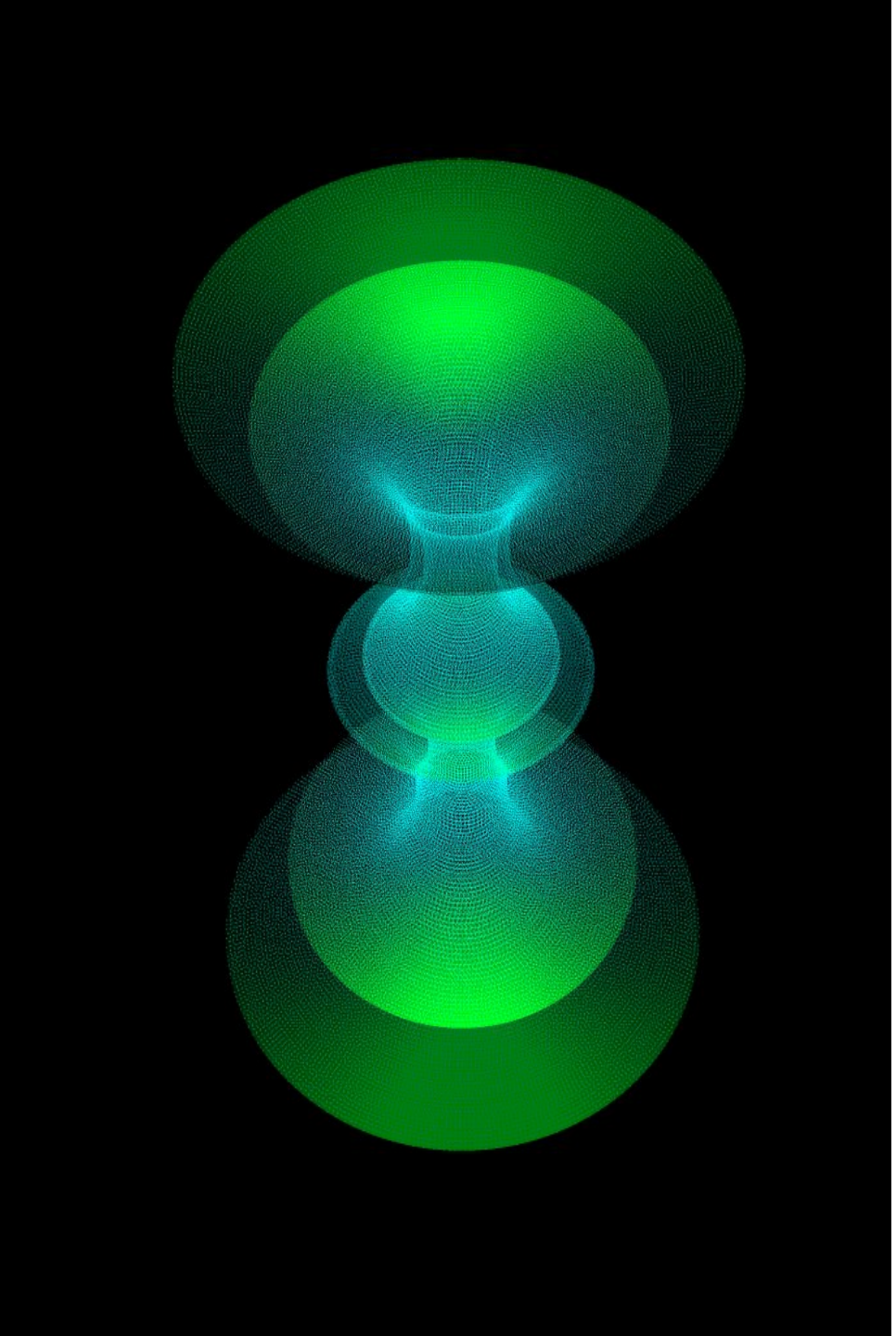


Black Hole States

ROBERTO EMPARAN ICREA+ICCUB

UK THEORY MEETING - DURHAM

16 DECEMBER 2024

WITH ANA CLIMENT

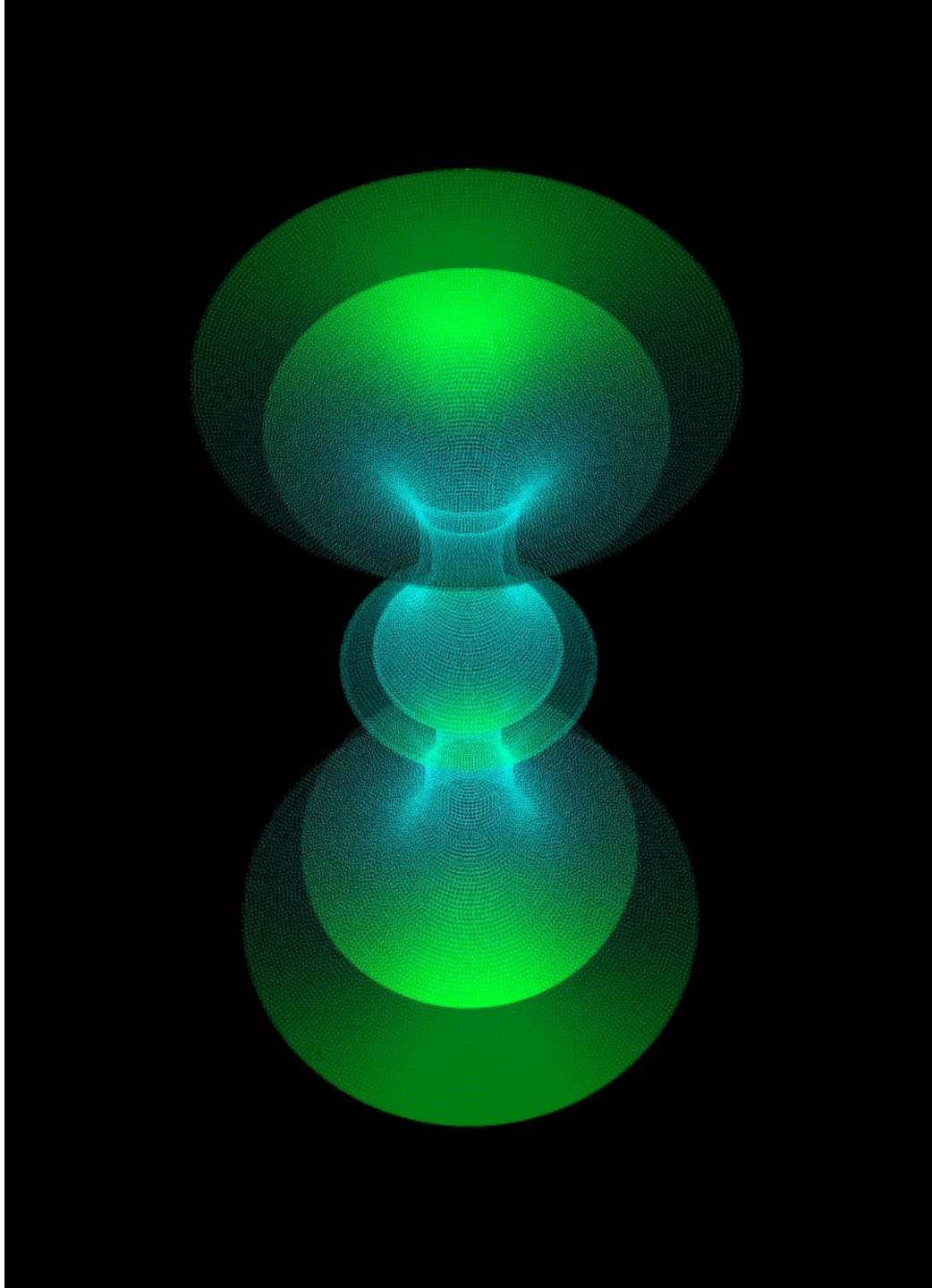
JAVIER M. MAGÁN

MARTÍN SASIETA

ALEJANDRO VILAR LÓPEZ

ARXIV 2401.08755

PRD 109 086024 (2024)



$$S = \frac{A}{4G\hbar}$$

and Black Hole states

Counting states – but not black holes

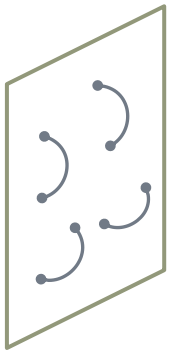
$$S = \frac{A}{4G\hbar}$$

from D-branes in String theory Strominger+Vafa 1996

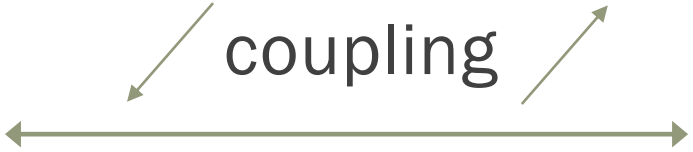
Non-gravitating states that are not black holes

Counting states – but not black holes

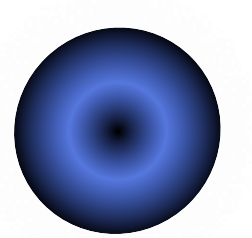
D-brane state



coupling



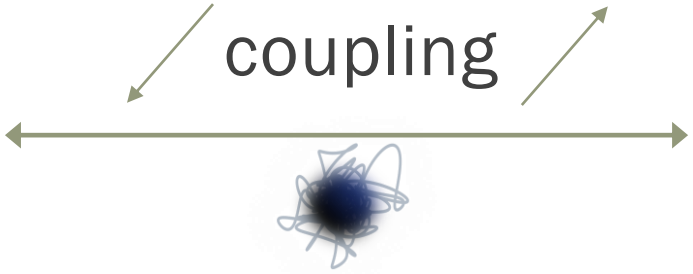
Extremal BPS charged black hole



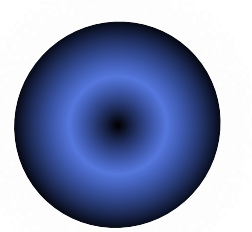
Susy protects number of states as coupling changes

Counting states – but not black holes

Fundamental string state



Neutral black hole



No susy protection, but smooth matching at transition string/BH

Black Hole states – what for?

