Beyond the Standard Model with Sp(2N) Gauge Theory: Meson Spectroscopy and Scattering

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`2202.05191, 2304.07191, 2405.01388, 2405.05765, 2405.06506`

`based on work with`: E.Bennett, Y.Dengler, N.Forzano, DK.Hong, H.Hsiao, S.Kulkarni, JW.Lee, CJD.Lin, B.Lucini, A.Maas, S.Mee, M.Nikolic, M.Piai, J.Pradler, F.Pressler, D.Vadacchino

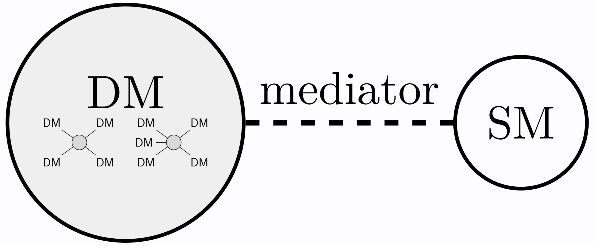
QCD-like gauge theories

- New non-SM gauge force with fermions
  - Composite Higgs Models: hyper-gluons and hyperquarks
  - Dark Matter Models: dark gluons and dark quarks
- Depending on the BSM model they can carry SM charges or not

- I will use the QCD nomenclature:
  - e.g. π: 0^- nonsinglet, ρ: 1^- nonsinglet, ...
  - these states are **not** the QCD hadrons
  - but they are similar in terms of their fermion structure

QCD-like Gauge Theories in Dark Matter Models

- With fermions: Global symmetries make Dark Matter stable
- With mediator: Dark sector coupled to SM
- Lightest dark Hadrons (here pions) are DM candidates



$$\mathcal{L}_{\mathrm{DM}} = -\frac{1}{2}\mathrm{Tr}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}_f(i\not{D} - m_f)\psi_f$$

- Non-vanishing self-scattering cross-section arise

$$\langle v\sigma_{\pi\pi\to\pi\pi}\rangle \neq 0$$

- Dark Matter Relic density driven by strong processes
    - Scattering cross-sections are required input

Theories With Multiple Fermion Representations

[1] Ferretti, Karateev [1312.5330]

$$\mathcal{L} = -\frac{1}{2}\mathrm{Tr}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}_i\left(i\not{D} - m_i\right)\psi_i + \bar{\Psi}_j\left(i\not{D} - m_j\right)\Psi_j$$

- Gauge theory of group G with field strength tensor $F_{\mu\nu}$
- Two species of fermions ψ and Ψ under different irreps of G

- Composite Higgs Models with partial top compositeness
  - Composite Higgs from one Goldstone sector
  - Composite Top partner from $\psi\psi\Psi$ or $\psi\Psi\Psi$
  - Global Symmetries must contain $SU(3)$ and $SU(2)\times U(1)$

Chiral Symmetry and Goldstone Bosons

- One breaking pattern for every fermion representation
  - complex: $SU(N_f) \times SU(N_f) \to SU(N_f)$
  - pseudoreal: $SU(2N_f) \times Sp(2N_f)$
  - real: $Sp(2N_f) \times SO(2N_f)$
- And one axial $U(1)$ for every representation
  - one linear combination broken by axial anomaly!
  - Additional $U(1)$ Goldstone at finite N_c for multirep!

Why Symplectic Gauge Theories?

- Fermions in real or pseudo-real representations
- **Dark Matter**: *Strongly Interactive Massive Particles*
  - Semi-annihilation process: $3\pi \to 2\pi$ sets DM relic density
  - $Sp(2N)$ groups with $N_f > 2$ favoured by χPT over $SU(N)$, $SO(N)$
- **Composite Higgs Models**: with partial top compositeness
  - two fundamental fermions: allow pseudo-Goldstone Higgs
  - three anti-symmetric fermions: top composite partner

BSM wishlist from the lattice

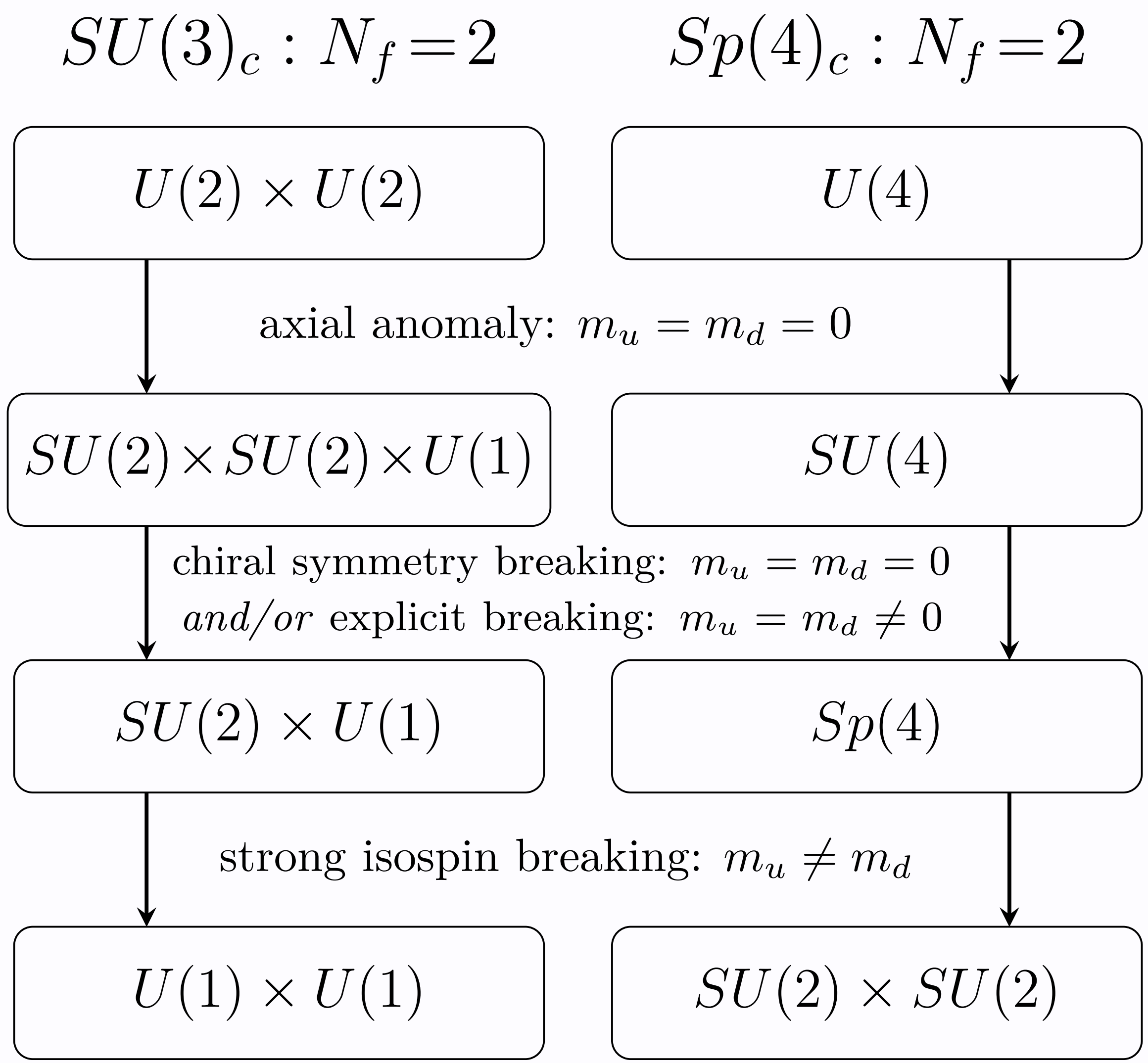
1. Masses and decay constants of dark hadrons
    - Non-singlet and singlet mesons, glueballs
2. Scattering of pions
    - $2\pi \rightarrow 2\pi$ for self-interaction crossection
    - $3\pi \rightarrow 2\pi$ for SIMP semi-annihilation
3. Applicability of χPT and related EFTs

Meson Spectrum of two-flavour Sp(4)

[1] Kosower (Phys.Lett.B. 1984)

[2] Hochberg et. al. [1411.3727] [1512.07917]

SIMPs from $Sp(4)$ gauge theory



- Pseudo-real representation: [1]
  $\Rightarrow$ more pseudo-Goldstones
  $\Rightarrow$ no fermionic bound states
- $N_f = 2$: exactly 5 Goldstones
  - Allows $3\pi \rightarrow 2\pi$ [2] DM semi-annihilation

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Sp(4) with two fermions is
a minimal SIMP DM realisation
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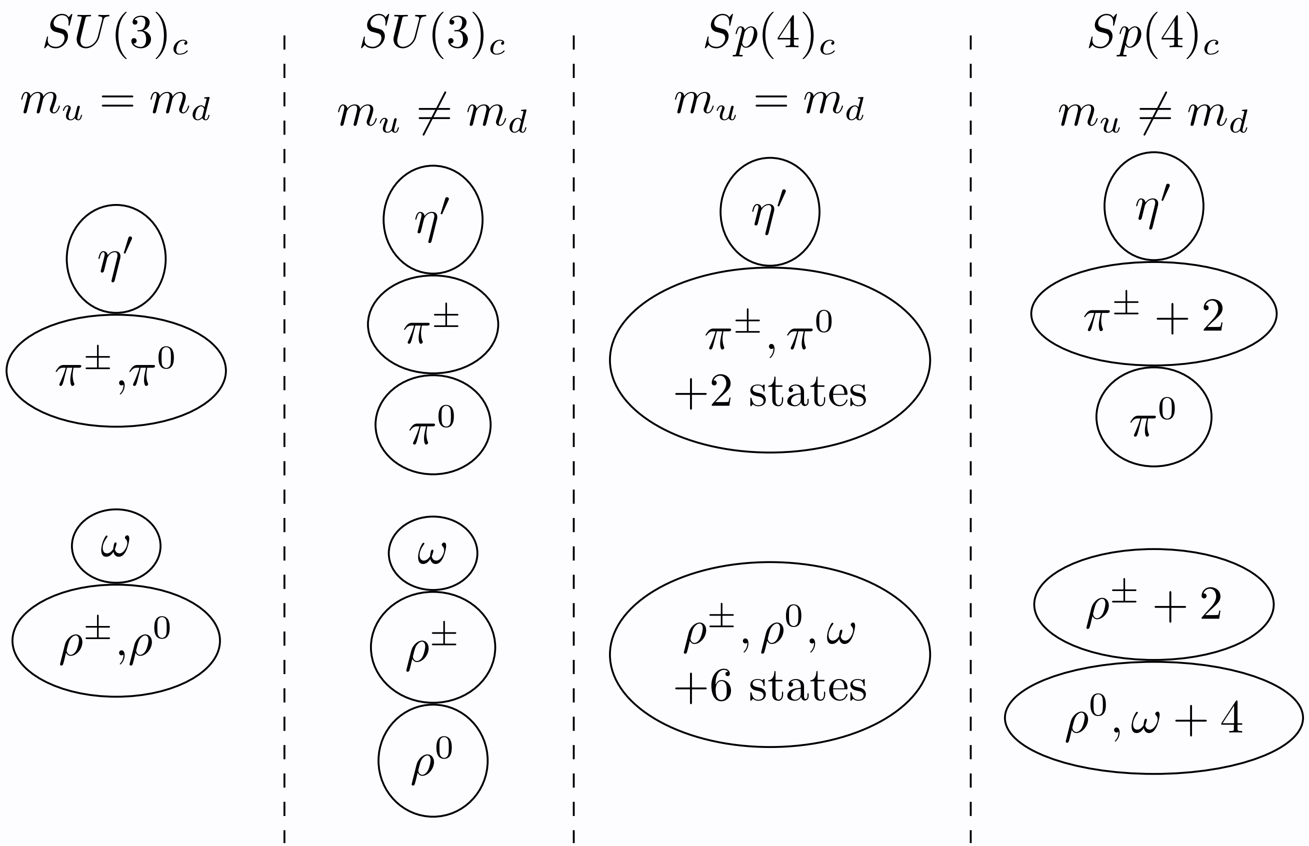
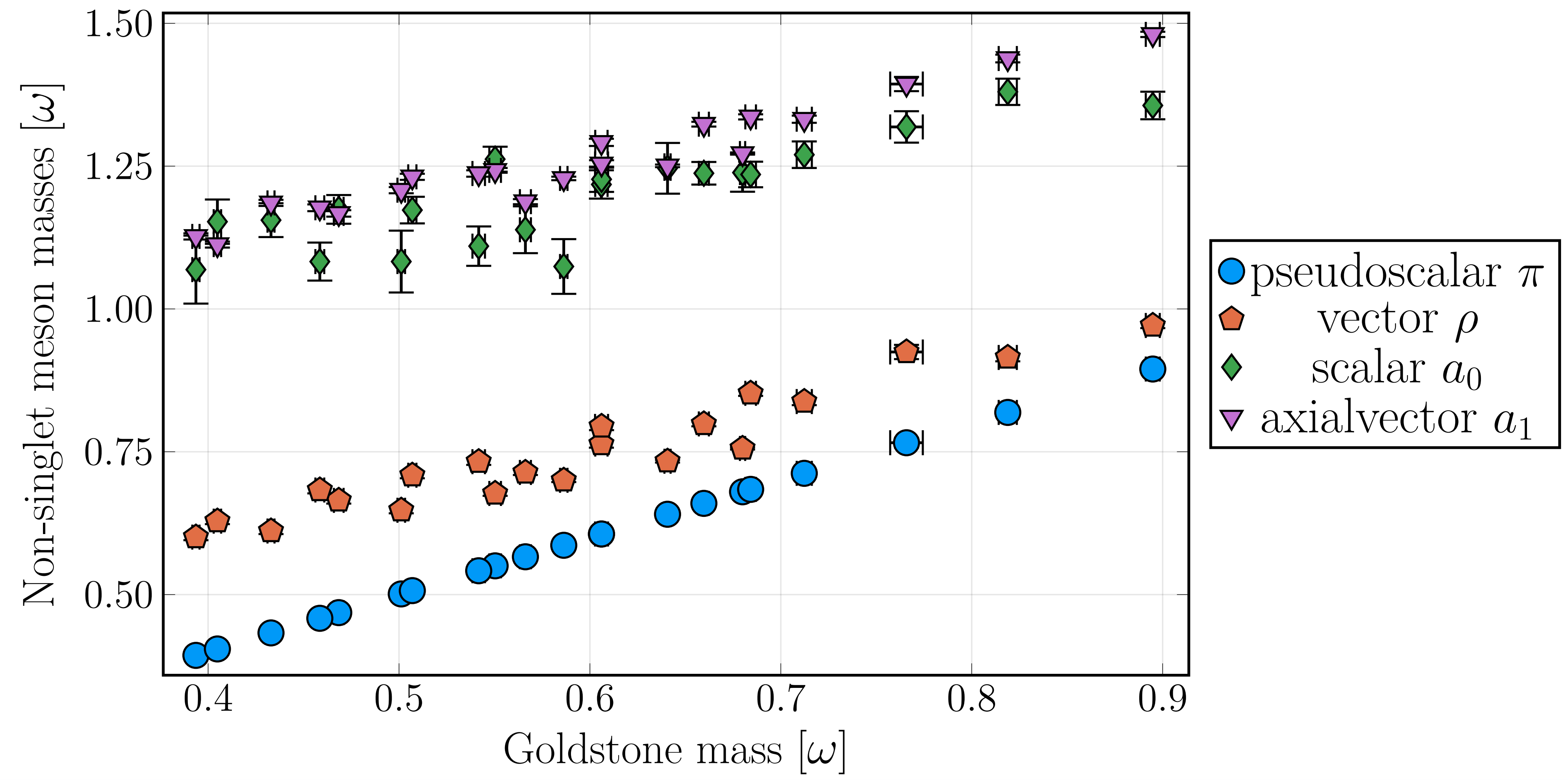
Pseudoscalar (PS) and vector (V) multiplets



