Spin information in the event record

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Spin and spin density

Scalars – Spin 0

Separate production and decay (neglect all non-factorisable contribs)

$$\mathcal{M} = \mathcal{M}^{\text{prod}} \left(\frac{i}{p^2 - m^2 + im\Gamma} \right) \mathcal{M}^{\text{decay}}$$

Narrow-width approximation

$$\left[\frac{1}{p^2 - m^2 + im\Gamma} \right]^2 \quad \longrightarrow \quad \frac{\pi}{m\Gamma}\, \delta\left(p^2 - m^2\right)$$

Production and decay matrix elements factorise trivially

$$\left|\mathcal{M}\right|^2 = \left|\mathcal{M}^{\text{prod}}\right|^2 \; - - - \; \left|\mathcal{M}^{\text{dec}}\right|^2$$

No spin correlations between production and decay

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Fermions – Spin $\frac{1}{2}$

Separate production and decay (neglect all non-factorisable contribs)

$$\mathcal{M} = \mathcal{M}_\alpha^{\text{prod}} \left(\frac{i\,(\not p \pm m)^{\alpha\beta}}{p^2 - m^2 + im\Gamma} \right) \mathcal{M}_\beta^{\text{decay}}$$

Narrow-width approximation as before deals with denominator.

Completeness relation for numerator

$$(\not p \pm m)^{\alpha\beta} = \tfrac{1}{2} \sum_s \left[\left(1 \pm \sqrt{\frac{m^2}{p^2}} \right) u_s^\alpha(p) \bar{u}_s^\beta(p) + \left(1 \mp \sqrt{\frac{m^2}{p^2}} \right) v_s^\alpha(p) \bar{v}_s^\beta(p) \right]$$

thus with, for an on-shell fermion with $p^2 = m^2$,

$$\mathcal{M}_s^{\text{prod}} = \mathcal{M}_\alpha^{\text{prod}}\, u_s^\alpha(p) \qquad \mathcal{M}_s^{\text{decay}} = \bar{u}_s^\beta(p)\, \mathcal{M}_\beta^{\text{decay}}$$

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Intermediate spin cannot be observed, can be different in $\mathcal{M}$ and $\mathcal{M}^*$

$$|\mathcal{M}|^2 \propto \sum_{s,s'} \left[\mathcal{M}_s^{\text{prod}} \mathcal{M}_{s'}^{\text{prod},*} \quad \longrightarrow \quad \mathcal{M}_s^{\text{decay}} \mathcal{M}_{s'}^{\text{decay},*} \right]$$

or, using spin density matrices,

$$|\mathcal{M}|^2 = \sum_{s,s'} \rho_{ss'}\, D^{ss'}$$

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Spin and spin density

Vector bosons – Spin 1

Decompose production and decay (neglect all non-factorisable contribs)

$$\mathcal{M} = \mathcal{M}_\mu^{\text{prod}} \left(\frac{i \left(-g^{\mu\nu} + \frac{p^\mu p^\nu}{m^2} \right)}{p^2 - m^2 + im\Gamma} \right) \mathcal{M}_\nu^{\text{decay}}$$

Again, using completeness relations,

$$\left(-g^{\mu\nu} + \frac{p^\mu p^\nu}{m^2} \right) = \sum_{\lambda=1}^{4} \varepsilon_\lambda^\mu(p)\, \varepsilon_\lambda^{*\nu}(p)$$

The unphysical fourth polarisation vanishes for $p^2 = m^2$. Define

$$\mathcal{M}_\lambda^{\text{prod}} = \mathcal{M}_\mu^{\text{prod}}\, \varepsilon_\lambda^\mu(p) \qquad \mathcal{M}_\lambda^{\text{decay}} = \varepsilon_\lambda^{*\nu}(p)\, \mathcal{M}_\nu^{\text{decay}}$$

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Properties of spin correlations

Intermediate spin/polarisation cannot be observed, can be different in $\mathcal{M}$ and $\mathcal{M}^*$.

$$|\mathcal{M}|^2 = \sum_{s,s'} \rho_{ss'}\, D^{ss'}$$

Production and decay spin densities, $\rho_{ss'}$ and $D^{ss'}$ are matrix valued, the intermediate particle does not have one specific spin state.

There are interference contributions between different spin states in $\mathcal{M}$ and $\mathcal{M}^*$. Their size depends on the process and the observable.

Often, but not always, the diagonal elements ($s = s'$) are dominant.

$$|\mathcal{M}|^2 \approx \sum_{s} \rho_s\, D^s$$

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Properties of spin/polarisation definition

Spin/polarisation is frame and basis dependent

- spin and polarisation are dependent on Lorentz frame
  - need to specify frame
  - need to specify polarisation basis and prefactor conventions
  - need to specify Dirac matrix representation

Spin/polarisation in event record

- event record operates on $|\mathcal{M}|^2$ with definitive intermediate propagating states
  $\Rightarrow$ **no representation of quantum interference**
- inclusion of off-diagonal elements a posteriori not trivial as linked to quantum interference between different spin states
  $\rightarrow$ need to communicate spin-density matrix in agreed-upon basis

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Examples (not exhaustive)

Herwig, Sherpa

- use spin-density formalism to effect spin correlations internally
  + consistent spin/polarisation def., incl. off-diagonal elements
  − less flexible to interface with tools outside the generator

Herwig+EvtGen

- customised interface
  + consistent spin/polarisation def., incl. off-diagonal elements
  − specific to those two generators

Tauola, TauSpinner

- reconstruction of spin info based on LO ME in TauSpinner
  + no spin info in event record needed, incl. off-diagonal elements
  − limited applicability when higher-order corrections in production or decay are included