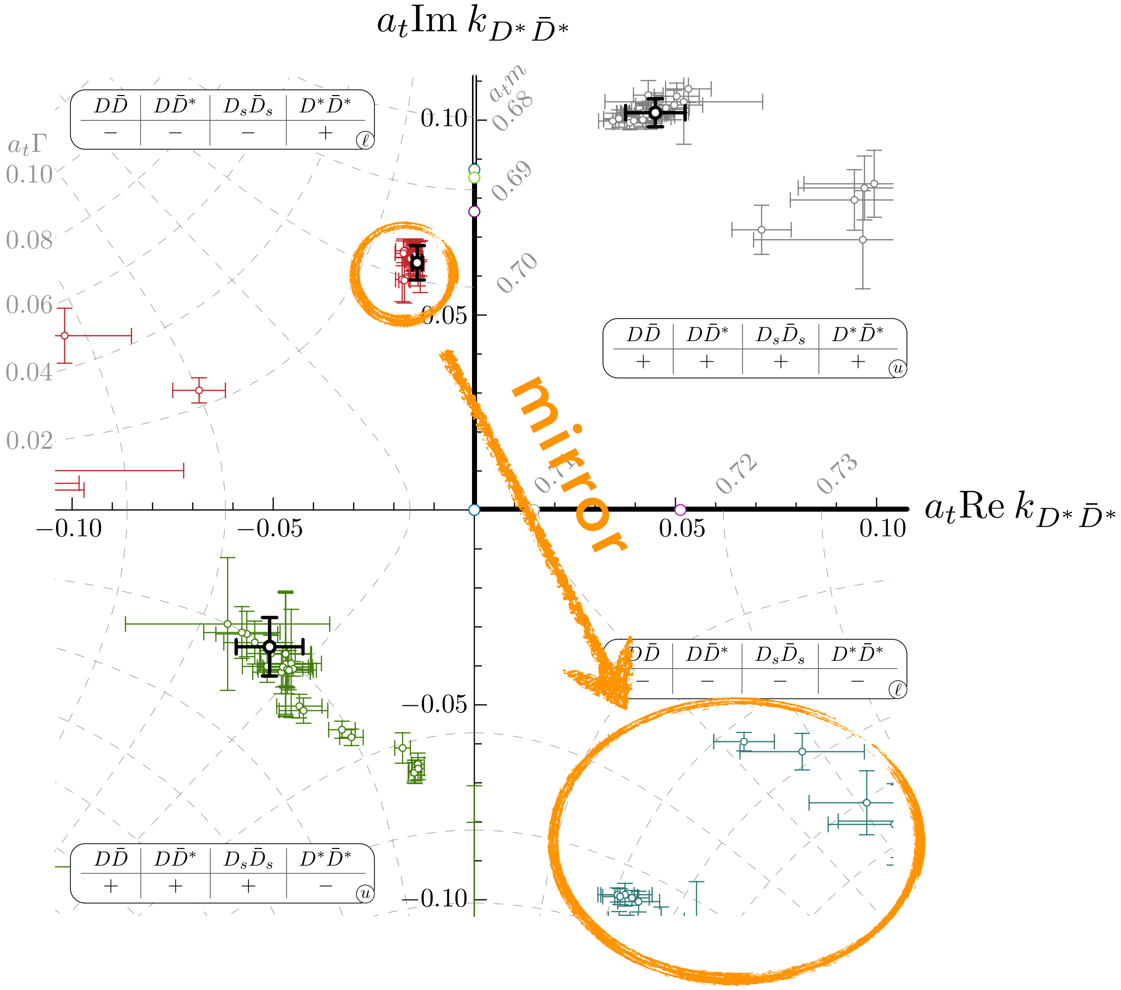


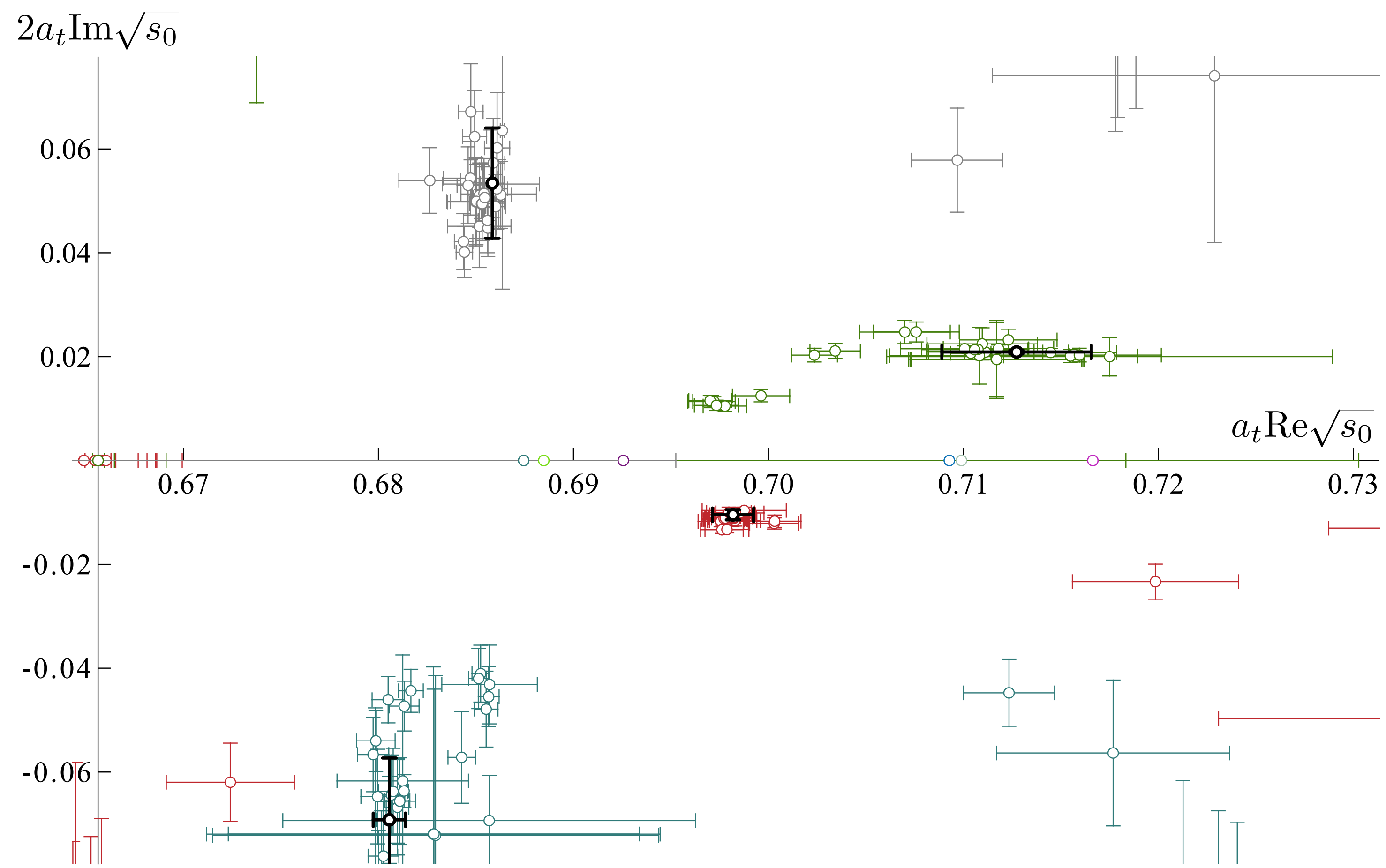
$a_t\mathrm{Im}\,k_{D^*\bar{D}^*}$
$a_t\mathrm{Re}\,k_{D^*\bar{D}^*}$
$a_t\Gamma$
$a_t m$
$D\bar{D}$ | $D\bar{D}^*$ | $D_s\bar{D}_s$ | $D^*\bar{D}^*$



$2a_t\mathrm{Im}\sqrt{s_0}$
$a_t\mathrm{Re}\sqrt{s_0}$

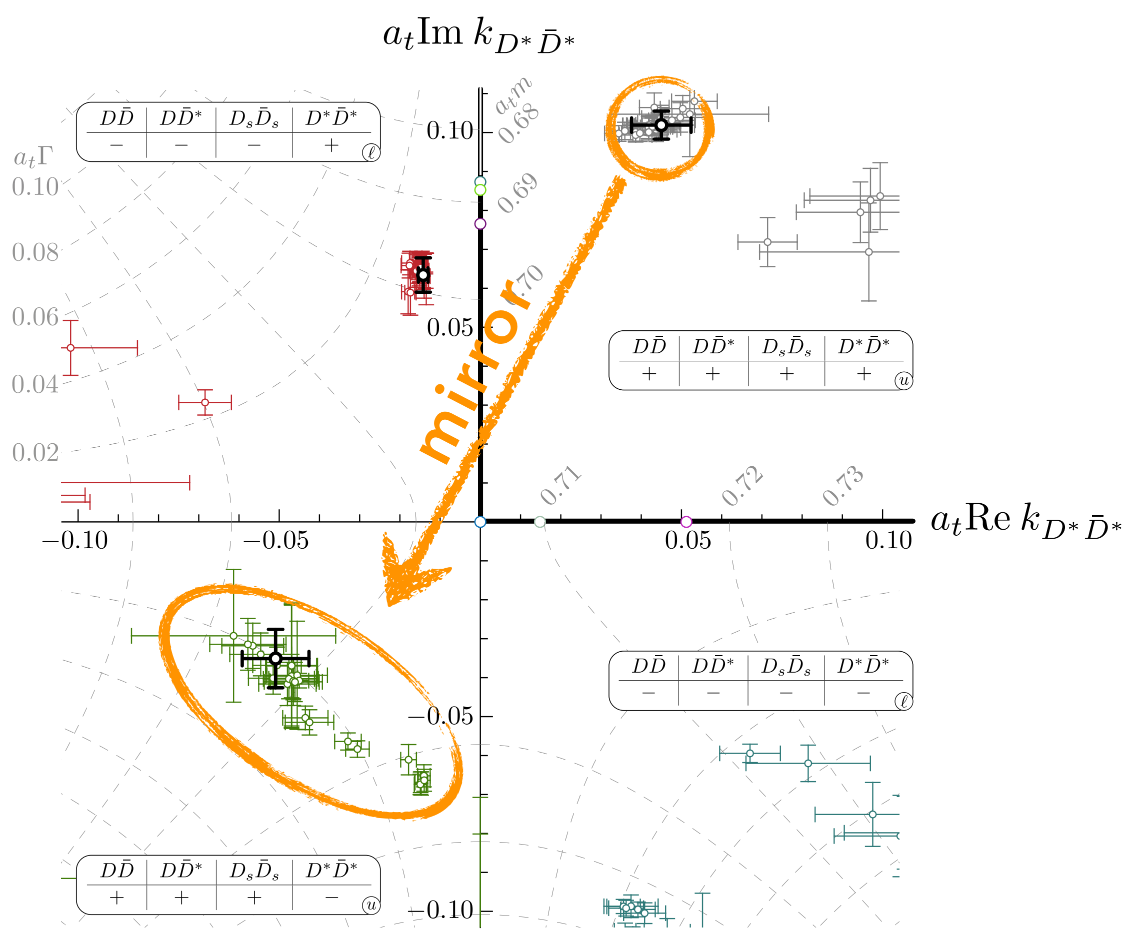
poles - tensor - more going on here, but mostly not physics

