RUHR UNIVERSITÄT BOCHUM

Structure of doubly charm exotic state T_{cc} from lattice QCD

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Based on *PRD* 105, 014024(2022); *PRL* 131, 131903 (2023); *PRD* 109 (2024), L071506

+ in preparation

in collaboration with

M. Abolnikov, X. Dong, M. Du, E. Epelbaum, A. Filin, A. Gasparyan, F.-K. Guo, C. Hanhart, L. Meng, A. Nefediev, J. Nieves and Q. Wang

T_{cc} : an ideal case for studying exotic properties

Aaij et al [LHCb] Nature Physics (2022)
Nature Comm.(2022)

$I = 0,\ J^P = 1^+$

— first exotic doubly charm state: $cc\bar{u}\bar{d}$

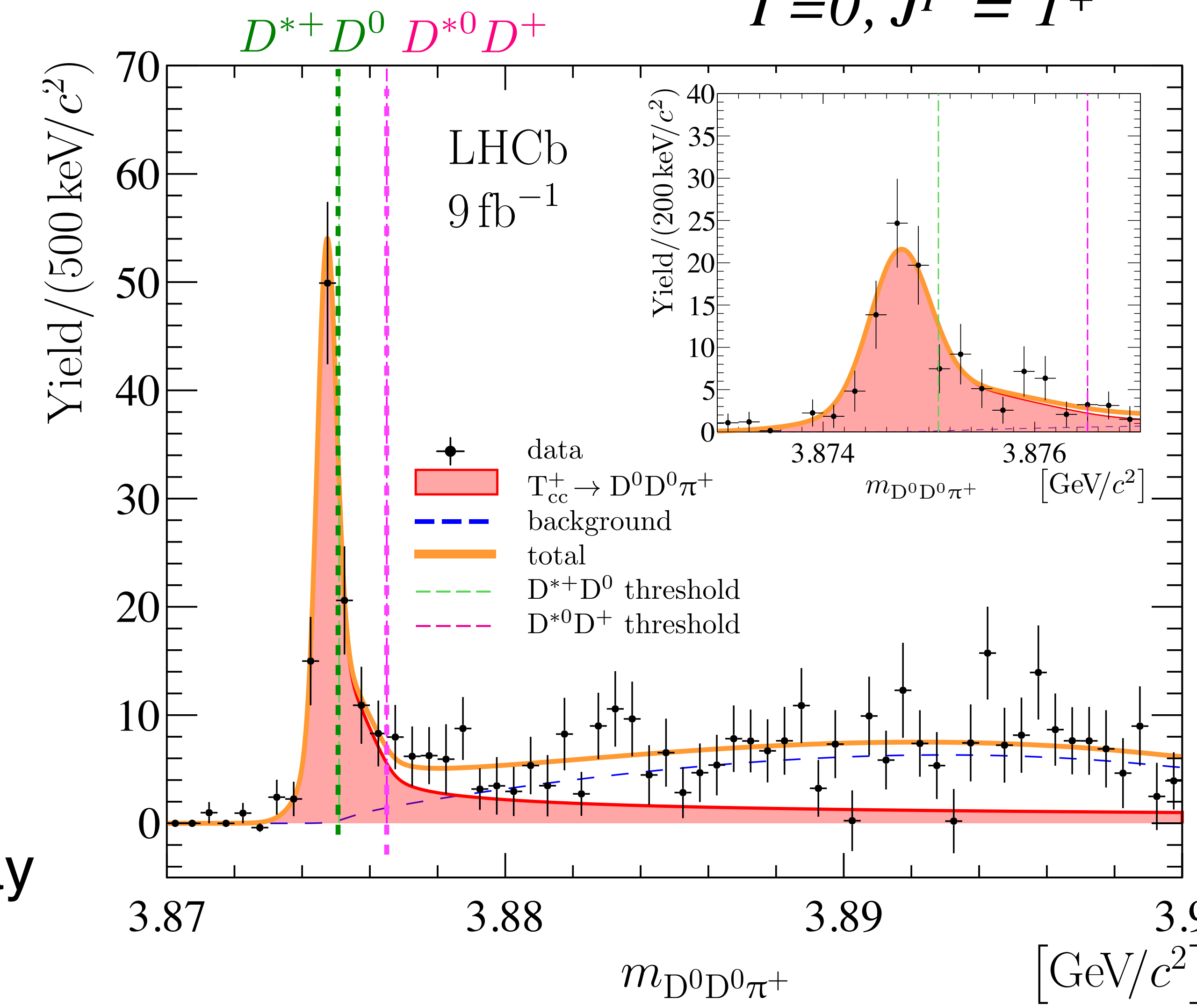
—Expansion in χEFT : $\chi = \frac{\sqrt{2\mu\Delta_M}}{\Lambda_\chi} < 0.1$

$$\Delta_M = m(D^+D^{*0}) - m(D^0D^{*+})$$

— No admixture of inelastic channels

— Width: almost entirely from the only strong decay

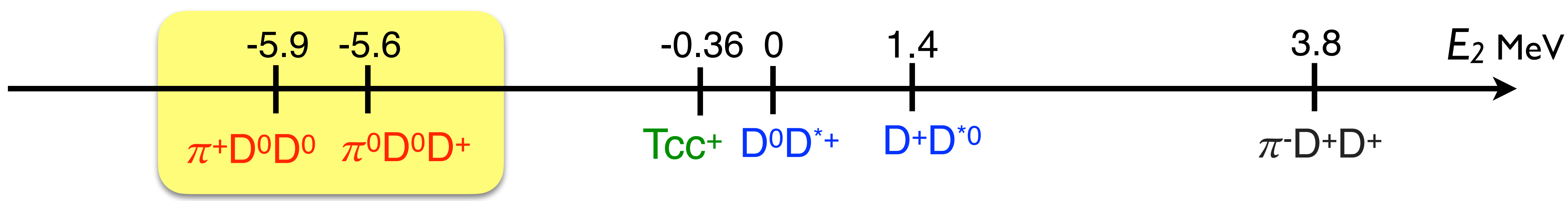
$$T_{cc}^+ \to D^0D^{*+} \to D^0D^0\pi^+/D^0D^+\pi^0$$



• unitarized Breit-Wigner fit

$$\delta m_{\rm pole} = -360 \pm 40^{+4}_{-0}\ \mathrm{keV}$$

$$\Gamma_{\rm pole} = 48 \pm 2^{+0}_{-14}\ \mathrm{keV}$$



T_{cc} on lattice

- HAL QCD Collaboration at mπ = 146 MeV: Lyu et al, *PRL* 131 161901 (2023)

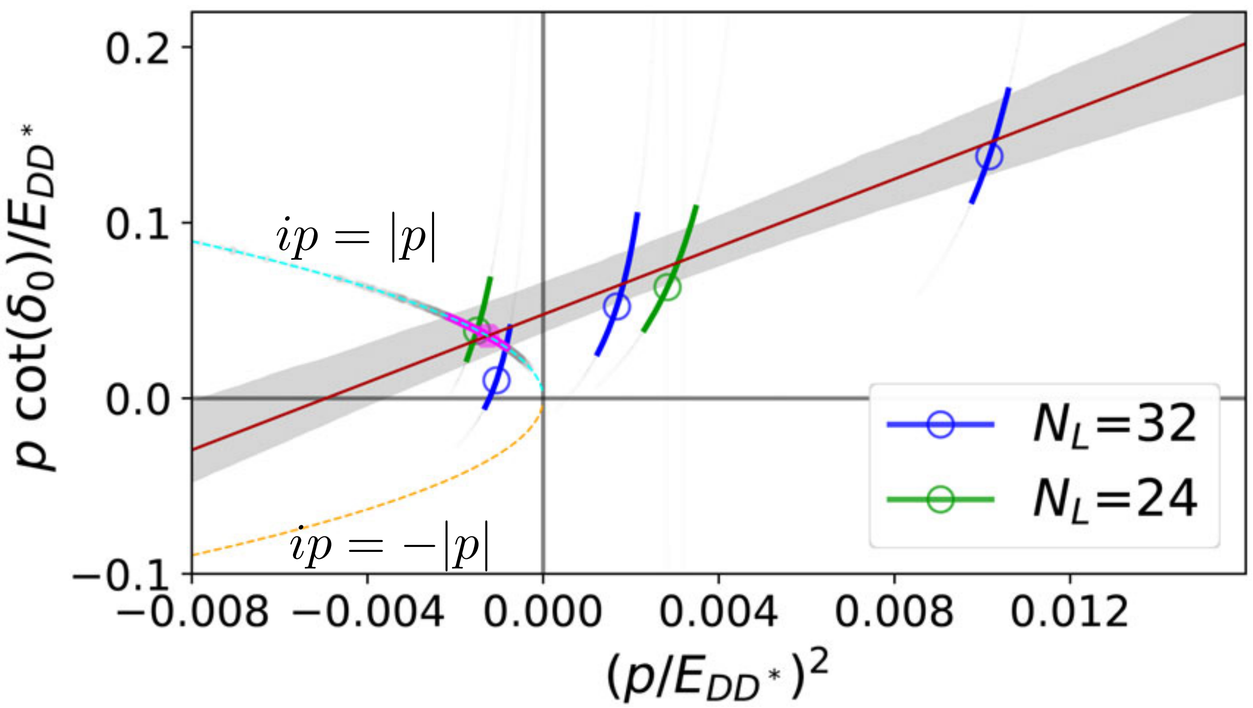
— calculate the DD* scattering potential ⇒ phase shifts above the two-body threshold

$E_{\rm pole} = -59\,^{+53\,+2}_{-99\,-67}$ keV DD* virtual state

- Lüscher method based analyses.

— DD* spectra at mπ = 280 MeV Padmanath and Prelovsek, PRL 129, 032002 (2022)

see also Collins et al *PRD* 109 (2024) 9, 094509



— DD* phase shifts parameterised using the ERE:

$$p\cot\delta = \frac{1}{a} + \frac{1}{2}rp^2 + \mathcal{O}(p^4)$$

$E_{\rm pole} = -9.9^{+3.6}_{-7.2}$ MeV DD* virtual state

— DD*-D*D* coupled channel parameterization at mπ = 391 MeV David Wilson talk on Tuesday

T. Whyte, D. Wilson, and C. Thomas 2405.15741v1 (2024)

$E_{\rm pole} = (-62 \pm 34)$ MeV DD* virtual state

$E_{\rm pole} = (-49 \pm 35 + i(11 \pm 13)/2)$ MeV D*D* resonance

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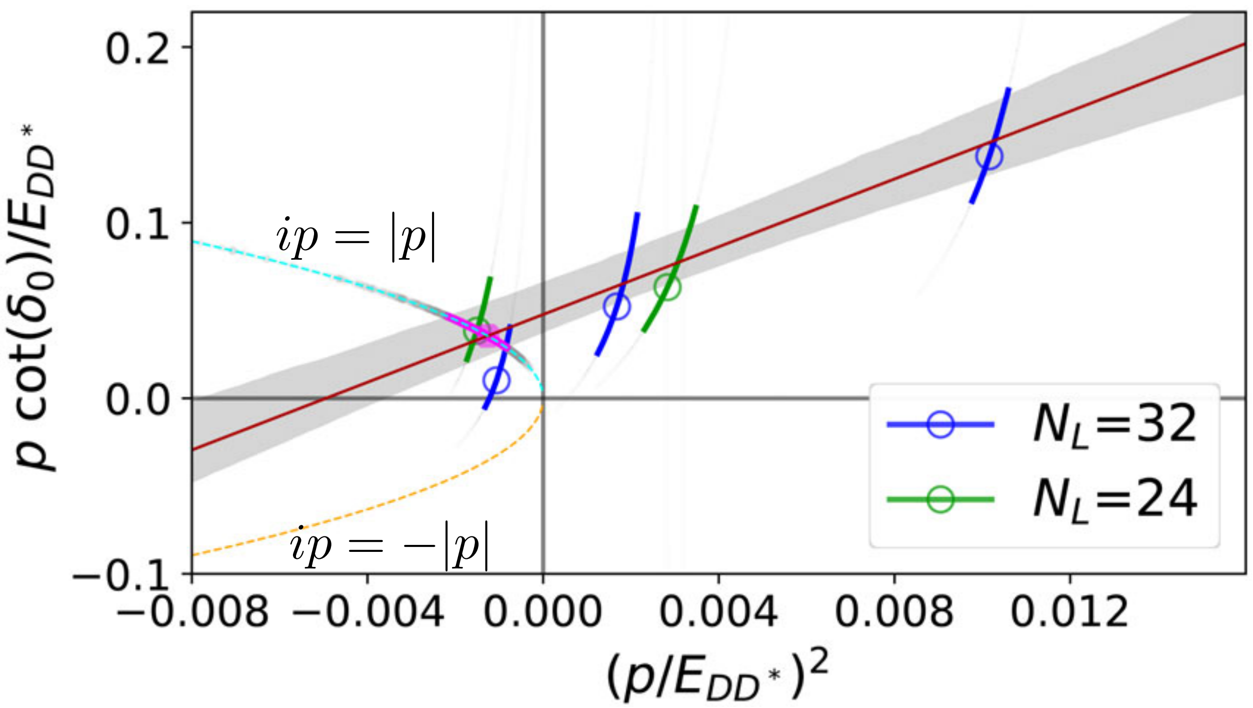

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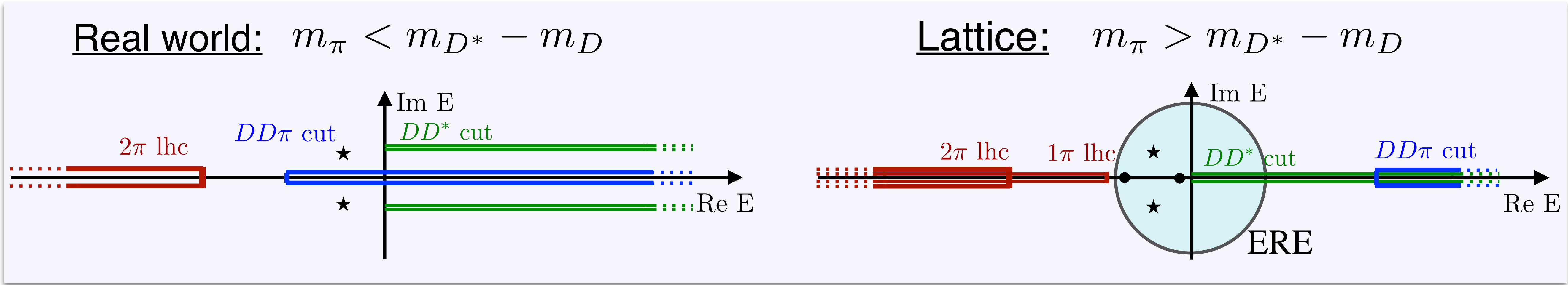
Main assumption: no nearby left-hand cuts!

Analytic structure of the DD* scattering amplitude

– Cut structure depends on the (light-quark or) pion mass



3-body DDπ cut

Left-hand cuts

⇒ Prominent role for the T_{cc} width

M. Du et al *PRD* 105, 014024(2022)

⇒ Constraints on the ERE applicability range

M. Du et al, *PRL* 131, 131903 (2023)

⇒ Invalidate Lüscher's QC at least below lhc

Raposo and Hansen (2023), L. Meng et al (2023), Green et al (2021), …

– The leading nearby cut is always associated with the one-pion exchange

⇒ Theoretical framework has to include it!