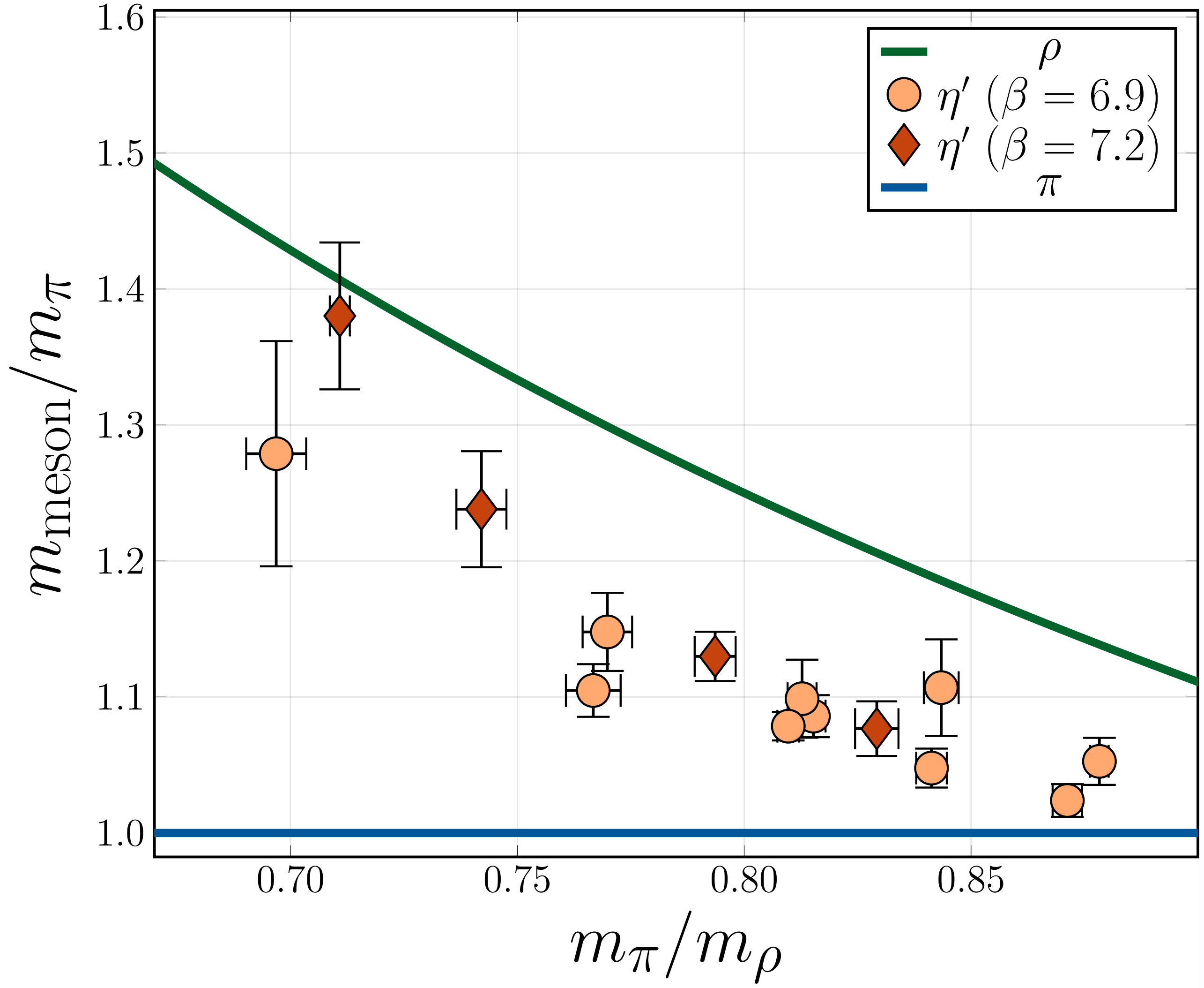
The same patterns persist for other channels.
Similar for anti-symmetric fermions

Non-singlet spectrum

Bennett et. al. [1909.12662]



The pseudoscalar and vector mesons are the lightest non-singlets.



The pseudoscalar singlet η' is surprisingly light!

- Phenomenologically relevant:
  - $m_\rho > m_{\eta'}$ different from QCD
  - relevant low-energy dof
  - η' relevant for $\pi\pi$ scattering
  - more accessible channels for decays into SM

Interesting! Is this surprising?

Bennett et. al. [2304.07191]

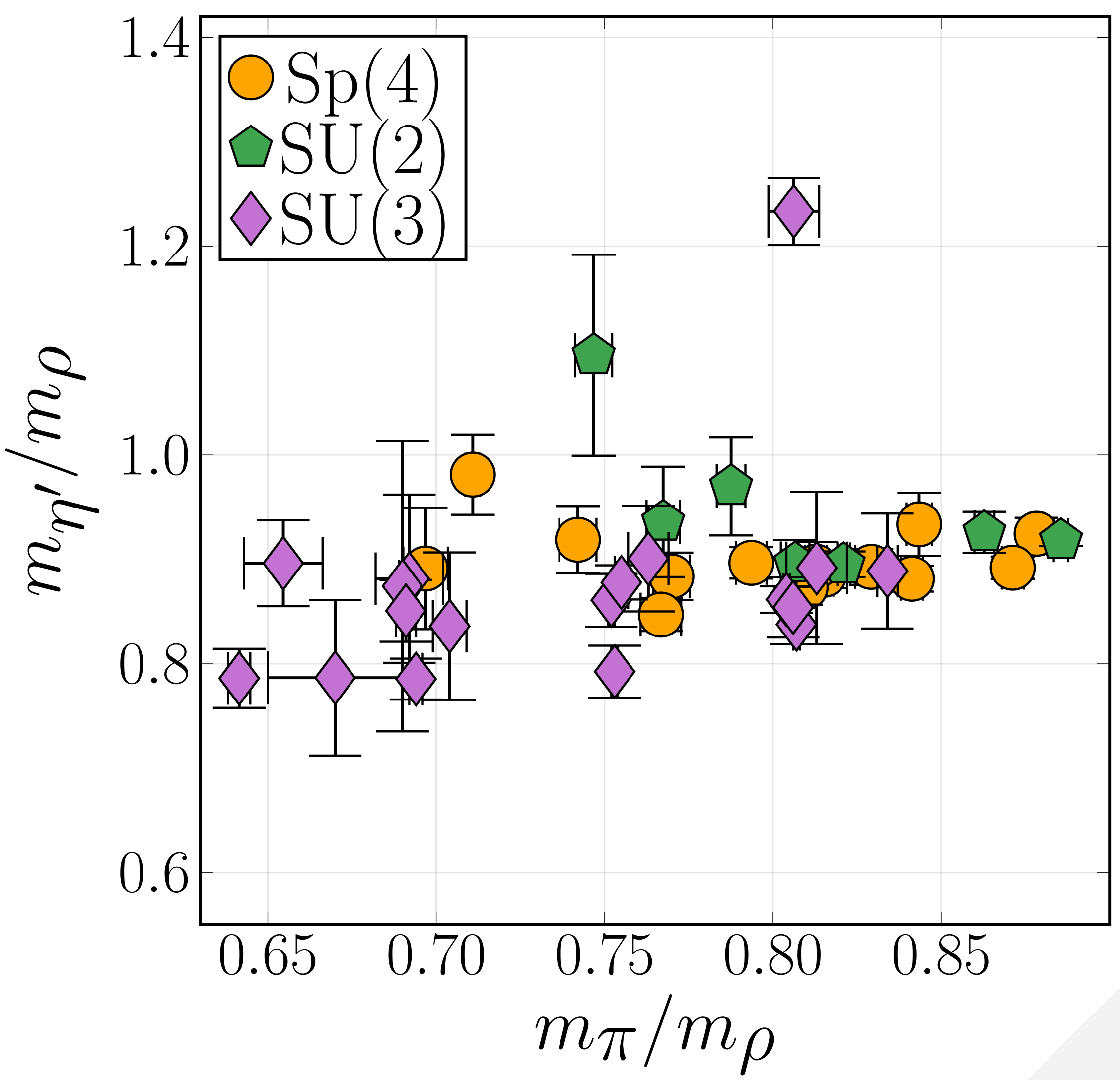
Consider different theories:

- Large N_c: $m_{\eta'} - m_\pi \propto N_f / N_c$
  - $N_f = 2$ could be "small"
  - $N_c = 4$ could be "large"

SU(2) and SU(3) comparison:

- Similarities:generic $N_f = 2$ feature?
- QCD: strong N_f dependence
- Differences may arise $m_\pi / m_\rho \to 0$

mass driven by flavour content!



Consequences for Dark Matter

- Mass hierarchies: limit χPT validity
  - inclusion of other states than π required, e.g. η' and ρ
  - additional tests needed (fermions are fairly heavy)
- Light unprotected states such as η' allow decays into SM
  - no protection from symmetry
  - could become relevant in thermal history of DM

Isospin-2 $\pi\pi$ scattering in two-flavour Sp(4)

Dark Matter Scattering on the Lattice

- Pions are in the 5-dimensional representations
- A two pion scattering is in one of three irreps

$$5 \times 5 = 14 \oplus 10 \oplus 1$$

- Corresponds to the usual QCD channels
  - $14 \Leftrightarrow$ isospin $I = 2$ in QCD, e.g. $\pi^+\pi^+$
  - $10 \Leftrightarrow$ isospin $I = 1$ in QCD, e.g. $\rho \to \pi\pi$
  - $0 \Leftrightarrow$ isospin $I = 0$ in QCD, e.g. $\sigma/f_0 \to \pi\pi$

Scattering information from the lattice

- Scattering phase shift $\delta_0(p)$ from finite volume energy

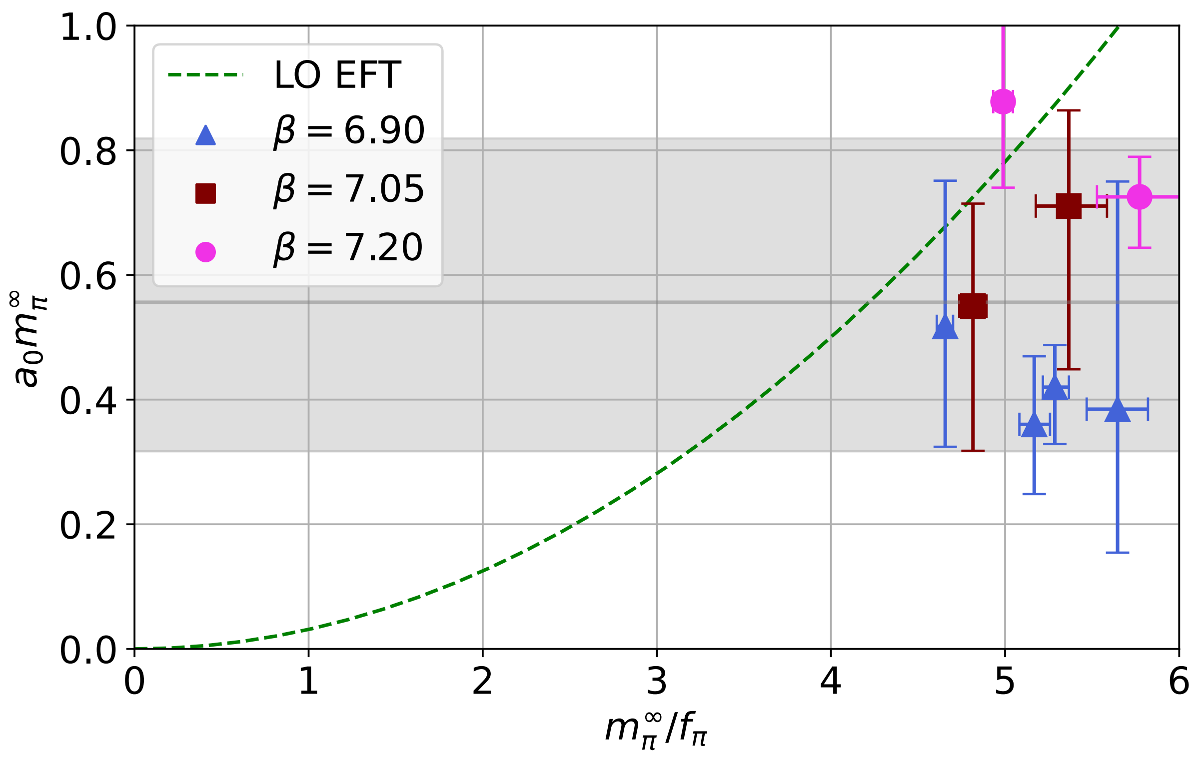
$$\tan(\delta_0(q)) = \frac{\pi^{\frac{3}{2}} q}{\mathcal{Z}_{00}^{\vec{0}}(1, q^2)}, \quad q = p^* \frac{L}{2\pi}$$

$$\cosh\left(\frac{E_{\pi\pi}}{2}\right) = \cosh(m_{\pi\pi}) + 2\sin\left(\frac{p^*}{2}\right)^2$$

- Low-velocity behaviour: Scattering length a_0

$$p \cot \delta_0 = -\frac{1}{a_0} + \frac{p^2}{2r_0^{-1}} + \mathcal{O}(p^4)$$

First investigation of isospin-2 scattering



- few lattice energy levels available $\Rightarrow$ systematics
- repulsive, roughly matches χPT
- first step towards other channels and resonances

Isosinglet Mesons in Mixed Representation Sp(4)