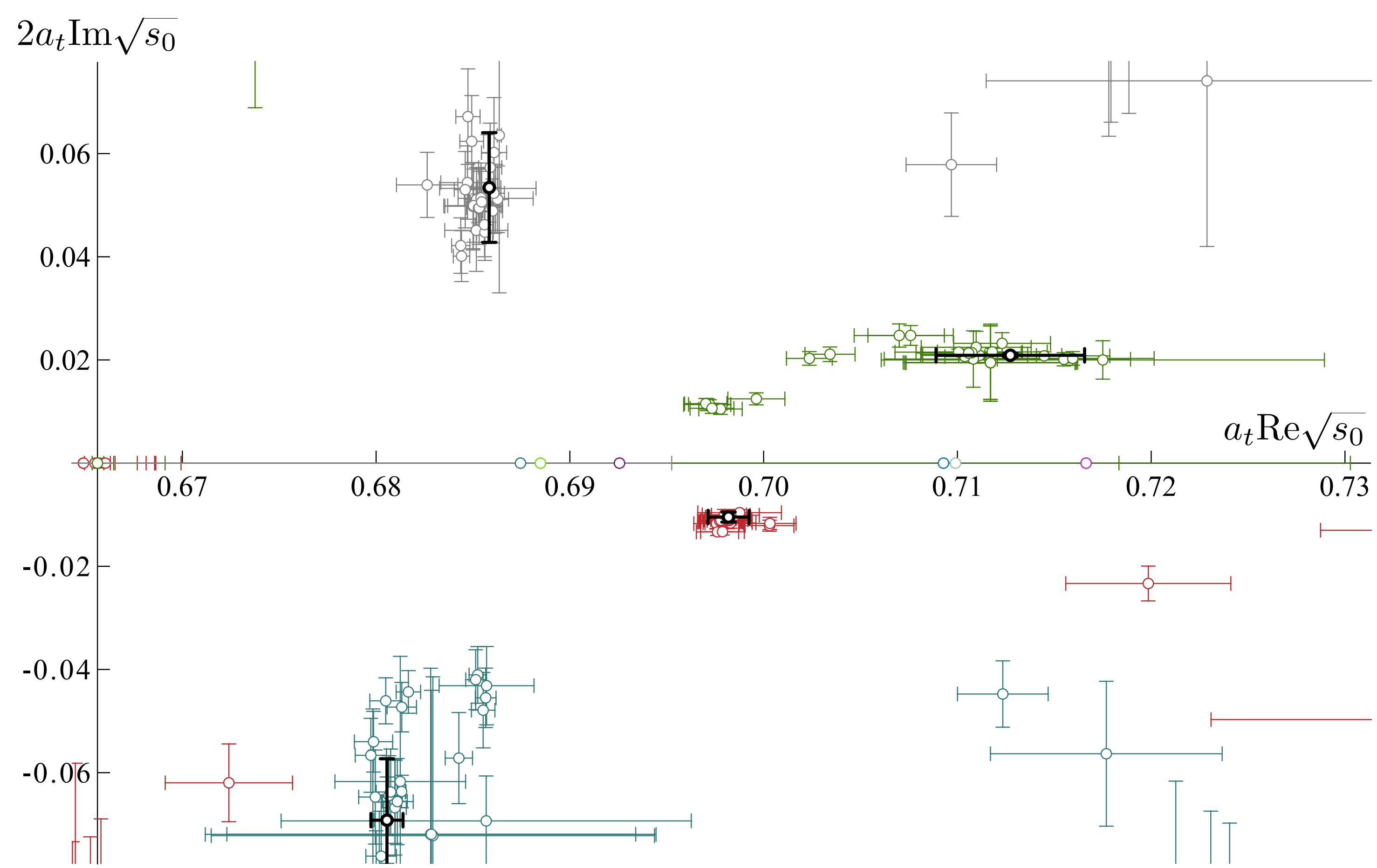




mirror pole - similar to a Flatté

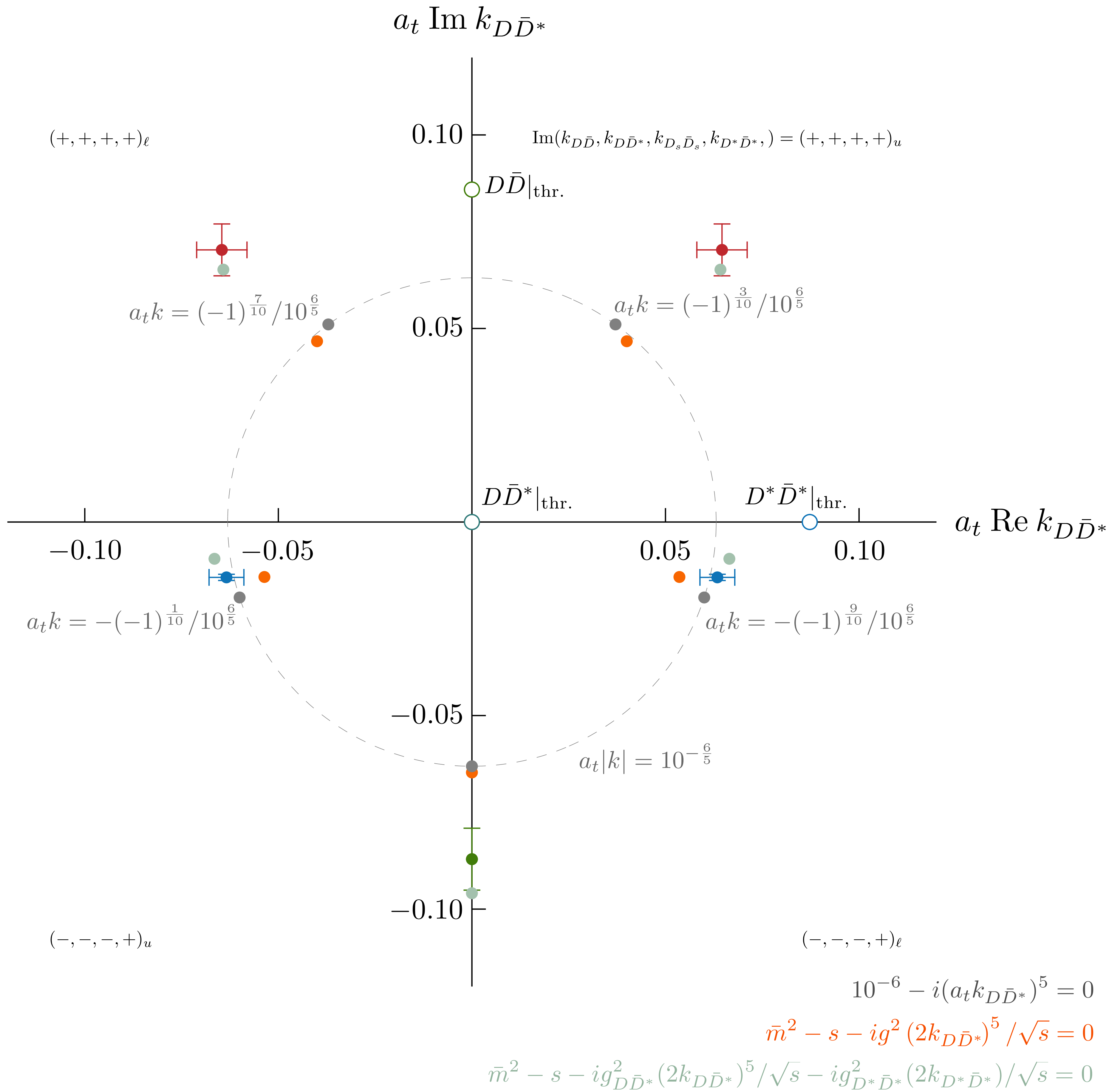
poles - tensor - more going on here, but mostly not physics



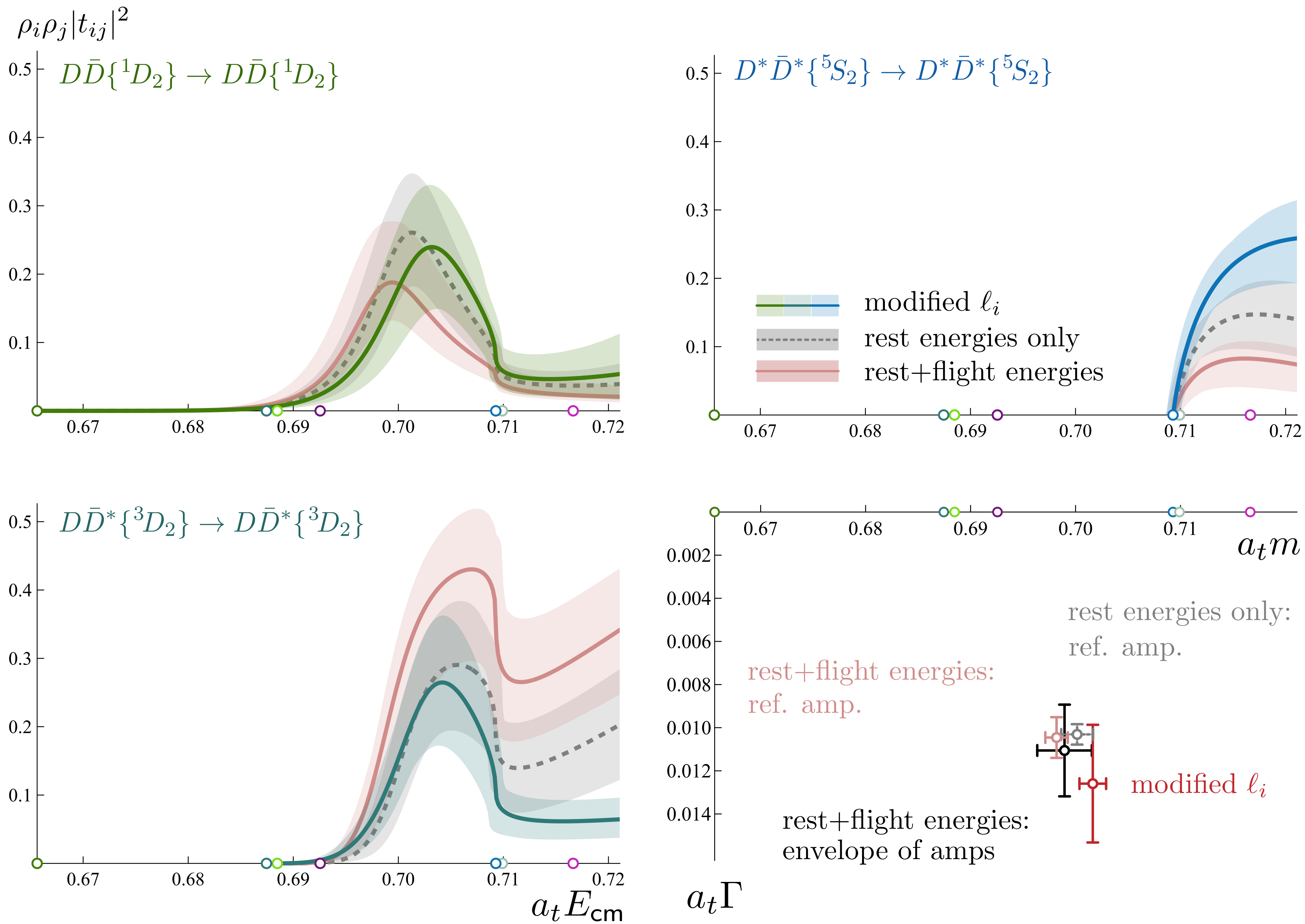


"green" pole is a mirror of the physical sheet pole

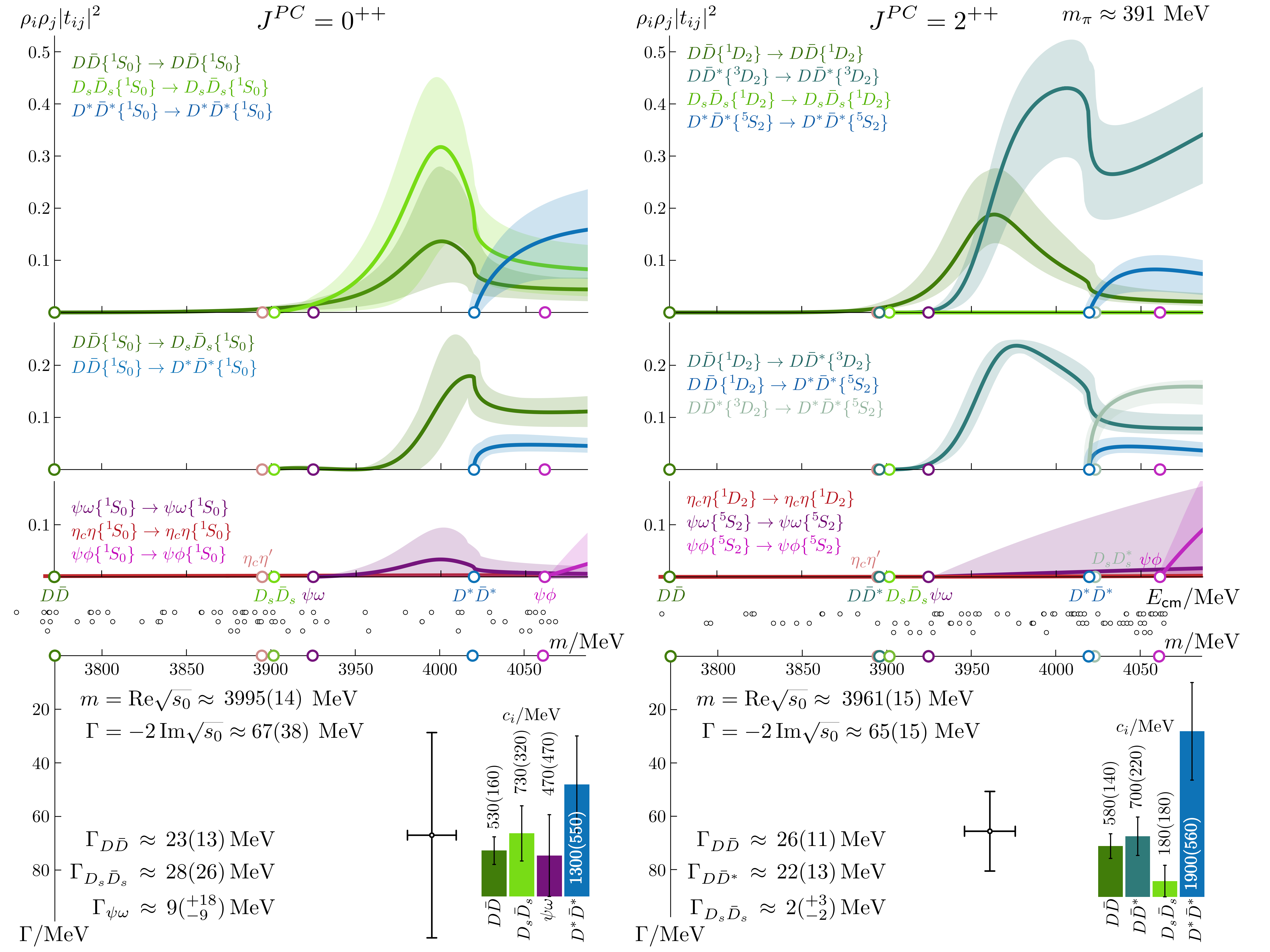
physical sheet pole arises because of the large g_{DD^*}


a_t Im $k_{D\bar{D}^*}$
0.10
0.05
−0.05
−0.10
a_t Re $k_{D\bar{D}^*}$
$(+,+,+,+)_\ell$
$\mathrm{Im}(k_{D\bar{D}}, k_{D\bar{D}^*}, k_{D_s\bar{D}_s}, k_{D^*\bar{D}^*},) = (+,+,+,+)_u$
$D\bar{D}|_{\mathrm{thr.}}$
$a_t k = (-1)^{\frac{7}{10}}/10^{\frac{6}{5}}$
$a_t k = (-1)^{\frac{3}{10}}/10^{\frac{6}{5}}$
$D\bar{D}^*|_{\mathrm{thr.}}$
$D^*\bar{D}^*|_{\mathrm{thr.}}$
$a_t k = -(-1)^{\frac{1}{10}}/10^{\frac{6}{5}}$
$a_t k = -(-1)^{\frac{9}{10}}/10^{\frac{6}{5}}$
$a_t|k| = 10^{-\frac{6}{5}}$
$(-,-,-,+)_u$
$(-,-,-,+)_\ell$
$10^{-6} - i(a_t k_{D\bar{D}^*})^5 = 0$
$\bar{m}^2 - s - ig^2\,(2k_{D\bar{D}^*})^5/\sqrt{s} = 0$
$\bar{m}^2 - s - ig^2_{D\bar{D}^*}(2k_{D\bar{D}^*})^5/\sqrt{s} - ig^2_{D^*\bar{D}^*}(2k_{D^*\bar{D}^*})/\sqrt{s} = 0$

using a modified DDbar* threshold factor



- different physical sheet pole
- no obvious nearby (+,+,+,-) sheet pole (there are some with a_t E >0.74)

