INTRODUCTION TO PHENOMENOLOGY WITH MASSIVE NEUTRINOS IN 2024

Concha Gonzalez-Garcia

(YITP-Stony Brook & ICREA-University of Barcelona)

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OUTLINE

- Historic Introduction to the SM of Massless Neutrinos
- Introducing ν mass: Dirac vs Majorana, Lepton mixing
- Mass induced Flavour Oscillations in Vaccum and in Matter
- Summary of Flavour Oscillation Observations:Status of 3ν global description

Discovery of ν's

- At end of 1800's radioactivity was discovered and three types identified: α, β, γ

  β : an electron comes out of the radioactive nucleus.

- Energy conservation $\Rightarrow$ e^- should have had a fixed energy

$$(A, Z) \to (A, Z+1) + e^- \;\Rightarrow\; E_e = M(A, Z+1) - M(A, Z)$$

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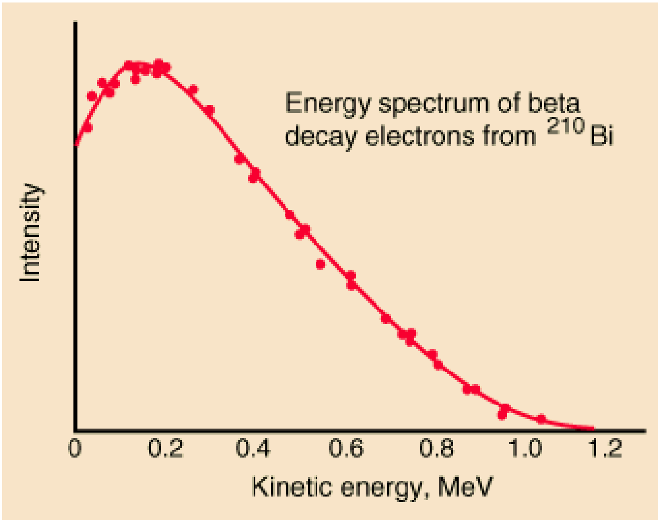
But 1914 James Chadwick showed that the electron energy spectrum is continuous



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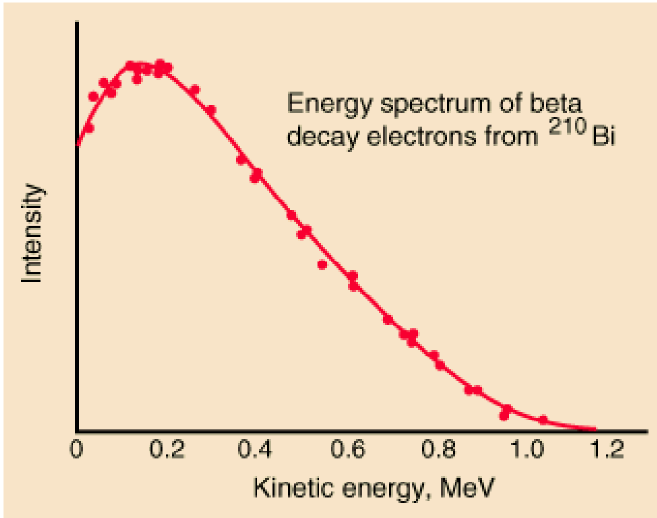

Do we throw away the energy conservation?

Bohr: *we have no argument, either empirical or theoretical, for upholding the energy principle in the case of β ray disintegrations*

Discovery of ν's

• The idea of the neutrino came in 1930, when W. Pauli tried a desperate saving operation of ”the energy conservation principle”.

In his letter addressed to the *Liebe Radioaktive Damen und Herren* (Dear Radioactive Ladies and Gentlemen), the participants of a meeting in Tubingen. He put forward the hypothesis that a new particle exists as *constituent of nuclei, the neutron* ν, able to explain the continuous spectrum of nuclear beta decay

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$$(A,Z) \to (A,Z+1) + e^- + \nu$$

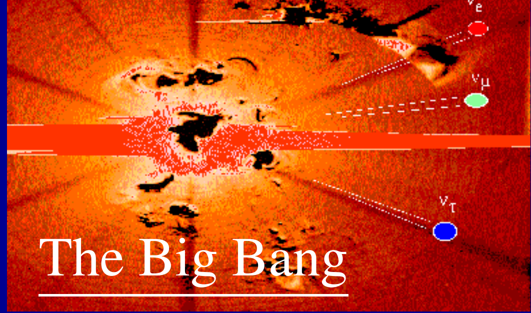
• The ν is light (in Pauli's words: m_ν *should be of the same order as the* m_e), neutral and has spin 1/2

Neutrino Detection

Fighting Pauli's "Curse":

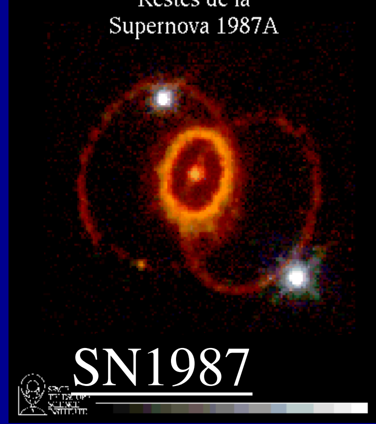
I have done a terrible thing, I have postulated a particle that cannot be detected.

Sources of ν's

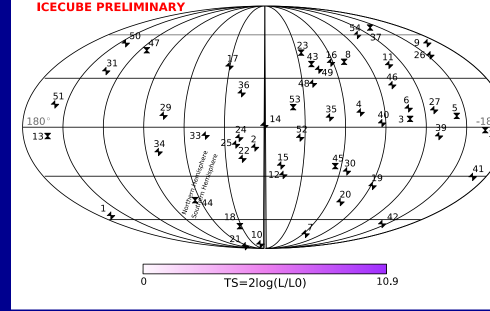


$\rho_\nu = 330/\text{cm}^3$

$p_\nu = 0.0004$ eV



$E_\nu \sim$ MeV



ExtraGalactic

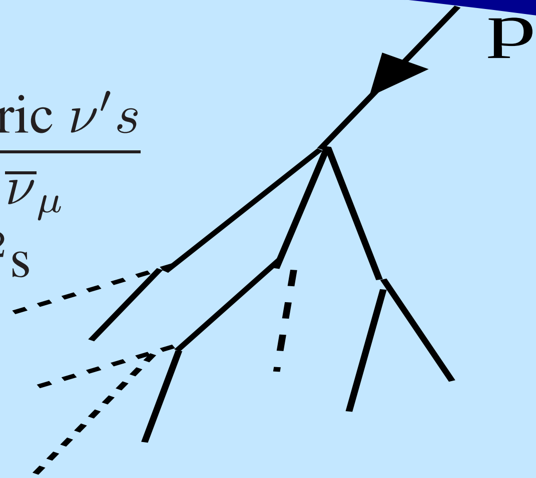
$E_\nu \gtrsim 30$ TeV

The Sun

ν_e

$\Phi_\nu^{Earth} = 6\times 10^{10} \nu/\text{cm}^2\text{s}$

$E_\nu \sim 0.1$–20 MeV



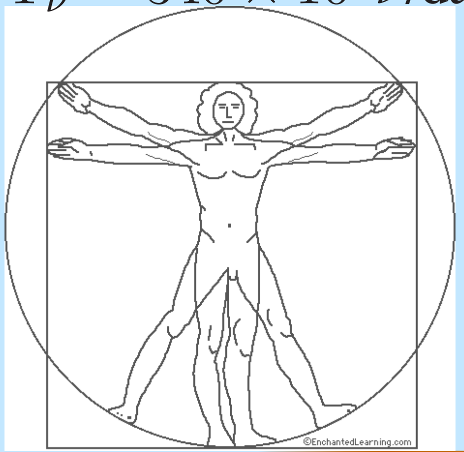
Atmospheric $\nu' s$

$\nu_e, \nu_\mu, \overline{\nu}_e, \overline{\nu}_\mu$

$\Phi_\nu \sim 1 \nu/\text{cm}^2\text{s}$

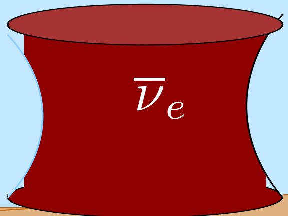
Human Body

$\Phi_\nu = 340\times 10^6 \nu/\text{day}$



Nuclear Reactors

$E_\nu \sim$ few MeV



Fermilab Discovering the Nature of Nature

CERN

KEK ～宇宙と物質の起源と構造を探る～

OAK RIDGE National Laboratory

Earth's radioactivity

$\Phi_\nu \sim 6\times 10^6 \nu/\text{cm}^2\text{s}$

Accelerators

$E_\nu \simeq 0.3$–30 GeV

NSS

$E_\nu \sim$ MeV

Neutrino Detection

Problem: Quantitatively: a ν sees a proton of area:

$$\sigma^{\nu p} \sim 10^{-38}\text{cm}^2\,\frac{E_\nu}{\text{GeV}}$$

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$$\Phi_\nu^{\mathrm{ATM}} = 1\, \nu/(\,\mathrm{cm}^2 \text{ second}) \;\; \mathrm{y} \quad \langle E_\nu \rangle = 1 \text{ GeV}$$

- How many interact?

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- How many interact? In a human body

$$N_{\text{int}} = \Phi_\nu \times \sigma^{\nu p} \times N_{\text{prot}}^{\text{human}} \times T_{\text{life}}^{\text{human}}$$

$$\left.\begin{array}{l} N_{\text{protons}}^{\text{human}} = \frac{M^{\text{human}}}{gr} \times N_A = 80\text{kg} \times N_A \sim 5 \times 10^{28}\text{protons} \\ T^{\text{human}} = 80\,\text{years} = 2 \times 10^9\ \text{sec} \end{array}\right\}$$

($M \times T \equiv$ Exposure)

$\text{Exposure}_{\text{human}}$

$\sim$ Ton $\times$ year

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$(M \times T \equiv \text{Exposure})$

$\text{Exposure}_{\text{human}} \sim \text{Ton} \times \text{year}$

$$N_{\text{int}} = (5 \times 10^{28})\,(2 \times 10^9) \times 10^{-38} \sim 1\,\text{interaction in life}$$

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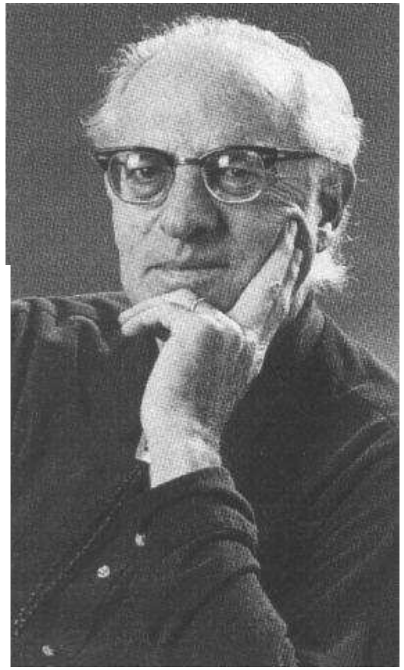
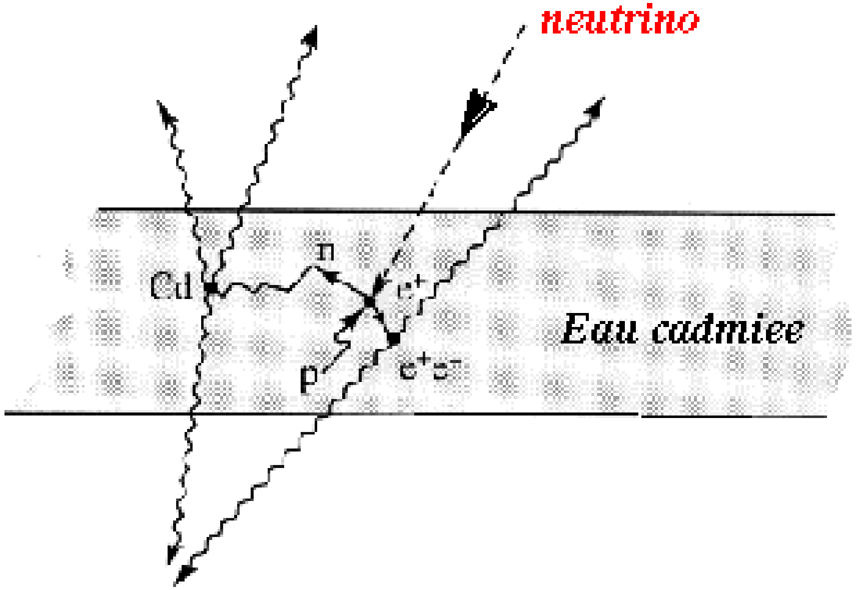
To detect neutrinos we need very intense source and/or
a hugh detector with Exposure $\sim$ KTon $\times$ year

First Neutrino Detection



In 1953 Frederick Reines and Clyde Cowan put a detector near a nuclear reactor (the most intense source available)

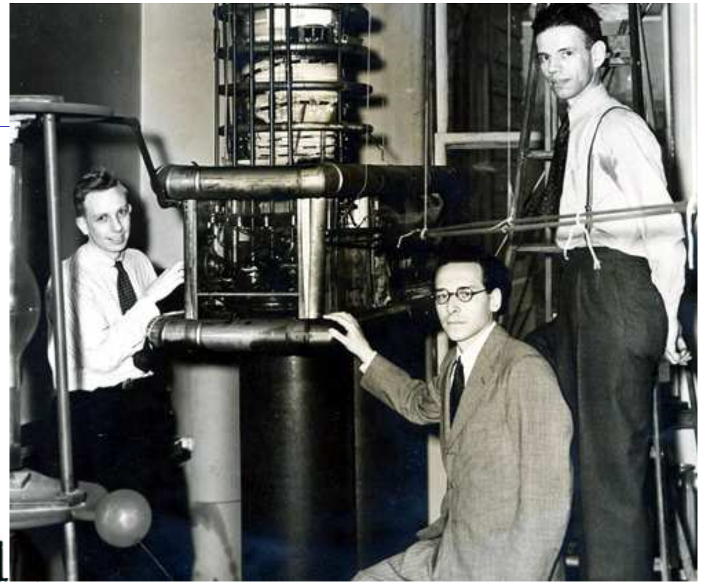
400 l of water
and Cadmium Chloride.



e^+ annihilates with e^- in the water and produces two γ's simultaneouoly.
neutron is captured by por the cadmium and a γ's is emitted 15 msec latter

Reines y Clyde saw clearly this signature: the first neutrino had been detected

Neutrinos = “Left-handed”



Helicity of Neutrinos*

M. Goldhaber, L. Grodzins, and A. W. Sunyar

Brookhaven National Laboratory, Upton, New York

(Received December 11, 1957)

A COMBINED analysis of circular polarization and resonant scattering of γ rays following orbital electron capture measures the helicity of the neutrino. We have carried out such a measurement with Eu^{152m}, which decays by orbital electron capture. If we assume the most plausible spin-parity assignment for this isomer compatible with its decay scheme,[1] 0−, we find that the neutrino is “left-handed,” i.e., $\boldsymbol{\sigma}_\nu \cdot \hat{p}_\nu = -1$ (negative helicity).