- BH info paradox and recovery of information
- Experience of infalling observer
- BH interior and singularity

Need finite (non-perturbative) gravitational coupling – hard for String Theory esp w/out SUSY

Black Hole states – what kind?

- Horizonless non-singular microstate geometries: fuzzballs
- Bag-of-gold states — with horizons and singularities

Completely different starting points, methods, and results—do they meet anywhere? BPS systems?

Black Hole states – what kind?

- Horizonless non-singular microstate geometries: fuzzballs
- Bag-of-gold states — with horizons and singularities

NB: this talk won't always require AdS/CFT, but often useful

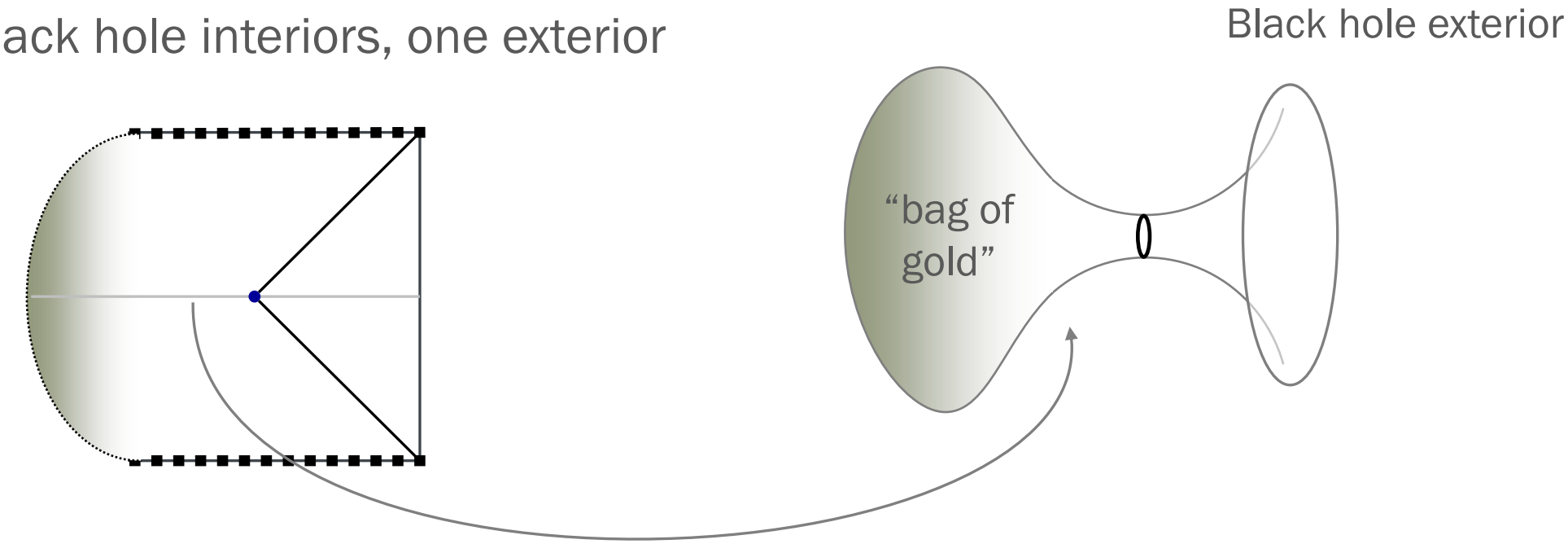
Black Hole states – what are they?

Microscopic pure states $|\Psi\rangle$ that are almost indistinguishable (for simple observables) from thermal state ρ_{th}

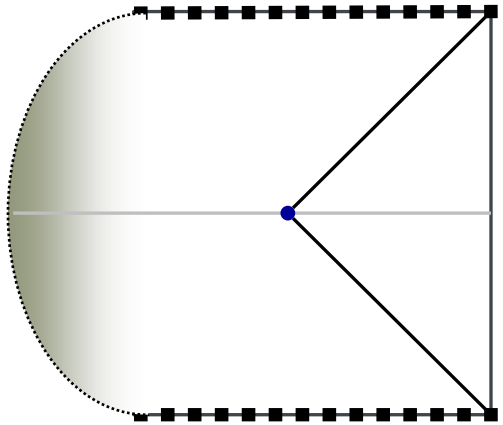
$$\langle\Psi|\mathcal{O}(t)|\Psi\rangle \to \mathrm{Tr}(\rho_{\mathrm{th}}\mathcal{O}), \qquad \langle\Psi|\mathcal{O}(t)\mathcal{O}(0)|\Psi\rangle \to \mathrm{Tr}(\rho_{\mathrm{th}}\mathcal{O}(t)\mathcal{O}(0))$$

Bags of gold

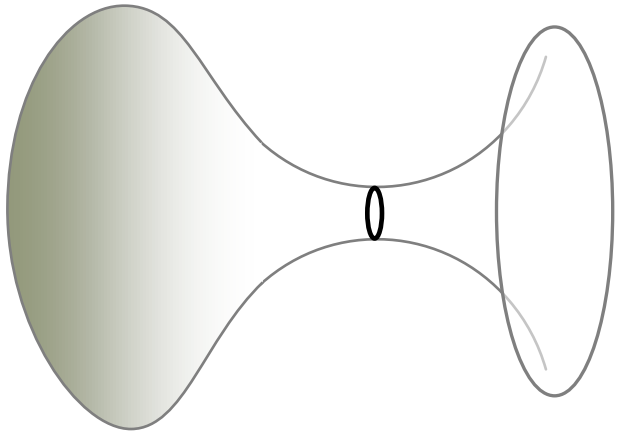
Many black hole interiors, one exterior



Bags of gold

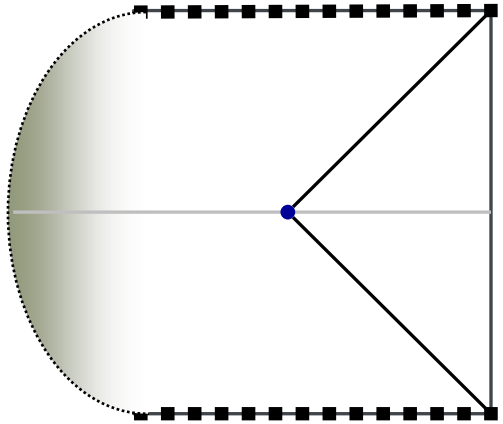


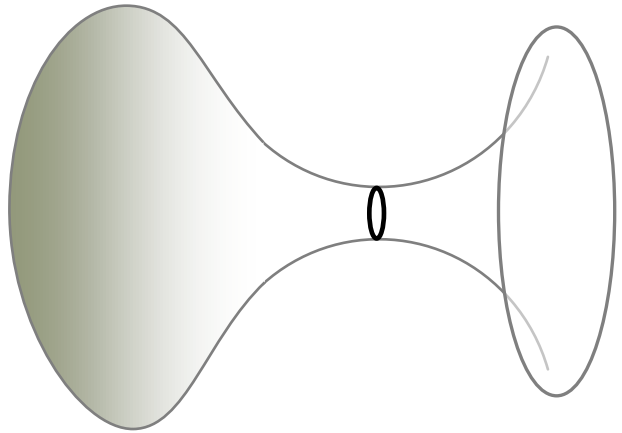
Black hole exterior



Almost indistinguishable (for simple observables) from black hole

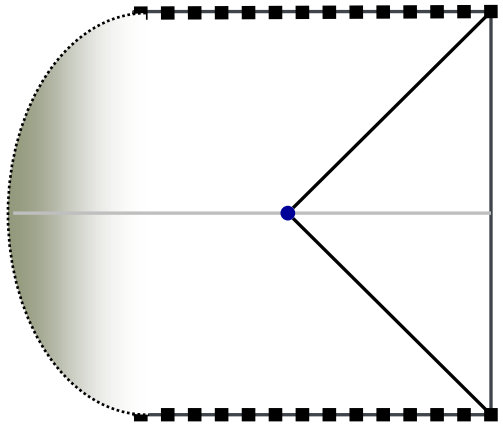
Bags of gold





Arbitrarily many internal states $\gg e^S$??

Bags of gold



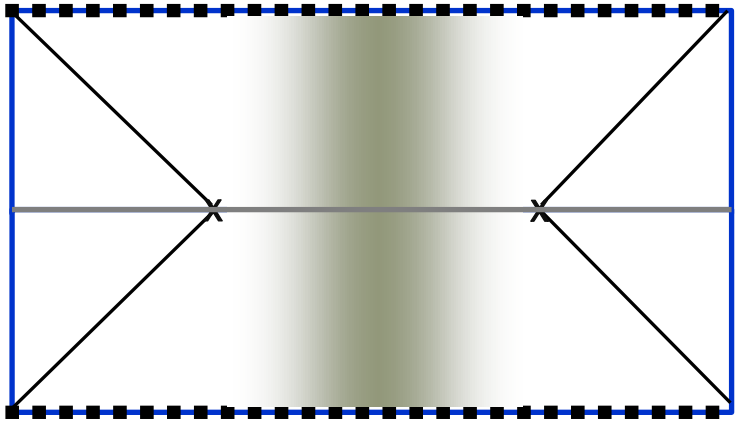
Arbitrarily many internal states $\gg e^S$??