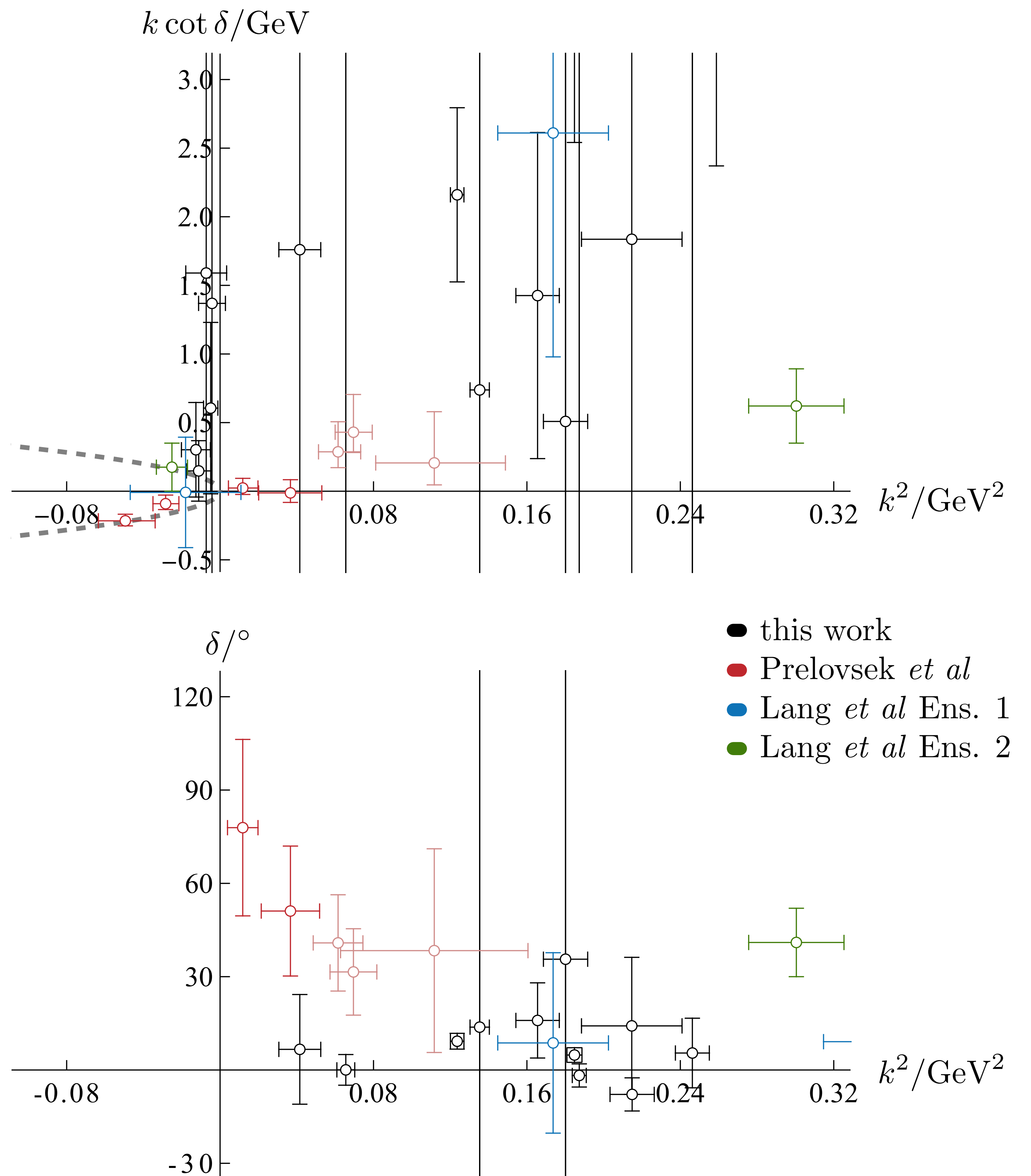


$\rho_i\rho_j|t_{ij}|^2$
$J^{PC}=0^{++}$
$D\bar{D}\{^1S_0\}\to D\bar{D}\{^1S_0\}$
$D_s\bar{D}_s\{^1S_0\}\to D_s\bar{D}_s\{^1S_0\}$
$D^*\bar{D}^*\{^1S_0\}\to D^*\bar{D}^*\{^1S_0\}$
$D\bar{D}\{^1S_0\}\to D_s\bar{D}_s\{^1S_0\}$
$D\bar{D}\{^1S_0\}\to D^*\bar{D}^*\{^1S_0\}$
$\psi\omega\{^1S_0\}\to\psi\omega\{^1S_0\}$
$\eta_c\eta\{^1S_0\}\to\eta_c\eta\{^1S_0\}$
$\psi\phi\{^1S_0\}\to\psi\phi\{^1S_0\}$
$\eta_c\eta'$
$D\bar{D}$ $D_s\bar{D}_s$ $\psi\omega$ $D^*\bar{D}^*$ $\psi\phi$
m/MeV
3800 3850 3900 3950 4000 4050
$m=\mathrm{Re}\sqrt{s_0}\approx 3995(14)$ MeV
$\Gamma=-2\,\mathrm{Im}\sqrt{s_0}\approx 67(38)$ MeV
$\Gamma_{D\bar{D}}\approx 23(13)$ MeV
$\Gamma_{D_s\bar{D}_s}\approx 28(26)$ MeV
$\Gamma_{\psi\omega}\approx 9(^{+18}_{-9})$ MeV
c_i/MeV
530(160) 730(320) 470(470) 1300(550)
$D\bar{D}$ $D_s\bar{D}_s$ $\psi\omega$ $D^*\bar{D}^*$
Γ/MeV
$\rho_i\rho_j|t_{ij}|^2$
$J^{PC}=2^{++}$
$m_\pi\approx 391$ MeV
$D\bar{D}\{^1D_2\}\to D\bar{D}\{^1D_2\}$
$D\bar{D}^*\{^3D_2\}\to D\bar{D}^*\{^3D_2\}$
$D_s\bar{D}_s\{^1D_2\}\to D_s\bar{D}_s\{^1D_2\}$
$D^*\bar{D}^*\{^5S_2\}\to D^*\bar{D}^*\{^5S_2\}$
$D\bar{D}\{^1D_2\}\to D\bar{D}^*\{^3D_2\}$
$D\bar{D}\{^1D_2\}\to D^*\bar{D}^*\{^5S_2\}$
$D\bar{D}^*\{^3D_2\}\to D^*\bar{D}^*\{^5S_2\}$
$\eta_c\eta\{^1D_2\}\to\eta_c\eta\{^1D_2\}$
$\psi\omega\{^5S_2\}\to\psi\omega\{^5S_2\}$
$\psi\phi\{^5S_2\}\to\psi\phi\{^5S_2\}$
$\eta_c\eta'$
$D_sD_s^*$ $\psi\phi$
$D\bar{D}$ $D\bar{D}^*$ $D_s\bar{D}_s$ $\psi\omega$ $D^*\bar{D}^*$
E_{cm}/MeV
m/MeV
3800 3850 3900 3950 4000 4050
$m=\mathrm{Re}\sqrt{s_0}\approx 3961(15)$ MeV
$\Gamma=-2\,\mathrm{Im}\sqrt{s_0}\approx 65(15)$ MeV
$\Gamma_{D\bar{D}}\approx 26(11)$ MeV
$\Gamma_{D\bar{D}^*}\approx 22(13)$ MeV
$\Gamma_{D_s\bar{D}_s}\approx 2(^{+3}_{-2})$ MeV
c_i/MeV
580(140) 700(220) 180(180) 1900(560)
$D\bar{D}$ $D\bar{D}^*$ $D_s\bar{D}_s$ $D^*\bar{D}^*$
Γ/MeV

Results from Prelovsek, Padmanath et al, suggest effects at DDbar and DsDsbar thresholds

- pion mass ~ 280 MeV
- light quark heavier than physical, strange quark lighter than physical

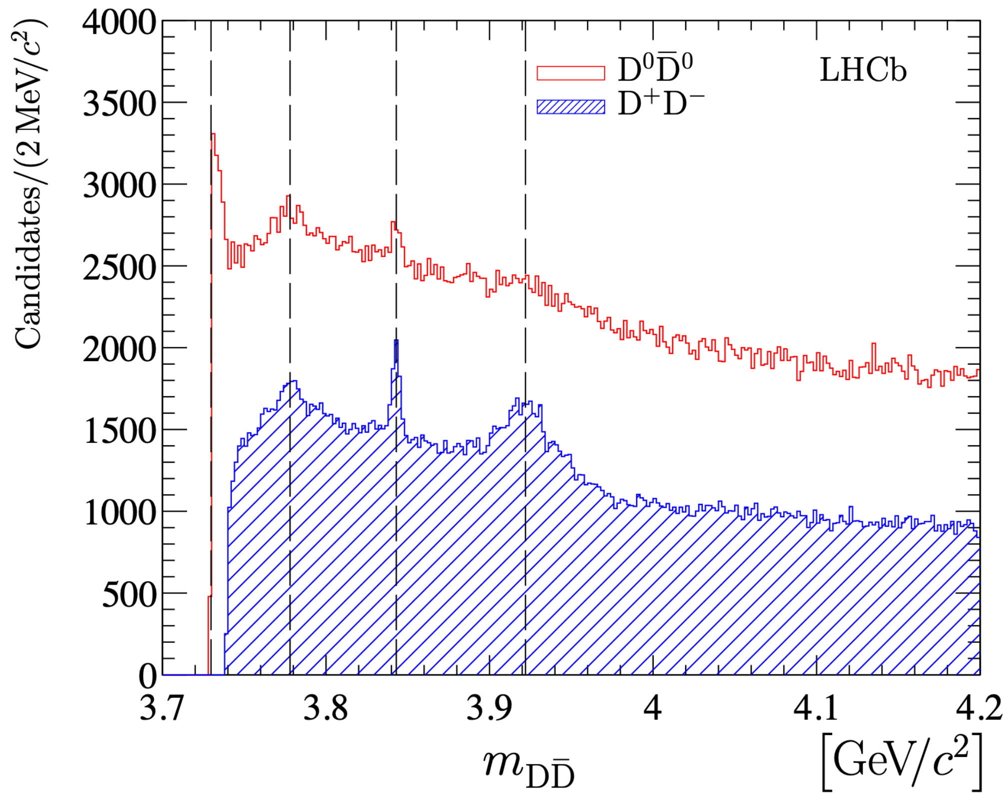
hard to justify such a large change due to the light quark mass (no one-pion-exchange term)



