Quantum computing for nuclear and particle physics

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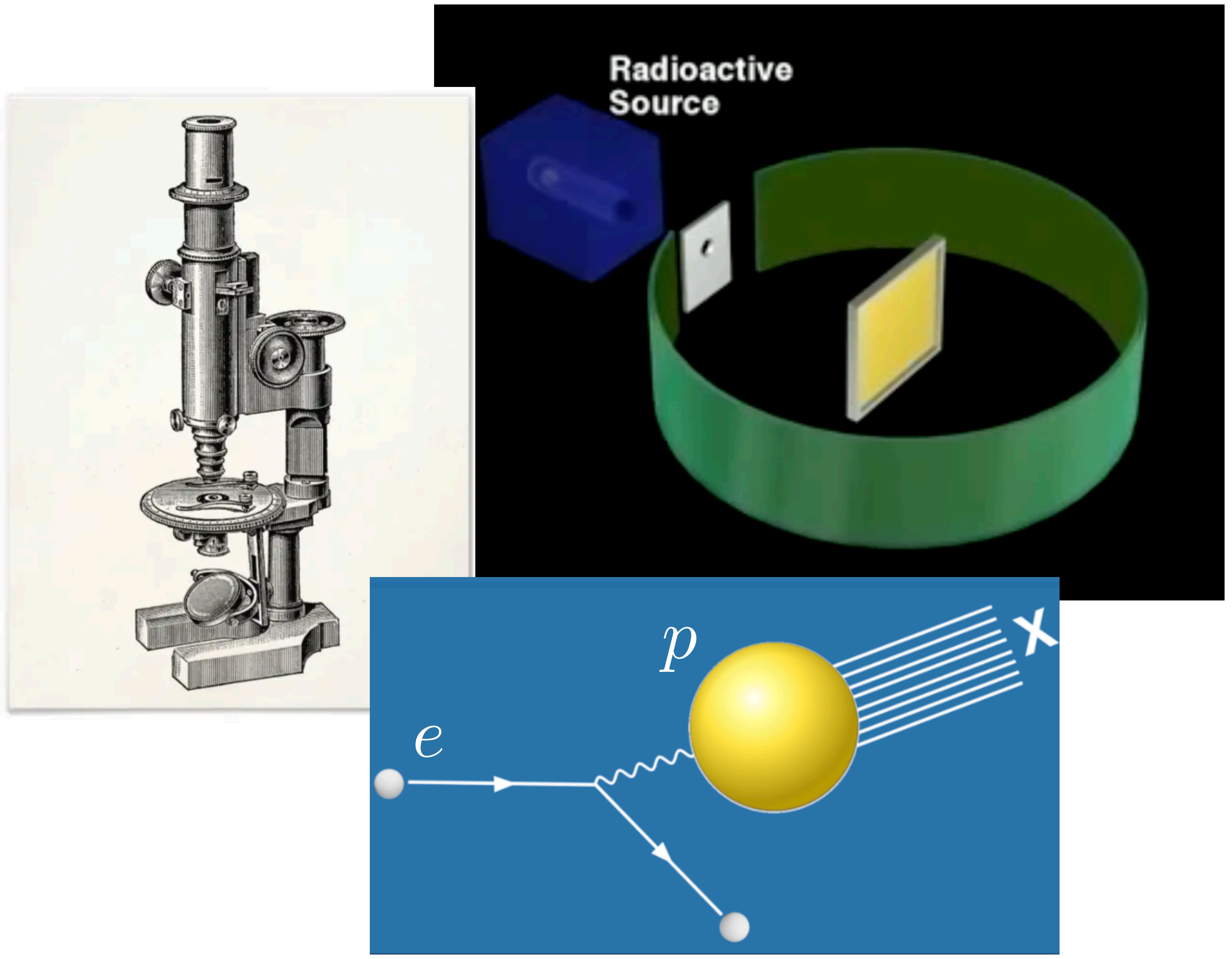
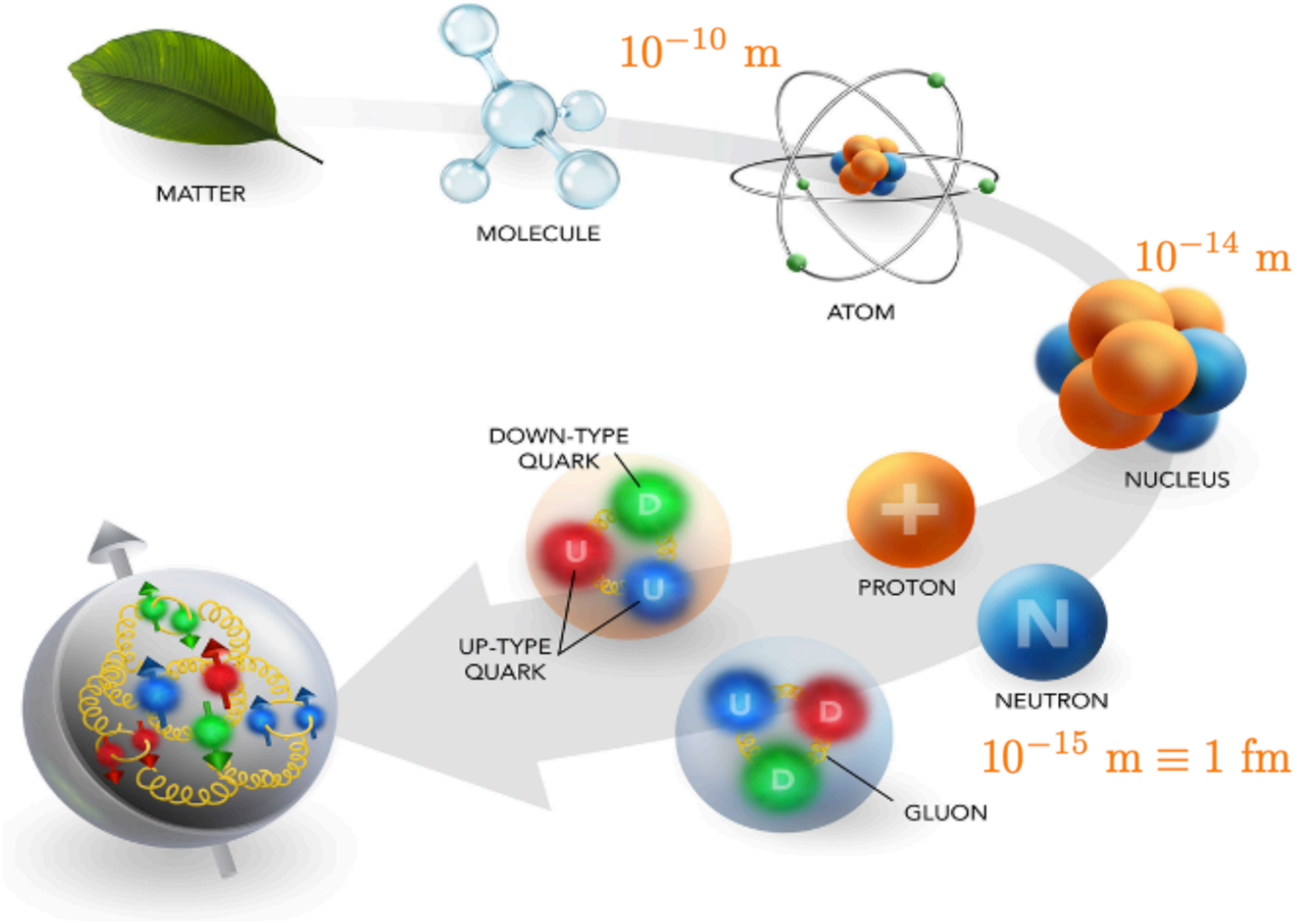
Annual Theory Meeting, Durham University, UK, 12/17/24

Stony Brook University

Probing the fundamental structure of matter

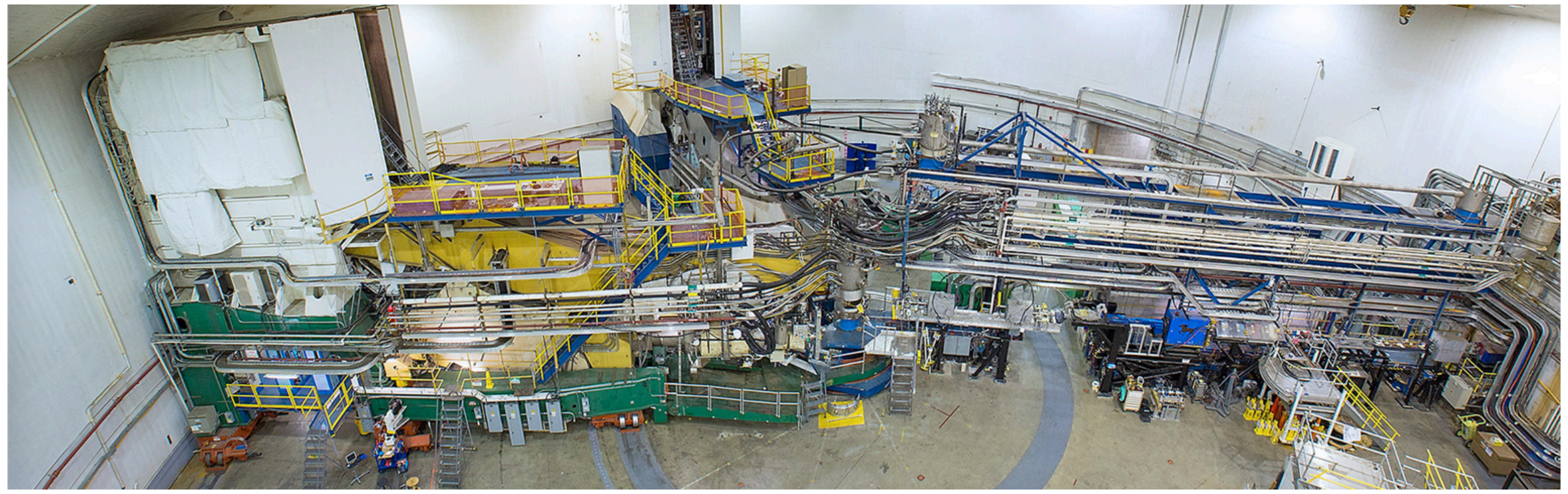




Quantum chromodynamics (QCD)

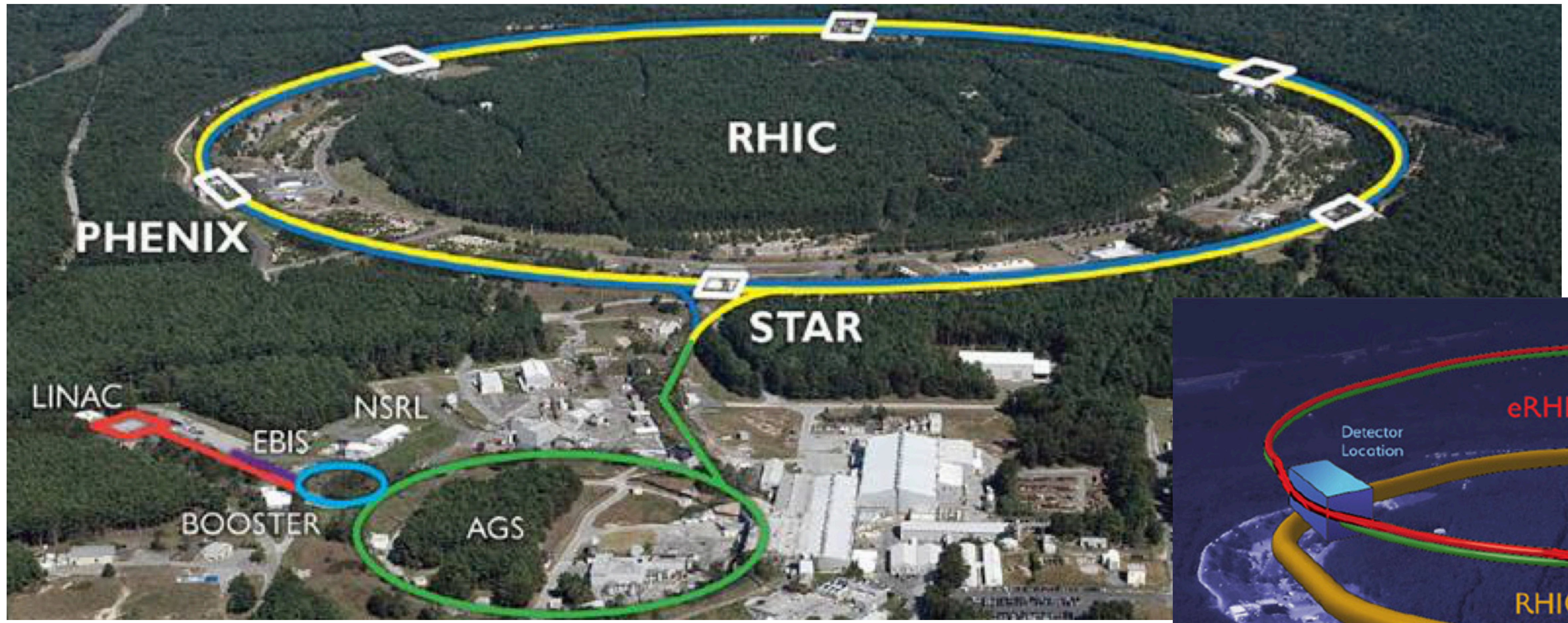
Large Hadron Collider CERN

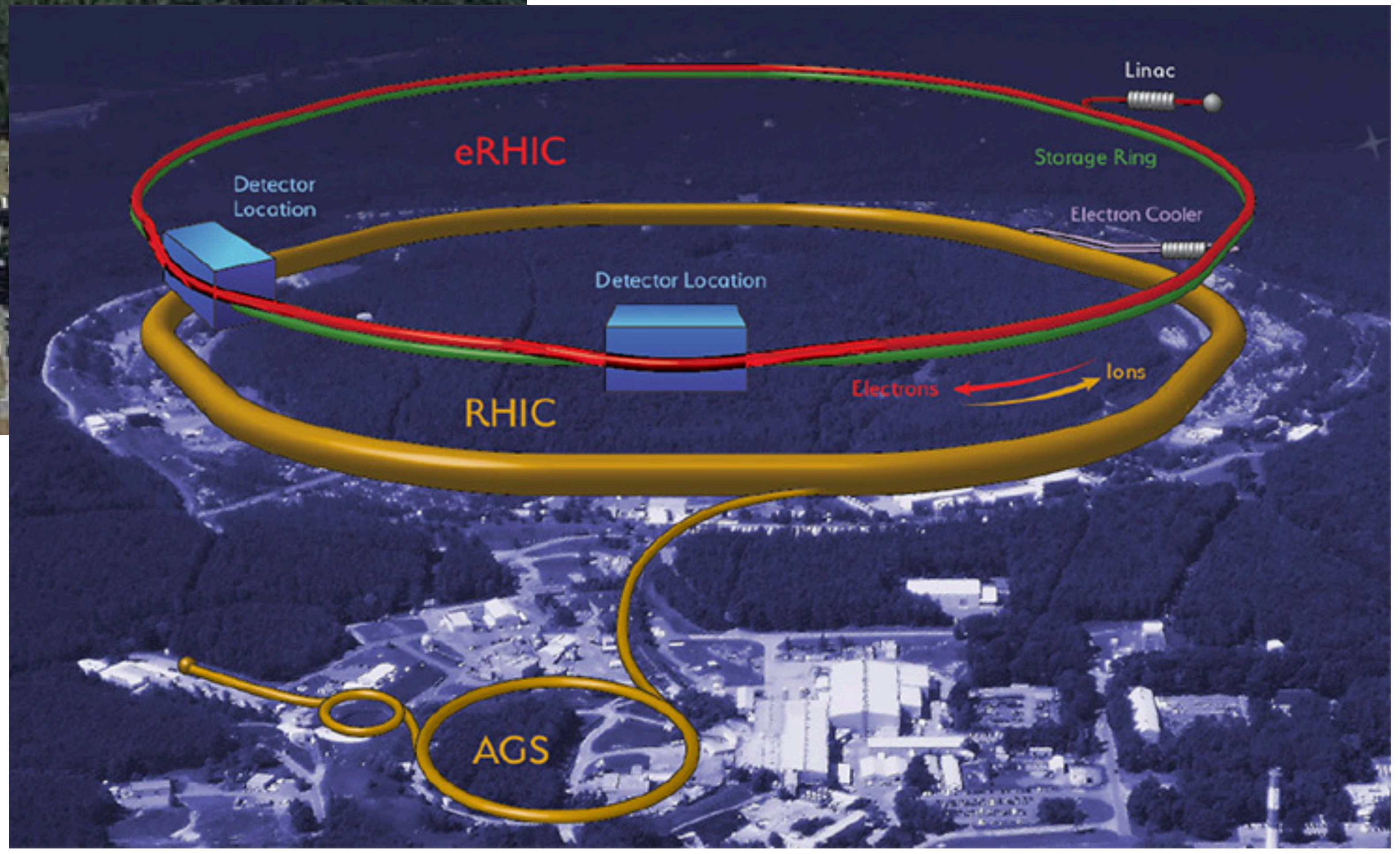
CEBAF JLab



RHIC & EIC

BNL

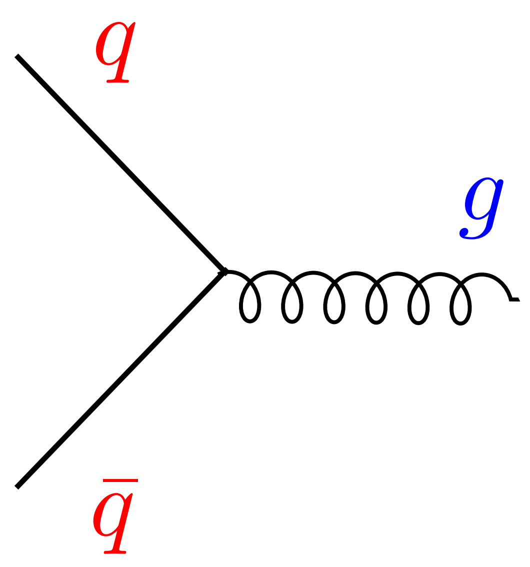


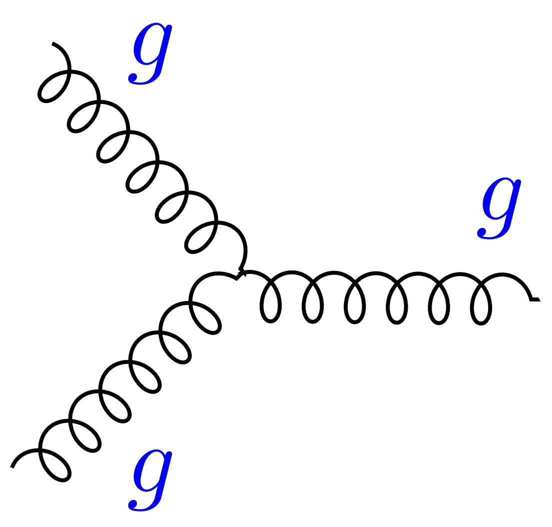


High-energy collider experiments

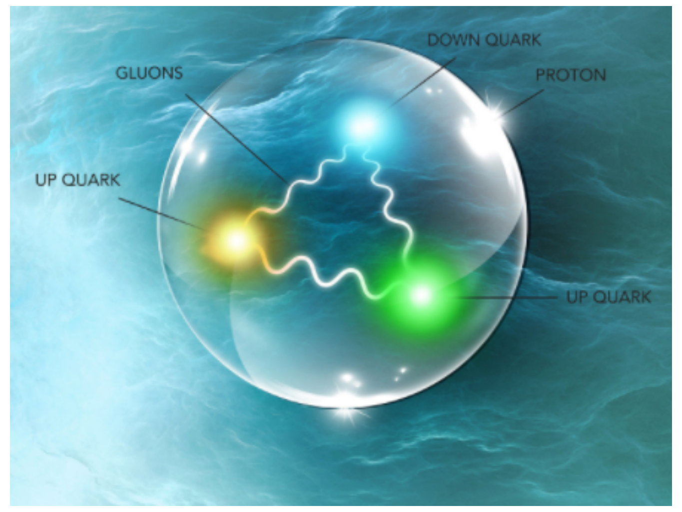
Quantum chromodynamics

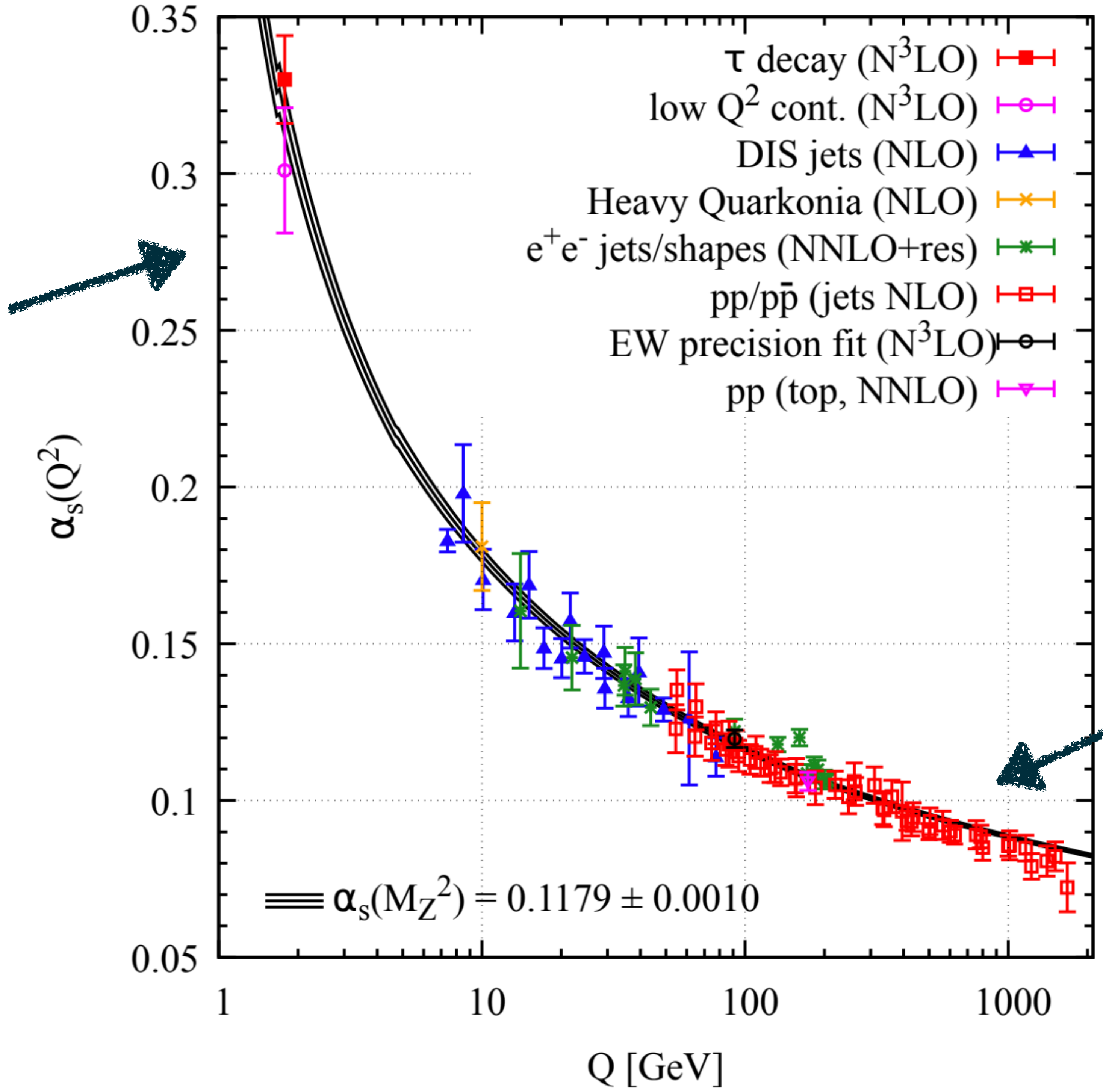
- Theory of the strong interaction between quarks and gluons
- The Lagrangian $\mathcal{L} = \bar{\psi}\left(i\partial^{\mu}\gamma_{\mu} - m\right)\psi + g_s\bar{\psi}\gamma^{\mu}T_a\psi A^a_{\mu} - \frac{1}{4}F^{\mu\nu}_a F^a_{\mu\nu}$





Confinement





Asymptotic freedom

Huston, Rabbertz, Zanderighi (PDG)

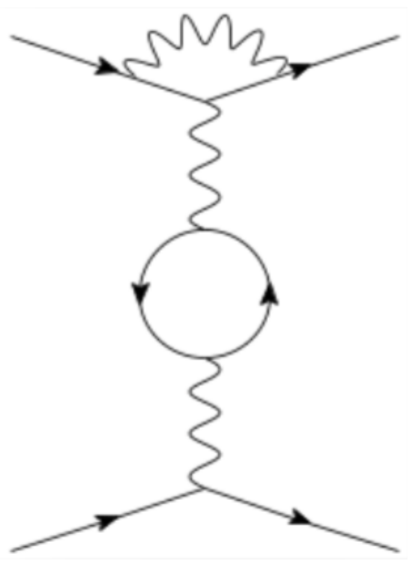
- QCD coupling constant $\alpha_s = \frac{g_s^2}{4\pi}$

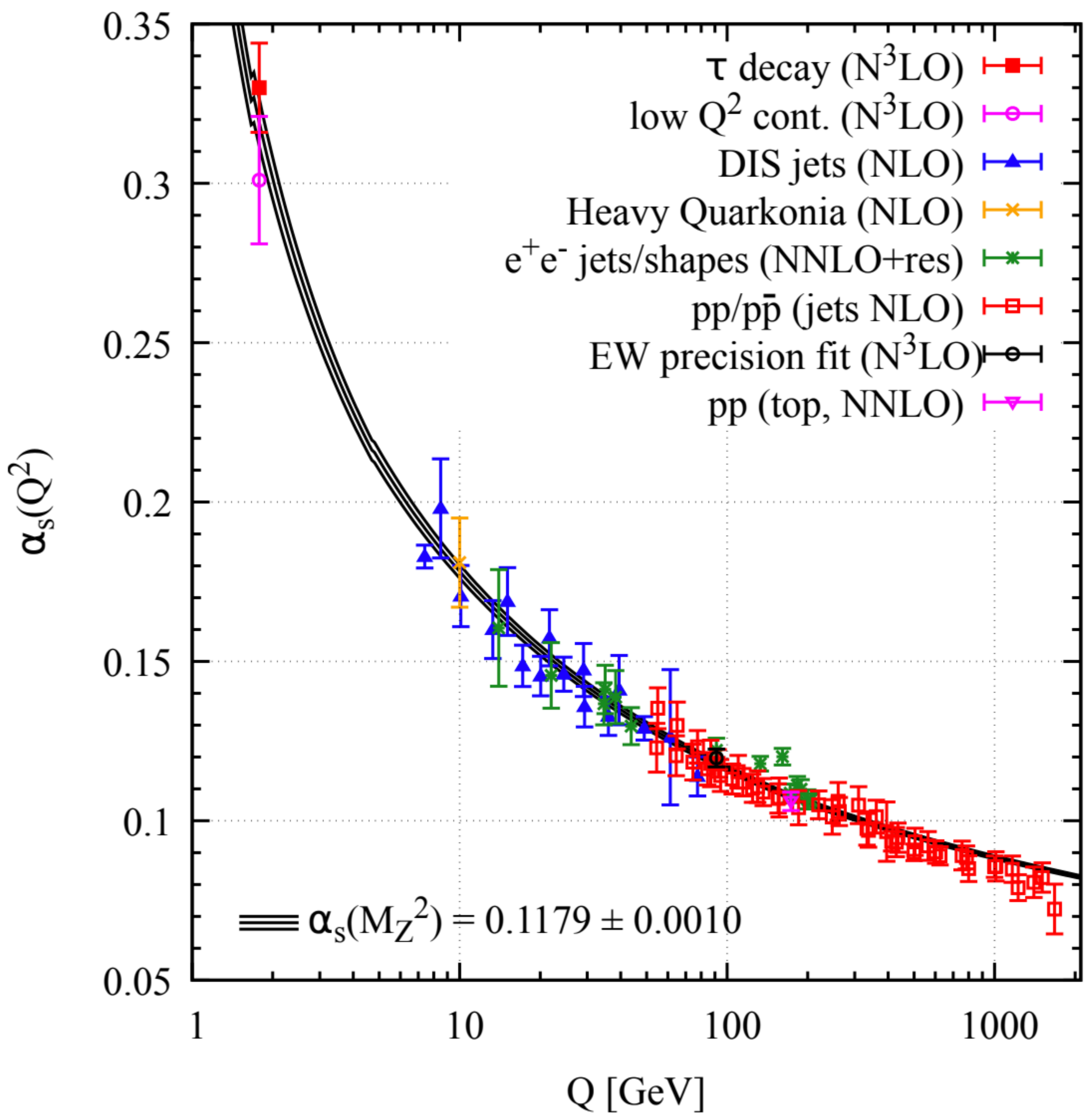
Nobel Prize 2004

Quantum chromodynamics

1. Perturbative QCD at high energies

$$\sigma = \sigma^{(0)} + \alpha_s\,\sigma^{(1)} + \alpha_s^2\,\sigma^{(2)} + \ldots$$

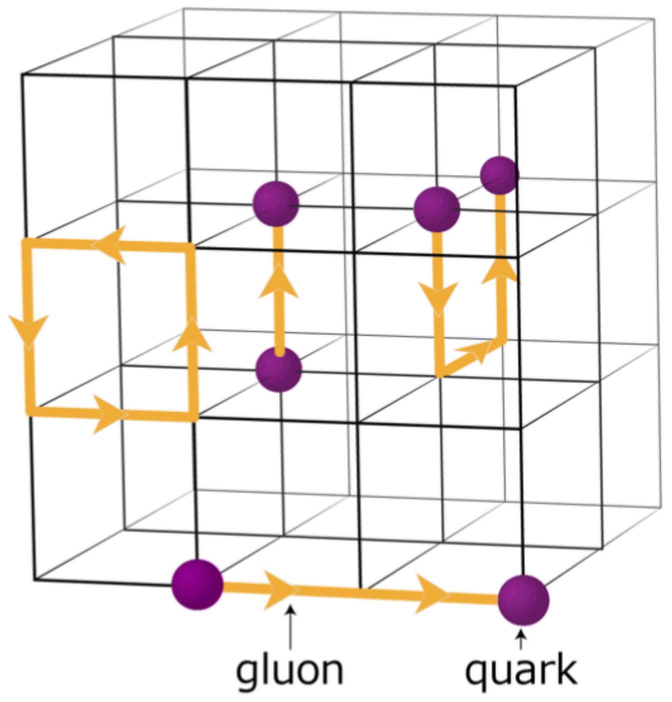




Huston, Rabbertz, Zanderighi (PDG)

2. Nonperturbative at low energies

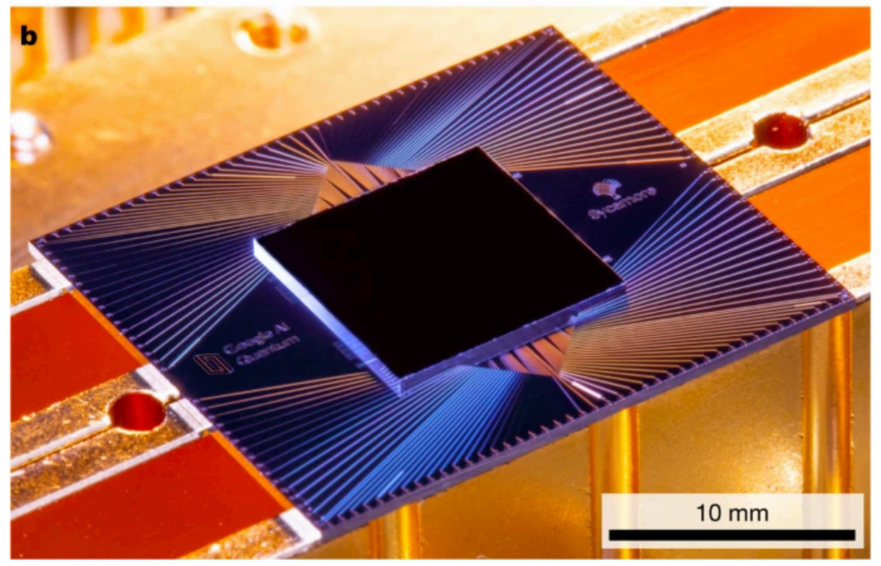
- Simulations using a lattice discretization
- Path integral vs. Hamiltonian formulation (imaginary vs. real time)
- Quantum computing for real-time dynamics?



