# Not Testing ``Entanglement” or ``Locality via Bell’s Inequality” at Colliders

**Herbi Dreiner**

Bethe Center for Theoretical Physics

Physikalisches Institut

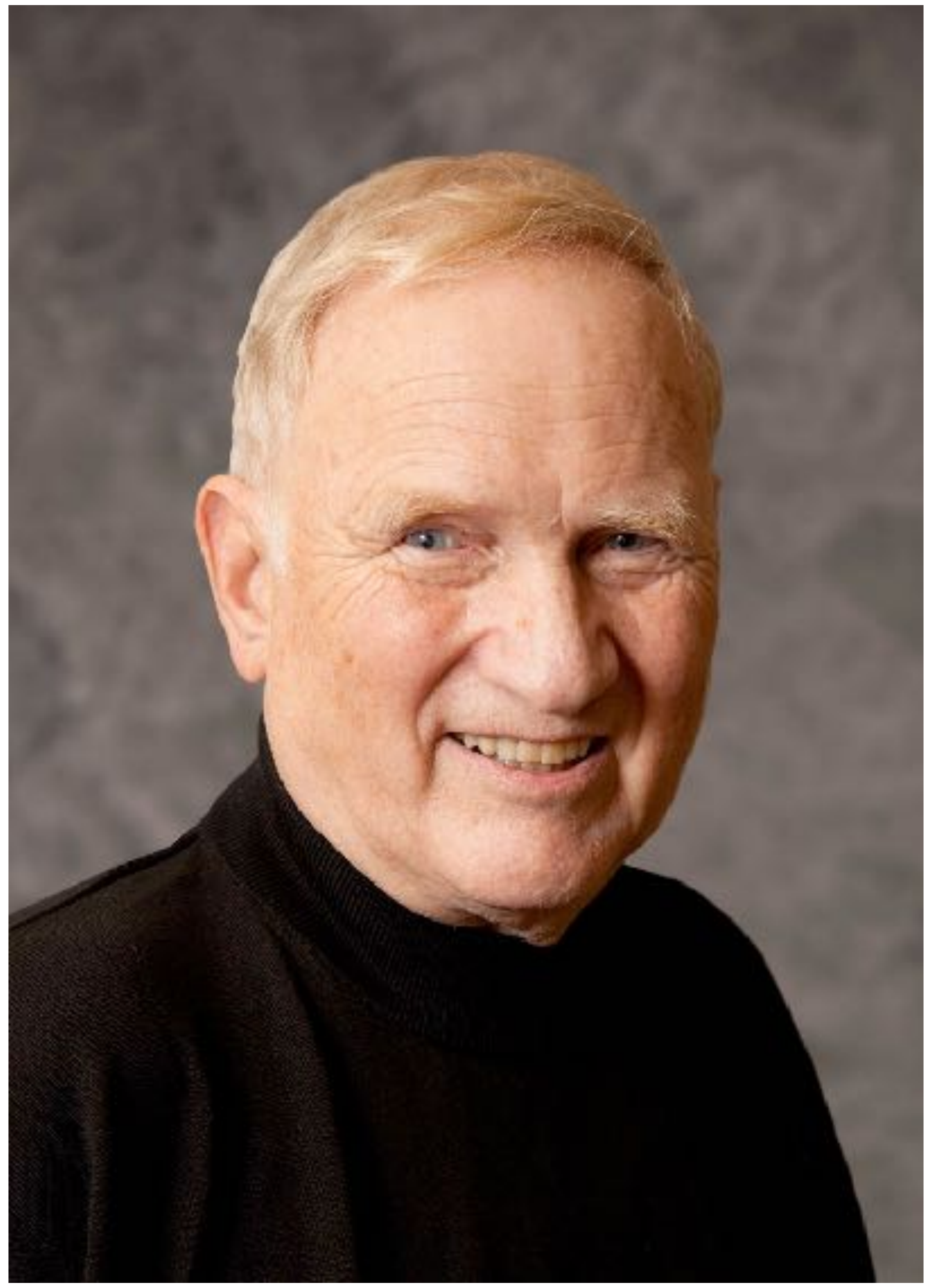
Universität Bonn

**Durham, Annual Christmas Meeting, Dec. 17th, 2024**

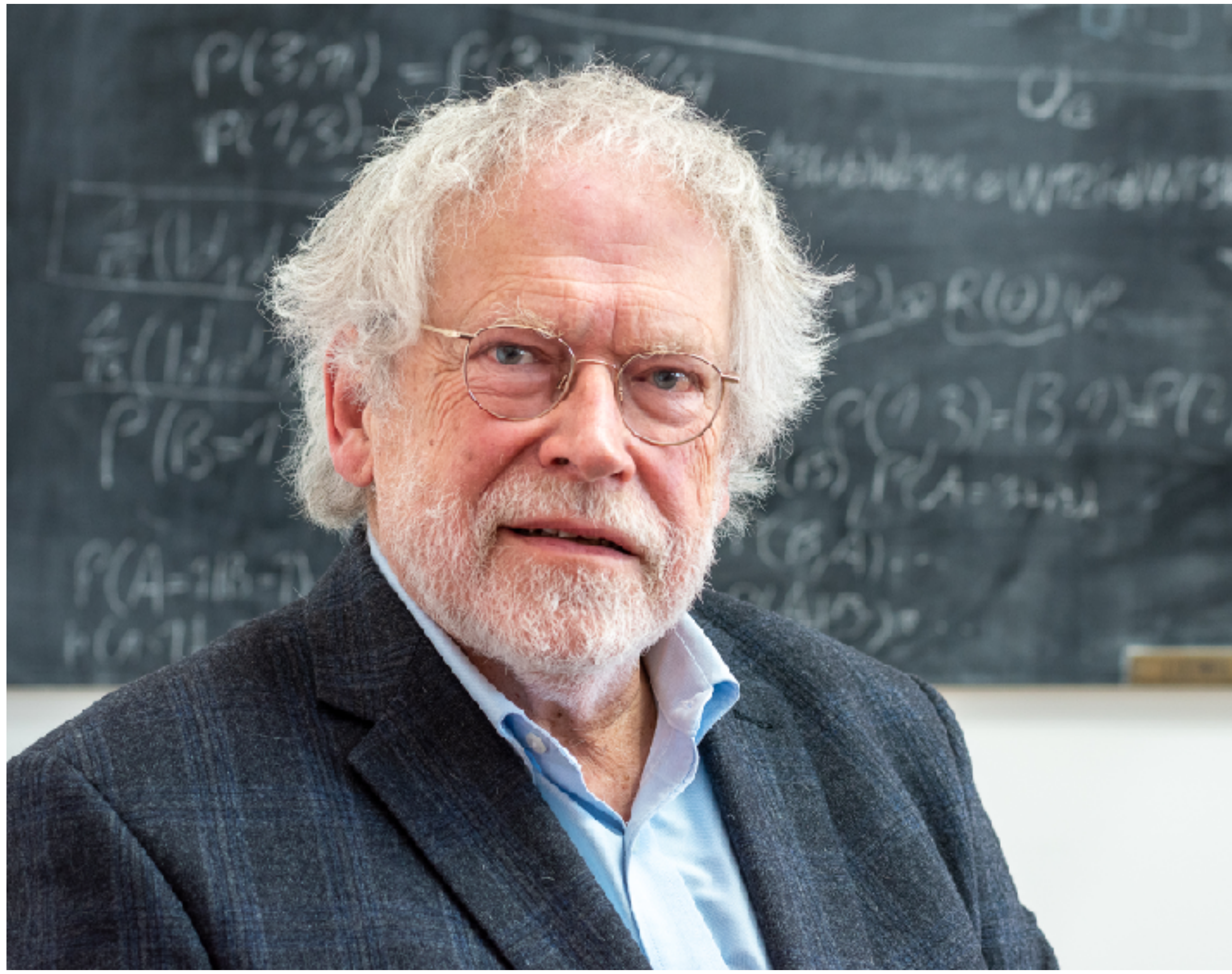
Work in collaboration with: Rhitaja Sengupta, Lorenzo Ubaldi, Steve Abel, and Michael Dittmar