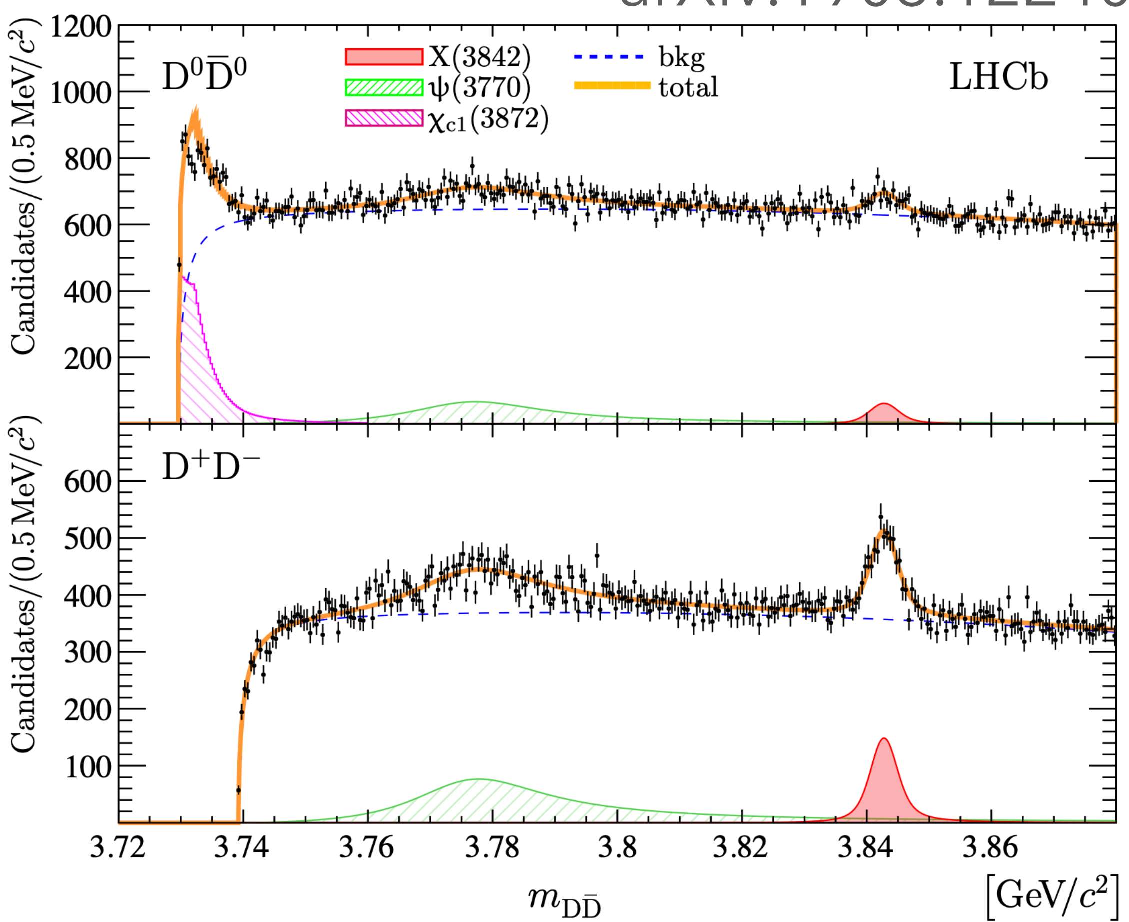
Many models with meson-meson components find strong effects in S-wave DDbar

Several suggestions of a near-threshold state in DDbar scattering
- γγ to DDbar (BaBar, Belle)
- near threshold structure partly due to Born/t-channel photon exchange
- see e.g. Guo & Meißner 2012, Wang et al 2021, Deineka et al 2022

Recent LHCb analyses find a peak at DDbar threshold but attribute this to "feed-down" from X(3872) decays



arXiv:1903.12240



Main messages from this work

Scalar and tensor charmonium scattering amplitudes have been determined

- at m_π=391 MeV, the **level counting is not** obviously **different from the quark model**

- large **coupled-channel** effects in OZI **connected D-meson channels**

- OZI **disconnected** channels look **small everywhere**

- we have extracted a **complete** unitary **S-matrix** and this naturally **connects** features seen in **different channels** and simplifies the overall picture

- a clear, as yet unobserved, 3++ resonance is present in DDbar*

- we do not find a near-threshold DDbar state (between 3700 and 3860 MeV)

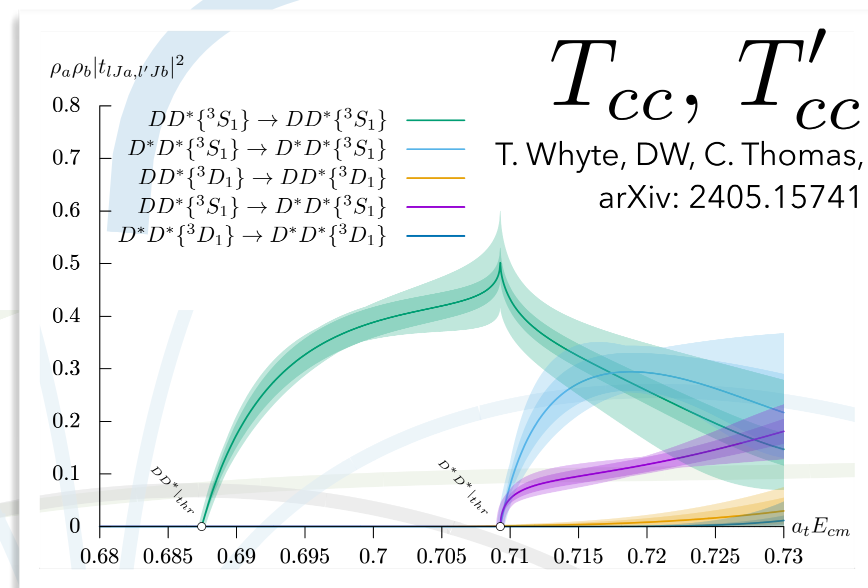
- these methods can also be applied to the X(3872) 1++ channel

Lattice QCD provides a **first-principles** tool to do **hadron spectroscopy**

Charmonium systems are difficult, but achievable
- overlapping effects in several J^{PC}
- many open channels
- quark mass dependence is readily accessible

These methods are widely applicable
- **T_{cc}** doubly-charmed systems, b-quarks
- form factors, radiative transitions (incl. resonances)