WARNING: Helicity versus Chirality

• We define the chiral projections $\mathcal{P}_{R,L} = \dfrac{1 \pm \gamma_5}{2}$ $\Rightarrow$ $\psi_L = \frac{1-\gamma_5}{2}\psi$ $\qquad \psi_R = \frac{1+\gamma_5}{2}\psi$

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- The Hamiltonian for a massive fermion ψ is $H = \overline{\psi}(x)(-i\sum_j \gamma^j \partial_j + m)\psi(x)$
- 4 states with $(E, \vec{p})$ $(\gamma^\mu p_\mu - m)u_s(\vec{p}) = 0$ $\quad (\gamma^\mu p_\mu + m)v_s(\vec{p}) = 0$ $\quad s = 1, 2$

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- Since $[H, \gamma_5] \neq 0$ and $[\vec{P}, \vec{J}] \neq 0$ $\quad [\ \vec{J} = \vec{L} + \frac{\vec{\Sigma}}{2} \quad (\Sigma^i = -\gamma^0\gamma^5\gamma^i)\ ]$

  $\Rightarrow$ Neither Chirality nor J_i can characterize the fermion simultaneously with $E, \vec{p}$

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- But $[H, \vec{J}.\vec{P}] = [\vec{P}, \vec{J}.\vec{P}]$=0 $\Rightarrow$ we can chose $u_1(\vec{p}) \equiv u_+(\vec{p})$ and $u_2(\vec{p}) \equiv u_-(\vec{p})$ (same for $v_{1,2}$) to be eigenstates of the helicity projector

$$\mathcal{P}_\pm = \tfrac{1}{2}\left(1 \pm 2\vec{J}\frac{\vec{P}}{|\vec{P}|}\right) = \tfrac{1}{2}\left(1 \pm \vec{\Sigma}\frac{\vec{P}}{|\vec{P}|}\right)$$

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- For massless fermions the Dirac equation can be written

$$\vec{\Sigma}\,\vec{P}\,\psi = -\gamma^0\gamma^5\vec{\gamma}\,\vec{p}\,\psi = -\gamma^0\gamma^5\gamma^0 E\,\psi = \gamma^5 E\psi \Rightarrow$$ For $m = 0$ $\mathcal{P}_\pm = \mathcal{P}_{R,L}$

Only for massless fermions Helicity and chirality states are the same.

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ν in the SM

• The SM is a gauge theory based on the symmetry group

$$SU(3)_C \times SU(2)_L \times U(1)_Y \Rightarrow SU(3)_C \times U(1)_{EM}$$

• 3 Generations of Fermions:

$(1,2,-\frac{1}{2})$	$(3,2,\frac{1}{6})$	$(1,1,-1)$	$(3,1,\frac{2}{3})$	$(3,1,-\frac{1}{3})$
L_L	Q_L^i	E_R	U_R^i	D_R^i
$\begin{pmatrix}\nu_e \\ e\end{pmatrix}_L$	$\begin{pmatrix}u^i \\ d^i\end{pmatrix}_L$	e_R	u_R^i	d_R^i
$\begin{pmatrix}\nu_\mu \\ \mu\end{pmatrix}_L$	$\begin{pmatrix}c^i \\ s^i\end{pmatrix}_L$	μ_R	c_R^i	s_R^i
$\begin{pmatrix}\nu_\tau \\ \tau\end{pmatrix}_L$	$\begin{pmatrix}t^i \\ b^i\end{pmatrix}_L$	τ_R	t_R^i	b_R^i

• Spin-0 particle ϕ: $(1,2,\frac{1}{2})$

$$\phi = \begin{pmatrix}\phi^+ \\ \phi^0\end{pmatrix} \overset{SSB}{\rightarrow} \frac{1}{\sqrt{2}}\begin{pmatrix}0 \\ v+h\end{pmatrix}$$

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$$Q_{EM} = T_{L3} + Y$$

- ν's are $T_{L3} = \frac{1}{2}$ components of L_L
- ν's have no strong or EM interactions
- No ν_R ($\equiv$ singlets of gauge group)

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SM Fermion Lagrangian

$$
\begin{aligned}
\mathcal{L} \;=\; & \sum_{k=1}^{3}\sum_{i,j=1}^{3} \overline{Q^i_{L,k}}\gamma^\mu \left(i\partial_\mu - g_s\frac{\lambda_{a,ij}}{2}G^a_\mu - g\frac{\tau_a}{2}\delta_{ij}W^a_\mu - g'\frac{1}{6}\delta_{ij}B_\mu\right) Q^j_{L,k} \\
& + \sum_{k=1}^{3}\sum_{i,j=1}^{3} \overline{U^i_{R,k}}\gamma^\mu \left(i\partial_\mu - g_s\frac{\lambda_{a,ij}}{2}G^a_\mu - g'\frac{2}{3}\delta_{ij}B_\mu\right) U^j_{R,k} \\
& + \sum_{k=1}^{3}\sum_{i,j=1}^{3} \overline{D^i_{R,k}}\gamma^\mu \left(i\partial_\mu - g_s\frac{\lambda_{a,ij}}{2}G^a_\mu + g'\frac{1}{3}\delta_{ij}B_\mu\right) D^j_{R,k} \\
& + \sum_{k=1}^{3} \overline{L_{L,k}}\gamma^\mu \left(i\partial_\mu - g\frac{\tau_i}{2}W^i_\mu + g'\frac{1}{2}B_\mu\right) L_{L,k} + \overline{E_{R,k}}\gamma^\mu \left(i\partial_\mu + g' B_\mu\right) E_{R,k} \\
& - \sum_{k,k'=1}^{3} \left(\lambda^u_{kk'}\overline{Q}_{L,k}(i\tau_2)\phi^* U_{R,k'} + \lambda^d_{kk'}\overline{Q}_{L,k}\phi D_{R,k'} + \lambda^l_{kk'}\overline{L}_{L,k}\phi E_{R,k'} + h.c.\right)
\end{aligned}
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& +\sum_{k=1}^{3}\overline{L_{L,k}}\gamma^\mu\left(i\partial_\mu - g\frac{\tau_i}{2}W^i_\mu + g'\frac{1}{2}B_\mu\right)L_{L,k} + \overline{E_{R,k}}\gamma^\mu\left(i\partial_\mu + g'B_\mu\right)E_{R,k} \\
& -\sum_{k,k'=1}^{3}\left(\lambda^u_{kk'}\overline{Q}_{L,k}(i\tau_2)\phi^* U_{R,k'} + \lambda^d_{kk'}\overline{Q}_{L,k}\phi D_{R,k'} + \lambda^l_{kk'}\overline{L}_{L,k}\phi E_{R,k'} + h.c.\right)
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- Invariant under global rotations

$Q_{L,k} \to e^{i\alpha_B/3}Q_{L,k}$ $\quad$ $U_{R,k} \to e^{i\alpha_B/3}U_{R,k}$ $\quad$ $D_{R,k} \to e^{i\alpha_B/3}D_{R,k}$ $\quad$ $L_{L,k} \to e^{i\alpha_{L_k}}L_{L,k}$ $\quad$ $E_{R,k} \to e^{i\alpha_{L_k}}E_{R,k}$

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\mathcal{L} \;=\; & \sum_{k=1}^{3}\sum_{i,j=1}^{3} \overline{Q^i_{L,k}}\gamma^\mu \left(i\partial_\mu - g_s\frac{\lambda_{a,ij}}{2}G^a_\mu - g\frac{\tau_a}{2}\delta_{ij}W^a_\mu - g'\frac{1}{6}\delta_{ij}B_\mu\right) Q^j_{L,k} \\
& + \sum_{k=1}^{3}\sum_{i,j=1}^{3} \overline{U^i_{R,k}}\gamma^\mu \left(i\partial_\mu - g_s\frac{\lambda_{a,ij}}{2}G^a_\mu - g'\frac{2}{3}\delta_{ij}B_\mu\right) U^j_{R,k} \\
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& + \sum_{k=1}^{3} \overline{L_{L,k}}\gamma^\mu \left(i\partial_\mu - g\frac{\tau_i}{2}W^i_\mu + g'\frac{1}{2}B_\mu\right) L_{L,k} + \overline{E_{R,k}}\gamma^\mu \left(i\partial_\mu + g' B_\mu\right) E_{R,k} \\
& - \sum_{k,k'=1}^{3} \left(\lambda^u_{kk'}\overline{Q}_{L,k}(i\tau_2)\phi^* U_{R,k'} + \lambda^d_{kk'}\overline{Q}_{L,k}\phi D_{R,k'} + \lambda^l_{kk'}\overline{L}_{L,k}\phi E_{R,k'} + h.c.\right)
\end{aligned}
$$

- Invariant under global rotations

$Q_{L,k} \to e^{i\alpha_B/3}Q_{L,k}$ $\quad U_{R,k} \to e^{i\alpha_B/3}U_{R,k}$ $\quad D_{R,k} \to e^{i\alpha_B/3}D_{R,k}$ $\quad L_{L,k} \to e^{i\alpha_{L_k}}L_{L,k}$ $\quad E_{R,k} \to e^{i\alpha_{L_k}}E_{R,k}$

$\Rightarrow$ *Accidental ($\equiv$ not imposed)* global symmetry: $U(1)_B \times U(1)_{L_e} \times U(1)_{L_\mu} \times U(1)_{L_\tau}$

$\Rightarrow$ Each lepton flavour, L_i, is conserved

$\Rightarrow$ Total lepton number $L = L_e + L_\mu + L_\tau$ is conserved

SM = massless $\nu's$ and LFC

- A fermion mass can be seen as at a Left-Right transition

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In SM ν's are *Strictly* Massless & Lepton Flavours are *Strictly* Conserved

- We have observed with high (or good) precision:
  - ∗ Atmospheric ν_μ & $\bar{\nu}_\mu$ disappear most likely to ν_τ (**SK**,MINOS, ICECUBE)
  - ∗ Accel. ν_μ & $\bar{\nu}_\mu$ disappear at $L \sim 300/800$ Km (K2K, **T2K, MINOS, NOνA**)
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All this implies that L_α are violated

and There is Physics Beyond SM

Dirac versus Majorana Neutrinos

- In the SM neutral bosons can be of two type:
  - Their own antiparticle such as γ, π^0 ...
  - Different from their antiparticle such as K^0, $\bar{K}^0$...
- In the SM ν are the only *neutral fermions*

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$\Rightarrow$ And the charged conjugate neutrino field $\equiv$ the antineutrino field

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⇒ 4 chiral fields

$$\nu_L\ ,\ \nu_R\ ,\ (\nu_L)^C\ ,\ (\nu_R)^C \quad \text{with} \quad \nu = \nu_L + \nu_R \quad \text{and} \quad \nu^C = (\nu_L)^C + (\nu_R)^C$$

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The difference arises when including *a neutrino mass*

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${V^\nu_R}^\dagger M_D V^\nu = \text{diag}(m_1, m_2, m_3)$

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- $\mathcal{L}_{\text{mass}}^{(\text{Dirac})}$ involves the four chiral fields ν_L , ν_R , $(\nu_L)^C$, $(\nu_R)^C$