# Nobel Prize for Physics — 2022

Alain Aspect

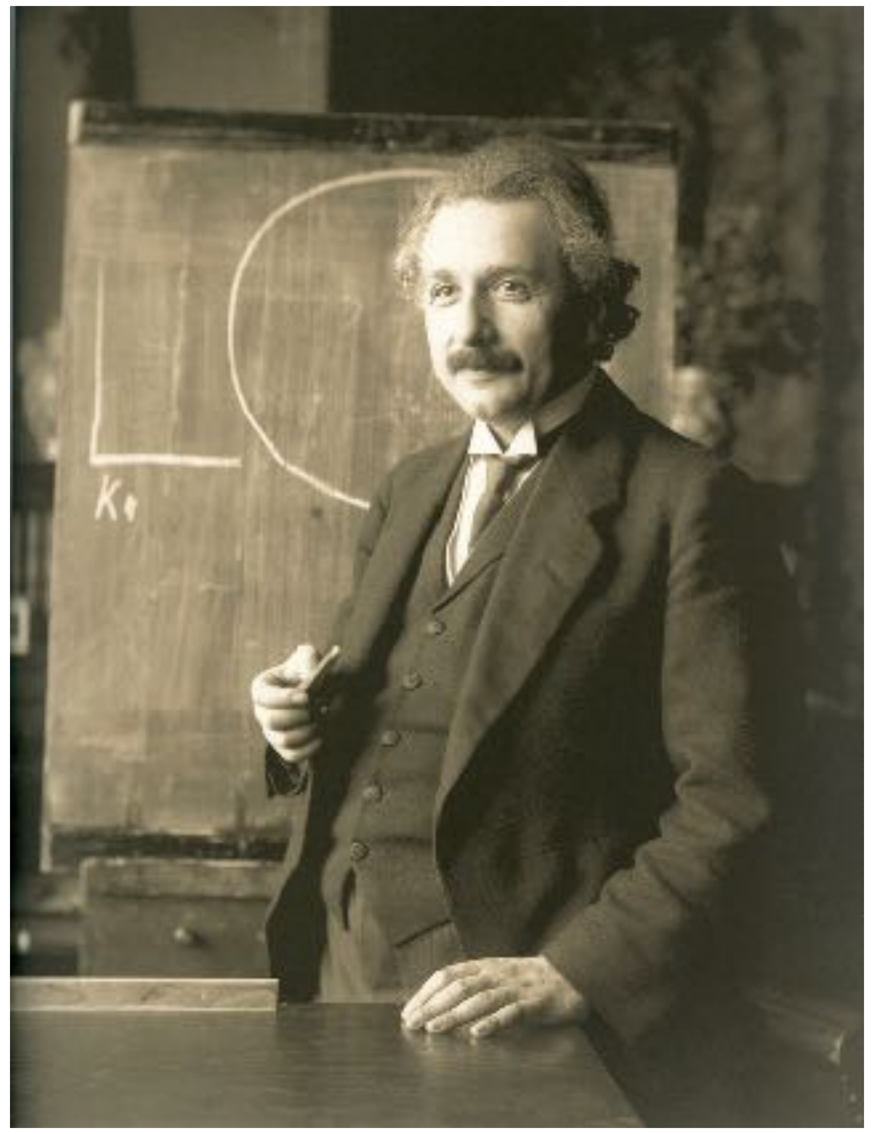


John Clauser



Anton Zeilinger

# Further important People



Albert Einstein

Wikipedia, public domain

David Bohm

https://www.theosophy-nw.org/theosnw/science/sc-pruyn.htm

John Stewart Bell



**Bell's Inequality**

modified Bohm,
Aharonov experiment

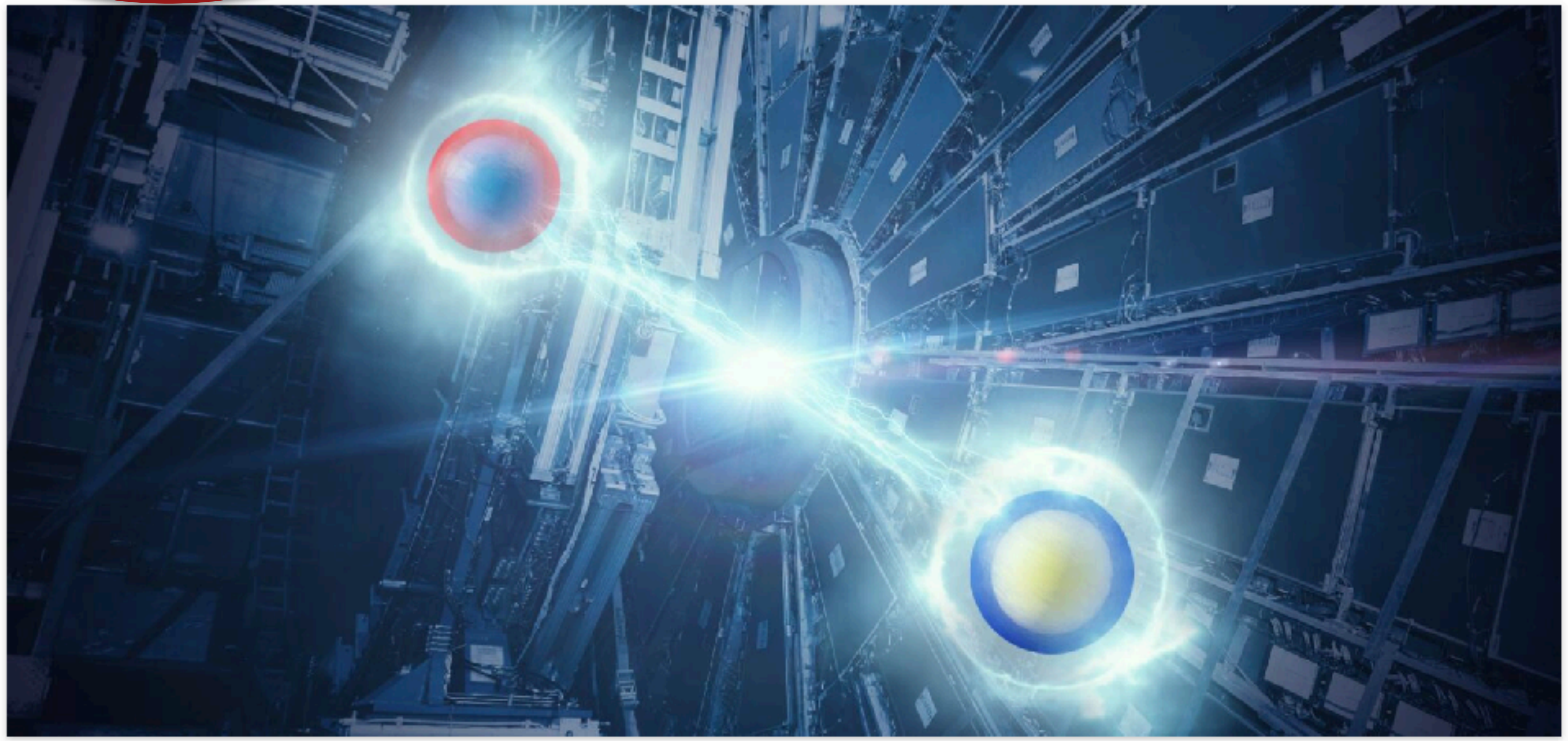
- Brand aktuell: kann man Verschränkung am LHC testen?

*Voir en français*

# LHC experiments at CERN observe quantum entanglement at the highest energy yet

**The results open up a new perspective on the complex world of quantum physics**

18 SEPTEMBER, 2024



Artist's impression of a quantum-entangled pair of top quarks. (Image: CERN)

Quantum entanglement is a fascinating feature of quantum physics – the theory of the very small. If two particles are quantum-entangled, the state of one particle is tied to that of the other, no matter how far apart the particles are. This mind-bending phenomenon, which has no analogue in classical physics, has been

18.9.2024

Bell's inequality directly connected to entanglement

# Outline

- Review Bell's Paper from 1964
  - Bell's inequality for entangled spin-½ pair
- Review our paper (Dittmar, HD, Abel) from 1992
  - Considered $Z^0 \longrightarrow \tau^+\tau^- \longrightarrow (\pi^+\bar{\nu}_\tau)(\pi^-\nu_\tau)$ at LEP
- Apply to $t\bar{t}$ production at the LHC

$$t \longrightarrow b + \ell^+ + \nu_\ell$$

- Conclusion: can not test locality via Bell's inequality or entanglement at colliders

Physics Vol. 1, No. 3, pp. 195–200, 1964 Physics Publishing Co. Printed in the United States

# ON THE EINSTEIN PODOLSKY ROSEN PARADOX*

J. S. BELL†
*Department of Physics, University of Wisconsin, Madison, Wisconsin*

(*Received* 4 *November* 1964)

## I. Introduction

THE paradox of Einstein, Podolsky and Rosen [1] was advanced as an argument that quantum mechanics could not be a complete theory but should be supplemented by additional variables. These additional variables were to restore to the theory causality and locality [2]. In this note that idea will be formulated mathematically and shown to be incompatible with the statistical predictions of quantum mechanics. It is the requirement of locality, or more precisely that the result of a measurement on one system be unaffected by operations on a distant system with which it has interacted in the past, that creates the essential difficulty. There have been attempts [3] to show that even without such a separability or locality requirement no "hidden variable" interpretation of quantum mechanics is possible. These attempts have been examined elsewhere [4] and found wanting. Moreover, a hidden variable interpretation of elementary quantum theory [5] has been explicitly constructed. That particular interpretation has indeed a grossly non-local structure. This is characteristic, according to the result to be proved here, of any such theory which reproduces exactly the quantum mechanical predictions.

## II. Formulation

With the example advocated by Bohm and Aharonov [6], the EPR argument is the following. Consider a pair of spin one-half particles formed somehow in the singlet spin state and moving freely in opposite directions. Measurements can be made, say by Stern-Gerlach magnets, on selected components of the spins $\vec{\sigma}_1$ and $\vec{\sigma}_2$. If measurement of the component $\vec{\sigma}_1 \cdot \vec{a}$, where $\vec{a}$ is some unit vector, yields the value

Physics Vol. 1, No. 3, pp. 195–200, 1964 Physics Publishing Co. Printed in the United States

I completed my PhD there 25 years later

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My b'day, I was exactly 2 yrs old

60th anniversary just passed

## I. Introduction

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# What did Bell say?


$A(\vec{a}, \lambda) = \pm 1$

$B(\vec{b}, \lambda) = \pm 1$

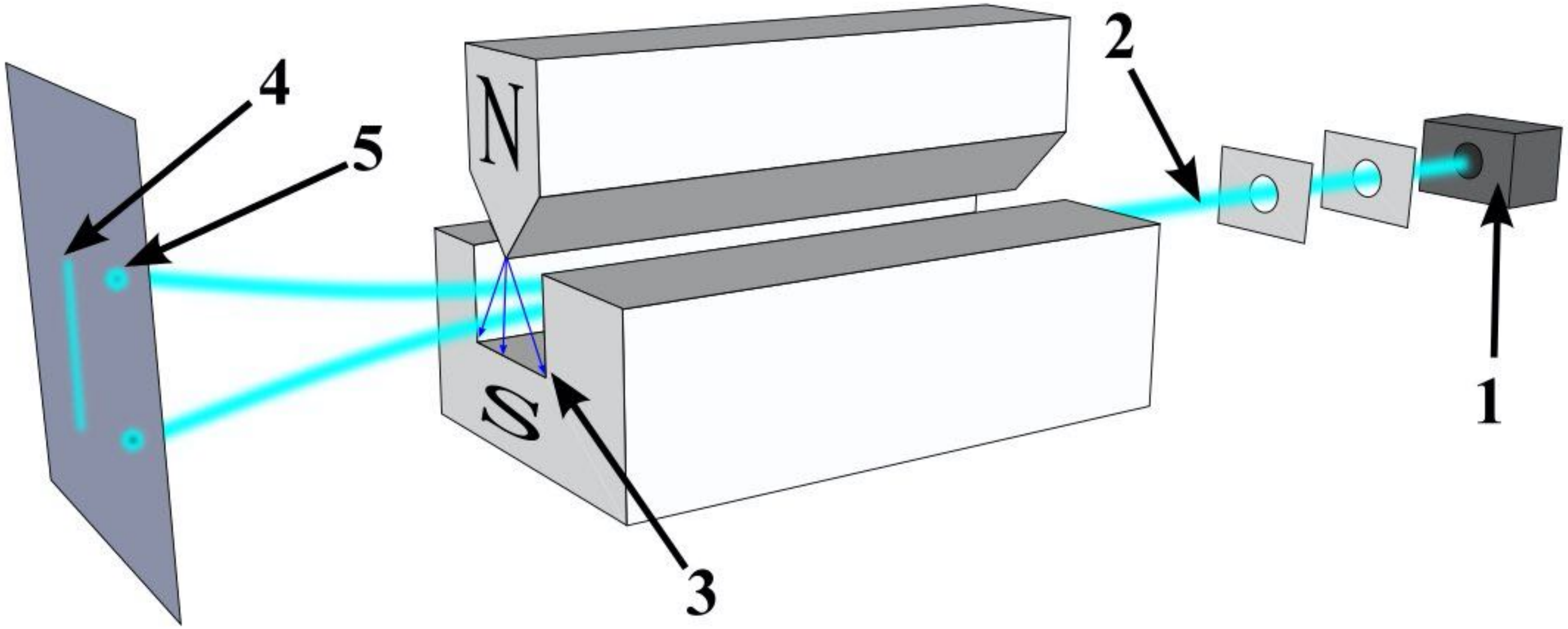
- Consider two spin-½ particles flying apart, spins anti-correlated

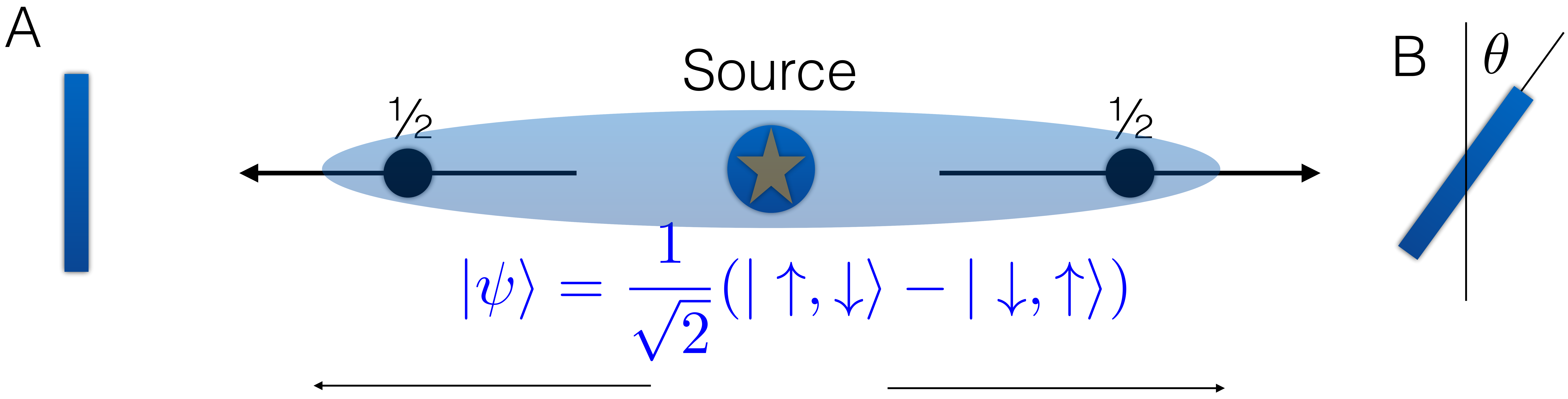
QM: $$|\psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow,\downarrow\rangle - |\downarrow,\uparrow\rangle)$$

$\vec{b} = \vec{a}$

- QM measurement of component: $\vec{\sigma}_A \cdot \vec{a}, \quad \pm 1 \Longrightarrow \vec{\sigma}_B \cdot \vec{a}, \quad \mp 1$

# Stern-Gerlach




A
Source
½
½
B
$\theta$
$|\psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow,\downarrow\rangle - |\downarrow,\uparrow\rangle)$

# QM Expectation

$$|\vec{a}| = |\vec{b}| = 1$$

- QM expectation value: $\langle \vec{\sigma}_1 \cdot \vec{a} \;\; \vec{\sigma}_2 \cdot \vec{b} \rangle = -\vec{a} \cdot \vec{b} = -\cos\theta$