χEFT approach at low energies

Weinberg (1992)

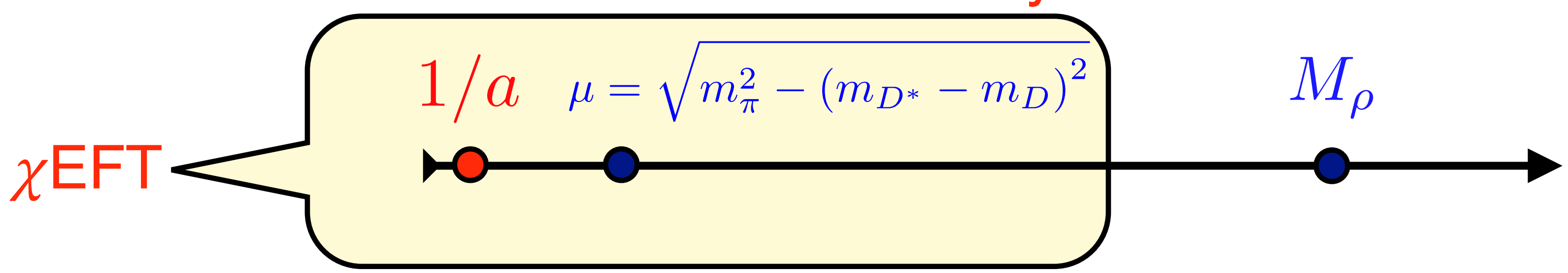
our works 2010-till now

see also AlFiky et al 2006

Fleming et al 2007

- DD* potential to a given order in $\chi = Q/\Lambda_h$

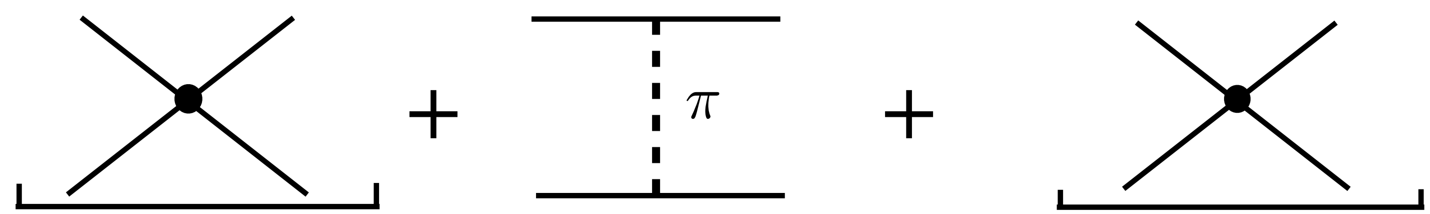
Hierarchy of scales



— Keep track of small scales ~ *1/a* due to shallow states ⇒ **resummation**

— Range effects can also come from D^0D^{*+} — D^+D^{*0} coupled channels

$$V = V_{\rm OPE}^{(0)} + V_{\rm cont}^{(0)} + V_{\rm cont}^{(2)} + \ldots =$$



$+\ldots$

$C^{(0)}$ Long range: OPE $C^{(2)}(p^2 + p'^2) + D^{(2)}(\xi^2 - 1)$

$$\xi = \frac{m_\pi}{m_\pi^{\rm ph}}$$

— Incorporates the relevant cuts

— Extention to incorporate DD*-D*D* coupled channels is straightforward

- Amplitudes are solutions of the integral equations

$$T_{\alpha\beta} = V_{\alpha\beta}^{\rm eff} - \sum_\gamma \int \frac{d^3q}{(2\pi)^3} V_{\alpha\gamma}^{\rm eff} G_\gamma T_{\gamma\beta}$$

→ Consistent with Unitarity and analyticity

G - Green functions

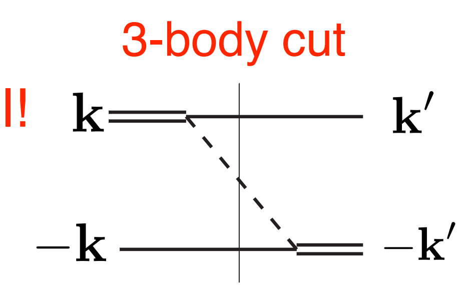
3-body DDπ cut

- OPE potential:

$$V_{\mathrm{DD}^*\to\mathrm{DD}^*}(\mathbf{k},\mathbf{k}',E) \propto \frac{g_c^2}{(4\pi f_\pi)^2}\,\tau_1\cdot\tau_2\,\frac{(\epsilon_1\cdot\vec{q})\,(\epsilon_2'^{\,*}\cdot\vec{q})}{2E_\pi(\mathbf{k}-\mathbf{k}')}\left(\frac{1}{D_{DD\pi}(\mathbf{k},\mathbf{k}',E)}+\frac{1}{D_{D^*D^*\pi}(\mathbf{k},\mathbf{k}',E)}\right)$$

$$D_{DD\pi}(k,k',E) = E_D(k)+E_D(k')+E_\pi(\mathbf{k}-\mathbf{k}')-E \quad \Rightarrow$$

goes on shell!



$$D_{DD\pi}(k,k',E) \to i\pi\delta(E_D(k)+E_D(k')+E_\pi(\mathbf{k}-\mathbf{k}')-E) \to \mathrm{Im\ part}$$

3-body cut condition

- For each $E \geq E_{\mathrm{thr}} \equiv 2m+m_\pi$, there are real values of k and k′ such that $D_{DD\pi}(k,k',E)=0$

  3-body branch point is (k=k′=0): $E \equiv 2m+m_\pi$

3-body DDπ cut

- OPE potential:

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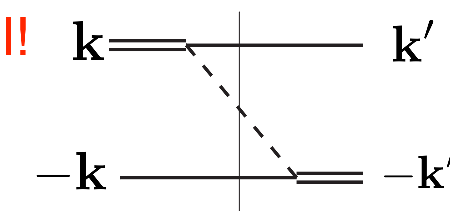

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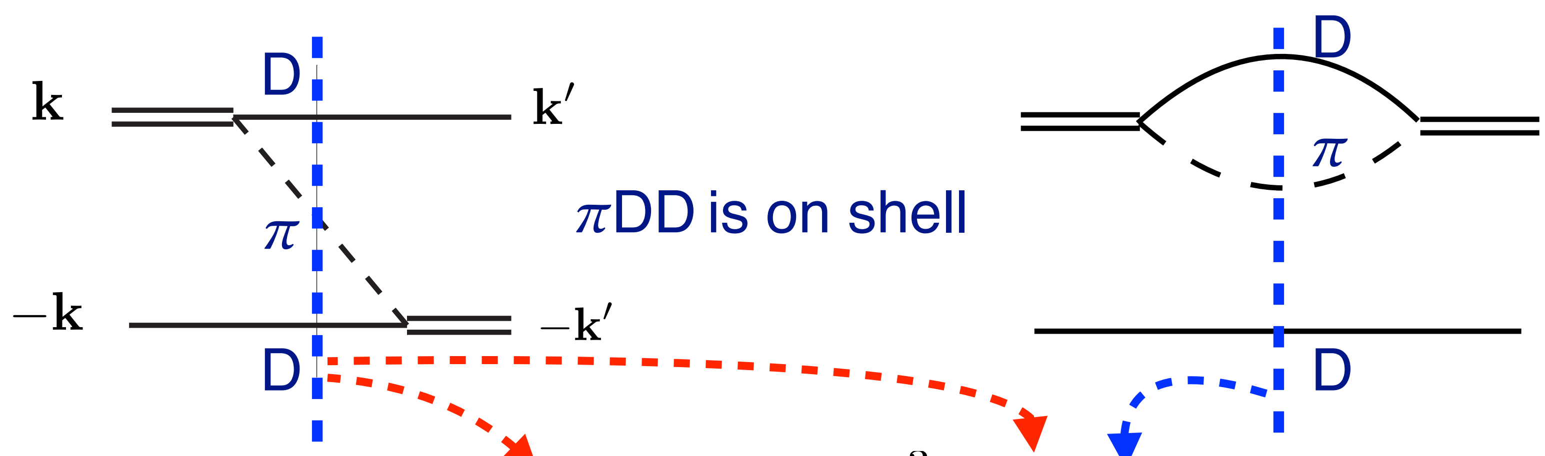
3-body branch point is (k=k$'$=0): $E \equiv 2m + m_\pi$

- 3-body cut stems from one-pion exchange (OPE) and self energies in the Green funct.



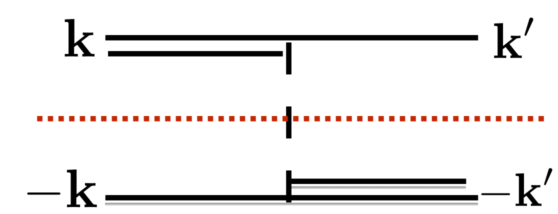
$$T_{\alpha\beta} = V_{\alpha\beta}^{\mathrm{eff}} - \sum_\gamma \int \frac{d^3q}{(2\pi)^3} V_{\alpha\gamma}^{\mathrm{eff}} G_\gamma T_{\gamma\beta}$$

Full analogy to the X(3872)

VB et al. PRD84 (2011)

Left-hand cut

— Leading singularity is from the on shell one-pion exchange



PWD of the static potential:

$$V_{l=0}(k,k') \propto \int dz \frac{1}{(\mathbf{k}-\mathbf{k}')^2+\mu^2} = \frac{1}{2kk'}\log\frac{(k+k')^2+\mu^2}{(k-k')^2+\mu^2} \xrightarrow[k=k'=p]{\text{on shell}} \frac{1}{2p^2}\log\frac{4p^2+\mu^2}{\mu^2}$$

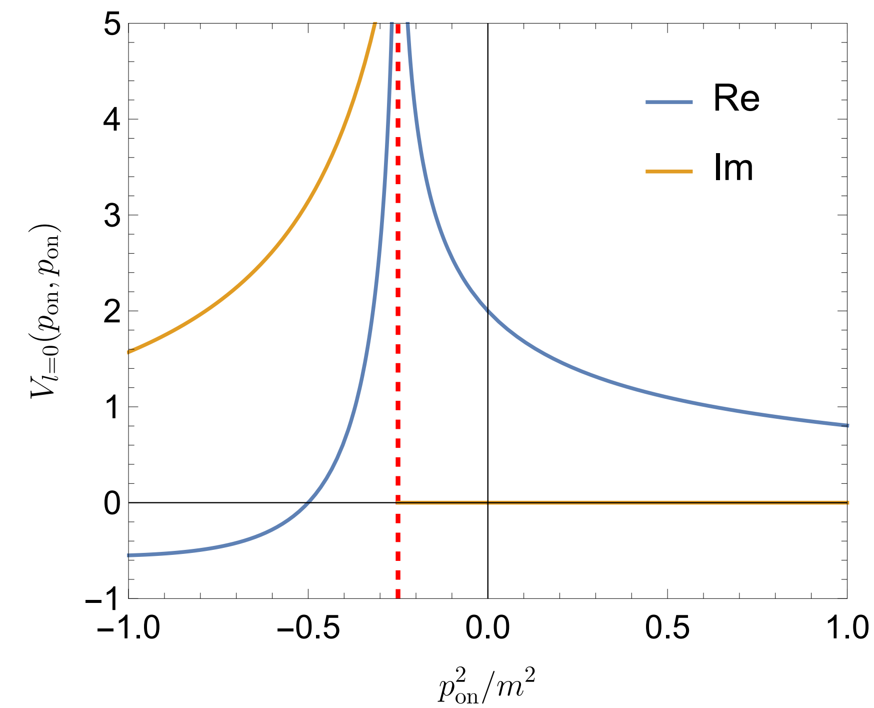
$\Rightarrow$ left-hand cut (lhc) branch point is at

$$(p_{\rm lhc}^{1\pi})^2 = -\frac{\mu^2}{4}$$

For DD*

$\mu^2 = m_\pi^2 - \Delta M^2$

$\Delta M = m_{D^*} - m_D$



Sc. amplitude is complex for E below the lhc
$\Rightarrow$ Lüscher's method breaks down

Meng et al, *PRD* 109 (2024), Raposo and Hansen 2311.18793 (2023), Green et al, *PRL* 127 (2021)

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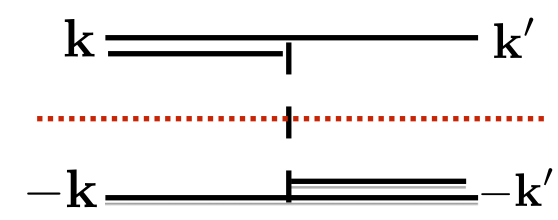

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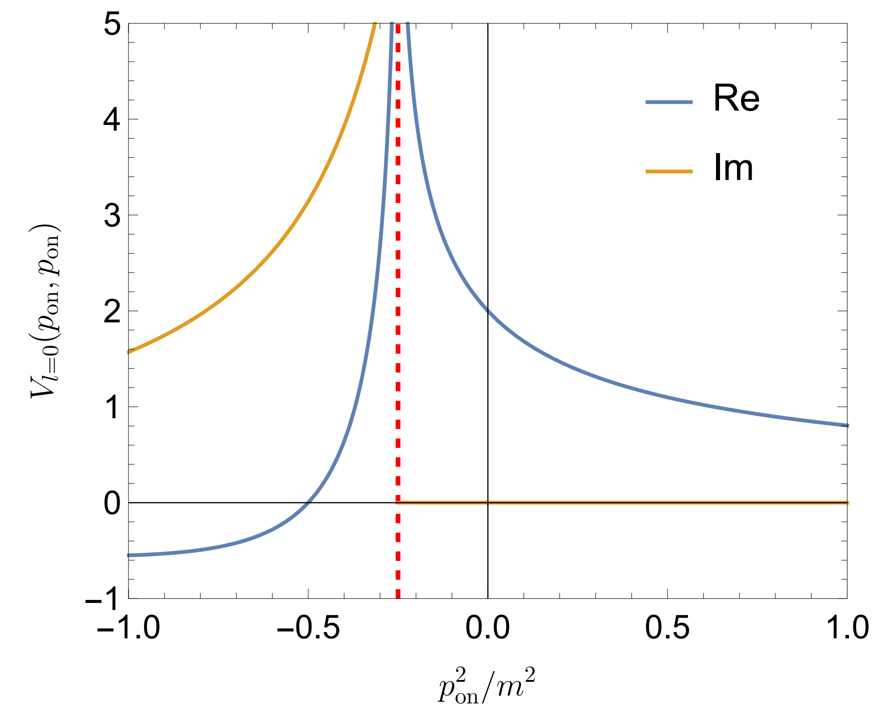
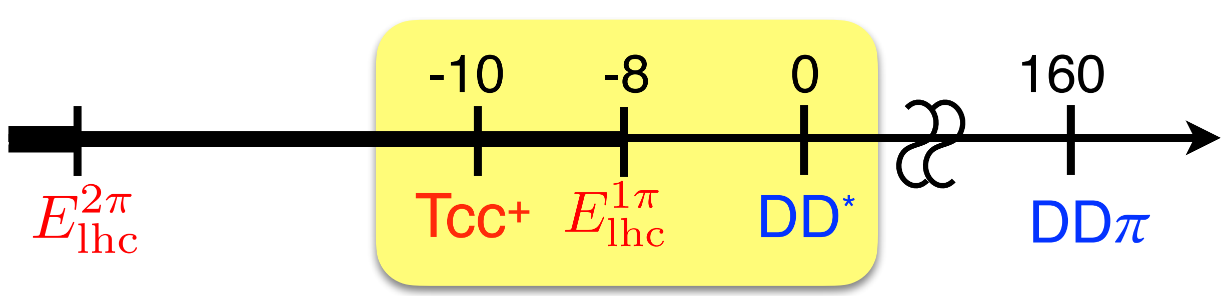


Sc. amplitude is complex for E below the lhc
⇒ Lüscher's method breaks down

Meng et al, *PRD* 109 (2024), Raposo and Hansen 2311.18793 (2023), Green et al, *PRL* 127 (2021)

At mπ = 280 MeV

$$E_{\rm lhc}^{1\pi} = \frac{(p_{\rm lhc}^{1\pi})^2}{2\mu_{DD^*}} = -8\,\text{MeV} \Rightarrow E_{\rm lhc}^{1\pi}$$ sets the range of convergence of the ERE: $E \ll |E_{\rm lhc}^{1\pi}|$



⇒ ERE is not applicable

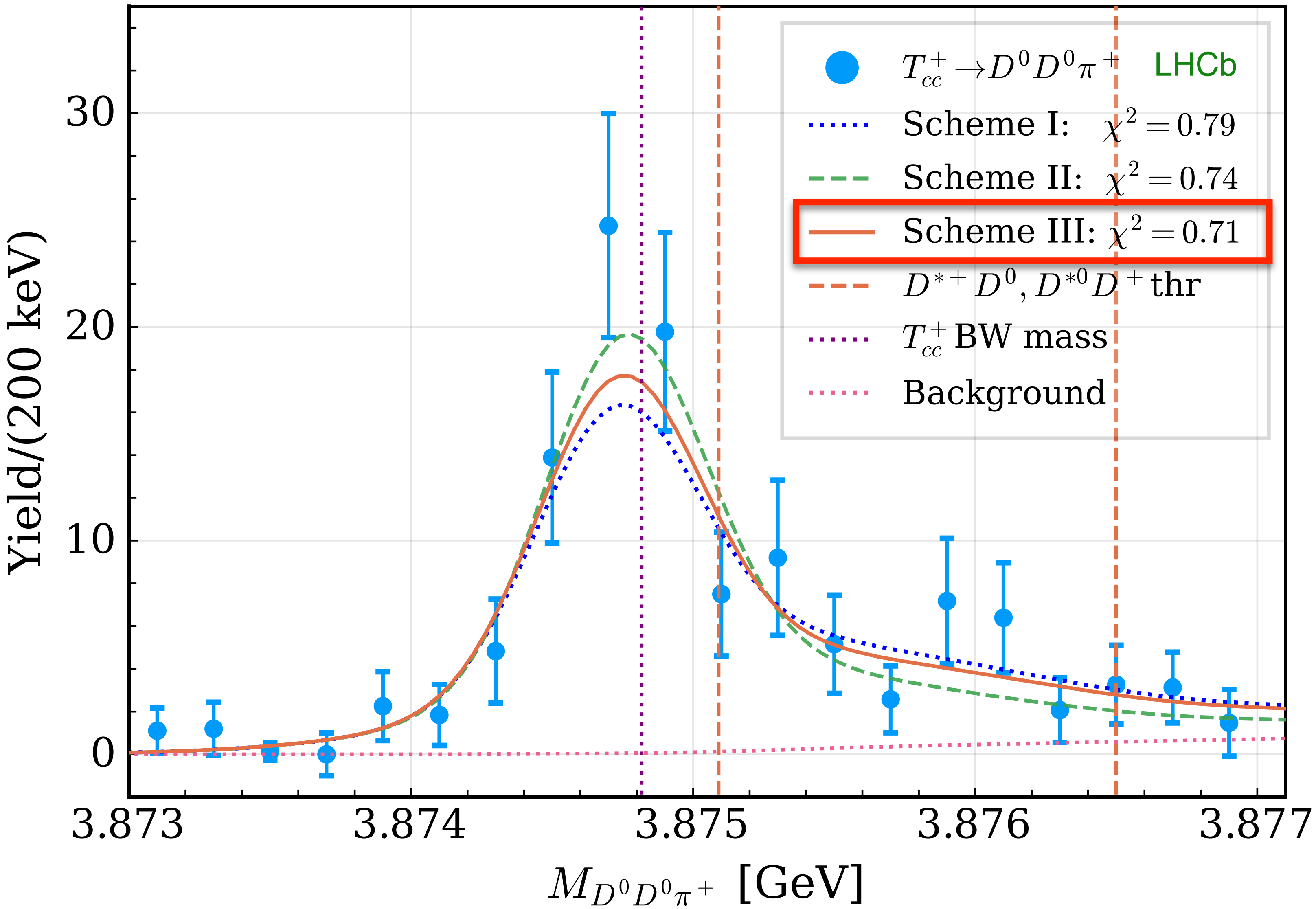
M. Du et al, *PRL* 131, 131903 (2023)

