The quenched glueball spectrum from smeared spectral densities

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Exhotic Hadrons Spectroscopy
Swansea University
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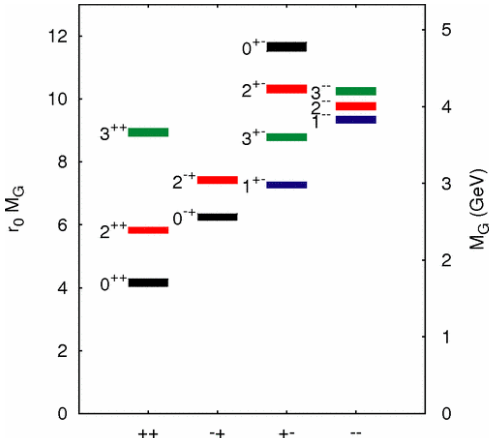
Glueball spectrum in pure $SU(3)$ Yang-Mills

- Gluballs are quarkless bound states predicted by QCD
- Conclusive experimental detection of glueballs is still missing
- Theory calculations heavily rely on lattice studies (quenched/unquenched)



[Y. Chen *et al.*, **PRD**, hep-lat/0510074]

News from experiment

BES III announced the determination of $X(2370)$ with $J^{PC} = 0^{-+}$.

Is it the pseudoscalar glueball?

PHYSICAL REVIEW LETTERS **132,** 181901 (2024)

Editors' Suggestion

Determination of Spin-Parity Quantum Numbers of $X(2370)$ as 0^{-+} from $J/\psi \to \gamma K_S^0 K_S^0 \eta'$

M. Ablikim *et al.*[*]
(BESIII Collaboration)

(Received 8 December 2023; revised 5 March 2024; accepted 28 March 2024; published 2 May 2024)

Based on $(10087 \pm 44) \times 10^6$ J/ψ events collected with the BESIII detector, a partial wave analysis of the decay $J/\psi \to \gamma K_S^0 K_S^0 \eta'$ is performed. The mass and width of the $X(2370)$ are measured to be $2395 \pm 11(\text{stat})^{+26}_{-94}(\text{syst})$ MeV$/c^2$ and $188^{+18}_{-17}(\text{stat})^{+124}_{-33}(\text{syst})$ MeV, respectively. The corresponding product branching fraction is $\mathcal{B}[J/\psi \to \gamma X(2370)] \times \mathcal{B}[X(2370) \to f_0(980)\eta'] \times \mathcal{B}[f_0(980) \to K_S^0 K_S^0] = (1.31 \pm 0.22(\text{stat})^{+2.85}_{-0.84}(\text{syst})) \times 10^{-5}$. The statistical significance of the $X(2370)$ is greater than 11.7σ and the spin parity is determined to be 0^{-+} for the first time. The measured mass and spin parity of the $X(2370)$ are consistent with the predictions of the lightest pseudoscalar glueball.

DOI: 10.1103/PhysRevLett.132.181901

What do we know?

- Experimental result is in good agreement with quenched lattice result ($m_G \approx 2.4\text{GeV}$).
- Unquenched lattice results tend to be in agreement with quenched results at least for pseudoscar and tensor channels. However no conclusive statement exist on this matter.
- No continuum exptrapolation results exist for unquenched lattice studies. However, dependence on the lattice specing seems weak.

What we don't know (yet)

- Mixing with $q\bar{q}$ states in unquenched setting.
- Unquenching suppression effects especially in the scalar channel.
- Systematic effects due to choice of operators in variational basis.
- The production rate of glueball states (a quenched results exists for the 0^{-+} channel [Gui L.C. *et. al.*, **PRD**,1906.03666]).