$\Rightarrow$ Total Lepton number is conserved *by construction* (not *accidentally*):

$$\left.\begin{array}{l} U(1)_L \;:\; \nu \to e^{i\alpha}\, \nu \quad \text{and} \quad \overline{\nu} \to e^{-i\alpha}\, \overline{\nu} \\ U(1)_L \;:\; \nu^C \to e^{-i\alpha}\, \nu^C \quad \text{and} \quad \overline{\nu^C} \to e^{i\alpha}\, \overline{\nu^C} \end{array}\right\} \Rightarrow \mathcal{L}_{\text{mass}}^{(\text{Dirac})} \to \mathcal{L}_{\text{mass}}^{(\text{Dirac})}$$

Adding ν Mass: Majorana Mass

• One does not introduce ν_R but uses that the field $(\nu_L)^c$ is right-handed, so that one can write a Lorentz-invariant mass term

$$\mathcal{L}_{\rm mass}^{\rm (Maj)} = -\frac{1}{2}\overline{\nu_L^c} M_M^\nu \nu_L + {\rm h.c.} \equiv -\frac{1}{2}\sum_k m_k \overline{\nu}_i^M \nu_i^M$$

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$\Rightarrow$ Breaks Total Lepton Number $\Rightarrow \mathcal{L}^{(\mathrm{Maj})}_{\mathrm{mass}}$ not generated at any order in the SM

ν Mass $\Rightarrow$ Lepton Mixing

- CC and mass for 3 charged leptons ℓ_i and N neutrinos in weak basis $\nu^W \equiv \begin{pmatrix} \nu_{L,e} \\ \nu_{L,\mu} \\ \nu_{L,\tau} \\ (\nu_{R,1})^C \\ \cdot \\ \cdot \\ \cdot \end{pmatrix}$

$$\mathcal{L}_{CC}+\mathcal{L}_M = -\frac{g}{\sqrt{2}}\sum_{i=1}^{3}\overline{\ell^W_{L,i}}\gamma^\mu \nu_i^W W_\mu^+ - \sum_{i,j=1}^{3}\overline{\ell^W_{L,i}} M_{\ell\, ij}\ell^W_{R,j} - \frac{1}{2}\sum_{i,j=1}^{N}\overline{\nu_i^{cW}} M_{\nu\, ij}\nu_j^W + \text{h.c.}$$

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$$U_{\text{LEP}}^{ij} = \sum_{k=1}^{3} P^\ell_{ii} V_L^{\ell\,\dagger\, ik}\, V^{\nu\, kj} P^\nu_{jj}$$

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- For example for 3 Dirac ν : 3 Mixing angles + 1 Dirac Phase

$$U_{\rm LEP} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{-i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{21} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

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- For 3 Majorana ν : 3 Mixing angles + 1 Dirac Phase + 2 Majorana Phases

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The New Minimal Standard Model

- Minimal Extension to allow for LFV ⇒ give Mass to the Neutrino

  $*$ Introduce ν_R AND impose L conservation ⇒ Dirac $\nu \neq \nu^c$:

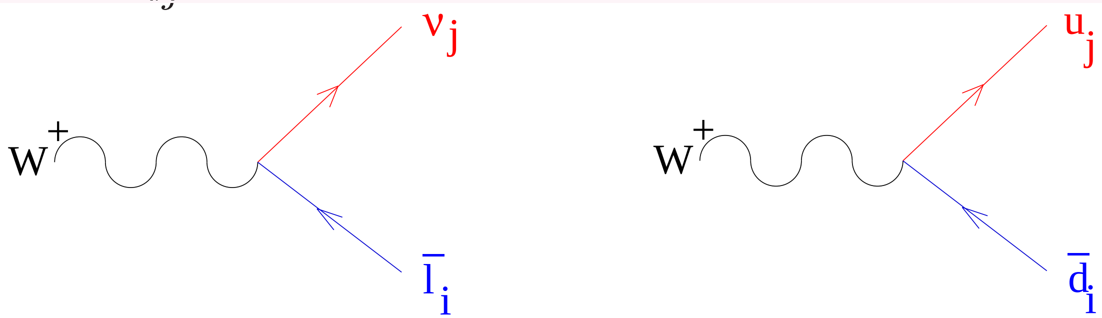
$$\mathcal{L} = \mathcal{L}_{SM} - M_\nu \overline{\nu_L} \nu_R + h.c.$$

  $*$ NOT impose L conservation ⇒ Majorana $\nu = \nu^c$

$$\mathcal{L} = \mathcal{L}_{SM} - \frac{1}{2} M_\nu \overline{\nu_L} \nu_L^C + h.c.$$

- The charged current interactions of leptons are not diagonal (same as quarks)

$$\frac{g}{\sqrt{2}} W_\mu^+ \sum_{ij} \left(U_{\rm LEP}^{ij} \, \overline{\ell^i} \, \gamma^\mu \, L \, \nu^j \; + \; U_{\rm CKM}^{ij} \, \overline{U^i} \, \gamma^\mu \, L \, D^j \right) \; + \; h.c.$$



Neutrino Mass Scale: Tritium β Decay

- Fermi proposed a kinematic search of ν_e mass from beta spectra in ${}^3H$ beta decay

$$ {}^3\mathrm{H} \rightarrow {}^3\,\mathrm{He} + e + \overline{\nu}_e $$

- For "allowed" nuclear transitions, the electron spectrum is given by phase space alone

$$ K(T) \equiv \sqrt{\frac{dN}{dT}\frac{1}{Cp\,E\,F(E)}} \propto \sqrt{(Q-T)\sqrt{(Q-T)^2 - m_{\nu_e}^2}} $$

$T = E_e - m_e$, Q= maximum kinetic energy, (for ${}^3H$ beta decay $Q = 18.6$ KeV)

Taking into account mixing $m_{\nu_e}^{\text{eff}} \equiv \sqrt{\sum m_{\nu_j}^2 |U_{ej}|^2}$

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- $m_\nu \neq 0 \Rightarrow$ distortion from the straight-line at the end point of the spectrum

$$m_\nu = 0 \;\Rightarrow\; T_{\max} = Q$$
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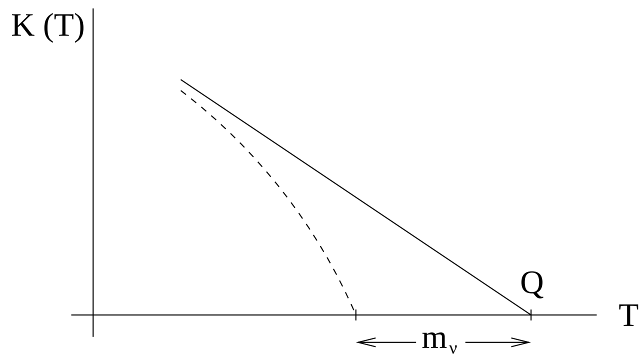

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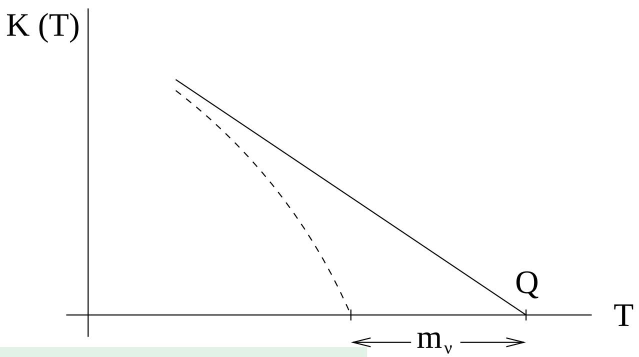
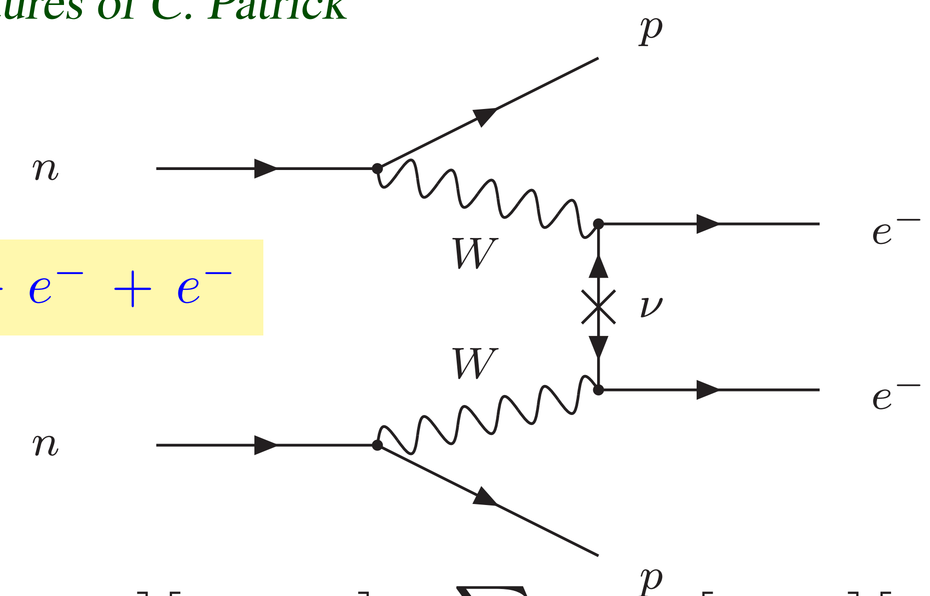


– At present only a bound: $m_{\nu_e}^{\text{eff}} < 0.8$ eV (at 90 % CL) (Katrin)

– Katrin operating can improve present sensitivity to $m_{\nu_e}^{\text{eff}} \sim 0.3\,\text{eV}$

Dirac or Majorana? ν-less Double-β Decay

Lectures of C. Patrick



$$(A, Z) \rightarrow (A, Z+2) + e^- + e^-$$

- Amplitude includes $\left[\overline{e}\gamma^\mu L\nu_e\right]\left[\overline{e}\gamma^\mu L\nu_e\right] = \sum_{ij} U_{ei}U_{ej}\left[\overline{e}\gamma^\mu \nu_i\right]\left[\overline{e}\gamma^\mu \nu_j\right]$

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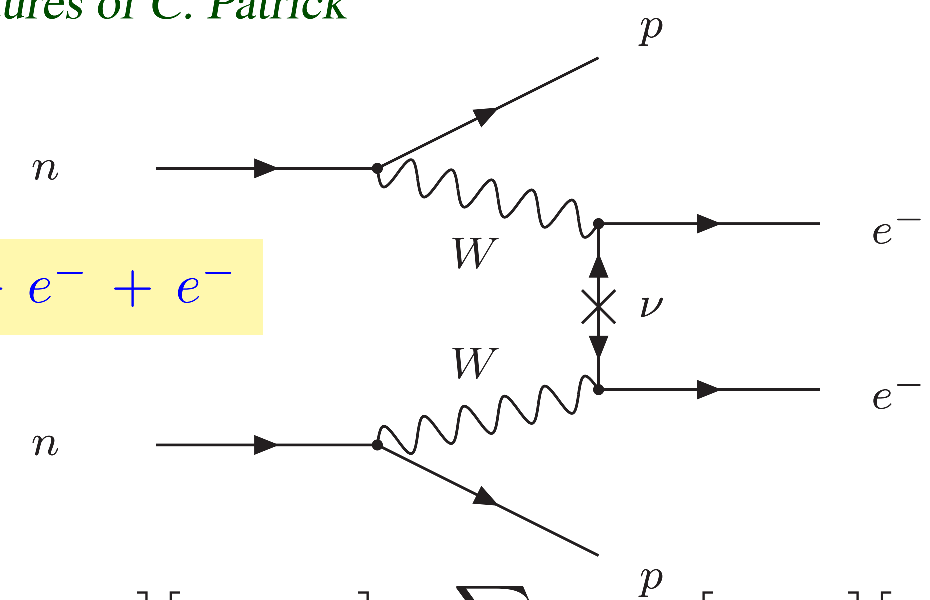

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  - If ν_i Dirac $\Rightarrow$ ν_i *annihilates* a *neutrino* and *creates* an *antineutrino*

    $\Rightarrow$ no same state $\Rightarrow$ Amplitude = 0
  - If ν_i Majorana $\Rightarrow$ $\nu_i = \nu_i^c$ *annihilates and creates* a *neutrino=antineutrino*

    $\Rightarrow$ same state $\Rightarrow$ Amplitude $\propto \overbrace{\nu_i \; (\nu_i)}^{}{}^T \neq 0$

Dirac or Majorana? ν-less Double-β Decay

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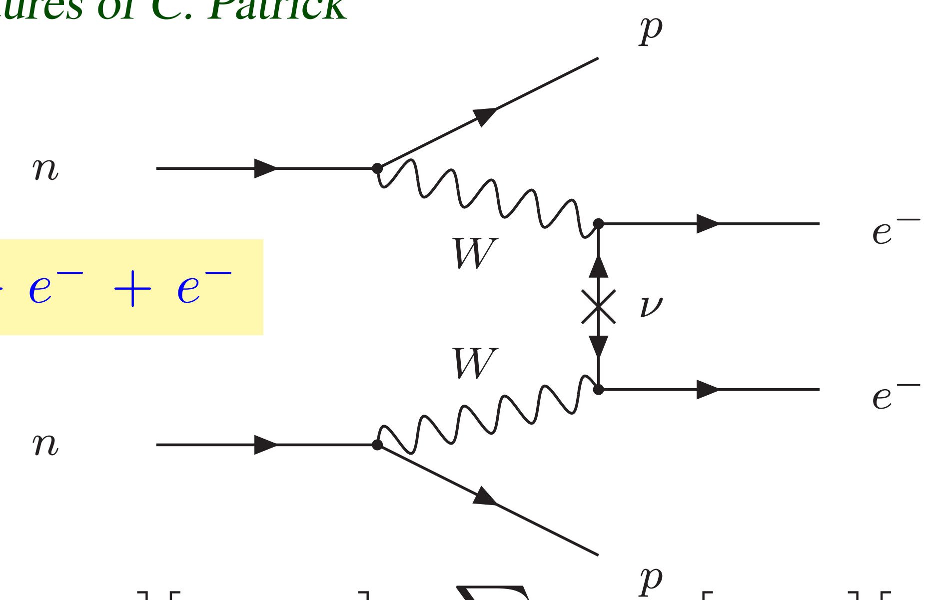