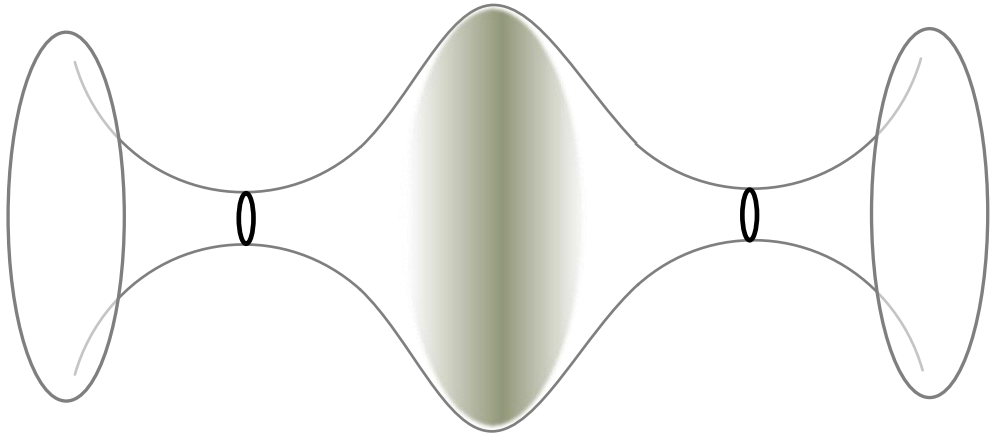
Exterior can be entangled with arbitrarily many interior states

BH can store arbitrary amount of info

→ no Page curve: BH info paradox

Two-sided





Constructing (and counting) BH states

Kourkoulou+Maldacena

Goel+Lam+Turiaci+Verlinde

Penington+Shenker+Stanford+Yang (PSSY)

Lin+Maldacena+Rozenberg+Shan

Chandra+Hartman

Boruch+Iliesiu+Lin+Yan

Balasubramanian+Lawrence+Magán+Sasieta

Climent+RE+Magán+Sasieta+Vilar-López

and many others

Quantum states of Gravity from GPI

The Gravitational Path Integral constructs Hilbert spaces of states

Marolf+Maxfield

- Define quantum states $|\Psi_a\rangle$

- Compute their products (overlaps) $\langle\Psi_a|\Psi_b\rangle$

Effective tool – no ultraviolet detail

Surprisingly powerful but very peculiar

Bag-of-gold states = Bogs

Fully gravitational picture

Extremely universal – charge, rotation, susy or not, quantum corrections

Very general argument that *Bogs are not orthogonal independent states*

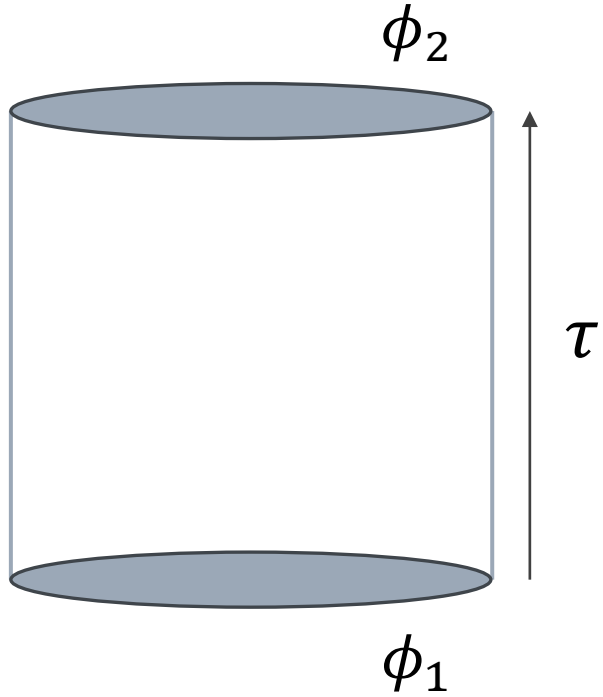
Can be realized within AdS/CFT, but basic Bog idea is more general

Quantum States from Path Integrals

FROM QUANTUM FIELD THEORY TO EUCLIDEAN QUANTUM GRAVITY

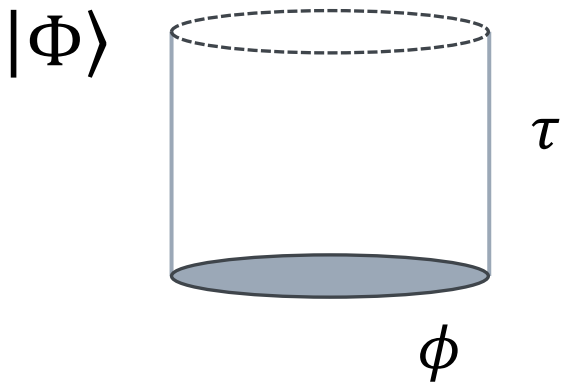
Amplitudes from Path Integral (PI)

$$\langle \phi_2 | e^{-\tau H} | \phi_1 \rangle = \int_{\phi(0)=\phi_1}^{\phi(\tau)=\phi_2} \mathcal{D}\phi \; e^{-I_E[\phi]}$$



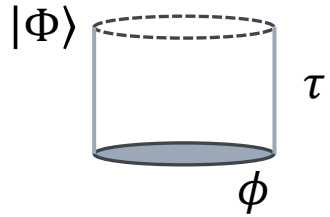
Cutting the PI: State preparation

$$|\Phi\rangle = |\phi(\tau)\rangle = e^{-\tau H}|\phi\rangle$$

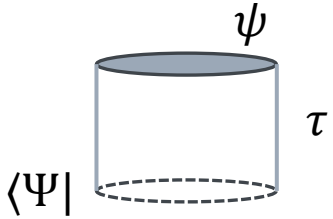


State overlaps

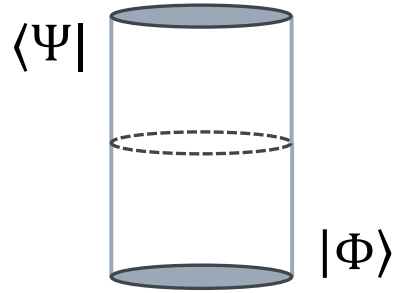
$$|\Phi\rangle = e^{-\tau H}|\phi\rangle$$



$$\langle\Psi| = \langle\psi|e^{-\tau H}$$

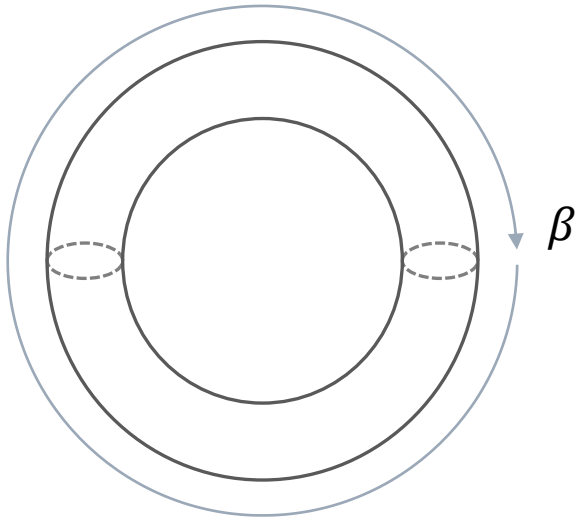


$$\langle\Psi|\Phi\rangle =$$



Thermal states

Imaginary time periodicity



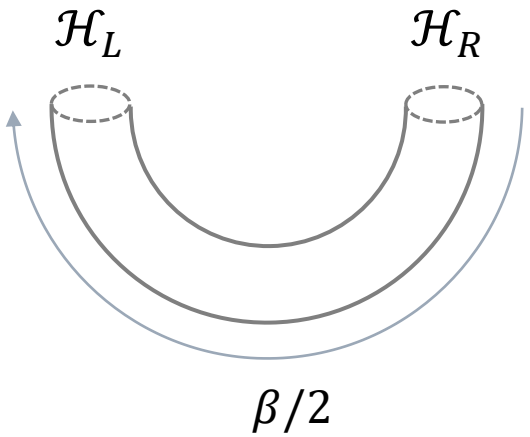
$$\int_{\phi(0)=\phi(\beta)} \mathcal{D}\phi \; e^{-I_E[\phi]}$$

$$= \sum_i \langle E_i | e^{-\beta H} | E_i \rangle$$

$$= \mathrm{Tr}\; e^{-\beta H}$$

Thermofield Double State – TFD

Cut open the path integral

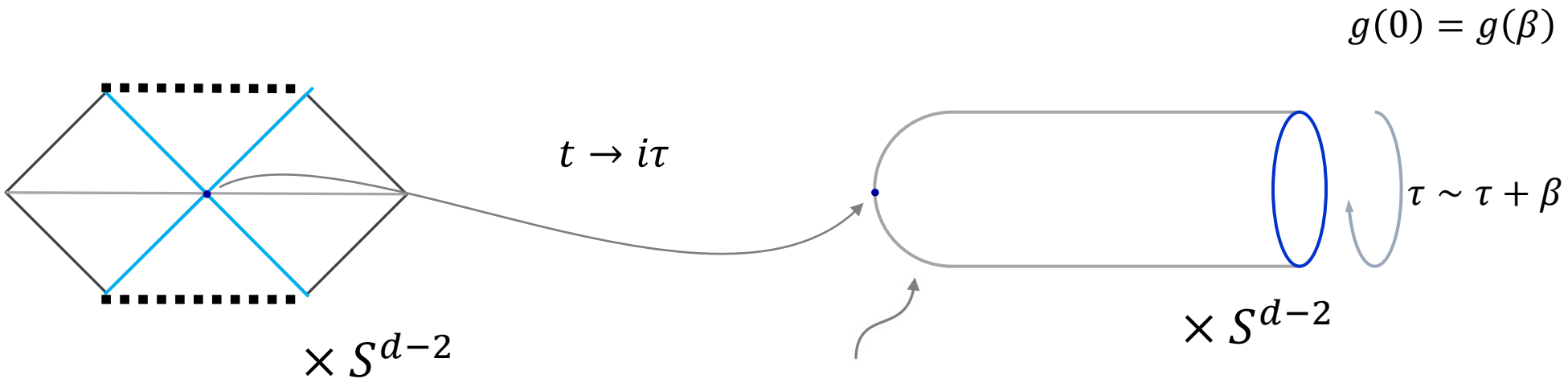


$$|\mathrm{TFD}\rangle = \frac{1}{\sqrt{Z}} \sum_i e^{-\beta H/2} |E_i\rangle_L \otimes |E_i\rangle_R$$