Applications

App I: LO χEFT analysis of $D^0D^0\pi^+$ data by LHCb

M. Du, VB, X. Dong, A. Filin, F.-K. Guo, C. Hanhart, A. Nefediev, J. Nieves and Q. Wang *PRD* 105, 014024(2022)

with resolution



The pole

III
full 3-body unitarity:
OPE + dynamical D^* width
$-356^{+39}_{-38} - i(28 \pm 1)$
0.71

— Coupled $D^0D^{*+} - D^+D^{*0}$ scattering

—1 parameter + overal normalization

Re part of the T_{cc} pole: Inconclusive about the role of 3-body effects with current exp. precision

Can be reanalysed if more precise data emerge

Im part of the T_{cc} pole: Controlled by 3-body effects

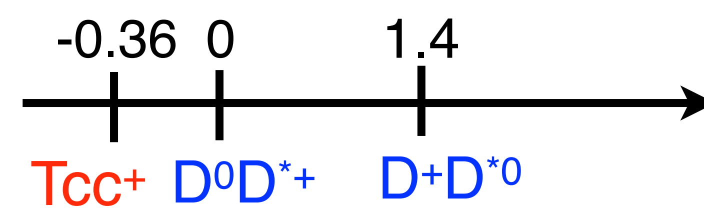
$$\Gamma_{T_{cc}}^{3-\text{body}} = 56 \pm 2\,\text{keV}$$

Low-energy parameters

Du et al. PRD 105, 014024 (2022)

Scattering amplitude in the 1st (close to the pole) channel :

$$T_{D^{*+}D^0 \to D^{*+}D^0}(k) = -\frac{2\pi}{\mu_{c0}} \left(\frac{1}{a_0} + \frac{1}{2} r_0 k^2 - ik + \mathcal{O}(k^4) \right)^{-1}$$



$$r_0' = r_0 - \Delta r$$

Eff. range in the 1st channel

$$\Delta r = -\sqrt{\frac{\mu_2}{2\mu_1^2 \delta_2}} \simeq -3.8 \text{ fm}$$

Negative "correction" from 2nd $D^{*0}D^+$ channel caused by isospin breaking δ_2

$$\delta_2 = m_{\text{thr2}} - m_{\text{thr1}}$$

VB et al., PLB 833 (2022)

a_0 [fm]	r_0 [fm]	r_0' [fm]	X_A
$\left(-6.72^{+0.36}_{-0.45} \pm 0.27\right) - i\left(0.10^{+0.03}_{-0.03} \pm 0.03\right)$	$-2.40 \pm 0.01 \pm 0.85$	$1.38 \pm 0.01 \pm 0.85$	$0.84 \pm 0.01 \pm 0.06$

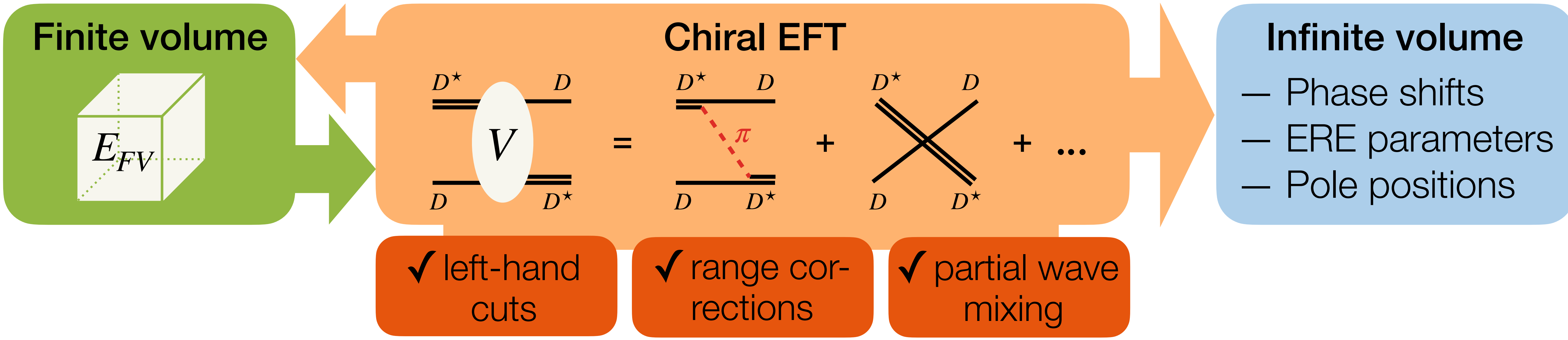
$r_0' \ll |a_0|$

- r_0' positive and is of natural size
- Contrib. to r_0' from OPE is ~ 0.4 fm

T_{cc} is consistent with a pure isoscalar molecule!

App II: Chiral EFT as an alternative to Lüscher

Meng, VB, Filin, Epelbaum and Gasparyan *PRD letter* 109, L071506 (2024)



— Construct regularized effective potential truncated to a given order

$$V = V^{(0)}_{\rm OPE} + V^{(0)}_{\rm cont} + V^{(2)}_{\rm cont} + \dots$$

$$V^{(0)+(2)}_{\rm cont}[{}^3S_1] = \left(C^{(0)}_{{}^3S_1} + C^{(2)}_{{}^3S_1}(p^2 + p'^2)\right)(\vec{\epsilon}\cdot\vec{\epsilon}'^*)$$

$$V^{(2)}_{\rm cont}[{}^3P_0] = C^{(2)}_{{}^3P_0}(\vec{p}'\cdot\vec{\epsilon}'^*)(\vec{p}\cdot\vec{\epsilon})$$

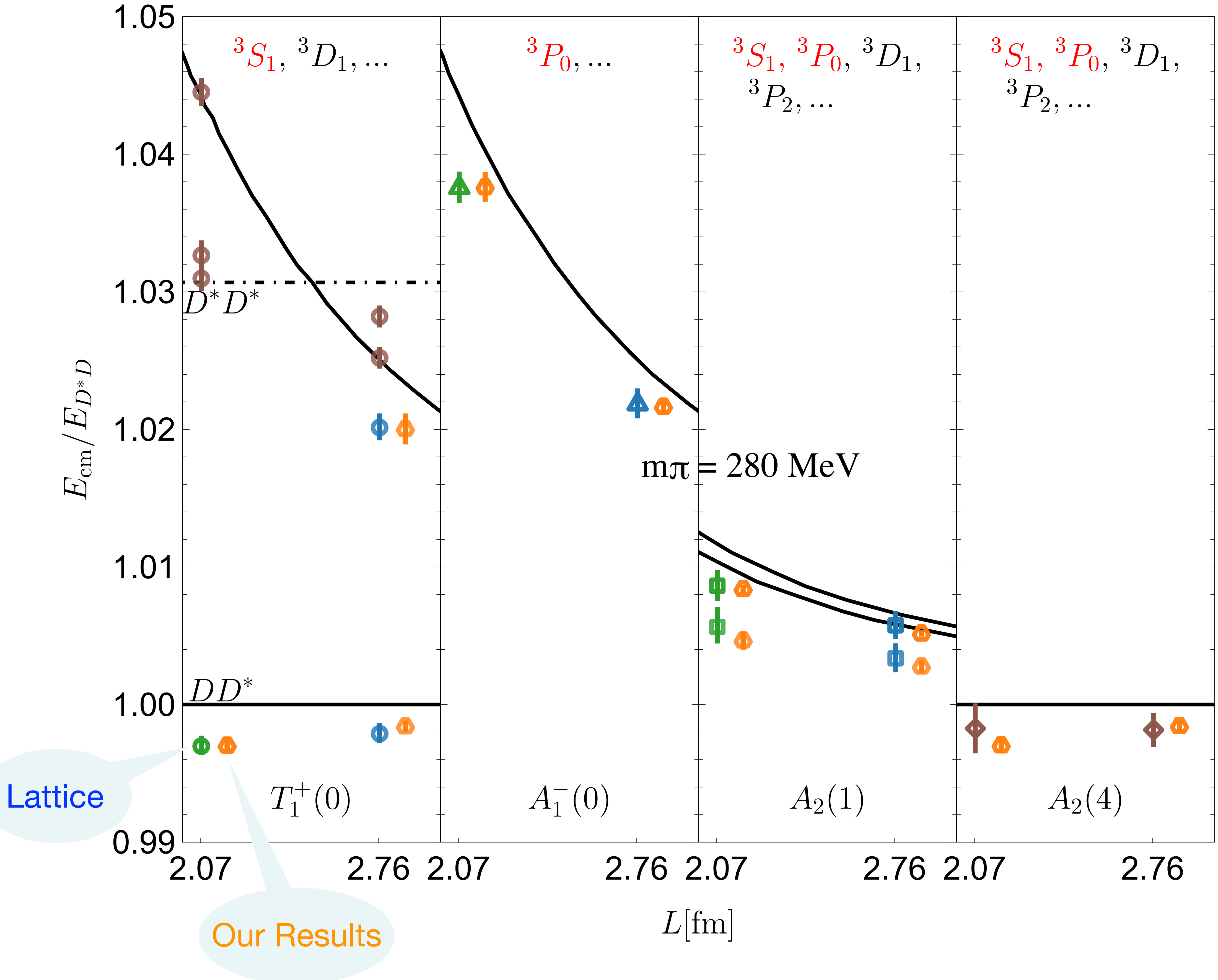
— Calculate E_{FV} in each irrep as a solution of the eigenvalue problem

$$\det\left[\mathbb{G}^{-1}(E) - \mathbb{V}(E)\right] = 0 \qquad \mathbb{G}_{\mathbf{n},\mathbf{n}'} = \mathcal{J}\frac{\delta_{\mathbf{n}',\mathbf{n}}}{L^3}\frac{1}{4E_D(\tilde{p}_{\mathbf{n}})E_{D^*}(\tilde{p}_{\mathbf{n}})}\frac{1}{E - E_D(\tilde{p}_{\mathbf{n}}) - E_{D^*}(\tilde{p}_{\mathbf{n}})}$$

— Adjust LEC's C's from best fits to E_{FV}: C's are independent of the volume size L

— Employ the EFT potential to calculate infinite volume amplitudes using LSE

DD* Finite Volume Energy Levels at $m_\pi = 280$ MeV



Lattice

Padmanath and Prelovsek, PRL 129, 032002 (2022)

Our Results

3 Param's:

2 in $^3S_1$

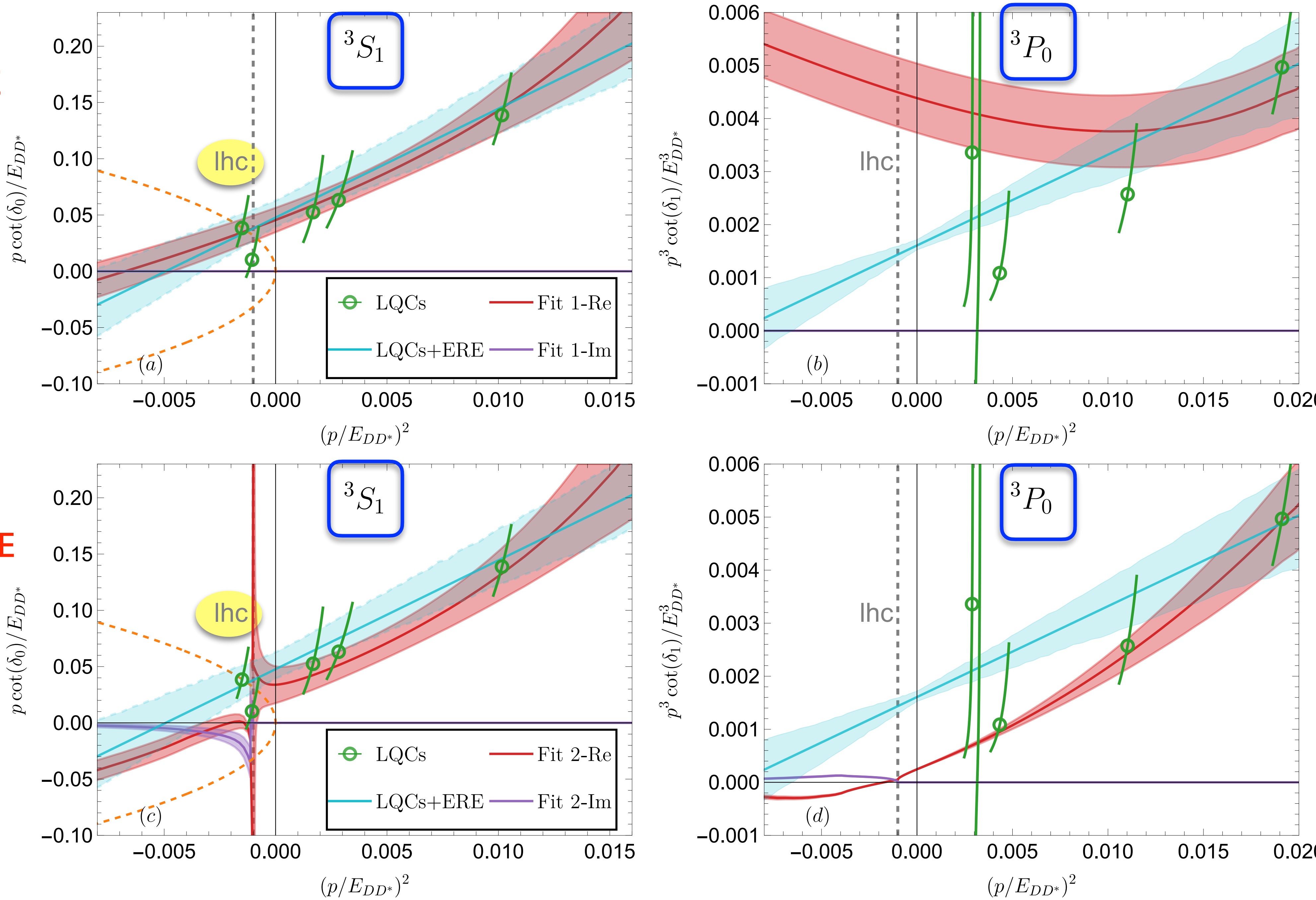
1 in $^3P_0$

Predict IFV phase shifts and Tcc pole

L. Meng et al *PRD* 109, L071506 (2024)

No OPE

With OPE



- $^3P_0$ shape controlled by OPE
- $^3S_1$ near lhc controlled by OPE

	$a_{^3S_1}$ [fm]	$r_{^3S_1}$ [fm]	$\delta m_{T_{cc}}$ [MeV]	$a_{^3P_0}$ [fm^3]	$r_{^3P_0}$ [fm^{-1}]	χ^2/dof	# of param's
LQCs+ERE fit [23]	1.04 ± 0.29	$0.96^{+0.18}_{-0.20}$	$-9.9^{+3.6}_{-7.2}$	$0.076^{+0.008}_{-0.009}$	6.9 ± 2.1	3.7/5	4
Fit 1: cont.	1.09 ± 0.35	0.75 ± 0.14	-10.6 ± 4.4	0.028 ± 0.004	-4.3 ± 0.05	5.52/6	3
Fit 2: cont.+OPE	1.46 ± 0.57	0.096 ± 0.53	$-6.6(\pm 1.5) - i4.0(\pm 3.7)$	0.497 ± 0.007	5.63 ± 0.19	2.95/6	3

[23] Padmanath and Prelovsek, PRL 129, 032002 (2022)