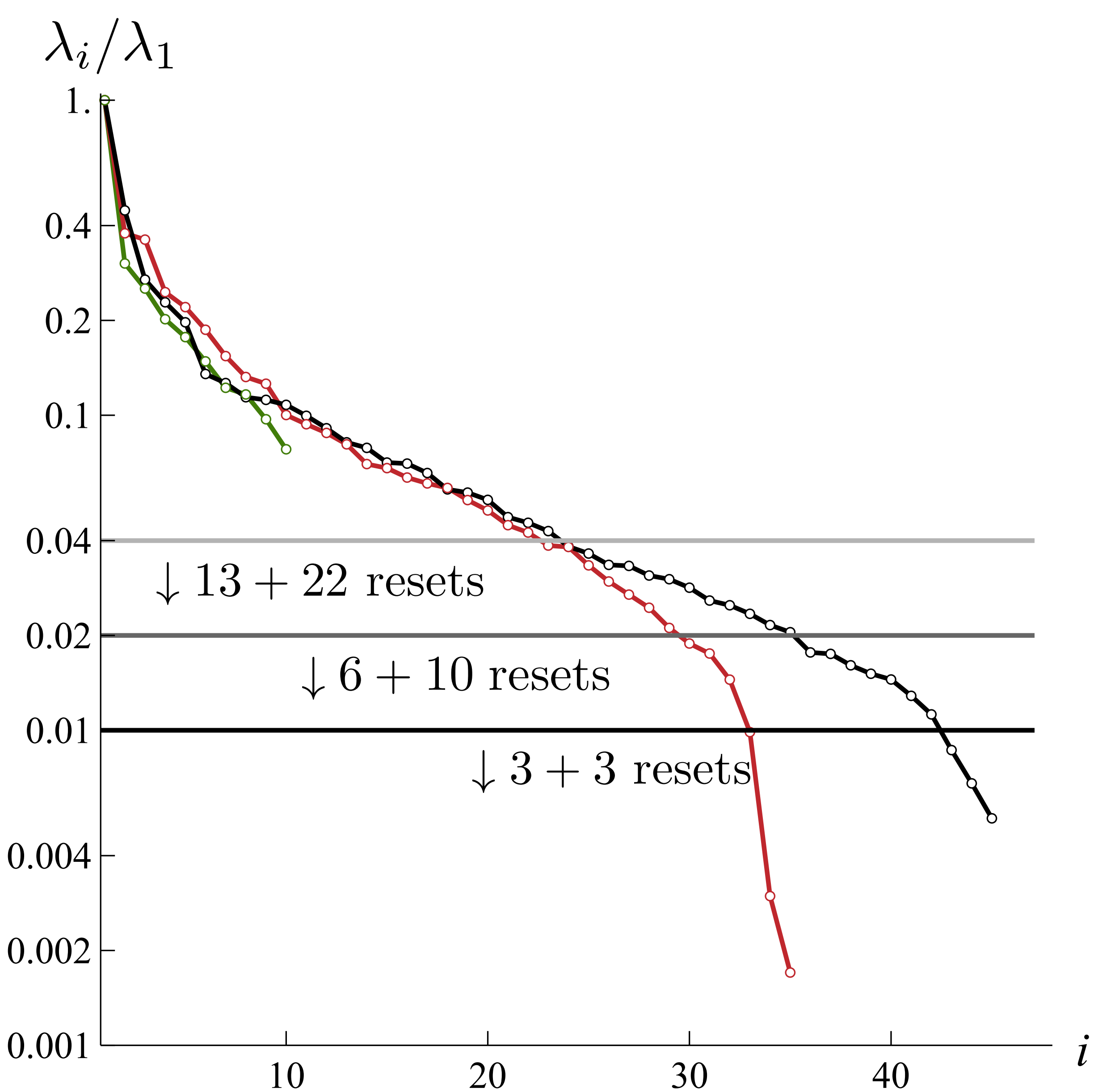
…



Control of 3+ body effects needed for
- lighter pion masses
- higher resonances

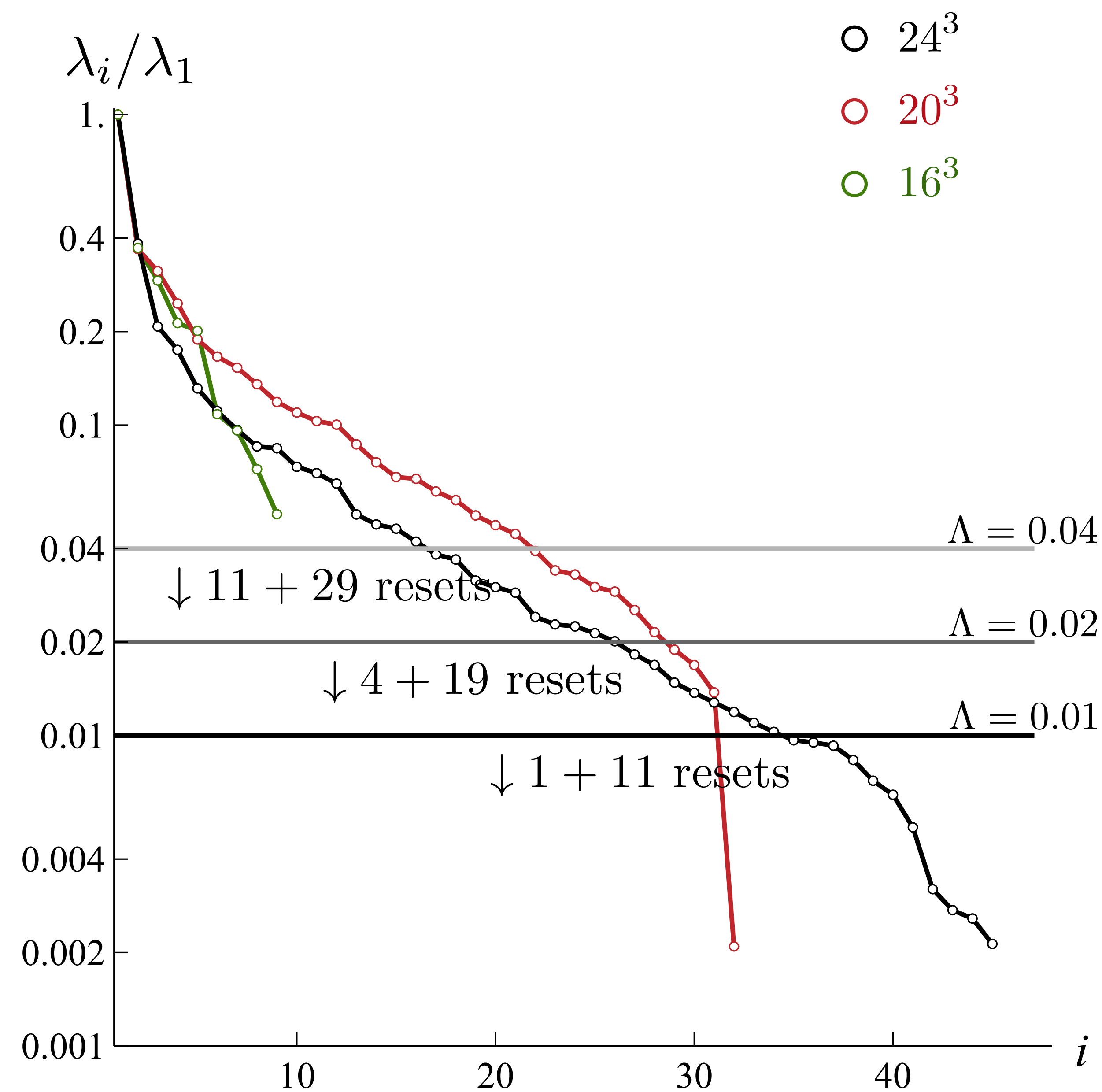
$$J^{PC} = 0^{++}$$

$[000]A_1^+$, $[001]A_1$, $[111]A_1$, $[002]A_1$

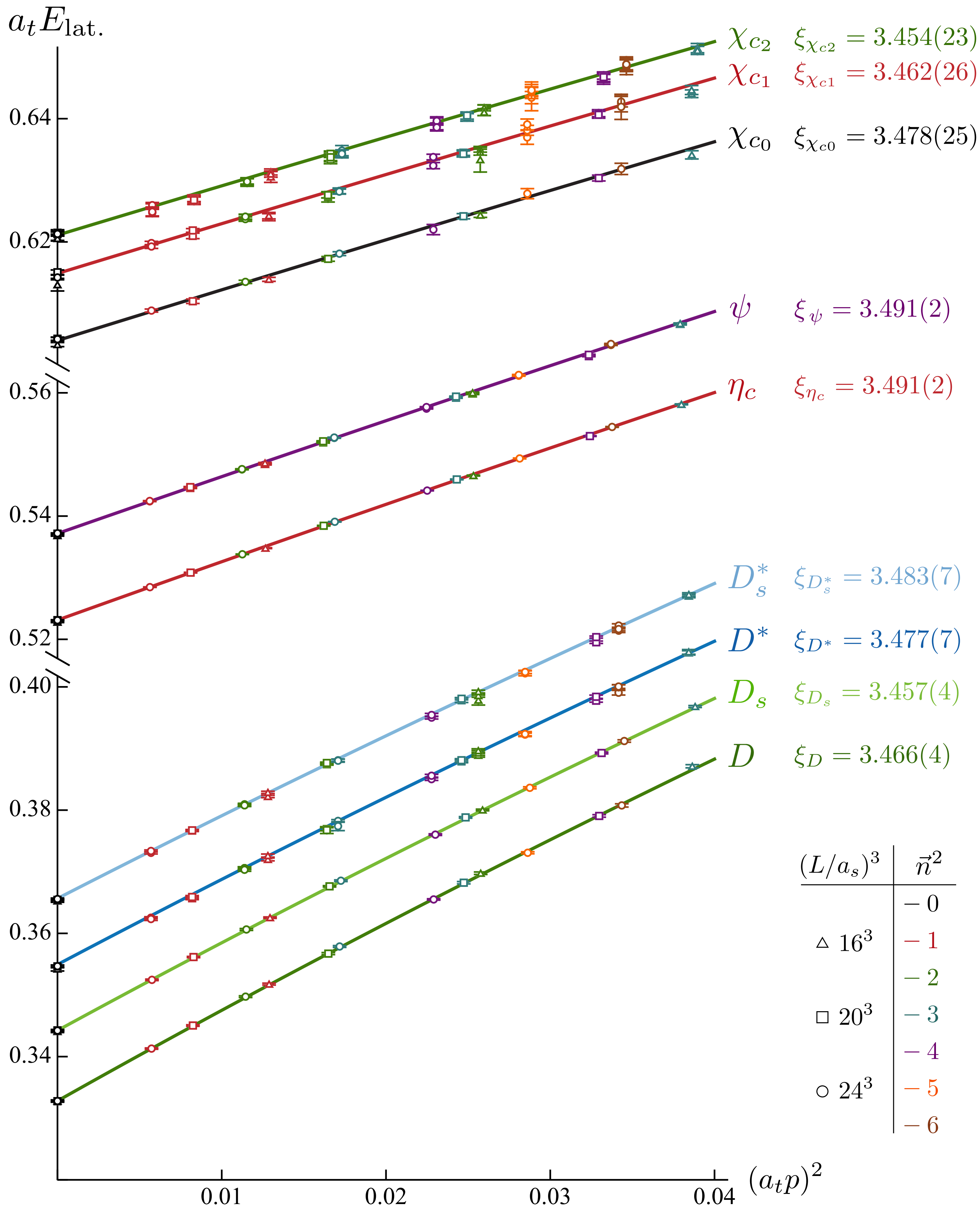


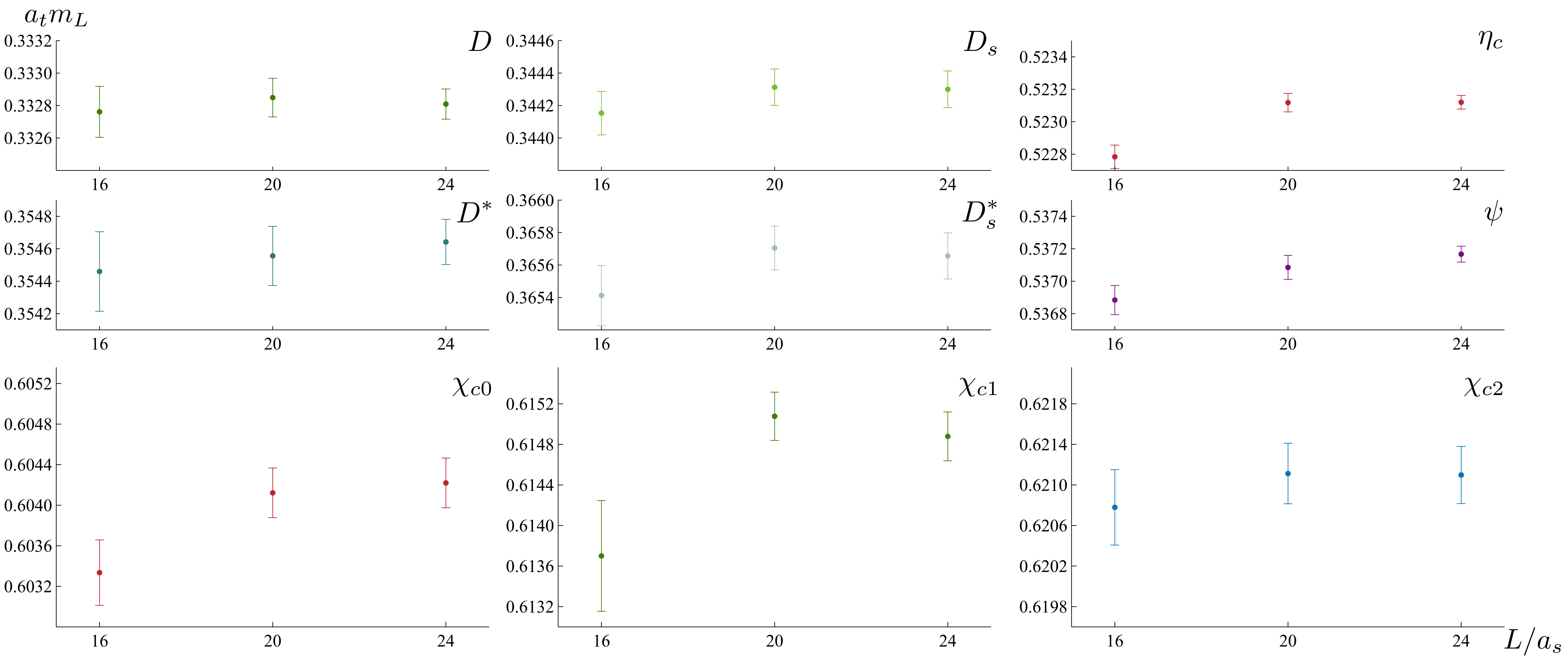
$$J^{PC} = 2^{++}$$

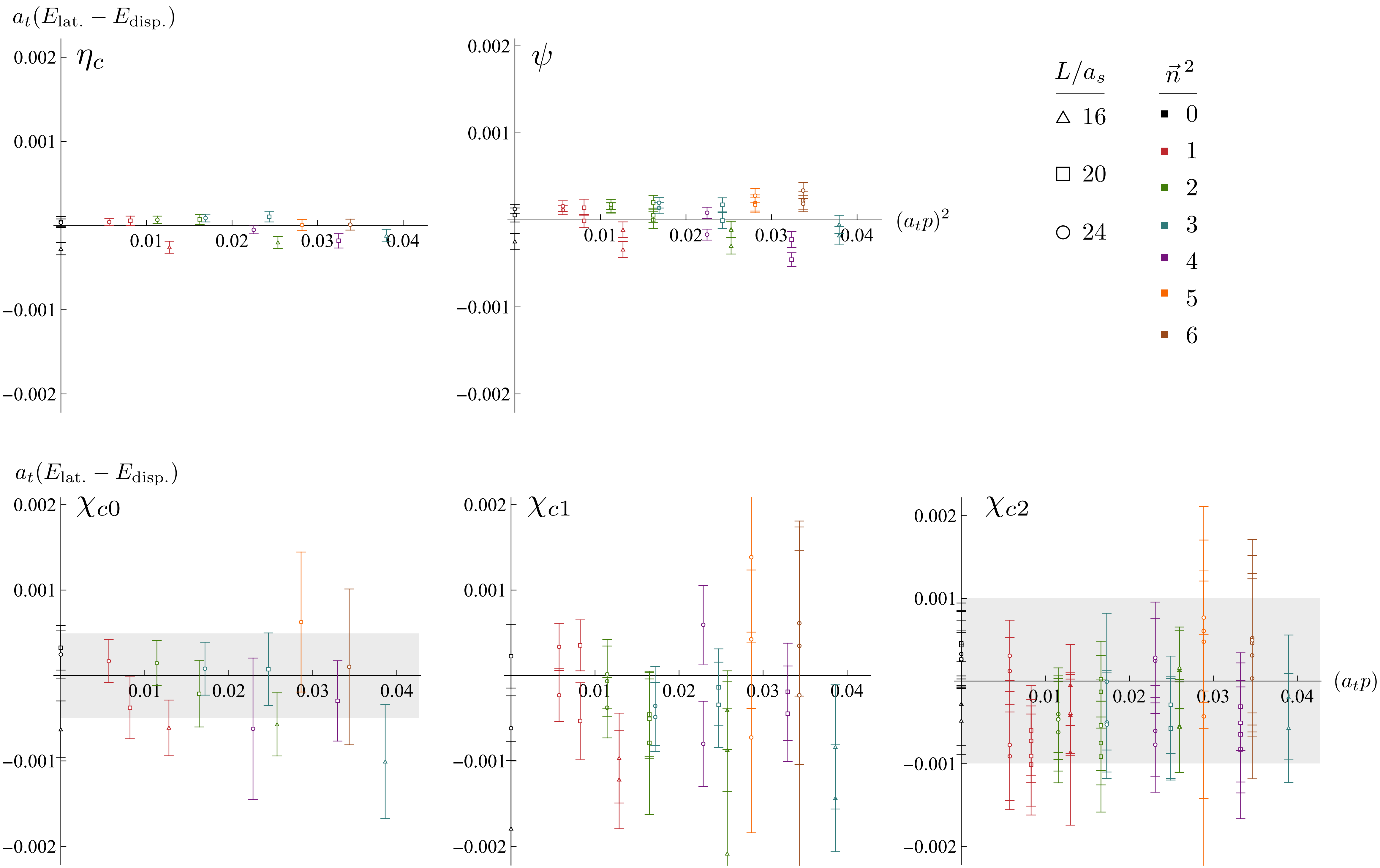
$[000]E^+$, $[000]T_2^+$, $[001]B_1$, $[002]B_1$

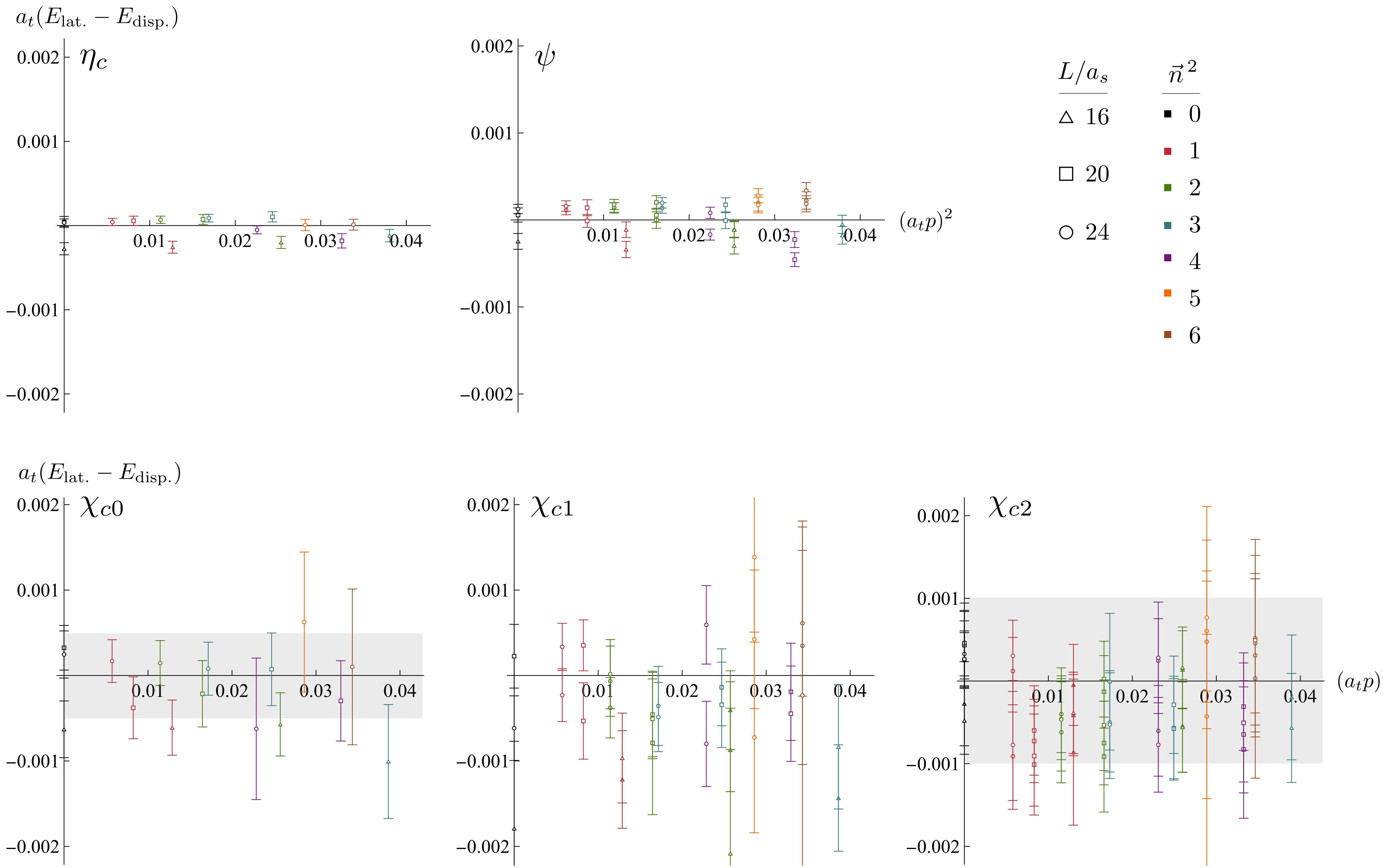


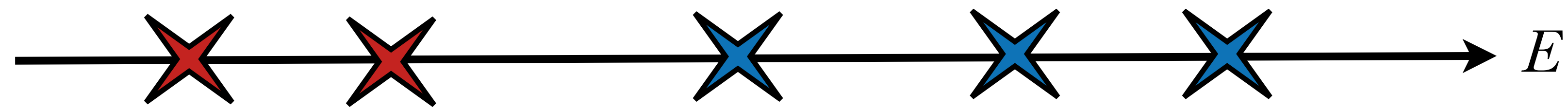
Large correlations are observed between energy levels on each ensemble