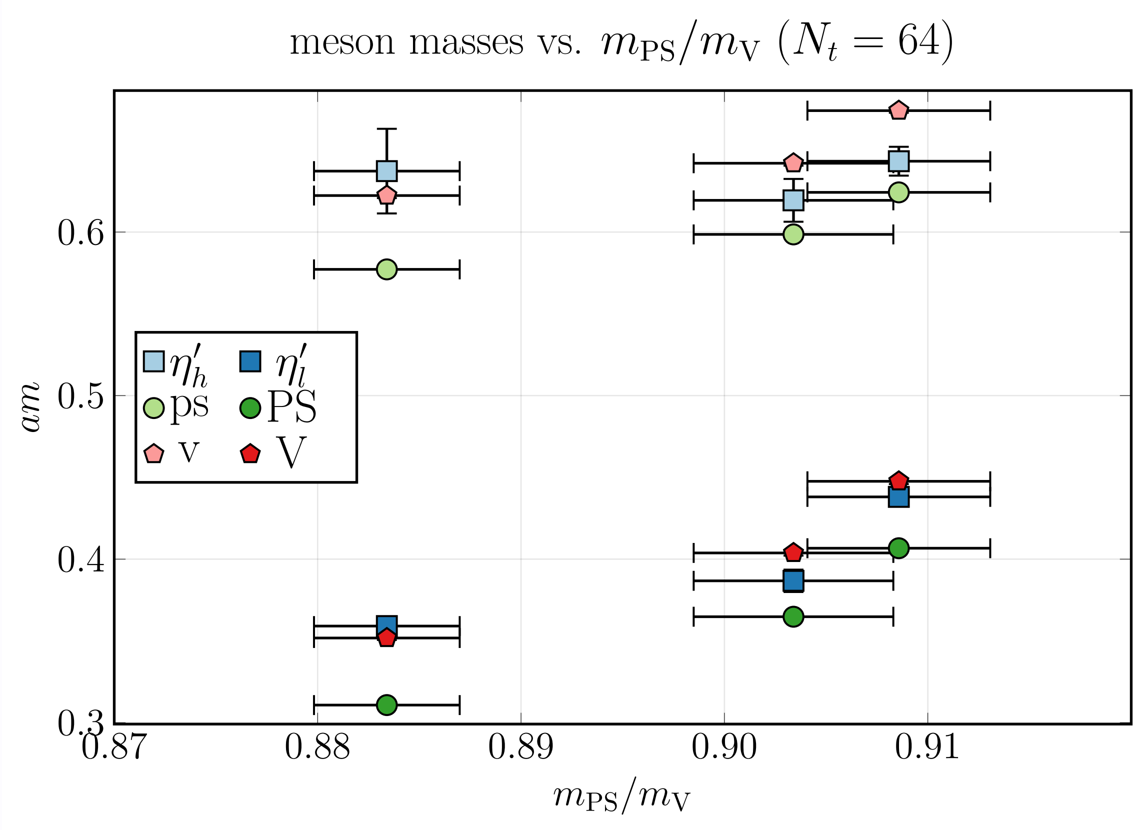
The axial $U(1)$ states: Isosinglet Pseudoscalars

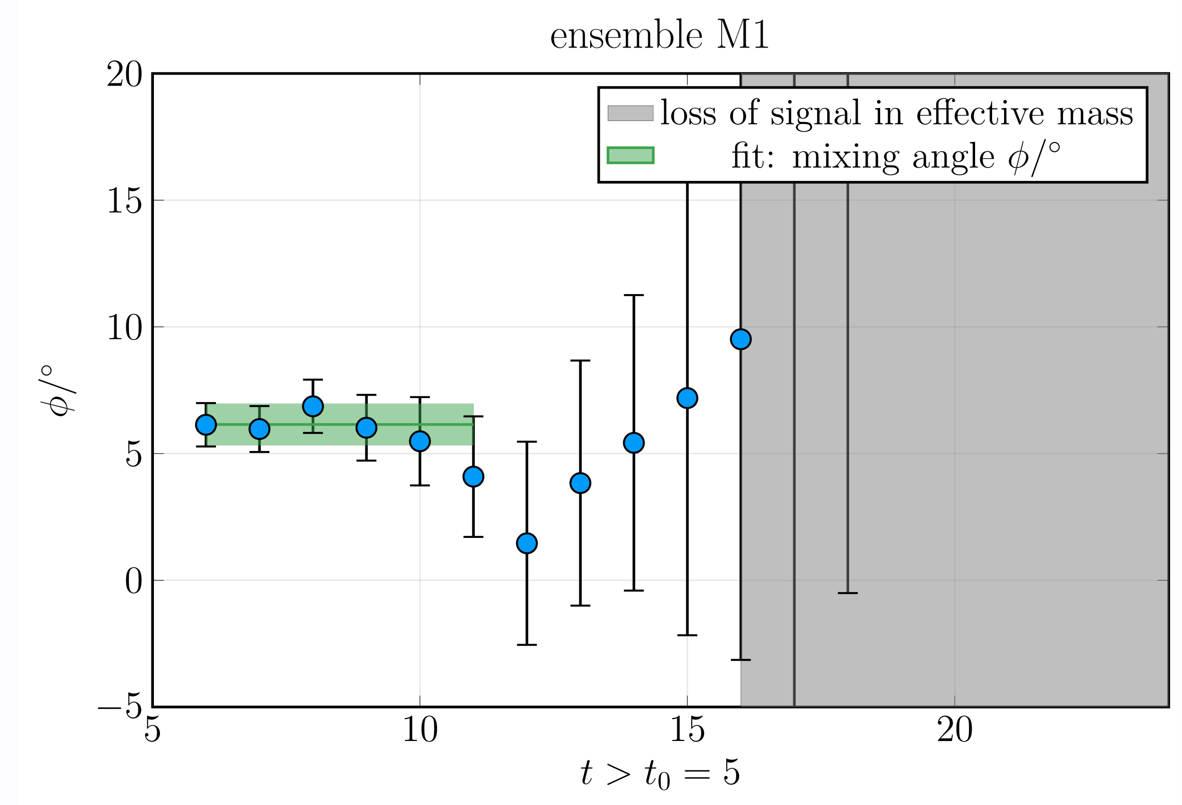
- Two fundamental, three anti-symmetric fermions
- pseudoscalar flavour-singlets: similar to η and η' of QCD
- probed by the following operators

$$O_{\eta^{\mathrm{f}}} = \left(\bar{\psi}_1\gamma_5\psi_1 + \bar{\psi}_2\gamma_5\psi_2\right)/\sqrt{2}$$
$$O_{\eta^{\mathrm{as}}} = \left(\bar{\Psi}_1\gamma_5\Psi_1 + \bar{\Psi}_2\gamma_5\Psi_2 + \bar{\Psi}_3\gamma_5\Psi_3\right)/\sqrt{3}$$

- These two states will mix: Light PNGB state η'_l + heavier state η'_h
  - mixing angle $\phi \neq 0$
  - Effective field theory in chiral limit is known

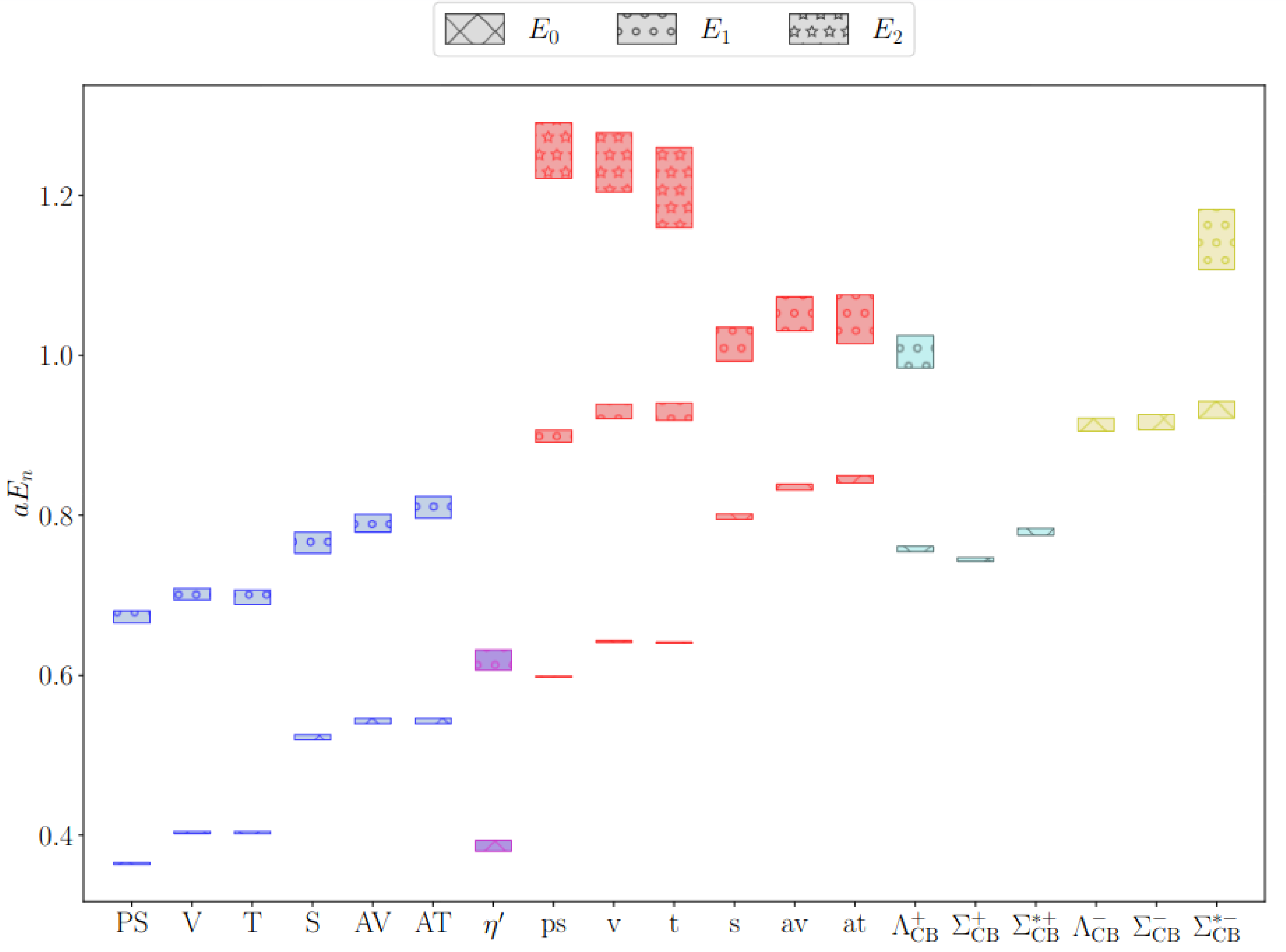
We are able to determine their masses and mixing angle!





- Encouraging proof of principle
- Caveat: Only heavy fermion masses accessible for now!

Example: Hadrons on a selected ensemble



Mesons and Baryons are accesible with our lattice methods

Summary

- Full light hadron spectrum of two-flavour $Sp(4)$
  - surprisingly light η' for moderately heavy fermions
  - first determination of isospin-2 $\pi\pi$ scattering
- Exploratory hadron spectrum of $Sp(4)$ with $N_f = 2^{(f)} + 3^{(as)}$

Outlook

- Full scattering analysis of $2\pi \to 2\pi$ and $3\pi \to 2\pi$ and resonances
- Better understanding of singlets and scattering states:
- Singlet spectroscopy closer to the chiral limit
- Lighter fermions for mixed-representation

Thank you

Back-up slides

Flavour symmetry

- Higher symmetry than QCD-like theories
- Mixing of left- and right-handed Weyl components

$$\Psi = \begin{pmatrix} u_L \\ d_L \\ -SCu_R^* \\ -SCd_R^* \end{pmatrix} = \begin{pmatrix} u_L \\ d_L \\ \tilde{u}_R \\ \tilde{d}_R \end{pmatrix} \qquad \begin{matrix} C \ldots \text{charge conj.} \\ S \ldots \text{colour matrix} \end{matrix}$$

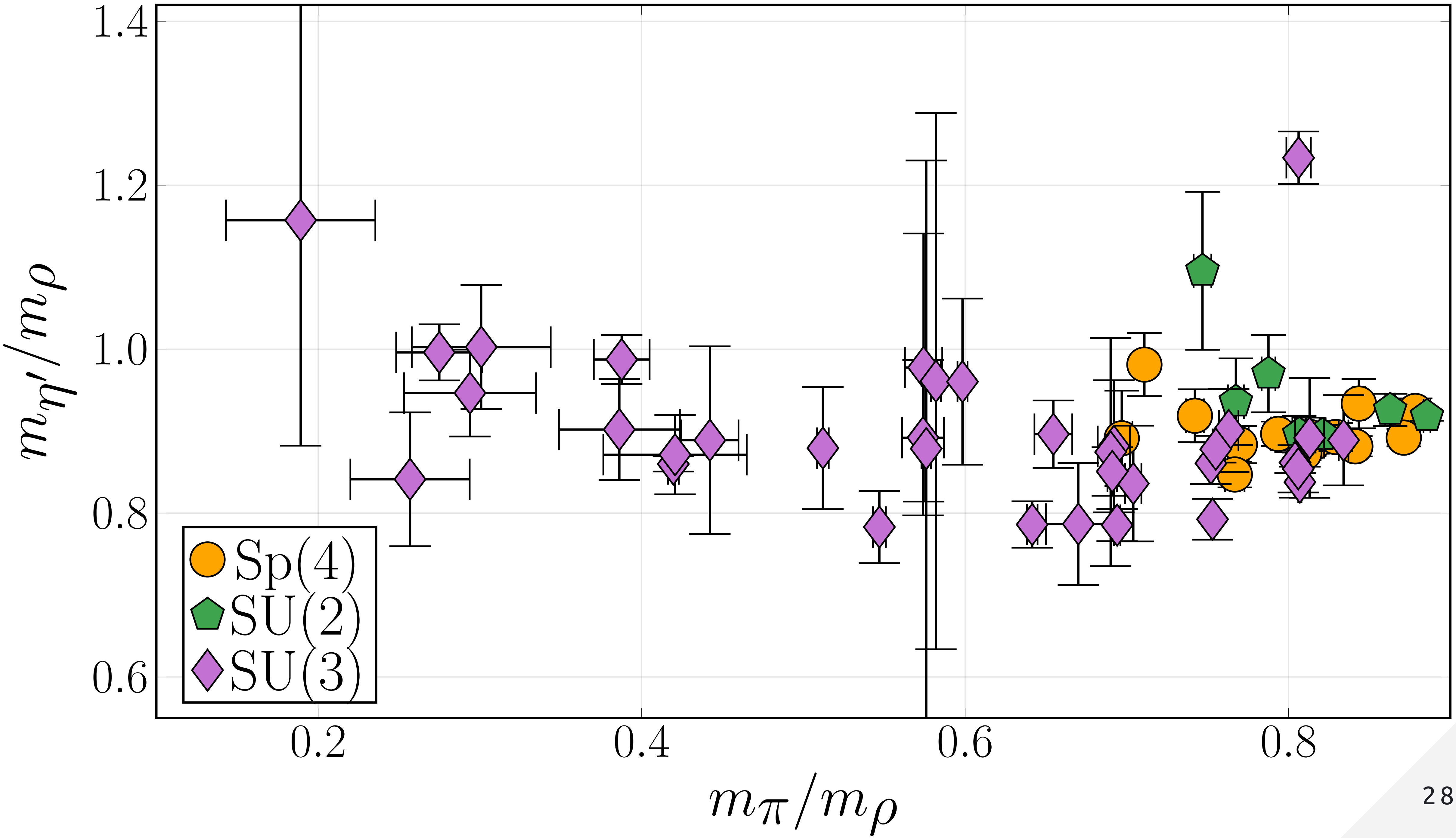
$$\mathcal{L}_{\text{DM}} = i\bar{\Psi}\not{D}\Psi - \frac{1}{2}\left(\Psi^T SCM\Psi + h.c.\right)$$