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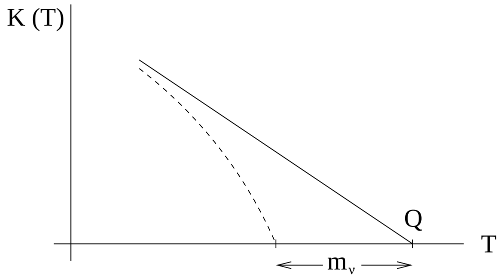
    $\Rightarrow$ same state $\Rightarrow$ Amplitude $\propto \overline{\nu_i\ (\nu_i})^T \neq 0$
- If Majorana m_ν only source of L-violation

$\Rightarrow$ Amplitude of ν-less-$\beta\beta$ decay is proportional to $\langle m_{ee}\rangle = \sum_j U_{ej}^2 m_j$

Probes of ν Mass Scale

Single β decay : Pure kinematics, Dirac or Majorana ν's, only model independent

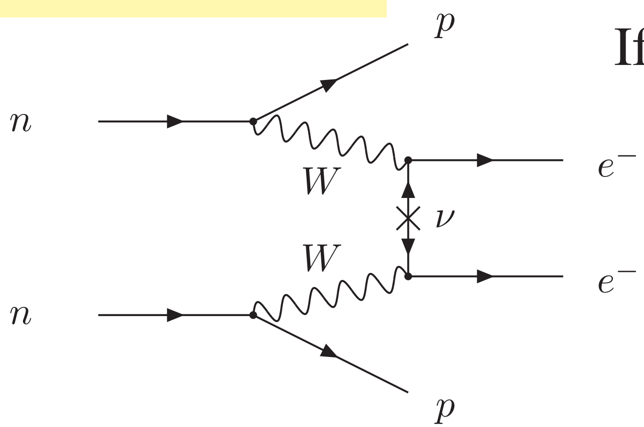


$$m^2_{\nu_e} = \sum m^2_j |U_{ej}|^2$$

Present bound: $m_{\nu_e} \leq 0.8$ eV (90% CL KATRIN 2021)

Katrin (20XX) Sensitivity to $m_{\nu_e} \sim 0.2\,\text{eV}$

ν-less Double-β decay: $\Leftrightarrow$ Majorana $\nu' s$



If m_ν only source of ΔL $\quad T^{0\nu}_{1/2} = \dfrac{m_e}{G_{0\nu}\, M^2_{\text{nucl}}\, m^2_{ee}}$

$$m_{ee} = |\sum U^2_{ej} m_j|$$

Present Bounds: $m_{ee} < 0.04{-}0.2$ eV

COSMOLOGY for Dirac or Majorana

m_ν affect growth of structures

$\sum m_i \leq ?$ (*Lecture by. E Di Valentino*)

Effects of ν Mass: Flavour Transitions

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- Mass basis (free propagation in space-time): ν_1, ν_2 and ν_3 $\dots$

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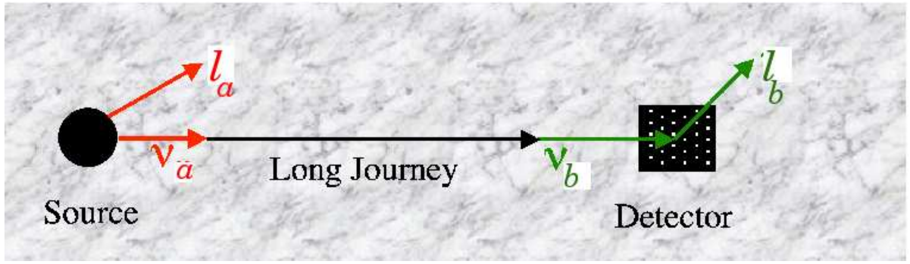
$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U_{\rm LEP} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \\ \ldots \end{pmatrix} \Rightarrow$$

Flavour not conserved in propagation



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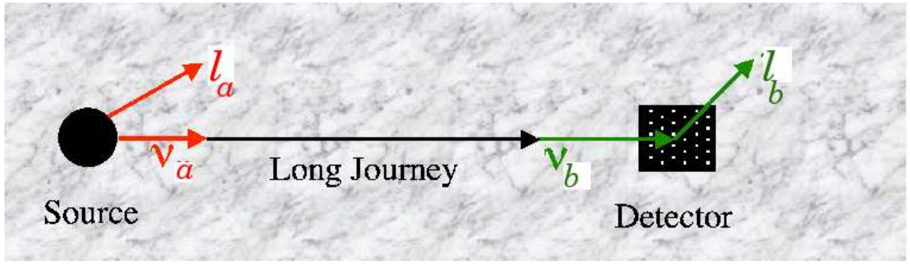
- Mass basis (free propagation in space-time): ν_1, ν_2 and ν_3 ...
- In general interaction eigenstates $\neq$ propagation eigenstates

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U_{\text{LEP}} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \\ \dots \end{pmatrix} \Rightarrow$$

Flavour not conserved in propagation



- The probability $P_{\alpha\beta}$ of producing neutrino with flavour α and detecting with flavour β has to depend on:
  - Misalignment between interaction and propagation states ($\equiv U$)
  - Difference between propagation eigenvalues
  - Propagation distance

Mass Induced Flavour Oscillations in Vacuum

- If neutrinos have mass, a weak eigenstate $|\nu_\alpha\rangle$ produced in $l_\alpha + N \to \nu_\alpha + N'$

  is a linear combination of the mass eigenstates ($|\nu_i\rangle$)

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- it can be detected with flavour β with probability

$$P_{\alpha\beta} = |\langle\nu_\beta|\nu_\alpha(t)\rangle|^2 = |\sum_{j=1}^{n}\sum_{i=1}^{n} U^*_{\alpha i} U_{\beta j} \langle\nu_j|\nu_i(t)\rangle|^2$$

Mass Induced Flavour Oscillations in Vacuum

- The probability

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- We call E_i the neutrino energy and m_i the neutrino mass

- Under the approximations:

  $(\mathbf{1})$ $|\nu\rangle$ is a *plane wave* $\Rightarrow$ $|\nu_i(t)\rangle = \mathrm{e}^{\,-i\,E_i t}|\nu_i(0)\rangle$ and using $\langle \nu_j|\nu_i\rangle = \delta_{ij}$

$$P_{\alpha\beta} = \delta_{\alpha\beta} - 4\sum_{i<j}^{n} \mathrm{Re}[U_{\alpha i} U^*_{\beta i} U^*_{\alpha j} U_{\beta j}] \sin^2\left(\frac{\Delta_{ij}}{2}\right) + 2\sum_{i<j} \mathrm{Im}[U_{\alpha i} U^*_{\beta i} U^*_{\alpha j} U_{\beta j}] \sin(\Delta_{ij})$$

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(**3**) Lowest order in mass $p_i \simeq p_j = p \simeq E$

$$\frac{\Delta_{ij}}{2} = \frac{(m_i^2 - m_j^2)\,L}{4\,E} = 1.27\frac{m_i^2 - m_j^2}{\mathrm{eV}^2}\,\frac{L/E}{\mathrm{Km/GeV}}$$

Mass Induced Flavour Oscillations in Vacuum

- The oscillation probability:

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– If $\alpha = \beta \Rightarrow$ $\mathrm{Im}[U_{\alpha i}U^*_{\alpha i}U^*_{\alpha j}U_{\alpha j}] = \mathrm{Im}[|U^\star_{\alpha i}|^2|U_{\alpha j}|^2] = 0$

$\Rightarrow$ CP violation observable only for $\beta \neq \alpha$

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- $P_{\alpha\beta}$ depends on Neutrino Parameters
  - $\Delta m^2_{ij} = m_i^2 - m_j^2$ The mass differences
  - $U_{\alpha j}$ The mixing angles (and Dirac phases)

and on Two *set-up* Parameters:
  - E The neutrino energy
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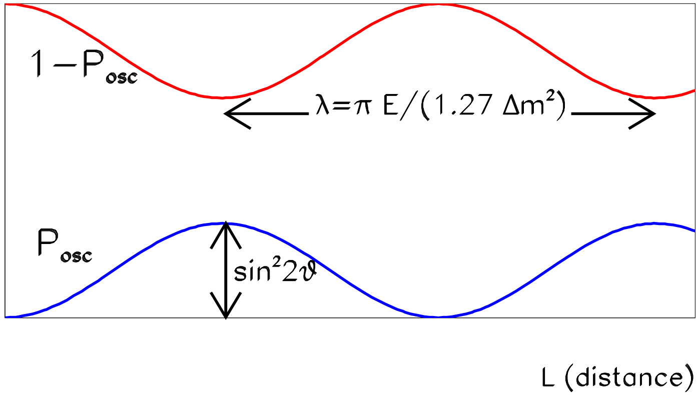
- No information on mass scale nor Majorana phases

2-ν Oscillations

- When oscillations between 2-ν dominate: $U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$

$P_{osc} = \sin^2(2\theta)\,\sin^2\left(\frac{\Delta m^2 L}{4E}\right)$ Appear

$P_{\alpha\alpha} = 1 - P_{osc}$ Disappear



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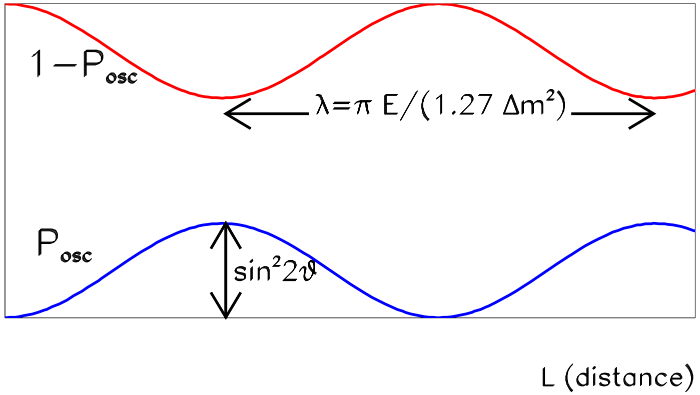


- P_{osc} is symmetric *independently* under $\Delta m^2 \to -\Delta m^2$ **or** $\theta \to -\theta + \frac{\pi}{2}$

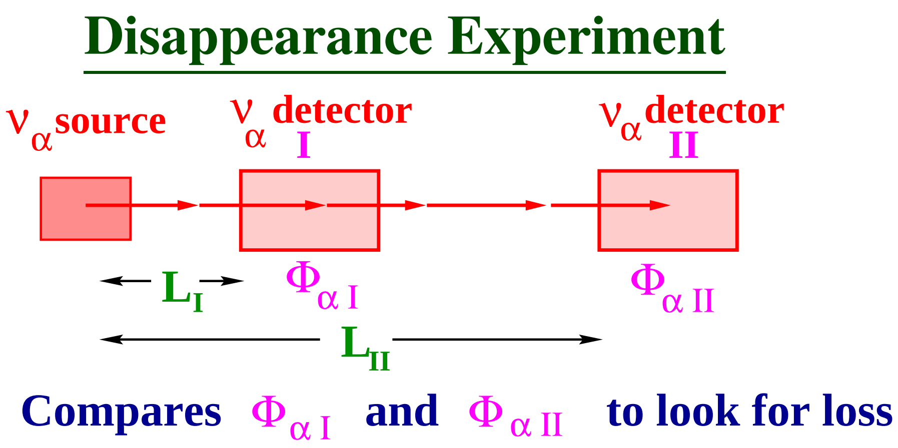
  $\Rightarrow$ No information on ordering ($\equiv$ signΔm^2) nor octant of θ

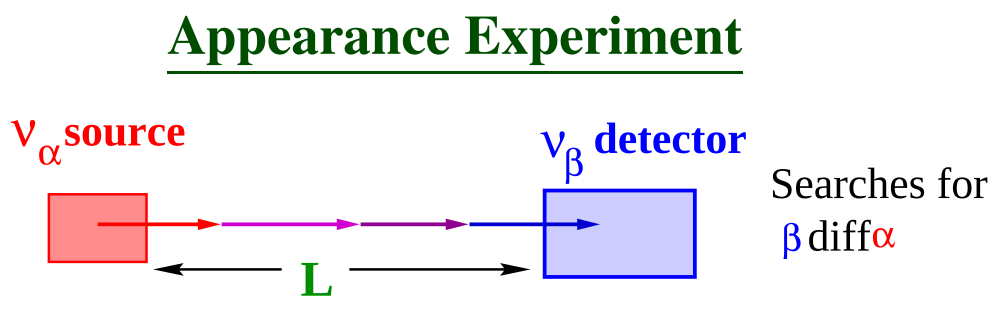
- U is real $\Rightarrow$ no CP violation