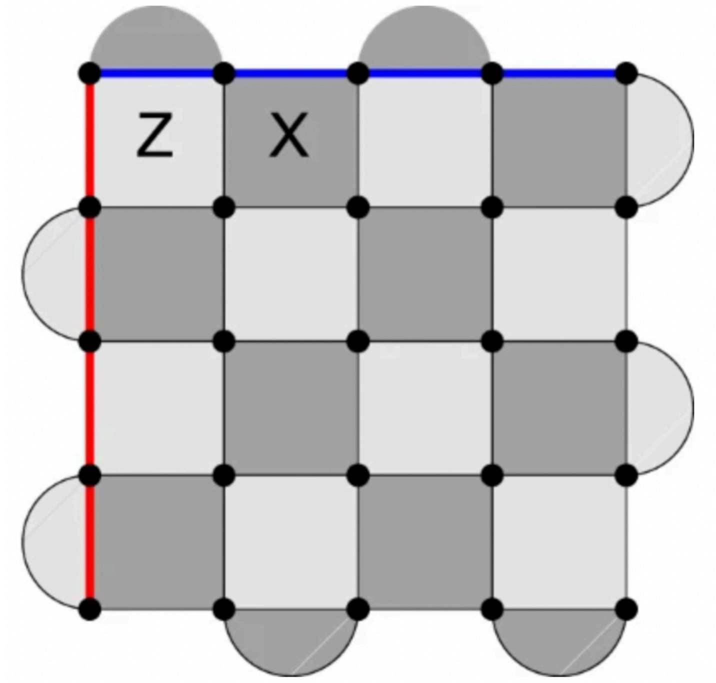
Wilson; Kogut, Susskind `70s

Quantum computing & fundamental physics

- Quantum advantage for random circuit sampling (2019)



- Progress toward long-lived logical qubits using quantum error correction (2023, 24)



Quantum computing & fundamental physics

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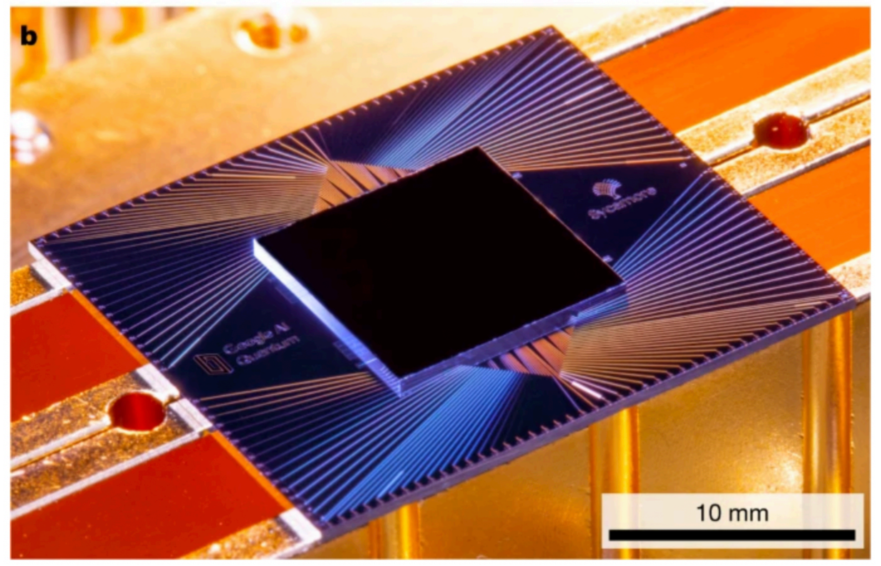
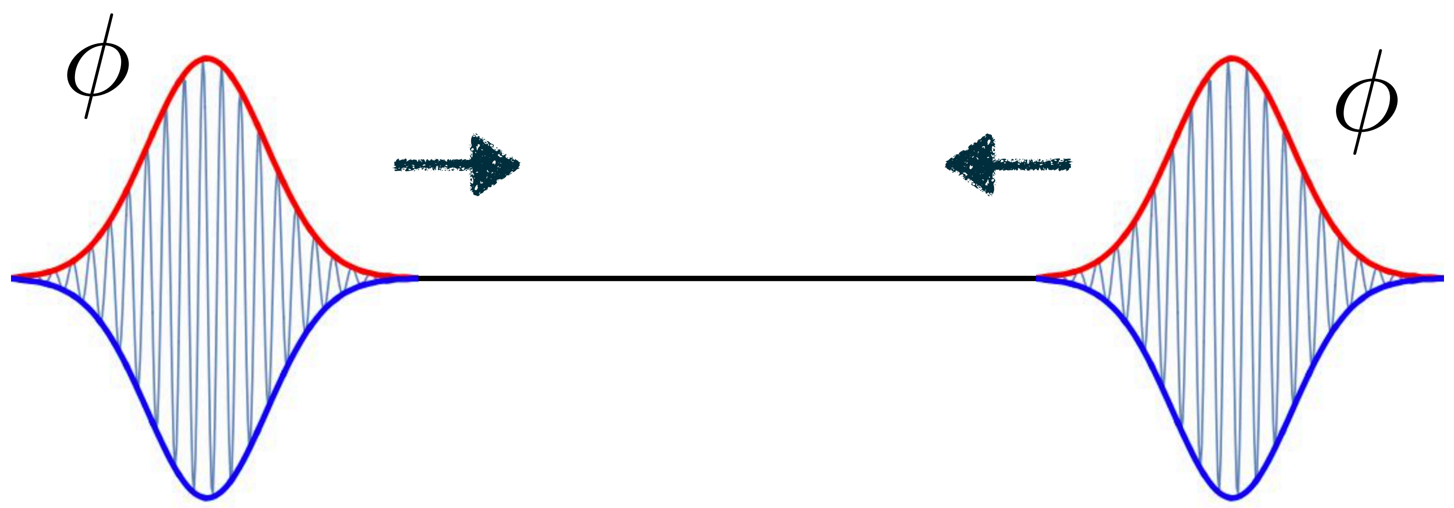


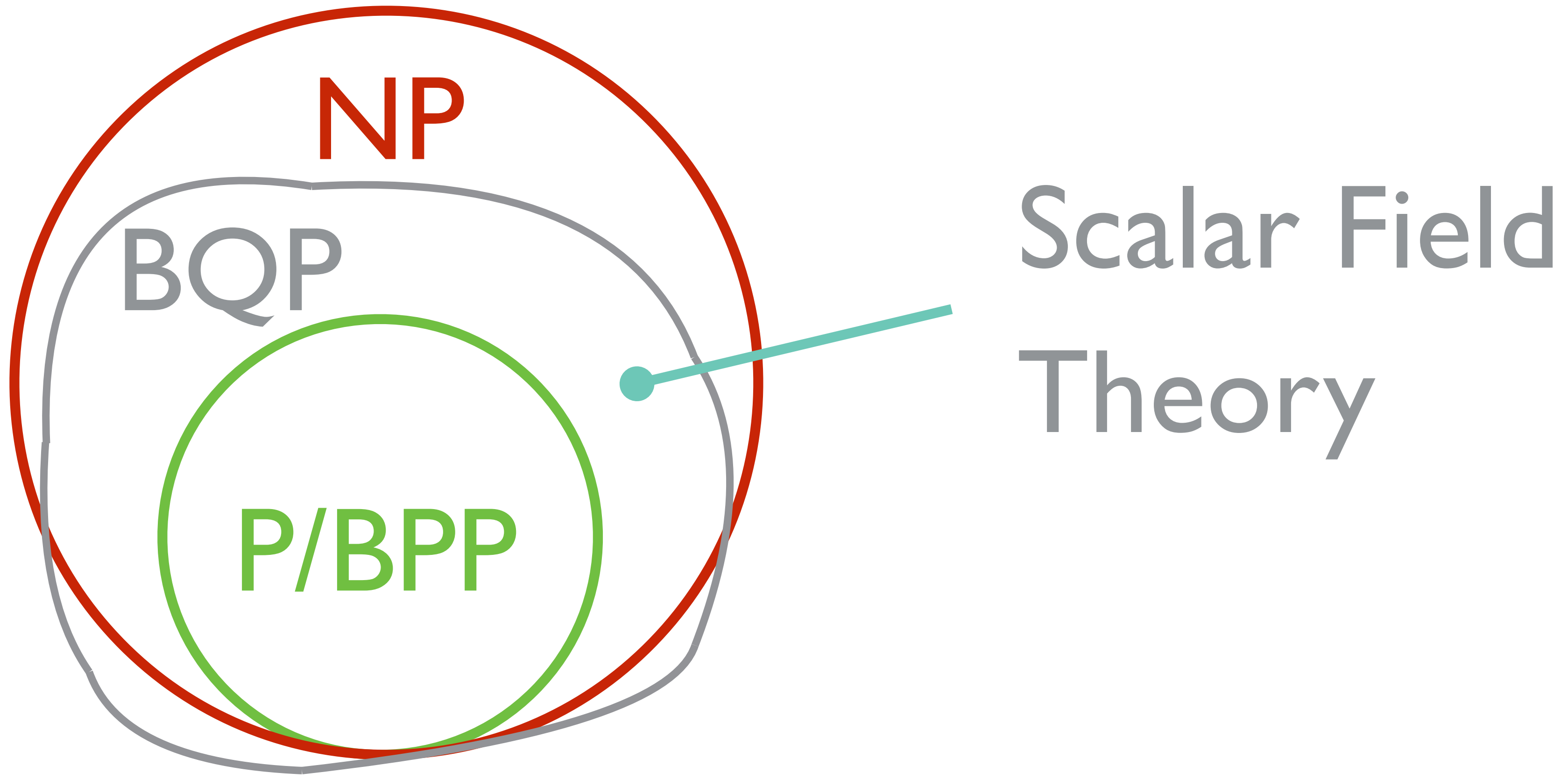
- Progress toward long-lived logical qubits using quantum error correction (2023, 24)

- Scalar field theory $\left|\langle X \left| U\left(t, t_0\right)\right| \phi\phi\rangle\right|^2$

Jordan, Lee, Preskill `10-`17



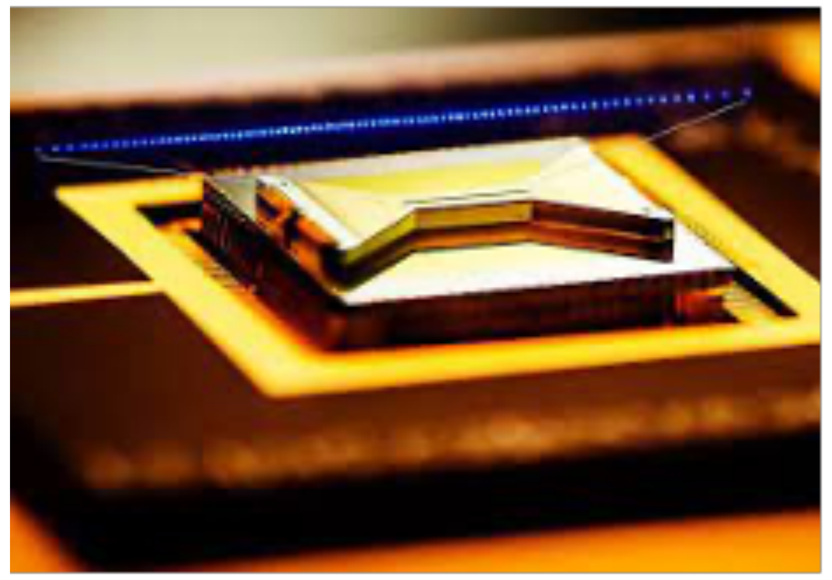
- Is QCD scattering in BQP?

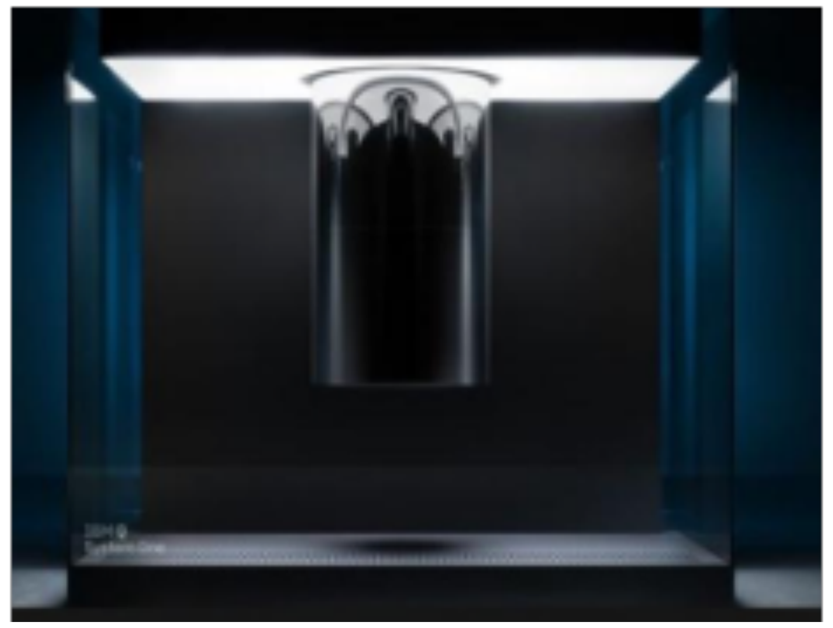


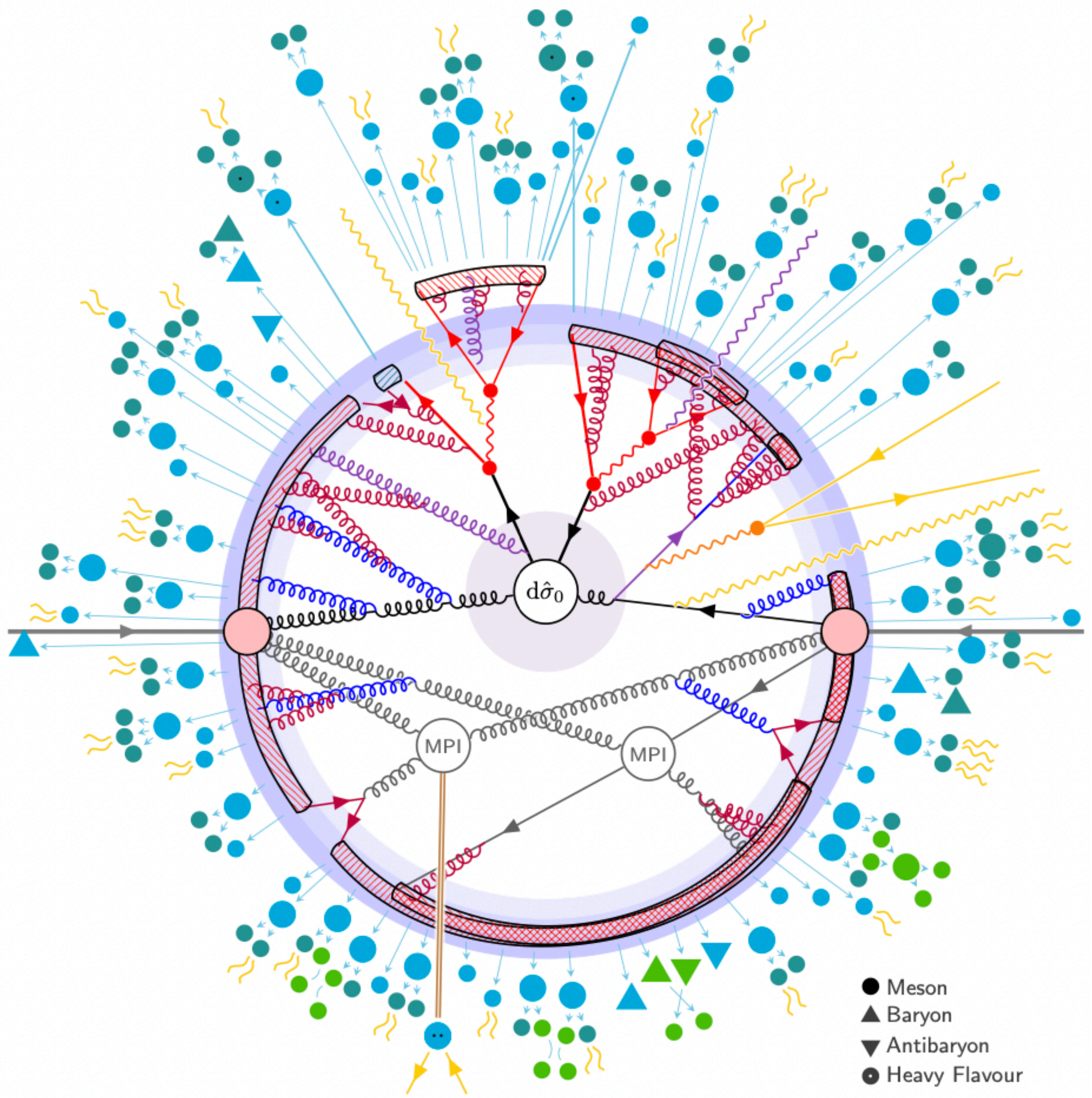
Toward simulating scattering processes

- Need to simulate classically intractable real-time dynamics

- Compton scattering amplitudes, in-medium correlation functions or full scattering events







$$\sim \int \mathrm{d}t\, \mathrm{d}\vec{x}\, e^{iq\cdot x} \langle P_f | \mathcal{T}[\mathcal{O}(t,\vec{x})\mathcal{O}(0)] | P_i \rangle$$

QCD factorization

Outline

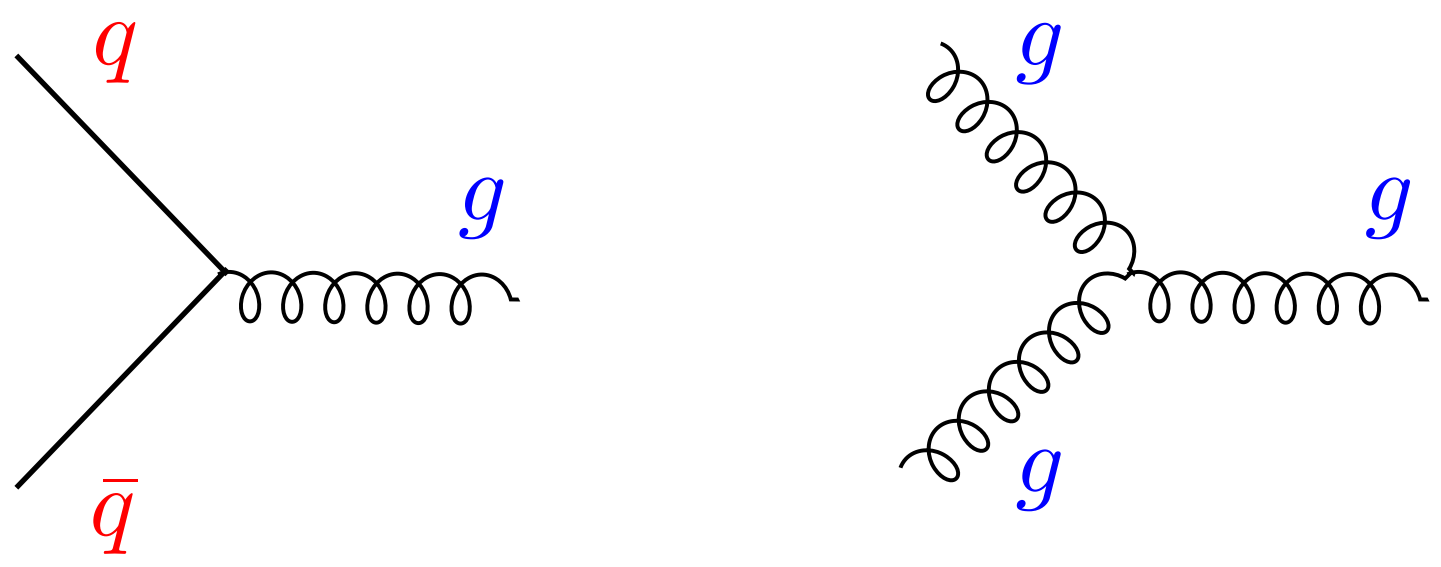
Qubits, qudits & qumodes

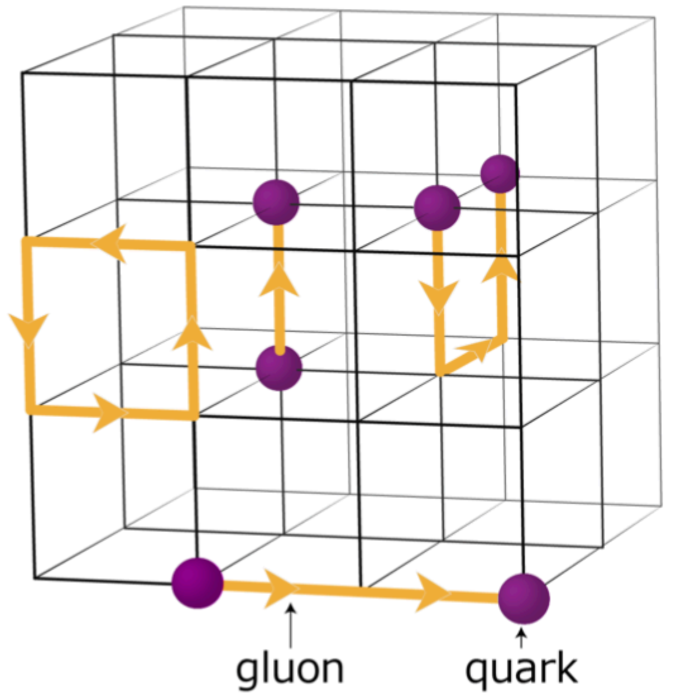
Lattice models and non-Abelian theories

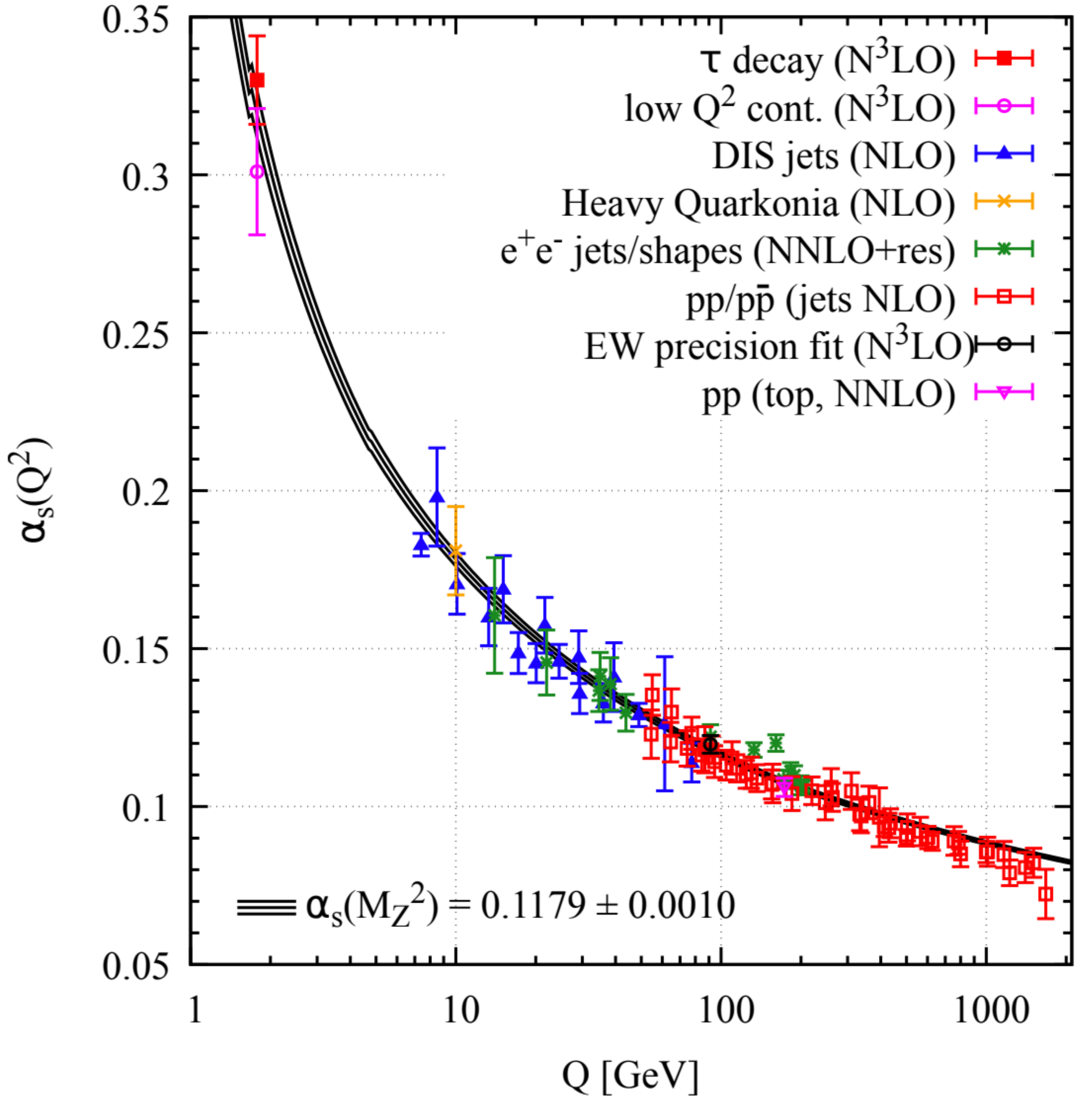
Applications in fundamental physics

Quantum chromodynamics

- Theory of the strong interaction between quarks and gluons
- The Lagrangian $\mathcal{L} = \bar{\psi}\left(i\partial^\mu\gamma_\mu - m\right)\psi + g_s\bar{\psi}\gamma^\mu T_a \psi A^a_\mu - \frac{1}{4}F_a^{\mu\nu}F^a_{\mu\nu}$







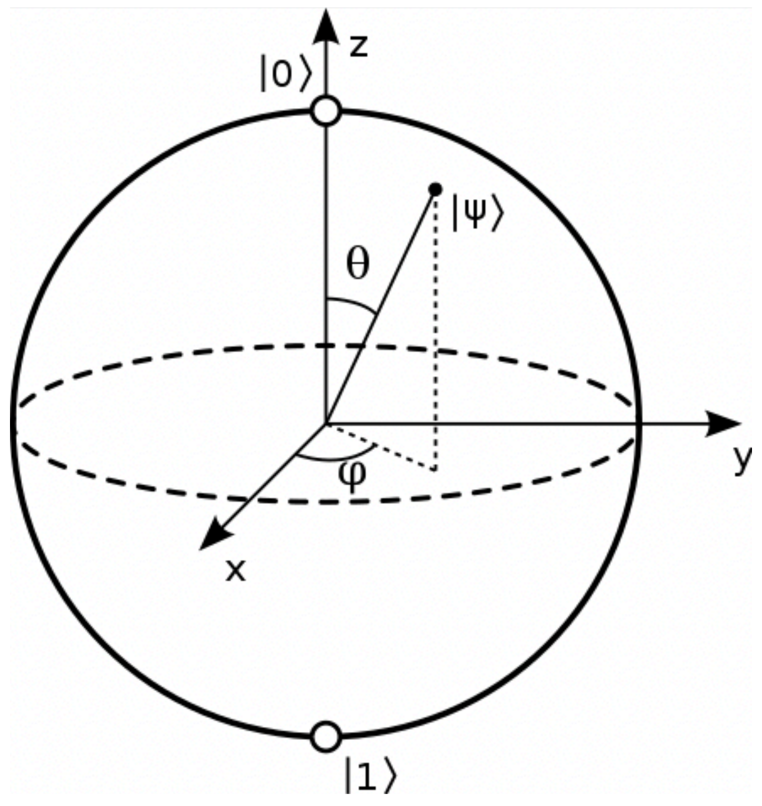
Huston, Rabbertz, Zanderighi (PDG)

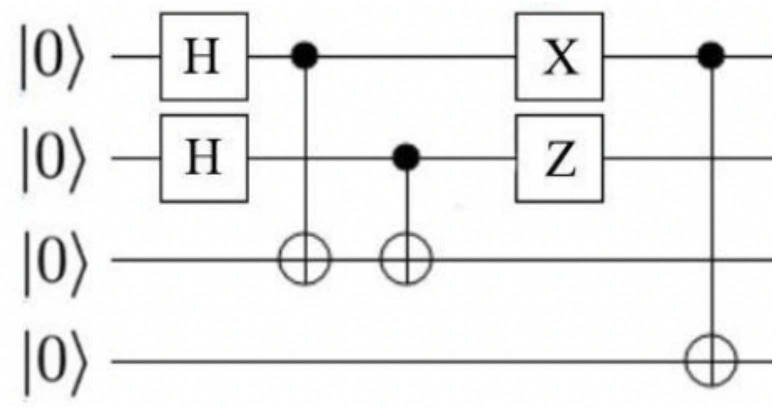
Simulate quarks and gluons with different quantum resources?