Maximally entangled state

Gravitational Partition Function

$$Z[\beta] = \int_{g(0)=g(\beta)} \mathcal{D}g \; e^{-I_{EH}[g]}$$



Black Magic

$$Z[\beta] = \int_{g(0)=g(\beta)} \mathcal{D}g\, e^{-I_{EH}[g]} \approx e^{-I_{EH}[g_{cl}]}$$

semiclassical saddle-point
g_{cl} = Euclidean black hole

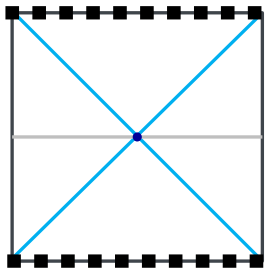
$$S = (\beta\partial_\beta - 1) I_{EH}[g_{cl}] = \frac{A}{4G}$$

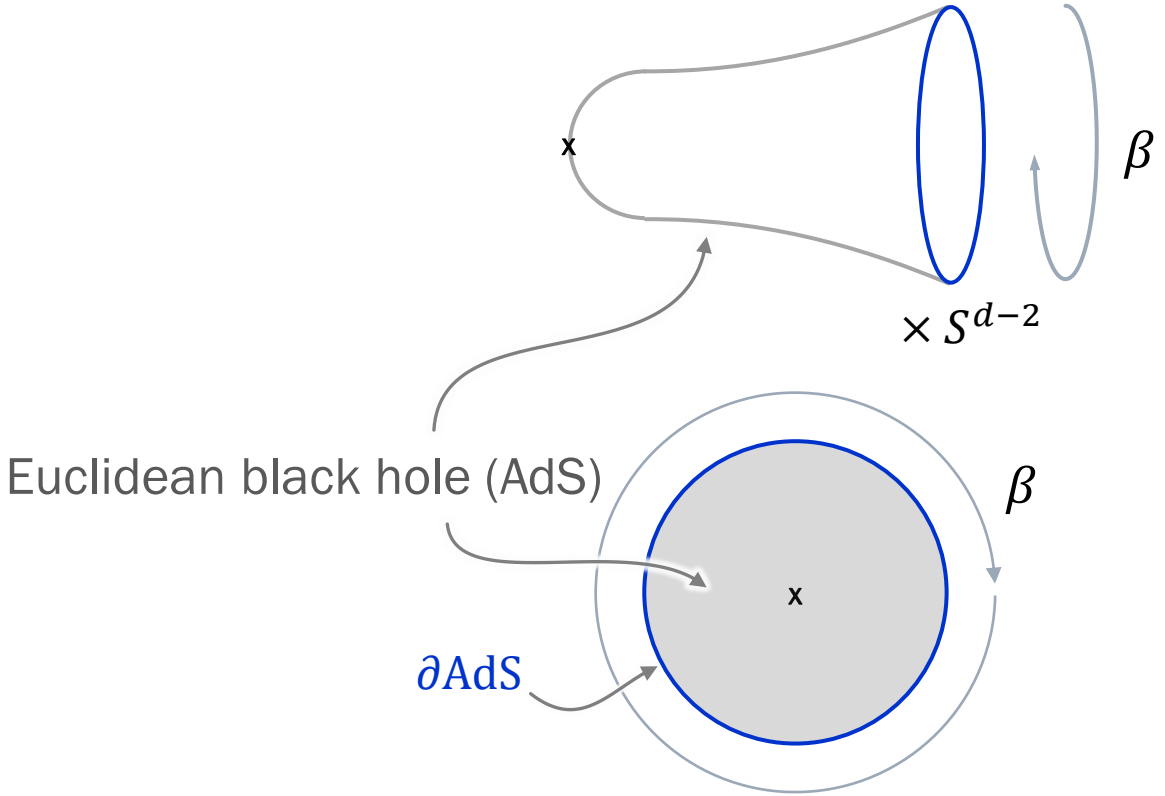
Gibbons+Hawking

- $I_{EH}[g_{cl}]$ = Euclidean action of ***classical*** field configuration: zero-loop
- Not a trace over states: Trace = sum over all states running in a loop: one-loop contribution
- ***Not a sum over microstates*** – but still gives non-zero & correct $S = A/4G$

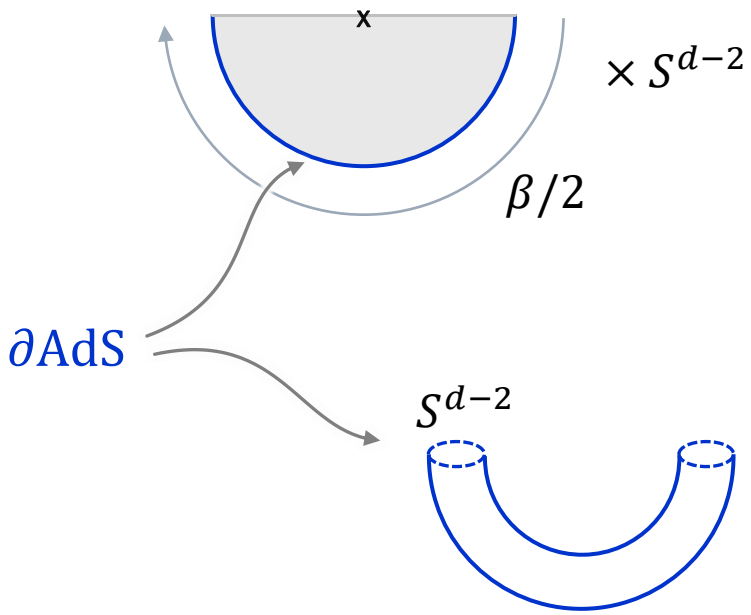
Gravitational Partition Function in AdS

$$\int_{g(0)=g(\beta)} \mathcal{D}g\, e^{-I_{EH}[g]}$$





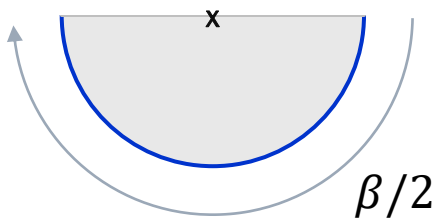
Thermal quantum states from cut GPI



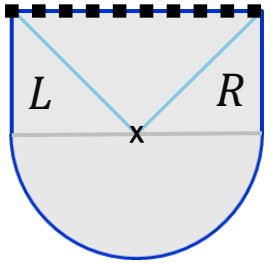
$$|\mathrm{TFD}\rangle = \frac{1}{\sqrt{Z}} \sum_i e^{-\beta H/2} |E_i\rangle_L \otimes |E_i\rangle_R$$

Black Hole as Thermofield double

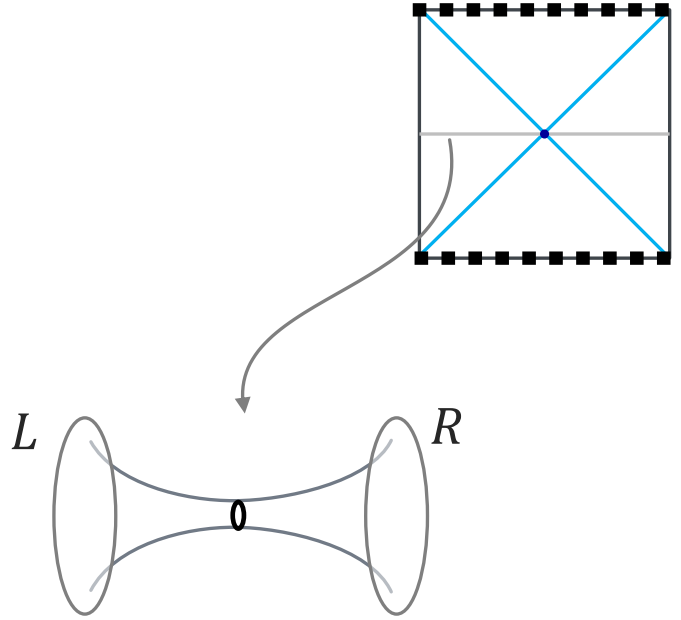
$|\mathrm{TFD}\rangle =$



Lorentzian evolution

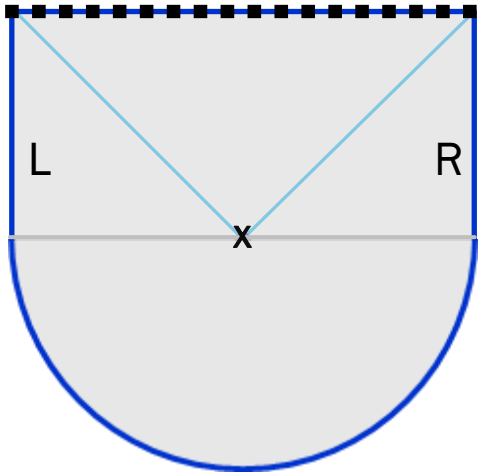


AdS black hole



Maldacena 2001

Thermofield double

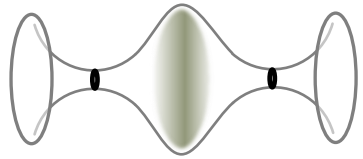


$$|\mathrm{TFD}\rangle = \frac{1}{\sqrt{Z}} \sum_i e^{-\beta H/2} |E_i\rangle_L \otimes |E_i\rangle_R$$