- Mass matrix M proportional to symplectic invariant tensor
- generators τ_a in fundamental : $S\tau_a S = -\tau_a^T$

Dark Matter interaction with SM: Freeze out

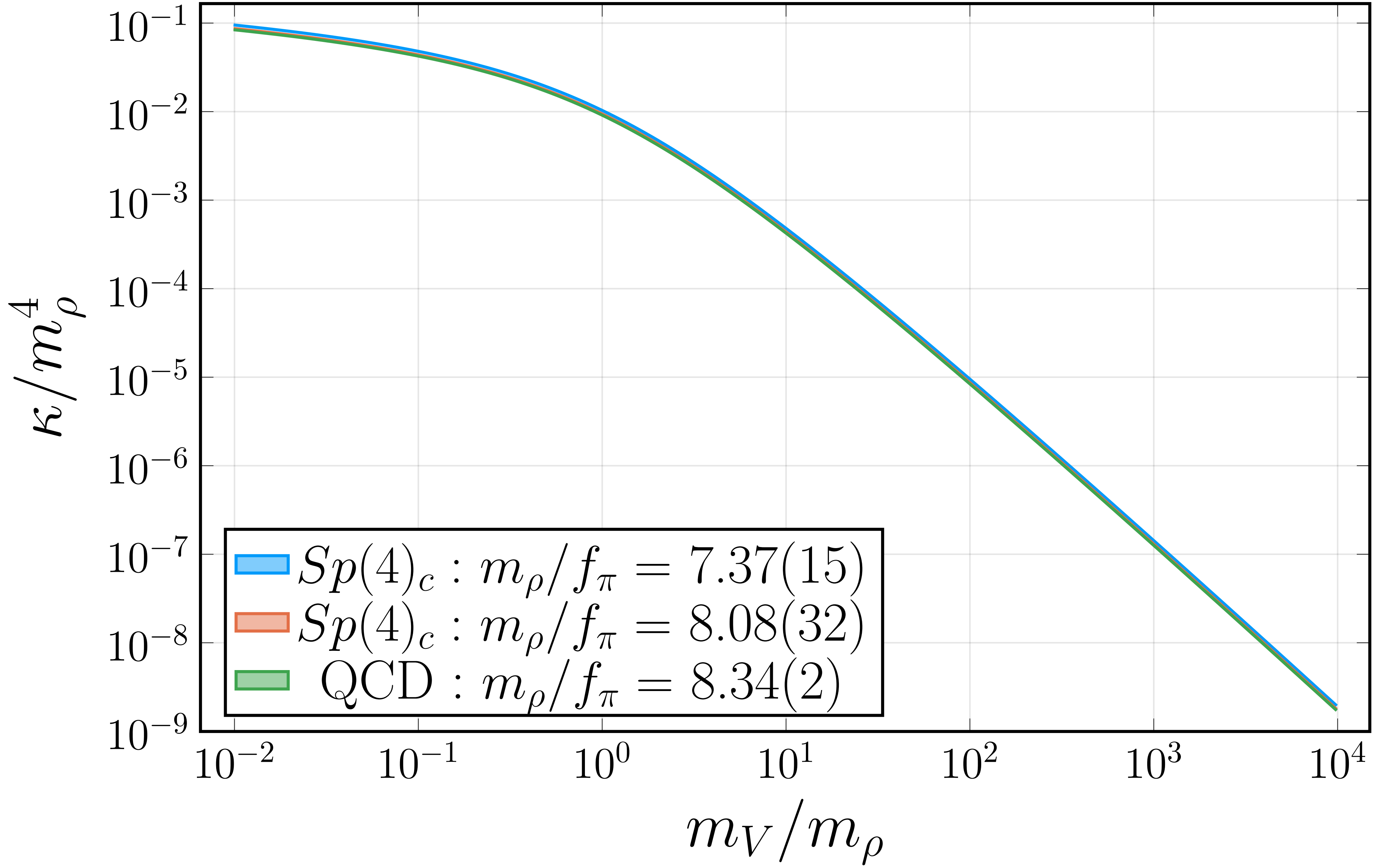
- Thermal interactions in early universe
- Assume $\mathrm{SM} \rightleftharpoons \mathrm{DM}$

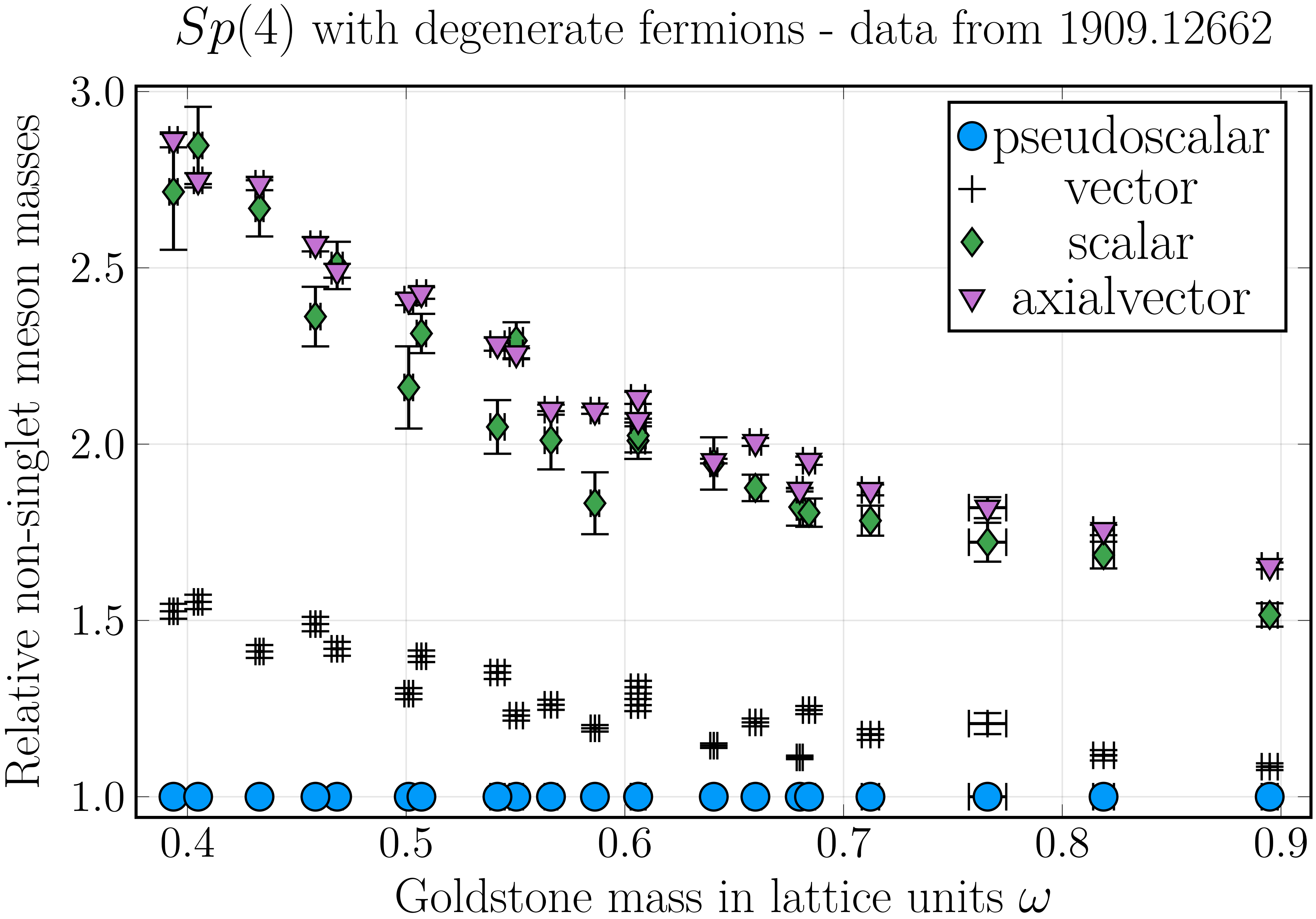
1. Early universe: equilibrium
2. $\mathrm{SM} \to \mathrm{DM}$ kinematically suppressed
   DM number drops*
3. Not enough DM for more annihilation
   DM number stays constant $\Rightarrow$ *freeze-out* [1]



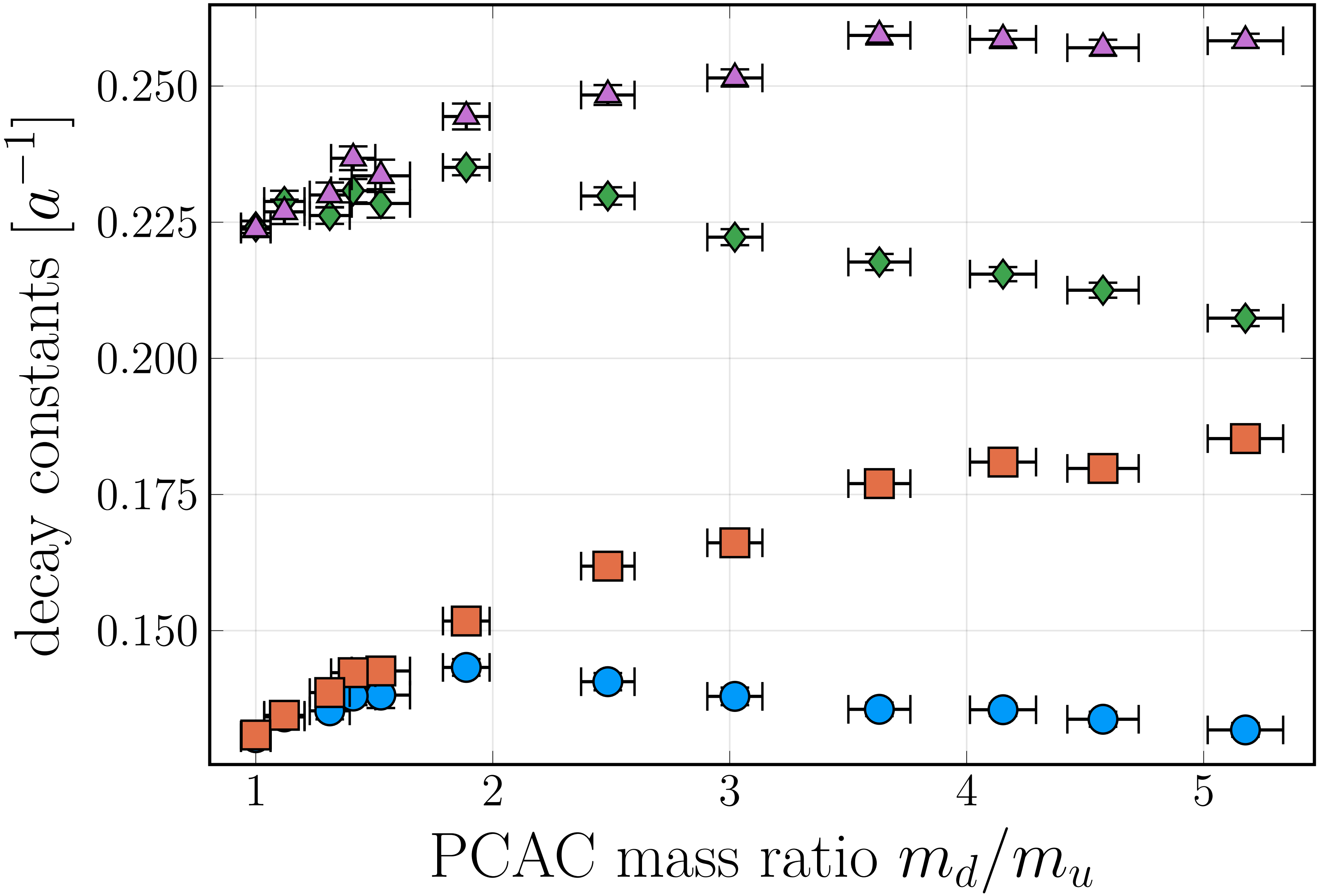
1.4
1.2
1.0
0.8
0.6
0.2
0.4
0.6
0.8
$m_{\eta'}/m_\rho$
m_π/m_ρ
Sp(4)
SU(2)
SU(3)