$a_t E_{\text{lat.}}$
0.64
0.62
0.56
0.54
0.52
0.40
0.38
0.36
0.34
0.01
0.02
0.03
0.04
$(a_t p)^2$
χ_{c2} $\xi_{\chi_{c2}} = 3.454(23)$
χ_{c1} $\xi_{\chi_{c1}} = 3.462(26)$
χ_{c0} $\xi_{\chi_{c0}} = 3.478(25)$
ψ $\xi_{\psi} = 3.491(2)$
η_c $\xi_{\eta_c} = 3.491(2)$
D_s^* $\xi_{D_s^*} = 3.483(7)$
D^* $\xi_{D^*} = 3.477(7)$
D_s $\xi_{D_s} = 3.457(4)$
D $\xi_{D} = 3.466(4)$
$(L/a_s)^3$
$\vec{n}^2$
16^3
20^3
24^3
– 0
– 1
– 2
– 3
– 4
– 5
– 6


$a_t m_L$
D
D_s
η_c
D^*
D_s^*
ψ
χ_{c0}
χ_{c1}
χ_{c2}
0.3332
0.3330
0.3328
0.3326
0.3446
0.3444
0.3442
0.3440
0.5234
0.5232
0.5230
0.5228
0.3548
0.3546
0.3544
0.3542
0.3660
0.3658
0.3656
0.3654
0.5374
0.5372
0.5370
0.5368
0.6052
0.6048
0.6044
0.6040
0.6036
0.6032
0.6152
0.6148
0.6144
0.6140
0.6136
0.6132
0.6218
0.6214
0.6210
0.6206
0.6202
0.6198
16
20
24
L/a_s


$a_t(E_{\text{lat.}} - E_{\text{disp.}})$
η_c
ψ
χ_{c0}
χ_{c1}
χ_{c2}
0.002
0.001
−0.001
−0.002
0.01
0.02
0.03
0.04
$(a_t p)^2$
L/a_s
△ 16
□ 20
○ 24
$\vec{n}^2$
0
1
2
3
4
5
6


$a_t(E_{\rm lat.} - E_{\rm disp.})$
η_c
ψ
$(a_t p)^2$
L/a_s
16
20
24
$\vec{n}^{\,2}$
0
1
2
3
4
5
6
$a_t(E_{\rm lat.} - E_{\rm disp.})$
χ_{c0}
χ_{c1}
χ_{c2}
$(a_t p)^2$
0.002
0.001
−0.001
−0.002
0.01
0.02
0.03
0.04

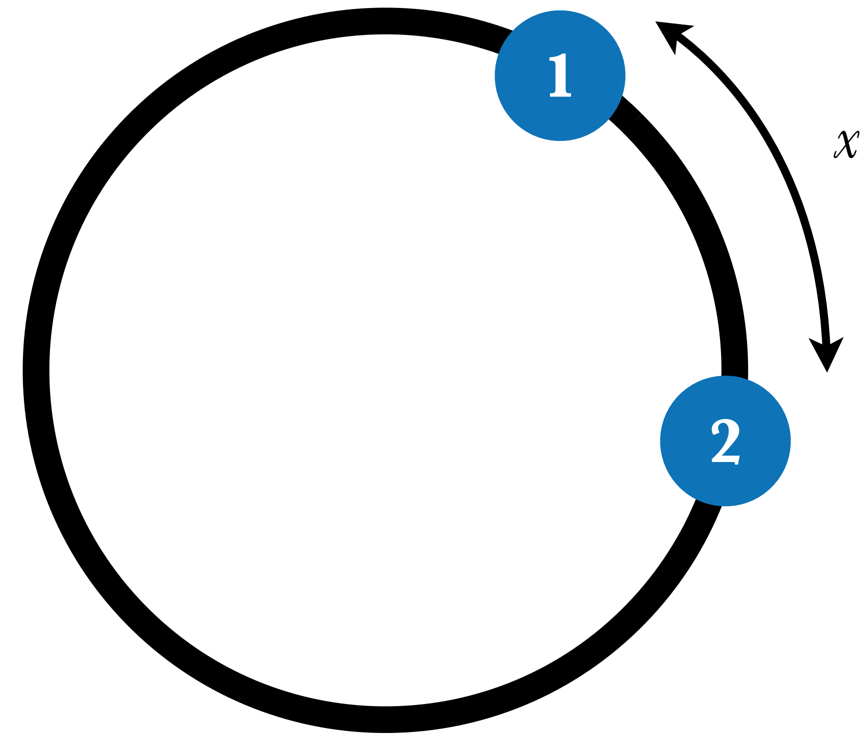


1-dimensional QM, periodic BC, two interacting particles: $V(x_1 - x_2) \neq 0$

$$\psi(0) = \psi(L), \quad \left.\frac{\partial\psi}{\partial x}\right|_{x=0} = \left.\frac{\partial\psi}{\partial x}\right|_{x=L}$$

$$\sin\left(\frac{pL}{2} + \delta(p)\right) = 0$$

$$p = \frac{2\pi n}{L} - \frac{2}{L}\delta(p)$$



Phase shifts via Lüscher's method: $$\tan\delta_1 = \frac{\pi^{3/2} q}{\mathcal{Z}_{00}(1;q^2)}$$

$$\mathcal{Z}_{00}(1;q^2) = \sum_{n\in\mathbb{Z}^3} \frac{1}{|\vec{n}|^2 - q^2}$$

Lüscher 1986, 1991

generalisation to a 3-dimensional strongly-coupled QFT

→ powerful non-trivial mapping from finite vol spectrum to infinite volume phase

See also Kim, Sachrajda, Sharpe: Nucl. Phys. B727 (2005) (arXiv:hep-lat/0507006)

Review by Briceno, Dudek, Young: Rev. Mod. Phys. 90, 025001 (arXiv:1706.06223)

Phase shifts via the Lüscher method: $$\tan\delta_1 = \frac{\pi^{3/2} q}{\mathcal{Z}_{00}(1;q^2)}$$

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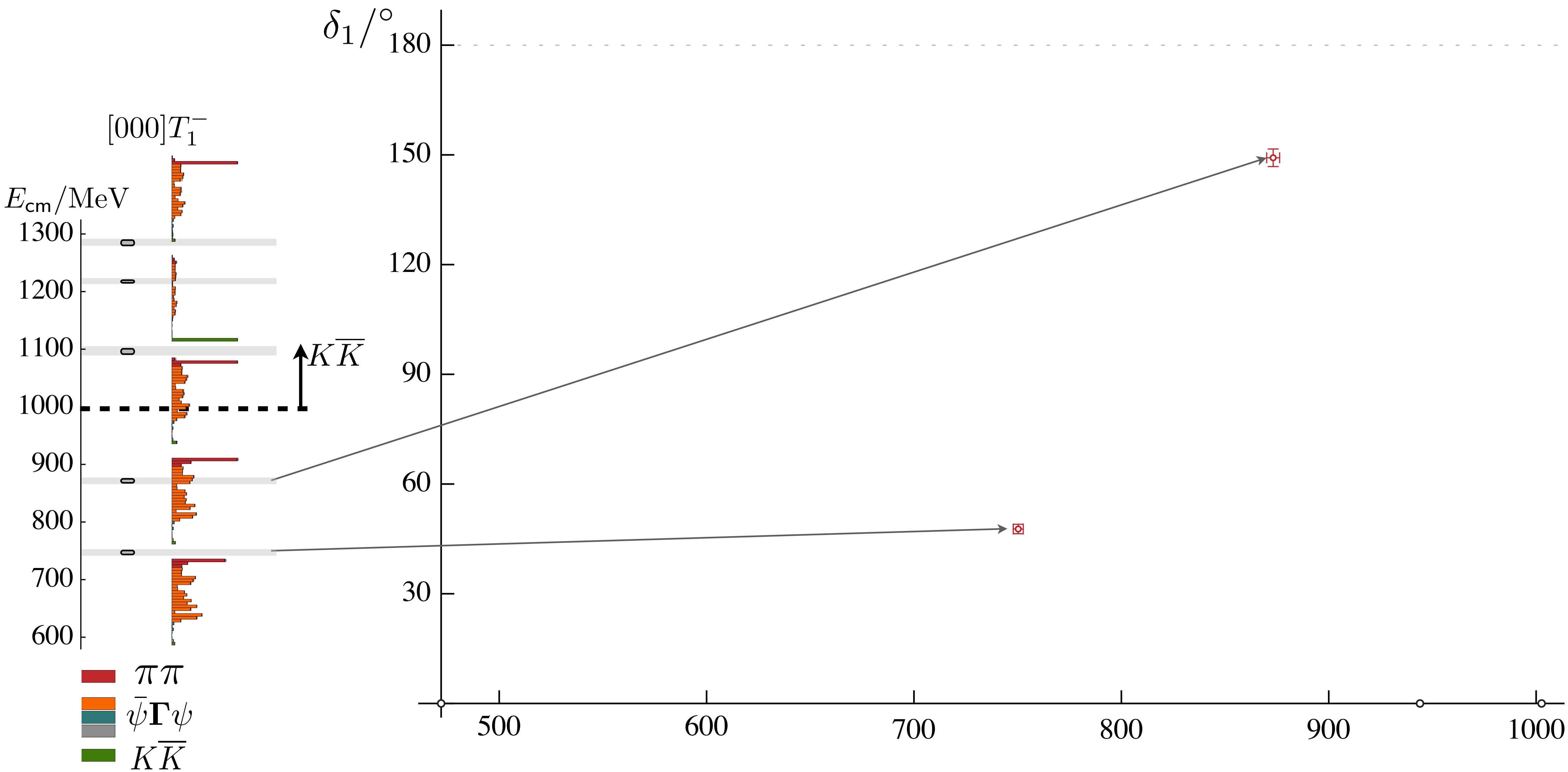
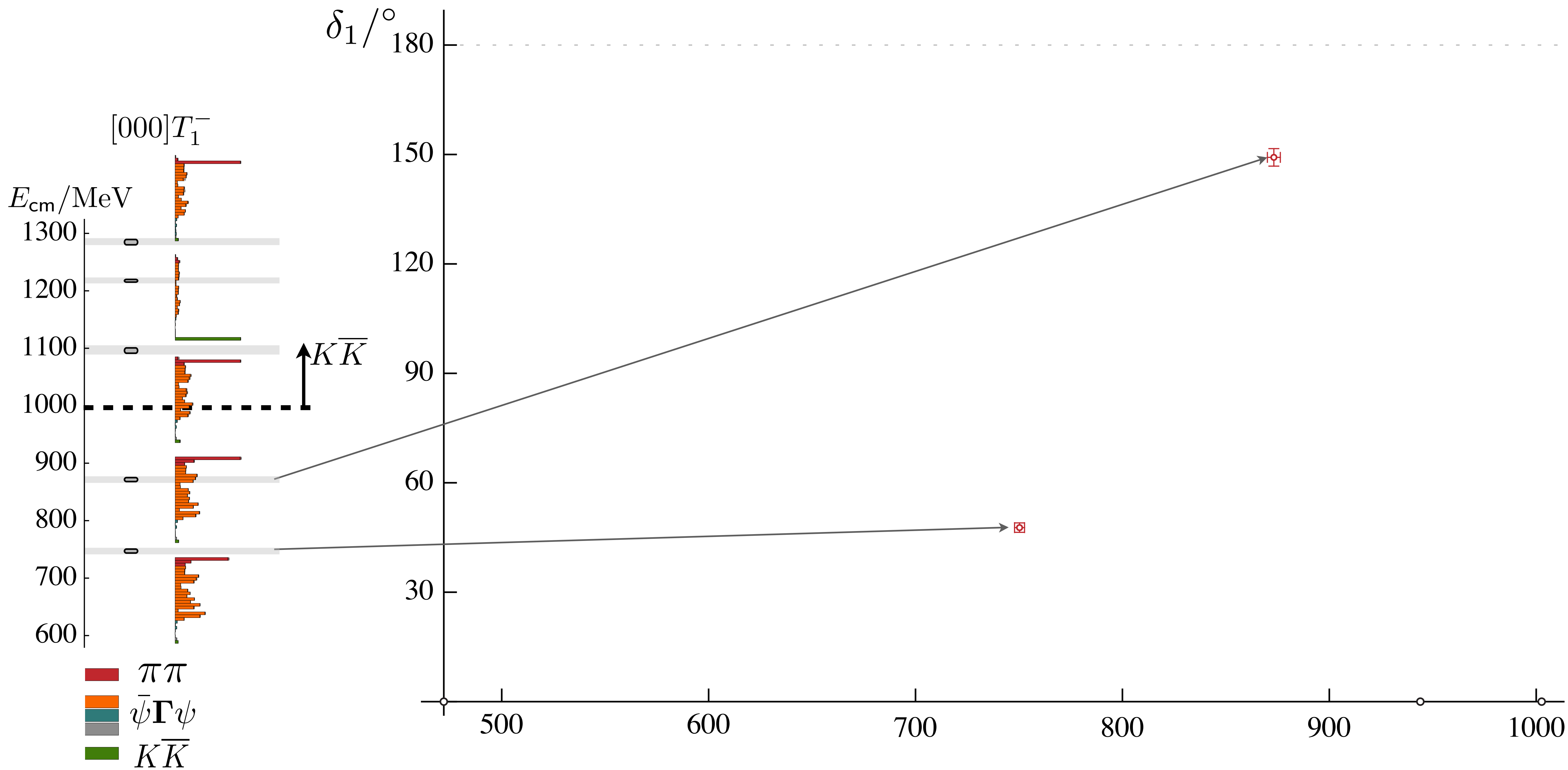