- States are **entangled** — information connected despite separation

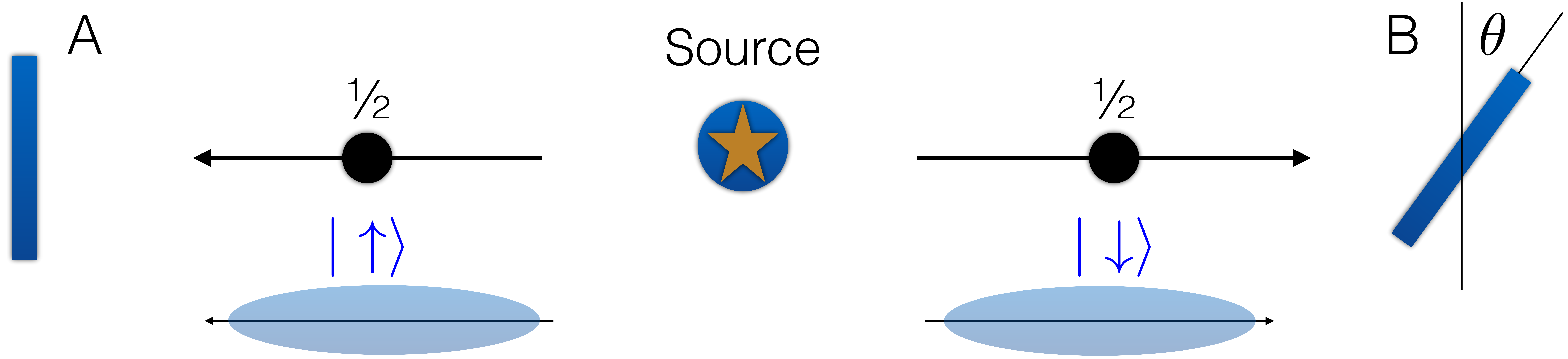
- State on the left is only determined AFTER measurement on the right (assuming measurement on the right is first)

# What was Bell interested in?


$$\mathrm{A}(\vec{a},\lambda) = \pm 1$$

$$\mathrm{B}(\vec{b},\lambda) = \pm 1$$

- Can this be described by a local hidden variable theory (LHVT)?
- What is a LHVT? — “Common sense”
- Measurements at A and B are independent of each other
- And the results as well




- Emitted at source with **definite** spin, just spins are anti-correlated
- If results are independent: a person or a machine can sit at source and toss out the pairs
- Definite result is achieved through additional, unknown, hidden variables

$$\lambda_i \to \lambda, \qquad \rho(\lambda) \quad \text{prob. dist.}$$

- Local Hidden Variable Theory (LHVT) expectation value:

$$P(\vec{a}, \vec{b}) = \int d\lambda \, \rho(\lambda) \, A(\vec{a}, \lambda) B(\vec{b}, \lambda)$$

$$\int d\lambda \, \rho(\lambda) = 1$$

prob. dist.

# Hidden Variable Theory of Spin

- No problem for LHVT to reproduce several special cases

  1) Spin measurement on a single particle $\vec{\sigma} \cdot \vec{a} \longrightarrow \mathrm{sign}(\vec{\lambda} \cdot \vec{a}')$

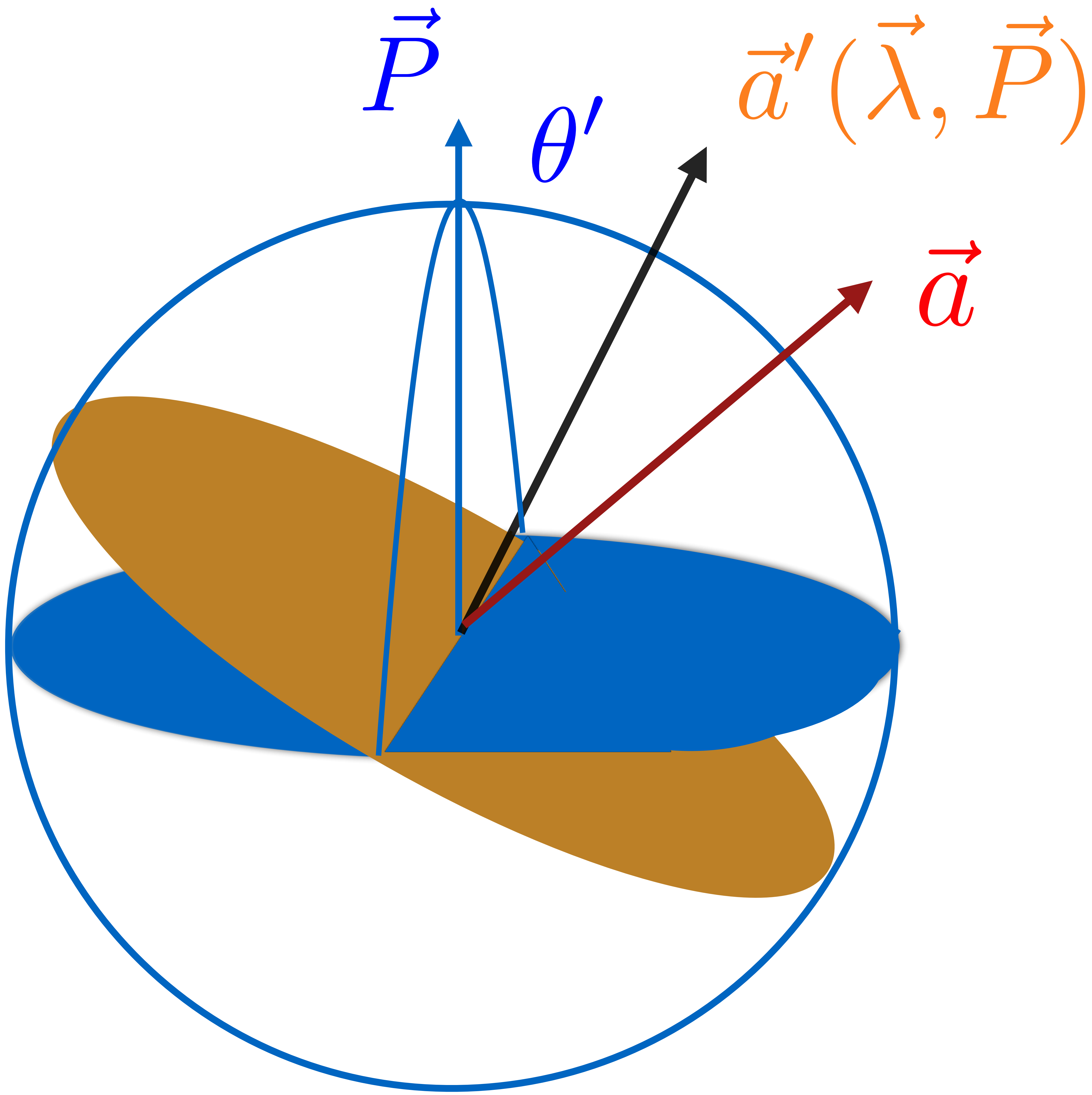
# Hidden Variable Account of Single Particle Spin Measurement

(Bell 1964)

- pure spin-½ state, polarization denoted by $\vec{P}, \quad |\vec{P}| = 1$
- let hidden variables be unit vector: $\vec{\lambda}$ , $\quad |\vec{\lambda}| = 1$
- with uniform distribution over hemisphere: $\lambda \cdot \vec{P} > 0$
- now measure spin component in direction $\vec{a}$ : $\quad \vec{\sigma} \cdot \vec{a}$
- result of measurement: $\longrightarrow \quad \text{sign}\, \vec{\lambda} \cdot \vec{a}'$

$$|\vec{a}'(\vec{\lambda}, \vec{P})| = 1\,, \qquad \vec{a}' \neq \vec{a}$$

# Hidden Variable Account of Single Particle Spin Measurement




$\theta$ : angle $\vec{P}$ and $\vec{a}$

avg. over $\vec{\lambda}$, expectation value (overlapping hemispheres)

$$\langle \vec{\sigma} \cdot \vec{a} \rangle = 1 - \frac{\theta'}{\pi}$$

Now rotate $\vec{a}'$ towards $\vec{a}$ until

$$1 - \frac{\theta'}{\pi} = \cos\theta$$

$$\Rightarrow \quad \langle \vec{\sigma} \cdot \vec{a} \rangle = \cos\theta$$

# Hidden Variable Theory of Spin

- No problem for LHVT to reproduce several special cases

1) Spin measurement on a single particle $\vec{\sigma} \cdot \vec{a} \longrightarrow \mathrm{sign}(\vec{\lambda} \cdot \vec{a}')$

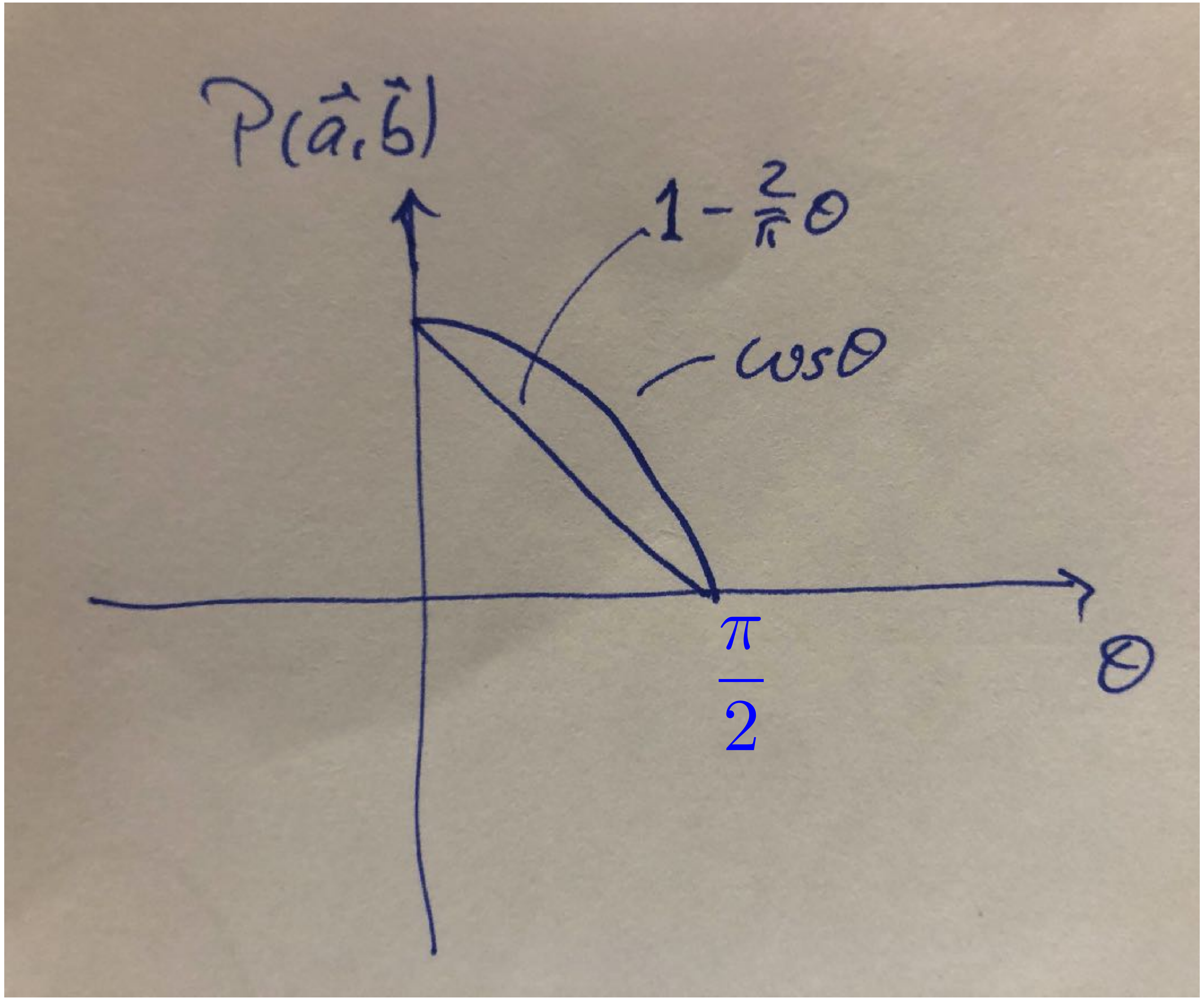
2) Using this in $P(\vec{a}, \vec{b})$ can easily reproduce simple cases