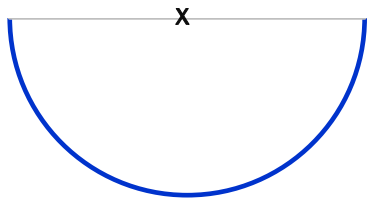
A specific (micro)state of the dual CFT

Dual geometry has a horizon and a singularity

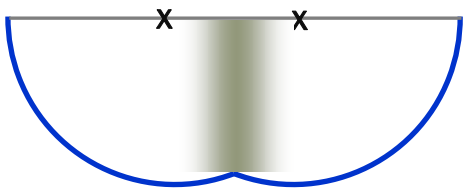
More states



$|\mathrm{TFD}\rangle =$



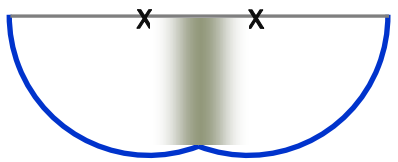
$|\Psi_m\rangle =$

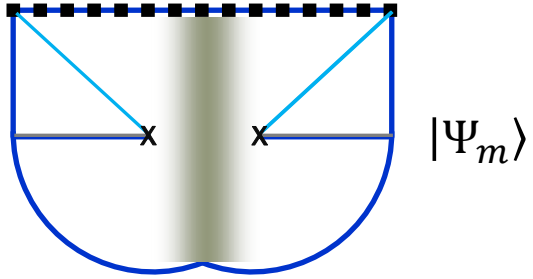


Introduce matter inside black hole

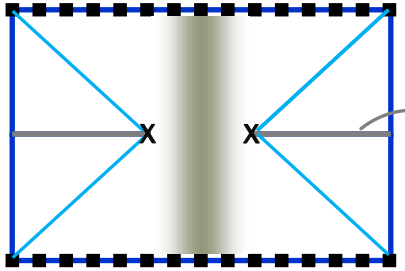
Heavy enough to backreact on geometry: enlarge interior

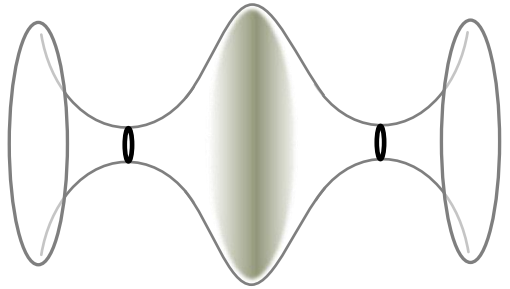
Bog states





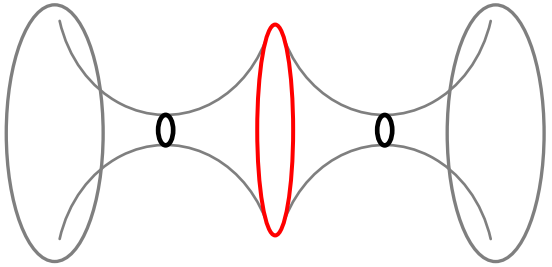
$|\Psi_m\rangle$

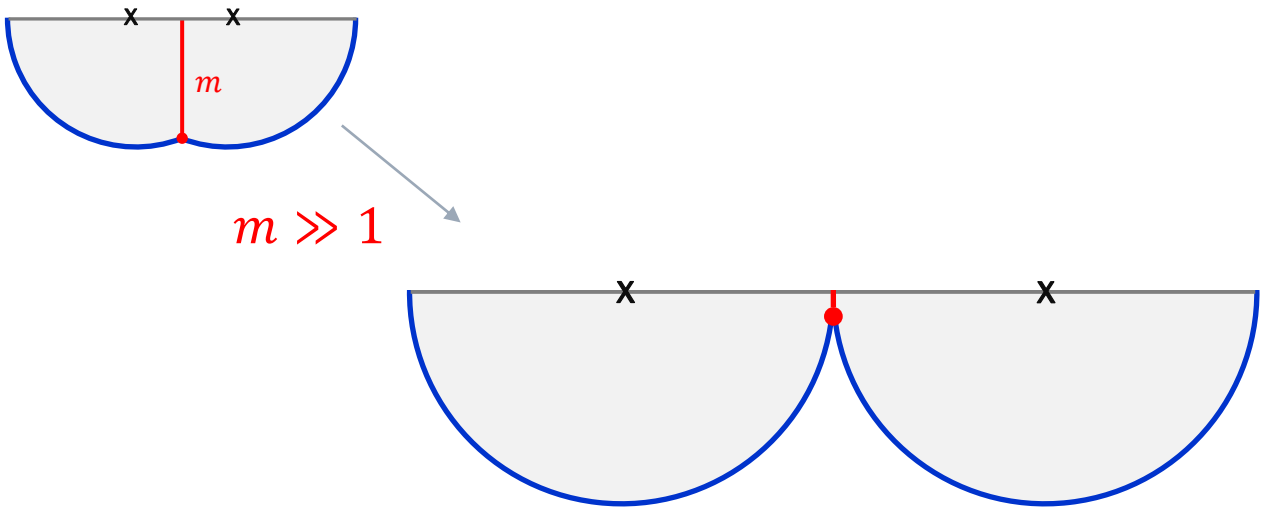


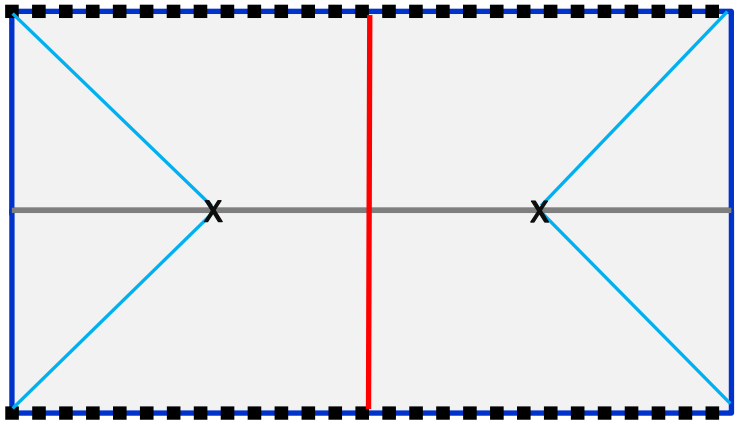


Interior bang/crunch cosmology

Heavy-shell states





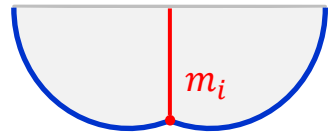


Shell close to the (would-be) boundary – little sensitivity to bulk black hole

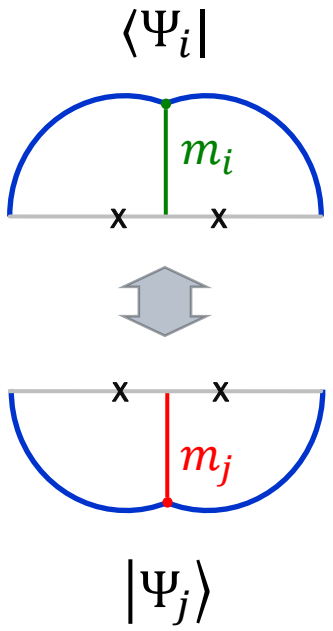
Computing

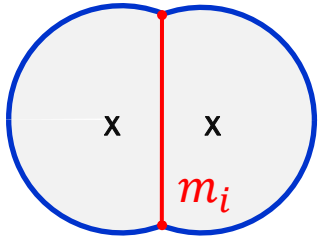
STATE OVERLAPS FROM WORMHOLES: UNIVERSALITY & STATISTICS

State overlaps

$|\Psi_i\rangle =$ 

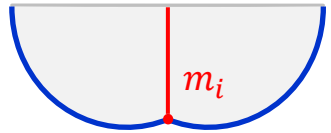
$G_{ij} = \langle\Psi_i|\Psi_j\rangle =$



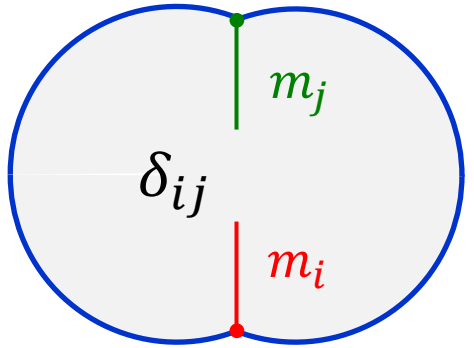
$\langle\Psi_i|\Psi_i\rangle =$ 

G: Gram matrix of state overlaps

Too many states?