Qubits, qudits and qumodes

Elementary units for computing

- **Qubits**
  - Superconducting circuits, cold atoms, trapped ions, topological qubits
  - Digital gate-based computing





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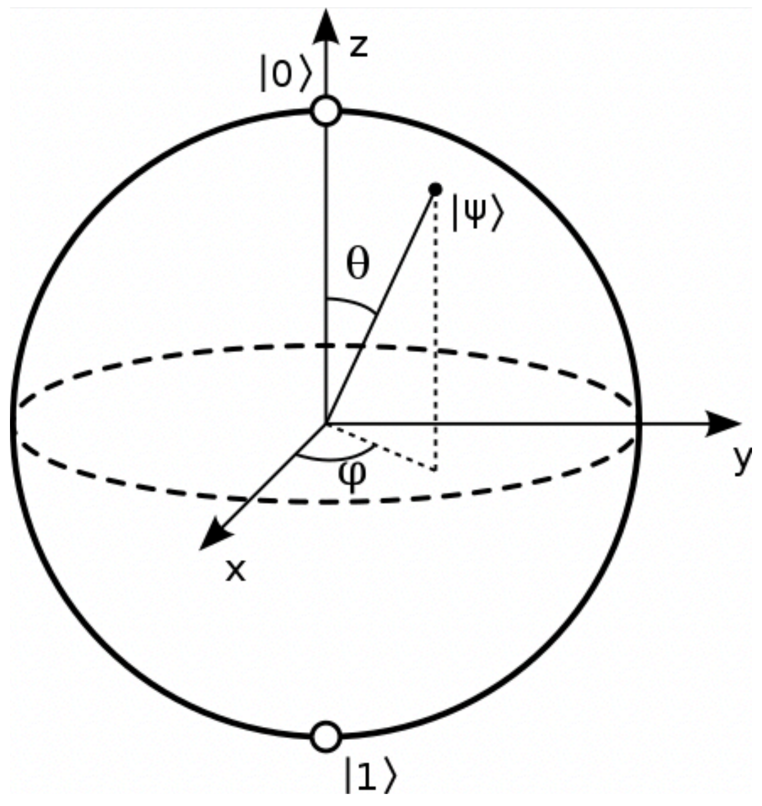
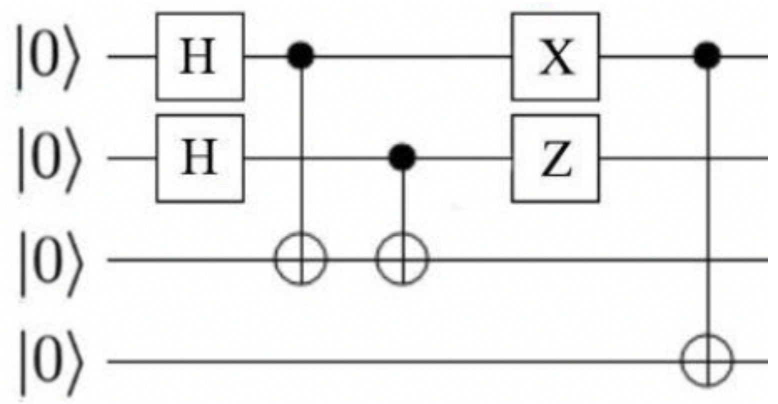
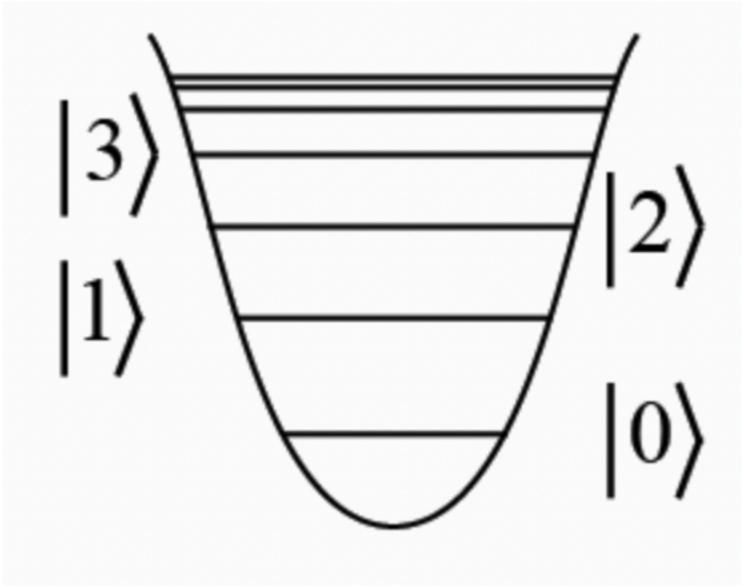




- **Qudits**
  - Multi-level $d > 2$ computational units
  - Gates e.g. $X_d,\ Z_d,\ C_2[R_d]$
  - Various hardware platforms



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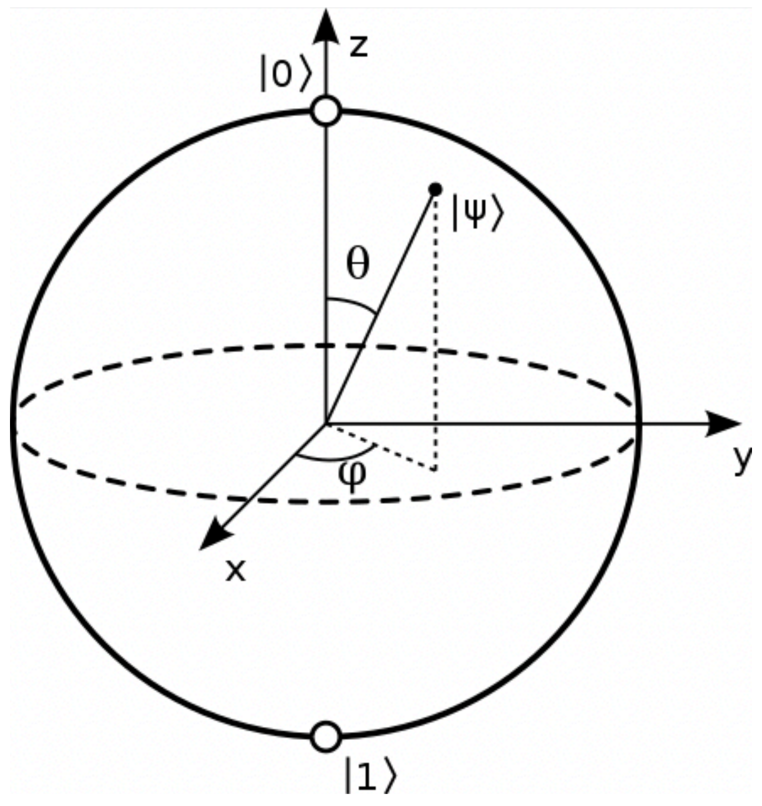
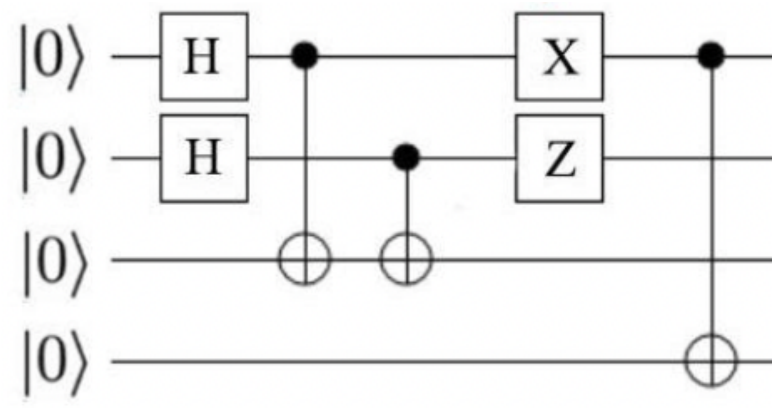
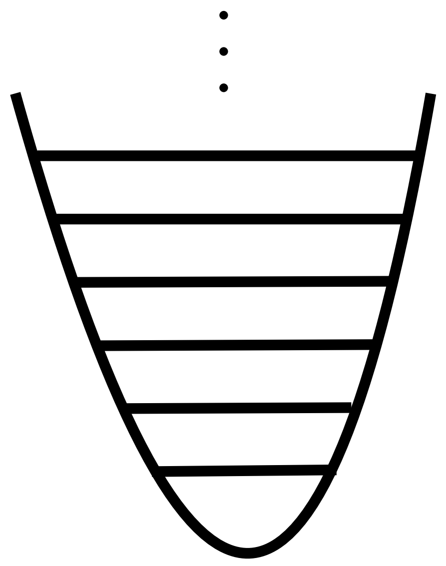
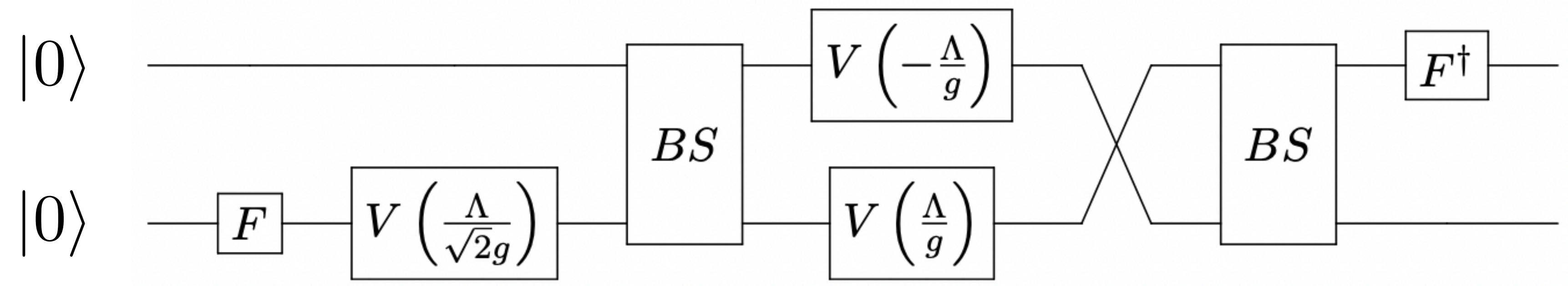




- **Qumodes**
  - Photonics, trapped ions, superconducting circuits
  - Infinite-dimensional Hilbert space
  - Gate based but with continuous variables



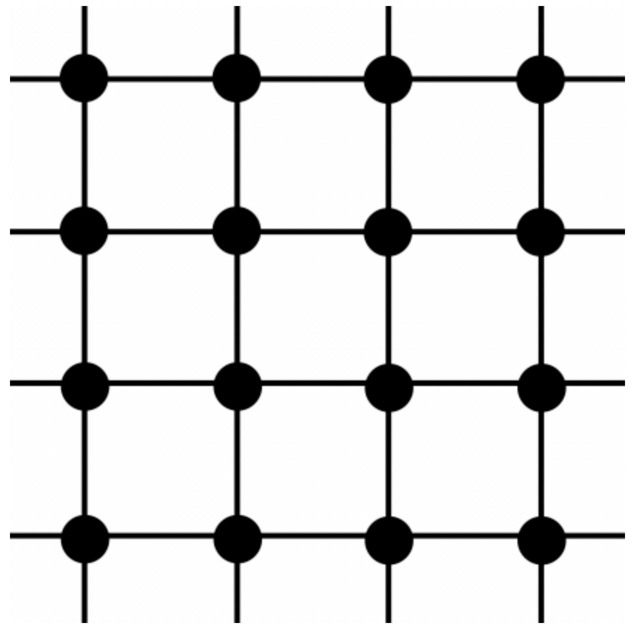


Quantum simulations for collider physics

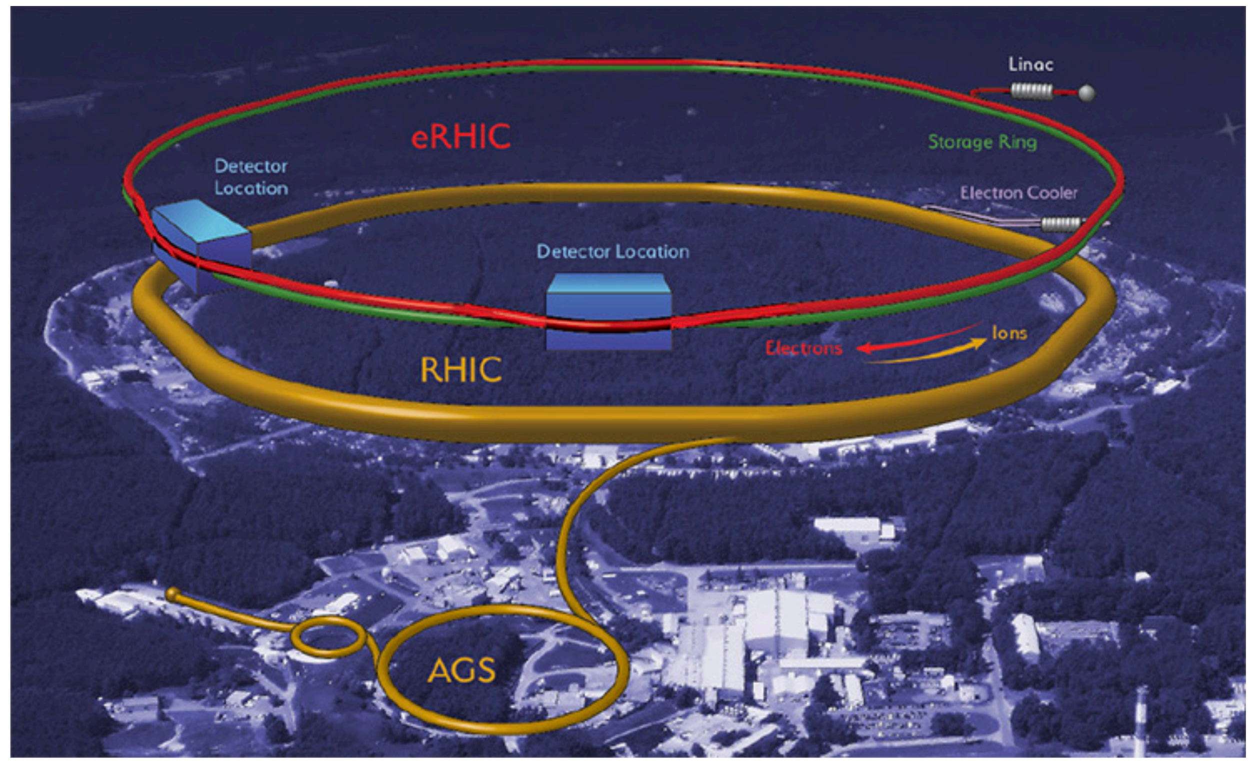
Kogut, Susskind `70s, Jordan, Lee, Preskill `11-`17

- Use the Hamiltonian formulation instead

$$\mathcal{H} = \frac{1}{2}\pi^2 + \frac{1}{2}\left(\nabla\phi\right)^2 + \frac{1}{2}m_0^2\phi^2 + \frac{\lambda}{4!}\phi^4$$



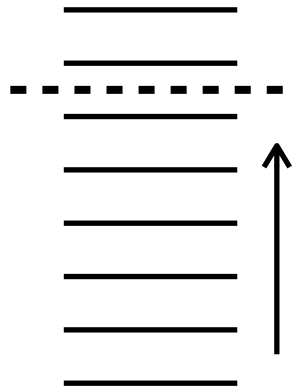
$\phi(x)$



  - Discretize spatial component $x = an\,, \quad n = 0, \ldots, L-1$

  - Truncate the local Hilbert space

Λ



Time-dependent n-point correlation functions $\langle \mathcal{O}(t_1)\mathcal{O}(t_2)\rangle$

- Take limits $L, \Lambda \to \infty\,, \; a \to 0$ **+** finite volume formalism

Toward quantum simulations for QCD

Marshall, Pooser, Siopsis, Weedbrock `15
Briceno, Edwards, Eaton, Gonzalez, Pfister, Siopsis `23
Abel, Spannowsky, Williams `24

- Qumodes or continuous variables well suited for bosonic modes, scalar ϕ^4, and gauge fields

ϕ, t continuous, $x = an$ discretized on a lattice

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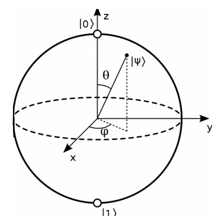
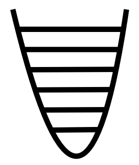
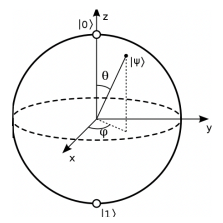
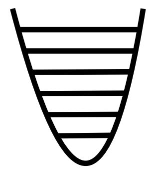
- QCD Lagrangian

$$\mathcal{L} = \bar{\psi}\left(i\partial^\mu\gamma_\mu - m\right)\psi + g_s\bar{\psi}\gamma^\mu T_a\psi A^a_\mu - \frac{1}{4}F^{\mu\nu}_a F^a_{\mu\nu}$$

Qubits 

Qumodes 

Qubit-qumode coupling  

→ **Develop a hybrid qubit/qumode approach**

$$\mathcal{H}^m_{\text{qumode}} \otimes \mathcal{H}^n_{\text{qubit}}$$

Quantum simulations with qumodes

see Lloyd, Braunstein `99

- Bosonic raising/lowering operators $\hat{a}, \hat{a}^\dagger$ and Fock states

$$|n\rangle = \frac{(a^\dagger)^n}{\sqrt{n!}}|0\rangle$$

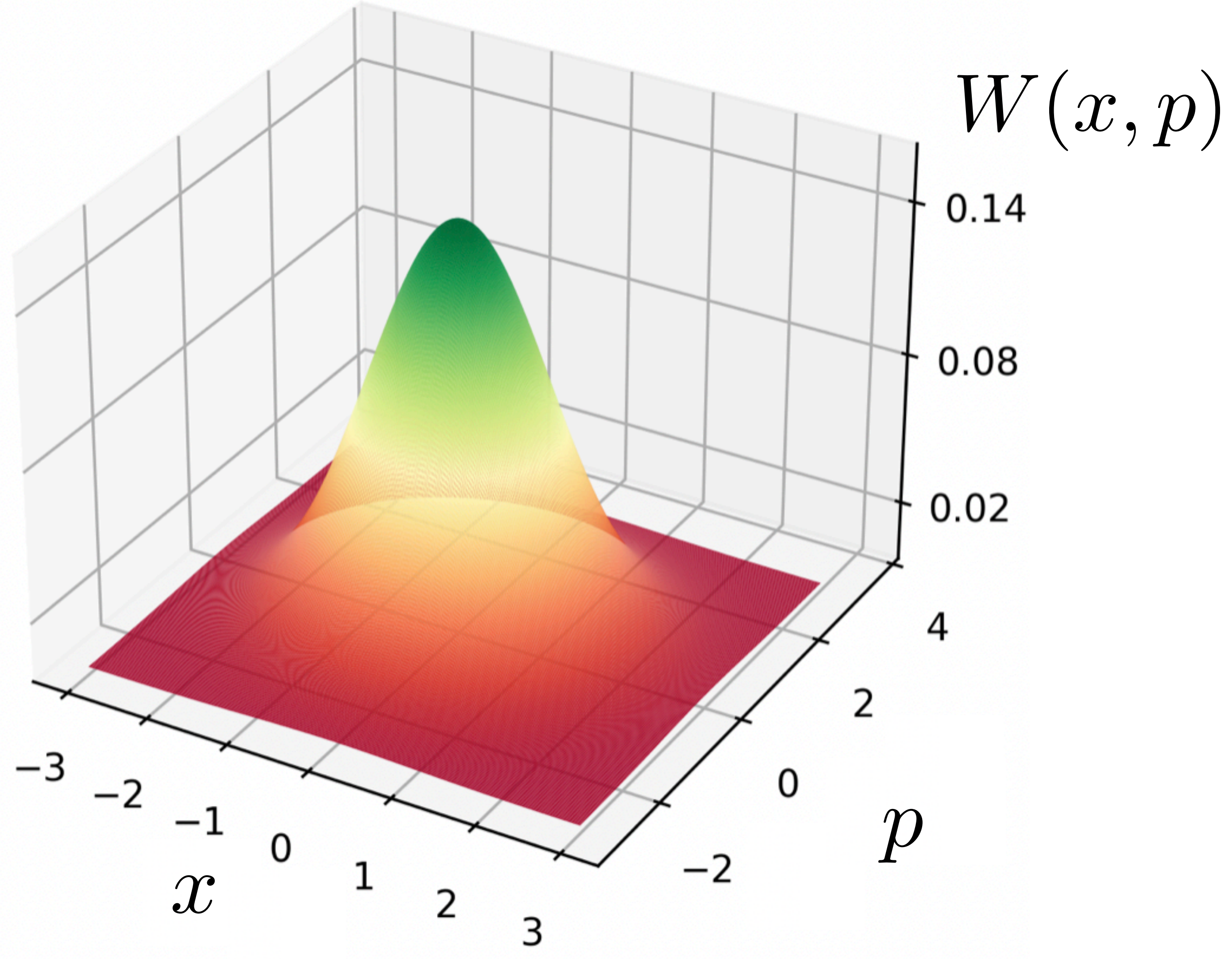
- Quadratures $\hat{X}, \hat{P}$ with $[\hat{X}, \hat{P}] = i\hbar$ and state

$$|\psi\rangle = \int_{\mathbb{R}} \psi(x)\,|x\rangle\,\mathrm{d}x$$

- Visualize using Wigner functions

$$W(x,p) = \frac{1}{\pi\hbar}\int_{-\infty}^{\infty} \psi^*(x+y)\psi(x-y)e^{2ipy/\hbar}\mathrm{d}y$$

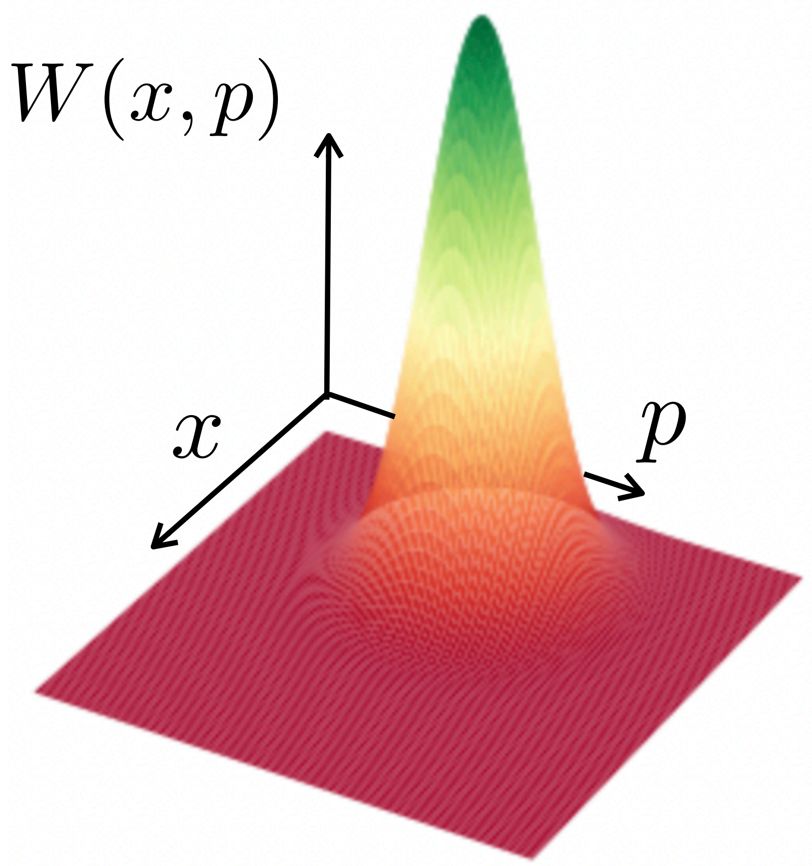
Ground state (in the Fock basis) $|0\rangle$