$$P(\vec{a}, \vec{a}) = -P(\vec{a}, -\vec{a}) = -1$$

$$P(\vec{a}, \vec{b}) = 0, \quad \text{if} \quad \vec{a} \cdot \vec{b} = 0$$

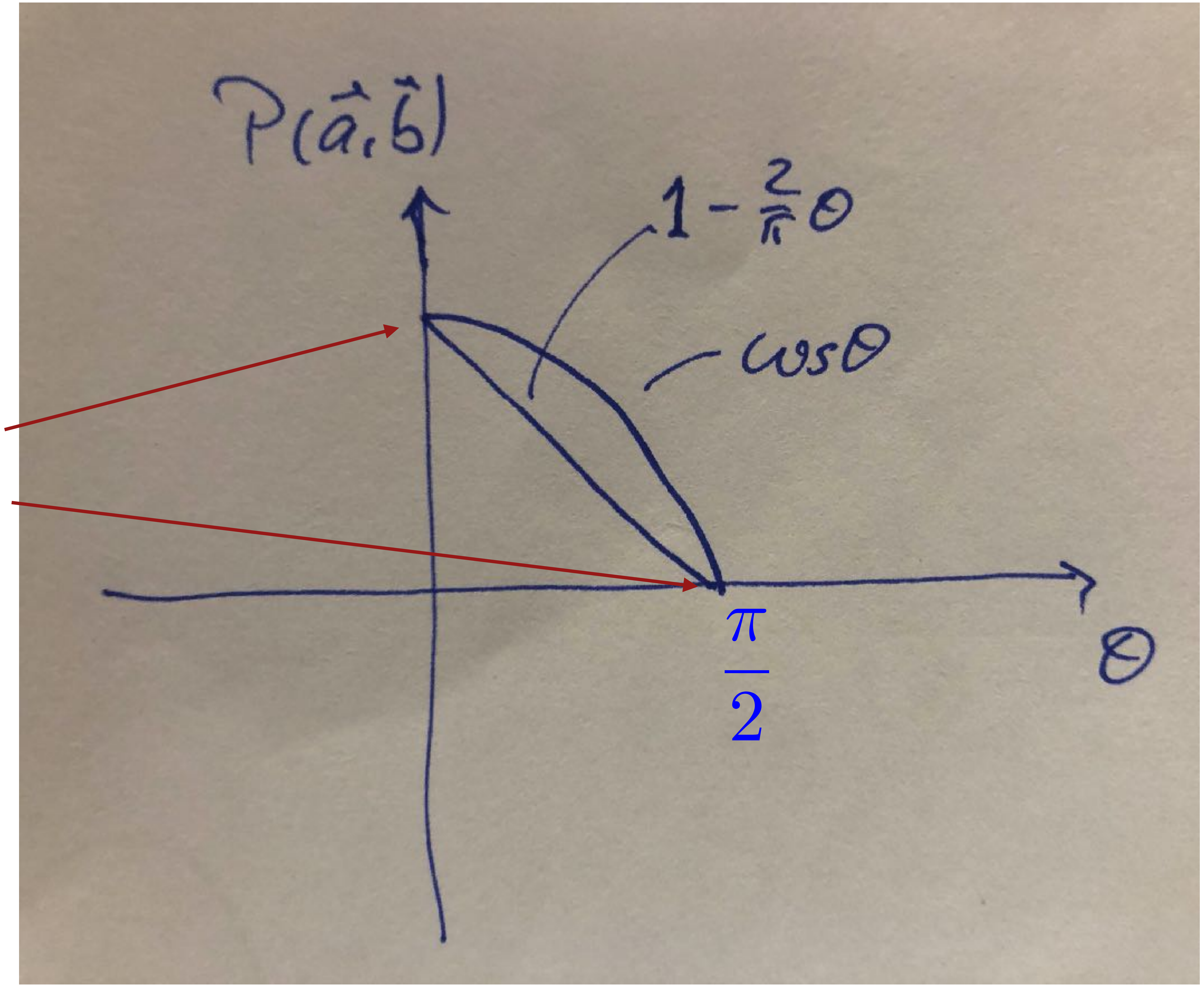
3) The most general case results in

$$P(\vec{a}, \vec{b}) = -1 + \frac{2}{\pi}\theta \qquad\qquad \vec{a} \cdot \vec{b} = \cos\theta$$




- You need 3 measurements to distinguish the 2 curves

LHVT agrees
with QM here
by construction




- You need 3 measurements to distinguish the 2 curves

# Bell's Inequality

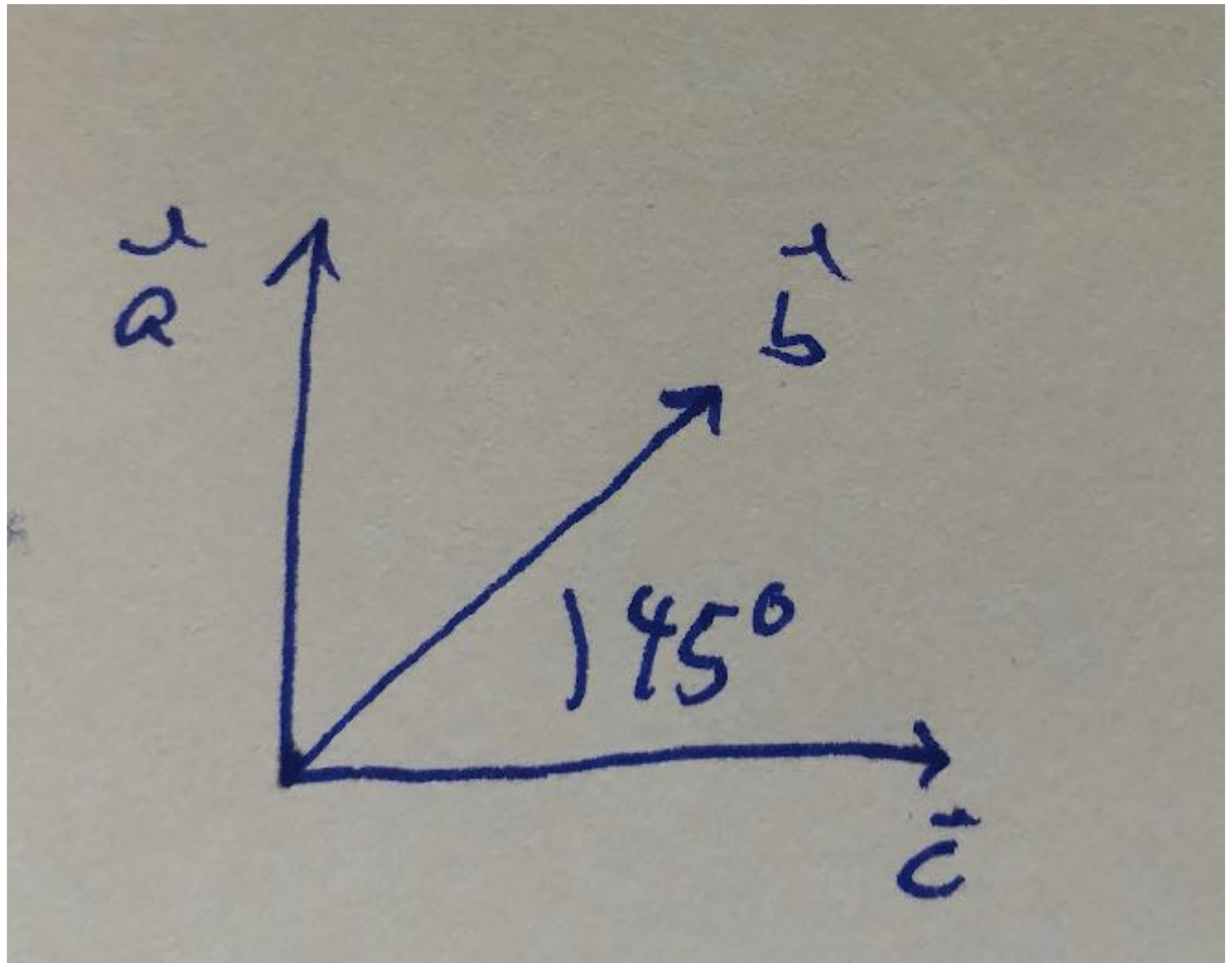
- Bell proved for all LHVT's

$$1 + P(\vec{b}, \vec{c}) \geq |P(\vec{a}, \vec{b}) - P(\vec{a}, \vec{c})|$$

- Note you need 3 settings

- QM expectation value violates this!

$$1 - \frac{1}{\sqrt{2}} \geq \frac{1}{\sqrt{2}}$$



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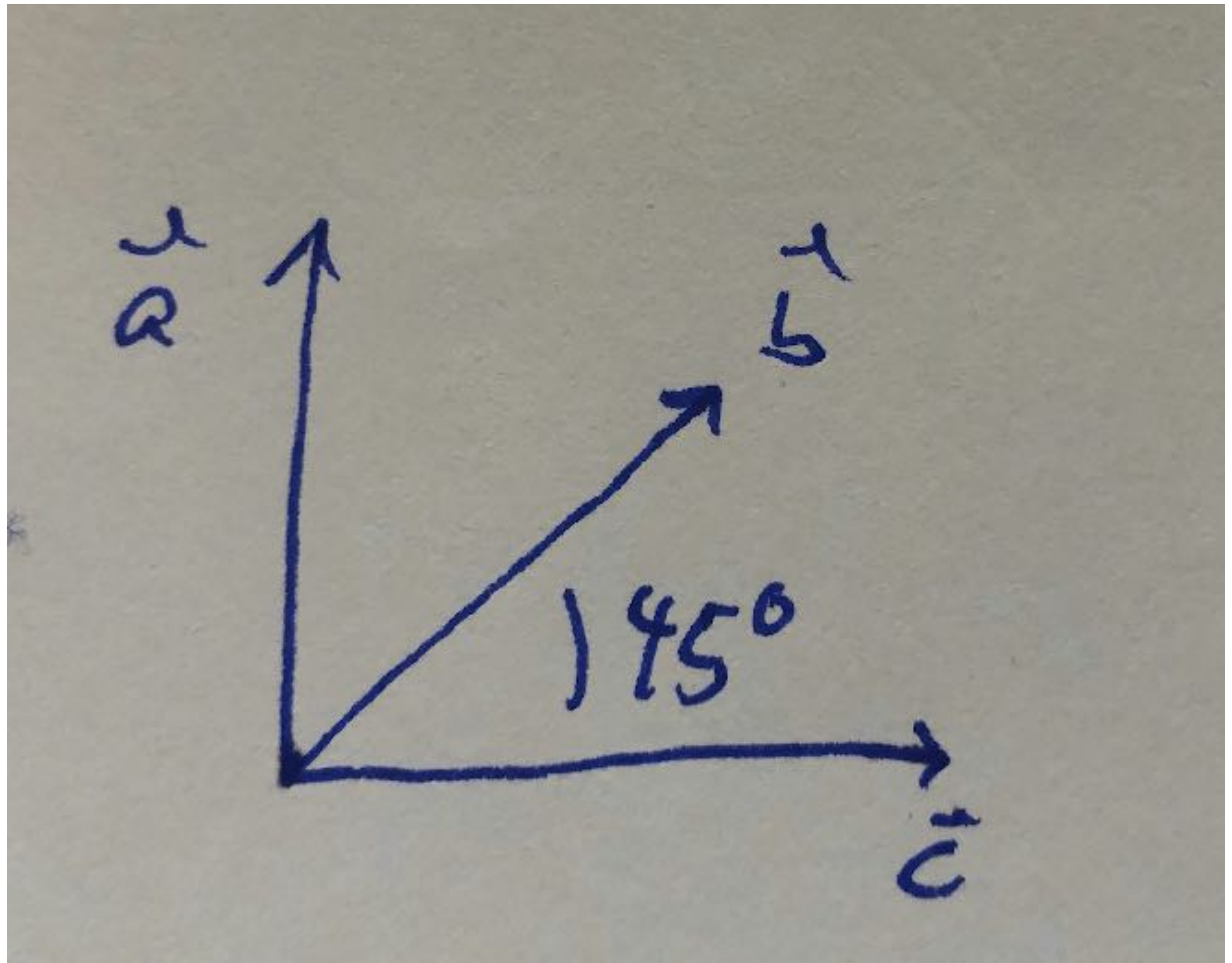

# Bell's Inequality: experiments

- All experimental tests of locality via Bell's inequality are with photons

- Measure 3 settings related to 2 independent spin components using polarimeters (Alain Aspect)

$$S_x\,, \quad S_y$$

- Would like to test this idea also at high energy and/or with fermions

Physics Letters B 280 (1992) 304–312
North-Holland

PHYSICS LETTERS B

# Testing locality at colliders via Bell's inequality?

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[a] *Department of Physics, University of Oxford, 1 Keble Road, Oxford OX1 3NP, UK*
[b] *Department of Physics, University of California, Riverside, CA 92521, USA*

(1992)

Received 5 December 1991; revised manuscript received 6 January 1992

We consider a measurement of correlated spins at LEP and show that it does *not* constitute a *general* test of local-realistic theories via Bell's inequality. The central point of the argument is that such tests, where the spins of two particles are inferred from a scattering distribution, can be described by a local hidden variable theory. We conclude that with present experimental techniques it is not possible to test locality via Bell's inequality at a collider experiment. Finally we suggest an improved fixed-target experiment as a viable test of Bell's inequality.

