







Science & Technology Facilities Council

Numerical relativity for fundamental physics

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Plan for the talk

- What is numerical relativity?
- What can it do? What can't it do?
- A fun(damental) example warp drive collapse







Main message:

Numerical relativity simulations can be used as a tool to investigate the consequences of fundamental physics in strong gravity regimes. They are a modern day gedanken experiment.



What is numerical relativity?

Curved spacetime



 $ds^{2} = \begin{pmatrix} dt & dx & dy & dz \end{pmatrix} \begin{pmatrix} g_{00} & g_{01} & g_{02} & g_{03} \\ g_{10} & g_{11} & g_{12} & g_{13} \\ g_{20} & g_{21} & g_{22} & g_{23} \\ g_{30} & g_{31} & g_{32} & g_{33} \end{pmatrix} \begin{pmatrix} dt \\ dx \\ dy \\ dz \end{pmatrix}$

"The spacetime metric"

 $g_{ab}(t, \vec{x})$

What is the goal of NR?

The metric

 $ds^2 = \begin{pmatrix} dt & dx & dy & dz \end{pmatrix}$



g_{00}	g_{01}	g_{02}	g_{03}
g_{10}	g_{11}	g_{12}	g_{13}
g_{20}	g_{21}	g_{22}	g_{23}
$\langle g_{30} \rangle$	g_{31}	g_{32}	$g_{33}/$
-			



 $g_{ab}(t, \vec{x})$



dt `

dx

dy

dz

The Einstein equation

$R_{ab} - R/2 g_{ab} = 8\pi T_{ab}$



 $f(\partial^2 g_{ab}, \partial g_{ab}, g_{ab})$ "Curvature"

"Energy-Momentum"



The Einstein equation tells us how the metric should look, given some energy/matter distribution



Four constraint equations for any time slice - non linear elliptic/Poisson equation



An evolution equation for all time - non linear hyperbolic/wave equation

 $R_{ab} - R/2 g_{ab} = 8\pi T_{ab}$

$\frac{\partial}{\partial x^2}$ + non linear terms = *f*(energy, momentum)

$$\frac{\partial^2 g}{\partial x^2} + \text{non linear terms} = f(\text{energy, momentum})$$

"Matter tells spacetime how to curve..."



The metric determines the motion of matter



 ∇^a (R_{ab} - $R/2 g_{ab}$) = ∇^a ($8\pi T_{ab}$) = 0

"...spacetime tells matter how to move."

Continuity equation

$$\frac{\partial \rho}{\partial t} + \underbrace{\nabla \cdot \mathbf{j}}_{g_{ab}} = \underbrace{\operatorname{source}}_{g_{ab}}$$

What is NR?

"local time"



Fill using Einstein equation $\partial_{tt}g_{\mu\nu} = f(\partial_t g_{\mu\nu}, g_{\mu\nu}, T_{\mu\nu})$ $\nabla_{\mu}T^{\mu\nu} = 0 \implies \partial_t T^{\mu\nu} = f(T_{\mu\nu}, g_{\mu\nu})$

What is NR?

"local time"



Fill using Einstein equation $\partial_t g_{\mu\nu} = \partial_{tt} g_{\mu\nu} = 0$ (a bit boring!)

What can it do?

The poster child for numerical relativity is a binary black hole merger



Science & Environment

Einstein's gravitational waves 'seen' from black holes

By Pallab Ghosh Science correspondent, BBC News

I1 February 2016







Why is NR necessary for the binary merger?