$|\Psi_i\rangle =$ 

$$G_{ij} = \langle \Psi_i | \Psi_j \rangle = \delta_{ij}$$



Infinite family of orthogonal states

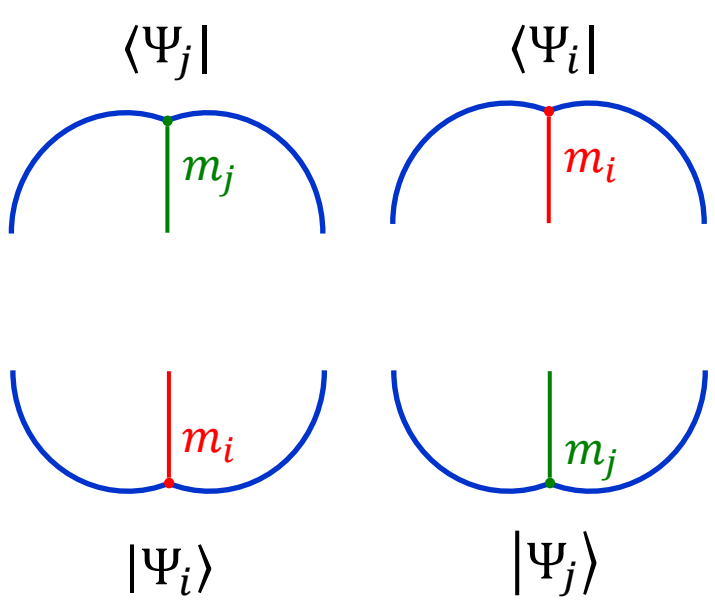
$$\dim(\mathcal{H}_{BH}) = \infty \text{ !?}$$

bag-of-gold problem

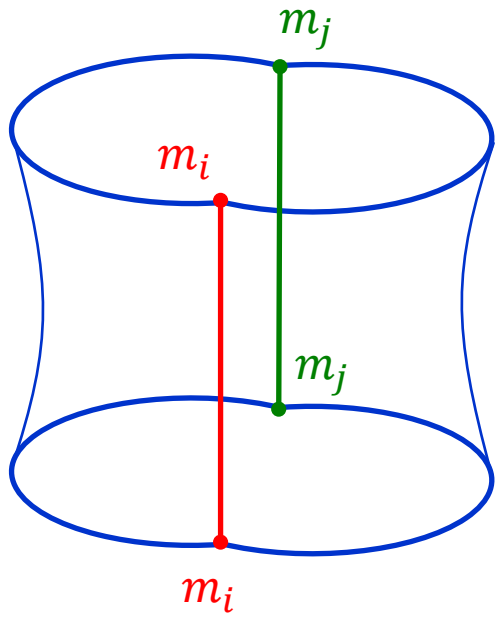
Products with Wormholes

average

$$\overline{G_{ij}G_{ji}} = \overline{\langle\Psi_i|\Psi_j\rangle\langle\Psi_j|\Psi_i\rangle}$$



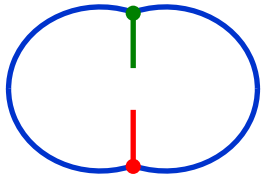
⇢



$\neq 0$

Wormholes ⇒ Statistical states

$\overline{G_{ij}} = 0$ for $i \neq j$

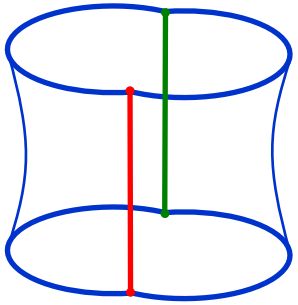


$= 0$

Not $\langle \Psi_i | \Psi_j \rangle = 0$

but $\overline{\langle \Psi_i | \Psi_j \rangle} = 0$

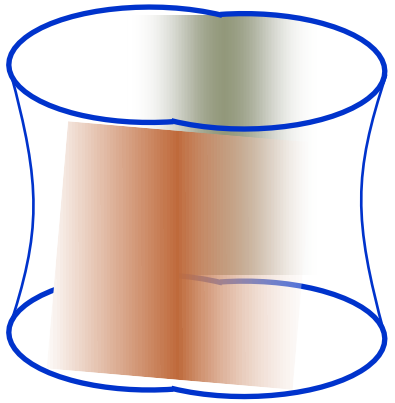
$\overline{G_{ij} G_{ji}} \neq 0$ for $i \neq j$



$\neq 0$

Bags of gold are *never* orthogonal

$$\overline{\langle \Psi_a | \Psi_b \rangle \langle \Psi_b | \Psi_a \rangle} \neq 0$$



$$\neq 0$$

Moments of G

$$\overline{G_{i_1 i_2} G_{i_2 i_3} \dots G_{i_n i_1}} = \frac{Z(n\beta, \mu_I)^2}{Z(\beta, \mu_i)^{2n}}$$

Heavy-shell universality

Depends only on BH properties

Moments

From grand-canonical to microcanonical BH window

$$\overline{G_{i_1 i_2} G_{i_2 i_3} \dots G_{i_n i_1}}\Big|_{\text{grcan}} = \frac{Z(n\beta, \mu_I)^2}{Z(\beta, \mu_i)^{2n}}$$

inverse Laplace transform

$$\overline{G_{i_1 i_2} G_{i_2 i_3} \dots G_{i_n i_1}}\Big|_{\text{micro}} = e^{-(n-1)\frac{2A}{4G_N}}$$

Counting

THE DIMENSION OF THE BLACK HOLE HILBERT SPACE

Dimension of set of states

$$F_\Omega = \{\, |\Psi_i\rangle \in \mathcal{H}, i = 1, \ldots, \Omega \,\}$$

$$d_\Omega = \dim F_\Omega = \min\{\Omega, \dim \mathcal{H}\}$$

$$= \operatorname{rank} G_{ij} \qquad\qquad G_{ij} = \langle \Psi_i | \Psi_j \rangle$$

Gram-Schmidt fails for BH heavy-shell states:

$$\overline{G_{ij}} = \delta_{ij}$$

Statistical counting

From statistical moments $\overline{G^n}$

Statistics forced by GPI wormholes

Borrow from random matrix techniques: resolvent

$$R(\lambda) = \mathrm{Tr}\left(\frac{1}{\lambda\mathbb{I} - G}\right) = \frac{\Omega}{\lambda} + \sum_{n=1}^{\infty} \frac{\mathrm{Tr}\, G^n}{\lambda^{n+1}} \quad \longrightarrow d_\Omega$$

Penington+al (PSSY) 2019

How many states? $(\exp A/4G_N)^2$

We had $d_\Omega = \dim F_\Omega = \min\{\Omega, \dim \mathcal{H}\}$

Resolvent for $\overline{G^n}$ gives $\overline{d_\Omega} = \min\{\Omega, e^{2A/4G_N}\}$

$\Rightarrow$ $\boxed{\dim \mathcal{H} = e^{2A/4G_N}}$

Two-sided black hole: $(\text{CFT})^2$

Universality of $\dim \mathcal{H} = (\exp S_{BH})^2$

Heavy shells can be constructed for

- Rotating and charged black holes
- Near-extremal, susy or not
- Quantum-corrected: $\log A$ and $\log T$
- Higher-curvature theories

Heavy-shell states $\Rightarrow$ $\dim \mathcal{H} = e^{2S_{BH}}$

Outlook

GPI RELOADED – WITH WORMHOLE STATISTICS

Gravitational Path Integral can do a lot

- Construct BH states and count their dimension
- Heavy-shell states $\Rightarrow \dim \mathcal{H} = e^{2S_{BH}}$
- Works for all cases where Gibbons-Hawking gives an entropy

Gravitational Path Integral is EFT

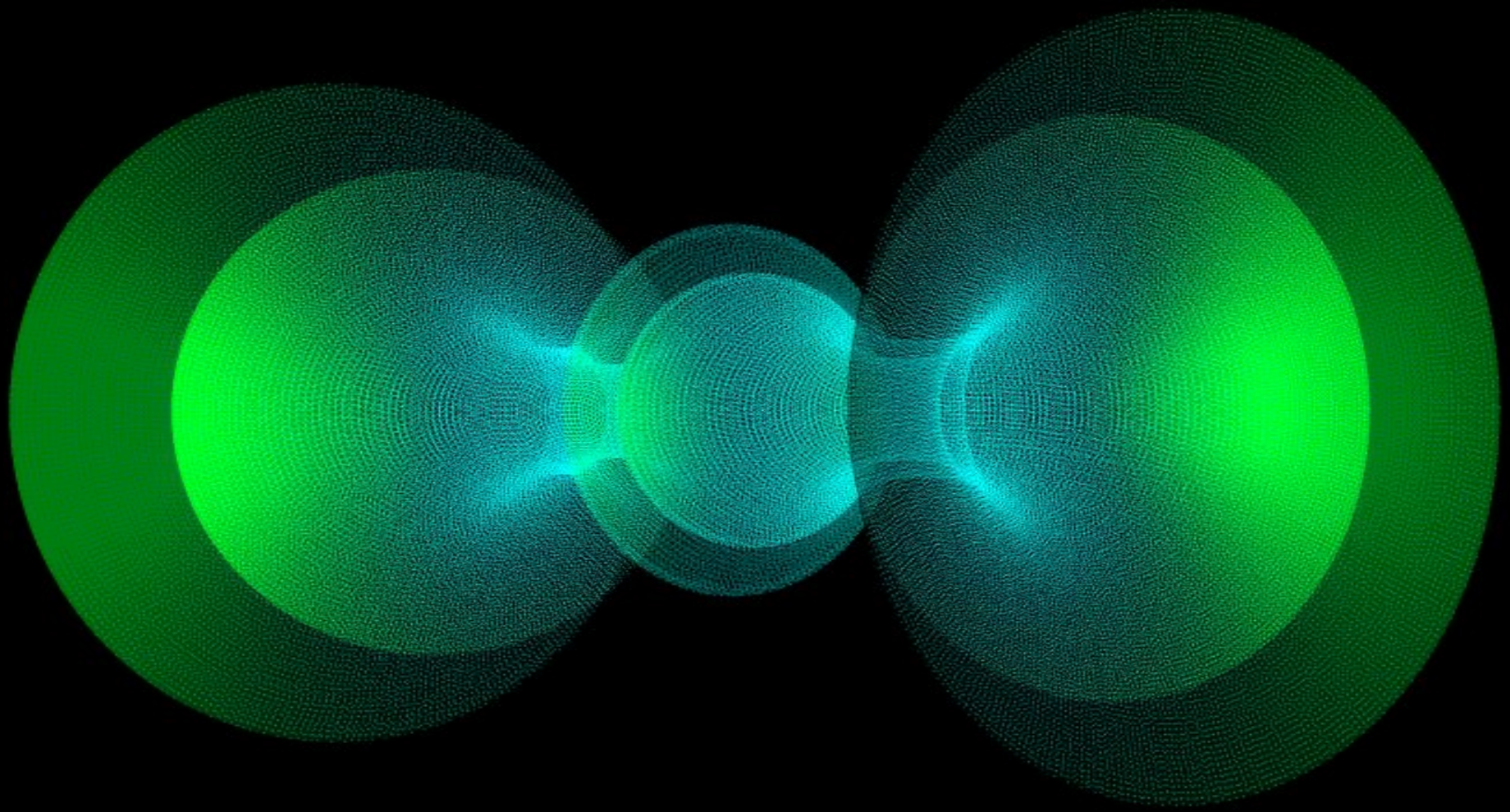
- The GPI is an effective tool with assumed (natural) rules
- Can this be derived from a microscopic theory?
- From what theory? Unlikely from (perturbative) string theory.