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➜ **Further work needed to improve lattice QCD calculations of the glueball spectrum**

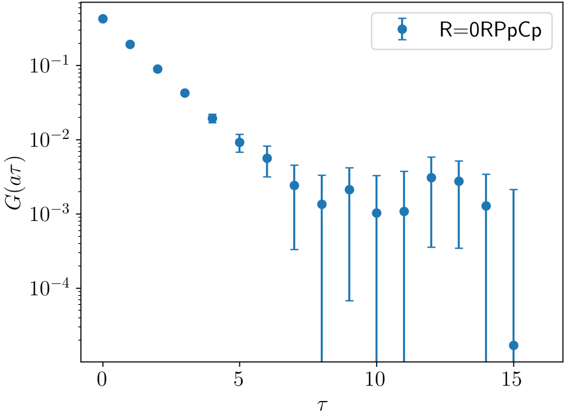
Glueballs on the lattice

Glueballs masses can be extracted from lattice correlation functions

$$G(a\tau) = \langle \Phi(a\tau)\Phi(0)\rangle_{\text{conn.}} = \sum_n |A_n|^2 e^{-a\tau\omega_n}$$

$$A_n = \frac{\langle n|\Phi(0)|0\rangle}{\sqrt{2\omega_n}}$$ → energy state overlap

Bad signal/noise ratio



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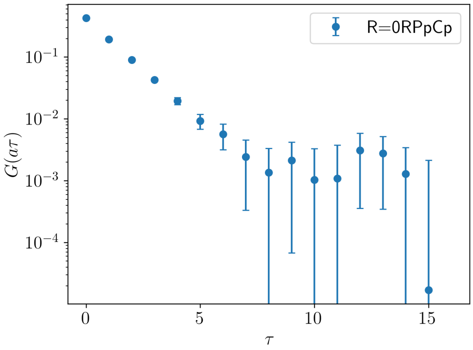
"We face an impasse: if t is small the estimated mass is not the true mass and if t is large the statistical error may be so large that nothing may be measured" G. Parisi



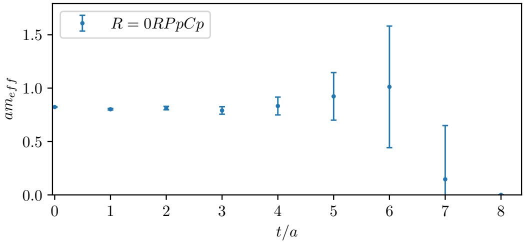
Variational method

The situation is improved considering a large set of operators with different smearing/blocking

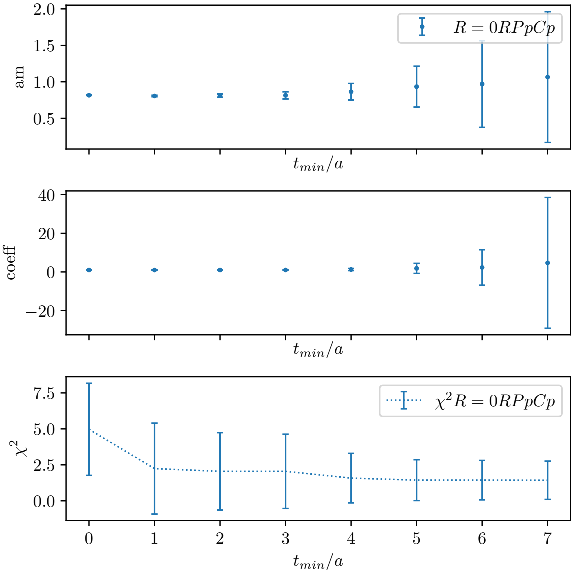
$$\sum_j C_{ij}(t_0)v_j = \sum_j \lambda_j(t_0)C_{ij}(0)v_j$$

$$am_{eff}(t_0) = \ln\left(\frac{v_i C_{ij}(t)v_j}{v_i C_{ij}(t-1)v_j}\right)$$

$$C_{ii}(a\tau) = |A_n|^2 \cosh(am_i\tau - \frac{N_L}{2})$$



- The "standard" method led to impressive results over the years
- Variational method help disentangle states
- Pratically can only use few lattice times



Can we use spectral functions?