DYNAMICAL **SPACETIME**



NO PERTURBATIVE EXPANSION

GRAVITATIONAL BACKREACTION (STRONG GRAVITY)



Numerical simulations play a key role in understanding black hole populations





Credit: LIGO Collaboration

And maybe dark matter too

(Somewhat unexpectedly) there can be a significant enhancement of light DM around equal mass binary BHs

density of dark matter



Dark matter

This can result in a significant dephasing, with potential to introduce bias in parameter estimation



J. Aurrekoetxea, KC, J Bamber, P Ferreira 2023 *Phys.Rev.Lett.* 132 (2024) 21, 211401

Which may include exotic **compact objects**

Our understanding of the potential to miss or mischaracterise these signals is improving

density of dark matter



R Croft, T Helfer, B Ge, M Radia, T Evstafyeva, E Lim, U Sperhake, KC Class.Quant.Grav. 40 (2023) 6,065001

See in particular:

Tamara Evstafyeva et. al., Phys.Rev.Lett. 133 (2024) 13, 131401 Nils Siemonsen Phys.Rev.Lett. 133 (2024) 3,031401



Also modified gravity

(Somewhat unexpectedly) we are able to simulate **beyond GR theories** (well posed formulations can be found)

density/curvature contribution of additional gravitational scalar

y/M



L Areste Salo, KC, P Figueras 2022 Phys.Rev.Lett. 129 (2022) 26, 261104

See in particular: P Figueras, A Held, A Kovacs 2022 Well-posed initial value formulation of general effective field theories of gravity gr-qc <u>2407.08775</u>

Modified gravity

0.4

Simulations in the rh_{22}^+/M early phases of testing and development, but dephasing also has the potential to create systematic bias

 rh_{22}^+/M

-0.2

-0.4

gravitational waves



L Areste Salo, KC, P Figueras 2022 Phys.Rev.Lett. 129 (2022) 26, 261104

Modified gravity

Simulations in the early phases of testing and development, but dephasing also has the potential to create systematic bias



Maxence Corman, Luis Lehner, William E. East, Guillaume Dideron Phys.Rev.D 110 (2024) 8, 084048



But numerical relativity is not just for gravitational waves!

Historical precedent for using these simulations as "numerical experiments" in strong gravity



log(black hole mass formed)

Universality and scaling in gravitational collapse of a massless scalar field Matthew W. Choptuik Phys.Rev.Lett. 70 (1993) 9-12

Historical precedent for using these simulations as "numerical experiments" in strong gravity

Berger, Moncrief and Garfinkle's work on the nature of BKL singularities



D. Garfinkle & F Pretorius, Spike behavior in the approach to spacetime singularities *Phys.Rev.D* 102 (2020) 12, 124067

Highly relevant for the beginning of the Universe, about which we know very little



NR provides a numerical "experiment" for very inhomogeneous universes

Field



Density perturbations

J. Aurrekoetxea, KC, F Muia Oscillon formation during inflationary preheating with general relativity Phys.Rev.D 108 (2023) 2, 023501

Studying bouncing cosmologies with Numerical Relativity



The robustness of slow contraction and the shape of the scalar field potential Timo Kist, Anna Ijjas JCAP 08 (2022) 08, 046

Do modifications to gravity change the story?



Higher dimensional spacetimes - do corrections to GR protect cosmic censorship?



P Figueras, A Kovacs, S Yao 2024 in prep

And warp drives...?





What can't it do?



Similarity of scales is a feature and a bug

DYNAMICAL **SPACETIME**



NO PERTURBATIVE EXPANSION

GRAVITATIONAL BACKREACTION (STRONG GRAVITY)

Finite space of simulations



periodic boundary conditions



Initial conditions problem in cosmological inflation revisited D. Garfinkle, A Ijjas, P Steinhardt *Phys.Lett.B* 843 (2023) 138028

Effect on conclusions?

Finite time of simulations





Numerical relativity part

Transients can dominate over the physical effect of interest



energy density of a vector field during a superradiant phase

Need a well posed formulation of the equations

e.g. EsGB gravity

L Areste Salo, KC, P Figueras 2022 Phys.Rev.Lett. 129 (2022) 26, 261104

Too much parameter space?

Pratten et. al. 2021 Assessing gravitational-wave binary black hole candidates with Bayesian odds

+ additional params?

A fun(damental) example

A warp drive metric exists

Alcubierre, The warp drive: hyper-fast travel within general relativity, CQG 1994

What happens if the warp drive containment field fails?

Could we see gravitational waves from alien warp drive ships?

What happens if the warp drive containment field fails?