Quantum simulations with qumodes

see Lloyd, Braunstein `99

Universal gate set



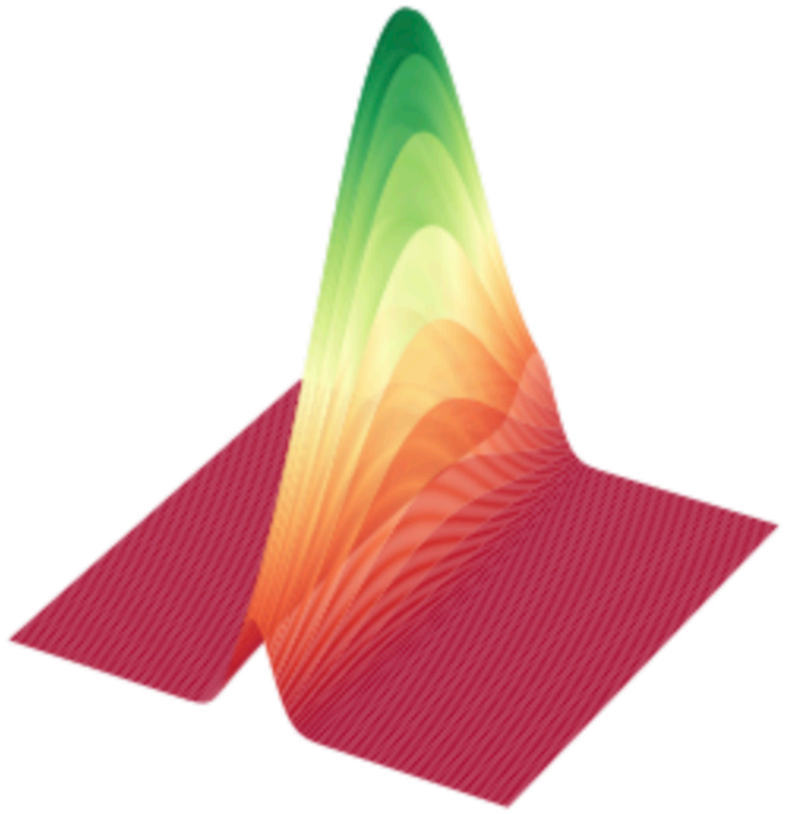
- Displacement $D(z) = e^{z\hat{a}^\dagger - z^*\hat{a}}$
- Rotation $R(\theta) = e^{i\theta\hat{a}^\dagger\hat{a}}$
- Two-qumode beam splitter

$$U_{\text{bs}}(z) = e^{z\hat{a}\hat{b}^\dagger - z^*\hat{a}^\dagger\hat{b}}$$

$$z = \theta e^{i\phi}$$

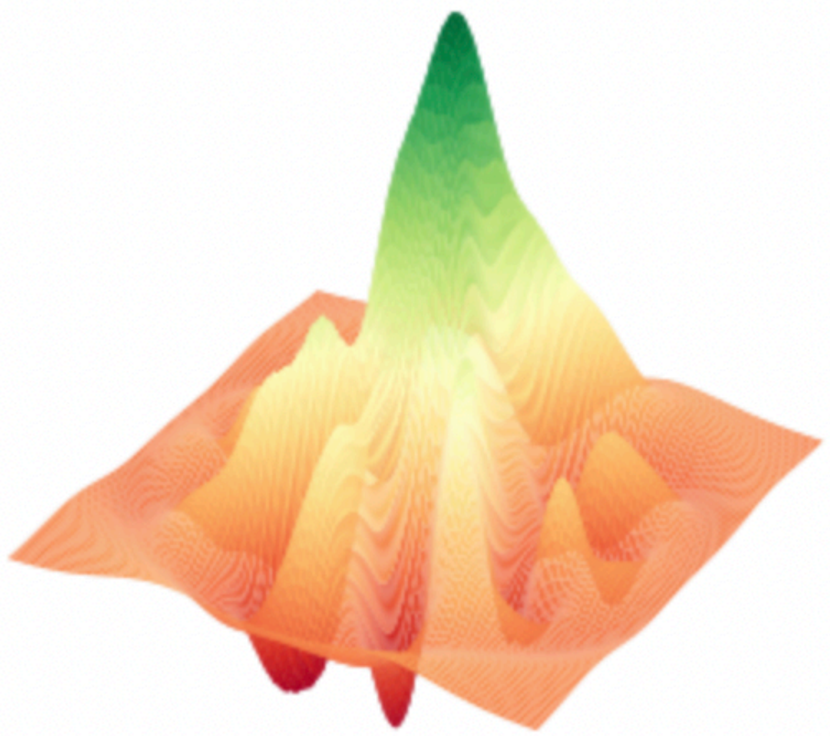
- Squeezing

$$S(z) = e^{\frac{1}{2}(z^*\hat{a}^2 - z\hat{a}^{\dagger 2})}$$



- Non-Gaussian operations

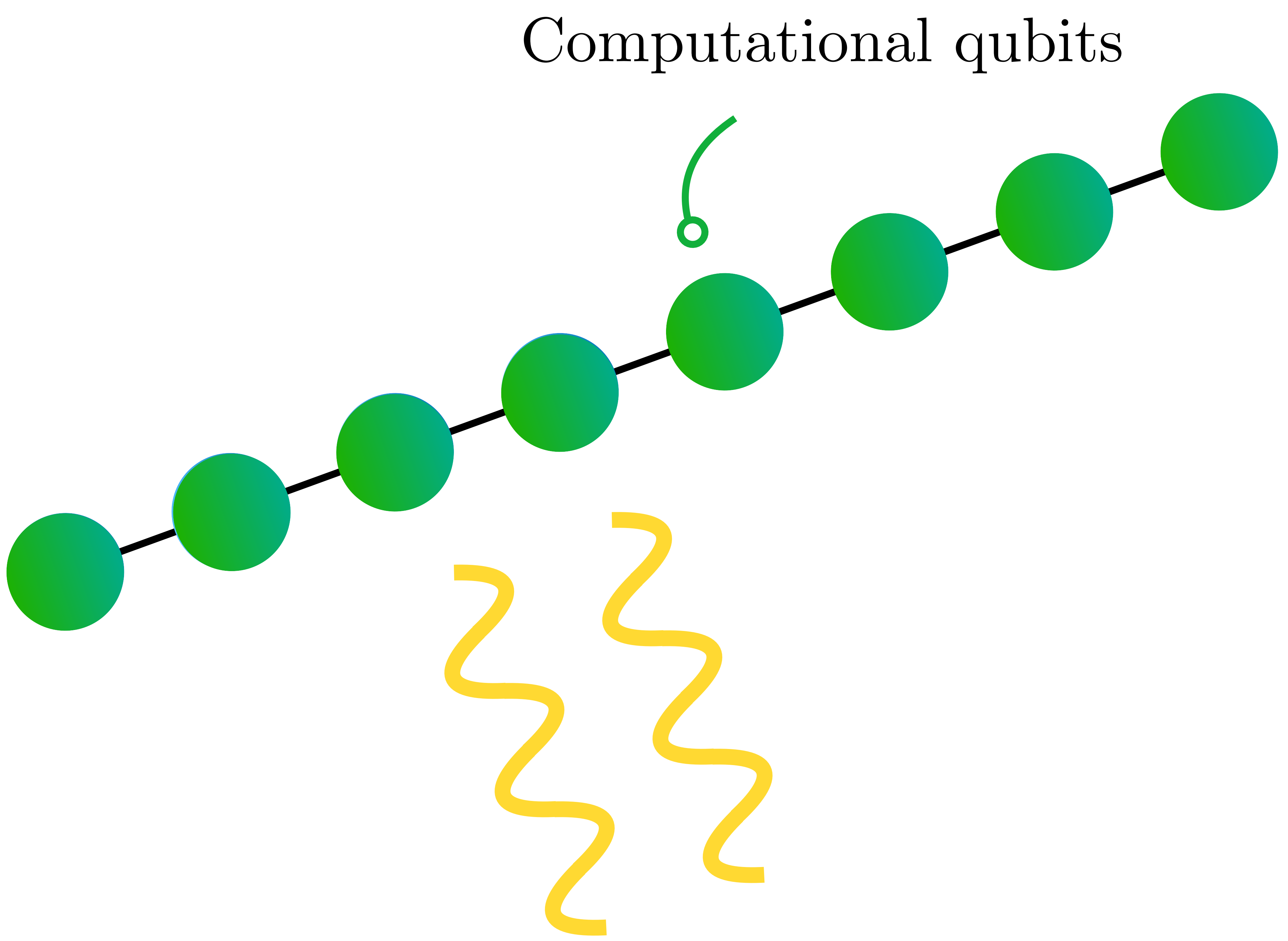
e.g. Kerr gate



Qubit & qumodes with trapped ions

Araz, Grau, Montgomery, FR `24

- **Qubits:** electronic states of the trapped ion

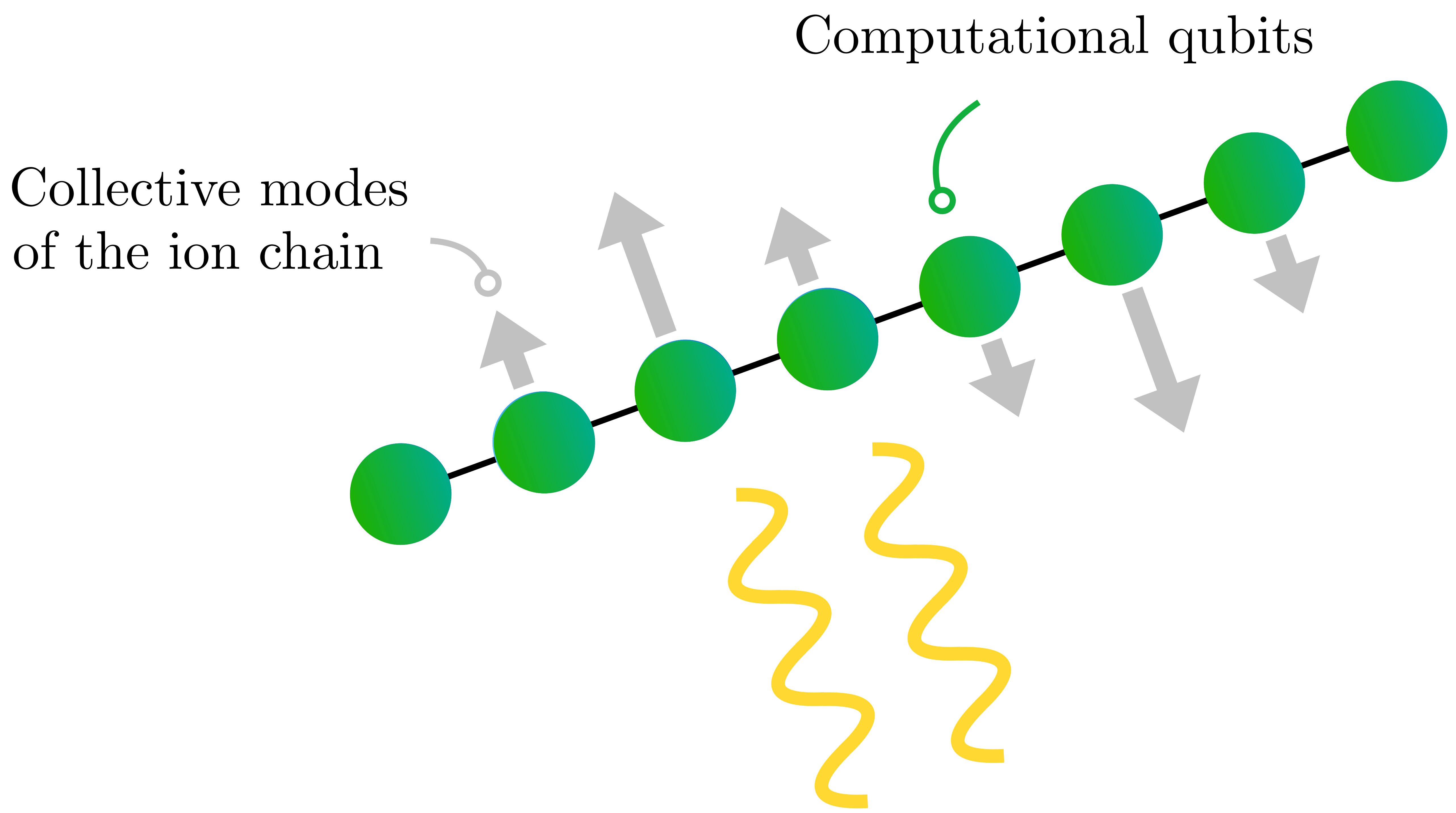


Circuit QED, see Bosonic Qiskit, Girvin, Wiebe et al. `22

Qubit & qumodes with trapped ions

- **Qubits:** electronic states of the trapped ion
- **Qumodes:** collective vibrational modes of the ion chain (phonons)

Araz, Grau, Montgomery, FR `24



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Qubit & qumodes with trapped ions

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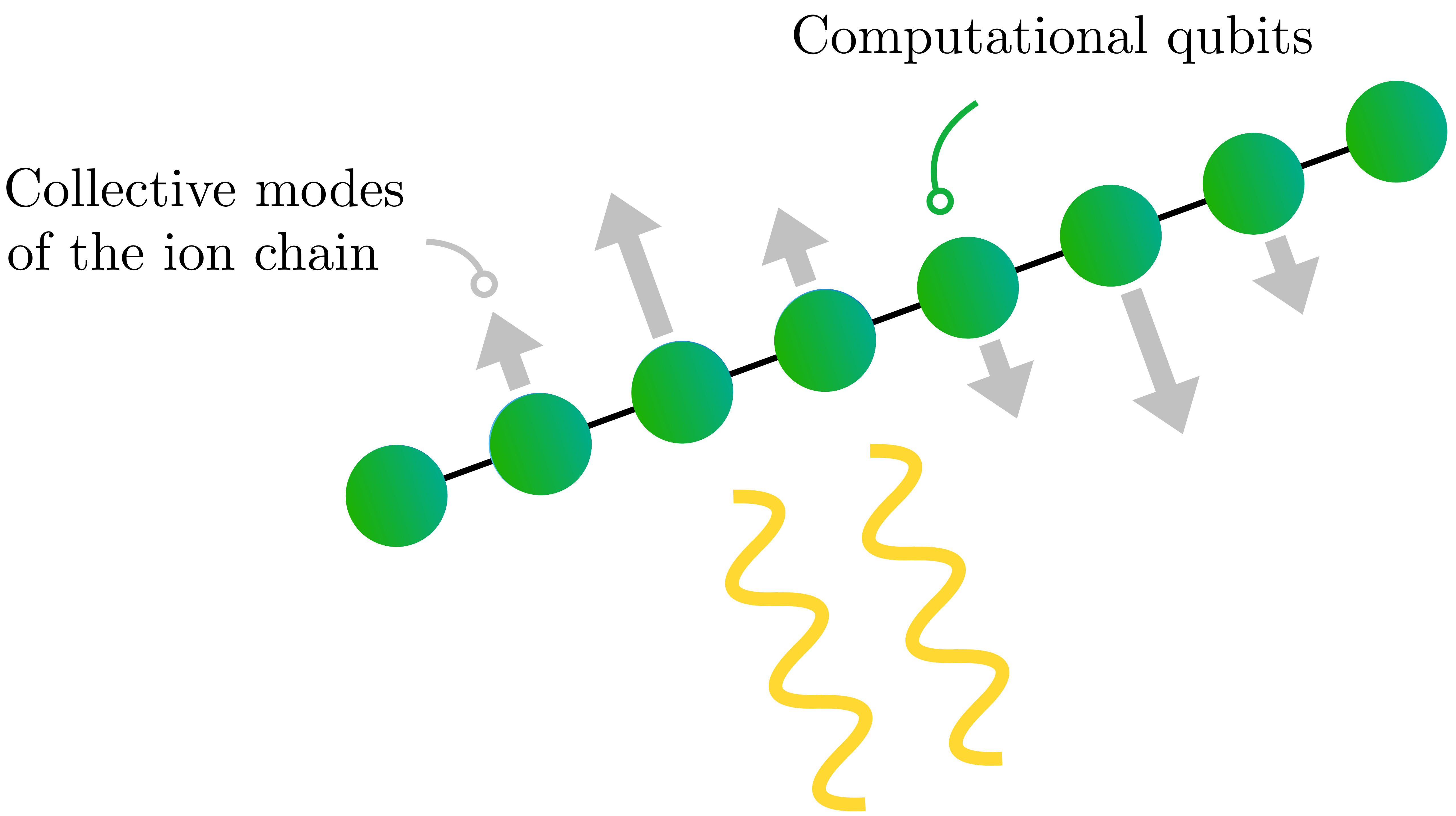
- **Qubits:** electronic states of the trapped ion
- **Qumodes:** collective vibrational modes of the ion chain (phonons)
- Collective modes are more resilient to environmental noise compared to local modes
- N ions → N axial and 2N radial modes



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Qubit & qumodes with trapped ions

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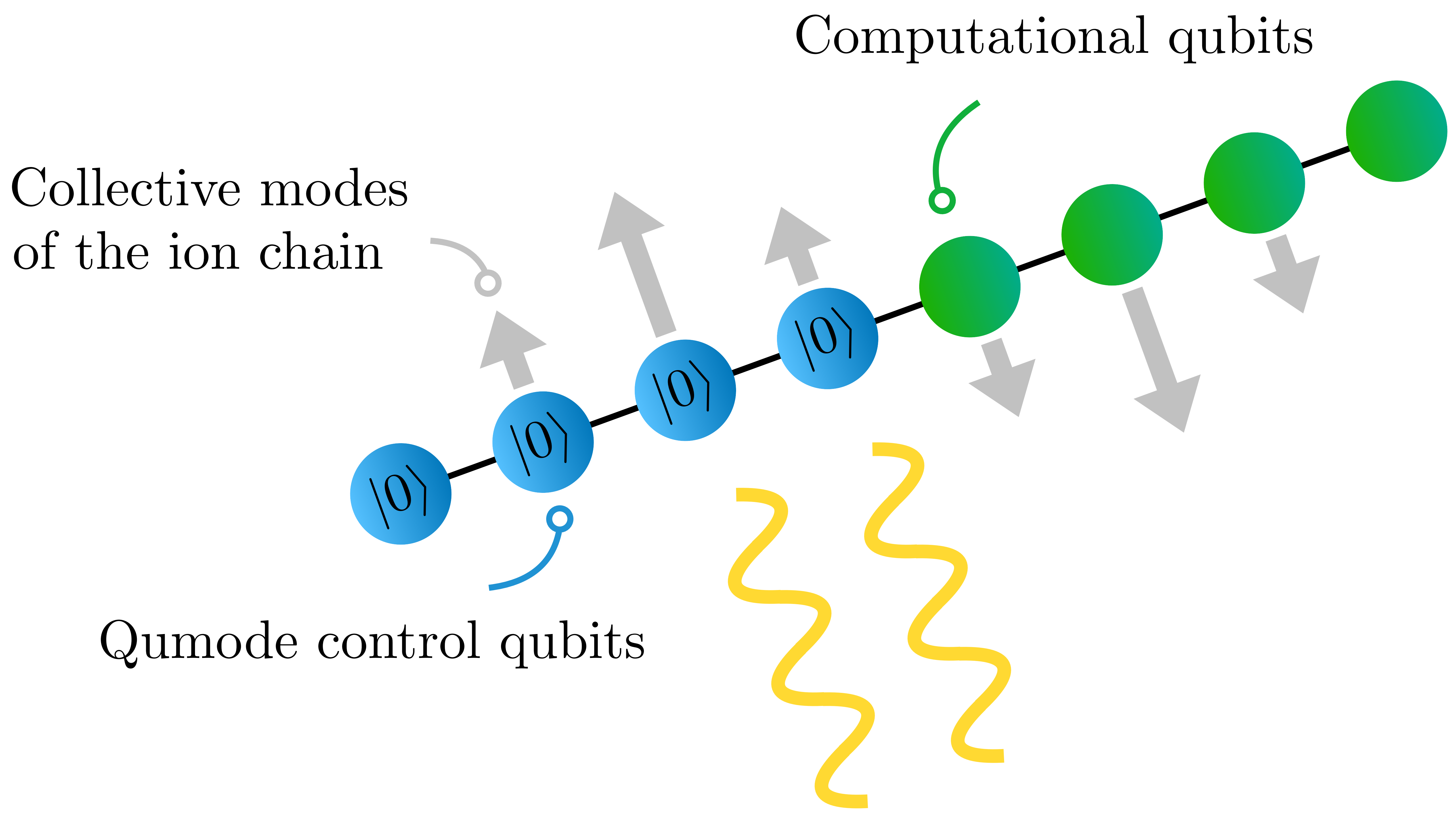
- **Qubits:** electronic states of the trapped ion

- **Qumodes:** collective vibrational modes of the ion chain (phonons)

- Laser interacts with qubits

→ Need several qubits to control and readout qumodes

→ But keep as many "computational qubits" as possible



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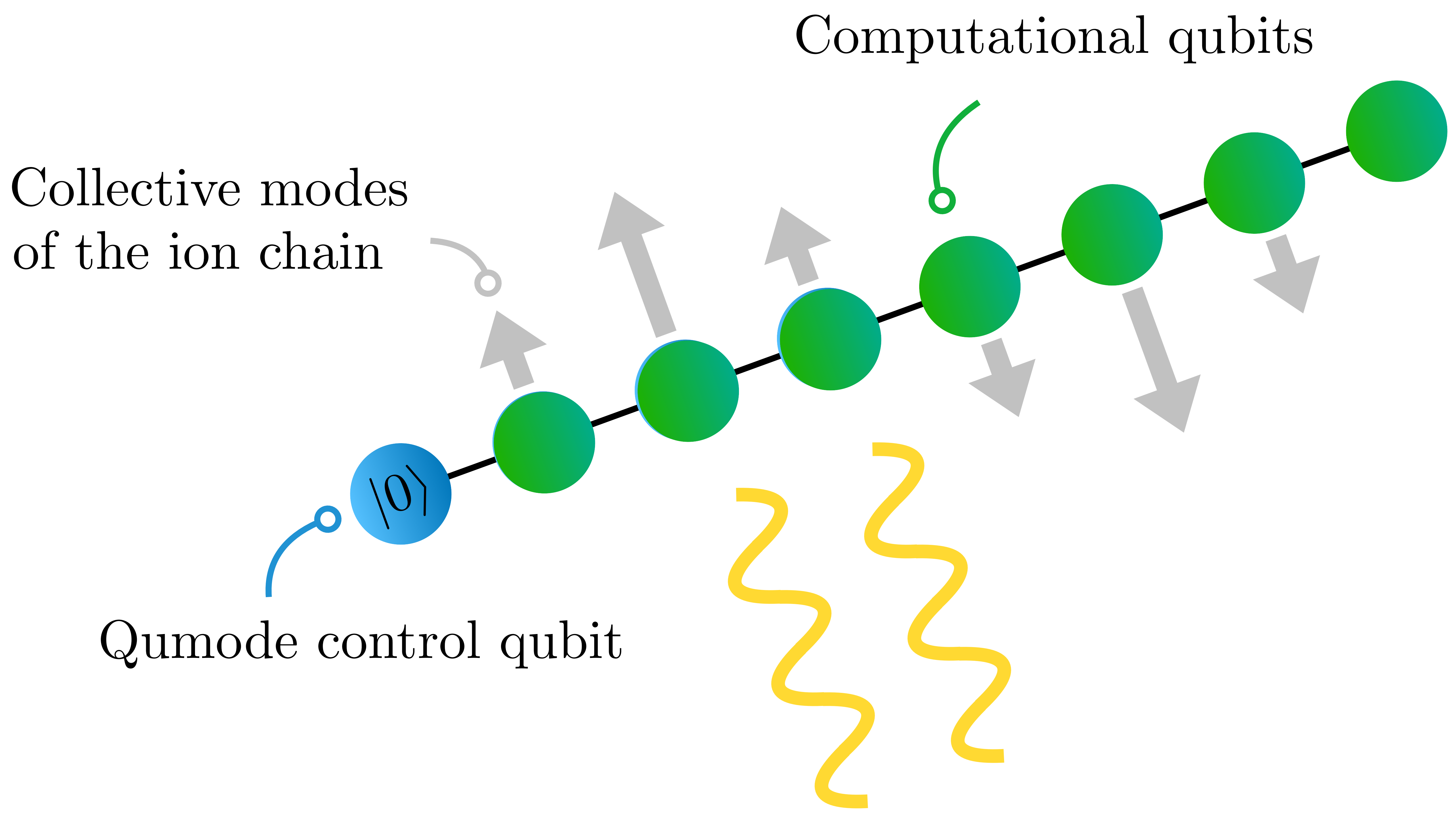
- Laser interacts with qubits

→ Need several qubits to control and readout qumodes

→ Feasibility studies with a minimal number of control qubits



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Qubit & qumodes with trapped ions

Araz, Grau, Montgomery, FR `24

Type	Operation	Short	Operator	Estimated gate time	Estimated fidelity	Ref.
Qubit gates	Pauli operators		σ^i	2 µs	99.999%	[92]
	Rotation	$\mathrm{R}_i(\theta)$	$e^{i\theta\sigma^i/2}$	2 µs	99.999%	[92]
	Controlled NOT	CNOT	$e^{i\frac{\pi}{4}(\mathbb{I}_1-\sigma_1^z)(\mathbb{I}_2-\sigma_2^x)}$	30 µs	99.9%	[93]
Qumode gates	Rotation	$\mathrm{R}(\theta)$	$e^{i\theta\hat{a}^\dagger\hat{a}}$	200 µs*	99%*	[94]
	Displacement	$\mathrm{D}(z)$	$e^{z\hat{a}^\dagger-z^*\hat{a}}$	10 µs	99%	[95]
	Single-mode squeezing	$\mathrm{S}(z)$	$e^{(z^*\hat{a}\hat{a}-z\hat{a}^\dagger\hat{a}^\dagger)/2}$	3 µs	98%	[96]
	Beam splitter	$\mathrm{BS}(z)$	$e^{z\hat{a}^\dagger\hat{b}-z^*\hat{a}\hat{b}^\dagger}$	250 µs	99%	[68]
	Kerr	$\mathrm{K}(z)$	$e^{i\theta(\hat{a}^\dagger\hat{a})^2}$	10 ms*	95%*	[97]
	Cross-Kerr	$\mathrm{CK}(z)$	$e^{i\theta\hat{a}^\dagger\hat{a}\hat{b}^\dagger\hat{b}}$	800 µs	97%	[98]
Hybrid gates	Red sideband	$\mathrm{RSB}(z)$	$e^{iz\hat{a}\sigma^++iz^*\hat{a}^\dagger\sigma^-}$	200 µs	99.9%	[68]
	Blue sideband	$\mathrm{BSB}(z)$	$e^{iz\hat{a}^\dagger\sigma^++iz^*\hat{a}\sigma^-}$	200 µs	99.9%	[68]
	Controlled rotation	$\mathrm{CR}(\theta)$	$e^{i\theta\sigma^z\hat{a}^\dagger\hat{a}}$	200 µs*	99%*	[13]
	Controlled displacement	$\mathrm{CD}(z)$	$e^{\sigma^z(z\hat{a}^\dagger-z^*\hat{a})}$	800 µs	95%*	[99]
	Controlled squeezing	$\mathrm{CS}(z)$	$e^{\sigma^z(z^*\hat{a}\hat{a}-z\hat{a}^\dagger\hat{a}^\dagger)/2}$	120 µs*	99%*	[13]
	Controlled beam splitter	$\mathrm{CBS}(z)$	$e^{\sigma^z(z\hat{a}^\dagger\hat{b}-z^*\hat{a}\hat{b}^\dagger)}$	250 µs	99%	[68]
Measurements	Qubit Pauli strings		σ^i	145 µs	99.99%	[100]
	Average phonon number		$\hat{N}$	200 µs	97%	[101]
	Qumode PNR		$\|n\rangle\langle n\|$	400 µs	99%	[102]
	Qumode homodyne		$\hat{X},\hat{P}$	200 µs	95%*	[102]
	Hybrid		$\sigma^i\hat{X},\ \sigma^i\hat{P}$	200 µs†	95%†	

$^{171}Yb^+$ ions in a linear Paul trap

where $z=\theta e^{i\phi}$

Qubit & qumodes with trapped ions

Araz, Grau, Montgomery, FR `24

Type	Operation	Short	Operator	Estimated gate time	Estimated fidelity	Ref.
Qubit gates	Pauli operators		σ^i	2 μs	99.999%	[92]
	Rotation	$\mathrm{R}_i(\theta)$	$e^{i\theta\sigma^i/2}$	2 μs	99.999%	[92]
	Controlled NOT	CNOT	$e^{i\frac{\pi}{4}(\mathbb{I}_1-\sigma_1^z)(\mathbb{I}_2-\sigma_2^x)}$	30 μs	99.9%	[93]
Qumode gates	Rotation	$\mathrm{R}(\theta)$	$e^{i\theta\hat{a}^\dagger\hat{a}}$	200 μs*	99%*	[94]
	Displacement	$\mathrm{D}(z)$	$e^{z\hat{a}^\dagger-z^*\hat{a}}$	10 μs	99%	[95]
	Single-mode squeezing	$\mathrm{S}(z)$	$e^{(z^*\hat{a}\hat{a}-z\hat{a}^\dagger\hat{a}^\dagger)/2}$	3 μs	98%	[96]
	Beam splitter	$\mathrm{BS}(z)$	$e^{z\hat{a}^\dagger\hat{b}-z^*\hat{a}\hat{b}^\dagger}$	250 μs	99%	[68]
	Kerr	$\mathrm{K}(z)$	$e^{i\theta(\hat{a}^\dagger\hat{a})^2}$	10 ms*	95%*	[97]
	Cross-Kerr	$\mathrm{CK}(z)$	$e^{i\theta\hat{a}^\dagger\hat{a}\hat{b}^\dagger\hat{b}}$	800 μs	97%	[98]
Hybrid gates	Red sideband	$\mathrm{RSB}(z)$	$e^{iz\hat{a}\sigma^++iz^*\hat{a}^\dagger\sigma^-}$	200 μs	99.9%	[68]
	Blue sideband	$\mathrm{BSB}(z)$	$e^{iz\hat{a}^\dagger\sigma^++iz^*\hat{a}\sigma^-}$	200 μs	99.9%	[68]
	Controlled rotation	$\mathrm{CR}(\theta)$	$e^{i\theta\sigma^z\hat{a}^\dagger\hat{a}}$	200 μs*	99%*	[13]
	Controlled displacement	$\mathrm{CD}(z)$	$e^{\sigma^z(z\hat{a}^\dagger-z^*\hat{a})}$	800 μs	95%*	[99]
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Measurements	Qubit Pauli strings		σ^i	145 μs	99.99%	[100]
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	Qumode homodyne		$\hat{X},\hat{P}$	200 μs	95%*	[102]
	Hybrid		$\sigma^i\hat{X},\ \sigma^i\hat{P}$	200 μs†	95%†	

$^{171}Yb^+$ ions in a linear Paul trap

where $z=\theta e^{i\phi}$

Qubit & qumodes with trapped ions

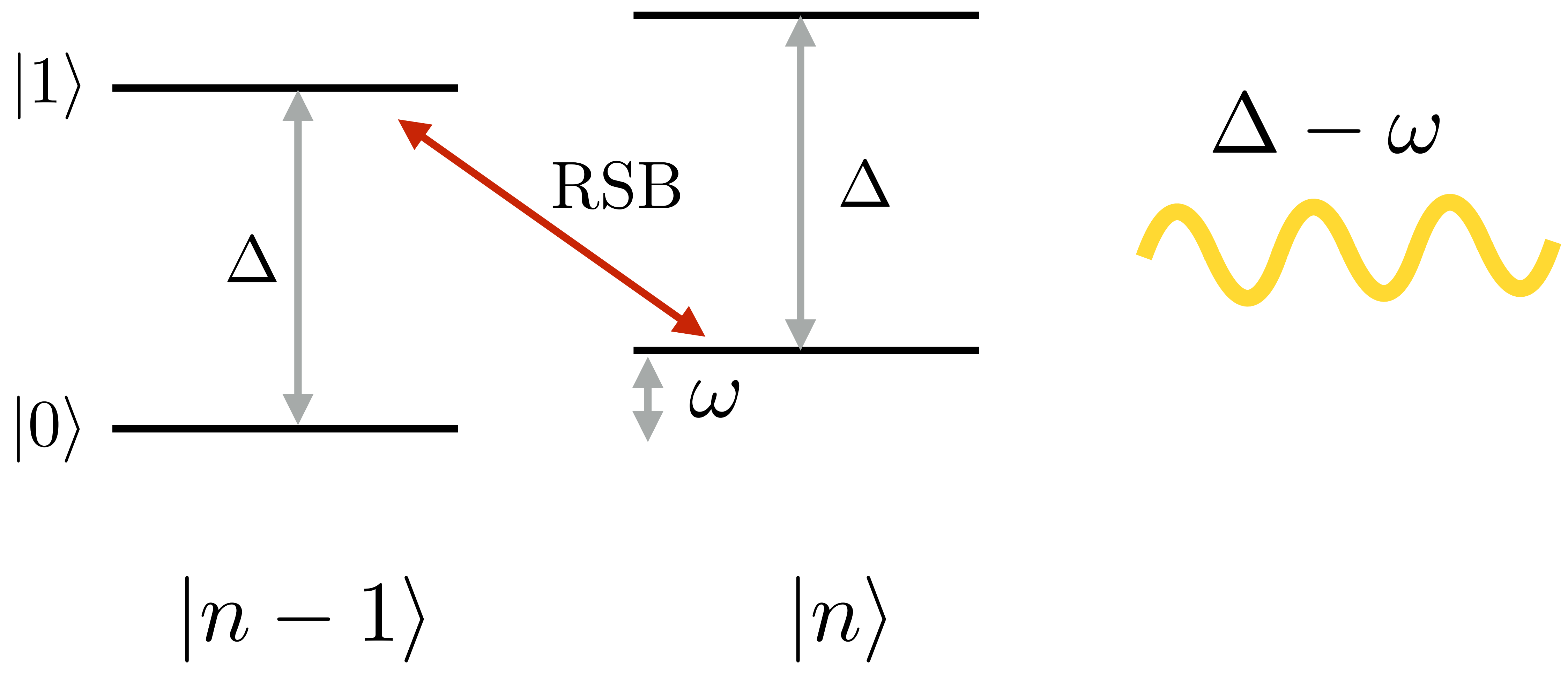
Araz, Grau, Montgomery, FR `24

- Hybrid gate example

  Red sideband gate $U_{\text{rsb}}(z) = e^{iz\hat{a}\sigma^+ + iz^*\hat{a}^\dagger\sigma^-}$