Writing the Euclidean correlator in the Källen-Lehmann representation

$$G(a\tau) = \int_{\omega_{\min}}^{\infty} d\omega \; \rho(\omega) e^{-a\omega\tau}$$

- ➜ For lattice correlators this leads to a **ill-posed inverse problem**
- ➜ Need a method to regularise the problem. Also, finite volume (L) means

$$\rho_L(\omega_n) = \sum_n \frac{|\langle n|\Phi(0)|0\rangle|^2}{2\omega_n} \delta(\omega - \omega_n).$$

Spectral reconstruction

The central idea is that we can calculate numerically

$$K(\omega_n, \omega) = \sum_{\tau=0}^{\tau_{\max}} g_t(\omega_n) e^{-\omega\tau}.$$

Then, spectral functions can be reconstructed applying linear combination of coefficients to LQCD correlators:

$$\hat{\rho}(\omega_n) = \int_0^\infty d\omega\ \rho_L(\omega) K(\omega, \omega_n) = \sum_{\tau=0}^{\tau_{\max}} g_t(\omega_n) \int_0^\infty d\omega\ \rho_L(\omega) e^{-\omega\tau}$$
$$\simeq \sum_{\tau=0}^{\tau_{\max}} g_t(\omega_n) G(\tau)$$

In an **ideal** scenario: $K(\omega_n, \omega) = \delta(\omega - \omega_n)$, so that

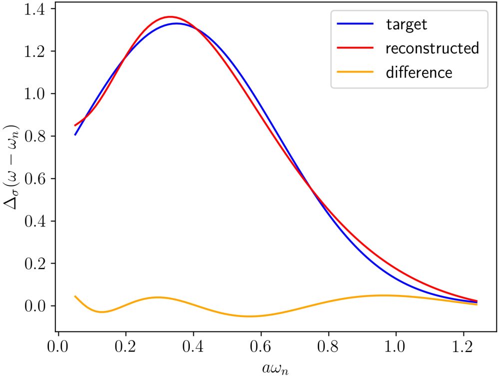
$$\hat{\rho}(\omega_n) = \int_0^\infty d\omega\ \delta(\omega - \omega_n) \rho_L(\omega)$$

HLT method [Hansen, Lupo, Tantalo, **PRD**, 1903.06476]

We can use Backus-Gilbert regularisation to extract **smeared** spectral function from the lattice correlation function

$$\Delta_\sigma(\omega; \boldsymbol{g}) = \sum_{\tau=1}^{\tau_{\max}} g_\tau(\sigma, \omega_n) e^{-a\tau\omega}$$

$$G(a\tau) = \int_{\omega_{\min}}^{\infty} d\omega \ \rho(\omega) e^{-a\omega\tau}$$

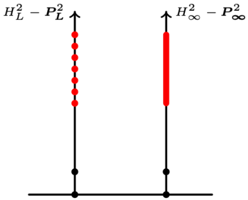


$\sigma/a = 0.3, \ a\omega_n = 0.35$

$$\rho_L^\sigma(\omega_n) = \int_0^\infty d\omega \ \rho_L(\omega) \Delta_\sigma(\omega - \omega_n) \simeq \sum_{\tau=1}^{\tau_{\max}} g_\tau(\sigma, \omega_n) G(a\tau).$$

Comments on the limits

$$\widehat{\rho}(\omega_n) = \lim_{\sigma\to 0} \lim_{L\to\infty} \widehat{\rho}_{\sigma,L}(\omega_n)$$

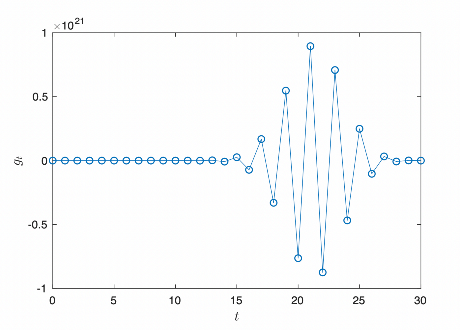


- Smearing allow to replace $\sum_n \delta_n$ with a smooth function. This is necessary to perform a meaningful infinite volume limit.
  [Hansen,Meyer,Robaina, **PRD**, 1704.08993]
- $\sigma \to 0$ is not strictly necessary, **if** we want to compare with an experimental result which can be equally smeared as our spectral density.
  [Hansen,Lupo,Tantalo, **PRD**, 1903.06476]