PHYSICAL REVIEW D VOLUME 55, NUMBER 1 1 JANUARY 1997

# How to find a Higgs boson with a mass between 155 and 180 GeV at the CERN LHC

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*Institute for Particle Physics (IPP), ETH Zürich, CH-8093 Zürich, Switzerland*

H. Dreiner
*Rutherford Laboratory, Chilton, Didcot, OX11 0QX, United Kingdom*

(1996)

(Received 13 August 1996)

We reconsider the signature of events with two charged leptons and missing energy as a signal for the detection of the standard model Higgs boson in the mass region $M(\text{Higgs})=155$–$180$ GeV. It is shown that a few simple experimental criteria allow us to distinguish events originating from the Higgs boson decaying to $H\to W^+W^-$ from the nonresonant production of $W^+W^-X$ at the CERN LHC. With this set of cuts, signal to background ratios of about one to one are obtained, allowing a 5–10$\sigma$ detection with about 5 fb$^{-1}$ of luminosity. This corresponds to less than one year of running at the initial lower luminosity $\mathcal{L}=10^{33}$ cm$^{-2}$s$^{-1}$. This is significantly better than for the hitherto considered Higgs boson detection mode $H\to Z^0Z^0{*}\to 2\ell^+2\ell^-$, where in this mass range about 100 fb$^{-1}$ of integrated luminosity are required for a 5$\sigma$ signal. [S0556-2821(97)02701-X]

PACS number(s): 14.80.Bn, 13.85.−t

# Abel, Dittmar and Dreiner (1992)

- Our idea: at LEP use the decay

$$Z^0 \longrightarrow \tau^+ + \tau^- \longrightarrow (\pi^+ + \bar{\nu}_\tau) + (\pi^- + \nu_\tau)$$

tau-spins are correlated

weak decay of tau,
pion momentum correlated
with tau spin

- Consider: $P(\hat{p}_{\pi^-}, \hat{p}_{\pi^+})$; $\quad \hat{p}_{\pi^\pm}$ unit $\pi^\pm$ momenta in $\tau^\pm$ rest frame avg over Z-pol's

- Compute in SM:

$$\frac{d\sigma}{d\cos\theta_{\pi\pi}}(e^+e^- \to \pi^+\pi^-\nu_\tau\bar{\nu}_\tau) = A(1 - \frac{1}{3}\cos\theta_{\pi\pi})$$

$$P_{\rm QM}(\cos\theta_{\pi\pi}) = \frac{d\sigma/d\cos\theta_{\pi\pi}(e^+e^- \to \pi^+\pi^-\nu_\tau\bar{\nu}_\tau)}{\sigma(e^+e^- \to \pi^+\pi^-\nu_\tau\bar{\nu}_\tau)}$$

$$= \frac{1}{2}(1 - \frac{1}{3}\cos\theta_{\pi\pi})$$

- Insert into Bell's inequality: satisfies it for all angles!!!

$$1 + P(\cos\theta_{ac}) \geq |P(\cos\theta_{ab}) - P(\cos\theta_{bc})|$$

$$1 + \frac{1}{2}(1 - \frac{1}{3}\cos\theta_{ac}) \geq | - \frac{1}{6}\cos\theta_{ab} + \frac{1}{6}\cos\theta_{bc}|$$

$$9 - \frac{1}{3}\cos\theta_{ac} \geq |\cos\theta_{ab} - \cos\theta_{bc}|$$

# Recall the Logic

- All LHVT's satisfy Bell's inequality

**LHVT —> Bell's inequality**

- QM can or can not satisfy Bell's inequality

- The art is to find a setup where QM violates Bell's inequality

**non(Bell's inequality) —> non(LHVT)**

# Why does it fail as a test of locality, despite anti-correlated spins?

- Simple answer: in decay $\tau^- \to \pi^- + \nu_\tau$ only measure $\tau$-helicity

- This is just one component of spin: $(\vec{S}_\tau)_z$

$$P(S_z^+ = \uparrow, S_z^- = \uparrow) \;\; \text{parallel}$$

$$P(S_z^+ = \uparrow, S_z^- = \downarrow) \;\; \text{anti-parallel}$$

- According to Bell analysis should be able to reproduce experiment with LHVT

- That is in fact the case!

- Consider

SM diff. X-Section

$$\frac{d\sigma}{d\cos\theta_{\pi\pi}}(e^+e^- \to \pi^+\pi^-\nu_\tau\bar{\nu}_\tau) = f(\hat{p}_{\pi^+}, \hat{p}_{\pi^-})$$

- Now let the hidden variables for each $\tau$ be a set of unit vectors

$$\hat{\lambda}_e, \hat{\lambda}_\mu, \hat{\lambda}_\pi, \hat{\lambda}_\rho, \ldots$$

- If tau decays as $\tau^- \to \pi^- + \nu_\tau$ then $\hat{\lambda}_\pi$ tells it to decay such that

$$\hat{p}_\pi = \hat{\lambda}_\pi$$

- Let $F(\hat{\lambda}_{\pi^+}, \hat{\lambda}_{\pi^-})$ be original prob. distribution of hidden variables

- Identify $F(\hat{\lambda}_{\pi^+}, \hat{\lambda}_{\pi^-}) = f(\hat{\lambda}_{\pi^+}, \hat{\lambda}_{\pi^-})$ $\quad (= \frac{d\sigma}{d\Omega_+ d\Omega_-})$

- Have an LHVT which **exactly** reproduces **all** experimental results

- Essential that

$$[(\hat{p}_{\pi^+})_i, (\hat{p}_{\pi^-})_j] = 0$$

$$[(\hat{p}_{\pi^\pm})_i, (\hat{p}_{\pi^\pm})_j] = 0$$

$$\forall i, j$$

- Only then does QM provide the function $f(\hat{\lambda}_{\pi^+}, \hat{\lambda}_{\pi^-})$

- For non-commuting spins (2 photon case),
QM does **not** provide function $f(S_x^i, S_y^i, \ldots)$

# Main Statement — Theorem

For all experiments where the correlated observables **commute** we can construct an LHVT using the QM function, which exactly reproduces the data.

In collider experiments we measure 4-momenta. These all commute. Ergo: all results can be reproduced by an LHVT.

# Examples at the LHC from the Recent Literature

$$gg \to H^0 \to W^+W^- \,, \quad W^+ \to \ell^+ + \nu_\ell$$

$$gg \to H^0 \to Z^0 Z^0 \,, \quad Z^0 \to \ell^+ + \ell^-$$

$$gg,\, q\bar{q} \to t\bar{t} \,, \quad t \to W^+ + b$$

$$gg,\, q\bar{q} \to \tau^+\tau^- \,, \quad \tau^\pm \to \pi^\pm + \nu_\tau$$

- In all cases measure final state momenta, i.e. can write an LHVT reproducing the data using the SM diff. X-section.

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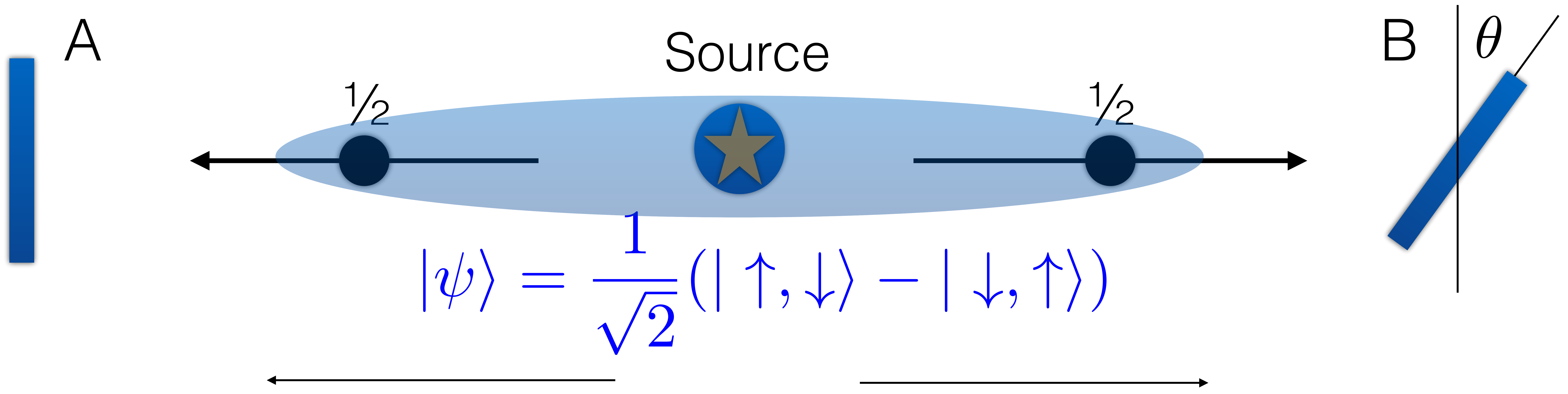
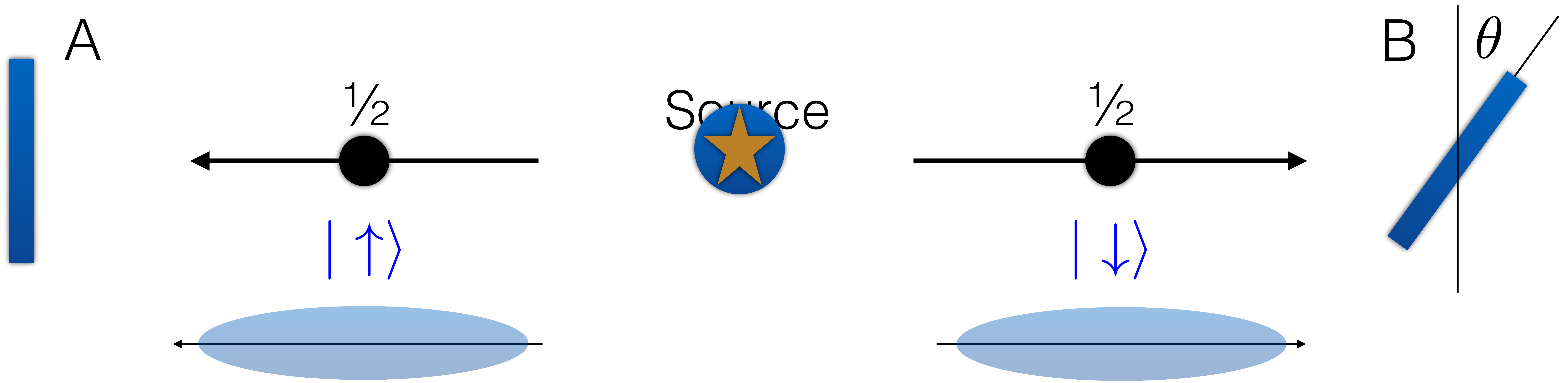
# A Word on Logic: entanglement vs local realism



- You can describe the data exactly with an LHVT, i.e. with a **non-entangled** state!

- Thus you have simply chosen a poor set-up to test locality.

- So: you are NOT testing locality, at all!

- You are also NOT testing entanglement!

# Where our Paper was purposely(?) misunderstood

- Can show on general symmetry grounds that **QM expectation value** must have form (s-wave plus p-wave)

$$P_{\pi\pi}(\theta) = c_1 + c_2 \langle (\hat{p}_{\pi^+} \cdot \sigma_{\tau^+})(\hat{p}_{\pi^-} \cdot \sigma_{\tau^-}) \rangle_{\text{QM}}$$

QM expectation value to observe to $\tau^+$ spin in the $\hat{p}_{\pi^+}$ direction **and** the $\tau^-$ spin in the $\hat{p}_{\pi^-}$ direction

$$P_{\sigma^\tau \sigma^\tau}(\theta) = \langle (\vec{a} \cdot \sigma_{\tau^+})(\vec{b} \cdot \sigma_{\tau^-}) \rangle_{\text{QM}}$$

(★) $$P_{\sigma^\tau \sigma^\tau}(\theta) = \frac{P_{\pi\pi}(\theta) - c_1}{c_2} = -\cos\theta$$

math. construction which violates Bell's inequality

**but: SO WHAT?!?!**

- In deriving Eq.(★) I have made blatant use of QM. Thus I am assuming QM … to "test" QM? - No, I am not even testing QM. I am not testing anything.

- It is a meaningless function which happens to mathematically violate Bell's inequality. It makes **NO statement about LOCALITY**.
  (or entanglement!)

- Yes, you are testing an **unknown** subclass of LHVT's, but you are doing this also if you drop 2 watermelons from the tower of Pisa.

- In our paper the computation of Eq.(★) was just to illustrate what people were doing.

- In our paper, we conclude by saying: “It is the second cornerstone in our claim that **it is not possible to test the completeness of QM in a collider experiment**.”

- The first cornerstone was the direct proof that Bell’s inequality is NOT violated.

- The statement: “this is not a general test of locality via Bell’s inequality” is logically true but highly misleading as it is NO test of locality. The data is described by a local theory!