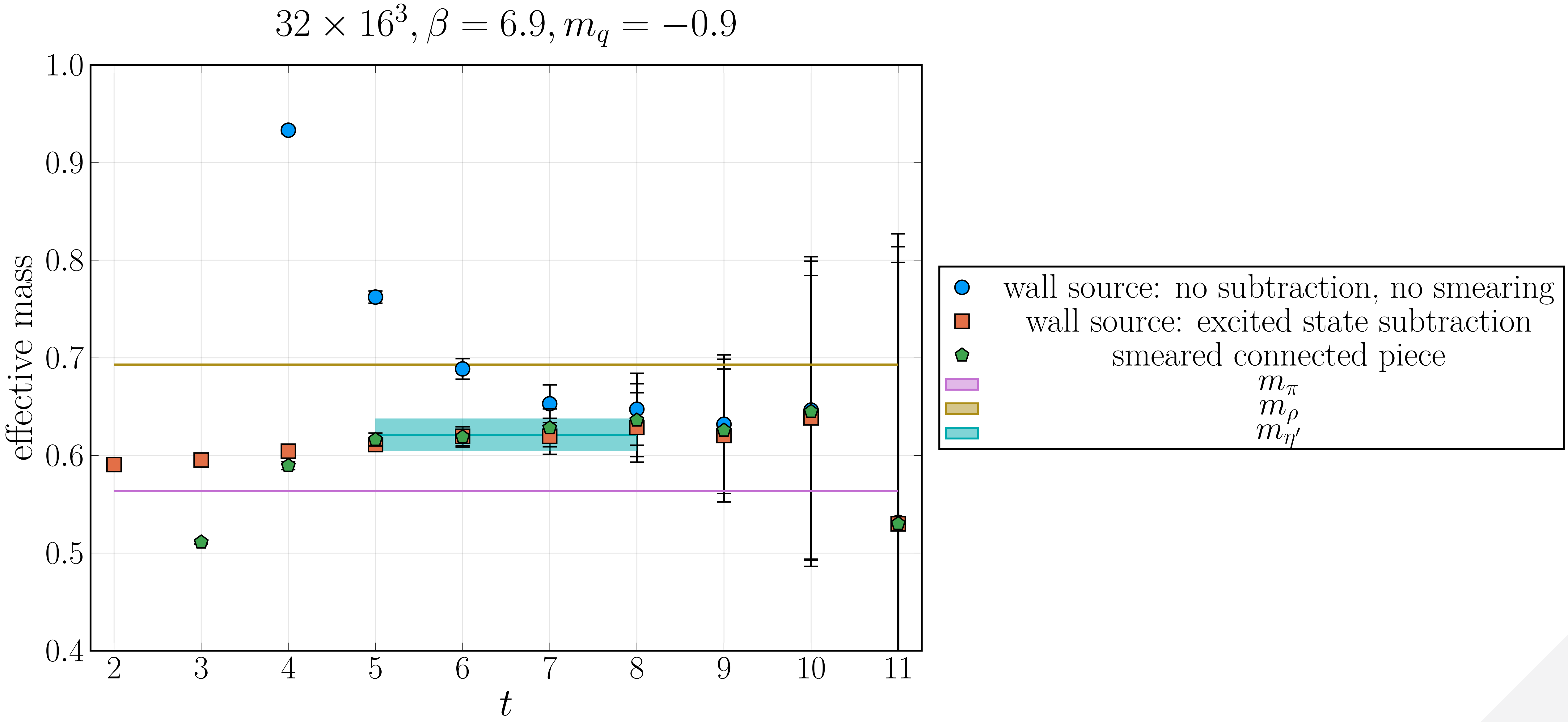
$U(1)'$ breaking parameter κ against dark photon mass m_V




$Sp(4)$ with degenerate fermions - data from 1909.12662
Relative non-singlet meson masses
Goldstone mass in lattice units ω
pseudoscalar
vector
scalar
axialvector
3.0
2.5
2.0
1.5
1.0
0.4
0.5
0.6
0.7
0.8
0.9

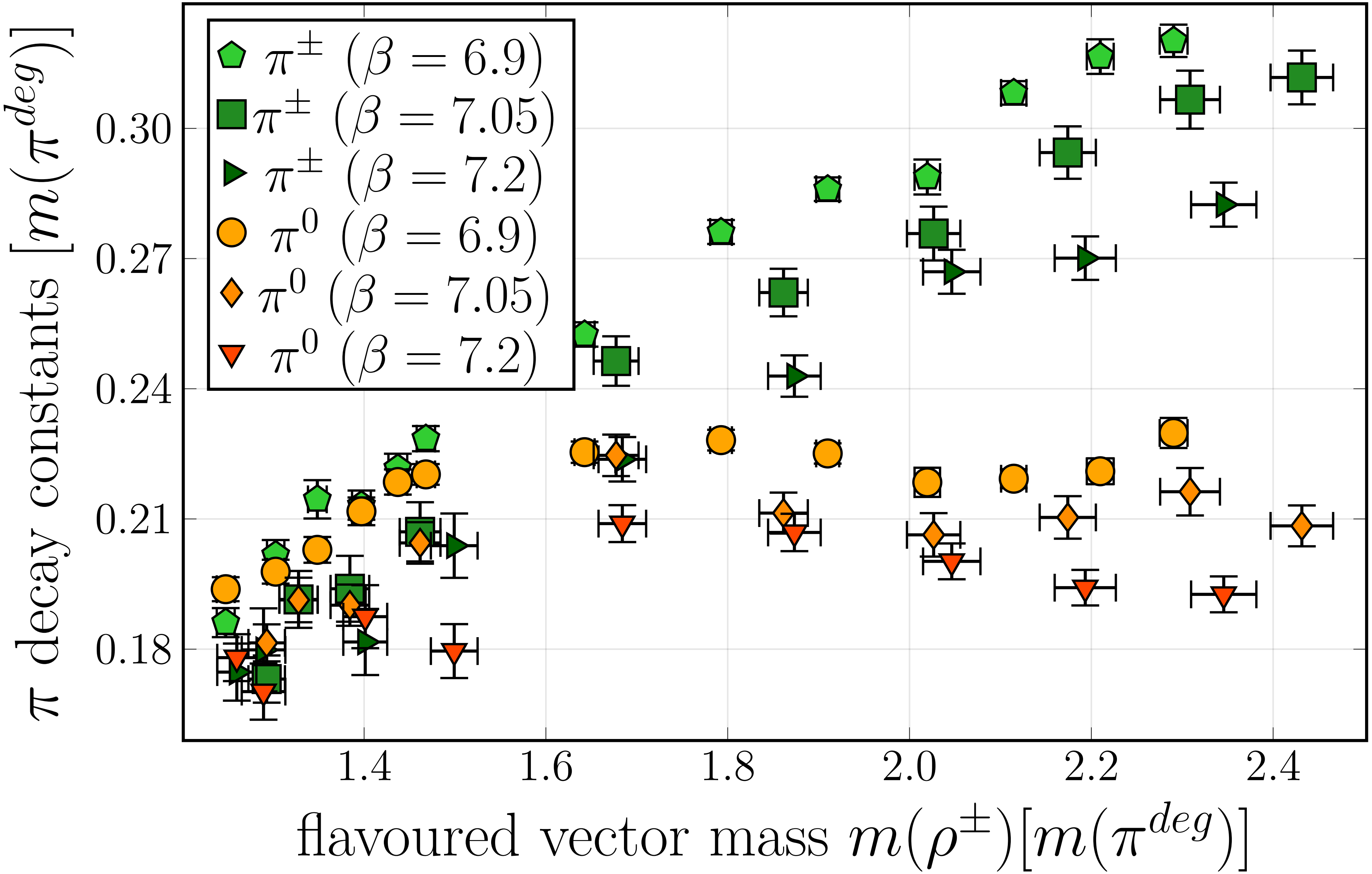
$$\beta{=}6.9 \qquad \left(\frac{m(\rho)}{m(\pi)}\right)_{deg} = 1.143(5)$$





$32 \times 16^3, \beta = 6.9, m_q = -0.9$
effective mass
t
1.0
0.9
0.8
0.7
0.6
0.5
0.4
2
3
4
5
6
7
8
9
10
11
wall source: no subtraction, no smearing
wall source: excited state subtraction
smeared connected piece
m_π
m_ρ
$m_{\eta'}$

Different $\beta = 6.9, 7.05, 7.2$: $\left(\frac{m(\rho)}{m(\pi)}\right)_{deg}$=1.247(10) - 1.29(2)



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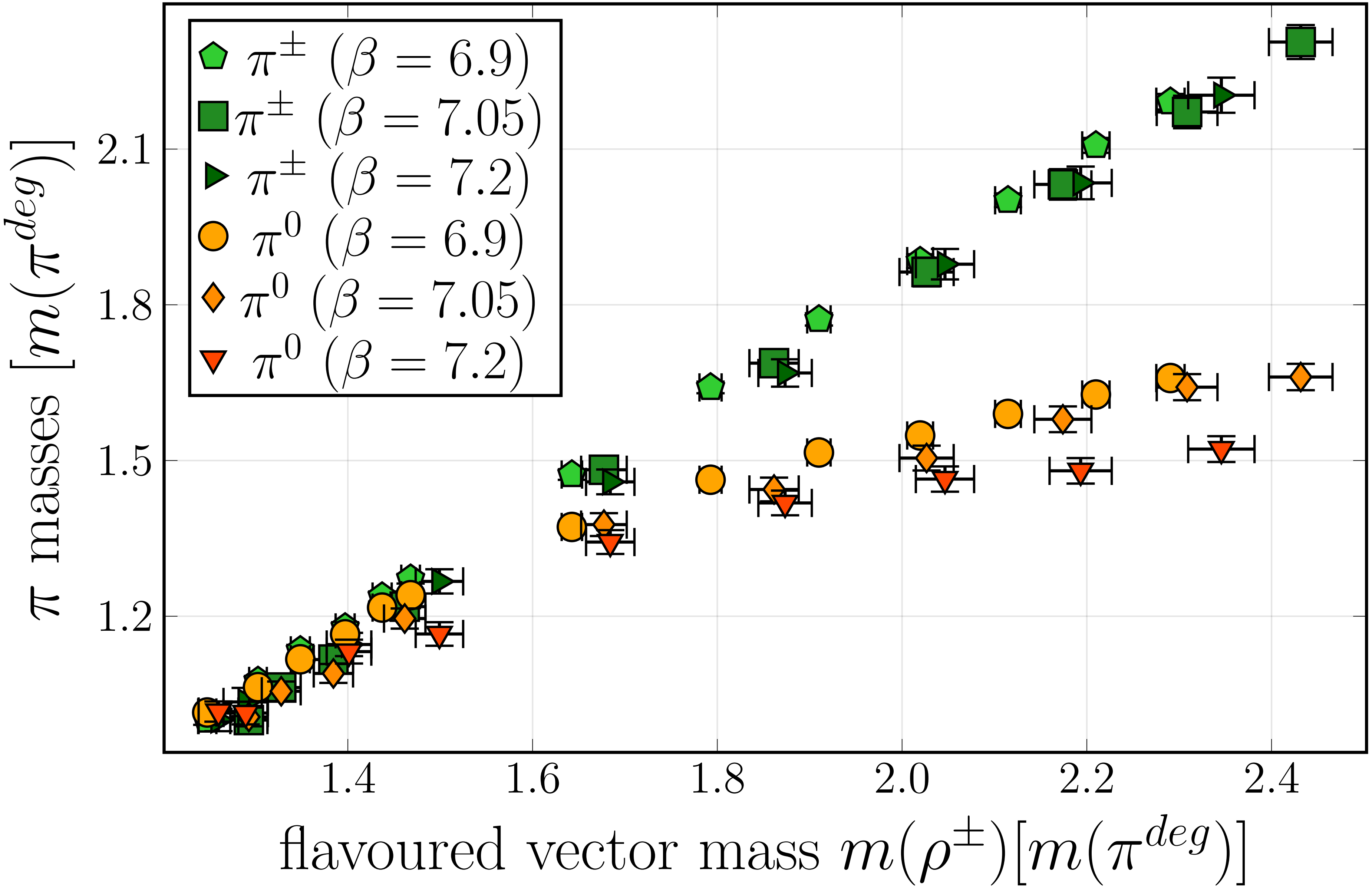

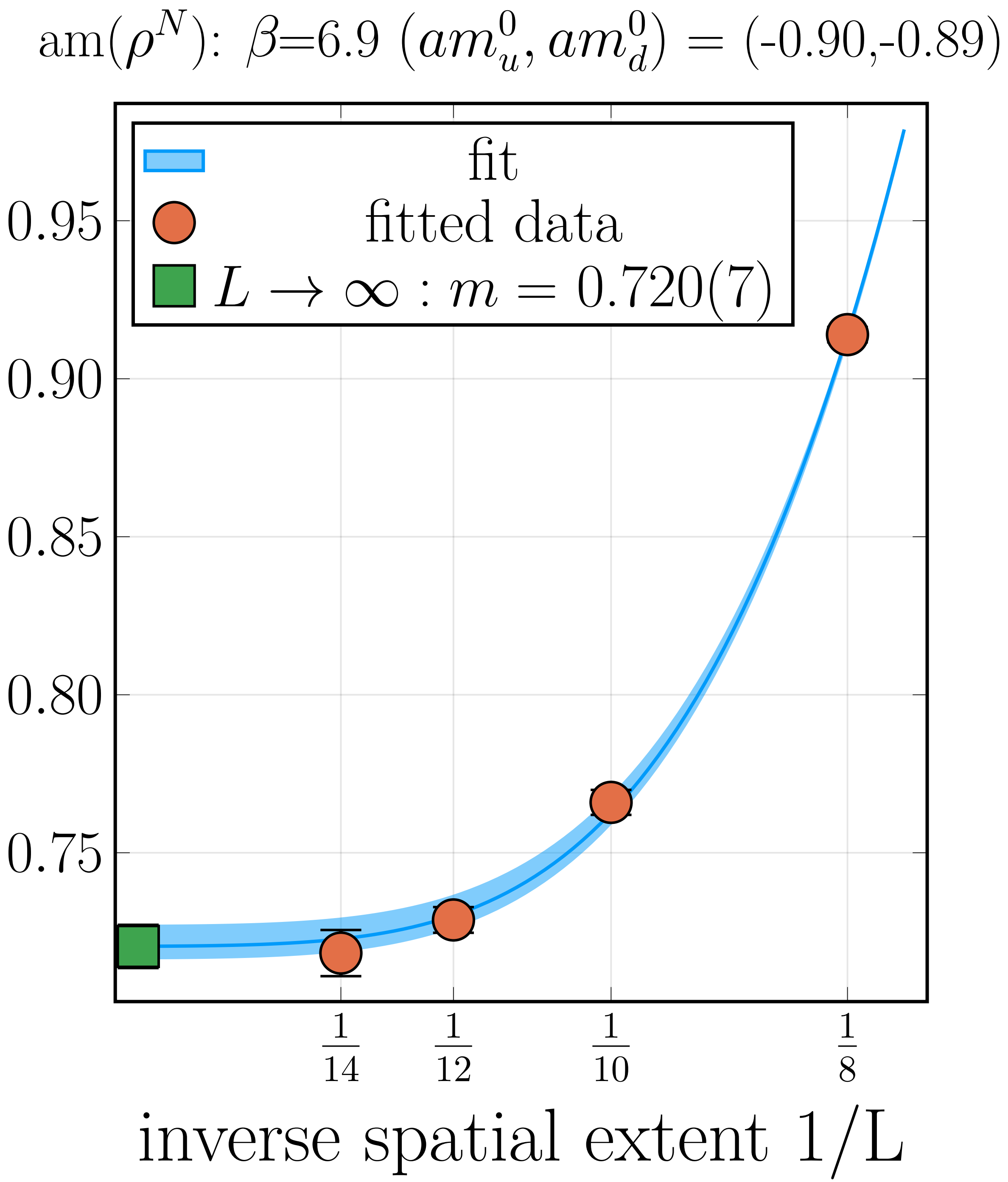

am(ρ^N): β=6.9 (am_u^0, am_d^0) = (-0.90,-0.89)
fit
fitted data
$L \to \infty : m = 0.720(7)$
0.95
0.90
0.85
0.80
0.75
$\frac{1}{14}$
$\frac{1}{12}$
$\frac{1}{10}$
$\frac{1}{8}$
inverse spatial extent 1/L