Could we see gravitational waves from alien warp drive ships?

WHAT NO ONE HAS SEEN BEFORE: GRAVITATIONAL WAVEFORMS FROM WARP DRIVE COLLAPSE

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 Version July 25, 2024

ABSTRACT

Despite originating in science fiction, warp drives have a concrete description in general relativity, with Alcubierre first proposing a spacetime metric that supported faster-than-light travel. Whilst there are numerous practical barriers to their implementation in real life, including a requirement for negative energy, computationally, one can simulate their evolution in time given an equation of state describing the matter. In this work, we study the signatures arising from a warp drive 'containment failure', assuming a stiff equation of state for the fluid. We compute the emitted gravitational-wave signal and track the energy fluxes of the fluid. Apart from its rather speculative application to the search for extraterrestrial life in gravitational-wave detector data, this work is interesting as a study of the dynamical evolution and stability of spacetimes that violate the null energy condition. Our work highlights the importance of exploring strange new spacetimes, to (boldly) simulate what no one has seen before.

The Open Journal of Astrophysics 7 (July). <u>https://doi.org/10.33232/001c.121868</u>, <u>arXiv:2406.02466</u>

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ABSTRACT

BUY A PAPER FUNERAL NOTICES MYNEWSASSISTANT SHOP HOROSCOPES CROSSWORDS STAR WINS CASINO OUR PAPER DISC

Boffins admit we already have tech to detect UFOs using Star Trek-style Warp Drives

EXCLUSIVE: Experts believe that technology already available to us could be used to detect specific gravitational waves, meaning we could be one step away from spotting alien spaceships

Deutscher Forscher will Warp-Antriebe von Aliens finden

Foto: Thomas Roese, imago images/Everett Collection

Konstantin Marrach

28.06.2024 - 20:34 Uhr

Der Weltraum. Unendliche Weiten. Dies sind die Abenteuer der Wissenschaft, die ganz tief ins All blickt und dabei vielleicht sogar Außerirdische entdecken kann.

Was nach Science Fiction klingt, ist tatsächlich

🗎 bild.de

A warp drive metric exists

Alcubierre, The warp drive: hyper-fast travel within general relativity, CQG 1994

A "designer spacetime" that needs exotic matter

$T_{\mu\nu}(t=0) = G^{alcubierre}_{\mu\nu}(t=0)/(8\pi)$

f(matter) = g(curvature)

The solution requires matter that violates the Null Energy Condition

We need to decide how the matter evolves

Local energy conservation tells us how 4 components evolve:

$$\begin{split} \nabla^{\mu}T_{\mu\nu} &= 0 \\ \implies \partial_{t}\rho = f(\rho, S_{i}, S_{ij}, g_{\mu\nu}) \quad \text{energy density} \\ \implies \partial_{t}S_{i} &= f(\rho, S_{i}, S_{ij}, g_{\mu\nu}) \quad \text{momentum density} \end{split}$$

Need equation of state to close the system (we choose $\bar{p} = \rho$, decaying anisotropic stresses)

ensity m (we ses)

What no one has seen before...

K (Trace of extrinsic curvature of spatial slice)

Evolution of the exotic matter from the bubble

Energy density of warp fluid

See: Negative Mass in General Relativity (Bondi 1957)

Matter energy radiation is both negative and positive

Energy flux of gravitational waves

Energy flux of warp fluid

Final result is a loss of negative energy, so a positive final mass

Total energy flux of warp fluid

One can of course extract the gravitational wave signals

Weyl4 scalar (~ Second time derivative of GWs)

Can it be detected?

Frequency ~ 1/R

Can it be detected?

Image credit: Christopher Berry (aliens added)

Maybe it's not the destination that matters... it's the journey

And now, the conclusion...

- Exciting times ahead in numerical relativity studies!
- NR simulations can be used as a tool to investigate the consequences of fundamental physics in strong gravity regimes. They are a modern day gedanken experiment.
- Potential to understand fundamental properties of gravity and extensions to GR (and discover aliens).

Questions?