Backus-Gilbert regularisation

Hansen, Lupo, Tantalo, **PRD**, 1903.06476

$$A_n[\boldsymbol{g}] = \int_{\omega_0}^{\infty} d\omega \; w_n(\omega) \left| \overline{\Delta}_\sigma(\omega;\boldsymbol{g}) - \Delta_\sigma(\omega - \omega_n) \right|^2 .$$



[Hansen, Lupo, Tantalo, **PRD**, 1903.06476]

Backus-Gilbert regularisation

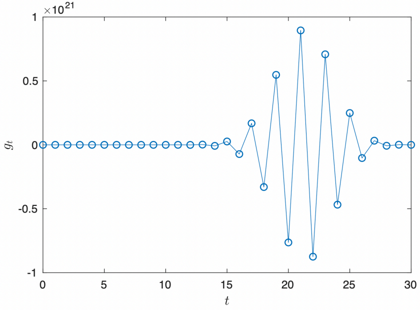
Hansen, Lupo, Tantalo **PRD**, 1903.06476

$$W_n[\boldsymbol{g}] = \frac{A_n[\boldsymbol{g}]}{A_n[\boldsymbol{0}]} + \lambda \frac{B[\boldsymbol{g}]}{B_{\text{norm}}}, \quad \left.\frac{\partial W_n[\boldsymbol{g}]}{\partial g_\tau}\right|_{g_\tau^{\boldsymbol{p}} = g_\tau^*} = 0$$

$$A_n[\boldsymbol{g}] = \int_{\omega_0}^{\infty} d\omega \; w_n(\omega) \left| \overline{\Delta}_\sigma(\omega; \boldsymbol{g}) - \Delta_\sigma(\omega - \omega_n) \right|^2.$$

$$B[\boldsymbol{g}] = \sum_{\tau_1, \tau_2 = 1}^{\tau_{\max}} g_{\tau_1} g_{\tau_2} \, \text{Cov}(\tau_1, \tau_2),$$

$$\boldsymbol{p} = (\alpha, \lambda, \omega_{\min}, \tau_{\max},)$$



[Hansen, Lupo, Tantalo, **PRD**, 1903.06476]

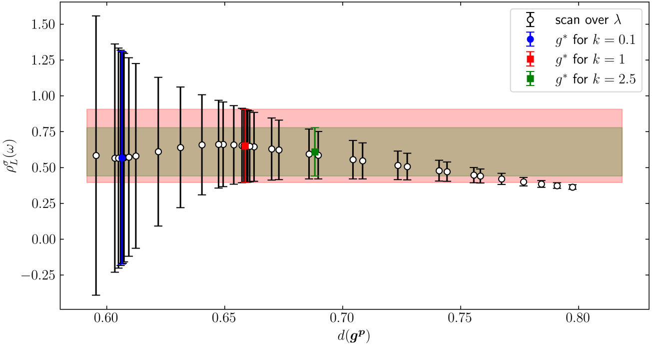
Stability analysis

- Method introduced in
  [Bulava *et al.*, JHEP, 2111.12774]
- Choose final result in statistically dominated region

$$\frac{A[\boldsymbol{g}^*]}{A[0]} = kB[\boldsymbol{g}^*]$$

- Final results need to be extrapolated

$$\rho(\omega) = \lim_{\sigma\to 0} \lim_{L\to\infty} \rho_L^\sigma(\omega)$$



$$d(\boldsymbol{g^p}) = \sqrt{A[\boldsymbol{g}]/A[0]}$$

Ensembles details

We are currently at a very pleriminary stage and plan to soon include more values of β and other representations $A_1^{-+}, E^{++}, \ldots$