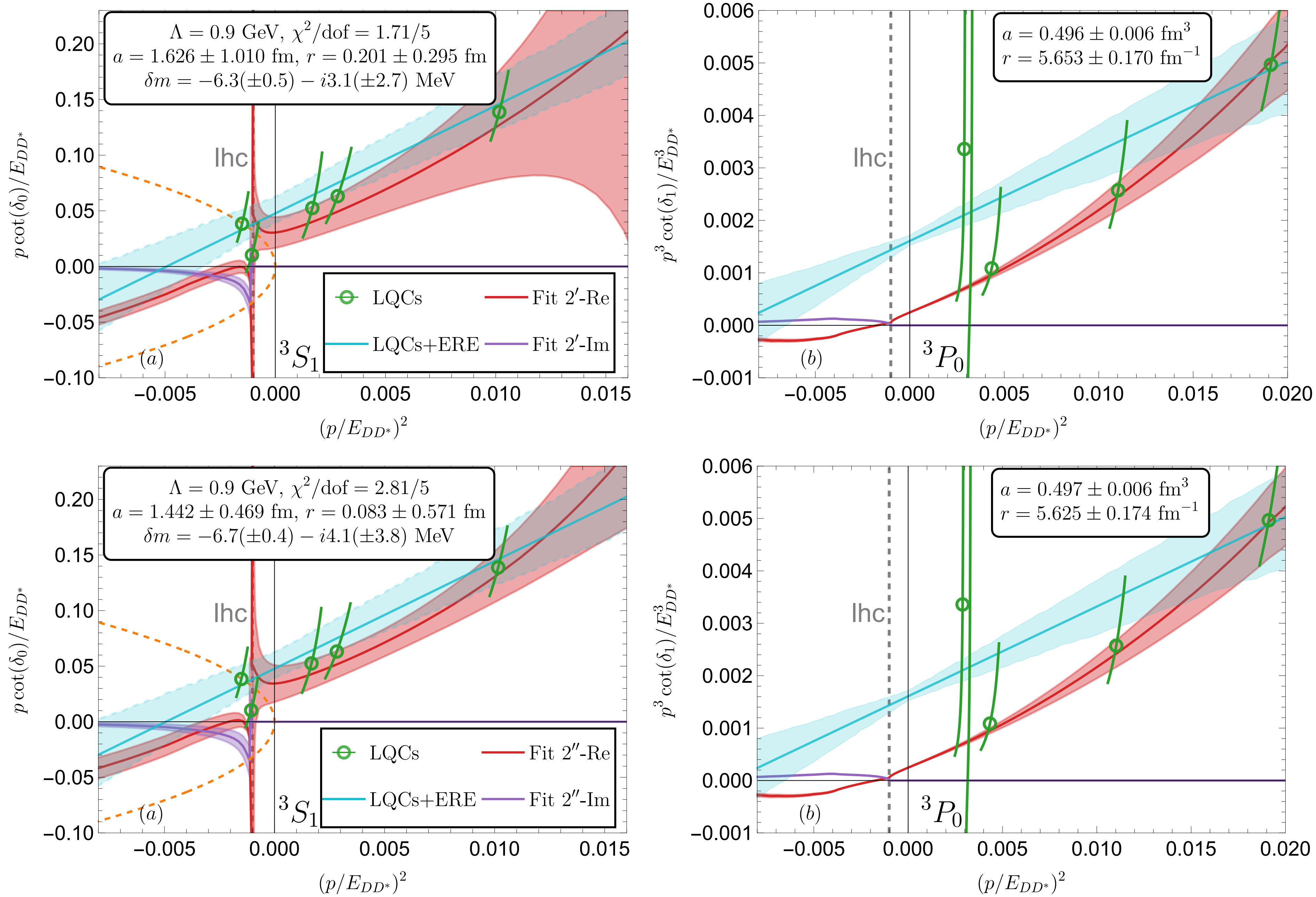
T_{cc} is a resonance state with 85% probability

Testing chiral truncation uncertainty at mπ = 280 MeV

- Additional contact terms

$$V^{(2)}_{\rm cont}[^3S_1 - {}^3D_1] = C^{(2)}_{SD}\, p'^2$$

$$V^{(2)}_{\rm cont}[^3P_2] = C^{(2)}_{^3P_2}$$



⇒ Some effect of the S-D term on phase shifts at larger momenta

⇒ The impact near the threshold and on the pole is minor

App III: pion-mass dependence of the Tcc pole

M. Abolnikov, VB, E. Epelbaum, A. Filin, C. Hanhart and L. Meng *in preparation*

Formulation: Employ knowledge of the NLO potential at $m_\pi = m_\pi^{\rm ph}$ and $m_\pi = 280$ MeV

$$V = V^{(0)}_{\rm OPE} + V^{(0)}_{\rm cont} + V^{(2)}_{\rm cont} + \ldots$$

$$V^{(0)+(2)}_{\rm cont}[{}^3S_1] = \underbrace{C^{(0)}_{{}^3S_1}}_{m_\pi = m_\pi^{\rm ph}} + \underbrace{C^{(2)}_{{}^3S_1}(p^2 + p'^2) + D^{(2)}_{{}^3S_1}}_{m_\pi = 280\ {\rm MeV}}(\xi^2 - 1) \qquad \xi = \frac{m_\pi}{m_\pi^{\rm ph}}$$

fixed at $m_\pi = m_\pi^{\rm ph}$ $m_\pi = 280$ MeV

- Calculate scattering amplitude for any m_π

Disclaimer:

- Setup in isospin limit: averaged π and D-meson masses, no EM effects
- Focus on near DD* threshold energy range: no DD*-D*D* coupled channels yet
- Lattice spacing dependence is weak for $ud\bar{c}\bar{c}$ P. Junnarkar, N. Mathur, and M. Padmanath, PRD 99, 034507 (2019)

  → Calculation in the continuum limit

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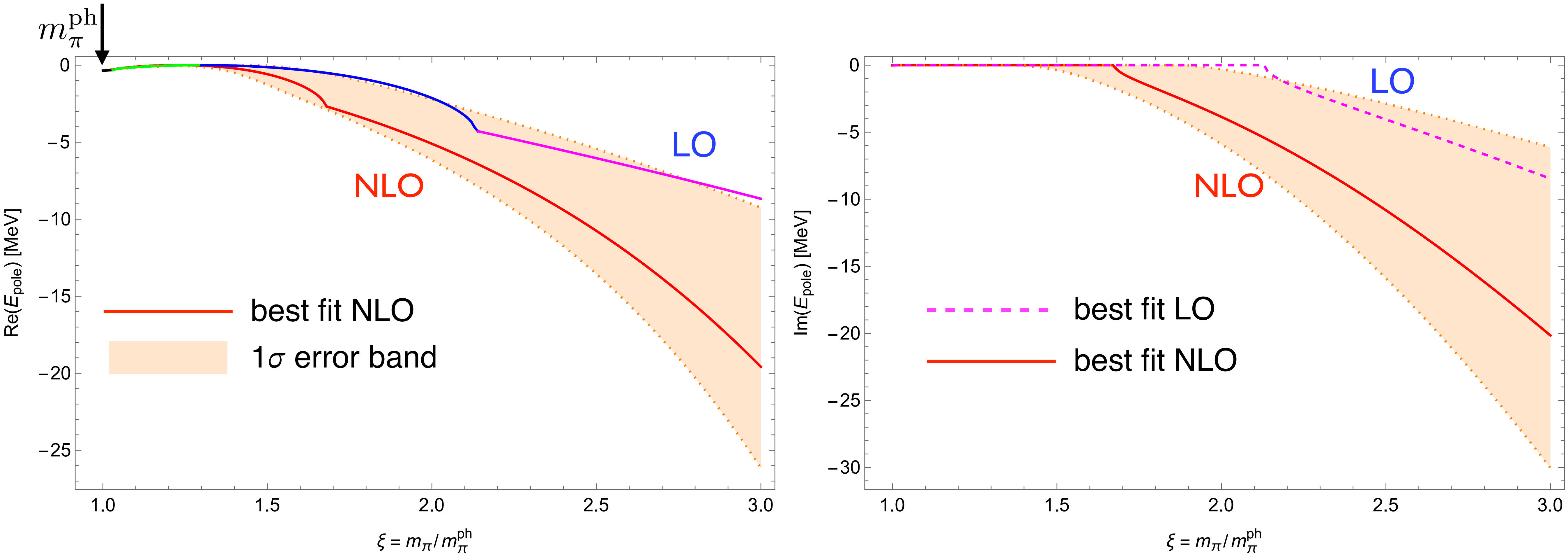

- Tcc pole transitions: quasi-bound → bound → virtual → resonance

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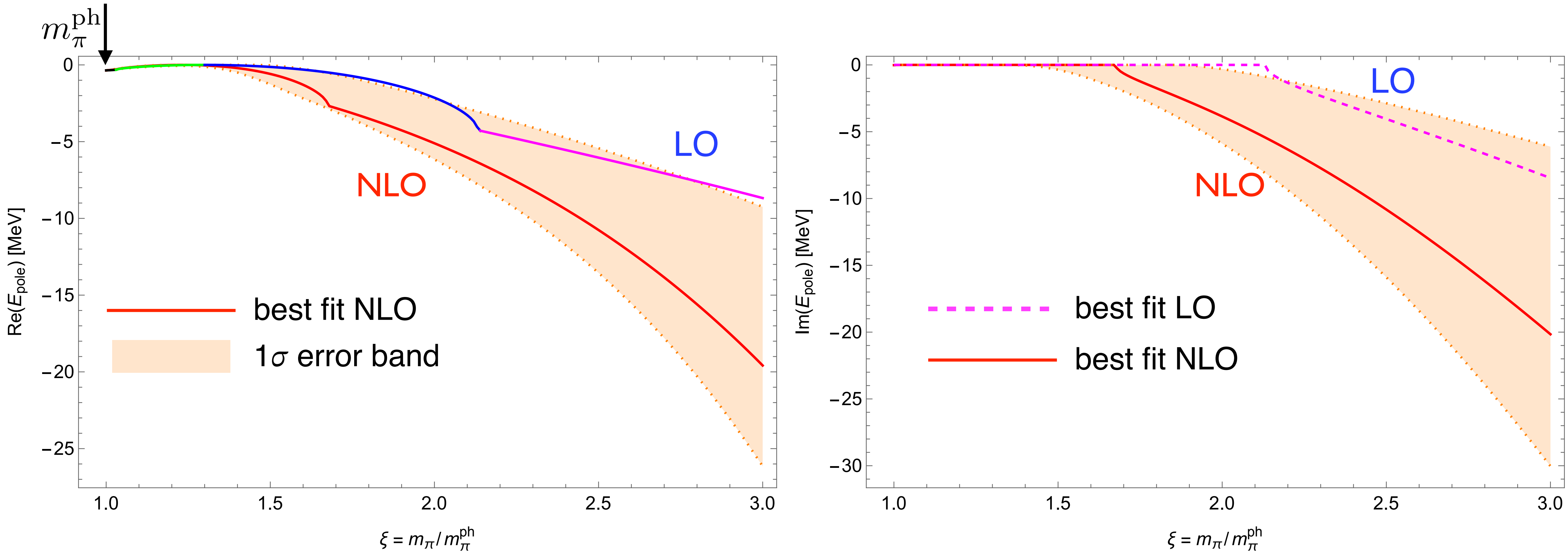
$$\xi = \frac{m_\pi}{m_\pi^{\rm ph}}$$

- Calculate scattering amplitude for any m_π



- Tcc pole transitions: quasi-bound → bound → virtual → resonance

- NLO is qualitatively consistent to LO; resonance is formed at smaller m_π

⇒ Trajectory consistent with hadronic molecule

Matuschek, VB, Guo, Hanhart, EPJA 57, 101 (2021)

Truncation uncertainty of chiral expansion

Add higher-order interactions + naturalness

$$V = {\color{red}V^{(0)}_{\rm OPE}} + V^{(0)}_{\rm cont} + V^{(2)}_{\rm cont} + V^{(4)}_{\rm cont}$$

$$V^{(0)+(2)}_{\rm cont}[{}^3S_1] = C^{(0)}_{{}^3S_1} + C^{(2)}_{{}^3S_1}(p^2 + p'^2) + D^{(2)}_{{}^3S_1}(\xi^2 - 1)$$

<u>*Fits*</u>

$$C_2 = \frac{\alpha_2}{F_\pi^2}\frac{1}{\Lambda_\chi^2}; \quad \alpha_2 = 0.2$$

$$D_2 = \frac{\tilde\alpha_2}{F_\pi^2}\left(\frac{m_\pi^{\rm ph}}{\Lambda_\chi}\right)^2; \quad \tilde\alpha_2 = 0.4$$

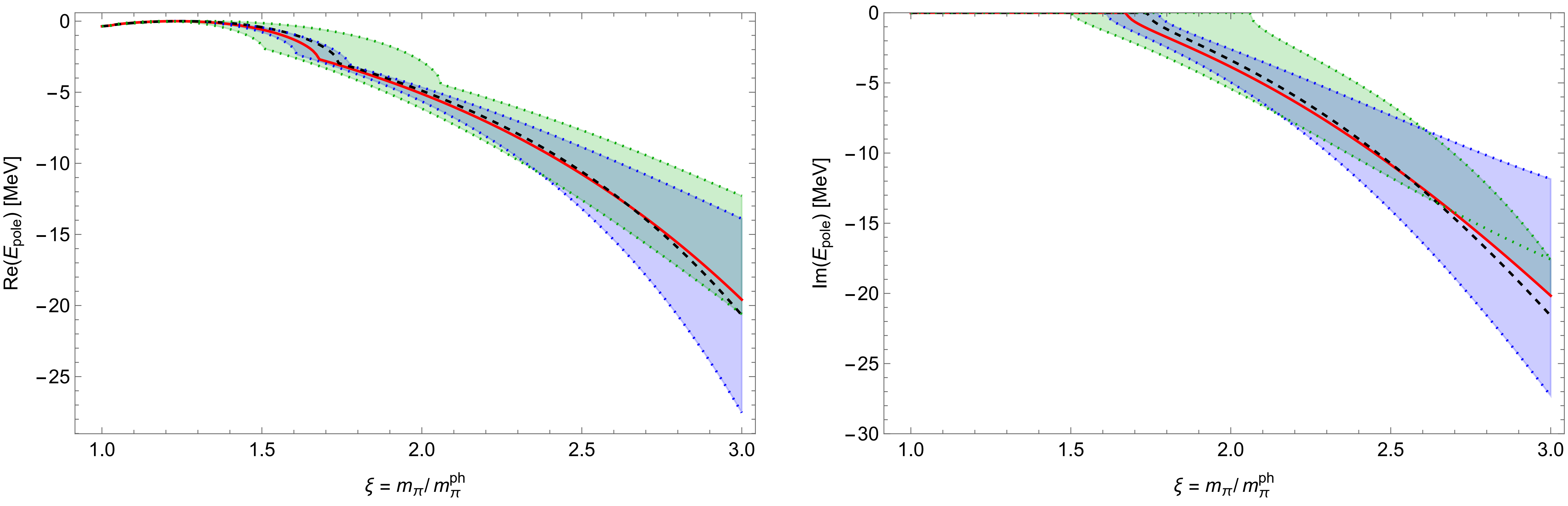
<u>*Naturalness*</u>

$${\color{green}D_4 = \frac{\alpha_4}{F_\pi^2}\left(\frac{m_\pi^{\rm ph}}{\Lambda_\chi^2}\right)^2; \quad \alpha_4 \in [-1, 1]}$$

$${\color{blue}\tilde D_4 = \frac{\tilde\alpha_4}{F_\pi^2}\left(\frac{m_\pi^{\rm ph}}{\Lambda_\chi}\right)^4; \quad \tilde\alpha_4 \in [-1, 1]}$$

$$V^{(4)}_{\rm cont} = {\color{green}D_4(\xi^2 - 1)(p^2 + p'^2)} + {\color{blue}\tilde D_4(\xi^4 - 1)}$$

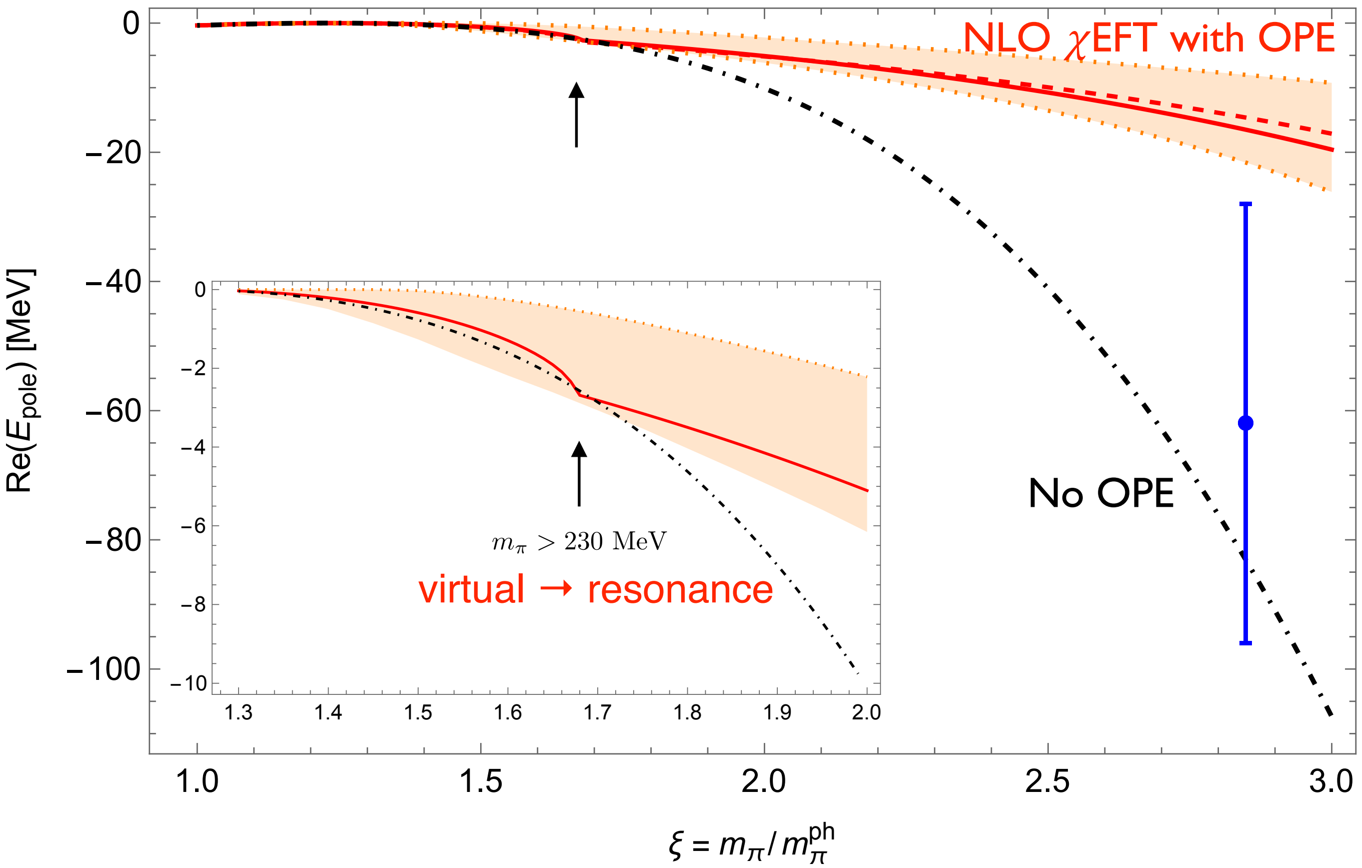
⇒ Comparable with statistical uncertainty