Phase shifts via the Lüscher method: $\tan\delta_1 = \frac{\pi^{3/2} q}{\mathcal{Z}_{00}(1;q^2)}$

$$\mathcal{Z}_{00}(1;q^2) = \sum_{n\in\mathbb{Z}^3} \frac{1}{|\vec{n}|^2 - q^2}$$



Phase shifts via the Lüscher method: $\tan\delta_1 = \frac{\pi^{3/2} q}{\mathcal{Z}_{00}(1;q^2)}$

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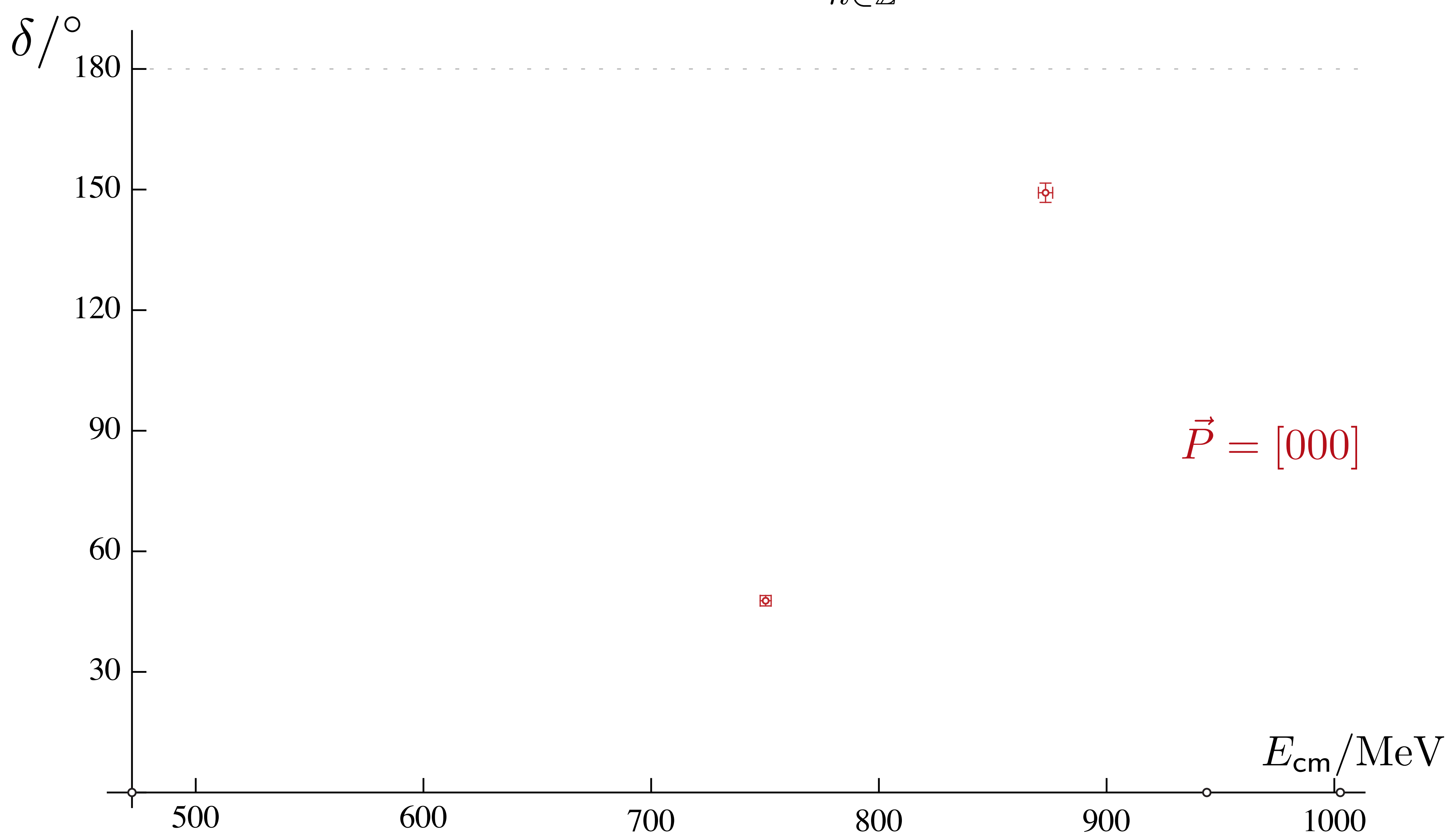
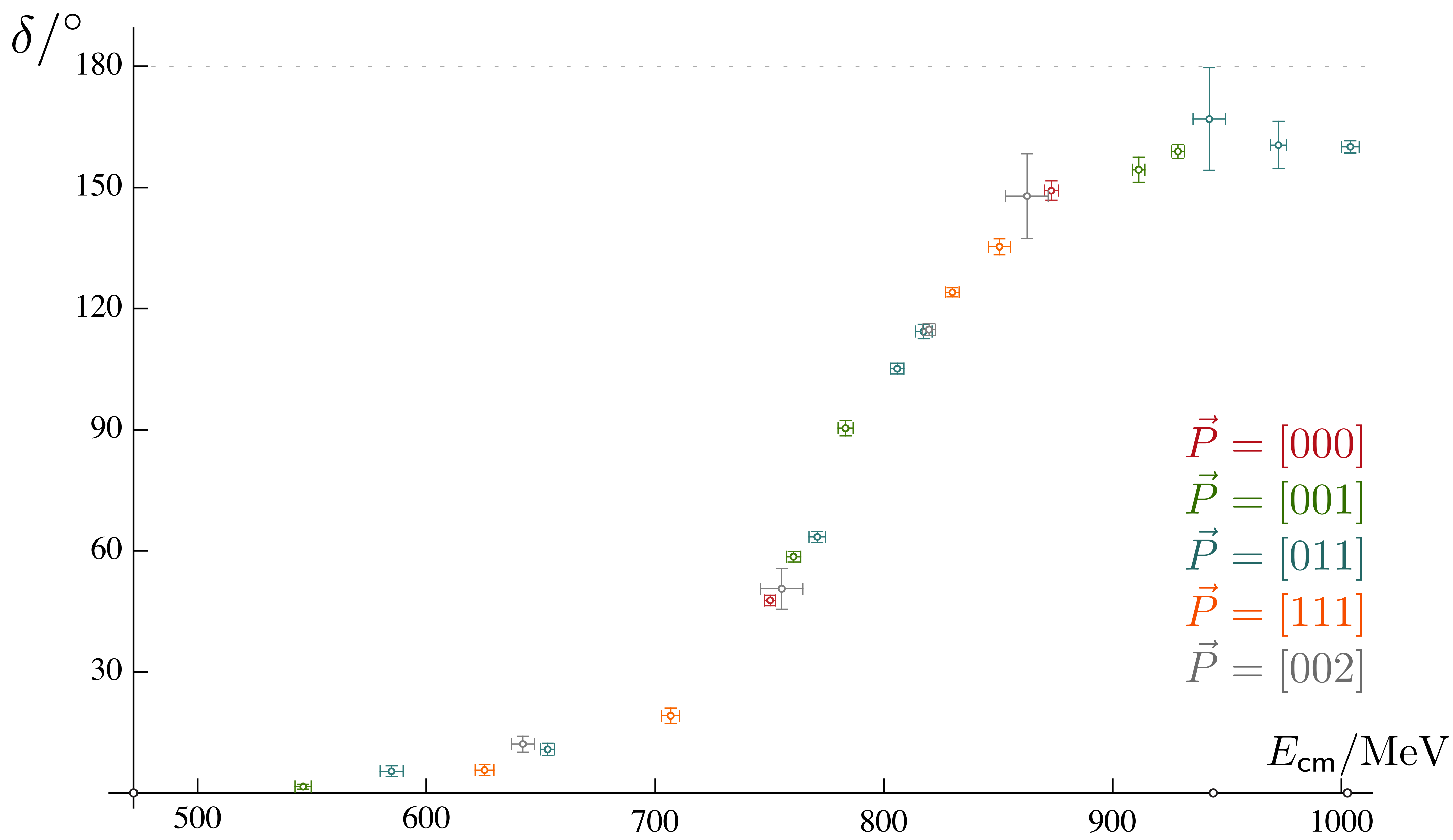
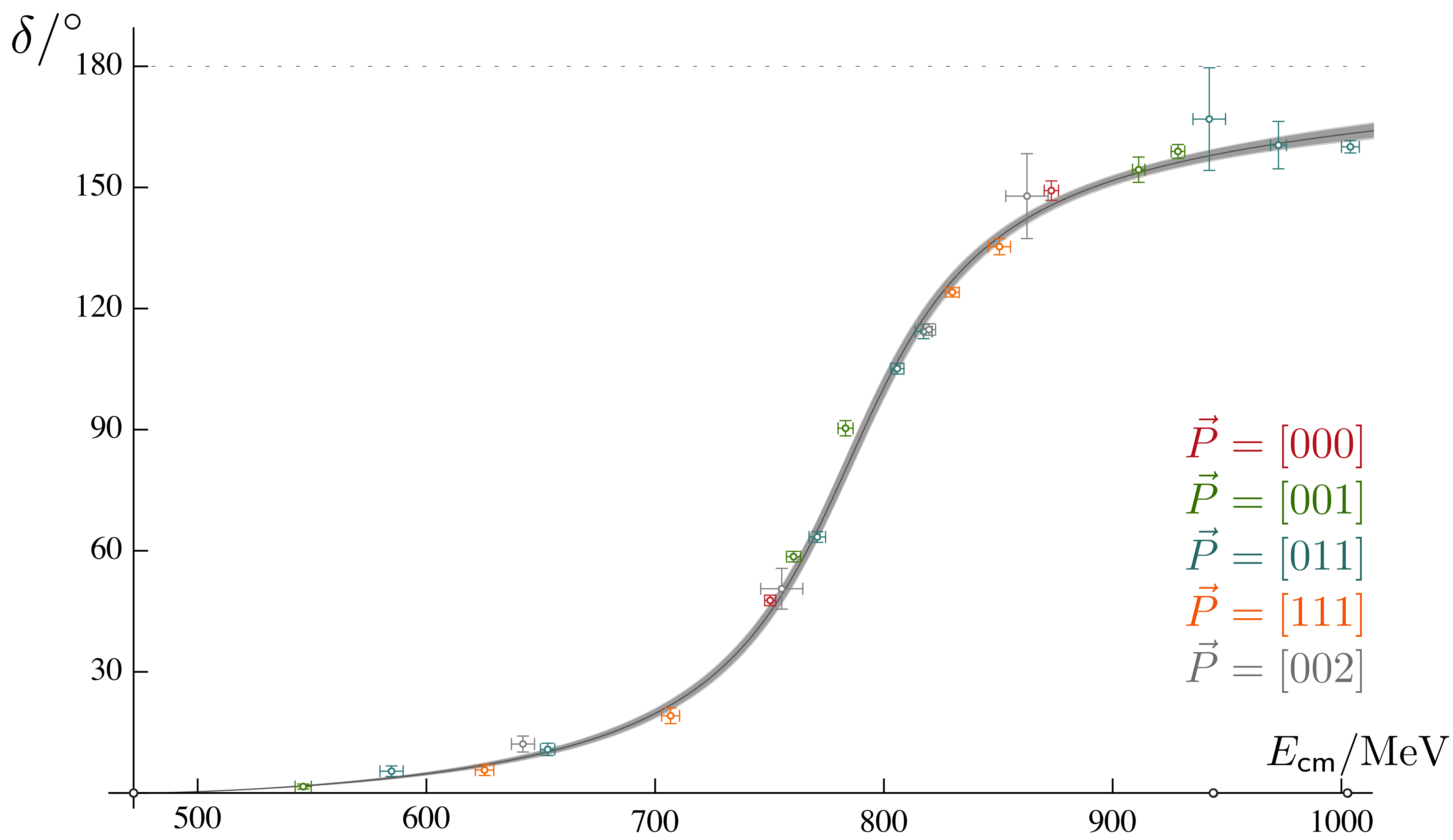




δ/°
180
150
120
90
60
30
500
600
700
800
900
1000
E_cm/MeV
P = [000]
P = [001]
P = [011]
P = [111]
P = [002]