  Red sideband detuned laser

  $$|0\rangle\,|n\rangle \leftrightarrow |1\rangle\,|n-1\rangle$$



- Preserves the total number of excitations
- Beam splitter using two detuned red sidebands

Qubit & qumodes with trapped ions

Araz, Grau, Montgomery, FR `24

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	Displacement	$\mathrm{D}(z)$	$e^{z\hat{a}^\dagger-z^*\hat{a}}$	10 μs	99%	[95]
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$^{171}Yb^+$ ions in a linear Paul trap

where $z = \theta e^{i\phi}$

Qubit & qumodes with transmons

Native transmon gates

Liu, Singh, Smith et al. `24
Crane, Smith, Tomesh et al. `24

Gate Name	Gate Operation
Mode gates	
$\mathrm{R}_i(\theta)$	$\exp\left(-i\theta\hat{n}_i\right)$
$\mathrm{D}_i(\alpha)$	$\exp\left(\alpha\hat{a}_i^\dagger - \alpha^*\hat{a}_i\right)$
$\mathrm{BS}_{i,j}(\varphi,\theta)$	$\exp\left(-i\theta\left(e^{i\varphi}\hat{a}_i^\dagger\hat{a}_j + e^{-i\varphi}\hat{a}_i\hat{a}_j^\dagger\right)\right)$
Transmon gates	
$\mathrm{R}_i^z(\theta)$	$\exp\left(-i\frac{\theta}{2}\hat{Z}_i\right)$
$\mathrm{R}_i^\varphi(\theta)$	$\exp\left(-i\frac{\theta}{2}\hat{\sigma}_i^\varphi\right)$ $\hat{\sigma}_i^\varphi = \hat{X}_i\cos\varphi + \hat{Y}_i\sin\varphi$
Transmon-Mode gates	
$\mathrm{CR}_{i,j}(\theta)$	$\exp\left(-i\frac{\theta}{2}\hat{Z}_i\hat{n}_j\right)$
$\mathrm{C\Pi}_{i,j}$	$\exp\left(-i\frac{\pi}{2}\hat{Z}_i\hat{n}_j\right)$
$\mathrm{CD}_{i,j}(\alpha)$	$\exp\left(\hat{Z}_i\left(\alpha\hat{a}_j^\dagger - \alpha^*\hat{a}_j\right)\right)$
$\mathrm{SNAP}_{i,j}(\vec{\theta})$	$\exp\left(-i\hat{Z}_i\sum_n\theta_n\left\|n\right\rangle\left\langle n\right\|_j\right)$
$\mathrm{SQR}_{i,j}(\vec{\theta},\vec{\varphi})$	$\sum_n \mathrm{R}_i^{\varphi_n}(\theta_n)\otimes\left\|n\right\rangle\left\langle n\right\|_j$

- Different native gate set
- Connectivity
- Coherence times

Trapped ions:

Qubits $\mathcal{O}(s)$, qumodes $\mathcal{O}(ms)$

Transmons:

Qubits $\mathcal{O}(ms)$, qumodes $\mathcal{O}(s)$

Outline

Qubits, qudits & qumodes

Lattice models and non-Abelian theories

Applications in fundamental physics

The many-body Jaynes-Cummings model

- 1-dimensional Hamiltonian with N lattice sites

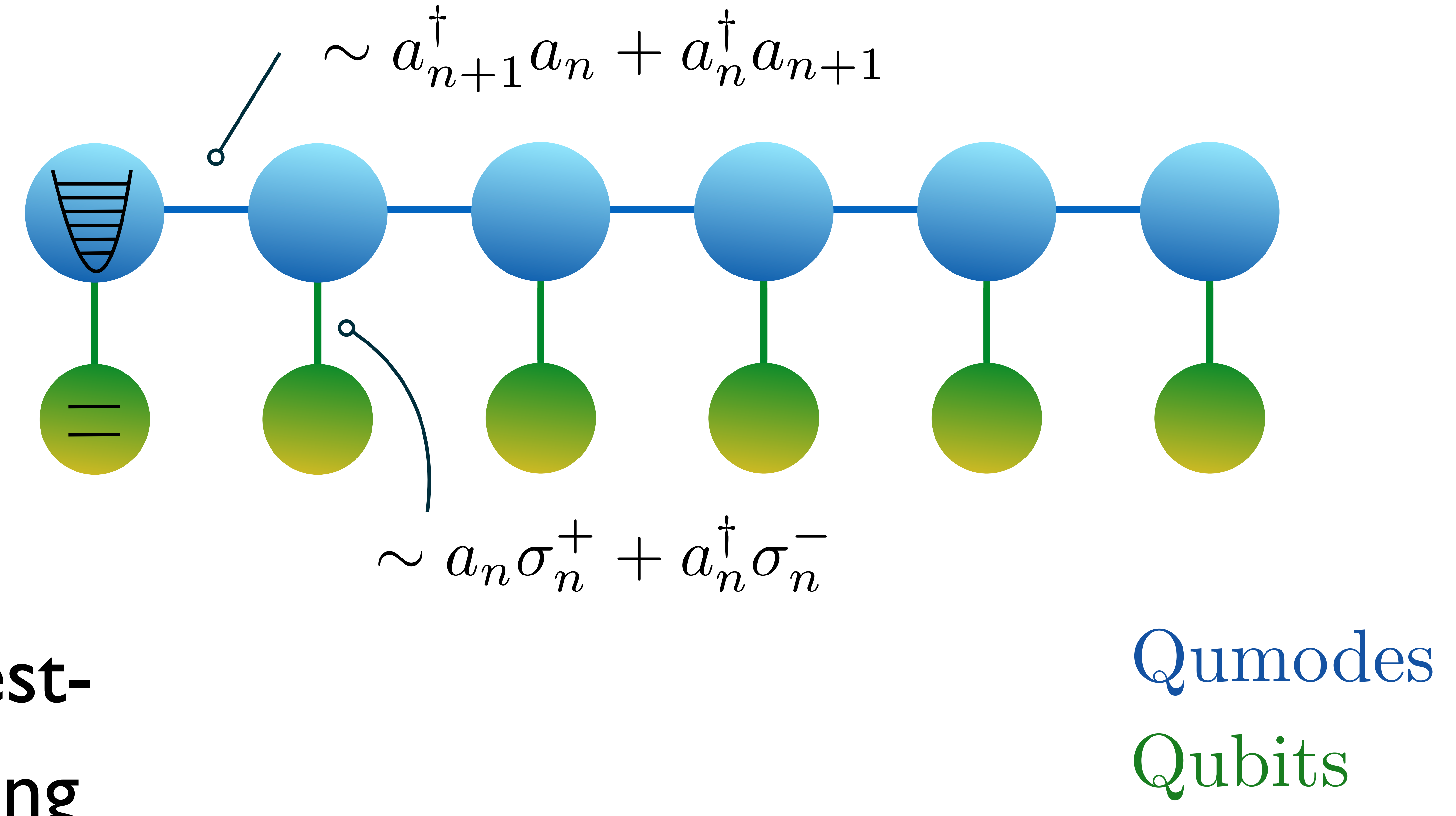
Araz, Grau, Montgomery, FR `24

$$\hat{H} = \sum_{n=1}^{N} \omega_c a_n^\dagger a_n \longrightarrow \text{N qumodes}$$

$$+ \sum_{n=1}^{N} \omega_a \sigma_n^+ \sigma_n^- \longrightarrow \text{N qubits}$$

$$+ \kappa \sum_{n=1}^{N} \left(a_{n+1}^\dagger a_n + a_n^\dagger a_{n+1} \right) \longrightarrow \text{Qumode nearest-neighbor hopping}$$

$$+ \eta \sum_{n=1}^{N} \left(a_n \sigma_n^+ + a_n^\dagger \sigma_n^- \right) \longrightarrow \text{Onsite qumode-qubit interaction}$$

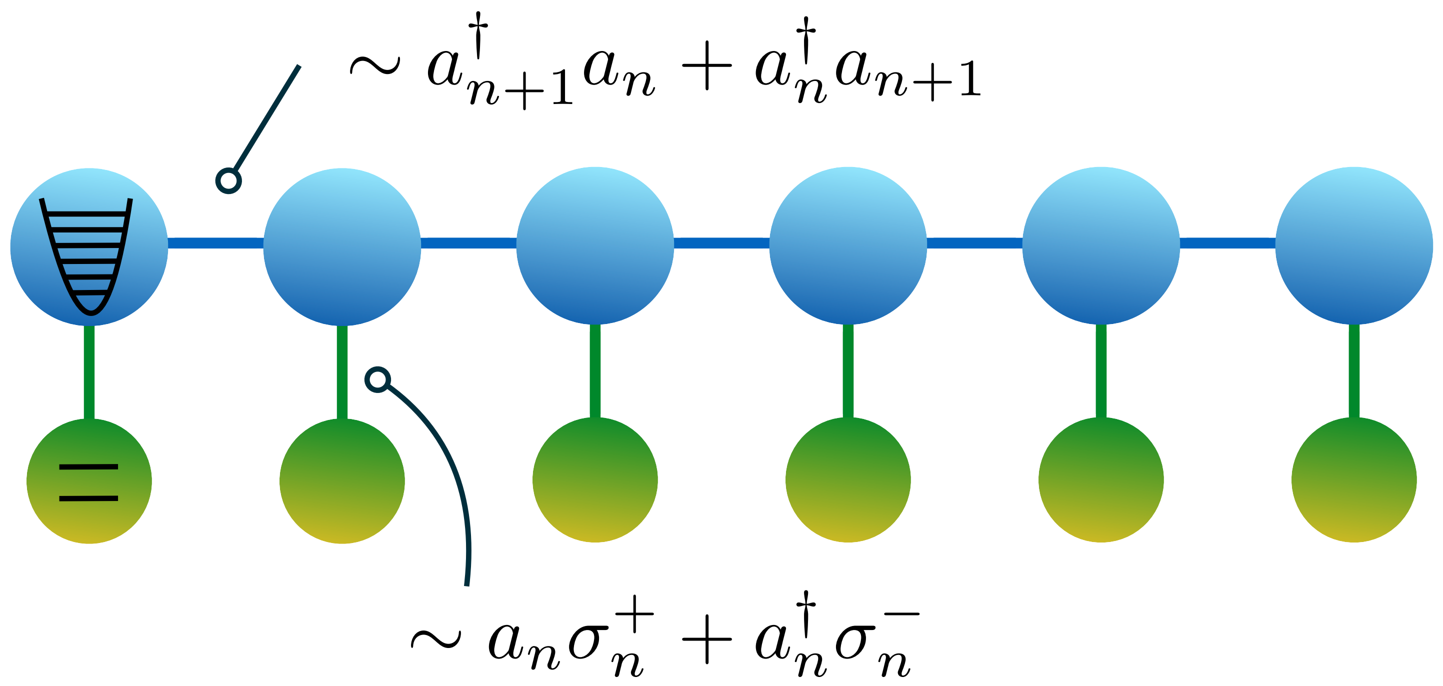


The many-body Jaynes-Cummings model

Araz, Grau, Montgomery, FR `24

- 1-dimensional Hamiltonian

$$\hat{H} = \sum_{n=1}^{N} \omega_c a_n^\dagger a_n + \sum_{n=1}^{N} \omega_a \sigma_n^+ \sigma_n^- + \kappa \sum_{n=1}^{N} \left(a_{n+1}^\dagger a_n + a_n^\dagger a_{n+1} \right) + \eta \sum_{n=1}^{N} \left(a_n \sigma_n^+ + a_n^\dagger \sigma_n^- \right)$$



- The total number of excitations is conserved

$$[\hat{H}, \hat{N}_{\mathrm{tot}}] = 0 \quad \text{where} \quad \hat{N}_{\mathrm{tot}} = \sum_{n=1}^{N} a_n^\dagger a_n + \sum_{n=1}^{N} \sigma^+ \sigma^-$$

- Related to the Schwinger model, U(1) gauge theory + fermions

The many-body Jaynes-Cummings model

- Real-time evolution

Araz, Grau, Montgomery, FR `24

$$\hat{H} = \sum_{n=1}^{N} \omega_c a_n^\dagger a_n \longrightarrow \text{Qumode rotation}$$

$$+ \sum_{n=1}^{N} \omega_a \sigma_n^+ \sigma_n^- \longrightarrow \text{Qubit} \quad \sim \mathbb{I} + \sigma_z$$

$$+ \kappa \sum_{n=1}^{N} \left(a_{n+1}^\dagger a_n + a_n^\dagger a_{n+1} \right) \longrightarrow \text{Qumode beam splitter}$$

$$+ \eta \sum_{n=1}^{N} \left(a_n \sigma_n^+ + a_n^\dagger \sigma_n^- \right) \longrightarrow \text{Red sideband}$$

Trotter decomposition

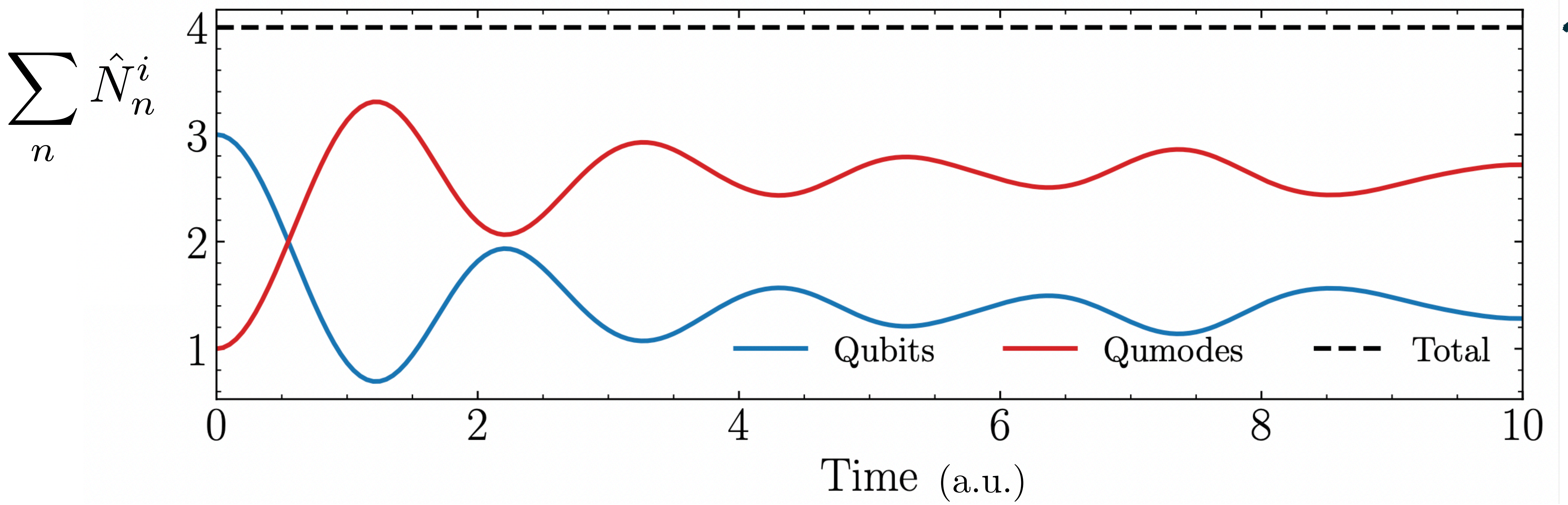
$$U_1(t) = \prod_j e^{-iH_j t}$$

The many-body Jaynes-Cummings model

Araz, Grau, Montgomery, FR `24

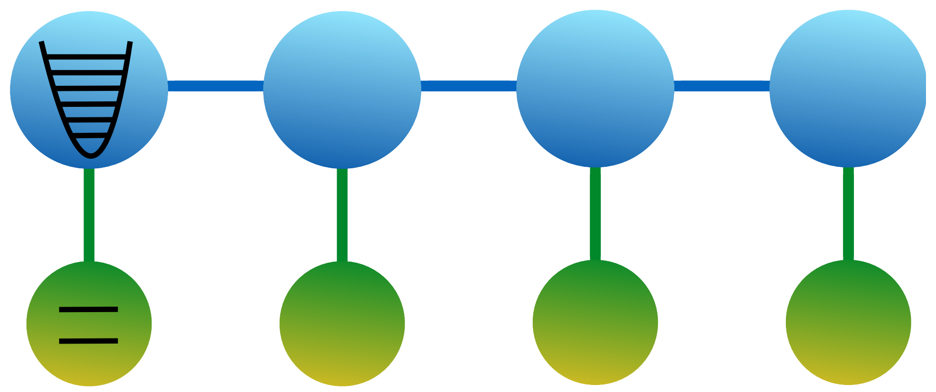
- Real-time evolution

$$H = \sum_{n=1}^{N} \omega_c a_n^\dagger a_n + \sum_{n=1}^{N} \omega_a \sigma_n^+ \sigma_n^- + \kappa \sum_{n=1}^{N} \left(a_{n+1}^\dagger a_n + a_n^\dagger a_{n+1} \right) + \eta \sum_{n=1}^{N} \left(a_n \sigma_n^+ + a_n^\dagger \sigma_n^- \right)$$



As expected for

$$[\hat{H}, \hat{N}_{\mathrm{tot}}] = 0$$



- Open boundary conditions
- 4 sites and qumodes truncated to 4 levels for the classical simulation

The many-body Jaynes-Cummings model

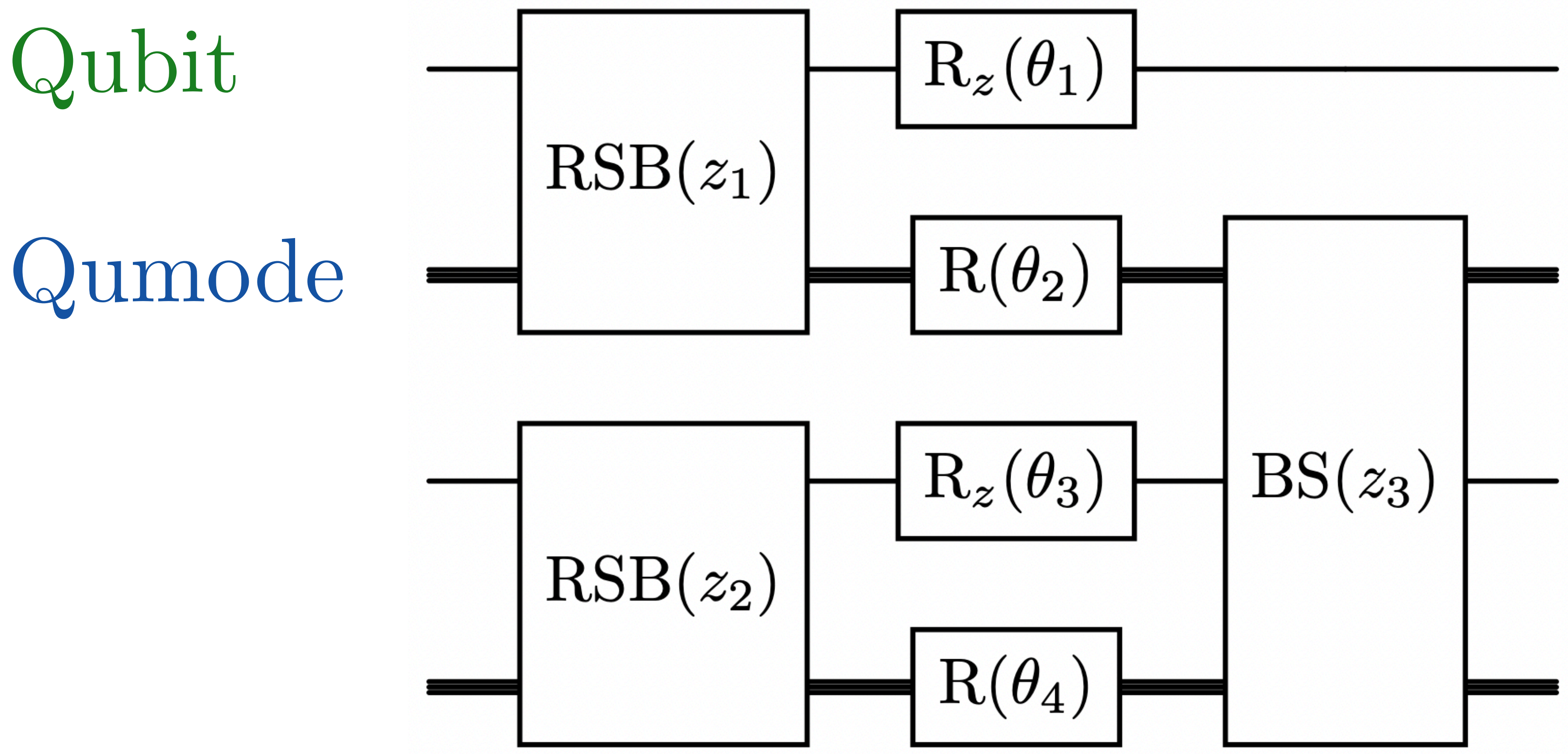
Araz, Grau, Montgomery, FR `24

- Ground state preparation

$$\hat{H} = \sum_{n=1}^{N} \omega_c a_n^\dagger a_n + \sum_{n=1}^{N} \omega_a \sigma_n^+ \sigma_n^- + \kappa \sum_{n=1}^{N} \left(a_{n+1}^\dagger a_n + a_n^\dagger a_{n+1} \right) + \eta \sum_{n=1}^{N} \left(a_n \sigma_n^+ + a_n^\dagger \sigma_n^- \right)$$

cf. Lloyd et al. `18, Siopsis et al. `23

- Hamiltonian-based VQA



1 layer with non-Gaussian red sideband gate

The many-body Jaynes-Cummings model

Araz, Grau, Montgomery, FR `24

- Ground state preparation

$$\hat{H} = \sum_{n=1}^{N} \omega_c a_n^\dagger a_n + \sum_{n=1}^{N} \omega_a \sigma_n^+ \sigma_n^- + \kappa \sum_{n=1}^{N} \left(a_{n+1}^\dagger a_n + a_n^\dagger a_{n+1} \right) + \eta \sum_{n=1}^{N} \left(a_n \sigma_n^+ + a_n^\dagger \sigma_n^- \right)$$

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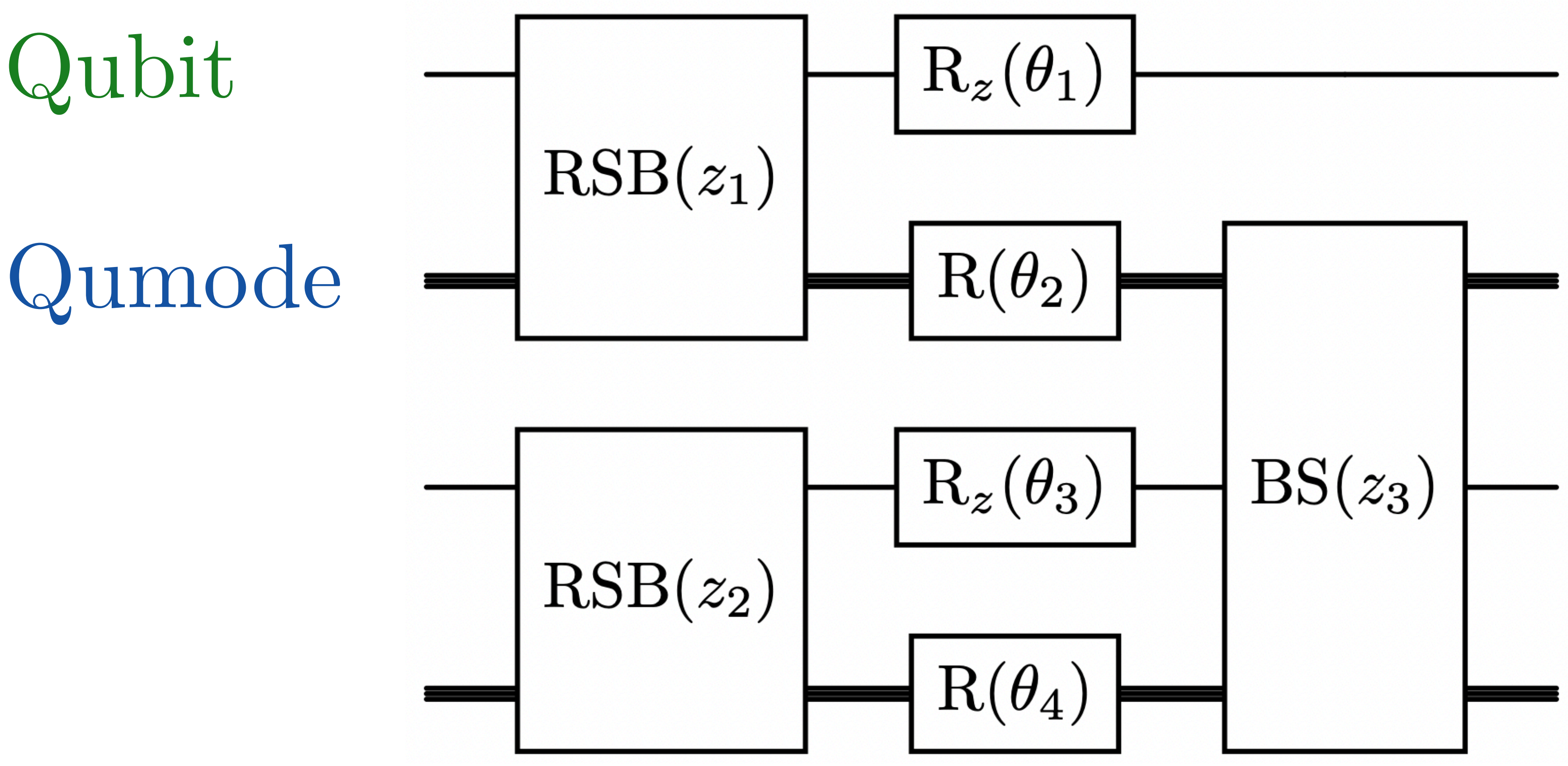
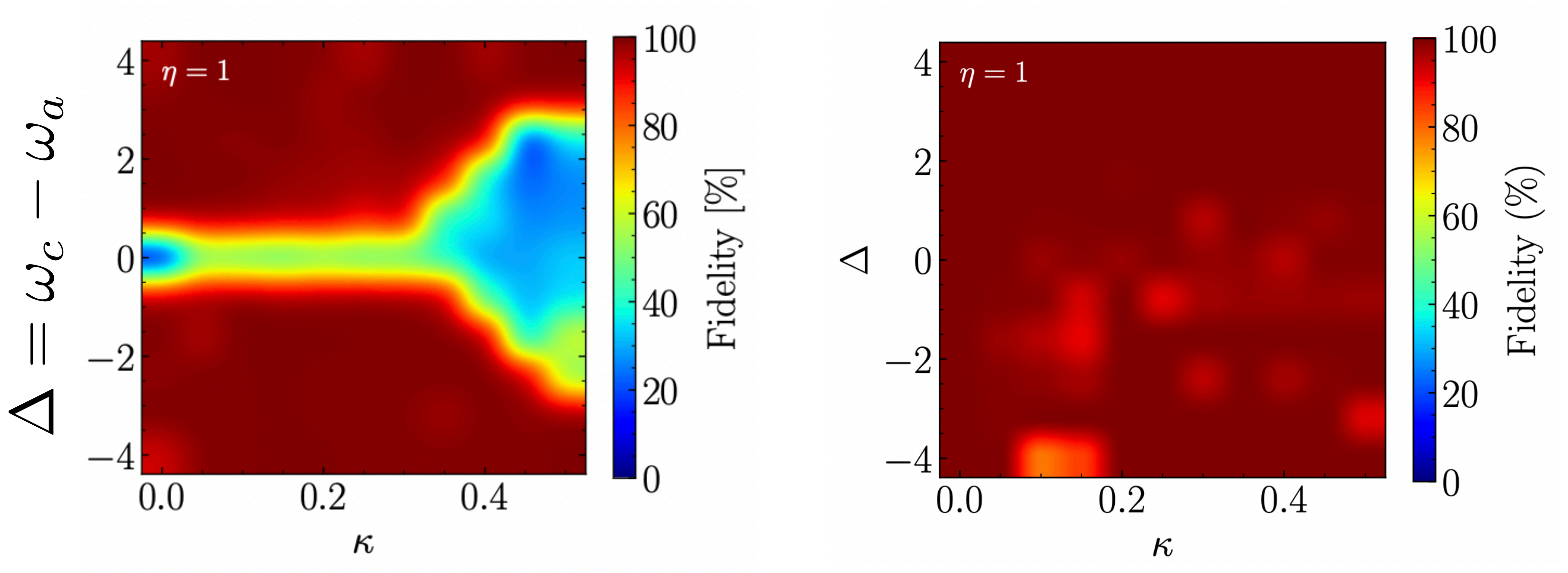
- Hamiltonian-based VQA