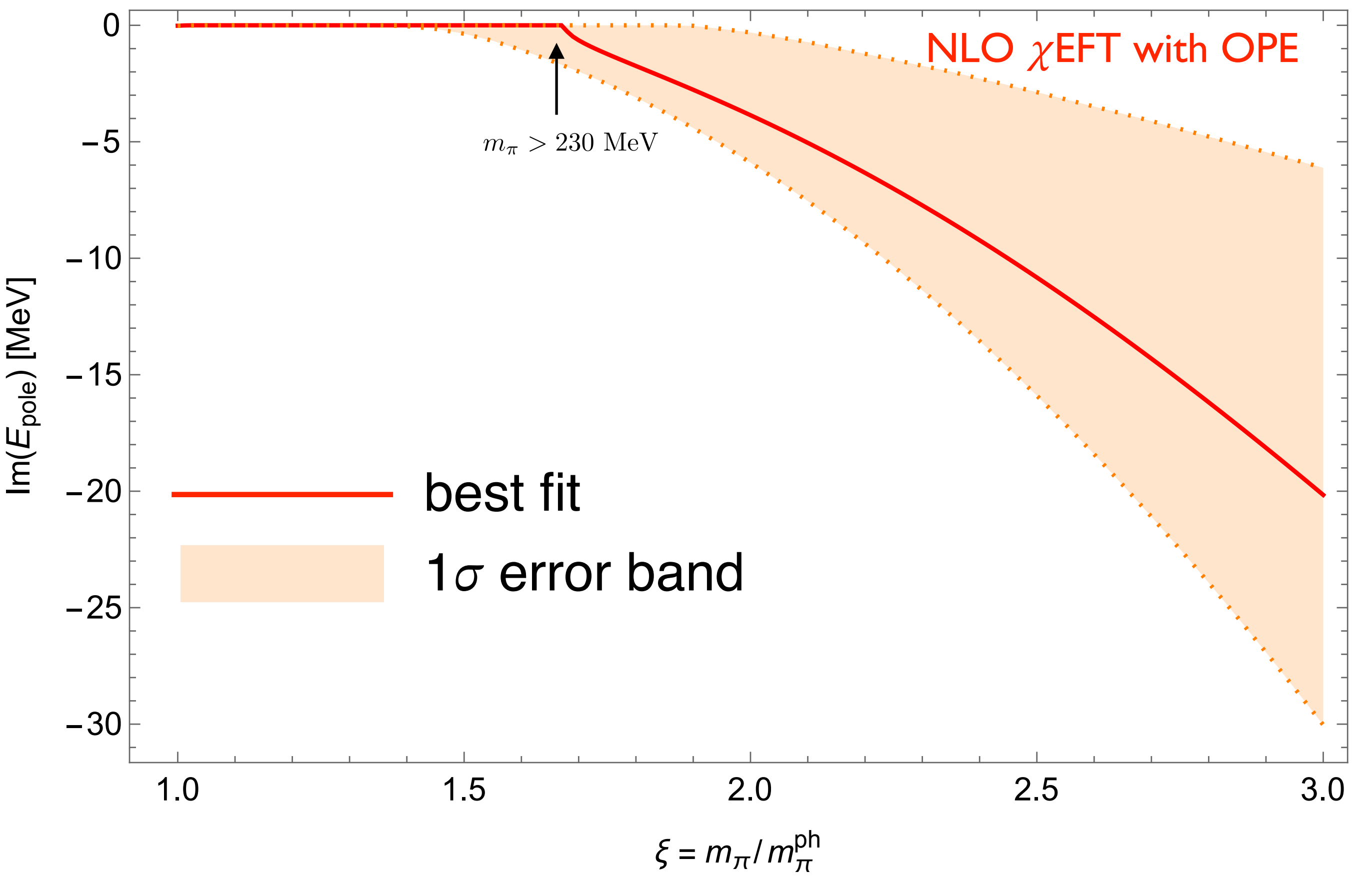


T_{cc} pole vs m_π: Contact vs Pionful

M. Abolnikov, VB, E. Epelbaum, A. Filin, C. Hanhart and L. Meng *in preparation*

$$V = V^{(0)}_{\rm OPE} + V^{(0)}_{\rm cont} + V^{(2)}_{\rm cont} \qquad V^{(0)+(2)}_{\rm cont}[{}^3S_1] = C^{(0)}_{{}^3S_1} + C^{(2)}_{{}^3S_1}(p^2 + p'^2) + D^{(2)}_{{}^3S_1}(\xi^2 - 1)$$



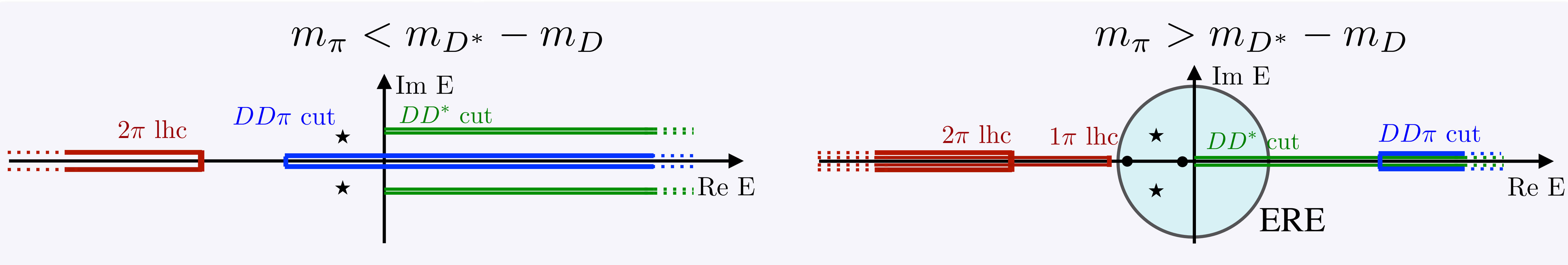


- Our pionless trajectory is consistent with new lattice data: virtual state at $m_\pi = 391$ MeV
  T. Whyte, D. Wilson, and C. Thomas arXiv:2405.15741v1

- But due to repulsion from the OPE, NLO χEFT yields a resonance for $m_\pi > 230$ MeV

Summary

- Systematic analysis of experimental and lattice data in χEFT



- χEFT as an alternative to Lüscher in the presence of left-hand cuts

  → DD* E_{FV} mπ = 280 MeV: Lüscher QC fails around lhc, ok above DD* thr, uncertainty unclear

  → plenty of possible applications

  → for other methods see Raposo, Hansen, arXiv:2311.18793; Bubna et al. JHEP 05 (2024) 168; Hansen et al. JHEP 06 (2024) 051

- Tcc properties and pole transitions: consistent with a molecule

  quasi-bound → bound → virtual → resonance

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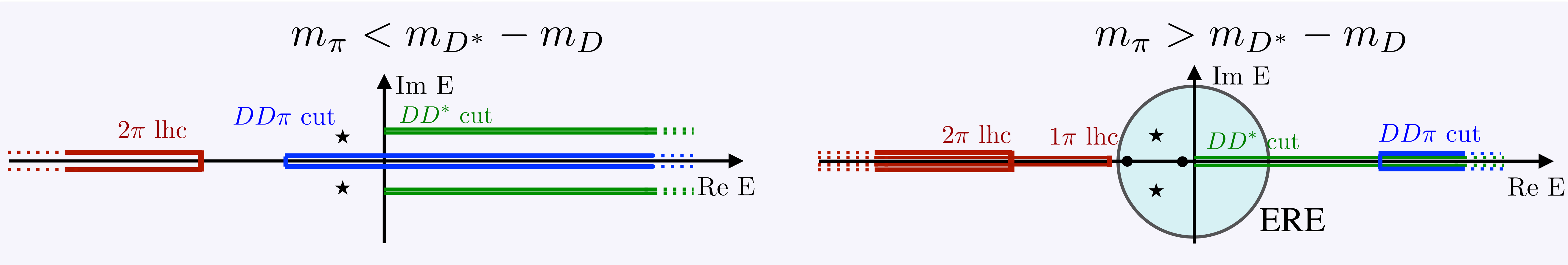

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- Tcc properties and pole transitions: consistent with a molecule

  quasi-bound → bound → virtual → resonance

Thanks for your attention!

Backup

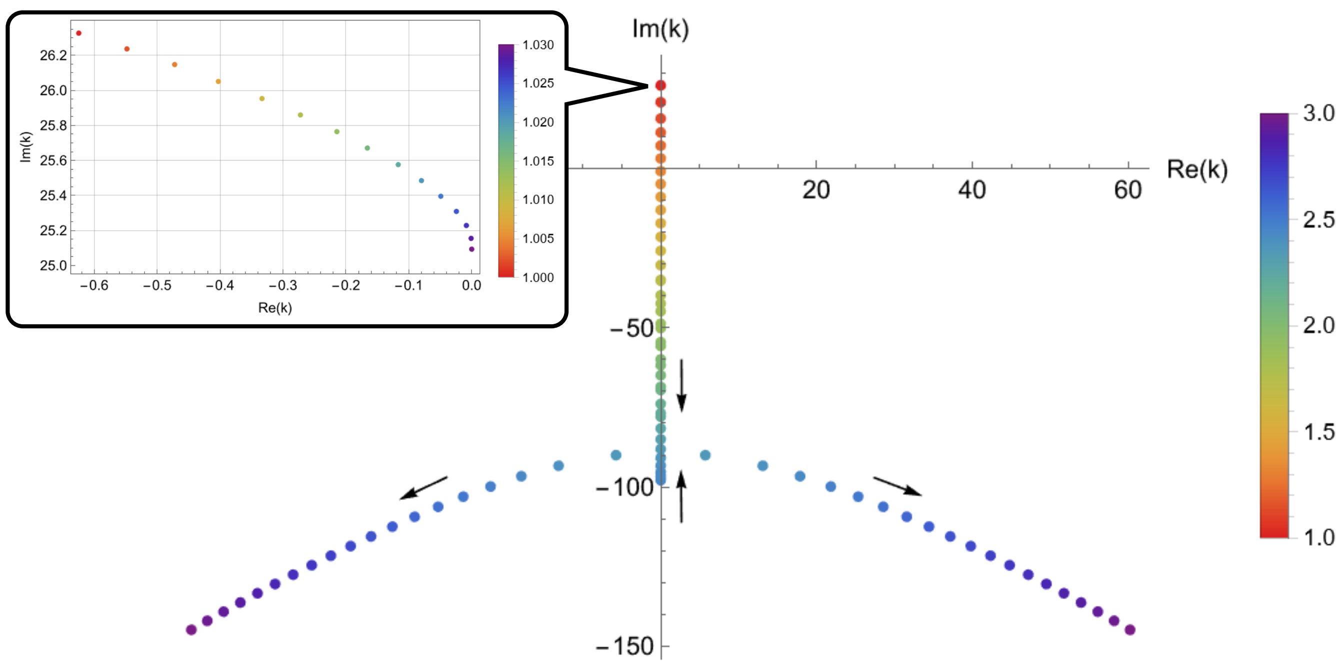
Pion-mass dependence of the Tcc pole at LO

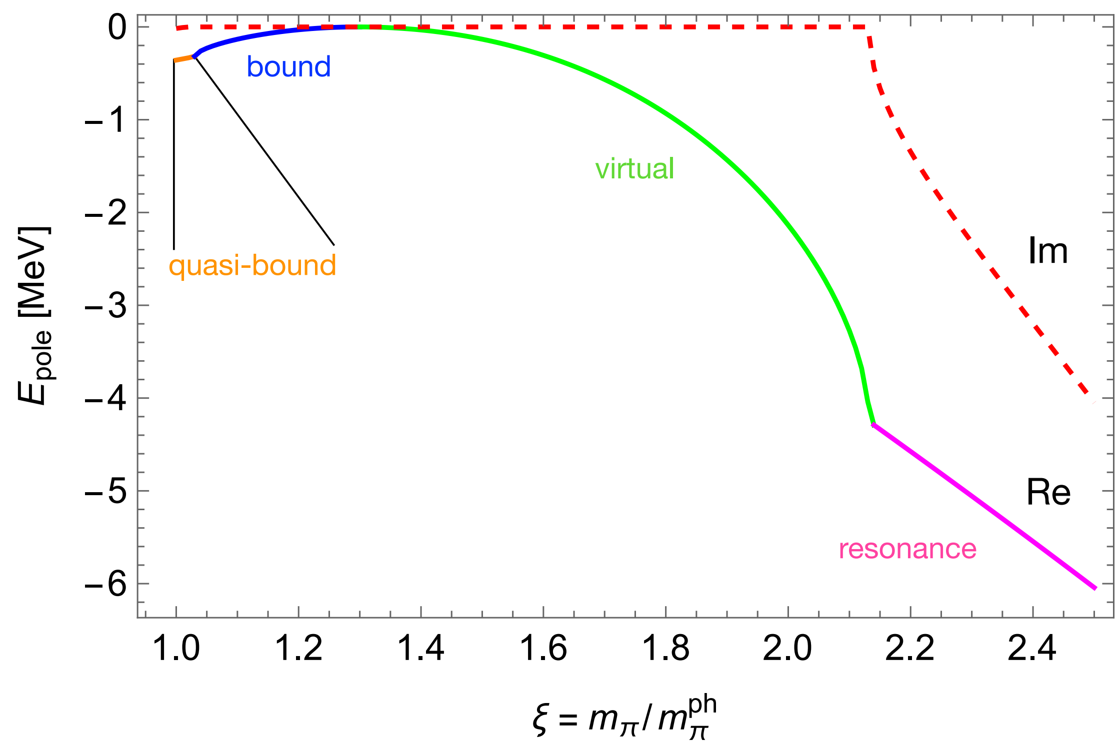
M. Abolnikov, VB, E. Epelbaum, A. Filin, C. Hanhart and L. Meng *in preparation*

Fixed from the Tcc pole at physical m_π^{ph}

LO results

$$V = V_{\text{OPE}}^{(0)} + C_{{}^3S_1}^{(0)}$$





Tcc pole transitions: quasi-bound → bound → virtual → resonance

Consistent with hadronic molecule

I. Matuschek, VB, F.-K. Guo, and C. Hanhart, Eur. Phys. J. A 57, 101 (2021)

The pion coupling from fits to data

Meng, VB, Filin, Epelbaum and Gasparyan *PRD letter* 109, L071506 (2024)

Two parameter fits to lattice + physical value:

- Linear extrapolation:

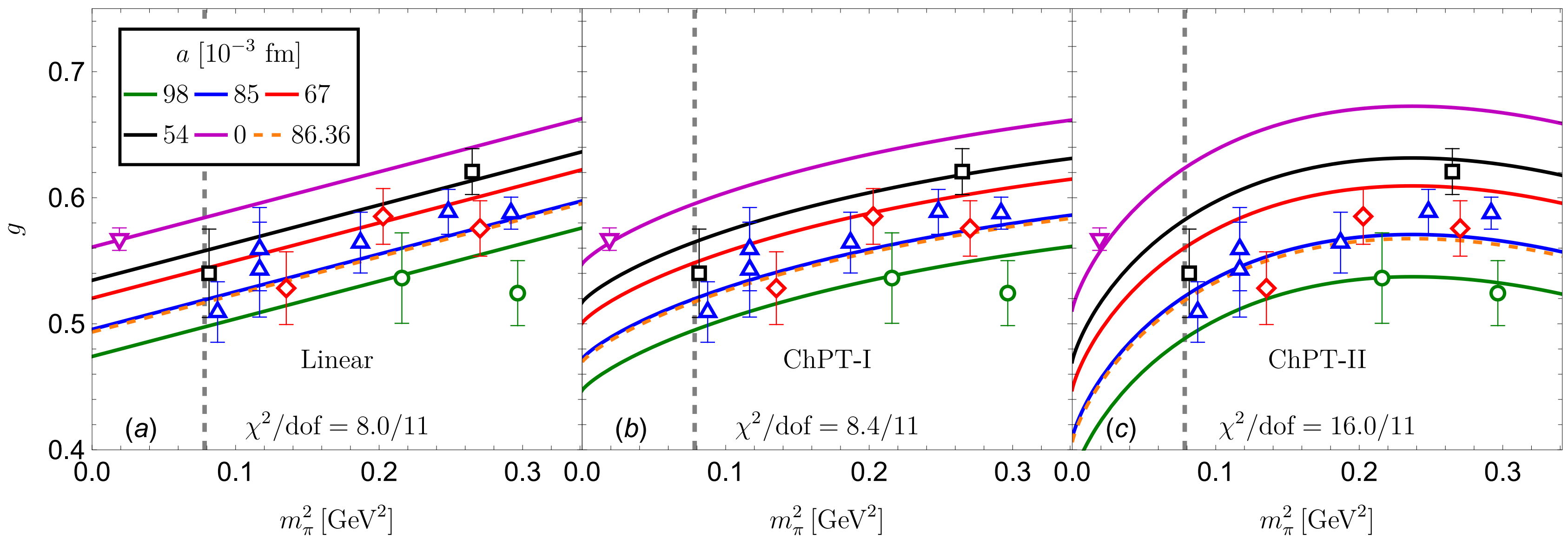
$$g(a, m_\pi) = g_0(1 + \alpha m_\pi^2 + \beta a^2)\,,$$