δ/°
180
150
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500
600
700
800
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1000
E_cm/MeV
P = [000]
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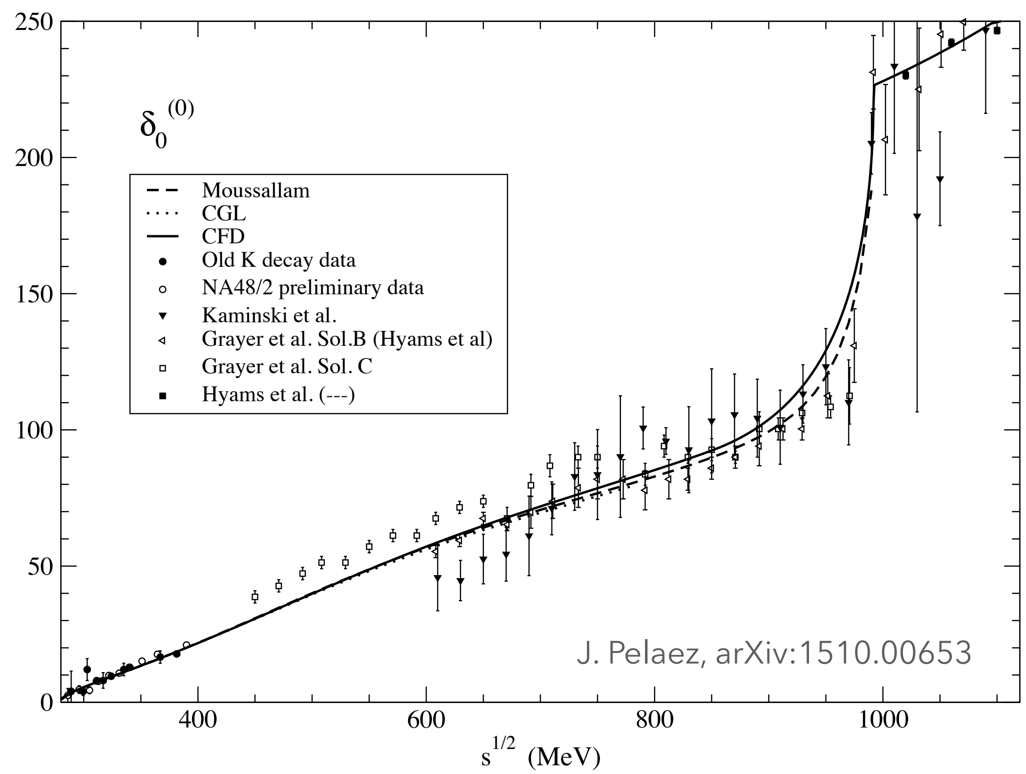
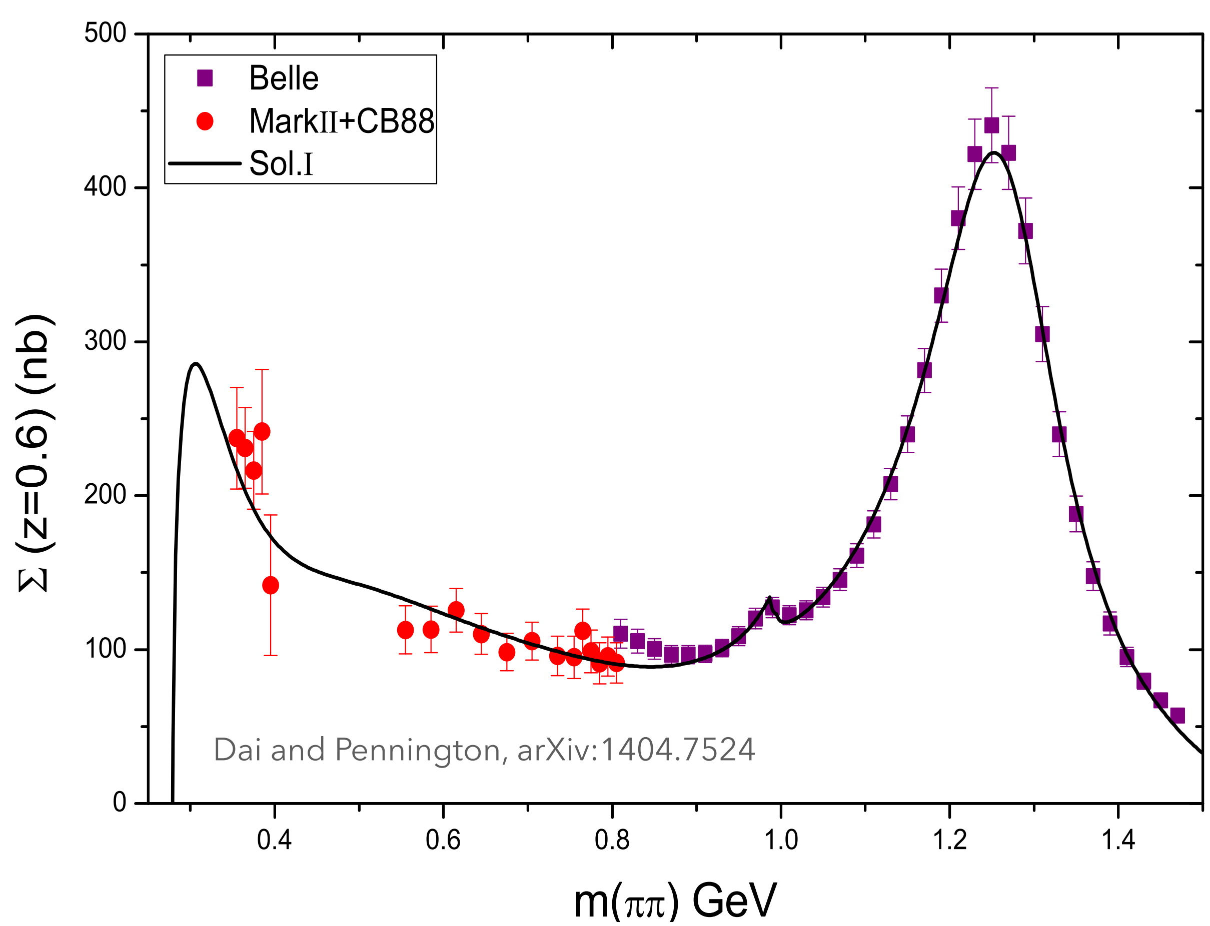
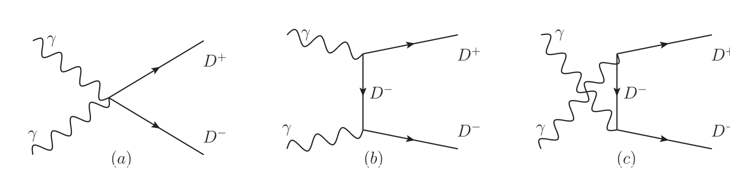


$$\pi\pi \to \pi\pi \quad (S-\text{wave})$$



$$\gamma\gamma \to \pi\pi$$

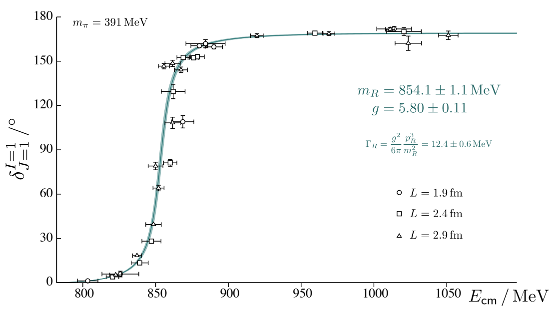


extra structure at threshold, not linked to a resonance or bound state

hadspec
hadspec.org

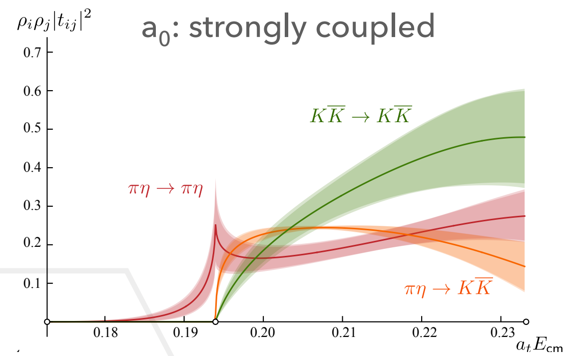
2012 2014 2016 2018 2020

applications: (a biased sample)



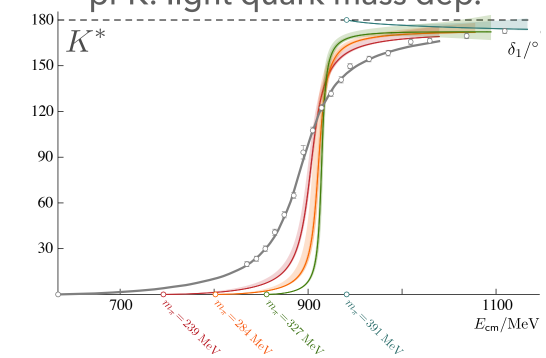
elastic scattering:
rho resonance

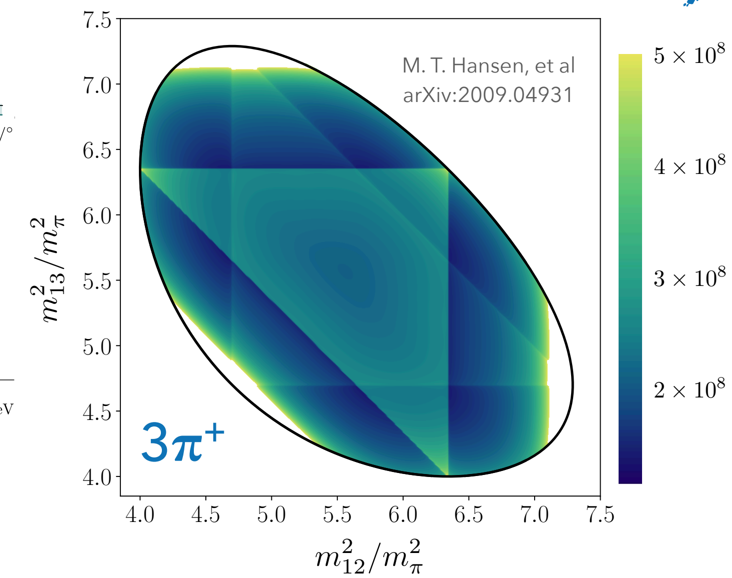
a$_0$: strongly coupled



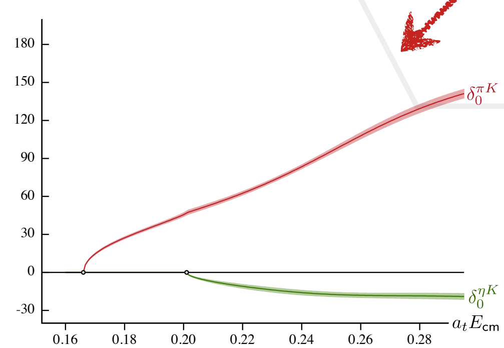
J. Dudek et al, PRD 93 (2016) 9, 094506

pi-K: light quark mass dep.

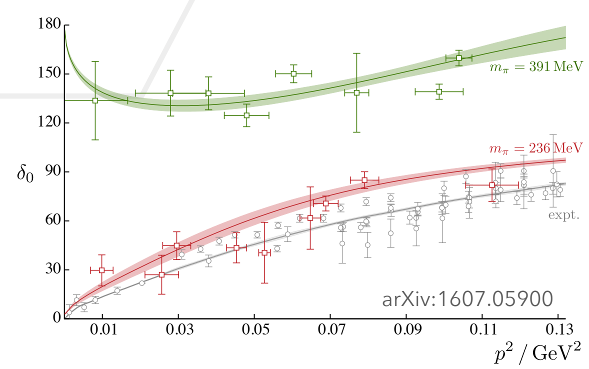




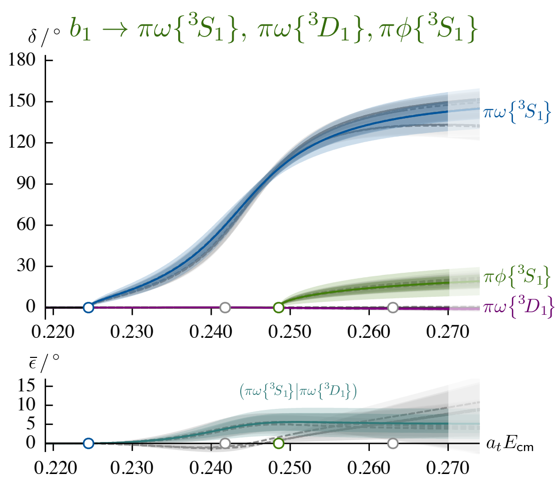
coupled-channel
scattering



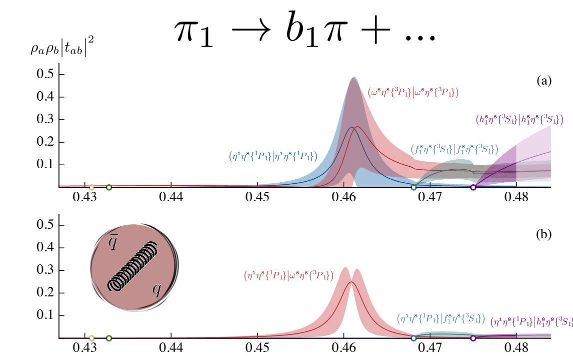
pi-K: almost decoupled,
first ever application



elastic scattering:
sigma resonance,
light quark mass dep.



scattering of
hadrons with spin



π_1 decays, SU(3) flavour,
11 active channels

formalism/theory developments:

pseudocalar two-body
coupled-channel
scattering

resonance
transition FFs

scattering of
hadrons with spin

three-body
scattering

form factors
of resonances

general three-body
scattering

more general processes:
two currents, ...