This only happens for 2ν vacuum oscillations

ν Oscillations: Experimental Probes

- Generically there are two types of experiments to search for ν oscillations :





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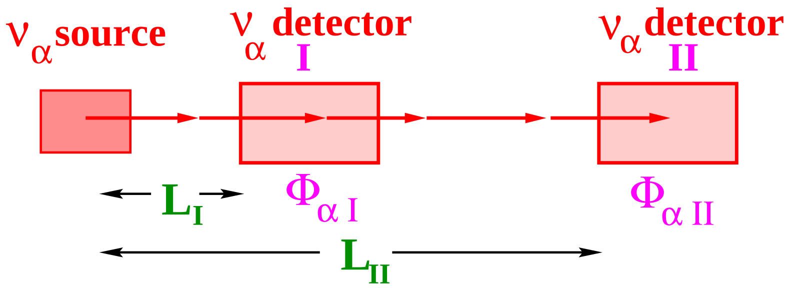
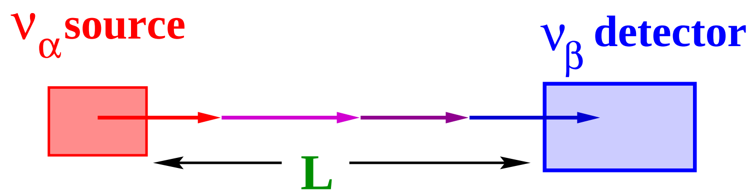


Compares $\Phi_{\alpha\, I}$ and $\Phi_{\alpha\, II}$ to look for loss

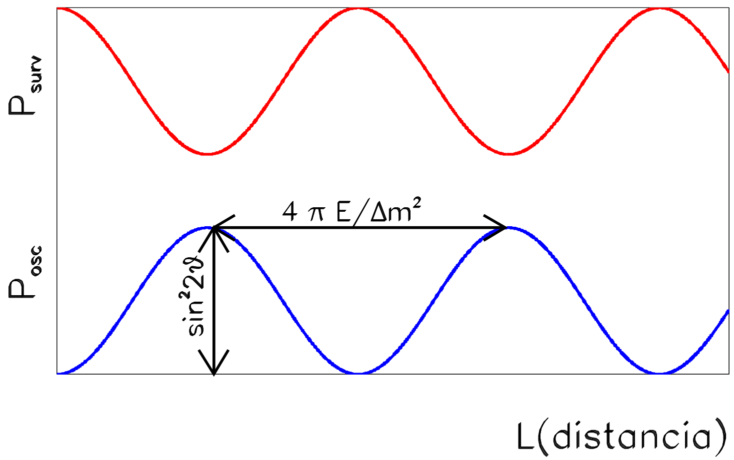
Appearance Experiment



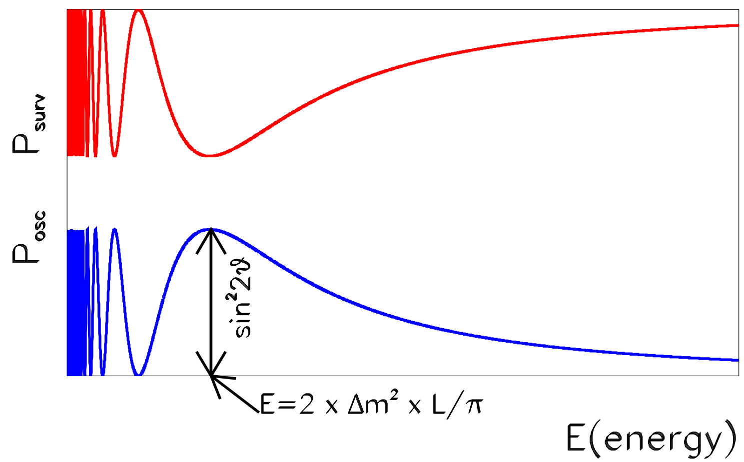
Searches for β diff α

- To verify mass-induced oscillations we can study the neutrino flavour

as function of the Distance to the source



As function of the neutrino Energy



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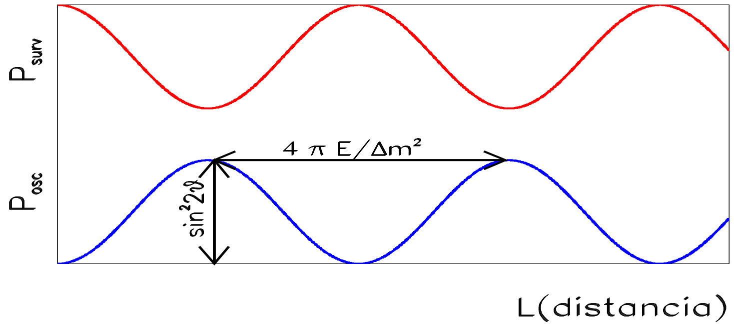

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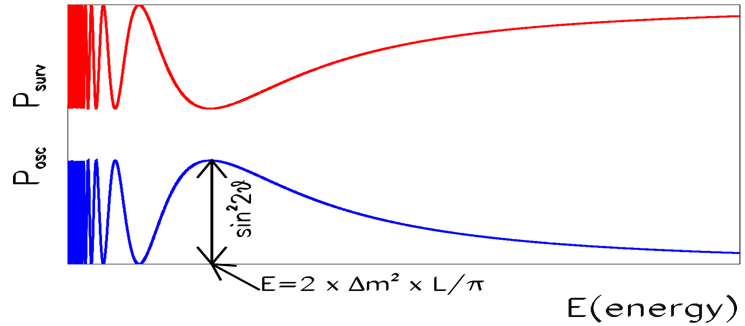

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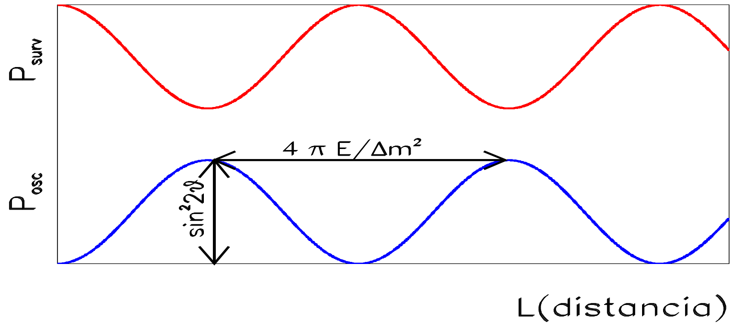

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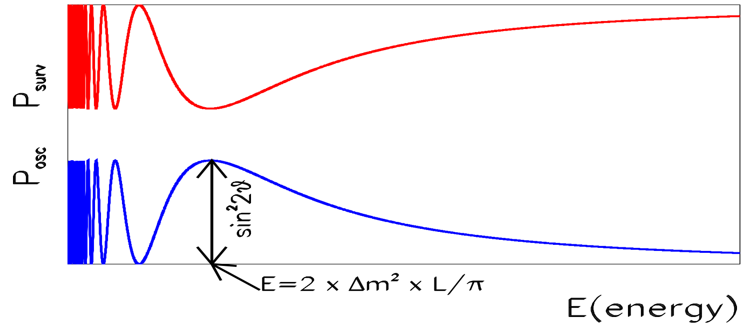
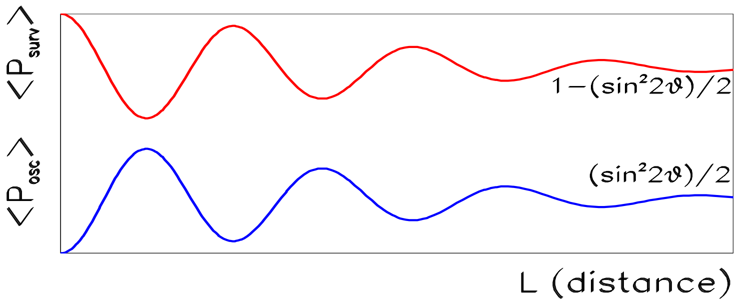
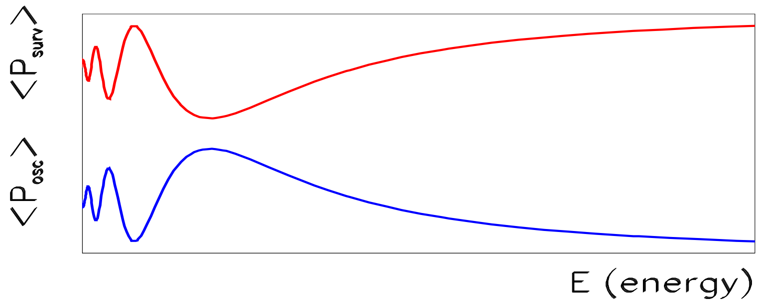


- In real experiments $\Rightarrow$ $\langle P_{\alpha\beta}\rangle = \int dE_\nu \frac{d\Phi}{dE_\nu}\sigma_{CC}(E_\nu)P_{\alpha\beta}(E_\nu)$





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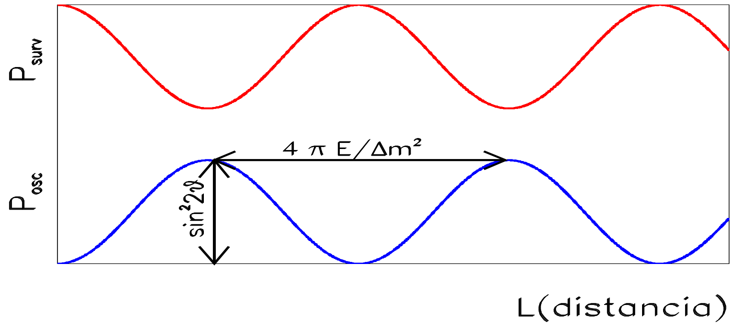

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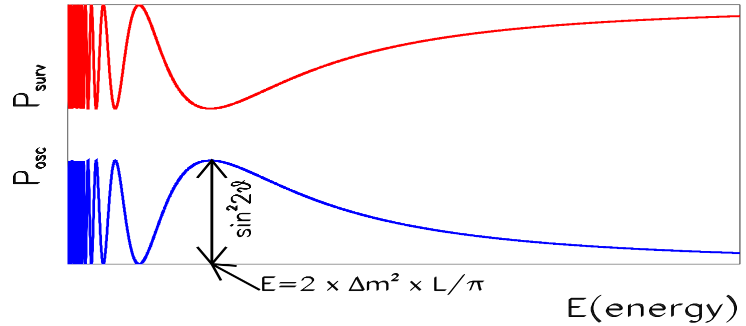

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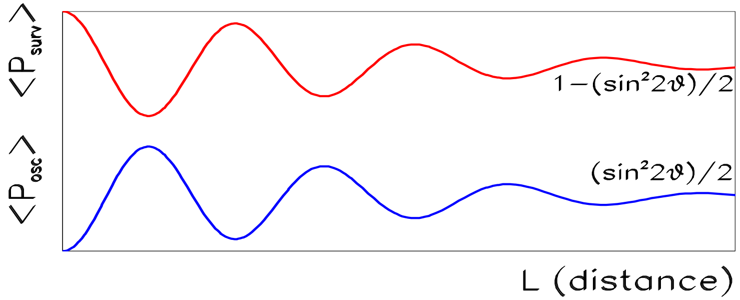

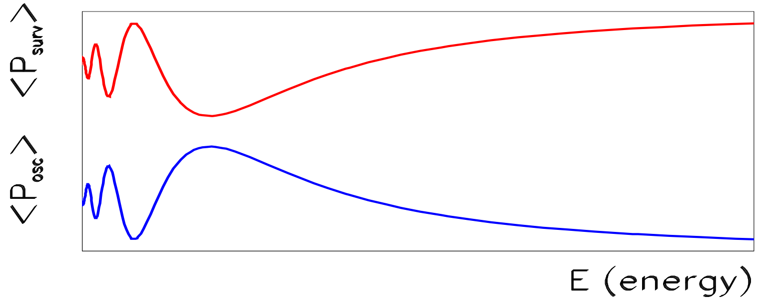

- Maximal sensitivity for $\Delta m^2 \sim E/L$

– $\Delta m^2 \ll E/L \Rightarrow \langle \sin^2(\Delta m^2 L/4E)\rangle \simeq 0 \rightarrow \langle P_{\alpha\neq\beta}\rangle \simeq 0$ & $\langle P_{\alpha\alpha}\rangle \simeq 1$

– $\Delta m^2 \gg E/L \Rightarrow \langle \sin^2(\Delta m^2 L/4E)\rangle \simeq \frac{1}{2} \rightarrow \langle P_{\alpha\neq\beta}\rangle \simeq \frac{\sin^2(2\theta)}{2} \leq \frac{1}{2}$ & $\langle P_{\alpha\alpha}\rangle \geq \frac{1}{2}$

Neutrinos in Matter:Effective Potentials

- In SM the characteristic ν-p interaction cross section

$$\sigma \sim \frac{G_F^2 E^2}{\pi} \sim 10^{-43}\text{cm}^2 \quad \text{at } \text{E}_\nu \sim \text{MeV}$$

- So if a beam of $\Phi_\nu \sim 10^{10} \nu' s$ was aimed at the Earth only 1 would be deflected

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- But that cross section is for *inelastic* scattering

  Does not contain forward elastic *coherent* scattering

- In *coherent* interactions $\Rightarrow$ ν and medium remain unchanged

  Interference of scattered and unscattered ν waves

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• Coherence ⇒ decoupling of ν evolution equation from equations of the medium.