1 layer with non-Gaussian red sideband gate



Fixed vs. variable $\langle N_{\text{tot}} \rangle$

$|\langle \psi_{\text{VQA}} | \psi \rangle|^2$

Qumodes and the O(3) model

Jha, FR, Siopsis, Thompson `23

- Hamiltonian 1+1d

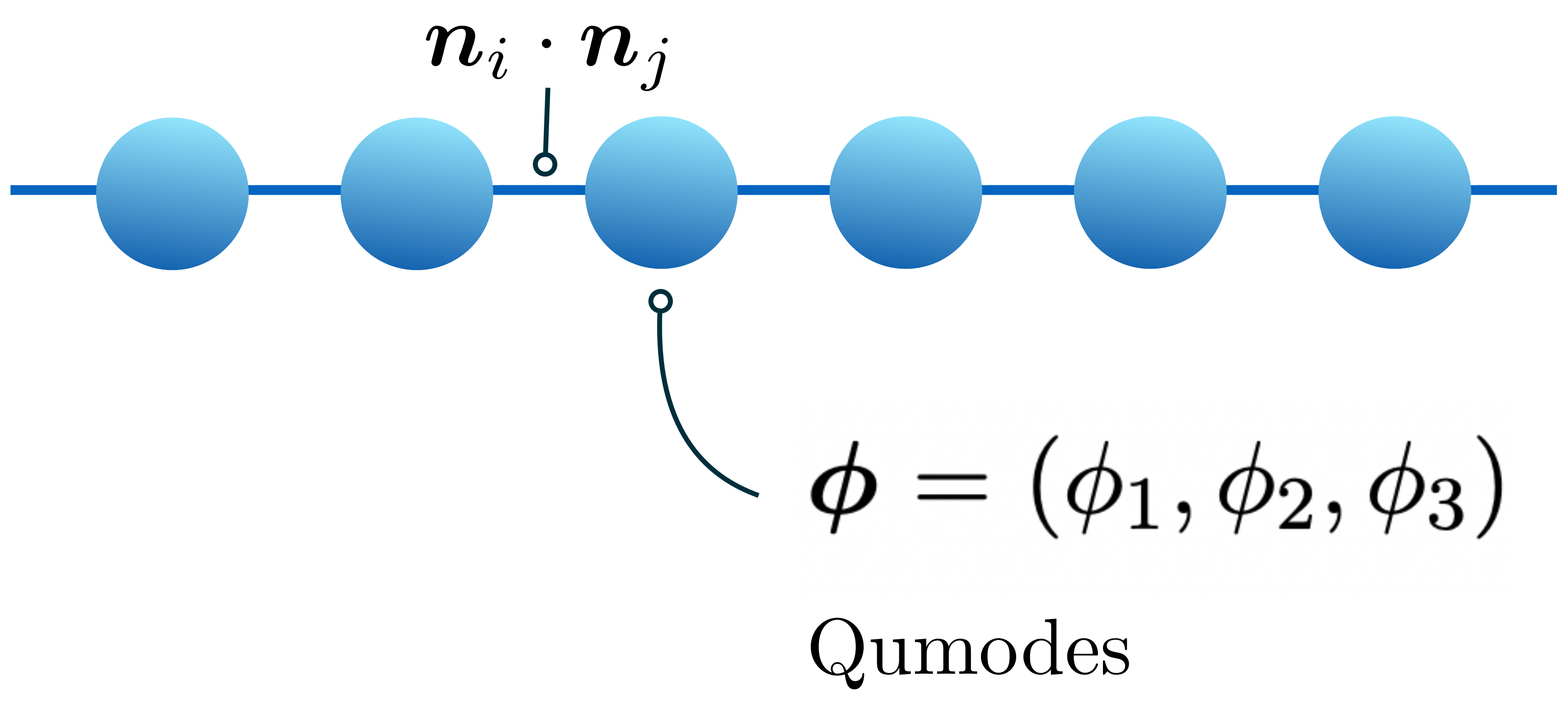
  Global non-abelian symmetry

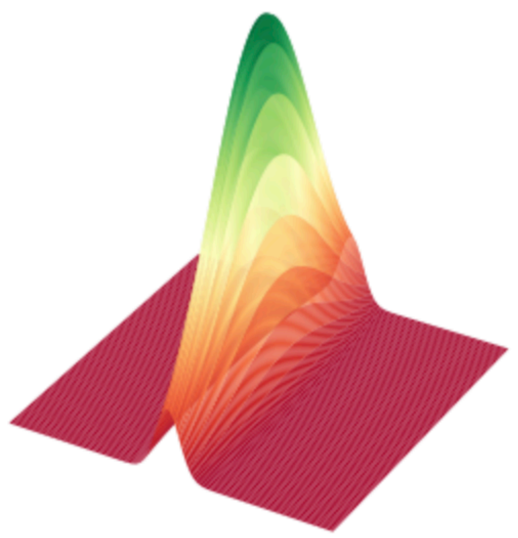
Angular momentum

$$H = \frac{1}{2g^2}\sum_i \boldsymbol{L}_i^2 - g^2 \sum_{\langle i,j\rangle} \boldsymbol{n}_i \cdot \boldsymbol{n}_j$$

Unit 3-vectors at sites i,j

- The fields are treated as continuous variables





Introduce 3 qumodes per lattice site and enforce $\phi_1^2 + \phi_2^2 + \phi_3^2 = 1$

Qumodes and the O(3) model

Jha, FR, Siopsis, Thompson `23

- Hamiltonian 1+1d

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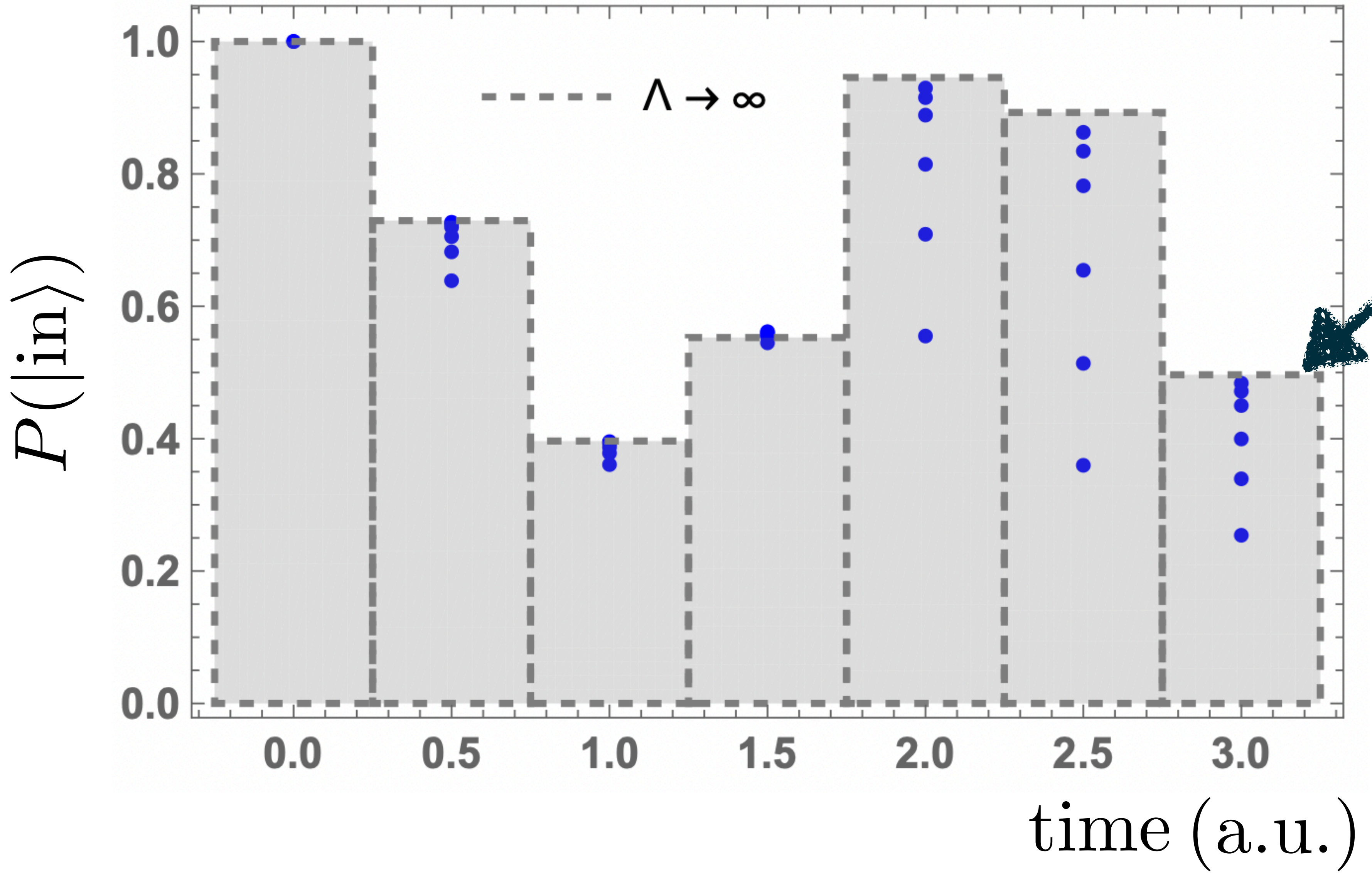
Unit 3-vectors at sites i,j

- Real-time evolution



Infinite squeezing limit

Qumodes and the O(3) model

Jha, FR, Siopsis, Thompson `23

- Hamiltonian 1+1d

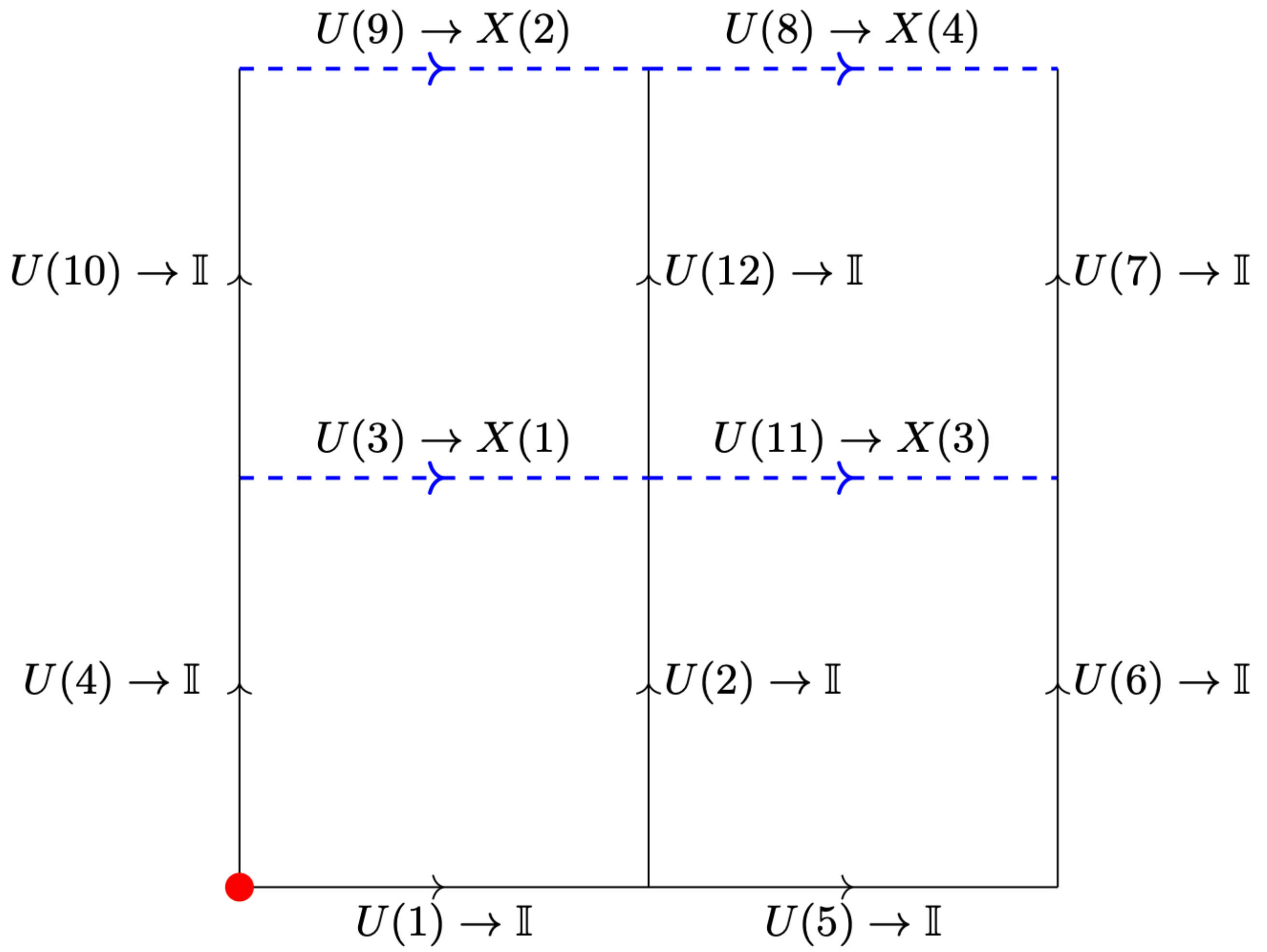
Angular momentum

$$H = \frac{1}{2g^2}\sum_i \boldsymbol{L}_i^2 - g^2 \sum_{\langle i,j \rangle} \boldsymbol{n}_i \cdot \boldsymbol{n}_j$$

Unit 3-vectors at sites i,j

Global non-abelian symmetry

- Extension to SU(2) gauge theories



Ale, Bauer, Jha, FR, Siopsis, Thompson `24

Alternatively Schwinger-boson formulation

Outline

Qubits, qudits & qumodes

Lattice models and non-Abelian theories

Applications in fundamental physics

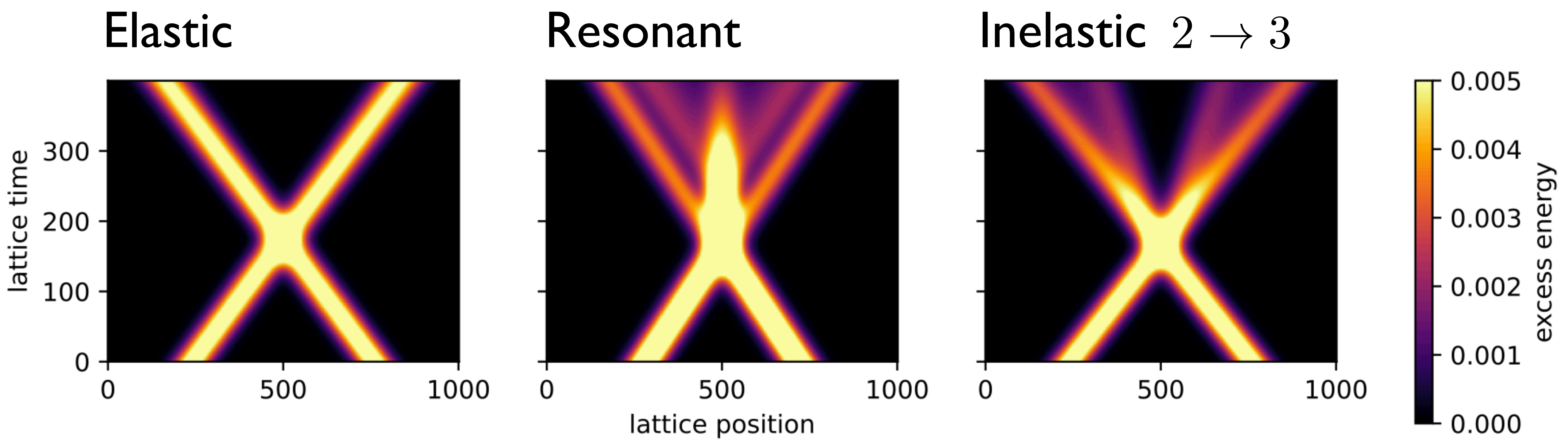
Applications in fundamental physics

Jha, Milsted, Neuenfeld, Preskill, Vieira `24
see also e.g. Savage et al., Davoudi et al.

- Full scattering process

$$|p_1, p_2\rangle_{\text{in}} = S_{11\to 11}(E;\eta)\,|p_1,p_2\rangle_{\text{out}} + \sum_{n=2}^{\infty}\prod_{i=1}^{n}\int \mathrm{dPS_i}\;\; S_{11\to X_n}(p_1,p_2,q_1,\dots,q_n;\eta)\,|q_1,\dots,q_n\rangle_{\text{out}}$$

- Tensor network simulations for Ising Field Theory in 1+1d with up to 2000 sites



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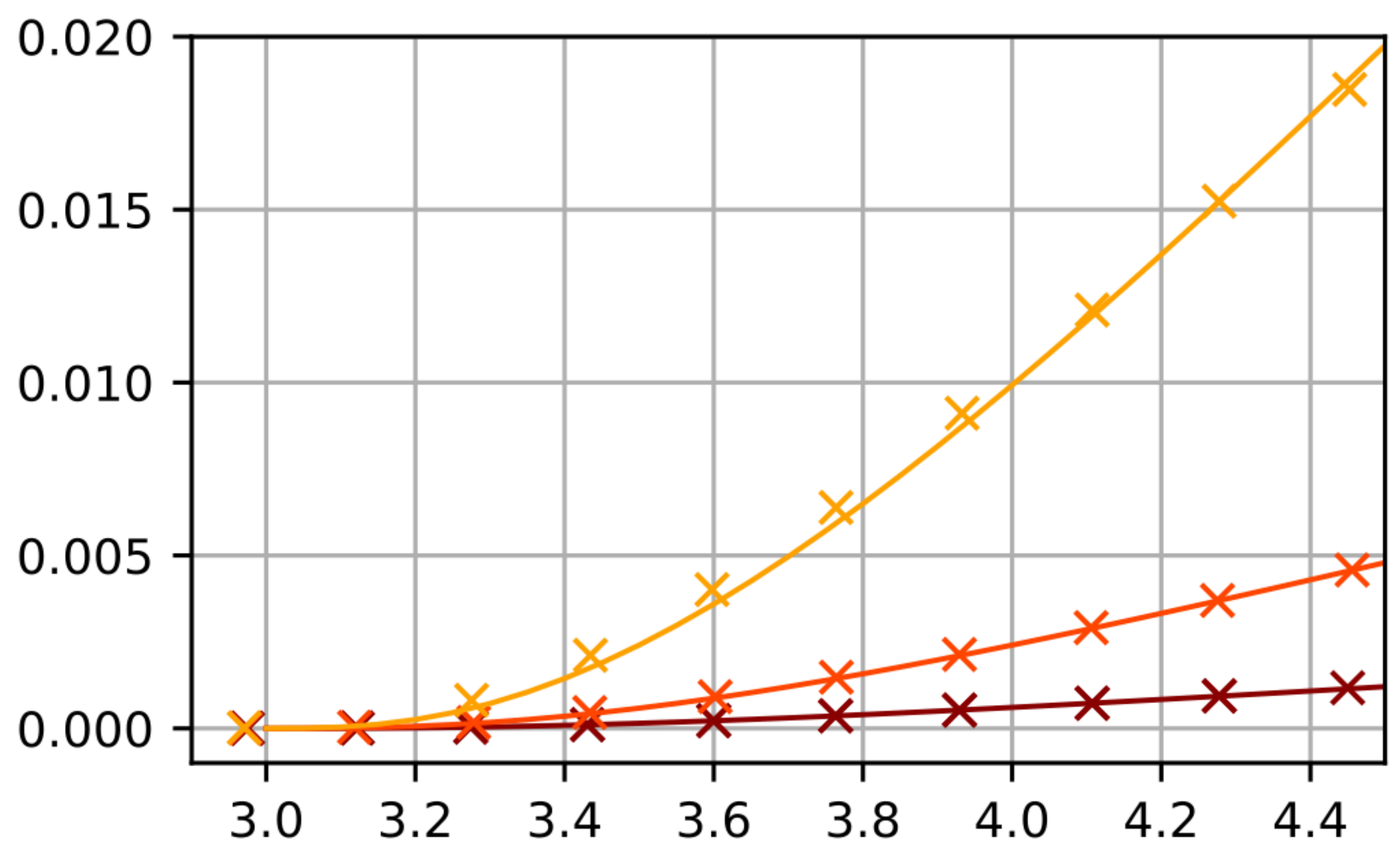
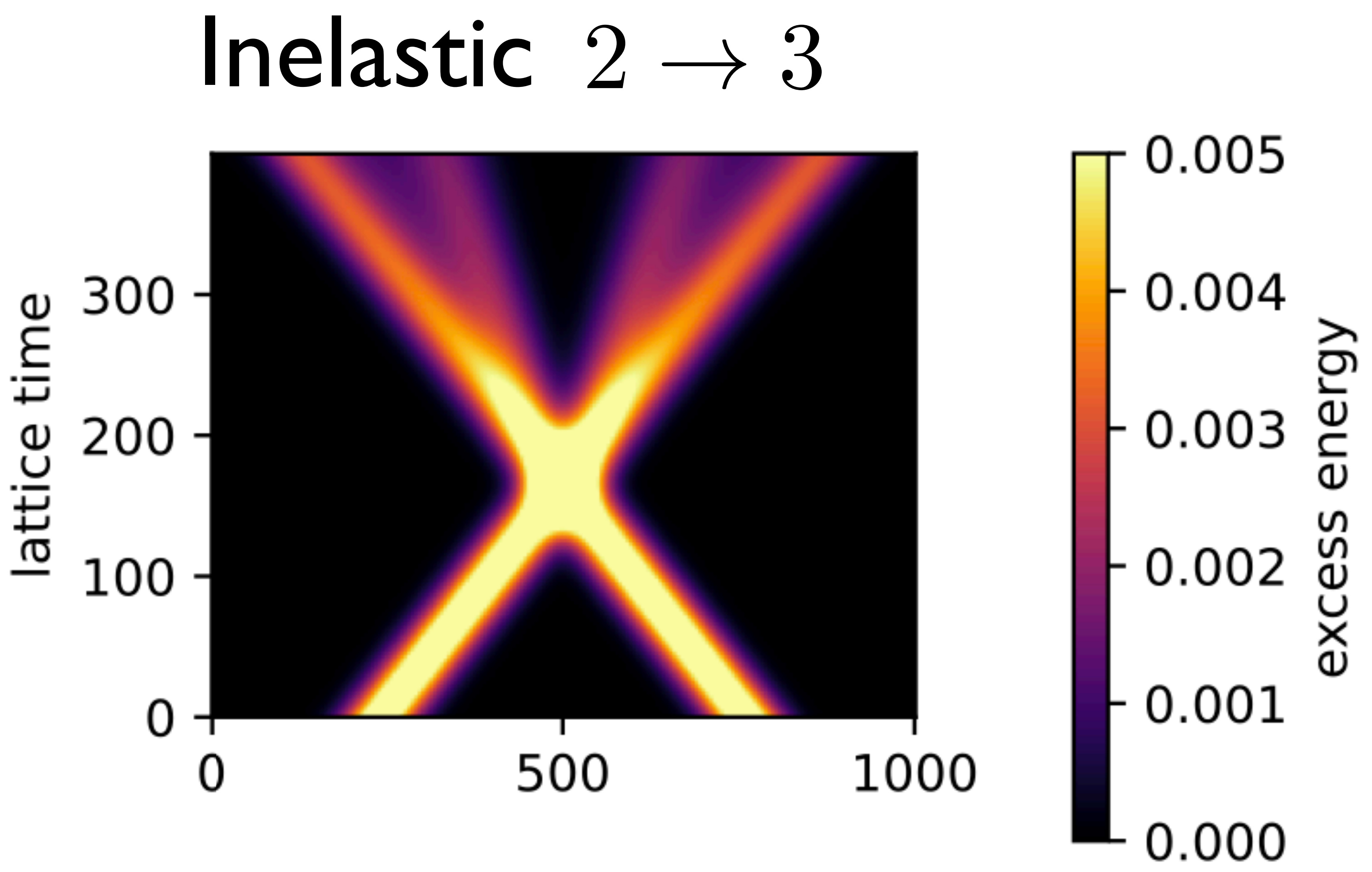


cf. Zamolodchikov, Ziyatdinov `11



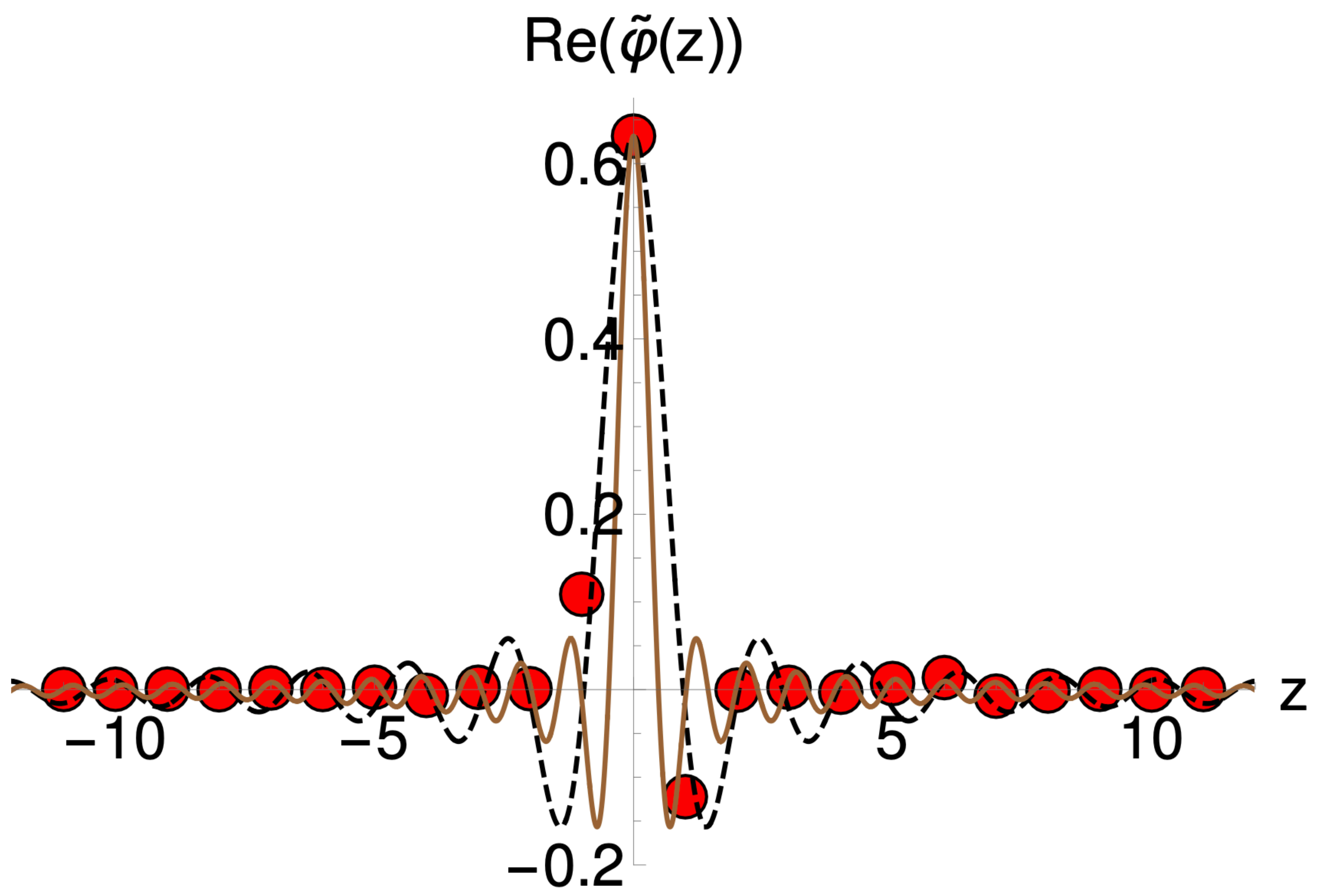
Toward parton distribution functions

Grieninger, Ikeda, Zahed `24
see also Xing et al., Rico et al.