- The data is described by a non-entangled state. You are **NOT** testing entanglement

# a few words on $t\bar{t}$ production

# Testing Bell Inequalities at the LHC with Top-Quark Pairs

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[2]*Dipartimento Politecnico di Ingegneria ed Architettura, Università degli Studi di Udine, Via della Scienze 206, 33100 Udine, Italy and INFN, Sezione di Trieste (Gruppo Collegato di Udine), via delle Scienze, 208, 33100 Udine, Italy*

(Received 17 February 2021; accepted 21 September 2021; published 15 October 2021)

Correlations between the spins of top-quark pairs produced at a collider can be used to probe quantum entanglement at energies never explored so far. We show how the measurement of a single observable can provide a test of the violation of a Bell inequality at the 98% C.L. with the statistical uncertainty of the data already collected at the Large Hadron Collider, and at the 99.99% C.L. with the higher luminosity of the next run. Detector acceptance, efficiency, and migration effects are taken into account. The test relies on the spin correlations alone and does not require the determination of probabilities—in contrast to all other tests of Bell inequalities.

DOI: 10.1103/PhysRevLett.127.161801

*Introduction.*—A characteristic property of a quantum system is the presence of quantum correlations (entanglement) among its constituents not accounted for by classical physics (for a review, see [1]), leading to the violation of

neutrino oscillations [19]. No test has so far been performed at the high energies made available by the LHC—even though some preliminary work has been done in [5,6] an[redacted] more recently in [20]. In particular, we build on the result[redacted]

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https://doi.org/10.1140/epjc/s10052-022-10245-9

THE EUROPEAN PHYSICAL JOURNAL C



Regular Article - Theoretical Physics

# Quantum tops at the LHC: from entanglement to Bell inequalities

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[2] Dipartimento di Fisica e Astronomia, Università di Bologna, via Irnerio 46, Bologna, Italy
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Received: 6 January 2022 / Accepted: 23 March 2022 / Published online: 2 April 2022


**Abstract** We present the prospects of detecting quantum entanglement and the violation of Bell inequalities in $t\bar{t}$ events at the LHC. We introduce a unique set of observables suitable for both measurements, and then perform the

ter of experiment. In 1964, Bell proved [1] classical theories obey correlation limits, i.e., Bell Inequalities (BIs), that QM can violate. In the last decades, several experiments have been performed and all results so far agree with QM predic-

Eur. Phys. J. C (2022) 82:666
https://doi.org/10.1140/epjc/s10052-022-10630-4

THE EUROPEAN PHYSICAL JOURNAL C

Regular Article - Theoretical Physics

# Improved tests of entanglement and Bell inequalities with LHC tops

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**Abstract** We discuss quantum entanglement in top pair production at the LHC. Near the $t\bar{t}$ threshold, entanglement observables are enhanced by suppressing the contribution of $q\bar{q}$ subprocesses, which is achieved by a simple cut on the velocity of the $t\bar{t}$ system in the laboratory frame. Furthermore, we design new observables that directly measure the relevant combinations of $t\bar{t}$ spin correlation coefficients involved in the measurement of entanglement and Bell inequalities. As a result, the statistical sensitivity is enhanced, up to a factor of 7 for Bell inequalities near threshold.

the qubits associated to the spin states of $t\bar{t}$ pairs produced at the LHC provide a suitable arena to investigate these matters since top quarks decay before their spins are randomised by strong radiation and the spin of the lepton produced in semileptonic decays $t \to \ell \nu b$ ($\ell = e, \mu$) is completely correlated to that of the mother top. Besides, owing to the large cross section, $\sigma = 832$ pb at 13 TeV [11] there is a good amount of statistics on $t\bar{t}$ production already at Run 2 with 139 $\text{fb}^{-1}$ of collected data, and there will be much more in the future at its high luminosity upgrade (HL-LHC). In Refs. [6–

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# Quantum entanglement and Bell inequality violation in semi-leptonic top decays

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Abstract: Quantum entanglement is a fundamental property of quantum mechanics. Recently, studies have explored entanglement in the $t\bar{t}$ system at the Large Hadron Collider (LHC) when both the top quark and anti-top quark decay leptonically. Entanglement is detected via correlations between the polarizations of the top and anti-top and these polarizations are measured through the angles of the decay products of the top and anti-top. In this work, we propose searching for evidence of quantum entanglement in the semi-leptonic decay channel where the final state includes one lepton, one neutrino, two $b$-flavor tagged jets, and two light jets from the $W$ decay. We find that this channel is both easier to reconstruct and has a larger effective quantity of data than the fully leptonic channel. As a result, the semi-leptonic channel is 60% more sensitive to quantum entanglement and a factor of 3 more sensitive to Bell inequality violation, compared to the leptonic channel. In 139 fb$^{-1}$ (3 ab$^{-1}$) of data at the LHC (HL-LHC), it should be feasible to measure entanglement at a precision of $\lesssim 3\%$ (0.7%). Detecting Bell inequality violation, on the other hand, is more challenging. With 300 fb$^{-1}$ (3 ab$^{-1}$) of integrated luminosity at the LHC Run-3 (HL-LHC),

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# Optimizing Entanglement and Bell Inequality Violation in Top Anti-Top Events

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## Abstract

A top quark and an anti-top quark produced together at colliders have correlated spins. These spins constitute a quantum state that can exhibit entanglement and violate Bell's inequality. In realistic collider experiments, most analyses allow the axes, as well the Lorentz frame to vary event-by-event, thus introducing a dependence on the choice of event-dependent basis leading us to adopt "fictitious states," rather than genuine quantum states. The basis dependence of fictitious states allows for an optimization procedure, which makes the usage of fictitious states advantageous in measuring entanglement and Bell inequality violation. In this work, we show analytically that the basis which diagonalizes the spin-spin correlations is optimal for maximizing spin correlations, entanglement, and Bell inequality violation. We show that the optimal basis is approximately the

# Observation of quantum entanglement with top quarks at the ATLAS detector

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Entanglement is a key feature of quantum mechanics[1–3], with applications in fields such as metrology, cryptography, quantum information and quantum computation[4–8]. It has been observed in a wide variety of systems and length scales, ranging from the microscopic[9–13] to the macroscopic[14–16]. However, entanglement remains largely unexplored at the highest accessible energy scales. Here we report the highest-energy observation of entanglement, in top–antitop quark events produced at the Large Hadron Collider, using a proton–proton collision dataset with a centre-of-mass energy of $\sqrt{s} = 13$ TeV and an integrated luminosity of 140 inverse femtobarns $(\text{fb})^{-1}$ recorded with the ATLAS experiment. Spin entanglement is detected from the measurement of a single observable $D$, inferred from the angle between the charged leptons in their parent top- and antitop-quark rest frames. The observable is measured in a narrow interval around the top–antitop quark production threshold, at which the entanglement detection is expected to be significant. It is reported in a fiducial phase space defined with stable particles to minimize the uncertainties that stem from the limitations of the Monte Carlo event generators and the parton shower model in modelling top-quark pair production. The entanglement marker is measured to be $D = -0.537 \pm 0.002$ (stat.) $\pm 0.019$ (syst.) for $340\ \text{GeV} < m_{t\bar{t}} < 380\ \text{GeV}$. The observed result is more than five standard deviations from a scenario without entanglement and hence constitutes the first observation of entanglement in a pair of quarks and the highest-energy observation of entanglement so far.

Particle colliders, such as the Large Hadron Collider (LHC) at CERN, probe fundamental particles and their interactions at the highest energies accessible in a laboratory, exceeded only by astrophysical

of quantum information about two particles in the same quantum state that exist in superposition. The spin quantum number of a fundamental fermion, a particle that can take spin values of ±1/2, is one of

# Atlas, *Nature* 633 (2024) 8030, 542

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega_+ d\Omega_-} = \frac{1 + \mathbf{B}^+ \cdot \hat{\mathbf{q}}_+ - \mathbf{B}^- \cdot \hat{\mathbf{q}}_- - \hat{\mathbf{q}}_+ \cdot C \cdot \hat{\mathbf{q}}_-}{(4\pi)^2},$$

$$t \to b\ell^+\nu_\ell$$

$$\bar{t} \to \bar{b}\ell^-\bar{\nu}_\ell$$

$$D \equiv \frac{1}{3} Tr[\mathbf{C}]$$

Atlas measurement: $D = -0.537 \pm 0.002\,(\text{stat.}) \pm 0.019\,(\text{syst.})$

Spin density matrix: $$\rho = \frac{1}{4}\left[ I_4 + \sum_i \left(B_i^+ \sigma^i \otimes I_2 + B_i^- I_2 \otimes \sigma^i\right) + \sum_{i,j} C_{ij} \sigma^i \otimes \sigma^j \right].$$

# Atlas, *Nature* 633 (2024) 8030, 542

Spin density matrix:

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``entanglement identifier”:

$$D \equiv \frac{1}{3} Tr[\mathbf{C}]$$

Interpreting in terms of QM spins:

$$D > -\frac{1}{3}$$

entanglement limit

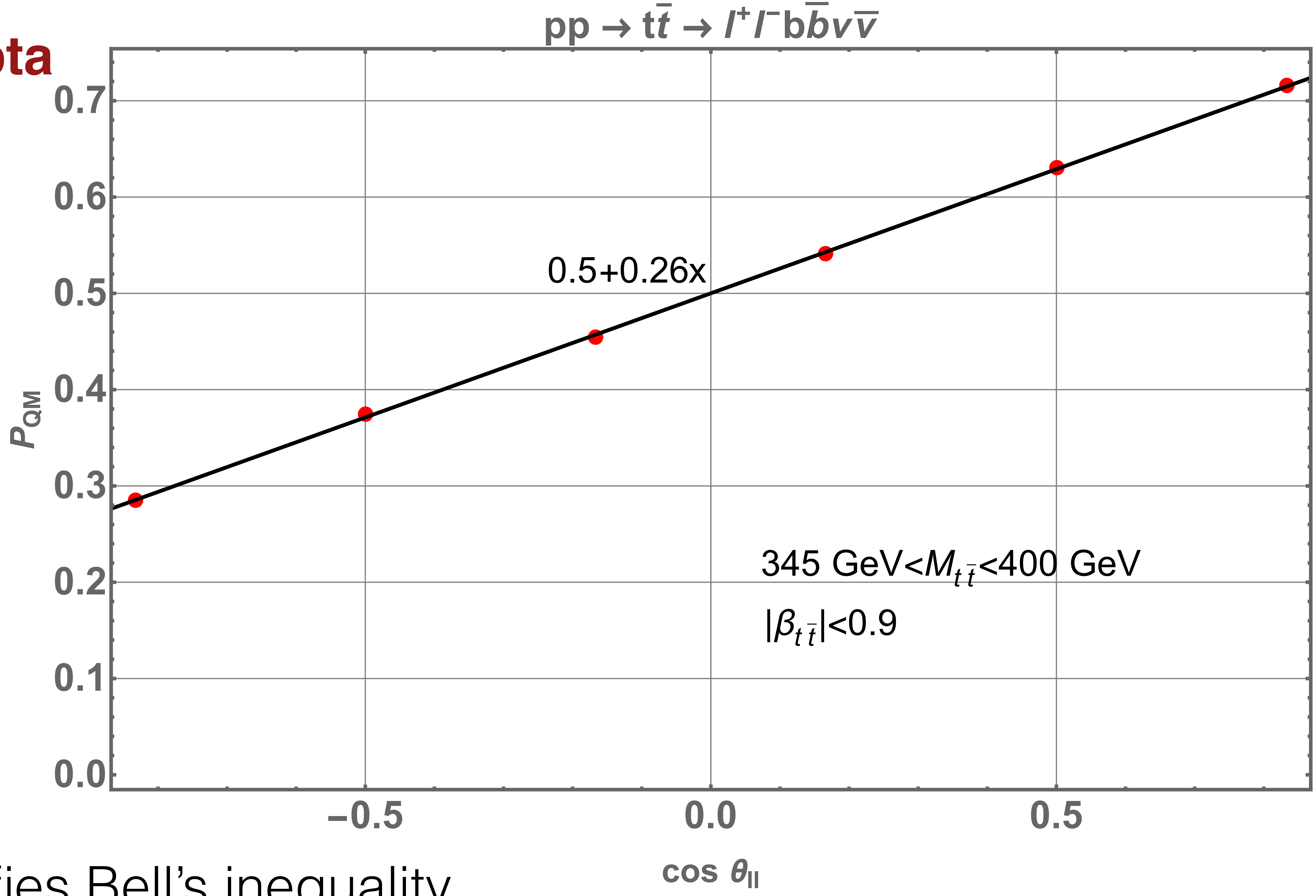
$$D = -0.537 \pm 0.002\,(\mathrm{stat.}) \pm 0.019\,(\mathrm{syst.})$$