Hadronic multiplets of $Sp(4)_c : N_f = 1 + 1$

- Global flavour symmetry breaks: $Sp(4)_F \to SU(2) \times SU(2)$ [1]

$$10 \to 6 \text{ (unflavoured)} + 4 \text{ (flavoured)}$$
$$5 \;\; \to 1 \text{ (unflavoured)} + 4 \text{ (flavoured)}$$

- *Phenomenological consequences*

$\Rightarrow$ one π is a singlet $\to$ not protected by symmetry

$\Rightarrow$ no vector singlets: no mixing of vector mediators (without further symmetry breaking)

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Singlets are particularly interasting for BSM phenomenology!
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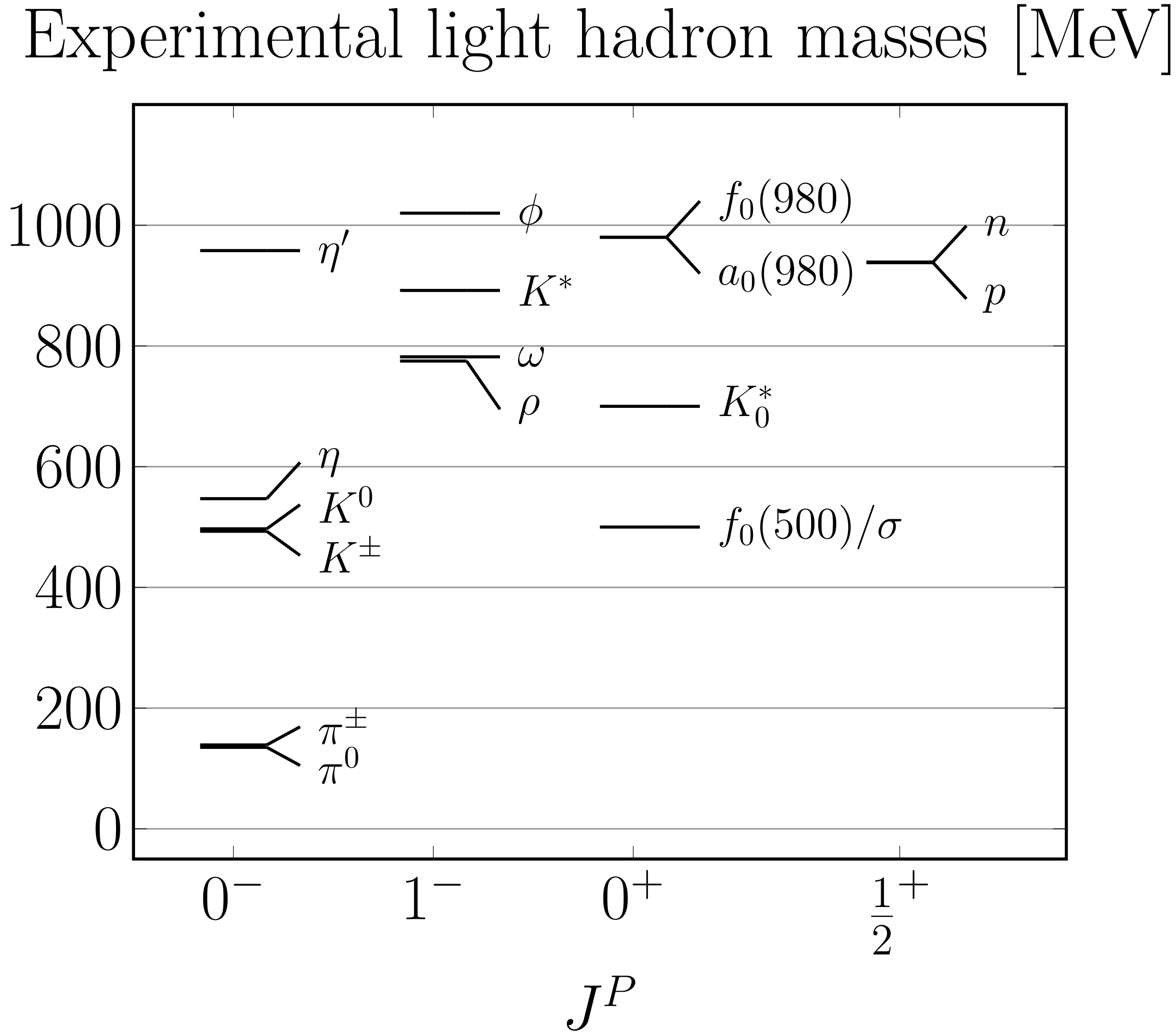
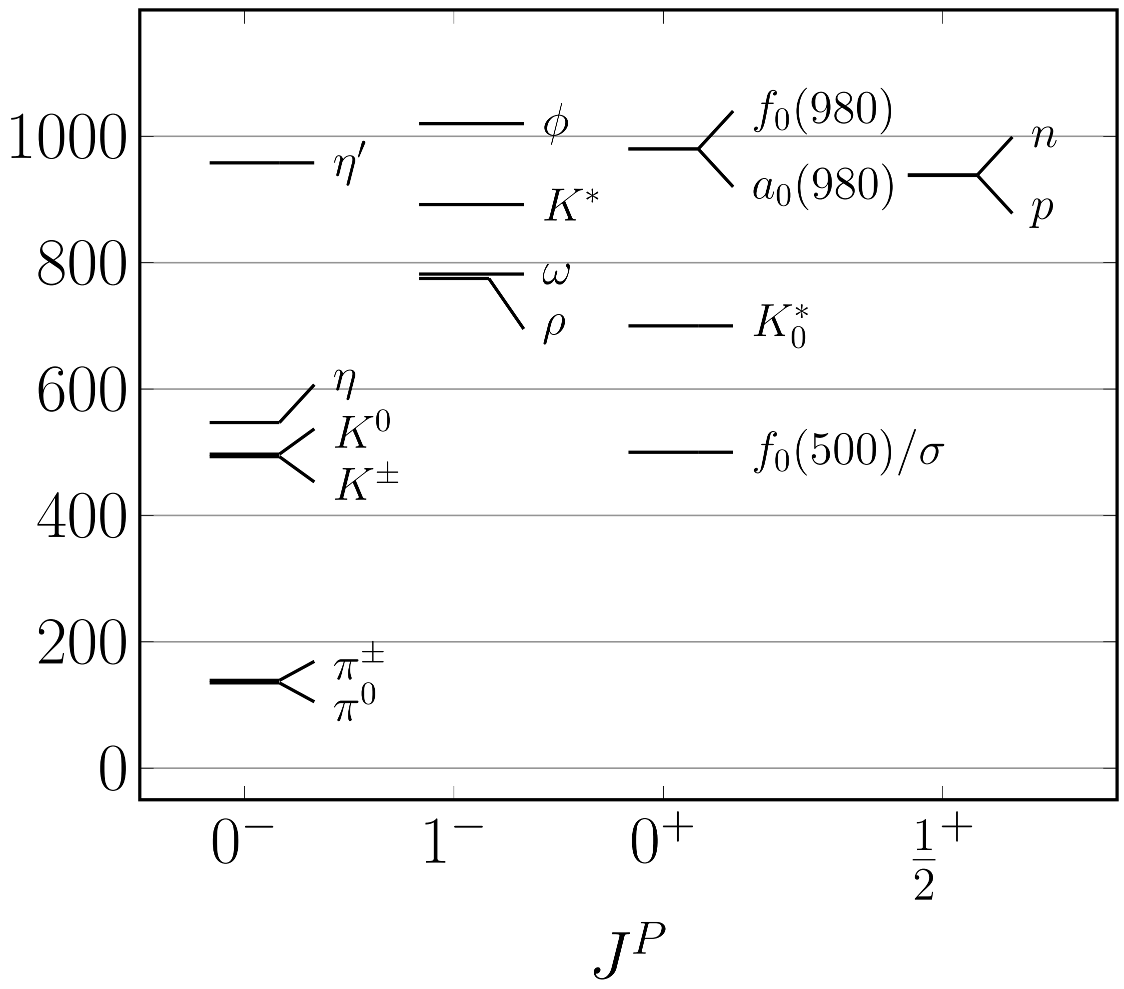


QCD Spectrum

- π, K, η light: pseudo-Goldstones
- Vectors and scalars light
- Light and broad 0^+ singlet f_0/σ
- Heavy 0^- singlet η'
  $\Rightarrow U(1)_A$ anomalously broken

Mesonic multiplets in QCD



- Approximate $SU(2)_F$, $SU(3)_F$
- Mesons in irreps:
  - $SU(2)_F$: triplets and singlets
  - $SU(3)_F$: octets and singlets

$$2 \otimes 2 = 3 \oplus 1$$

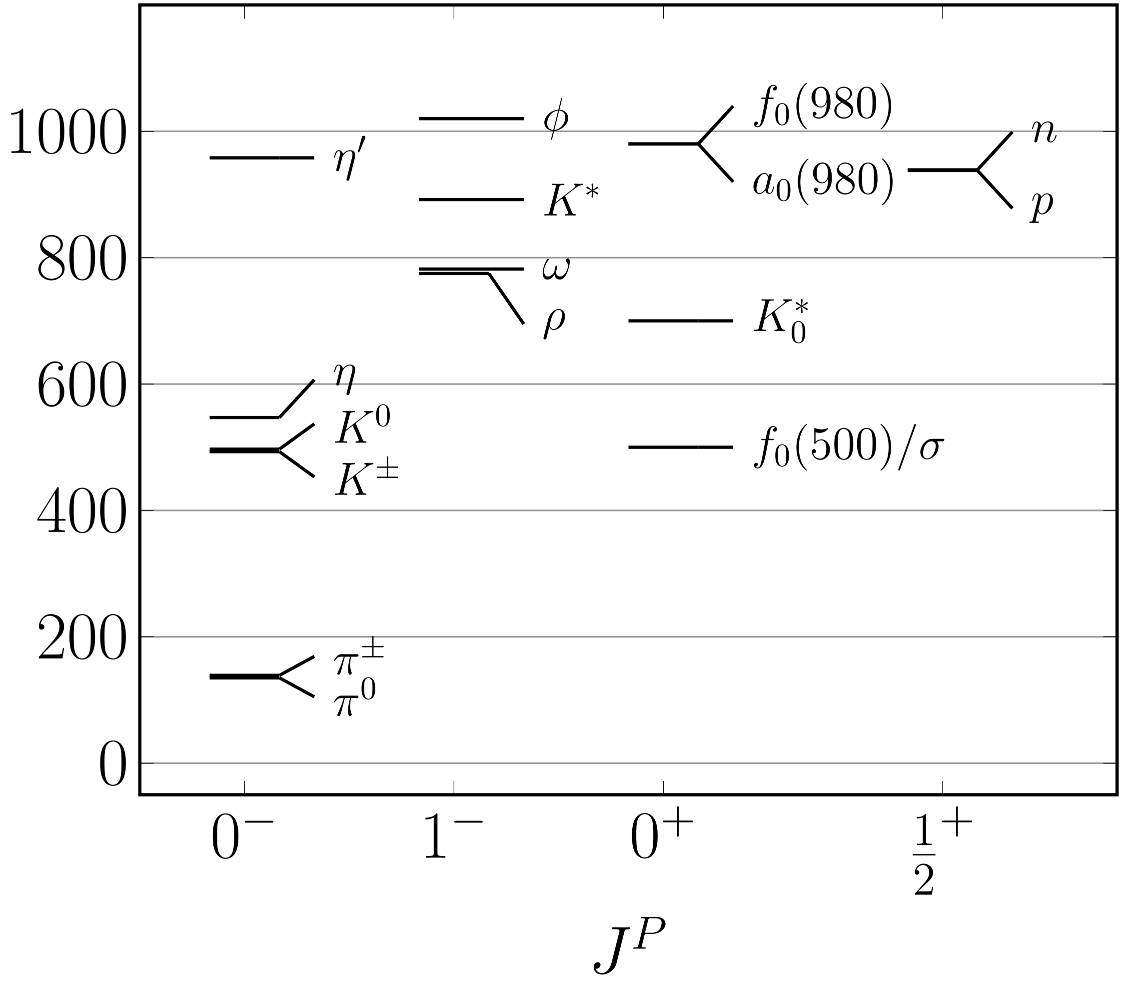
$$\bar{3} \otimes 3 = 8 \oplus 1$$

$$\text{octets} : (\pi, K, \eta), (\rho, K^*, \omega), \ldots$$

$$\text{triplets} : (\pi^\pm, \pi^0), (\rho^\pm, \rho^0), \ldots$$

$$\text{singlets} : \eta', f_0/\sigma, \phi, \ldots$$

Experimental light hadron masses [MeV]



Hadronic naming scheme

- dark fermions/quarks: u and d
- mesons named after equivalent QCD state: e.g. dark pions:

$$\pi^+ = \bar{d}\gamma_5 u \quad \pi^- = \bar{u}\gamma_5 d$$

$$\pi^0 = \bar{d}\gamma_5 d - \bar{u}\gamma_5 u$$

$$\eta' = \bar{d}\gamma_5 d + \bar{u}\gamma_5 u$$

The minimal Sp(4) SIMP model

$$\mathcal{L} = \mathcal{L}_{SM} + \mathcal{L}_{Sp(4)} + \mathcal{L}_{\text{mediator}}$$

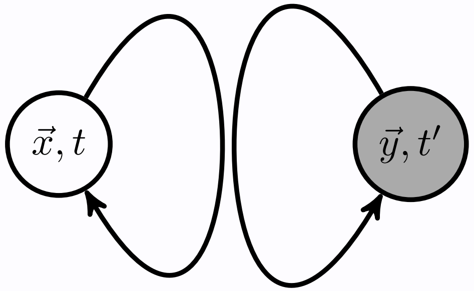
- $Sp(4)$ with $N_f = 2$ has exactly 5 Goldstones
- Dark hadrons DM candidates $\rightarrow$ non-perturbative
- Low energy effective theory (EFT) needed

- **Combine the methods with lattice field theory**
  - Derive low energy EFT for dark sector + mediator
  - Low energy constants (LECs) from lattice
  - Use EFT for astro/collider/direct detection pheno