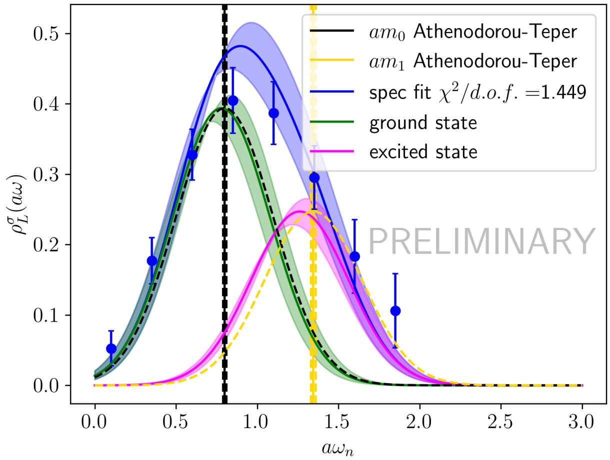
J^{PC}	β	$L^3 \times T$	N_{cnfg}
A_1^{++}	5.8941	$32^3 \times 32$	15000
A_1^{++}	6.0625	$32^3 \times 32$	15000

Fit of smeared spectral functions

[Athenodorou,Teper, 2007.06422, JHEP]

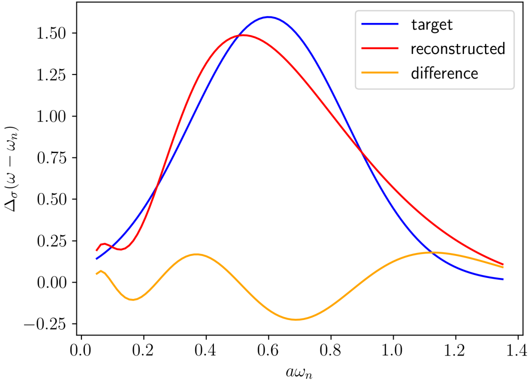
- Introduced in [Del Debbio, *et al.*, 2211.09581, *Eur.Phys.J.C.*]
- We can perform fit of spectral functions rather than correlators
- Minimise χ^2 function defined in terms of $\mathrm{Cov}[\rho^\sigma]$

$$f_k^\sigma(\omega) = \sum_k a_k \, e^{\frac{-(\omega-\omega_k)^2}{2\sigma^2}},$$