- Spin density matrix: $$\rho = \frac{1}{4}\left[ I_4 + \sum_i (B_i^+ \sigma^i \otimes I_2 + B_i^- I_2 \otimes \sigma^i) + \sum_{i,j} C_{ij} \sigma^i \otimes \sigma^j \right].$$

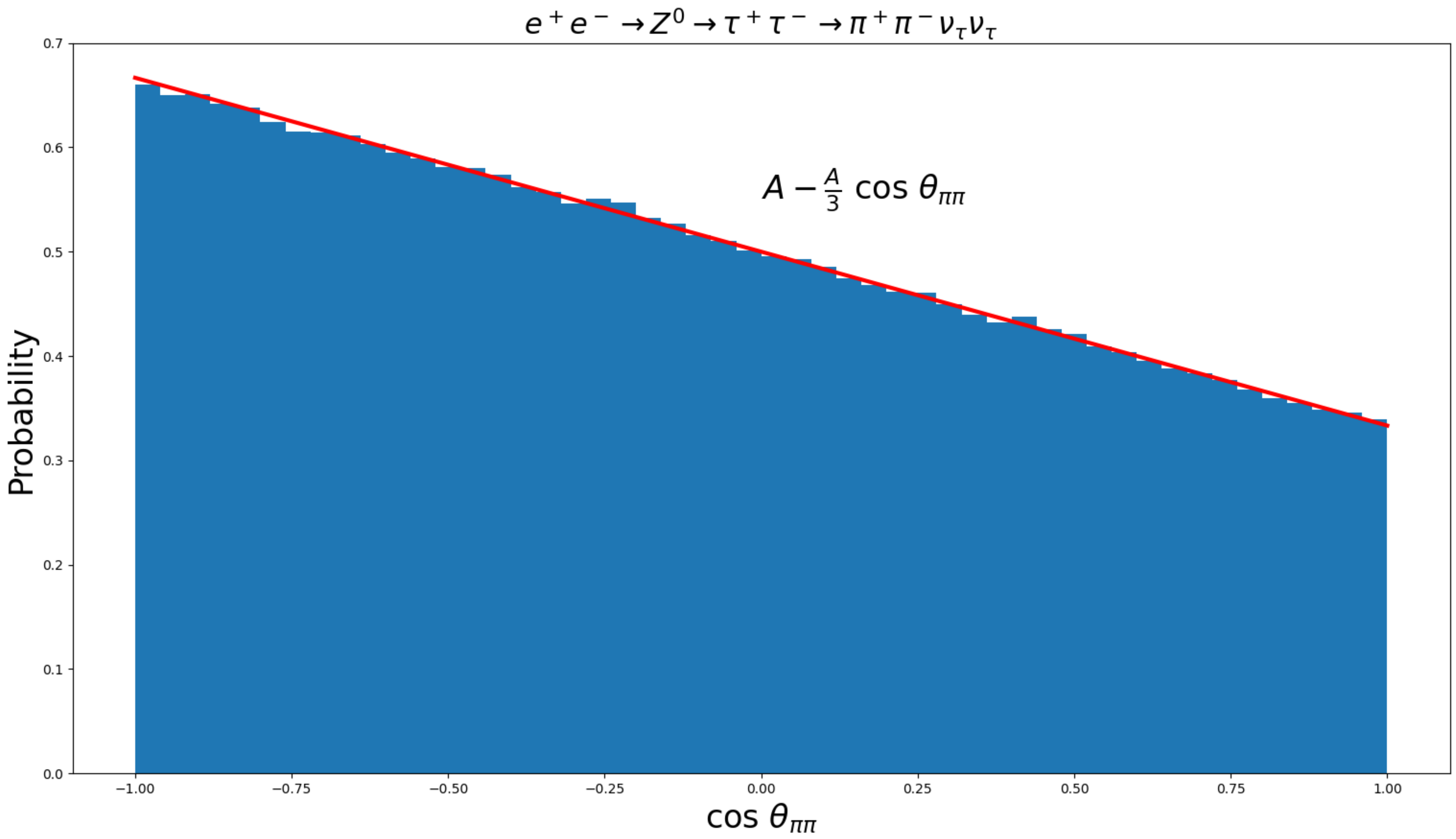
- parametrization of diff. Xsect.: $$\frac{1}{\sigma}\frac{d\sigma}{d\Omega_+ d\Omega_-} = \frac{1 + \mathbf{B}^+ \cdot \hat{\mathbf{q}}_+ - \mathbf{B}^- \cdot \hat{\mathbf{q}}_- - \hat{\mathbf{q}}_+ \cdot C \cdot \hat{\mathbf{q}}_-}{(4\pi)^2},$$

- connection inherently assumes QM (or QFT)

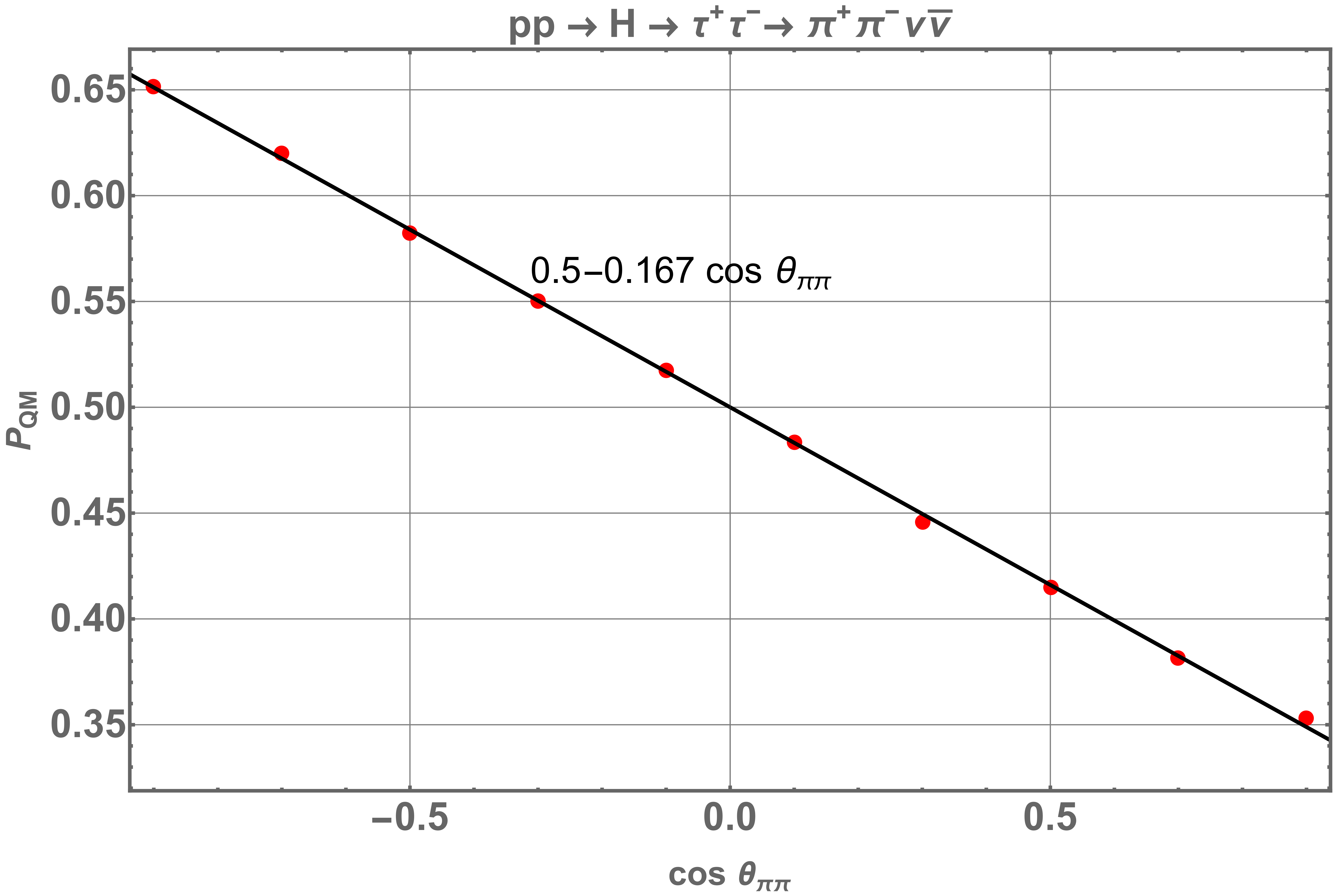
**Plot by Rhitaja Sengupta**




- always satisfies Bell's inequality




- always satisfies Bell's inequality

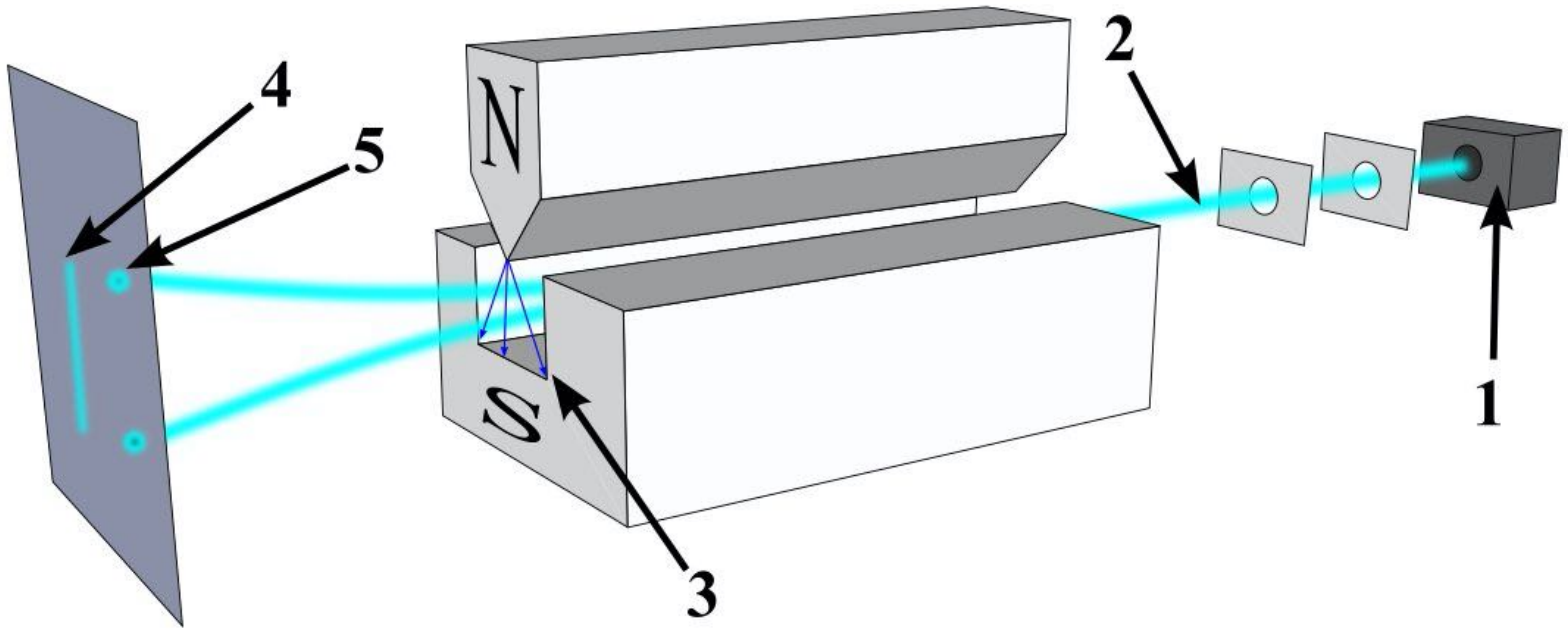



- always satisfies Bell's inequality

# Bell's Inequality — Stern-Gerlach

- Bell showed for the first time you can distinguish **experimentally** between entangled state and local realistic state

- Problems with this




- Impossible to measure spin-½ with Stern-Gerlach for charged particles, i.e. electrons (or taus)

Mott & Massey, Theory of Atomic Collisions (1949)



## 2. Magnetic moment of the electron

We have discussed so far only the magnetic moment of the atom. We shall not review here the evidence, derived from the anomalous Zeeman effect, from the gyromagnetic effect, etc., that the *electron* has a fourth degree of freedom, a magnetic moment $\epsilon h/4\pi mc$, and a mechanical moment $\frac{1}{2}h/2\pi$. We shall content ourselves with remarking that according to the Schrödinger theory the ground state of the hydrogen atom is not degenerate, and therefore, in order to account for the splitting in a magnetic field revealed by the Stern-Gerlach experiment, it is necessary to assume that the electron has a fourth degree of freedom.