- ChPT-I:

$$g(a, m_\pi) = g_0\left(1 - \frac{2g_0^2}{(4\pi f_0)^2} m_\pi^2 \ln m_\pi^2 + \alpha m_\pi^2 + \beta a^2\right)$$

- ChPT-II:

$$g(a, m_\pi) = g_0\left(1 - \frac{1 + 2g_0^2}{(4\pi f_0)^2} m_\pi^2 \ln m_\pi^2 + \alpha m_\pi^2 + \beta a^2\right)$$



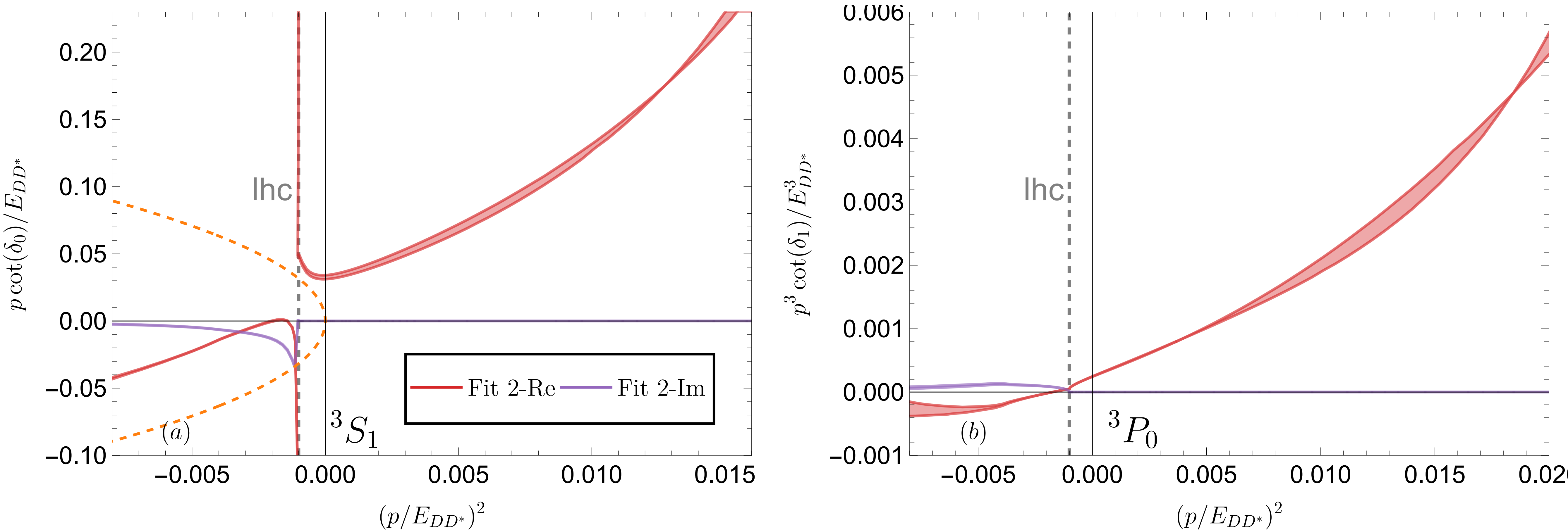
Lattice data: Becirevic and Sanfilippo Phys. Lett. B 721, 94 (2013)

	g_0	α [GeV^{-2}]	β [fm^{-2}]	g
Linear	0.561(9)	0.53(13)	−16.1(44)	0.517(15)
ChPT-I	0.547(8)	0.24(14)	−19.1(45)	0.517(15)
ChPT-II	0.511(8)	−0.59(15)	−27.6(48)	0.519(15)

$g \equiv g(0.08636, 0.280)$

Residual cutoff dependence

- Cutoff variation from 0.7 to 1.2 GeV

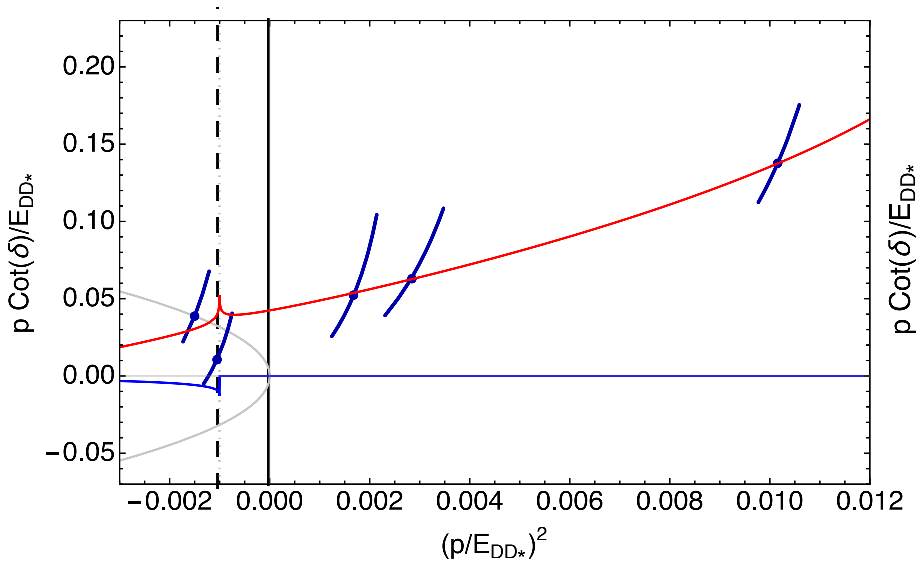


$\Rightarrow$ very small

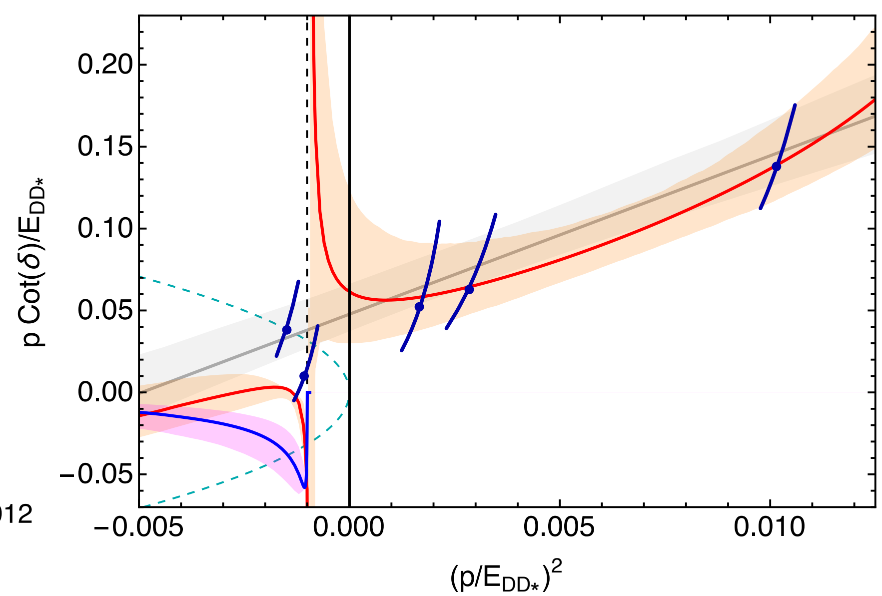
Dependence on the pion coupling

- Importance of lhc is controlled by its position and strength (residue)

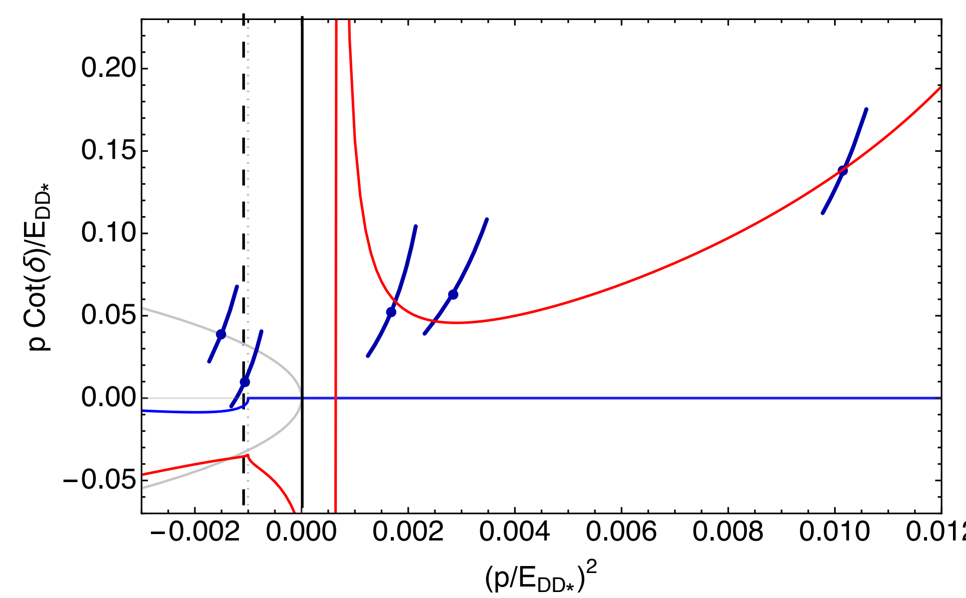
$\frac{1}{10} V^{\rm OPE}_{\rm DD^* \to DD^*}(m_\pi = 280 {\rm MeV})$



$V^{\rm OPE}_{\rm DD^* \to DD^*}(m_\pi = 280 {\rm MeV})$



$10\ V^{\rm OPE}_{\rm DD^* \to DD^*}(m_\pi = 280 {\rm MeV})$

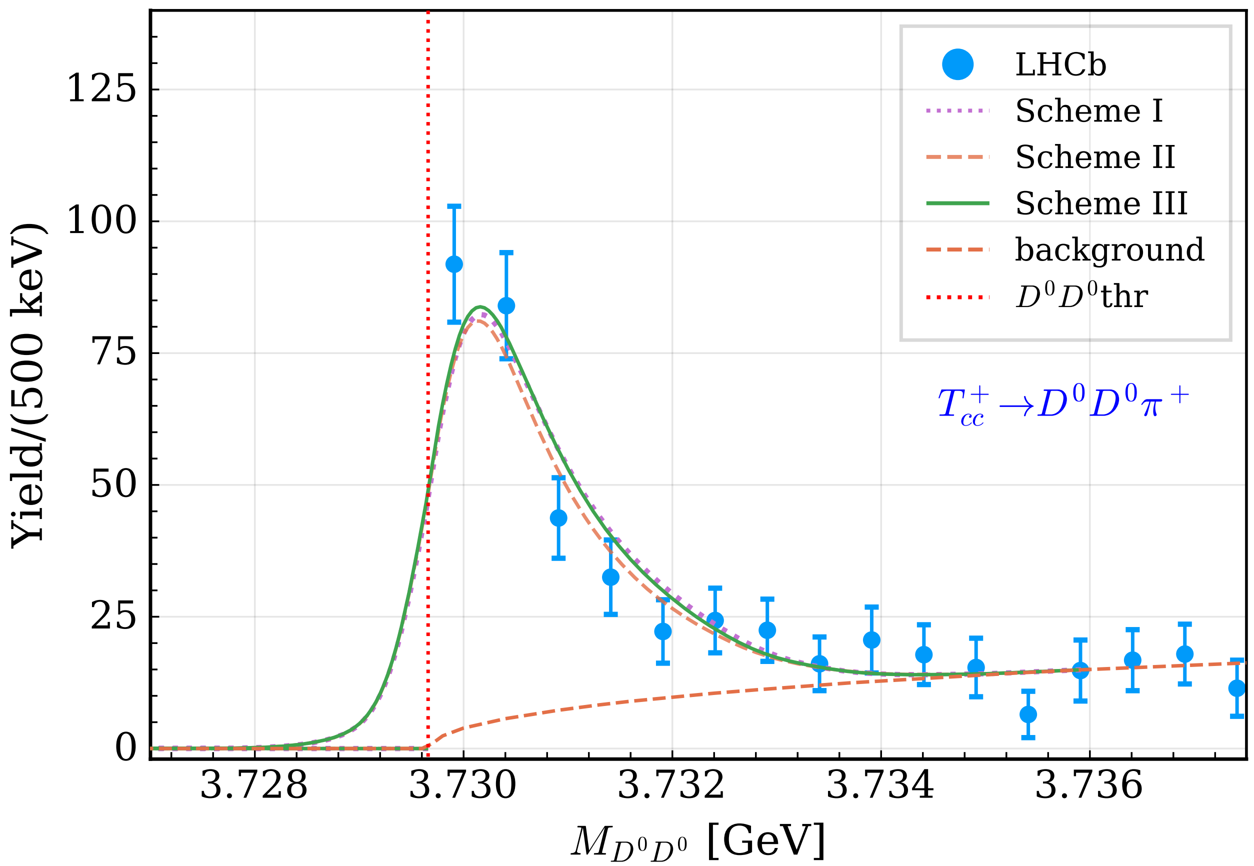


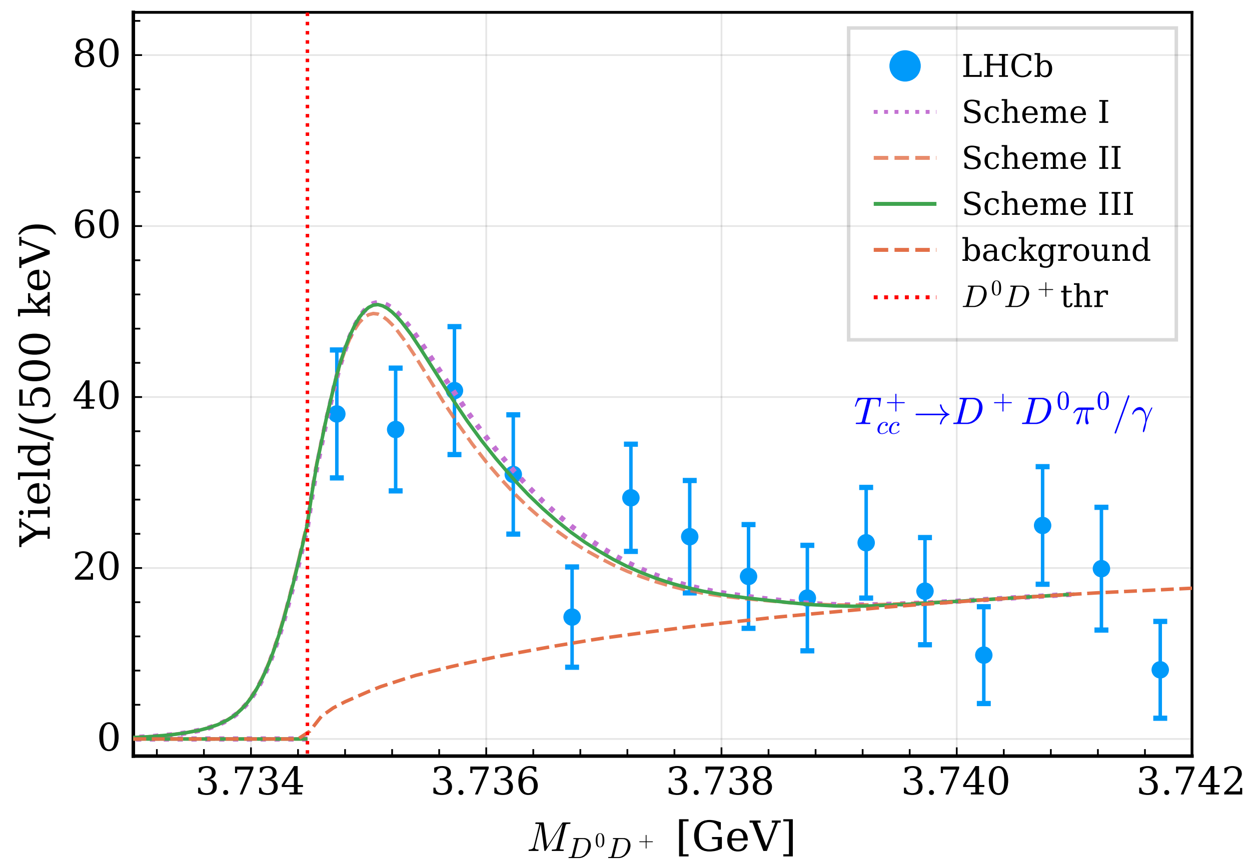
- The smaller the coupling the closer the fit is to the ERE