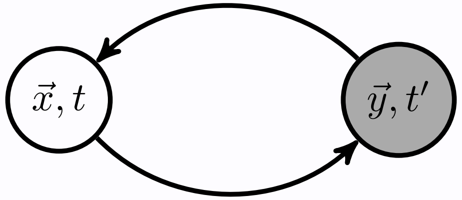
Calculating the meson correlator

- Evaluate diagrams in terms of fermion propagator D^{-1}

$$C(t-t') = \sum_{\vec{x},\vec{y}} \Bigg($$

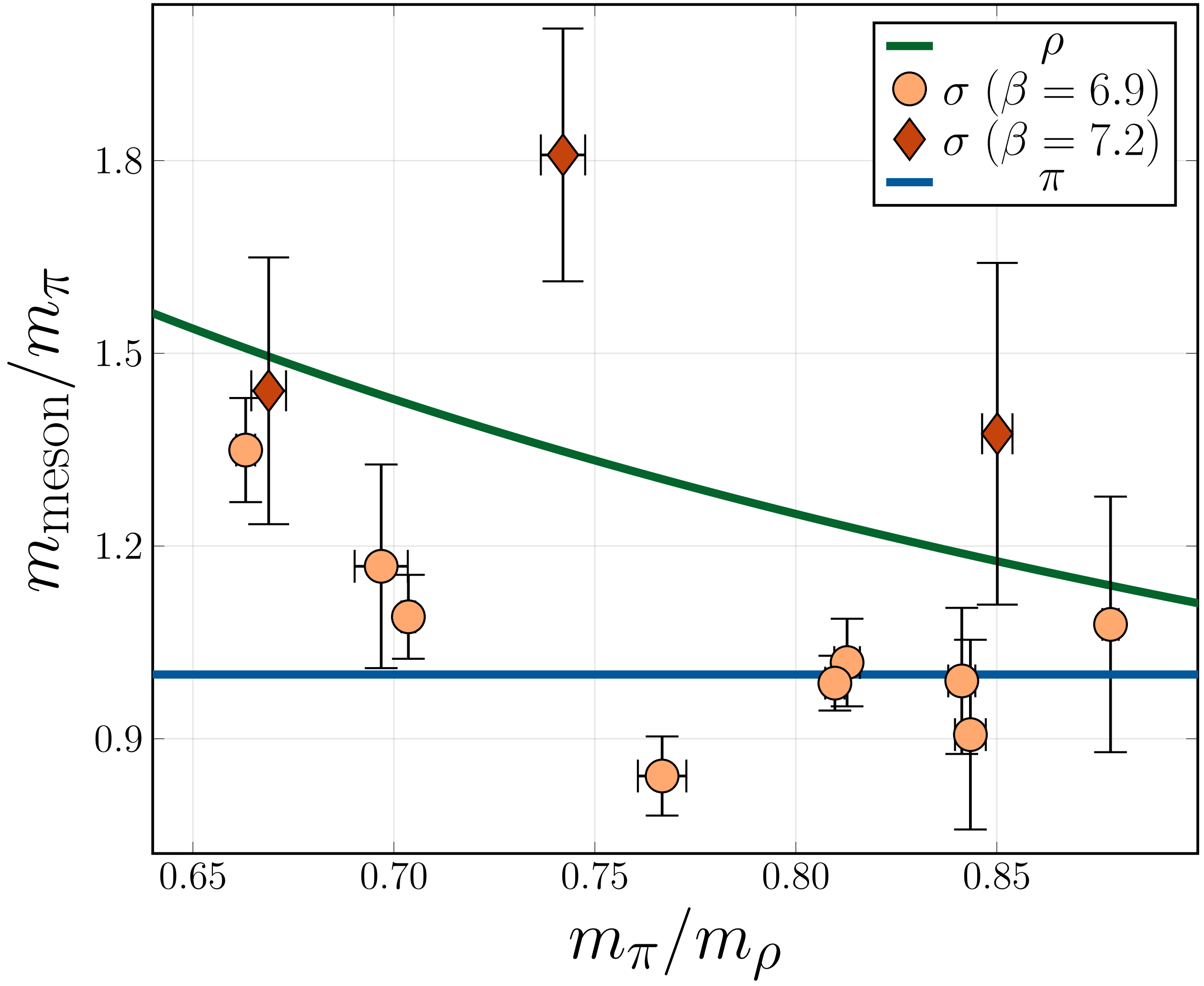


$$+$$



$$\Bigg) + \underbrace{\text{const.}}_{=|\langle 0|O|0\rangle|^2}$$

- Disconnected diagram (left) particularly challenging
  - only appears for singlets (gluonic propagation)
- Constant term arises for singlets
  - vacuum term for σ, fixed topological charge for η'



The scalar singlet σ

- Strong finite spacing effects!
- Potentially light state below $2m_\pi$ threshold
- Unclear systematics

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Overall, inconclusive results.
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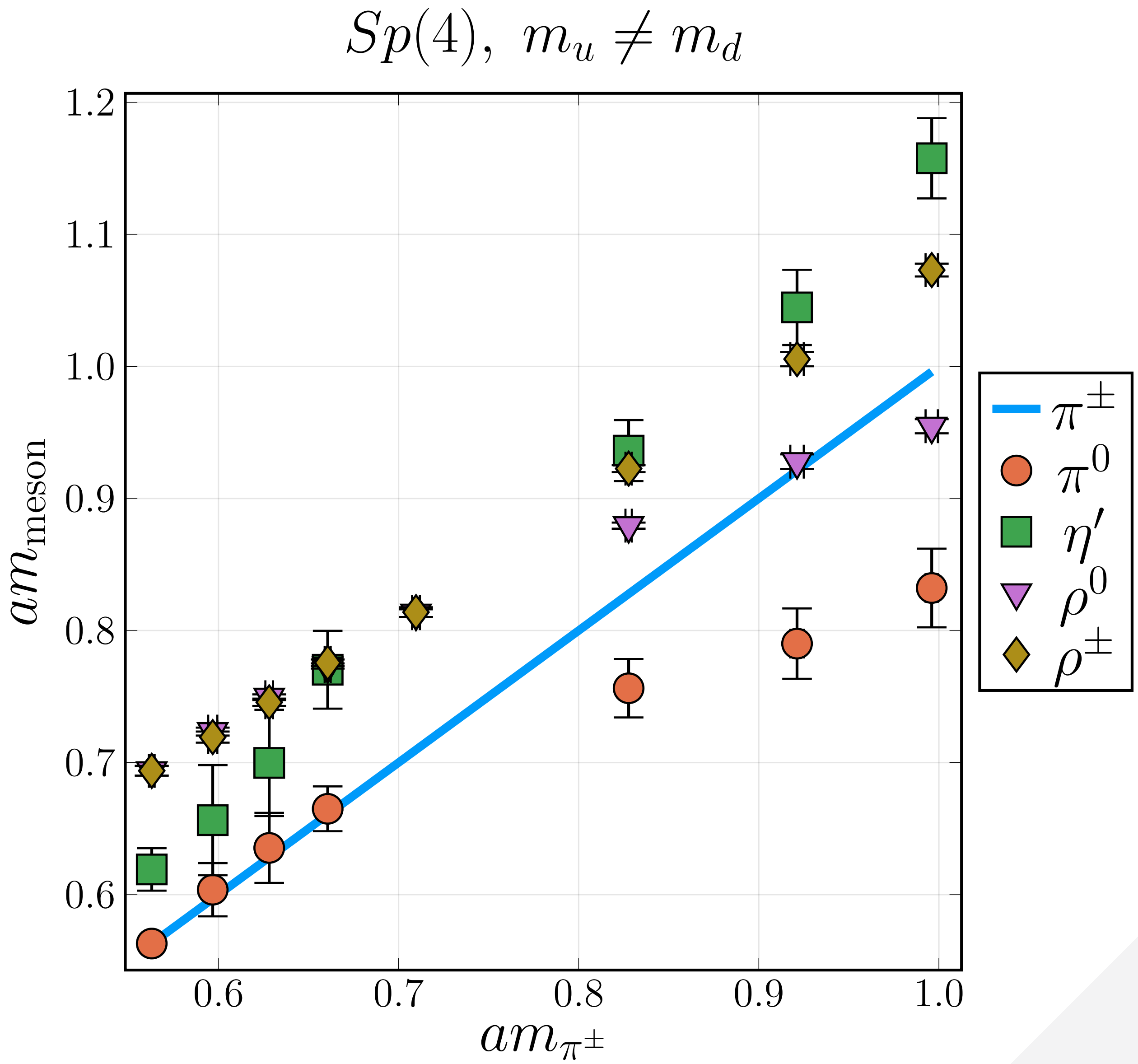
[1] Bennett et. al. [2304.07191]

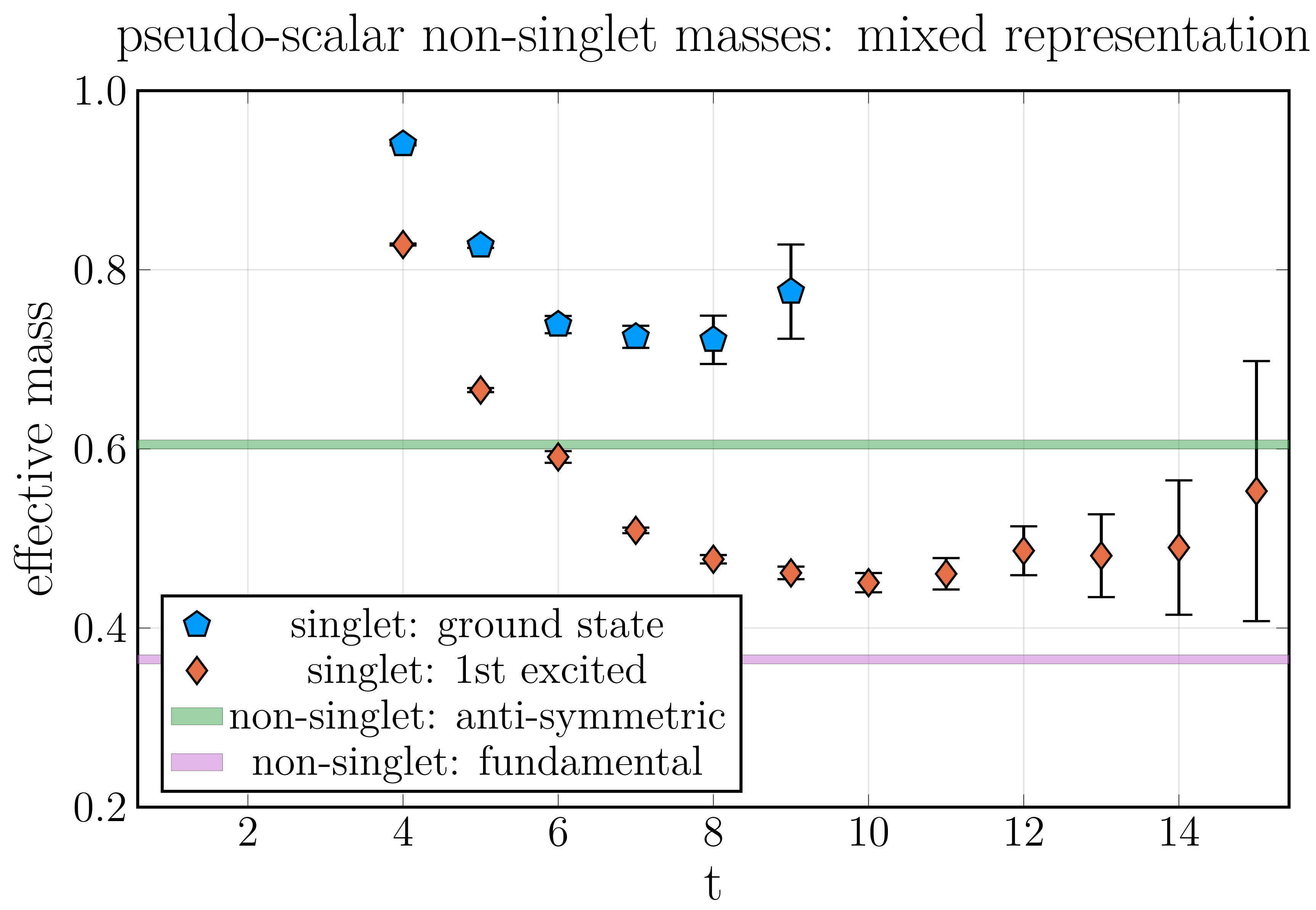
Kulkarni et. al. [2202.05191]

Non-degenerate fermions

- Different mass hierarchies
- Transition from a flavour-symmetric theory to a heavy-light system
- One light and one heavy pseudoscalar flavour-singlet



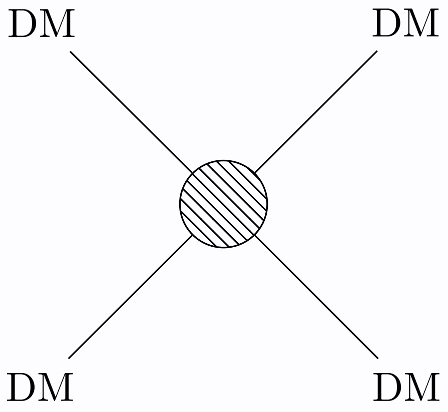
Pseudoscalar (non-)singlets: mixed representation



[1] see e.g. Bullock,Boylan-Kolchin [1707.04256], Tulin, Yu [1705.02358]

Dark Matter properties

- DM self-interaction phenomenologically allowed[1] and potentially relevant for small-scale structure problems
    - non-vanishing scattering cross-sections $\sigma_{2\mathrm{DM}\to 2\mathrm{DM}}$
    - velocity dependence of $\sigma_{2\mathrm{DM}\to 2\mathrm{DM}}$ preferred

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QCD-like Dark Matter can those provide self-interactions!
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Dark meson scattering: Determine DM relic density

- Any model must predict the current density of DM correctly
  - number density n can be calculated using Boltzmann equations

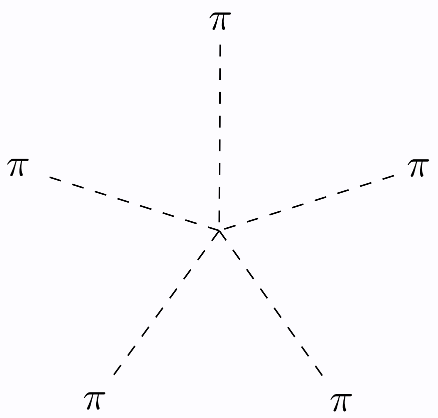
$$\partial_t n + 3Hn = f\left(\langle v\sigma_{\text{number changing}}\rangle\right)$$

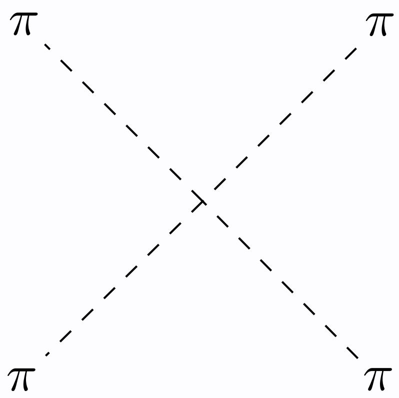
- Cross-sections $\langle \sigma v \rangle$ are input for Boltzmann equations
  - describe non-equilibrium dynamics
  - H is the Hubble rate