The present evidence that electrons have a magnetic moment is derived from their behaviour when bound in stationary states in atoms. For the study of collision problems it is necessary to inquire what meaning can be attached to the magnetic moment of a free electron. In the first place, just as in the case of the atom, it is impossible to determine the moment by means of a magnetometer experiment. This can be shown by the following argument, due to Bohr.† Let us suppose that the position of the electron is known with an accuracy $\Delta r$ and that we wish to determine the magnetic moment at a point distant $r$ from it. It will not be possible to deduce from our measurement anything about the magnetic moment of the electron unless

$$\Delta r \ll r. \tag{5}$$

The field $H$ that we wish to observe will be of order of magnitude

$$H \sim M/r^3.$$

If, however, the electron is in motion with velocity $v$, there will be a magnetic field due to its motion, of amount $\epsilon v/cr^2$; since we do not know $v$ exactly we cannot allow for this field exactly. From our measurements, therefore, of the magnetic field, it will not be possible to find out anything about the magnetic moment of the electron, unless

$$M/r^3 \gg \epsilon\Delta v/cr^2,$$

where $\Delta v$ is the uncertainty in our knowledge of $v$. Since by the uncertainty principle $\Delta r \Delta v > h/m$, this leads to

$$\Delta r \gg r,$$

which contradicts the inequality (5). We conclude therefore that it is not possible to measure the magnetic moment of an electron in this manner.

We shall now show that it is impossible, by means of a Stern-Gerlach

† Cf. Mott, *Proc. Roy. Soc.* A, **124** (1929), 440.



experiment, to determine the magnetic moment of a free electron, or to prepare a beam of electrons with the magnetic moments all pointing in the same direction. The argument is also due to Bohr.

In Fig. 9 a beam of electrons is supposed to travel parallel to the $z$-axis (i.e. perpendicular to the plane of the paper). The pole pieces of the magnet are shown, as are also the lines of force. The purpose of the

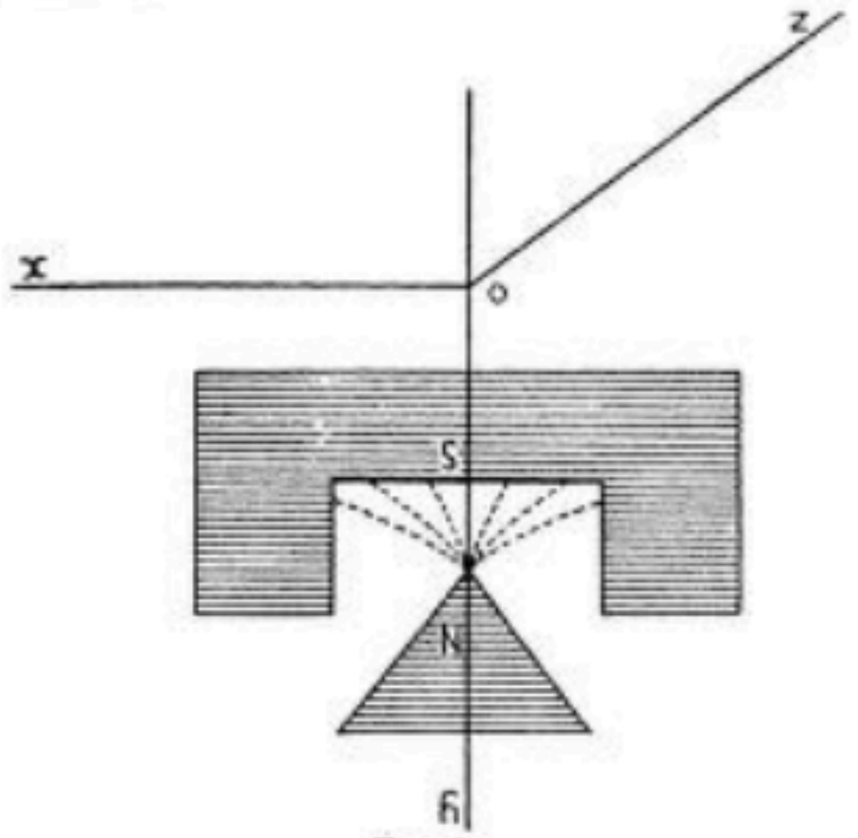


Fig. 9.

experiment is to observe a splitting in the $y$-direction. The force on an electron tending to split the beam will be

$$\pm M\frac{\partial H_y}{\partial y}. \tag{6}$$

Now all electrons will experience a force due to their motion through the field. Those moving in the plane $Oyz$ will experience a force in the direction $Ox$. This force is perpendicular to the direction of the splitting, and its only effect will be to displace the beams to the right or to the left. However, electrons which do not move in the plane $Oyz$ will experience a force in the direction $Oy$, because the lines of force in an inhomogeneous magnetic field cannot be straight, and there must be a component $H_x$ of $H$ along $Ox$. This force will have magnitude

$$\epsilon v H_x/c. \tag{7}$$

We can compare (7) with the force (6) tending to produce the splitting. $H_x$ at a point distant $\Delta x$ from the plane $Oyz$ will be equal to $\frac{\partial H_x}{\partial x}\Delta x$, and since $\operatorname{div} H$ vanishes, this is equal to $-\frac{\partial H_y}{\partial y}\Delta x$. The quantities (6) and (7) therefore stand in the ratio

$$\frac{\epsilon h}{4\pi mc}\frac{\partial H_y}{\partial y} : \frac{\epsilon v}{c}\frac{\partial H_y}{\partial y}\Delta x.$$

Dividing through by common factors this becomes

$$1 : 4\pi\Delta x/\lambda, \tag{7.1}$$

where $\lambda$ is the wave-length $h/mv$ of the waves that represent the electrons. Suppose now that $\pm\Delta x$ is the distance from the plane $Oyz$ of the two extremities of the beam. Since $\Delta x$ must be greater than $\lambda$, it is clear that the two extremities of the beam will be deflected in opposite directions through angles greater than the angle of splitting, which we hope to observe.

To see now that it is impossible to observe any splitting, let us consider the trace that the beam would make on a photographic plate. Suppose that it were possible to use finer beams than is allowed by the uncertainty principle, so that the thickness $\Delta y$ of the beam in the $y$-direction would be infinitely small. Before passing through the magnetic field, the cross-section of the beam would be as in Fig. 10($a$). Afterwards, it would be as in Fig. 10($b$), which shows the trace produced on a photographic plate. The tilting of the traces is produced by the Lorentz forces discussed above. If $ABC$, $A'B'$ are two lines parallel to $Oy$ and distant $\lambda$ apart, then by (7.1) we see that the tilting is so great that $AB > BC$. If $A\beta\gamma$ is drawn perpendicular to the traces, it follows that $A\beta > \beta\gamma$. But $A\beta < \lambda$, and hence $\beta\gamma$, the distance between the traces, is less than $\lambda$. Thus the maximum separation that can be produced is $\lambda$. But actually we cannot obtain a trace of breadth comparable with $\lambda$. Therefore it is impossible to observe any splitting.

From these arguments we must conclude that it is meaningless to assign to the free electron a magnetic moment. It is a property of the electron that when it is bound in an $S$ state in an atom, the atom has a magnetic moment. When we consider the relativistic treatment of the electron due to Dirac, we shall see that this magnetic moment is not in general equal to $\epsilon h/4\pi mc$, unless the velocities of the electron within the atom are small compared with that of light (§ 3.3). A single electron bound in its lowest state in the field of a nucleus of charge $Z\epsilon$ gives, according to Dirac's theory, a magnetic moment†

$$\tfrac{1}{3}[1+2\sqrt{(1-\gamma^2)}]\epsilon h/4\pi mc \quad (\gamma = 2\pi Z\epsilon^2/hc). \tag{8}$$

† This formula is due to Breit, *Nature*, **122** (1928), 649. Cf. § 3.3 of this chapter.



The statement that a free electron has four degrees of freedom is on a different footing, for it is hardly conceivable that an electron in an atom should have four degrees of freedom, and a free electron three. It is interesting to inquire, therefore, whether there is any conceivable experiment by which this fourth degree of freedom could be detected.

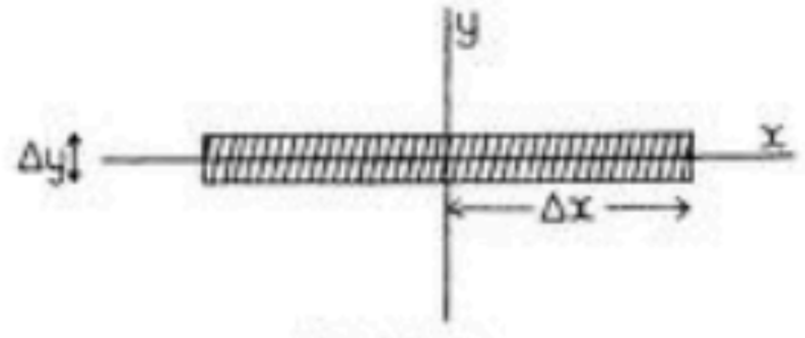



FIG. 10($a$).

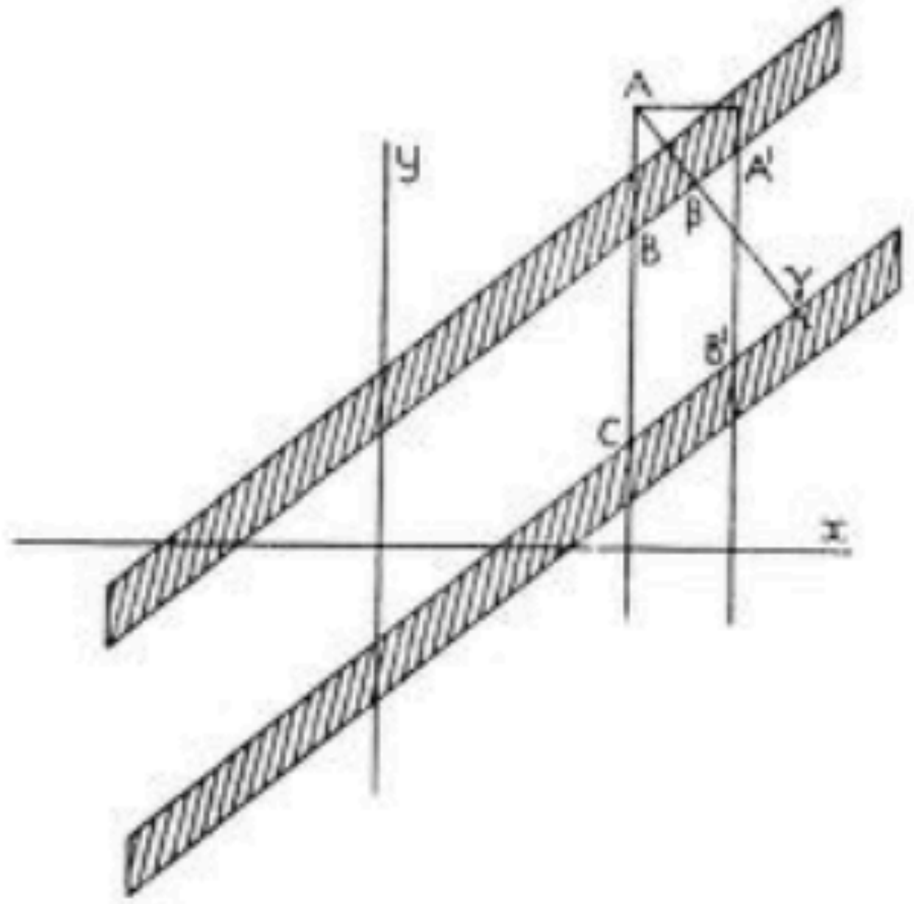



FIG. 10($b$).

We wish to know whether it is possible to prepare a beam of electrons that is in some sense 'polarized', and whether it would be possible to detect this polarization.

There is at present no certain experimental evidence on this point; theoretical considerations show, however, that it is possible, in principle, both to prepare a polarized beam and to detect the polarization. Let us consider the following experiment.† A beam of atoms is prepared, by means of a Stern-Gerlach experiment, with their axes all pointing in the same direction, say along the $z$-axis. Electrons are ejected from

† This method of preparing a polarized beam of electrons was first suggested by Fues and Hellmann, *Phys. Zeits.* **31** (1930), 465.

# Editorial

- Why are people doing this?
- QM well established on many fronts, including Aspect experiment
- Lamb shift
- (g-2) of the electron
- global electroweak fit
- QCD radiative corrections to production Xsections at the LHC

# Summary

- In a collider experiment measuring momenta always possible to construct a local hidden variable theory which explains the data

- Thus one is NOT testing locality via Bell's inequality

- Thus one is NOT testing for entanglement

# The End