Various predictions

D⁰D⁰ and D⁰D⁺ spectra

with resolution





Heavy quark spin partners

see also Albaladejo PLB 829 (2022) in contact EFT

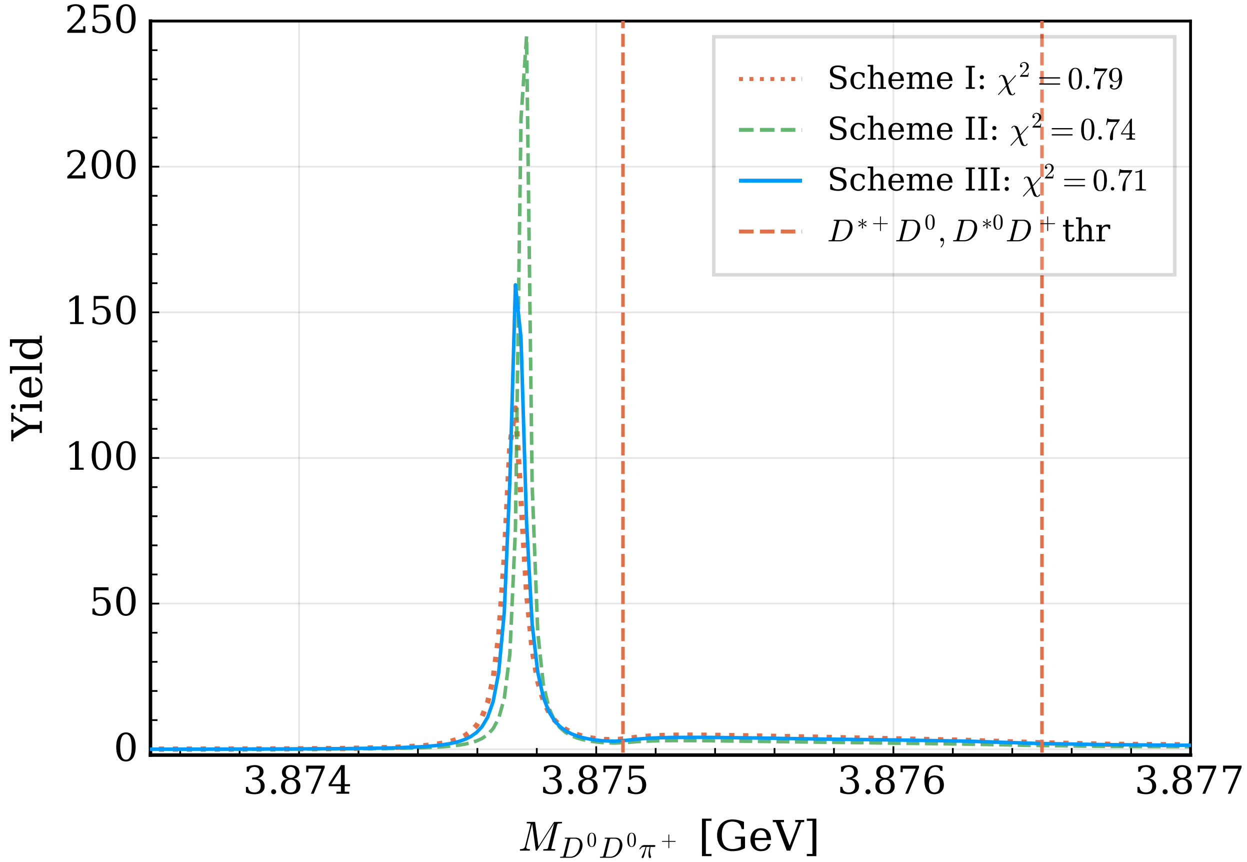
$$V^{I=0}(D^*D^* \to D^*D^*, 1^+) = V^{I=0}(D^*D \to D^*D, 1^+)$$

$$\delta_{cc}^{*+} = m_{T_{cc}^{*+}} - m_c^* - m_0^* = -503(40)\ \text{keV}$$

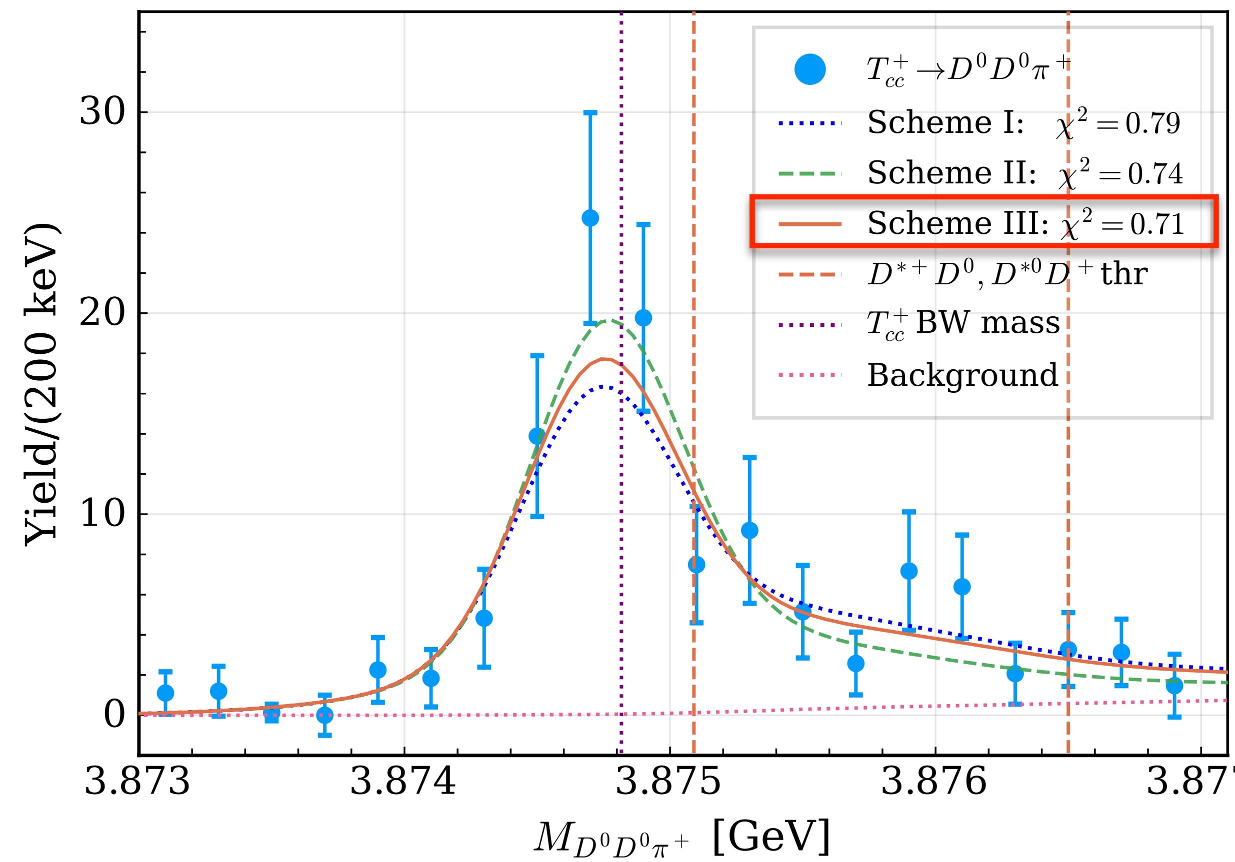
⇒ (quasi)bound D*D* state ~ 0.5 MeV below the threshold

Fits to the $D^0D^0\pi^+$ mass spectrum

w/o resolution



with resolution



Scheme	I	II	III
Description	2-body unitarity: No OPE, static D^* width	Incomplete 3-body unitarity: No OPE, dynamical D^* width	full 3-body unitarity: OPE + dynamical D^* width
Pole [keV]	$-368^{+43}_{-42} - i(37 \pm 0)$	$-333^{+41}_{-36} - i(18 \pm 1)$	$-356^{+39}_{-38} - i(28 \pm 1)$
χ^2	0.79	0.74	0.71

Real part of the pole: all Fits are consistent within 1σ —more precise data are needed

Width of Tcc+ : Accuracy requires 3-body effects

$\Gamma^{3-\text{body}}_{T_{cc}} = 56 \pm 2\,\text{keV}$ —remove OPE→ 36 keV —remove dynam.width→ 74 keV

Weinberg compositeness

Weinberg 1963-65

Physical coupling and ERE parameters via probability of a molecular component *X*

$$a = -2\,\frac{X}{1+X}\frac{1}{\gamma} + \mathcal{O}(1/\beta) \qquad r = -\frac{1-X}{X}\frac{1}{\gamma} + \mathcal{O}(1/\beta) \qquad g_R^2 = \frac{2\pi\gamma}{\mu^2} X + \mathcal{O}(1/\beta)$$

a < 0 — bound state

VB et al. PLB 2004

If $|a| \gg |r|, \;\; r \sim 1/\beta \quad \Longrightarrow \quad X \to 1$ ⟹ Molecule

If $|a| \ll |r|, \;\; r < 0 \quad \Longrightarrow \quad X \to 0$ ⟹ Compact state

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- Same information can be inferred from pole counting

Morgan 1992

— one near-thr. pole: $k_1 = -\frac{i}{a}\left[1 + \mathcal{O}\left(\frac{r}{a}\right)\right]$ $\quad |a| \gg |r|$ $\Longrightarrow$ Molecule

— two near-thr. poles $k_{1,2} = \pm i\sqrt{\frac{2}{a\,r}} + \frac{i}{r} + \mathcal{O}\left(\sqrt{\frac{a}{r^3}}\right)$ $\quad |a| \ll |r|$ $\Longrightarrow$ Compact state

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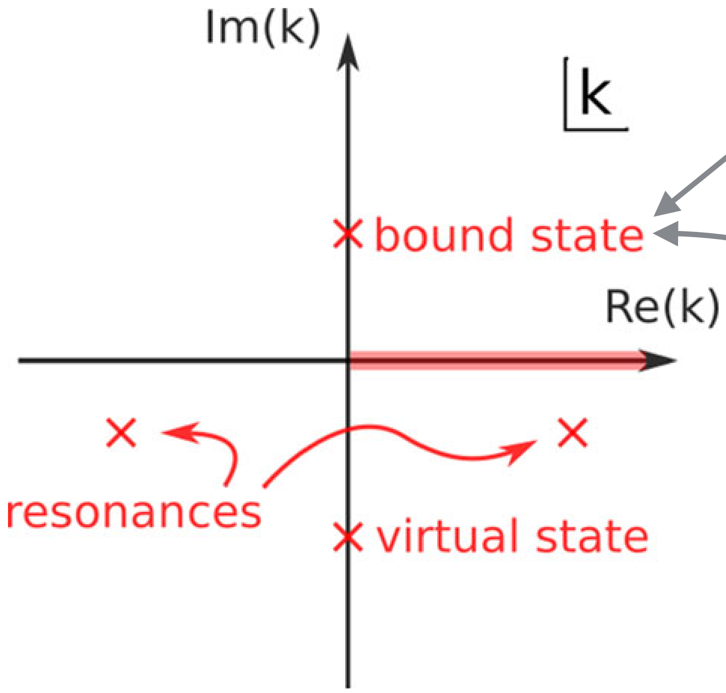
- Extensions mostly for resonances by Jido, Kamiya, Nieves, Oller, Oset, Sekihara,… — review Kamiya and Hyodo 2017

- Recent generalisations to virtual states, coupled-channels, … — Matuschek et al. EPJA 57 (2021), VB et al., PLB 833 (2022); see also talk by Kinugawa on Thursday

- Insights on range effects — Albaladejo, Nieves 2022, Li et al. 2022, Song et al 2022, Kinugawa, Hyodo 2022

Extensions beyond bound states

Matuschek, VB, Guo, Hanhart
EPJA 2021

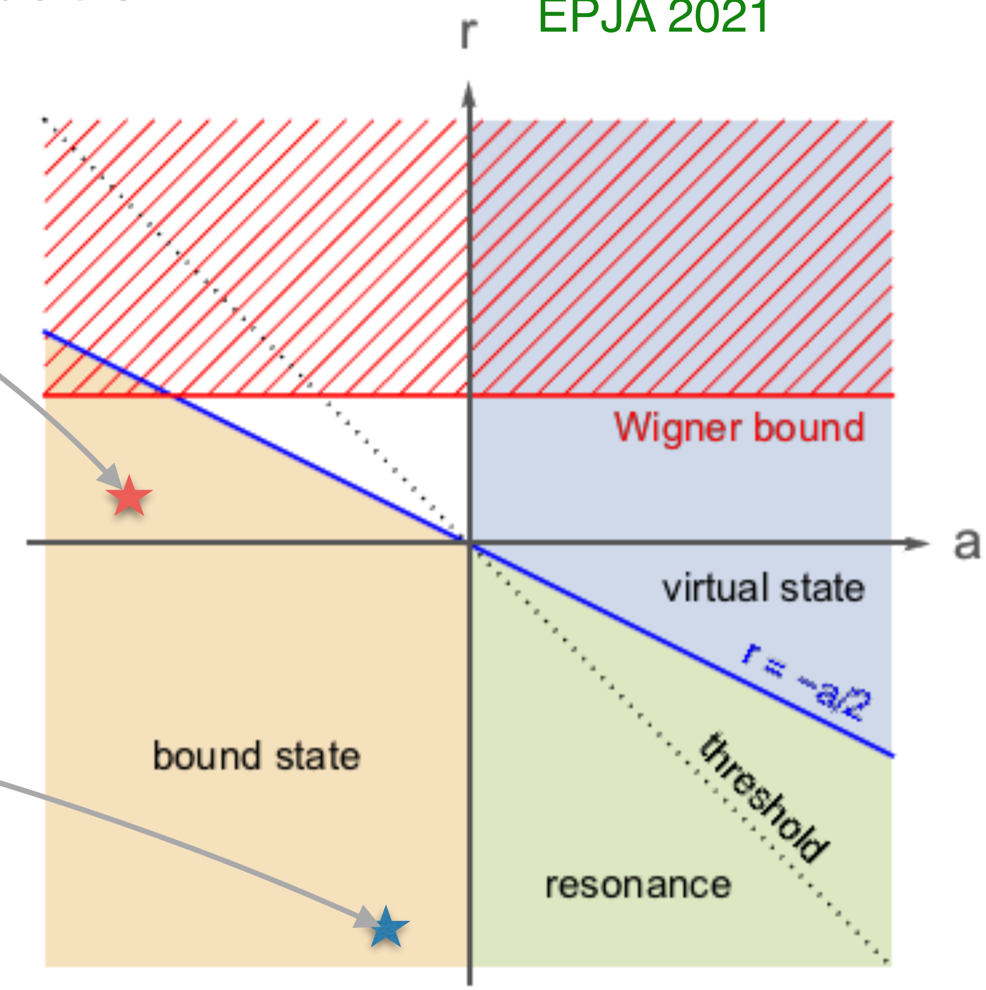


Molecular bound st.

$$|a| \gg |r|, \;\; r \sim 1/\beta$$

Compact bound st.

$$|a| \ll |r| \quad \text{and} \quad r < 0$$



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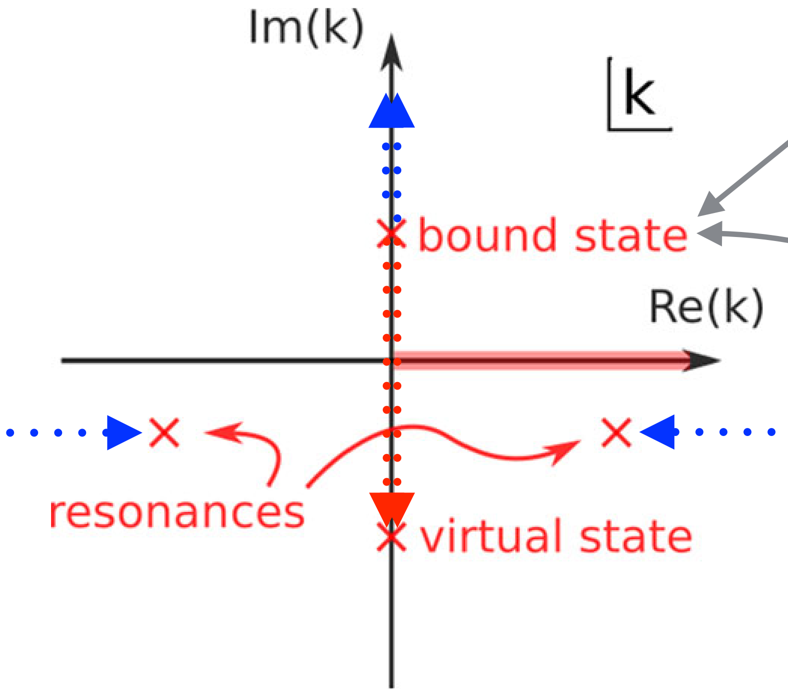

Molecular bound st.

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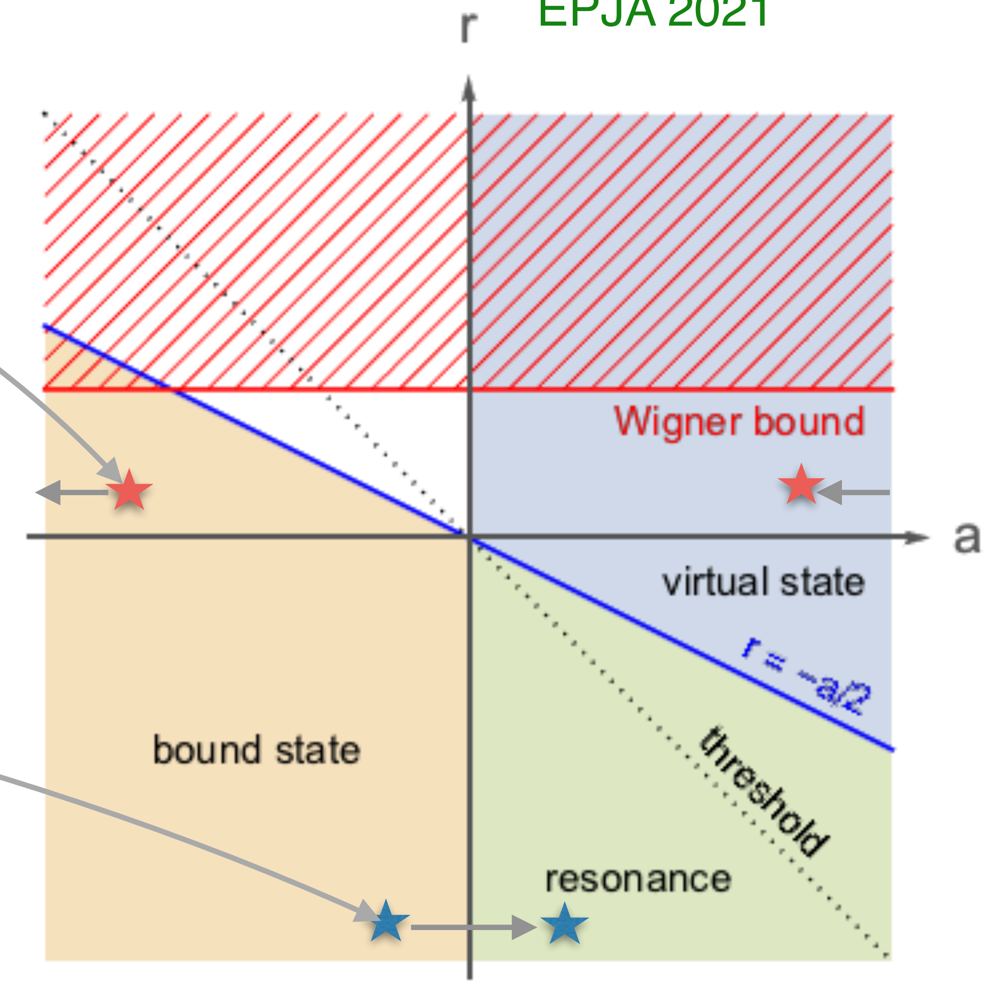
Compact bound st.