• The effect of the medium is described by an effective potential depending on density and composition of matter

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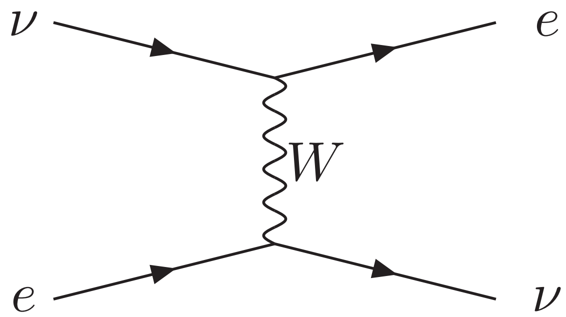

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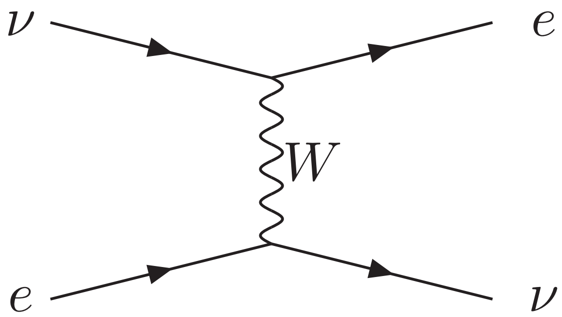

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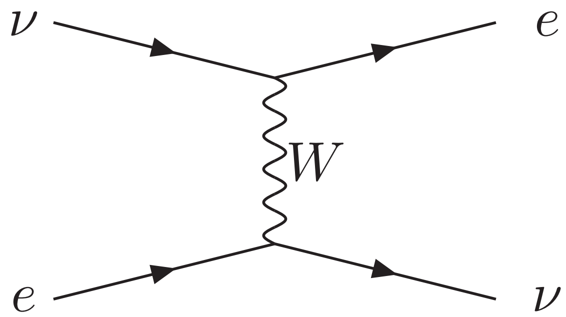

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- Other potentials for ν_e $(\overline{\nu}_e)$ for ν_α $(\overline{\nu}_\alpha)$ $\alpha = e, \mu, \tau$

medium	V_C	V_N
e^+ and e^-	$\pm\sqrt{2}G_F(N_e - N_{\bar{e}})$	$\mp\frac{G_F}{\sqrt{2}}(N_e - N_{\bar{e}})(1 - 4\sin^2\theta_W)$
p and $\bar{p}$	0	$\pm\frac{G_F}{\sqrt{2}}(N_p - N_{\bar{p}})(1 - 4\sin^2\theta_W)$
n and $\bar{n}$	0	$\mp\frac{G_F}{\sqrt{2}}(N_n - N_{\bar{n}})$
Neutral $(N_e = N_p)$	$\pm\sqrt{2}G_F N_e$	$\mp\frac{G_F}{\sqrt{2}} N_n$

Vacuum Oscillations Revisited

- ν oscillations can also be understood from the eq. of motion of weak eigenstates
- A state mixture of 2 neutrino species $|\nu_\alpha\rangle$ and $|\nu_\beta\rangle$ or equivalently of $|\nu_1\rangle$ and $|\nu_2\rangle$

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- Using (1) in Dirac Eq. we can factorize ϕ_i and α_x and get:

$$-i\frac{\partial \nu_1(x)}{\partial x} = \left\{E^2 - m_1^2\right\}^{1/2} \nu_1(x)$$
$$-i\frac{\partial \nu_2(x)}{\partial x} = \left\{E^2 - m_2^2\right\}^{1/2} \nu_2(x)$$

- In the relativistic limit $\sqrt{E^2 - m_i^2} \simeq E - \frac{m_i^2}{2E}$

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- An overall phase: $\nu_\alpha \to e^{i\eta x}\nu_\alpha$ and $\nu_\beta \to e^{i\eta x}\nu_\beta$ is unobservable

$\Rightarrow$ pieces proportional to $I = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$ do not affect evolution:

$$\Rightarrow -i\frac{\partial}{\partial x}\begin{pmatrix}\nu_\alpha\\ \nu_\beta\end{pmatrix} = -\begin{pmatrix} -\frac{\Delta m^2}{4E}\cos 2\theta & \frac{\Delta m^2}{4E}\sin 2\theta \\ \frac{\Delta m^2}{4E}\sin 2\theta & \frac{\Delta m^2}{4E}\cos 2\theta\end{pmatrix}\begin{pmatrix}\nu_\alpha\\ \nu_\beta\end{pmatrix}$$

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Can be rewritten as

$$\begin{aligned} \ddot{\nu}_\alpha + \omega^2 \nu_\alpha &= 0 \\ \ddot{\nu}_\beta + \omega^2 \nu_\beta &= 0 \end{aligned} \qquad \text{with} \qquad \omega = \frac{\Delta m^2}{4E}$$

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- For initial conditions: $\nu_\alpha(0) = 1$ and $\nu_\beta(0) = 0 \Rightarrow \begin{cases} A_1 = \sin^2\theta \quad A_2 = \cos^2\theta \\ B_1 = -B_2 = \sin\theta\cos\theta \end{cases}$

- And the flavour transition probability

$$P(\nu_\alpha \to \nu_\beta) = |\nu_\beta(L)|^2 = B_1^2 + B_2^2 + 2B_1B_2\cos(2\omega L) = \sin^2(2\theta)\,\sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

Neutrinos in Matter: Evolution Equation

Evolution Eq. for $|\nu\rangle = \nu_1|\nu_1\rangle + \nu_2|\nu_2\rangle \equiv \nu_\alpha|\nu_\alpha\rangle + \nu_\beta|\nu_\beta\rangle$

(a) In vacuum in the mass basis: $-i\frac{\partial}{\partial x}\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix} = \left\{E \times I - \begin{pmatrix}\frac{m_1^2}{2E} & 0\\ 0 & \frac{m_2^2}{2E}\end{pmatrix}\right\}\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix}$

(b) In vacuum in the weak basis

$$-i\frac{\partial}{\partial x}\begin{pmatrix}\nu_\alpha\\ \nu_\beta\end{pmatrix} = \left\{\left[E - \frac{m_1^2+m_2^2}{4E}\right] \times I - \begin{pmatrix}-\frac{\Delta m^2}{4E}\cos 2\theta & \frac{\Delta m^2}{4E}\sin 2\theta\\ \frac{\Delta m^2}{4E}\sin 2\theta & \frac{\Delta m^2}{4E}\cos 2\theta\end{pmatrix}\right\}\begin{pmatrix}\nu_\alpha\\ \nu_\beta\end{pmatrix}$$

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(c) In matter (e, p, n) in weak basis

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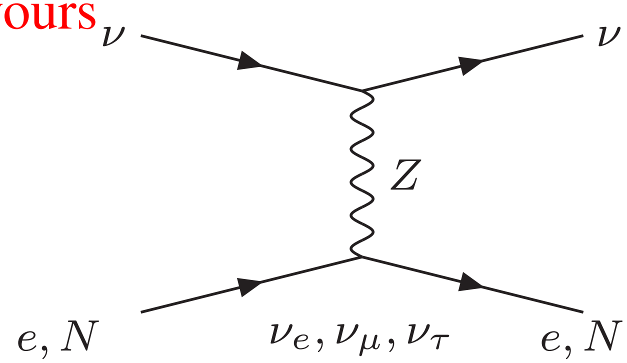
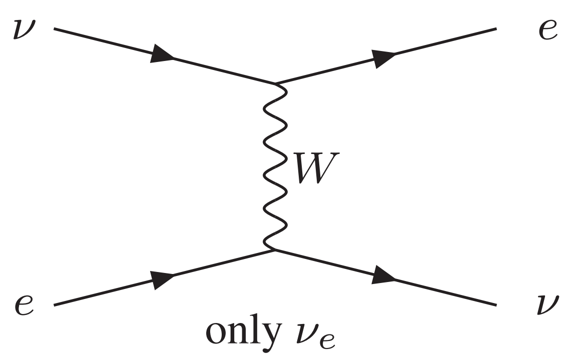
(c)$\neq$ (b) because different flavours have different interactions

For example $\alpha = e$, $\beta = \mu, \tau$:

$$V_{CC} = V_\alpha - V_\mu = \sqrt{2}G_F N_e$$

(opposite sign for $\overline{\nu}$)





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Diagonalizing:

$$-i\frac{\partial}{\partial x}\begin{pmatrix}\nu_\alpha\\ \nu_\beta\end{pmatrix} \equiv \left\{\left[E - \frac{\mu_1^2+\mu_2^2}{4E}\right]\times I - \begin{pmatrix}-\frac{\Delta\mu^2}{4E}\cos 2\theta_m & \frac{\Delta\mu^2}{4E}\sin 2\theta_m\\ \frac{\Delta\mu^2}{4E}\sin 2\theta_m & \frac{\Delta\mu^2}{4E}\cos 2\theta_m\end{pmatrix}\right\}\begin{pmatrix}\nu_\alpha\\ \nu_\beta\end{pmatrix}$$

Effective masses and mixing are different than in vacuum

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– The effective masses: $(A = 2E(V_\alpha - V_\beta))$

$$\mu_{1,2}^2(x) = \frac{m_1^2 + m_2^2}{2} + E(V_\alpha + V_\beta) \mp \frac{1}{2}\sqrt{(\Delta m^2 \cos 2\theta - A)^2 + (\Delta m^2 \sin 2\theta)^2}$$

$$\Delta\mu^2(x) = \sqrt{(\Delta m^2 \cos 2\theta - A)^2 + (\Delta m^2 \sin 2\theta)^2}$$

– The mixing angle in matter

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- For constant matter density ⇒ θ_m and μ_i are constant along ν evolution
  ⇒ the evolution is determined by masses and mixing in matter so

$$P_{\alpha\neq\beta} = \sin^2(2\theta_m)\,\sin^2\left(\frac{\Delta\mu^2 L}{2E}\right)$$

- Constant matter potential is a good approximation for LBL experiments.

$\Rightarrow$ If matter density varies along ν trajectory the effective masses and mixing vary too

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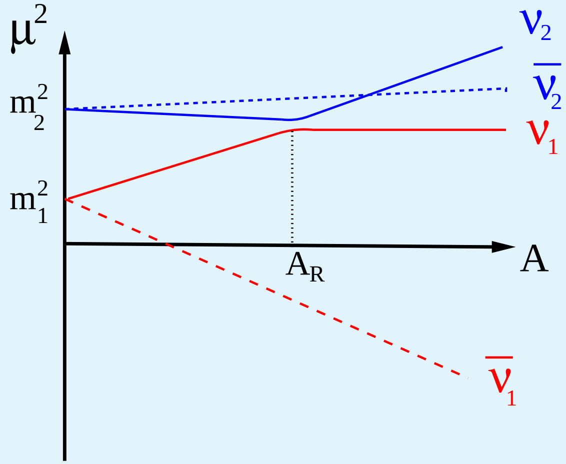

At *resonant* potential: $A_R = \Delta m^2 \cos 2\theta$

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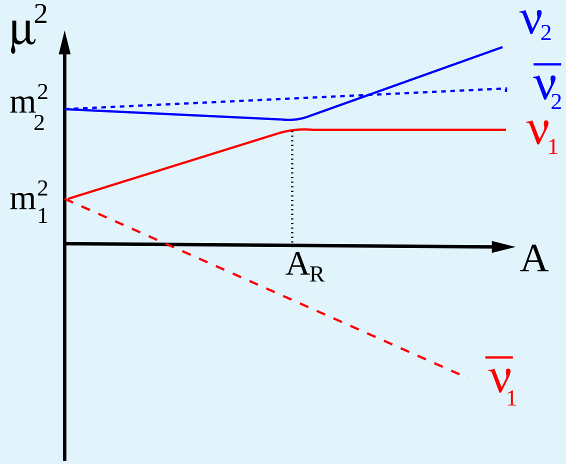

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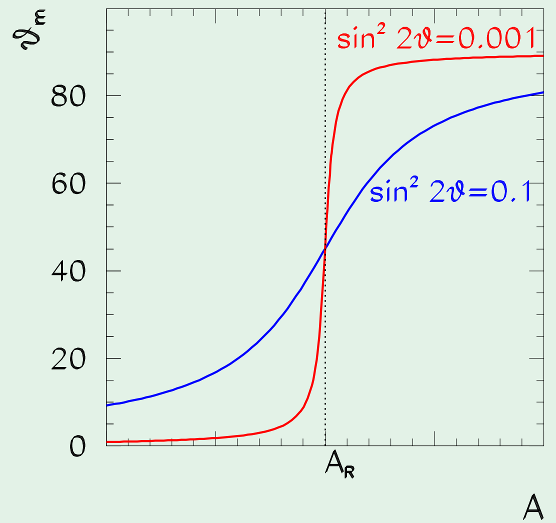


* At $A = 0$ (vacuum) $\Rightarrow \theta_m = \theta$
* At $A = A_R \Rightarrow \theta_m = \frac{\pi}{4}$
* At $A > A_R \Rightarrow \theta_m = \frac{\pi}{2} - \theta$
* At $A >> A_R \Rightarrow \theta_m = \frac{\pi}{2}$