- Real-time quasi-PDFs
- Schwinger model

$$\mathcal{L} = \overline{\psi}(i\not{D} - m)\psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}$$

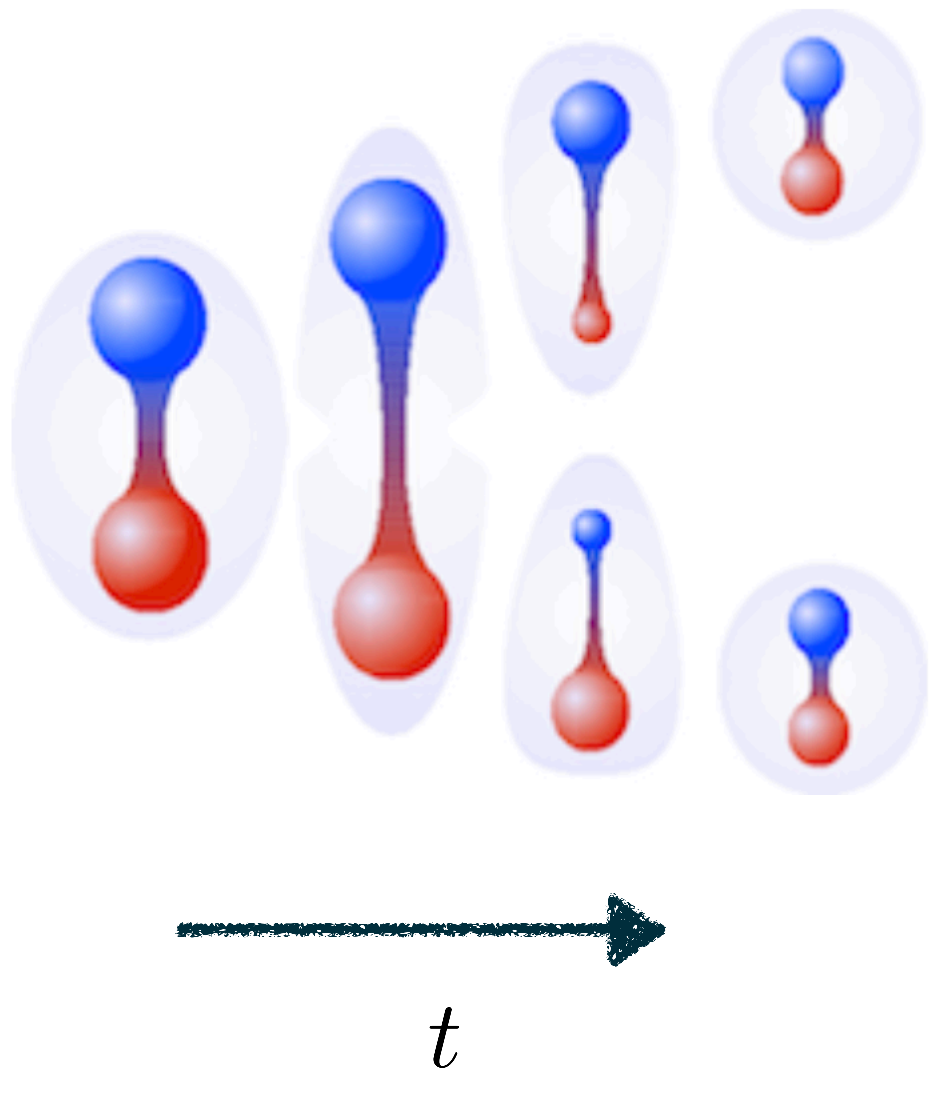
$$\frac{1}{2}\int_{-\infty}^{+\infty}\frac{dz}{2\pi}\,e^{-iz\zeta p^1}\langle\eta(0)|\,e^{-i\chi\mathbb{K}}\,\overline{\psi}(0,-z)[-z,+z]_S\gamma^+\gamma^5\psi(0,+z)\,e^{i\chi\mathbb{K}}\,|\eta(0)\rangle$$



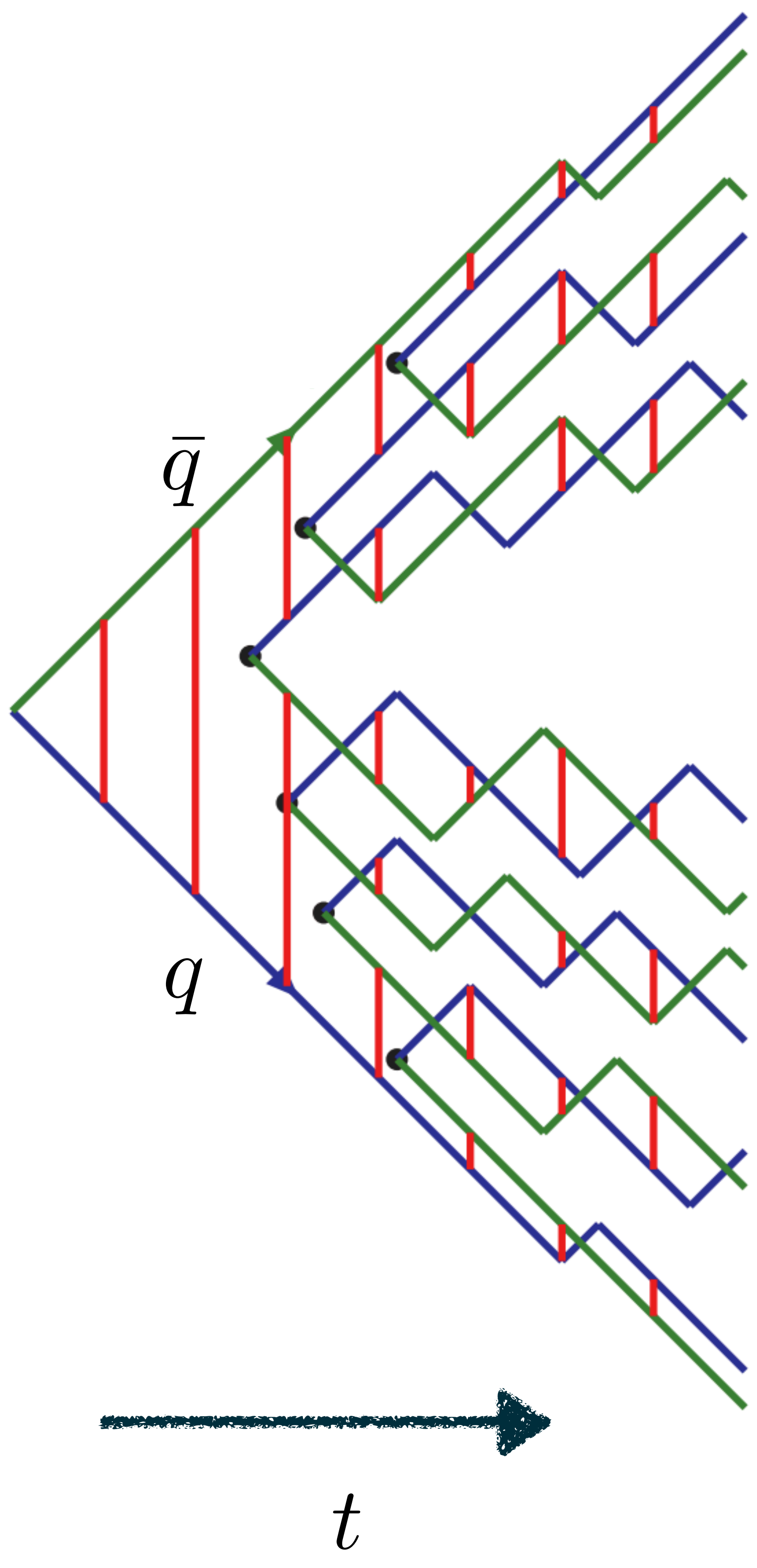
- Continuum limit requires a large lattice
- Extension to 3D hadron structure
- Fragmentation functions *Grieninger, Zahed `24*

Hadronization and string breaking

- QED in 1+1 dimensions *Schwinger `62*
- Model of hadronization
- Confining potential $V \sim r$



QCD string breaking



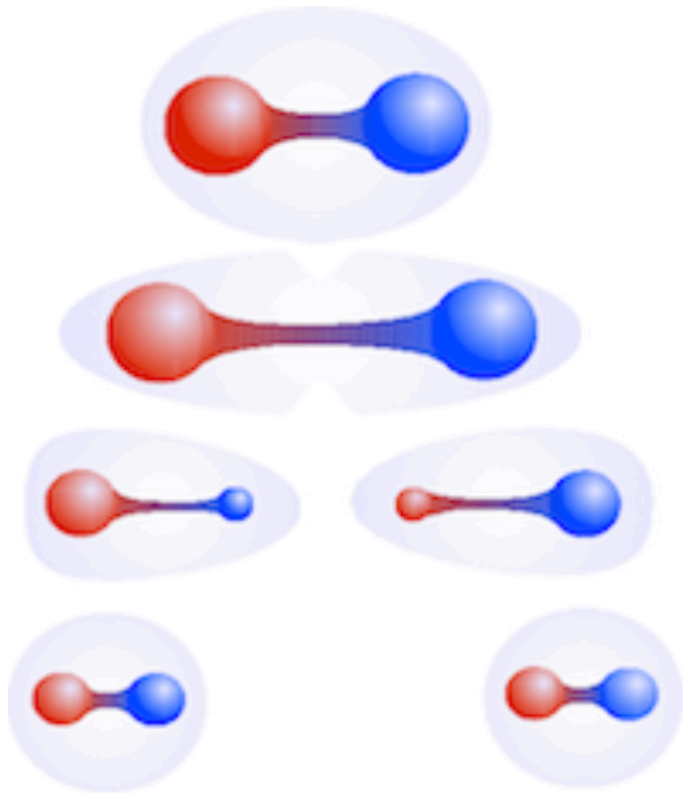
$$\mathcal{L} = \overline{\psi}(i\not{D} - m)\psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}$$

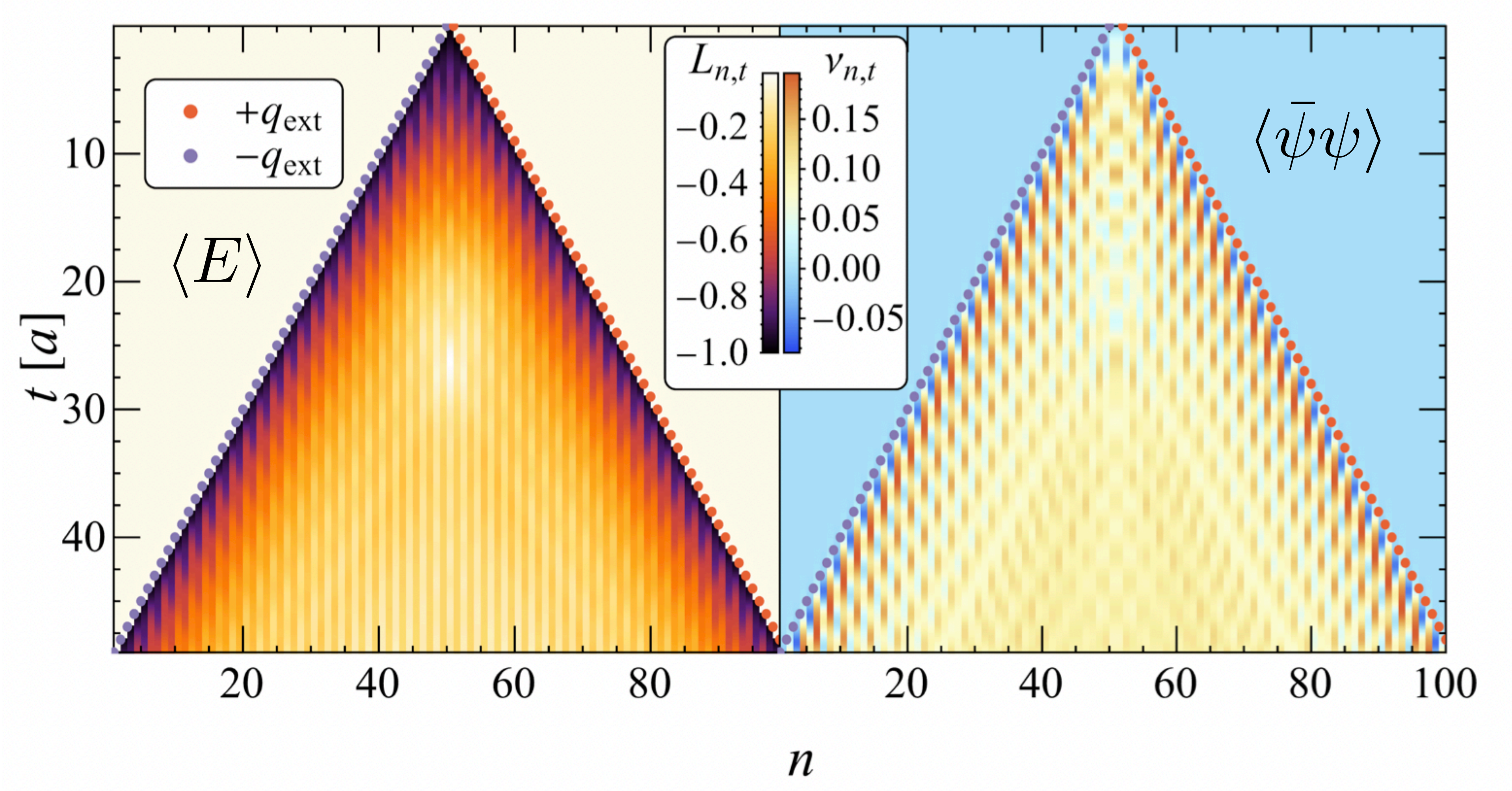
Pythia Lund string hadronization
Florio, Kharzeev et al. `23, Kharzeev, Zahed et al. `23

Hadronization and string breaking

- Full scattering process

Florio, Frenklakh, Kharzeev et al. `24



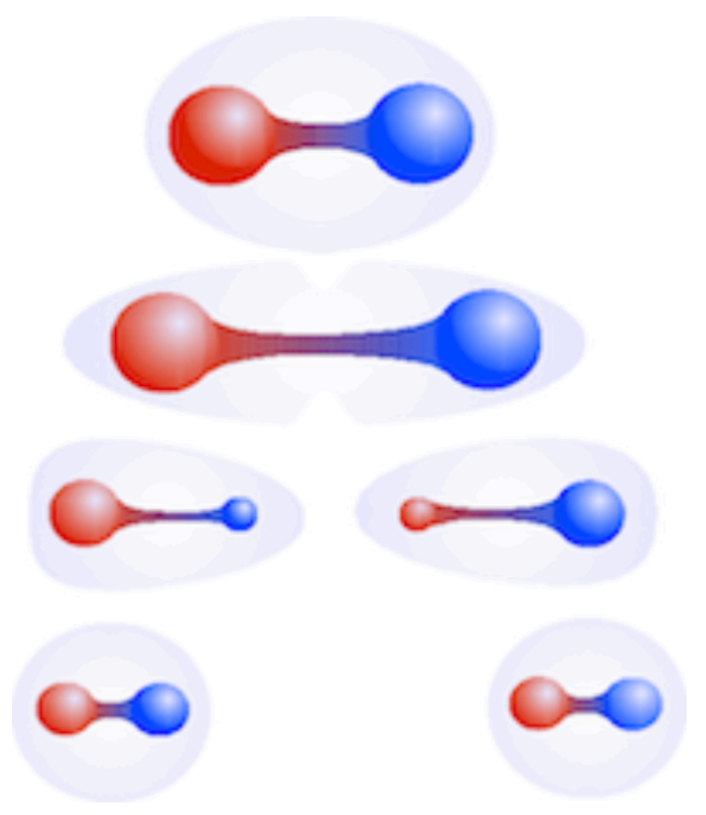


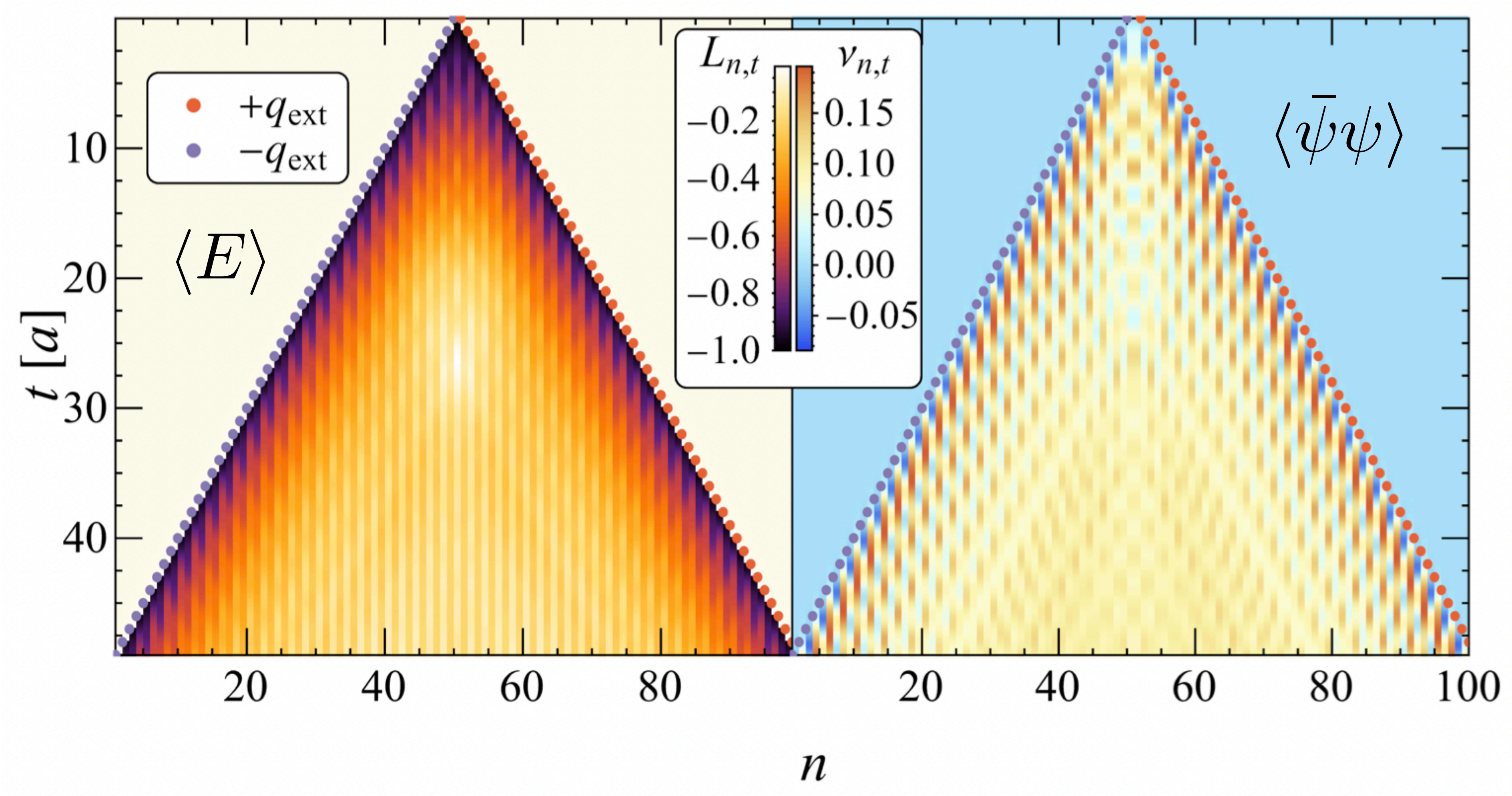
Studies of entanglement and thermalization