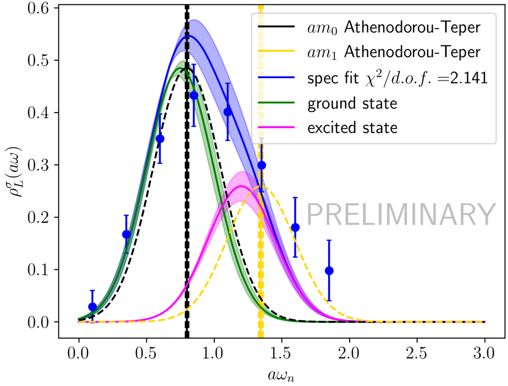


$\beta = 5.89$, $\sigma = 0.3/a$, $\chi^2_{red} = 1.45$

Reducing σ is challenging

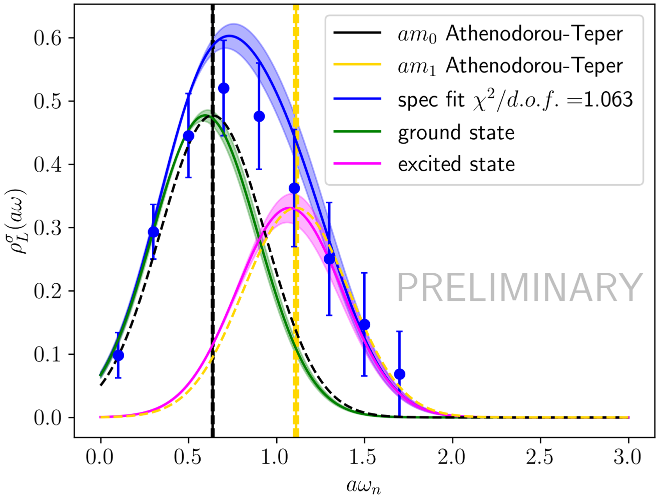


$\beta = 5.89,\ \sigma = 0.25/a,\ a\omega_n = 0.6$



$\beta = 5.89,\ \sigma = 0.25/a,\ \chi^2_{red} = 2.14$

Reducing the lattice spacing



$$\beta = 6.0625,\ \sigma = 0.3/a,\ \chi^2_{red} = 1.06$$

Preliminary results

	β	spectral fit	effective mass
am_0	5.89	$0.779(58)$	$0.799(10)$
am_1	5.89	$1.262(43)$	$1.345(14)$
am_0	6.0625	$0.595(13)$	$0.6365(43)$
am_1	6.0625	$1.074(14)$	$1.111(11)$

Effective mass results taken from [Athenodorou,Teper, 2007.06422, JHEP]

What's next

- Accuracy of final results depends on correlator's precision:

  Increase statistics to match literature standard

  Use multi-level (?)

- Extend analysis to other representations $A_1^{-+}, E^{++}, \ldots$ and different volumes.

- Perform continuum limit $a \to 0$.

- Repeat study in un-quenched setting where glueballs are allowed to decay:

  We want to study how the spectral density $\rho(\omega)$ changes as we change am_π, effectively going from a nearly quenched set-up to the physical scenario.

  Here the double limit $L \to \infty$ and $\sigma \to 0$ will be a crucial step.

Take home points

- The standard method of extracting glueball masses is particularly challenging.
- The investigation of the glueball spectrum using spectral densities gives an additional tool to determine the glueball energy states
- The limited precision of glueball correlators is a problem also in the spectral density picture.
- The preliminary results are encouraging and a full systematic study will appear (hopefully) soon!

BACKUP SLIDES

σ expansion of $\rho_\sigma(\omega)$

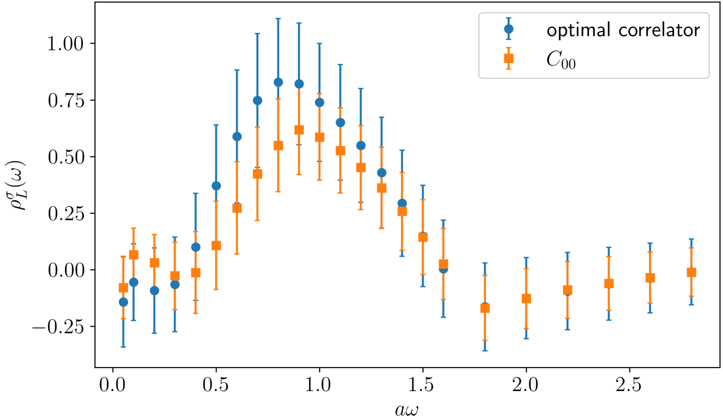
The $\sigma \to 0$ extrapolation is done following an expansion of $\rho(\omega)$ for small σ. Following [Bulava *et al.*, 2111.12774, JHEP]

$$\rho(\omega)_\sigma = \pi\rho(\omega) + \sigma^2 \frac{\pi}{2}\rho^{(2)}(\omega) \int_{-\infty}^{\infty} dx\ x^{2n+2}\Delta(x) + \mathcal{O}(\sigma^{2n+2}) \qquad \forall n \in \mathbb{N}$$

where $\Delta(x) = \frac{\exp(\frac{-x^2}{2})}{\sqrt{2\pi}}$, $x = \omega - \omega_n$ and $\rho(\omega)^{(2)}$ is the second derivative w.r.t. σ

Glueball smeared spectral functions

Studying the spectral functions allows to check contributions to the optimal correlators in the variational method



$\beta = 5.8941,\ \sigma = 0.15/a$