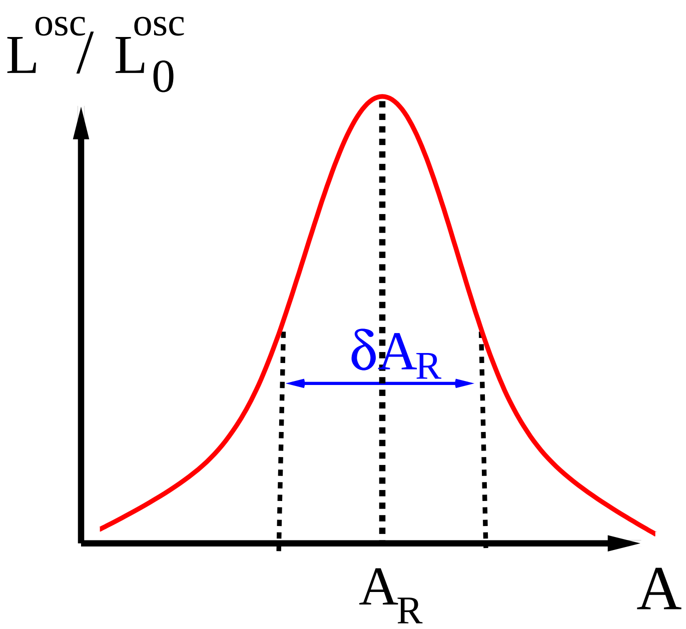
The oscillation length in vacuum

$$L_0^{osc} = \frac{4\pi E}{\Delta m^2}$$

The oscillation length in matter ($A = 2E(V_\alpha - V_\beta)$)

$$L^{osc} \equiv \frac{4\pi E}{\Delta\mu^2} = \frac{L_0^{osc}}{\sqrt{(\Delta m^2 \cos 2\theta - A)^2 + (\Delta m^2 \sin 2\theta)^2}}$$

L^{osc} presents a resonant behaviour



At the resonant density $A_R = \Delta m^2 \cos 2\theta$

$$L_R^{osc} = \frac{L_0^{osc}}{\sin 2\theta}$$

The width of the resonance in potential:

$$\delta V_R \equiv \frac{\delta A_R}{E} = \frac{\Delta m^2 \sin 2\theta}{E}$$

The width of the resonance in distance:

$$\delta r_R = \frac{\delta V_R}{|\frac{dV}{dr}|_R}$$

- The instantaneous mass and mixings in matter ($A = 2E(V_\alpha - V_\beta)$)

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- The evolution equation in instantaneous mass basis

$$i \begin{pmatrix} \dot\nu_1^m \\ \dot\nu_2^m \end{pmatrix} = \frac{1}{4E} \begin{pmatrix} -\Delta\mu^2(x) & -4\,i\,E\,\dot\theta_m(x) \\ 4\,i\,E\,\dot\theta_m(x) & \Delta\mu^2(x) \end{pmatrix} \begin{pmatrix} \nu_1^m \\ \nu_2^m \end{pmatrix}$$

$\Rightarrow$ It is not diagonal $\Rightarrow$ Instantaneous mass eigenstates $\neq$ eigenstates of evolution

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• For $\Delta\mu^2(x) \gg 4\, E\, \dot{\theta}_m(x)$ $\left[\frac{1}{V} \frac{dV}{dx}\Big|_R \ll \frac{\Delta m^2}{2E} \frac{\sin^2 2\theta}{\cos 2\theta} \right]$ $\equiv$ Slowly varying matter potent

$\Rightarrow$ ν_i^m behave approximately as *evolution eigenstates*

$\Rightarrow$ ν_i^m do not mix in the evolution This is the *adiabatic* transition approximation

• The instantaneous mass and mixings in matter ($A = 2E(V_\alpha - V_\beta)$)

$$\Delta\mu^2(x) = \sqrt{(\Delta m^2 \cos 2\theta - A)^2 + (\Delta m^2 \sin 2\theta)^2} \qquad \tan 2\theta_m = \frac{\Delta m^2 \sin 2\theta}{\Delta m^2 \cos 2\theta - A}$$

• The evolution equation in instantaneous mass basis

$$i \begin{pmatrix} \dot\nu_1^m \\ \dot\nu_2^m \end{pmatrix} = \frac{1}{4E} \begin{pmatrix} -\Delta\mu^2(x) & -4\,i\,E\,\dot\theta_m(x) \\ 4\,i\,E\,\dot\theta_m(x) & \Delta\mu^2(x) \end{pmatrix} \begin{pmatrix} \nu_1^m \\ \nu_2^m \end{pmatrix}$$

$\Rightarrow$ It is not diagonal $\Rightarrow$ Instantaneous mass eigenstates $\neq$ eigenstates of evolution

$\Rightarrow$ Transitions $\nu_1^m \to \nu_2^m$ can occur $\equiv$ *Non adiabaticity*

• For $\Delta\mu^2(x) \gg 4\,E\,\dot\theta_m(x)$ $\left[\frac{1}{V}\frac{dV}{dx}\Big|_R \ll \frac{\Delta m^2}{2E}\frac{\sin^2 2\theta}{\cos 2\theta}\right]$ $\equiv$ Slowly varying matter potent

$\Rightarrow$ ν_i^m behave approximately as *evolution eigenstates*

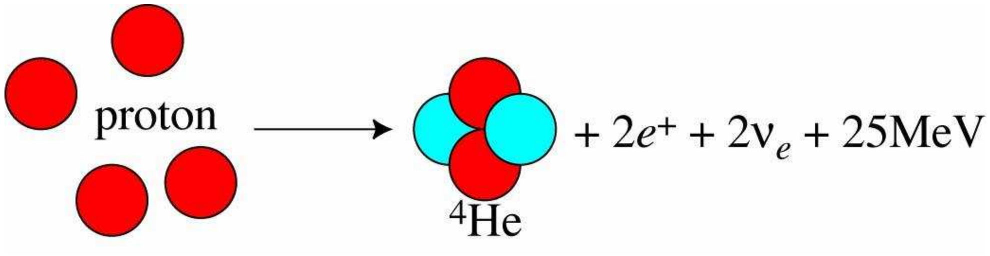
$\Rightarrow$ ν_i^m do not mix in the evolution This is the *adiabatic* transition approximation

The adiabaticity condition: $\frac{1}{V}\frac{dV}{dx}\Big|_R \ll \frac{\Delta m^2}{2E}\frac{\sin^2 2\theta}{\cos 2\theta}$ $\equiv$ $\delta r_R \gg L_R^{osc}/2\pi$

$\Rightarrow$ Many oscillations take place in the resonant region

Solar Neutrinos

- Sun shines by nuclear fusion of protons into He



And only ν_e are produced

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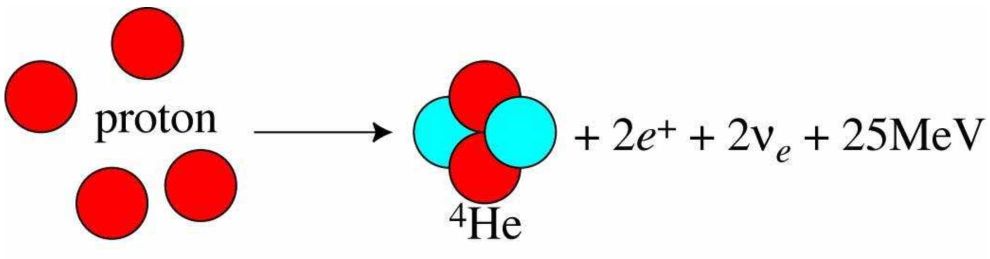
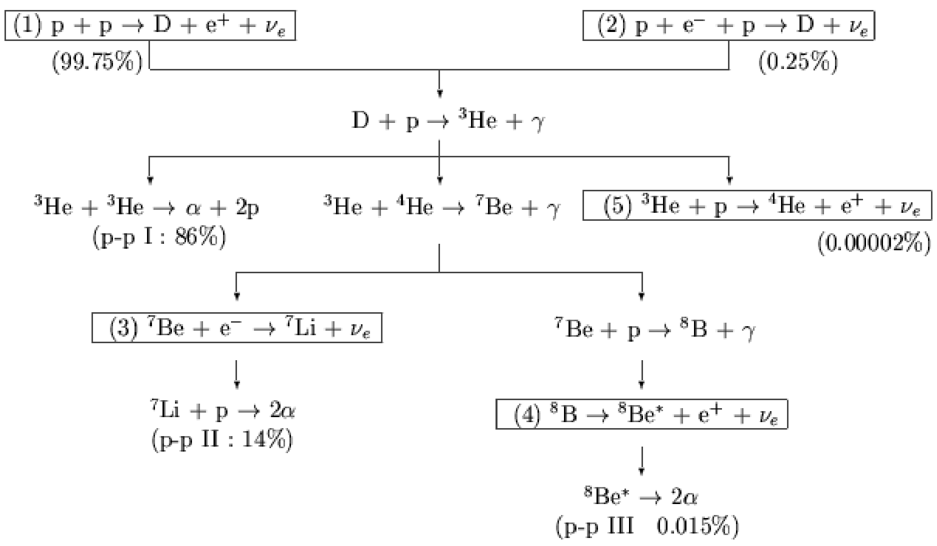


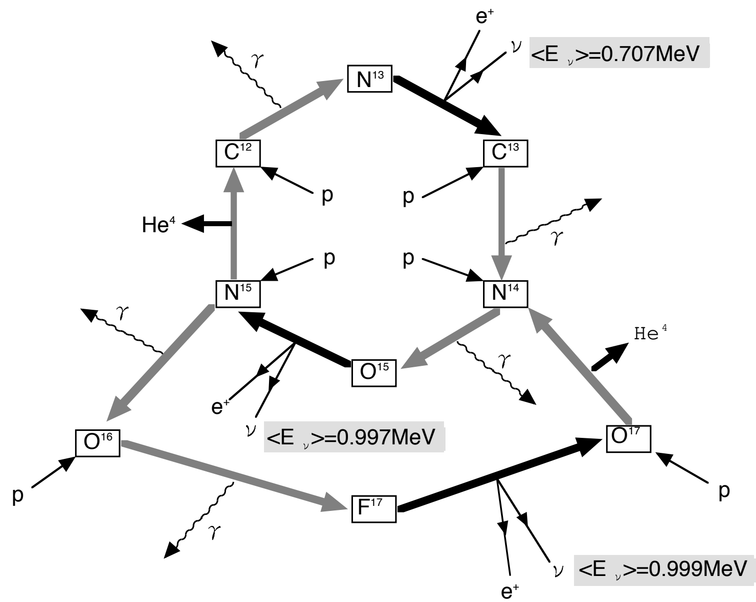
And only ν_e are produced

- Two main chains of nuclear reactions

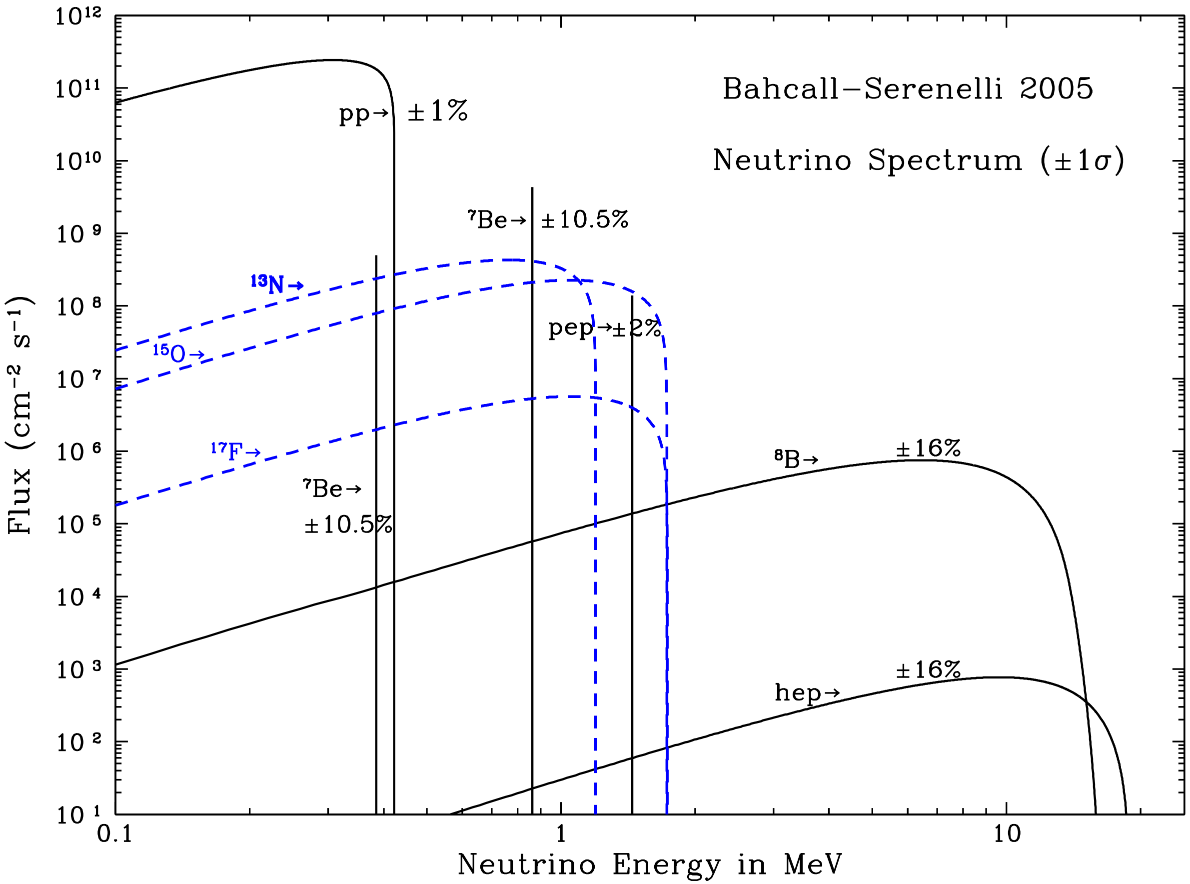
pp Chain :