[1] Hochberg et. al. [1402.5143] [2] Tulin, Yu [1705.02358], [3] Kondo et.al. [2205.08088] [4] Chu et.al. [2401.12283]

Relevant pion scattering channels

- $3\pi \to 2\pi$ (semi-annihilation) [1]
- $2\pi \to 2\pi$ (self-scattering)
  - self-scattering among DM [2]
  - resonant enhancements [3]
- $2n\pi \to 2\pi$ (multi-hadron bound states) [4]





Strongly Interacting Massive Particles (SIMPs)

- Depletion via $3\mathrm{DM} \to 2\mathrm{DM}$ [1], i.e. $3\pi \to 2\pi$
  - same as $KK \to 3\pi$ in QCD [2]
  - Early universe: $\mathrm{SM} \rightleftharpoons \mathrm{DM}$ equilibrium
  - Dark matter depletion process: ***freeze-out***
- LO ChiPT matches relic density at

$$m_\pi \approx \mathcal{O}(100)\mathrm{MeV} - \mathcal{O}(1)\mathrm{GeV}$$

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Dark Matter with 3DM → 2DM depletion and self-interactions
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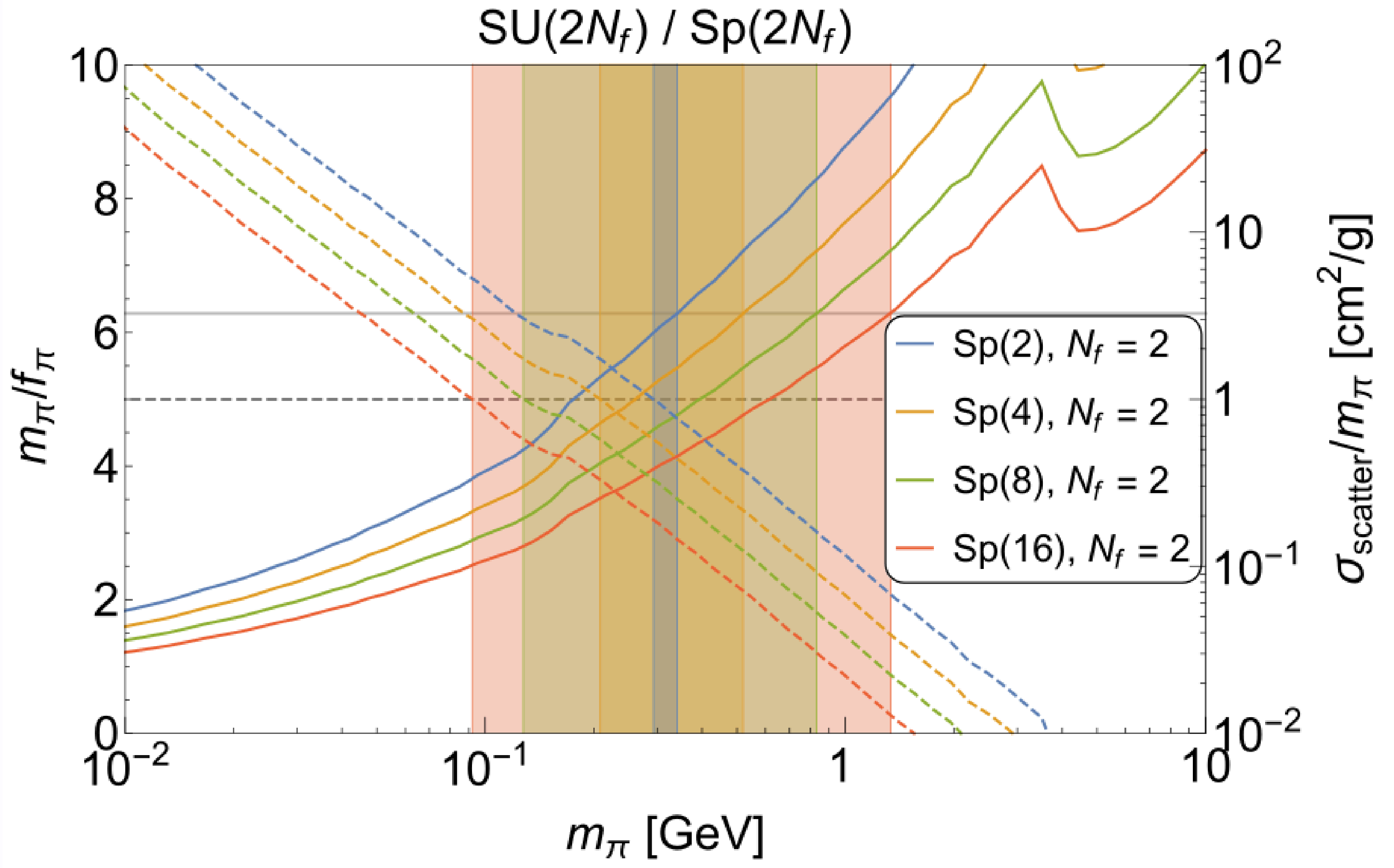
SIMP model at leading order in ChiPT



Relevant channels and EFT descriptions

- decay to Standard Model: $2\pi \to SM$ [1]
- involvement of vector mesons: $\pi\pi \to \pi\rho$, $3\pi \to \pi\rho$ [2]
- influence of light singlets: $\eta'\eta' \to \pi\pi, \pi\pi \to \eta'\pi, \ldots$ [3]

- The relevance depends on the spectrum
  - investigation of the meson spectrum important
  - lattice investigations inform EFT construction

Lagrangian of $Sp(4)_c$ with fermions

$$\mathcal{L}_{Sp(4)} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \sum_{f=u,d} \bar{\psi}_f (i \not{D} + m_f) \psi_f$$

- Higher symmetry than QCD-like theories

$$\Psi = \begin{pmatrix} u_L \\ d_L \\ -SCu_R^* \\ -SCd_R^* \end{pmatrix} = \begin{pmatrix} u_L \\ d_L \\ \tilde{u}_R \\ \tilde{d}_R \end{pmatrix}$$

C ... charge conj.
S ... colour matrix

$$\mathcal{L}_{Sp(4)} = i\bar{\Psi} \not{D} \Psi - \frac{1}{2} \left(\Psi^T SCM \Psi + h.c. \right) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

- generators τ_a in fundamental repr. : $S \tau_a S = -\tau_a^T$
- mass matrix M proportional to symplectic invariant

Meson multiplets of $Sp(4)_c$ with $N_f = 2$

- $Sp(2N_f)$ flavour symmetry between $2N_f$ Weyl components
- Extra gauge invariant states: $q^T \ldots q$ and $\bar{q} \ldots \bar{q}^T$

$$Sp(4)_F: \qquad 4 \otimes 4 = 1 \oplus 5 \oplus 10$$

```
The global symmetries lead to a richer meson multiplet structure!
```

Extra meson states:

Diquarks and Anti-Diquarks

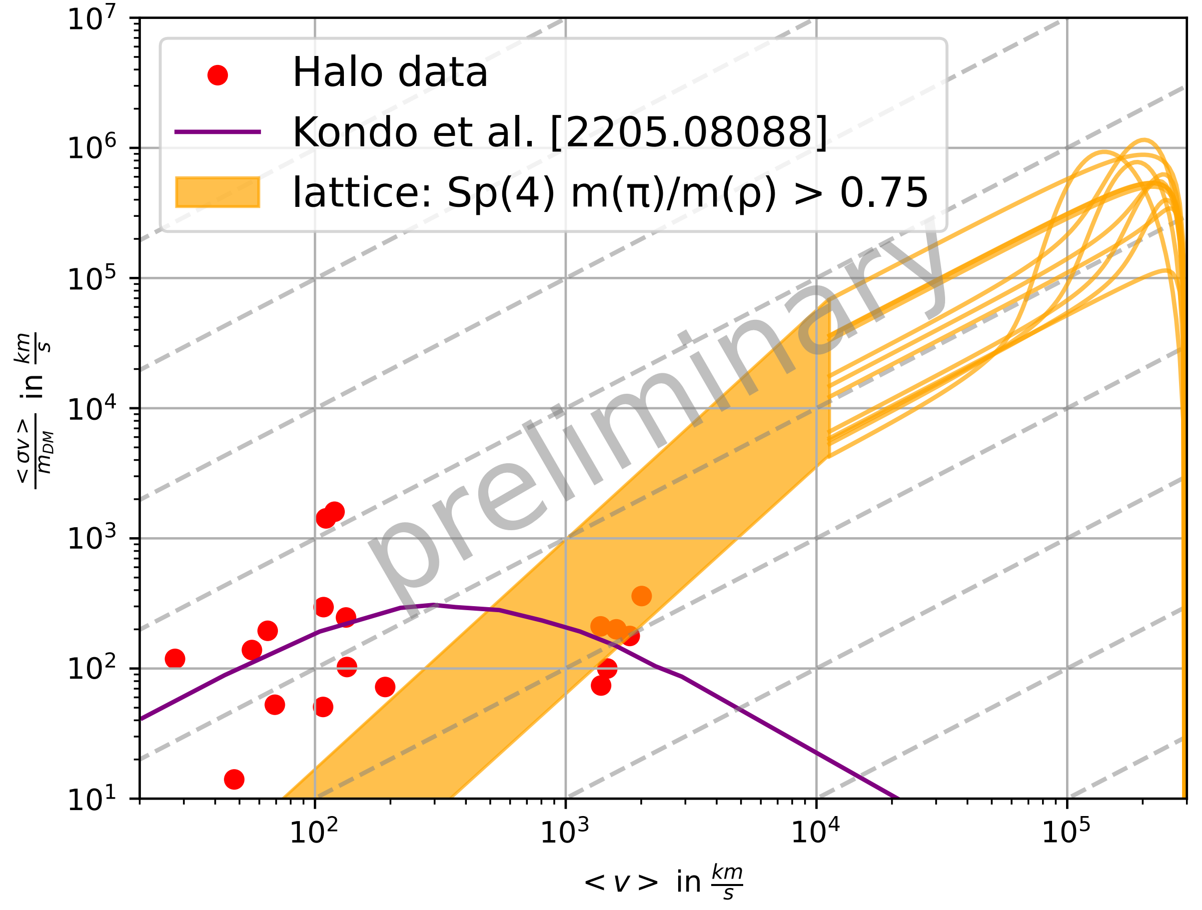
$$\pi_1 : \quad \bar{u}\gamma_5 d$$

$$\pi_2 : \quad \bar{d}\gamma_5 u$$

$$\pi_3 : \quad \frac{1}{\sqrt{2}}\left(\bar{u}\gamma_5 u - \bar{d}\gamma_5 d\right)$$

$$\pi_4 : \quad \bar{d}\gamma_5 S C \bar{u}^T$$

$$\pi_5 : \quad d^T S C \gamma_5 u$$



First investigation of isospin-2 scattering

- phase shift $\delta(p)$ gives velocity dependence $\langle \sigma v \rangle$
- compare with best fits to experimental halo data
- No velocity dependence in this channel!

Other mass scales than QCD are relevant!

- Lagrangian has two free parameters: g^2 and m_f
  - one overall energy scale
  - one scale for explicit chiral symmetry breaking
- Overall scale should allow sufficiently heavy DM
- m_f should lead to parametrically light m_π
  - both scales can deviate strongly from QCD!

```
Lattice investigations of a larger parameter space are useful!
```

Composite Higgs + Top Realisations

G_{HC}	ψ	χ	Restrictions	G/H
$\mathrm{SO}(N_{\mathrm{HC}})$	$5\times\mathbf{F}$	$6\times\mathbf{Spin}$	$N_{\mathrm{HC}}=7,9$	$\frac{\mathrm{SU}(5)}{\mathrm{SO}(5)}\frac{\mathrm{SU}(6)}{\mathrm{SO}(6)}\mathrm{U}(1)$
$\mathrm{SO}(N_{\mathrm{HC}})$	$5\times\mathbf{Spin}$	$6\times\mathbf{F}$	$N_{\mathrm{HC}}=7,9$	
$\mathrm{Sp}(2N_{\mathrm{HC}})$	$5\times\mathbf{A}_2$	$6\times\mathbf{F}$	$2N_{\mathrm{HC}}=4$	$\frac{\mathrm{SU}(5)}{\mathrm{SO}(5)}\frac{\mathrm{SU}(6)}{\mathrm{Sp}(6)}\mathrm{U}(1)$
$\mathrm{SU}(N_{\mathrm{HC}})$	$5\times\mathbf{A}_2$	$3\times(\mathbf{F},\overline{\mathbf{F}})$	$N_{\mathrm{HC}}=4$	$\frac{\mathrm{SU}(5)}{\mathrm{SO}(5)}\frac{\mathrm{SU}(3)\times\mathrm{SU}(3)'}{\mathrm{SU}(3)_D}\mathrm{U}(1)$
$\mathrm{SO}(N_{\mathrm{HC}})$	$5\times\mathbf{F}$	$3\times(\mathbf{Spin},\overline{\mathbf{Spin}})$	$N_{\mathrm{HC}}=10$	
$\mathrm{Sp}(2N_{\mathrm{HC}})$	$4\times\mathbf{F}$	$6\times\mathbf{A}_2$	$2N_{\mathrm{HC}}=4$	$\frac{\mathrm{SU}(4)}{\mathrm{Sp}(4)}\frac{\mathrm{SU}(6)}{\mathrm{SO}(6)}\mathrm{U}(1)$
$\mathrm{SO}(N_{\mathrm{HC}})$	$4\times\mathbf{Spin}$	$6\times\mathbf{F}$	$N_{\mathrm{HC}}=11$	
$\mathrm{SO}(N_{\mathrm{HC}})$	$4\times(\mathbf{Spin},\overline{\mathbf{Spin}})$	$6\times\mathbf{F}$	$N_{\mathrm{HC}}=10$	$\frac{\mathrm{SU}(4)\times\mathrm{SU}(4)'}{\mathrm{SU}(4)_D}\frac{\mathrm{SU}(6)}{\mathrm{SO}(6)}\mathrm{U}(1)$
$\mathrm{SU}(N_{\mathrm{HC}})$	$4\times(\mathbf{F},\overline{\mathbf{F}})$	$6\times\mathbf{A}_2$	$N_{\mathrm{HC}}=4$	
$\mathrm{SU}(N_{\mathrm{HC}})$	$4\times(\mathbf{F},\overline{\mathbf{F}})$	$3\times(\mathbf{A}_2,\overline{\mathbf{A}}_2)$	$N_{\mathrm{HC}}=5,6$	$\frac{\mathrm{SU}(4)\times\mathrm{SU}(4)'}{\mathrm{SU}(4)_D}\frac{\mathrm{SU}(3)\times\mathrm{SU}(3)'}{\mathrm{SU}(3)_D}\mathrm{U}(1)$

Table 6. Subclass of models that is likely to be outside of the conformal window, together with the coset they give rise to after spontaneous symmetry breaking.