  AdS/CFT?

Geometry and Randomness

- Wormholes are how gravity knows about finite $\dim \mathcal{H}_{BH}$
- But they introduce intrinsic randomness
- Semiclassical BH geometry seems to need chaotic microscopics

Is this all one needs/can do for
(non-susy) BH microscopics?

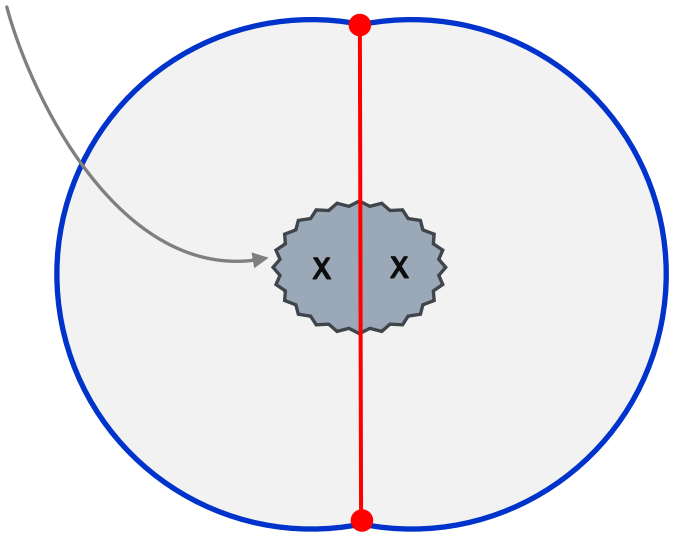


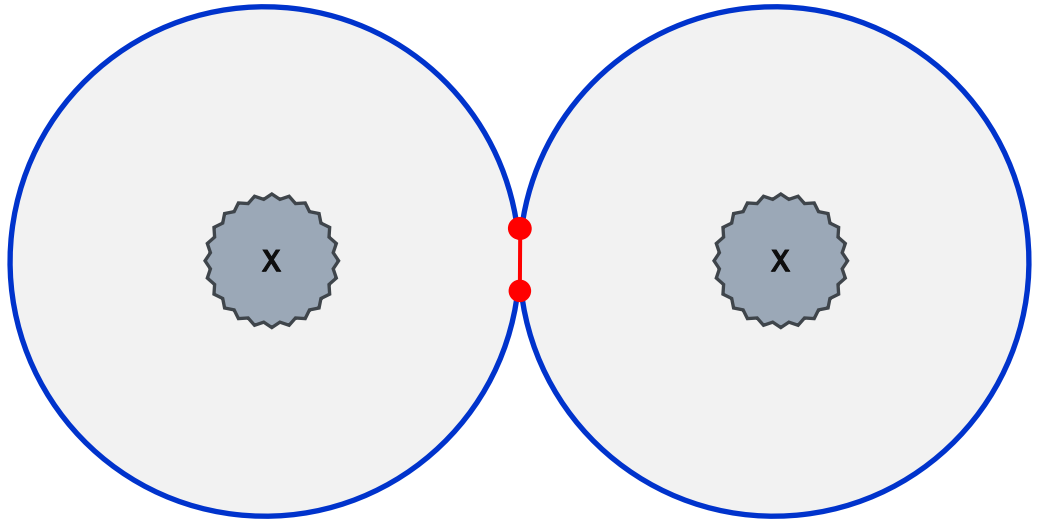
Thank you

Backup material

Near-extremal Microstates

near-extremal AdS_2 throat (JT Schwarzian)





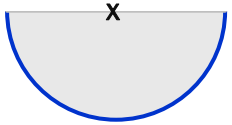
In-throat microstates (one JT Schwarzian)

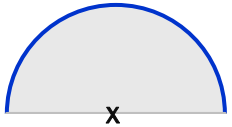
Sensitive to throat

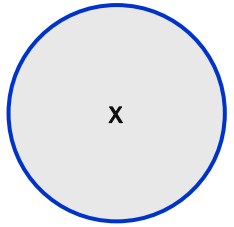
Out-throat microstates (two Schwarzians)

Universal

Products from path integral

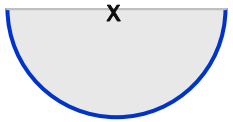
$|\mathrm{TFD}\rangle =$ 

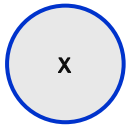
$\langle \mathrm{TFD}| =$ 

$\langle \mathrm{TFD}|\mathrm{TFD}\rangle =$ 

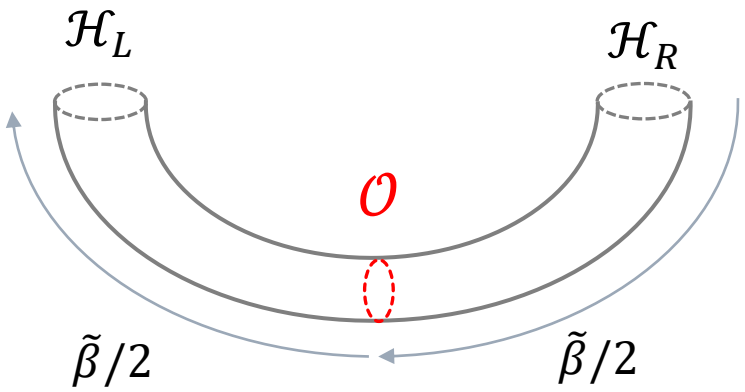
$\equiv Z_1[\beta]$

Not interpreted as partition function $e^{-\beta F}$
but as norm of state

Normalized $|\mathrm{TFD}\rangle =$ 

$\times \dfrac{1}{\sqrt{\quad}}$ 

Partially Entangled Thermal States

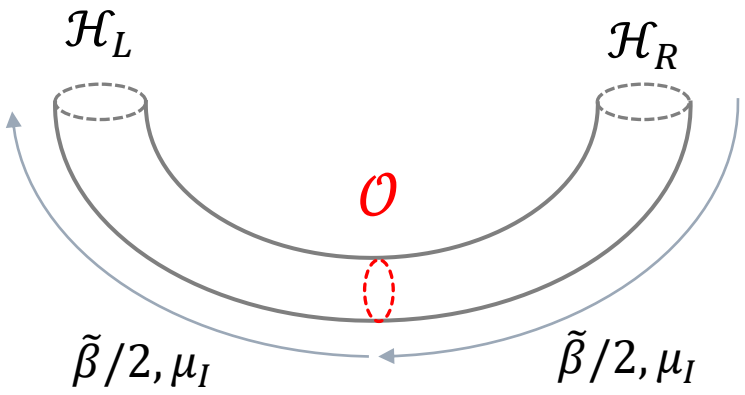


PETS

Goel+Lam+Turiaci+Verlinde

$$|\Psi\rangle = \frac{1}{\sqrt{Z_1}} \sum_i e^{-\tilde{\beta}H/2} \mathcal{O} e^{-\tilde{\beta}H/2} |E_i\rangle_L \otimes |E_i\rangle_R$$

Partially Entangled Grand-canonical States



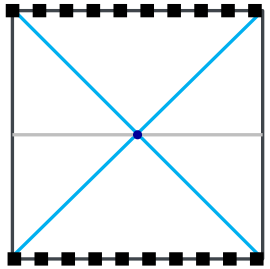
Add charge & rotation: **PEGS**

$$|\Psi\rangle = \frac{1}{\sqrt{Z_1}} \sum_i e^{-(\tilde{\beta}-\mu_I Q_I)H/2} \mathcal{O} e^{-(\tilde{\beta}-\mu_I Q_I)H/2} |E_i\rangle_L \otimes |E_i\rangle_R$$

Thermofield double = Eternal black hole

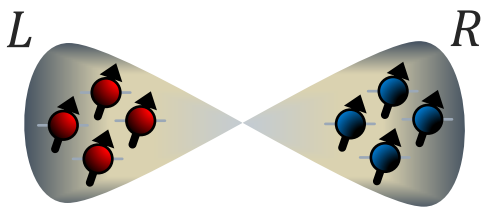
$$|\mathrm{TFD}\rangle = \frac{1}{\sqrt{Z}} \sum_i e^{-\beta E_i/2} |i\rangle_L |i\rangle_R$$

=



Bell/EPR pair

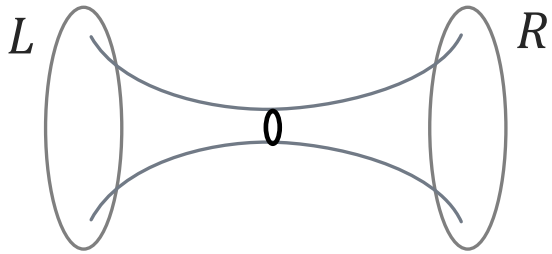
$$|\Psi\rangle = \frac{1}{\sqrt{2}} (|0\rangle_L \, |0\rangle_R + |1\rangle_L \, |1\rangle_R)$$