Hadronization and string breaking

- Full scattering process

Florio, Frenklakh, Kharzeev et al. `24

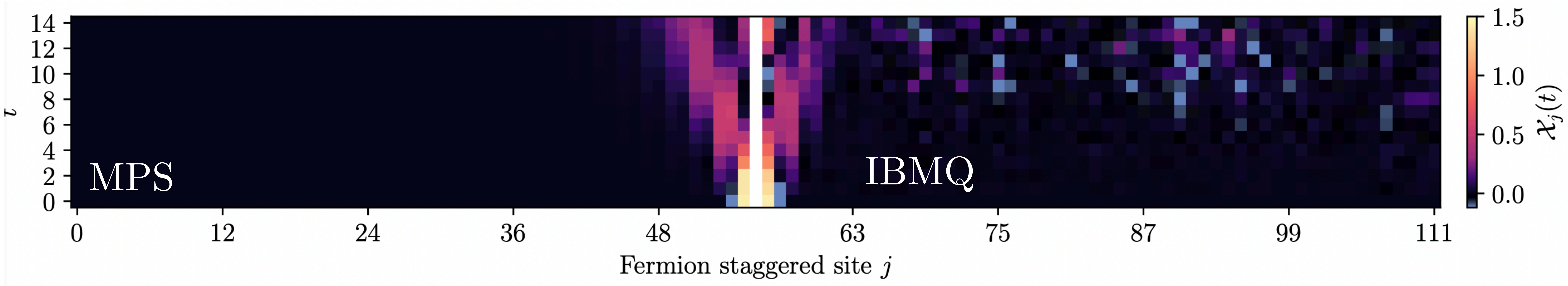




Studies of entanglement and thermalization

- Evolution of a hadron wave packet

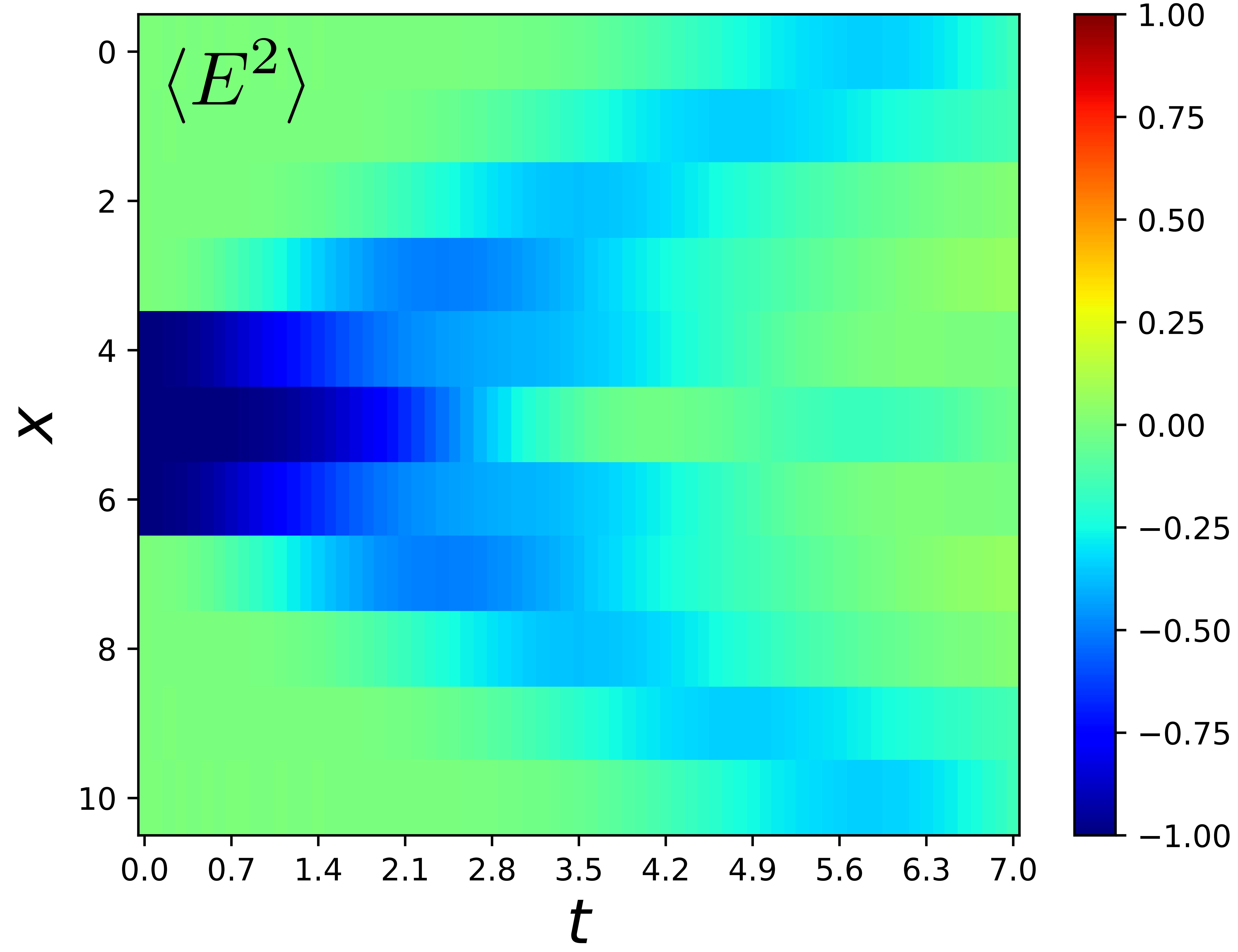
Farrell, Illa, Ciavarella, Savage `24

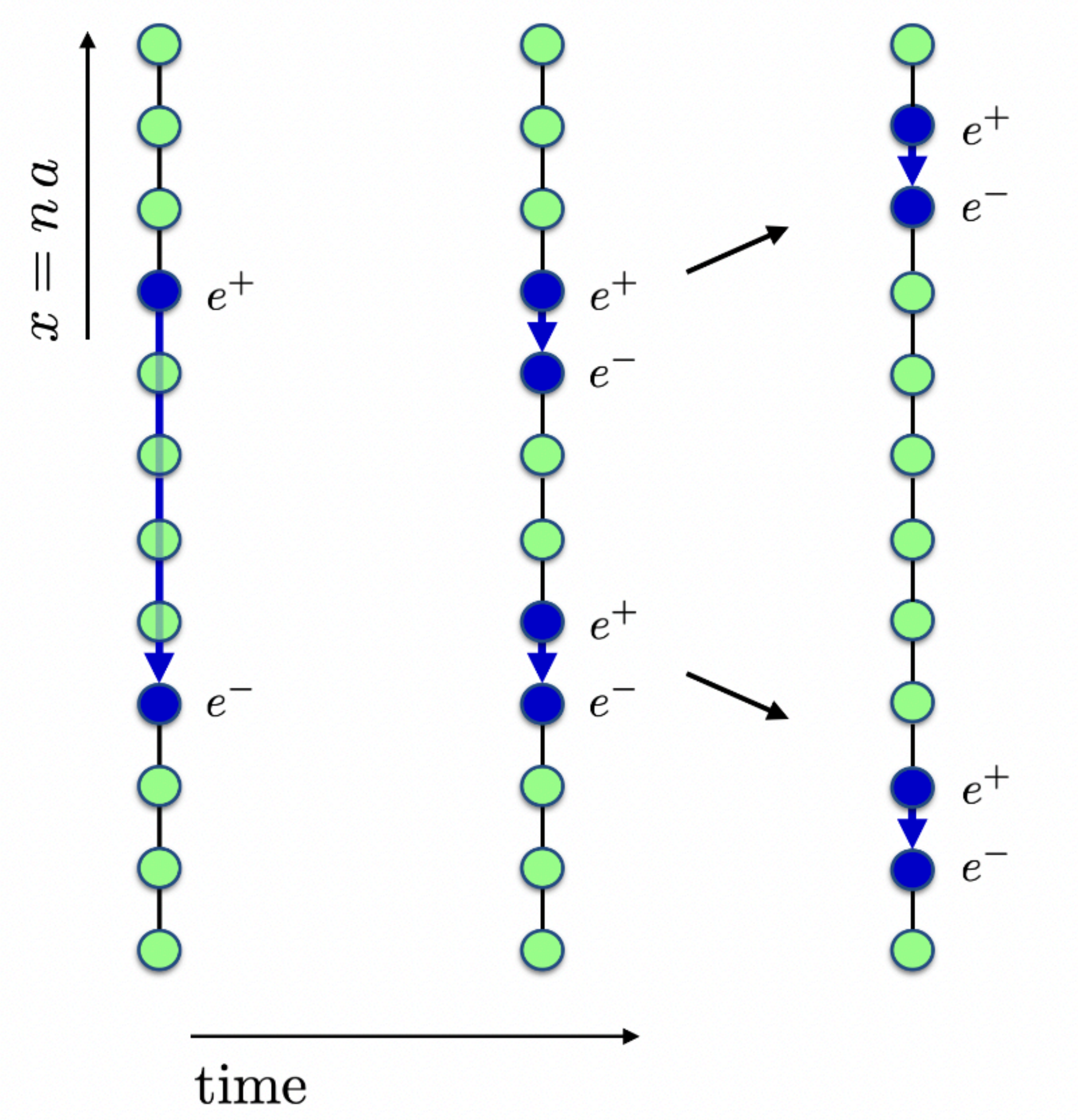


String breaking — medium modification

Vacuum

Lee, Mulligan, FR, Yao `24



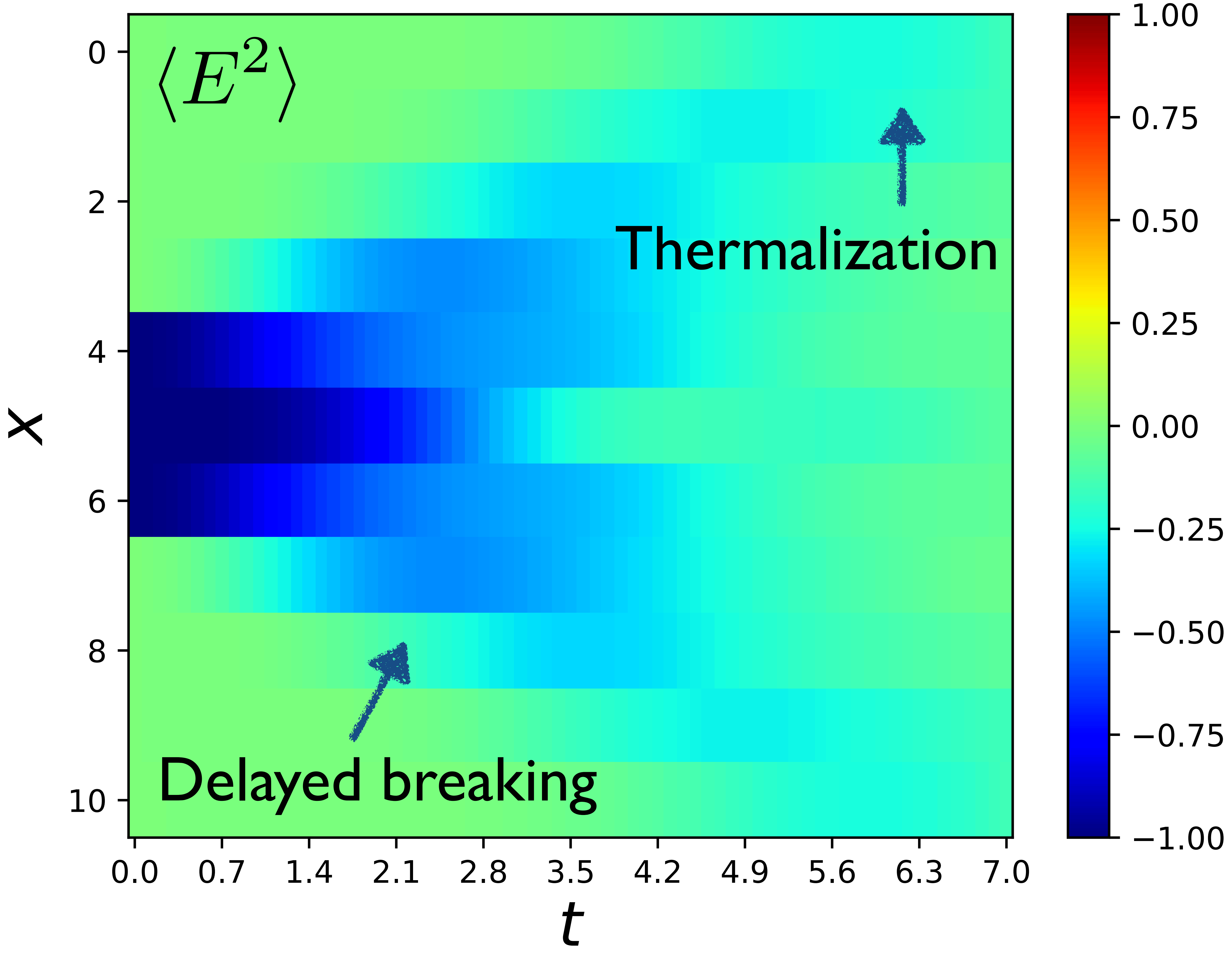


- String breaking in vacuum

String breaking — medium modification

Lee, Mulligan, FR, Yao `24

Medium



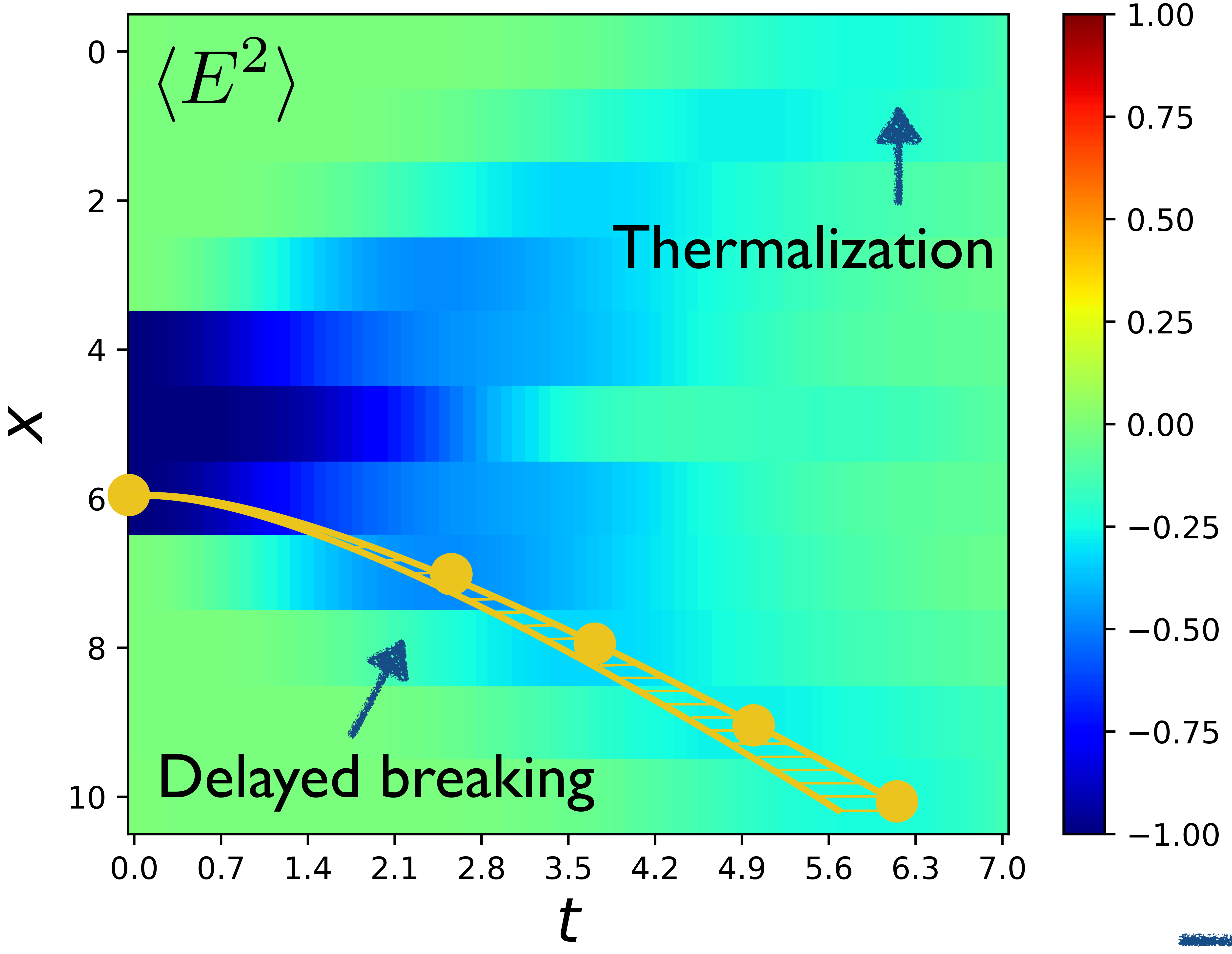
$$\frac{\mathrm{d}}{\mathrm{d}t}\rho_S = -i\left[H_S, \rho_S\right] + \sum_{j=1}^{m}\left(L_j \rho_S L_j^\dagger - \frac{1}{2}L_j^\dagger L_j \rho_S - \frac{1}{2}\rho_S L_j^\dagger L_j\right)$$

- String breaking in a thermal background
- Lindblad evolution

String breaking — medium modification

Medium

Lee, Mulligan, FR, Yao `24



$$\frac{\mathrm{d}}{\mathrm{d}t}\rho_S = -i\left[H_S, \rho_S\right] + \sum_{j=1}^{m}\left(L_j \rho_S L_j^\dagger - \frac{1}{2}L_j^\dagger L_j \rho_S - \frac{1}{2}\rho_S L_j^\dagger L_j\right)$$

- String breaking in a thermal background
- Lindblad evolution

→ Simulations may inform modeling of nuclear medium effects

Outline

Qubits, qudits & qumodes

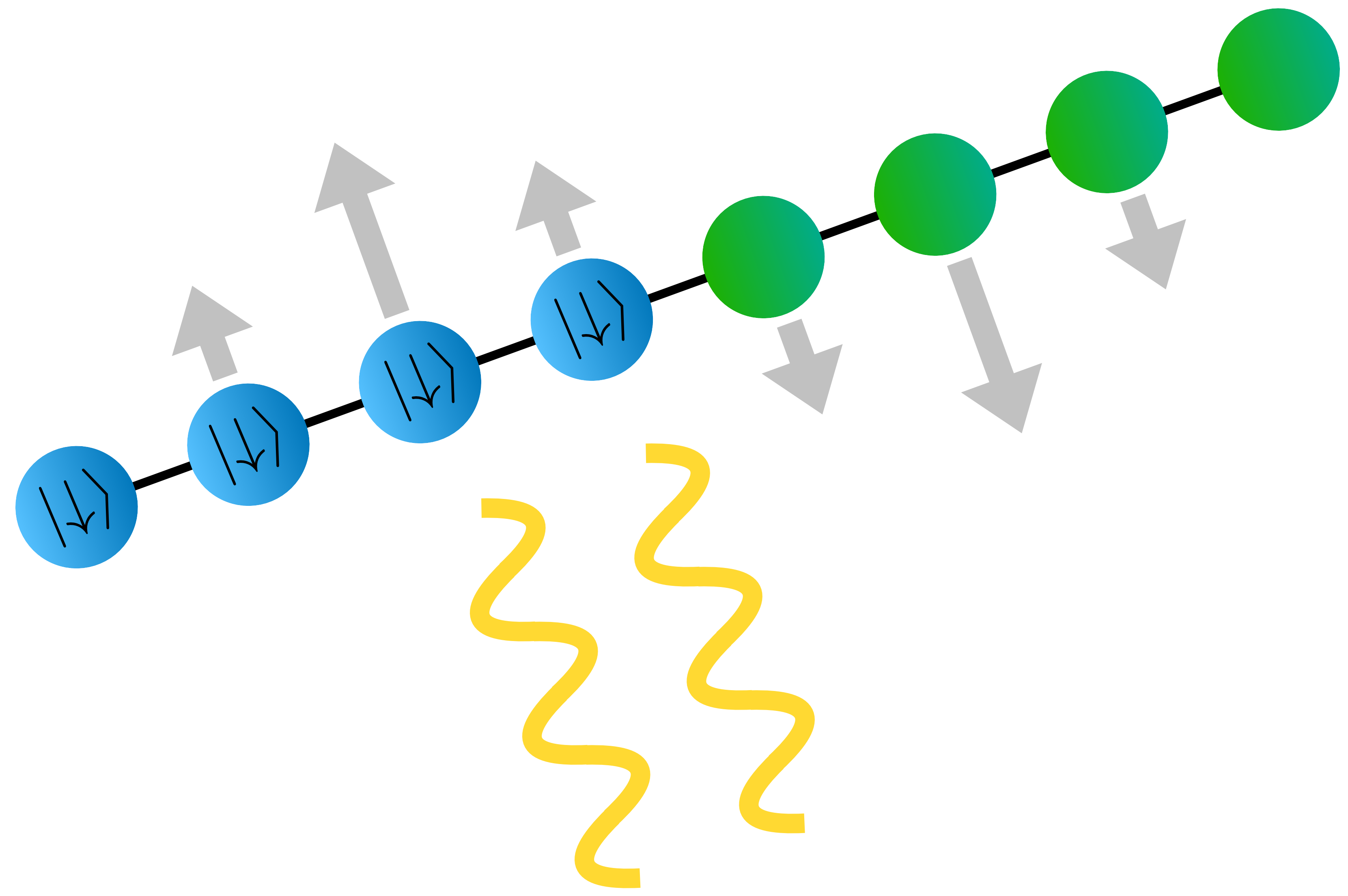
Lattice models and non-Abelian theories

Applications in fundamental physics

Conclusions

- Quantum computing well-suited to address challenges in fundamental physics
- Exploration of qubits, qudits, qumodes

- Small-scale simulations
- Continuum extrapolations
- Building up toward simulations of the Standard Model



Qubit & qumodes with trapped ions

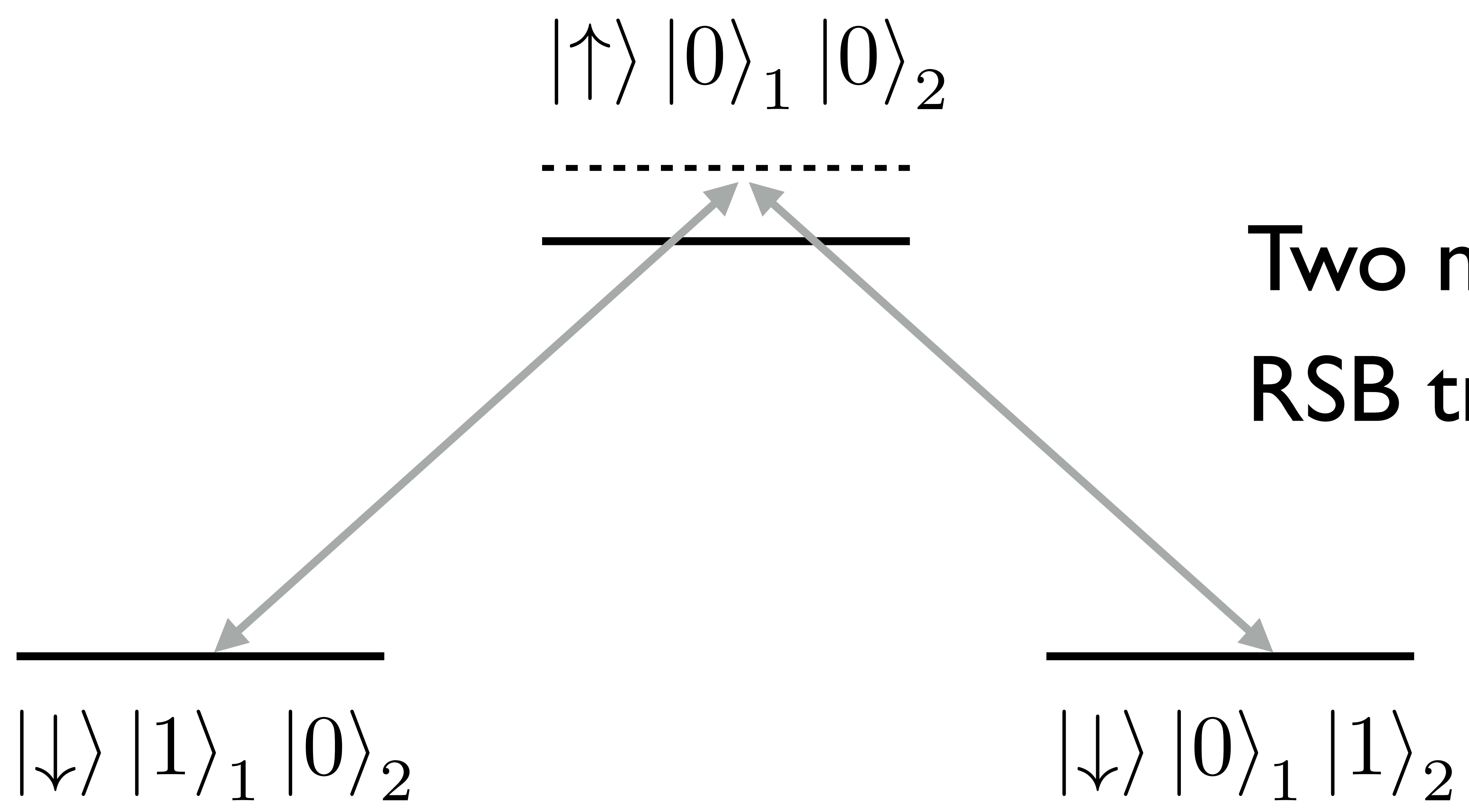
Araz, Grau, Montgomery, FR `24

- Two-qumode beam splitter *Katz, Monroe `22, Kim et al. `23*

- Laser does not interact with the two modes directly

$$U_{\rm bs}(z) = e^{z\hat{a}^\dagger \hat{b} - z^* \hat{a}\hat{b}^\dagger}$$

→ Potential bottleneck



Two non-resonant RSB transitions

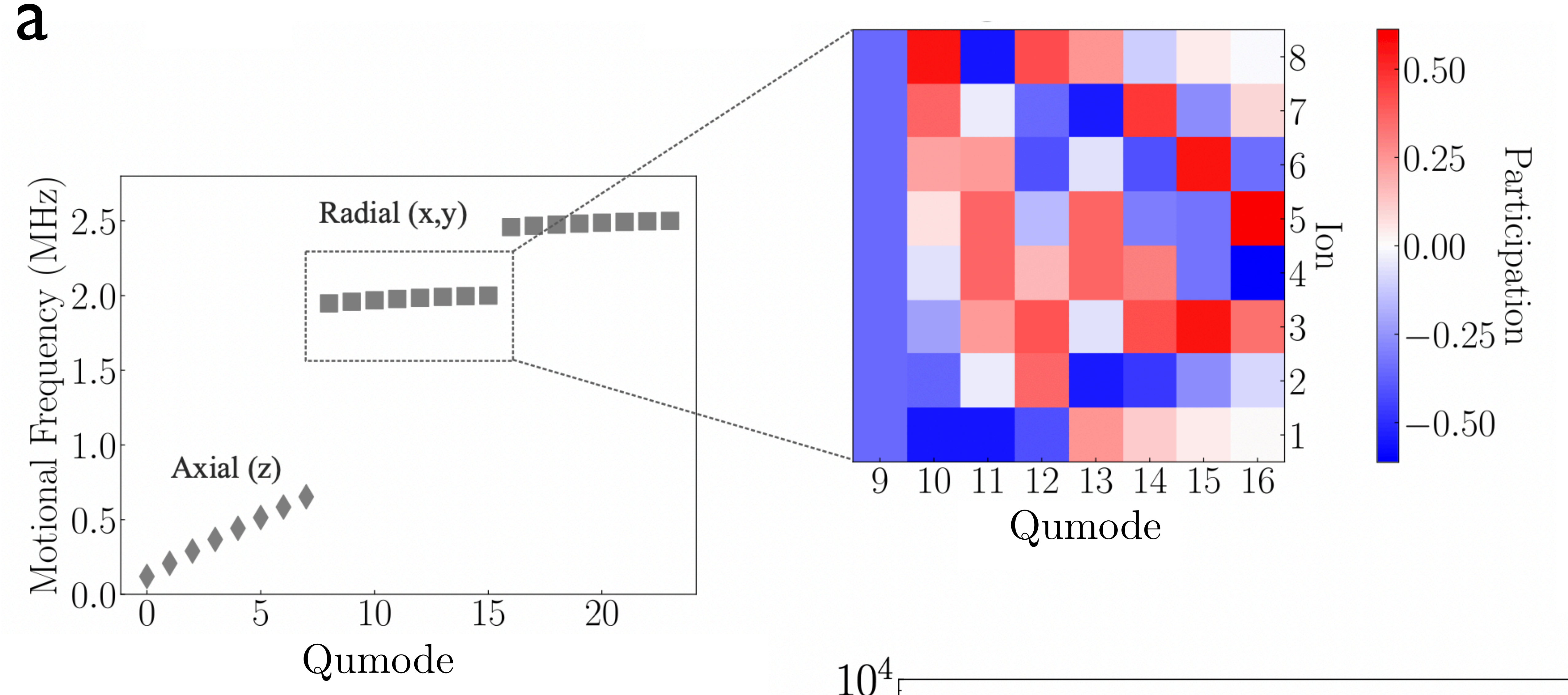
Control qubit

Qumodes 1&2

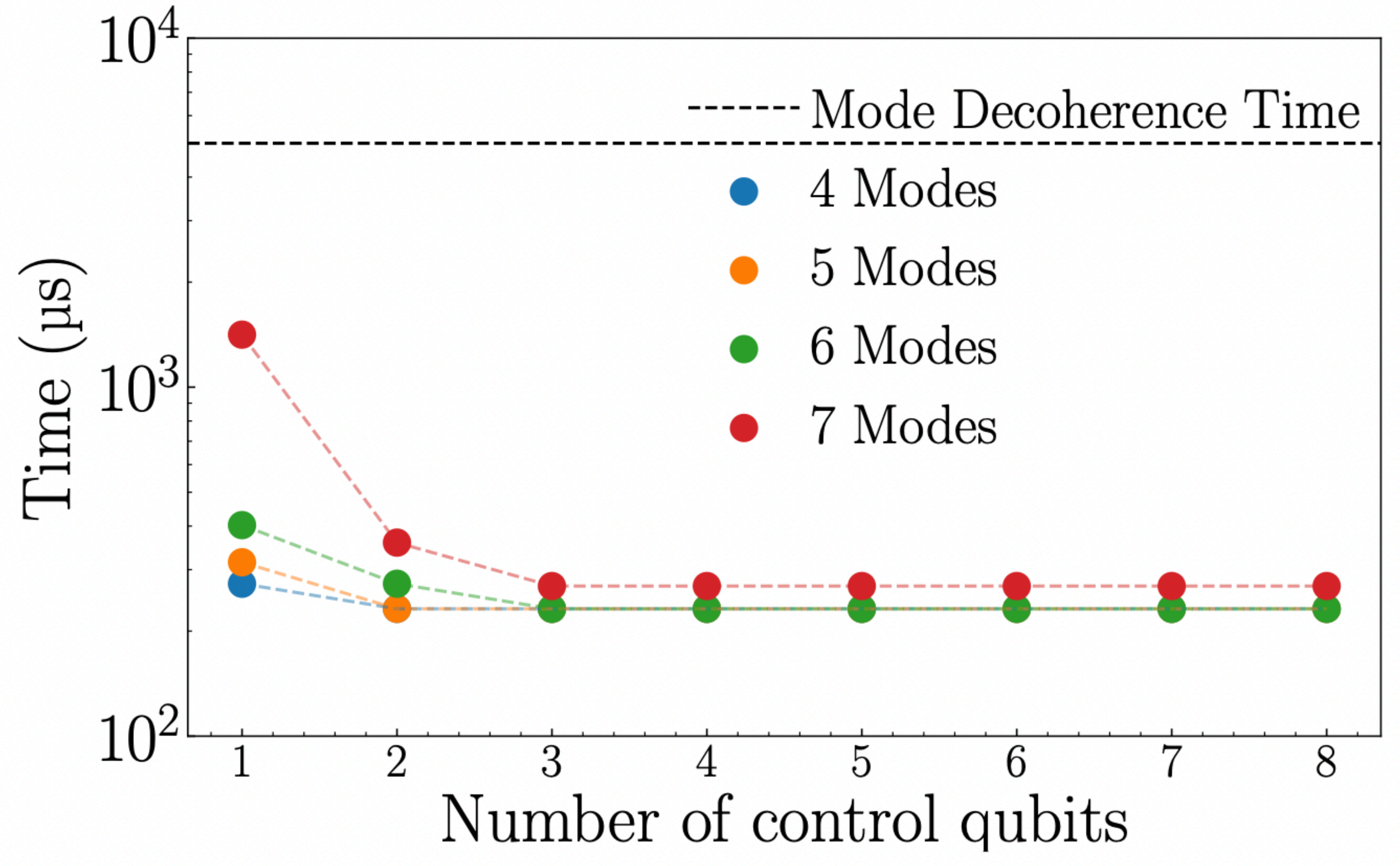
Qubit & qumodes with trapped ions

Araz, Grau, Montgomery, FR `24

- Eight $^{171}Yb^{+}$ ions in a linear Paul trap



- E.g. two-qumode beam splitter $U_{\rm bs}(z) = e^{z\hat{a}^\dagger\hat{b} - z^*\hat{a}\hat{b}^\dagger}$ →



→ Feasible with current platforms

The many-body Jaynes-Cummings model

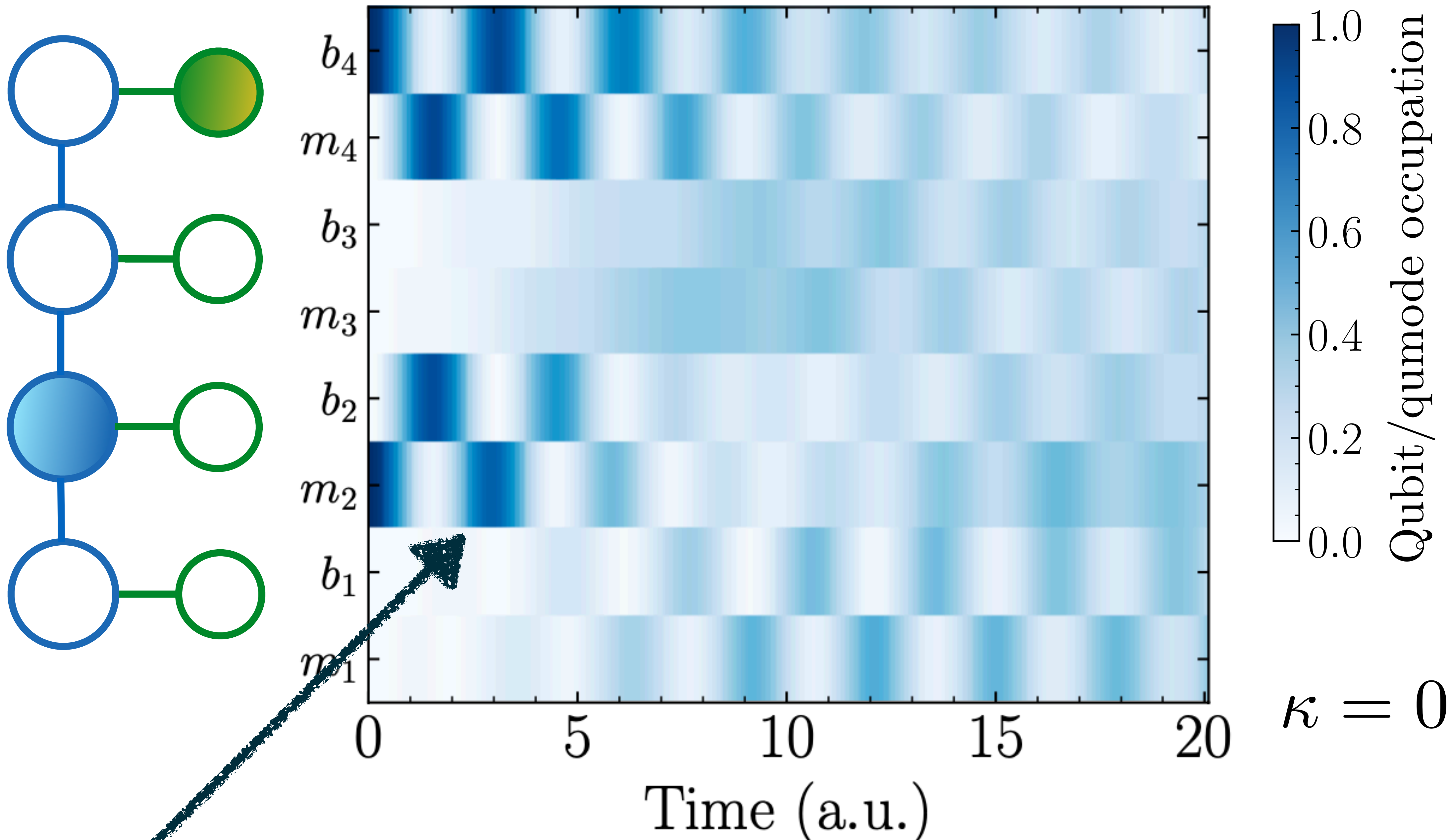
Araz, Grau, Montgomery, FR `24

- Real-time evolution

$$H = \sum_{n=1}^{N} \omega_c a_n^\dagger a_n + \sum_{n=1}^{N} \omega_a \sigma_n^+ \sigma_n^- + \kappa \sum_{n=1}^{N} \left(a_{n+1}^\dagger a_n + a_n^\dagger a_{n+1} \right) + \eta \sum_{n=1}^{N} \left(a_n \sigma_n^+ + a_n^\dagger \sigma_n^- \right)$$

- Lattice sites

Initial configuration



$\kappa = 0.2, \;\; \eta = 1$

Excitation initially localized

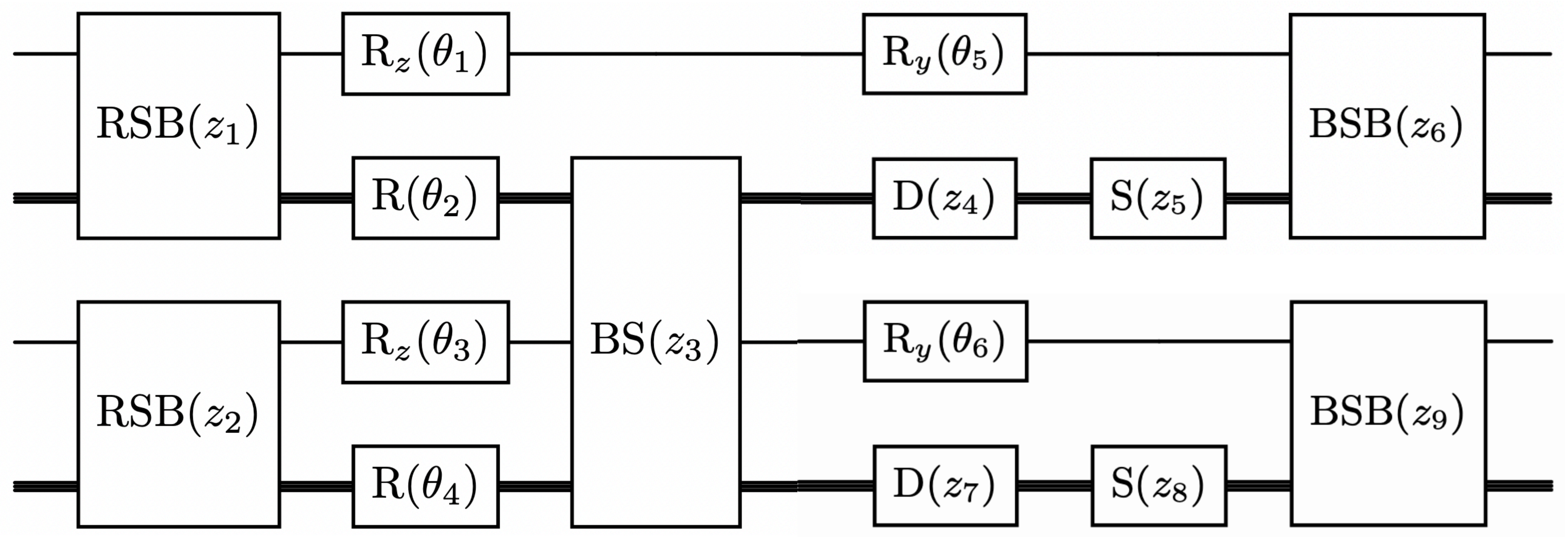
The many-body Jaynes-Cummings model

Araz, Grau, Montgomery, FR `24

- Ground state preparation

$$\hat{H} = \sum_{n=1}^{N} \omega_c a_n^\dagger a_n$$
$$+ \sum_{n=1}^{N} \omega_a \sigma_n^+ \sigma_n^-$$
$$+ \kappa \sum_{n=1}^{N} \left(a_{n+1}^\dagger a_n + a_n^\dagger a_{n+1} \right)$$
$$+ \eta \sum_{n=1}^{N} \left(a_n \sigma_n^+ + a_n^\dagger \sigma_n^- \right)$$

- VQA ansatz
- Include additional number-nonconserving gates



Hamiltonian ansatz

Can achieve variable $\langle \hat{N}_{\text{tot}} \rangle$