$|a| \ll |r|$ and $r < 0$



- Evolution of poles and analyticity → Extensions beyond bound states

Molecular pole: $k_1 = -\frac{i}{a}\left[1 + \mathcal{O}\left(\frac{r}{a}\right)\right]$ if sc. length changes sign → virtual state

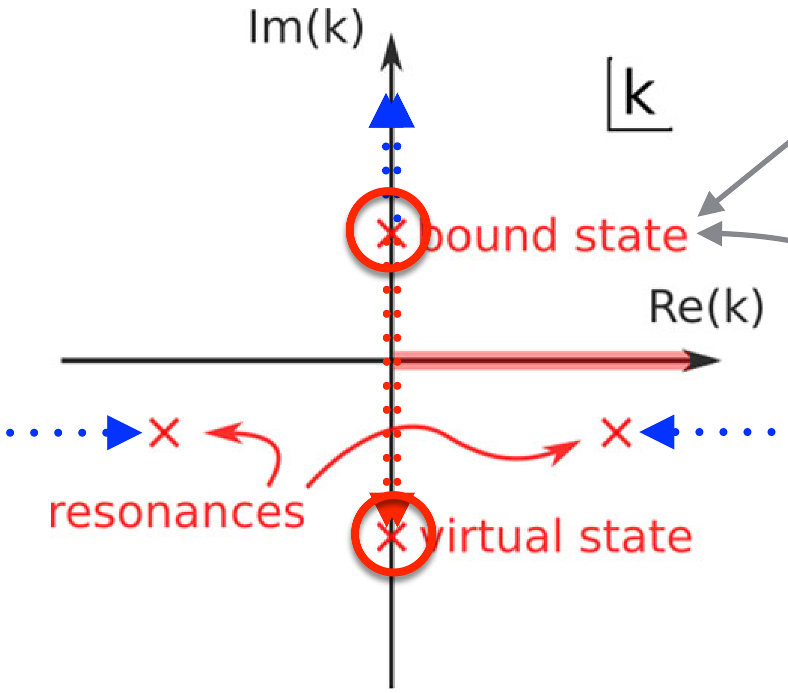
$|a| \gg |r|$

Compact pole: $k_{1,2} = \pm i\sqrt{\frac{2}{a\,r}} + \frac{i}{r} + \mathcal{O}\left(\sqrt{\frac{a}{r^3}}\right)$ if sc. length changes sign → turns to a resonance

$|a| \ll |r|$

Extensions beyond bound states

Matuschek, VB, Guo, Hanhart
EPJA 2021



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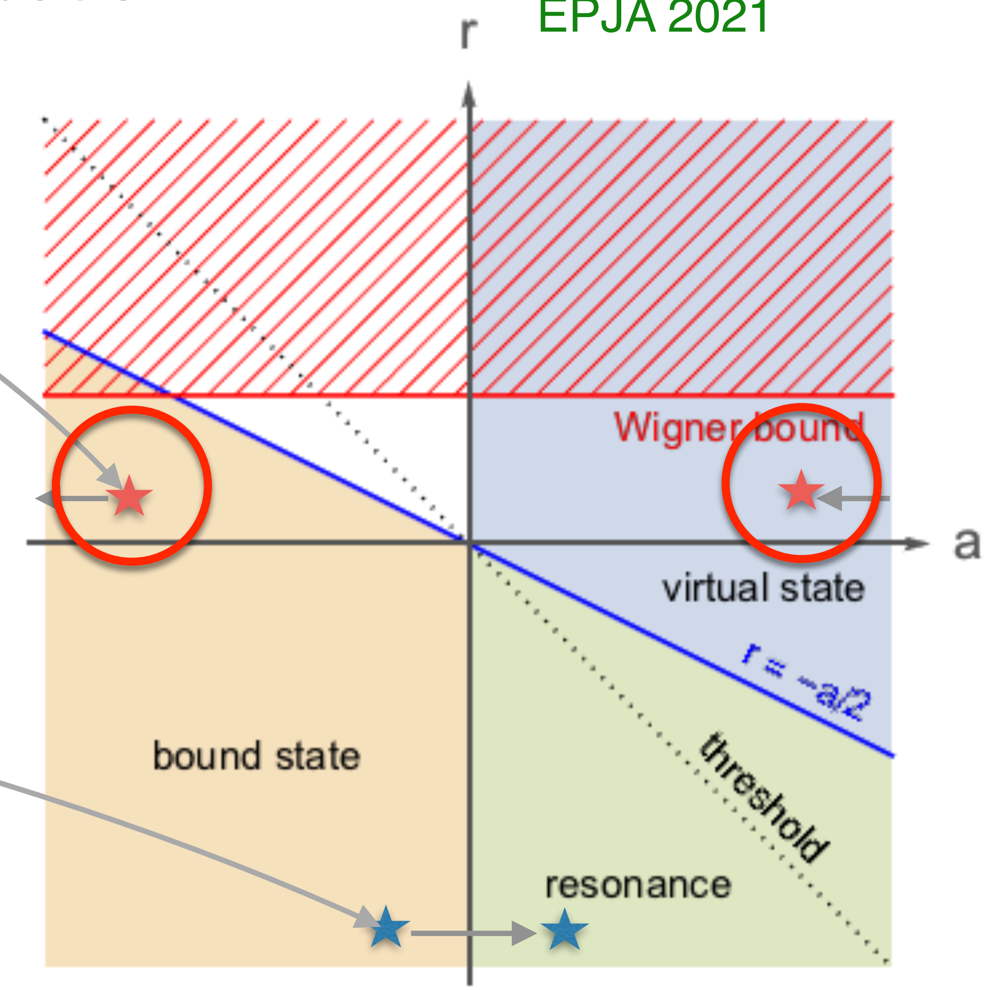

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Near thr. molecules

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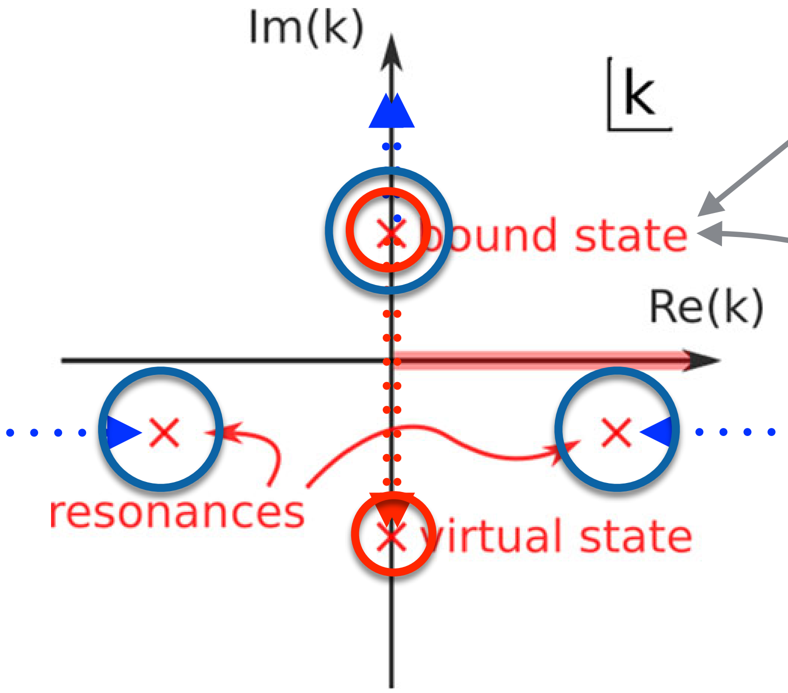
Extensions beyond bound states

Matuschek, VB, Guo, Hanhart
EPJA 2021



Molecular bound st.

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Compact bound st.

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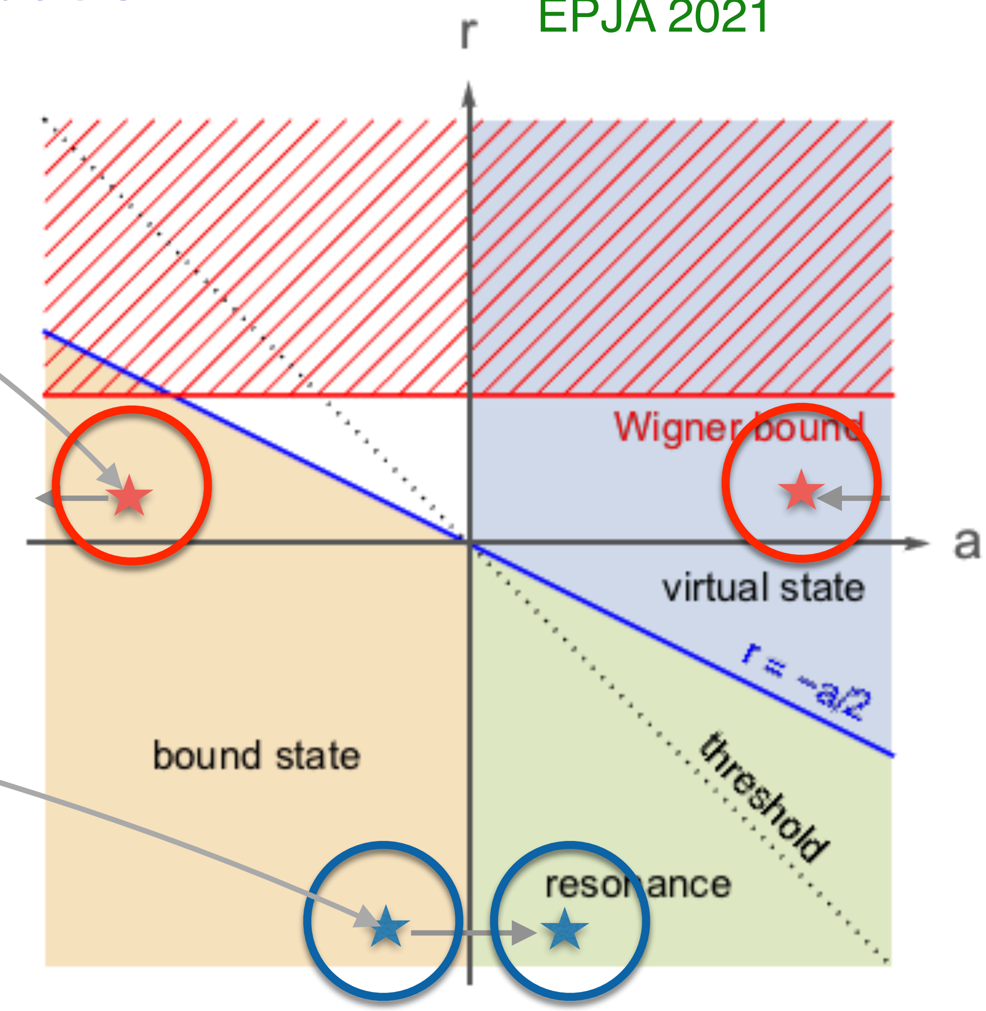

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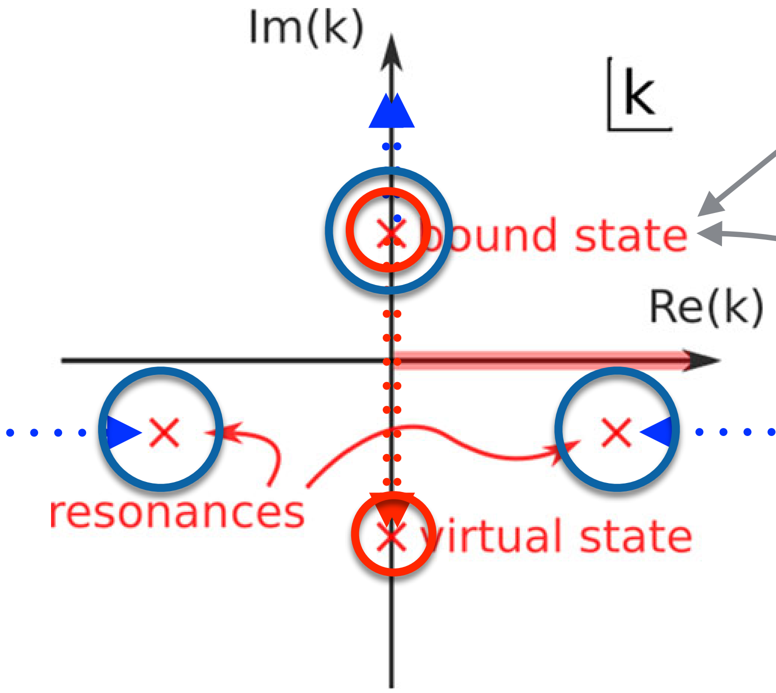
Near thr. compact states

Extensions beyond bound states

Matuschek, VB, Guo, Hanhart
EPJA 2021

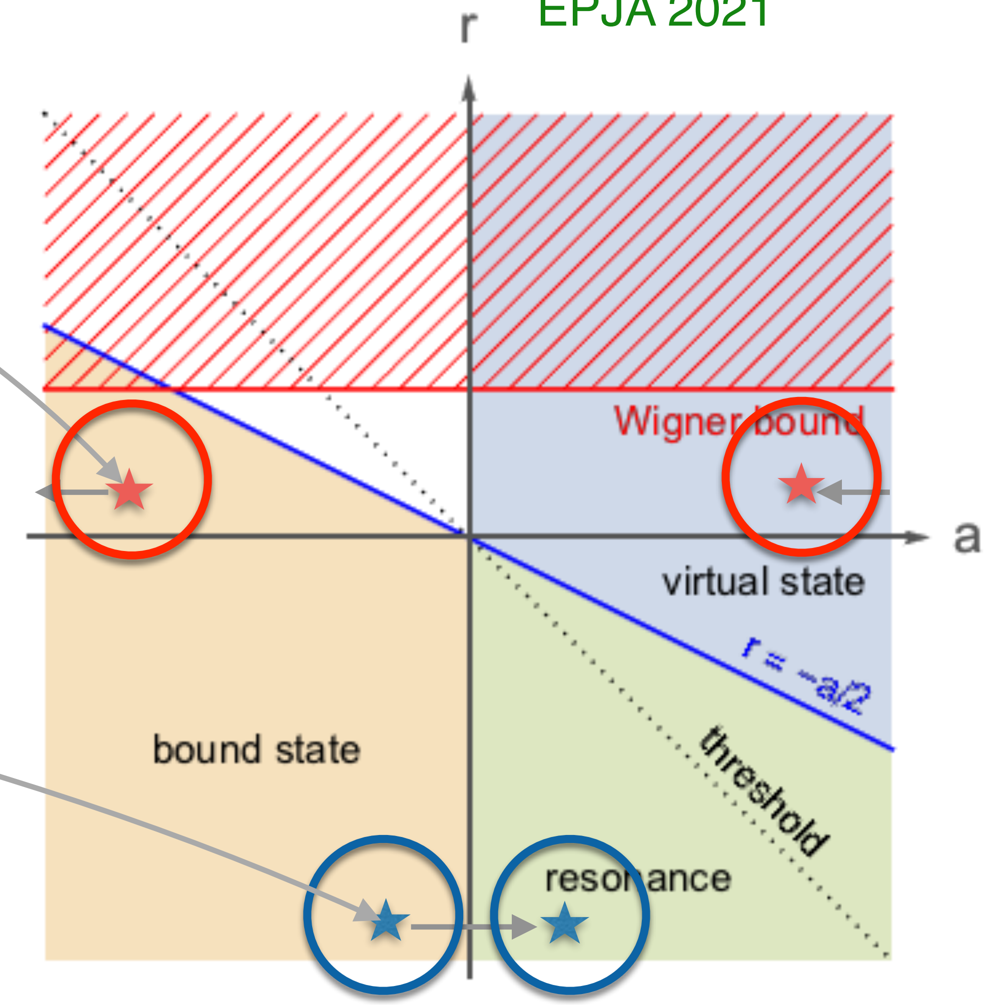


Molecular bound st.

$|a| \gg |r|, \;\; r \sim 1/\beta$

Compact bound st.

$|a| \ll |r|$ and $r < 0$



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$|a| \ll |r|$

Near thr. compact states

$$X_W = \sqrt{\frac{1}{1 + 2r/a}} \quad \Longrightarrow \quad \bar{X} = \sqrt{\frac{1}{1 + |2r/a|}}$$

both cases subsumed here

- $\bar{X}$ allows one to test compositeness for bound/virtual states and resonances