Correlation/connection, but no communication between sides

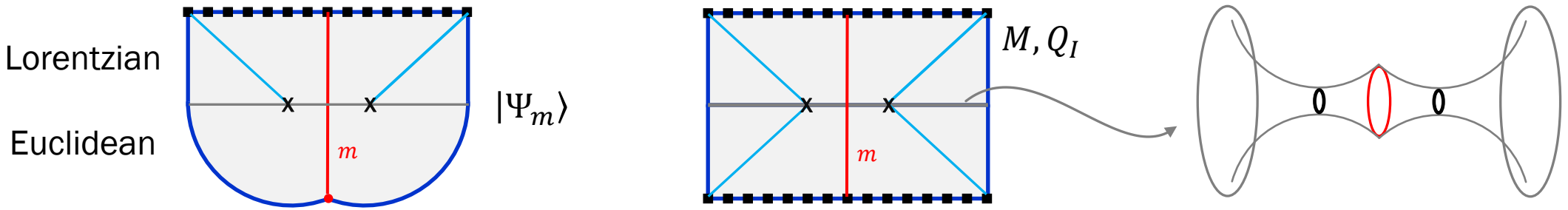
Thermal behavior when only one side is probed

ER bridge



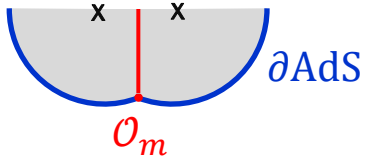
Maldacena+Susskind 2013

Shell states

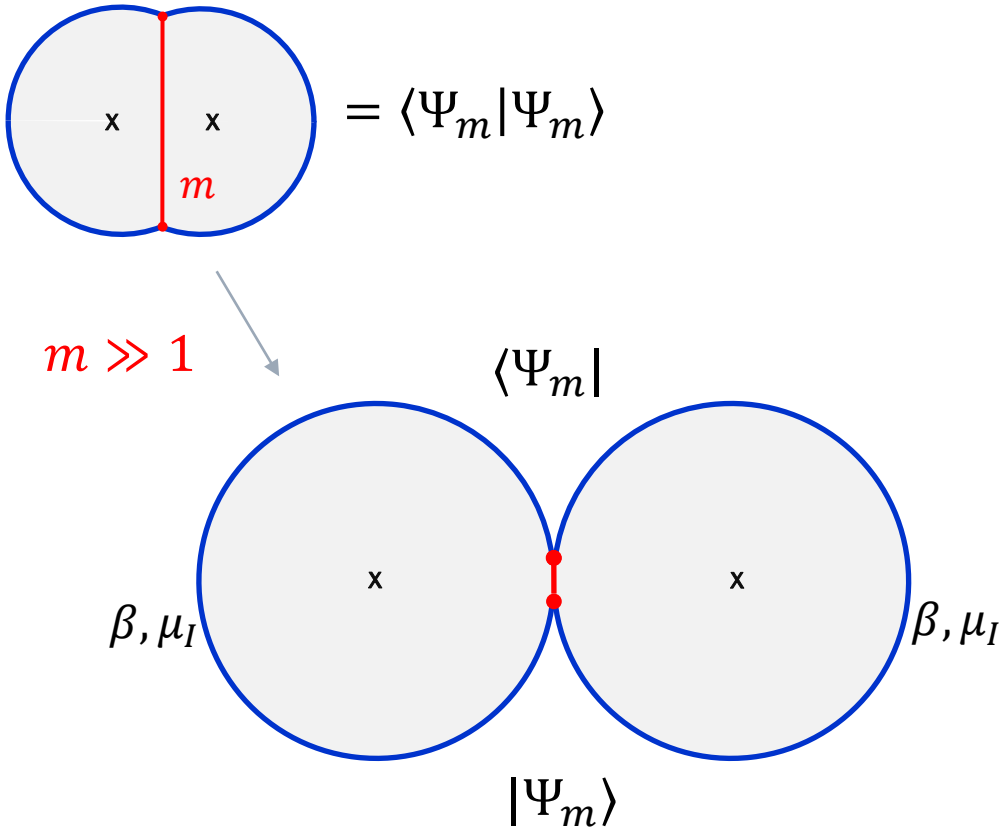


Particles in the bulk – a ‘shell of dust’ matter m

In AdS/CFT, operator $\mathcal{O}_m$ inserted at boundary



Heavy-shell states



$\langle \Psi_m | \Psi_m \rangle$ factorizes into $\approx Z_1[\beta, \mu_I]^2$

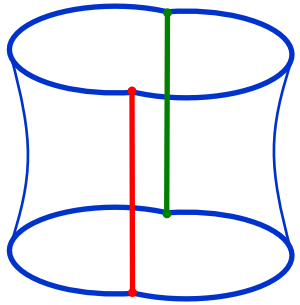
Shell does not affect horizon properties

Dependence on shell m drops out →

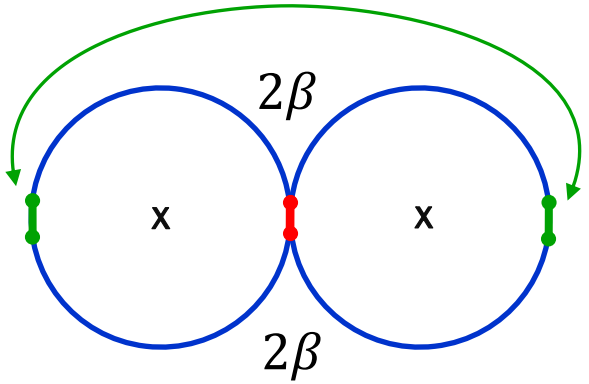
universality

Heavy-shell Wormholes

$$\overline{G_{ij}G_{ji}} =$$



$\rightarrow$

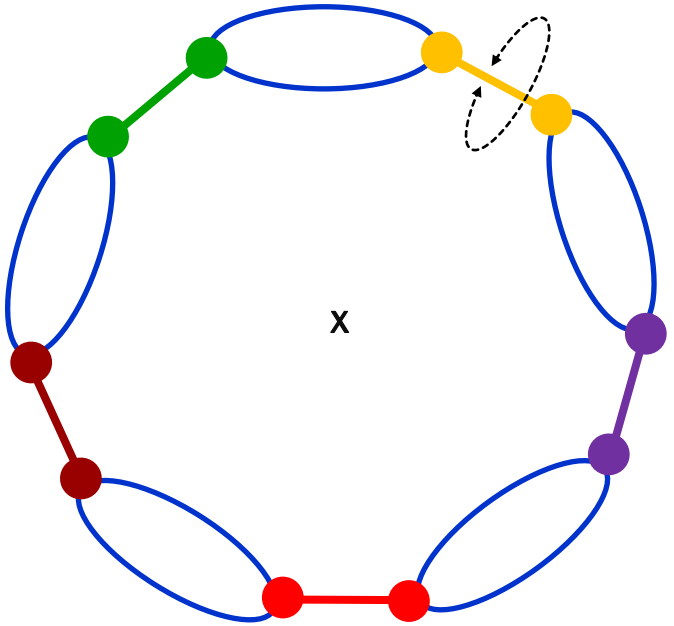


$$= \frac{Z(2\beta, \mu_I)^2}{Z(\beta, \mu_i)^4}$$

unaffected by shell m_i

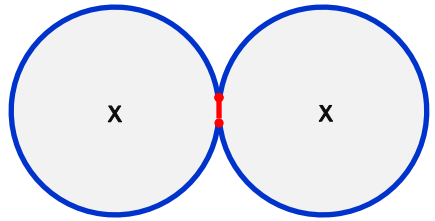
Multi-boundary Wormholes

$$\overline{G_{i_1 i_2} G_{i_2 i_3} \dots G_{i_5 i_1}} =$$



How many states? $(\exp S_{BH})^2$

More generally



$$\dim \mathcal{H} = e^{2S_{BH}}$$

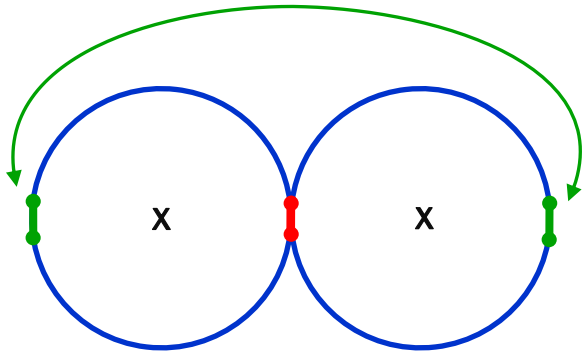
where S_{BH} is the value from Gibbons-Hawking Partition Function

Non-trivial consistency of the GPI

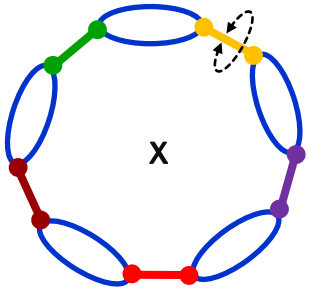
Bog states: remarks

Complete semiclassical analisis of overlaps needs multi-wormhole geometries, and weak interaction

Only possible for *large* Bogs with highly *localized matter*



Two-boundary wormhole



Five-boundary wormhole

Bog states: remarks

Microcanonical geometry? (better when BPS)

Complete semiclassical analisis of overlaps needs multi-wormhole geometries, and weak interaction

- Only possible for *large* Bogs with highly *localized matter*