CNO cycle:

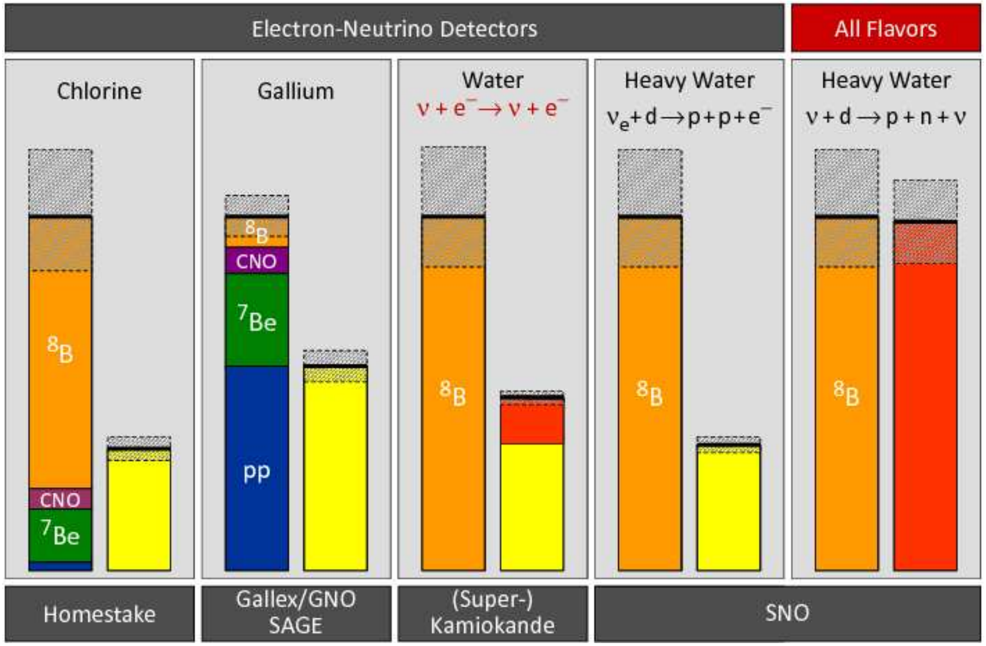


Solar Neutrinos: Fluxes



PP CHAIN	E_ν (MeV)
(pp)	
$p + p \rightarrow {}^2H + e^+ + \nu_e$	≤ 0.42
(pep)	
$p + e^- + p \rightarrow {}^2H + \nu_e$	1.552
(^{7}Be)	
${}^7Be + e^- \rightarrow {}^7Li + \nu_e$	$0.862 (90\%)$ 0.384 (10%)
(hep)	
${}^2He + p \rightarrow {}^4He + e^+ + \nu_e$	≤ 18.77
(^{8}B)	
${}^8B \rightarrow {}^8Be^* + e^+ + \nu_e$	≤ 15
CNO CHAIN	E_ν (MeV)
(^{13}N)	
${}^{13}N \rightarrow {}^{13}C + e^+ + \nu_e$	≤ 1.199
(^{15}O)	
${}^{15}O \rightarrow {}^{15}N + e^+ + \nu_e$	≤ 1.732
(^{17}F)	
${}^{17}F \rightarrow {}^{17}O + e^+ + \nu_e$	≤ 1.74

Solar Neutrinos: Results



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Deficit disappears in NC

$\Rightarrow$ Solar Model Independent Effect

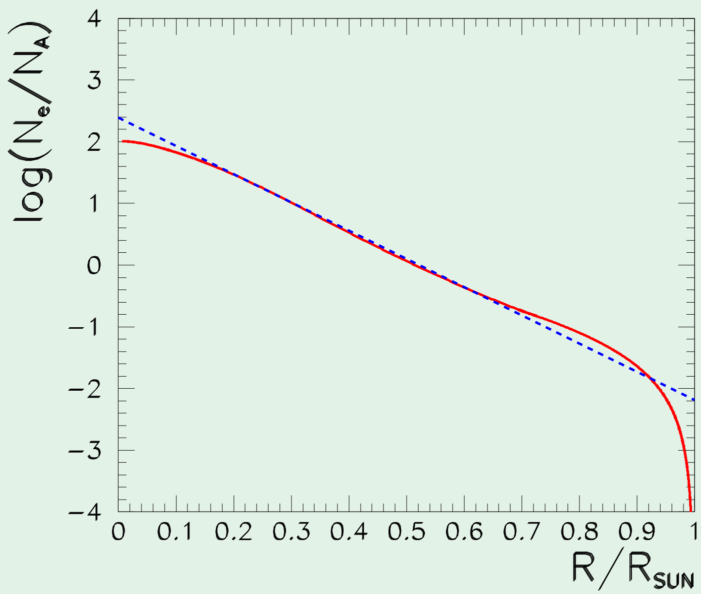
Deficit is energy dependent

Deficit $\Rightarrow P_{ee} \sim 30\% \, (< 0.5)$ for $E_\nu \gtrsim 0.8$ MeV

Neutrinos in The Sun : MSW Effect

- Solar neutrinos are ν_e produced in the core ($R \lesssim 0.3 R_\odot$) of the Sun

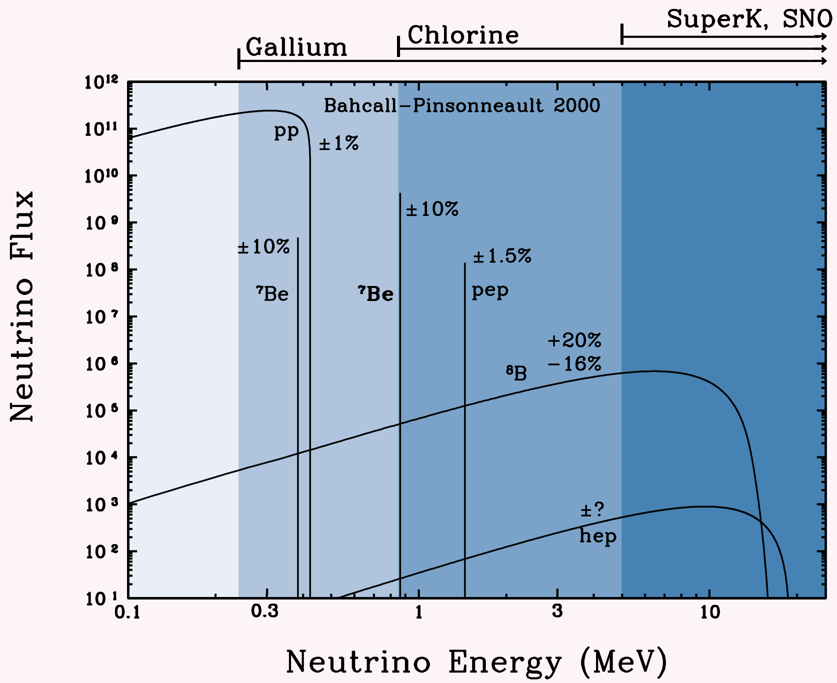
The solar matter density



$$V_{CC} = \sqrt{2} G_F N_e \sim 10^{-14} \frac{N_e}{N_A} \text{ eV}$$

At core: $V_{CC,0} \sim 10^{-14}$–10^{-12} eV

The energy spectrum of solar $\nu_e' s$



$E_\nu \sim 0.1$–10 MeV

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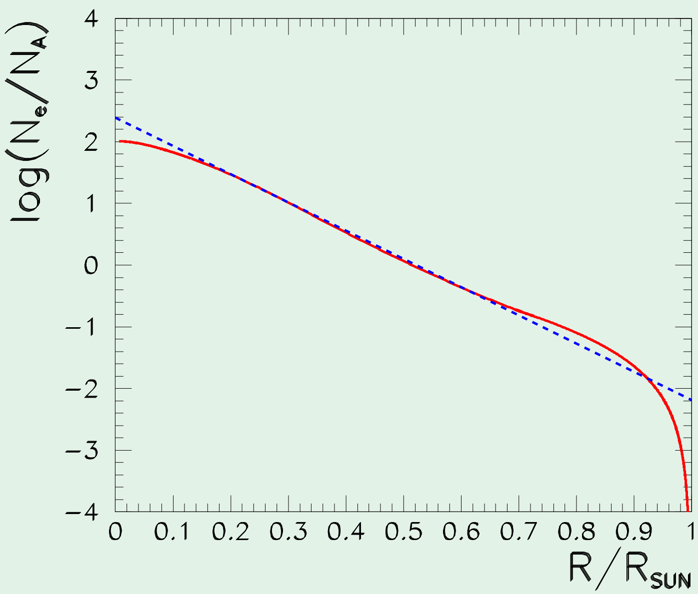

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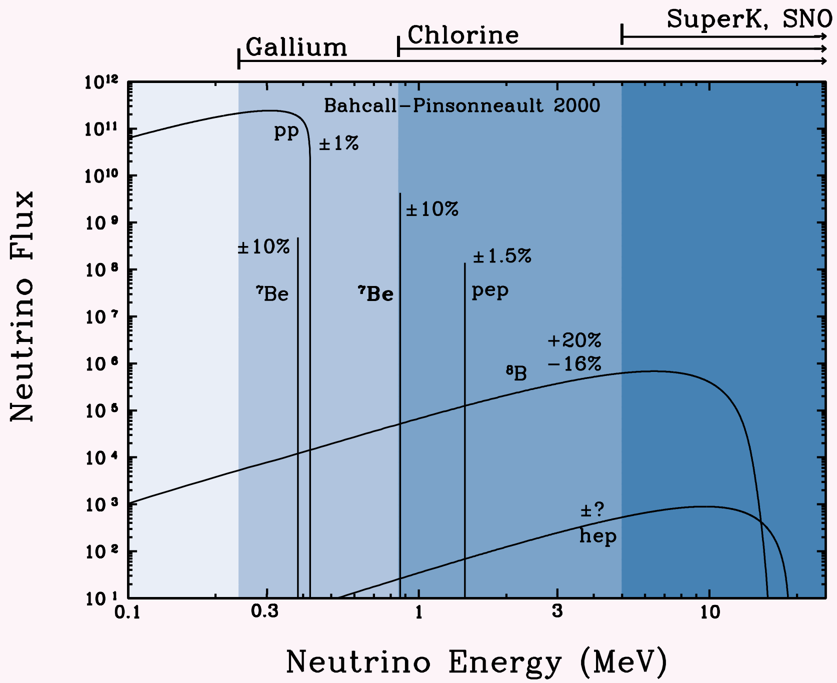

$E_\nu \sim 0.1$–10 MeV

- For $\nu_e \leftrightarrow \nu_{\mu(\tau)}$, in vacuum $\nu_e = \cos\theta\, \nu_1 + \sin\theta\, \nu_2$

- For 10^{-9} eV$^2 \lesssim \Delta m^2 \lesssim 10^{-4}$ eV^2 $\Rightarrow$ $2E_\nu V_{CC,0} > \Delta m^2 \cos 2\theta$

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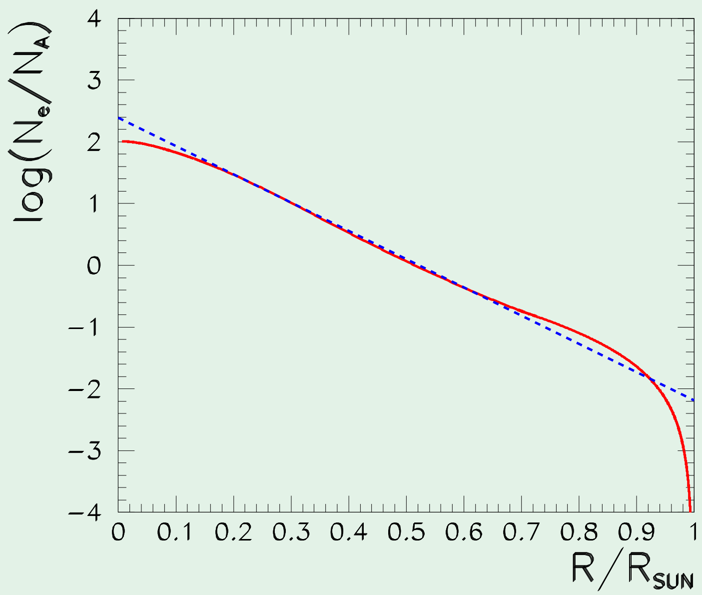

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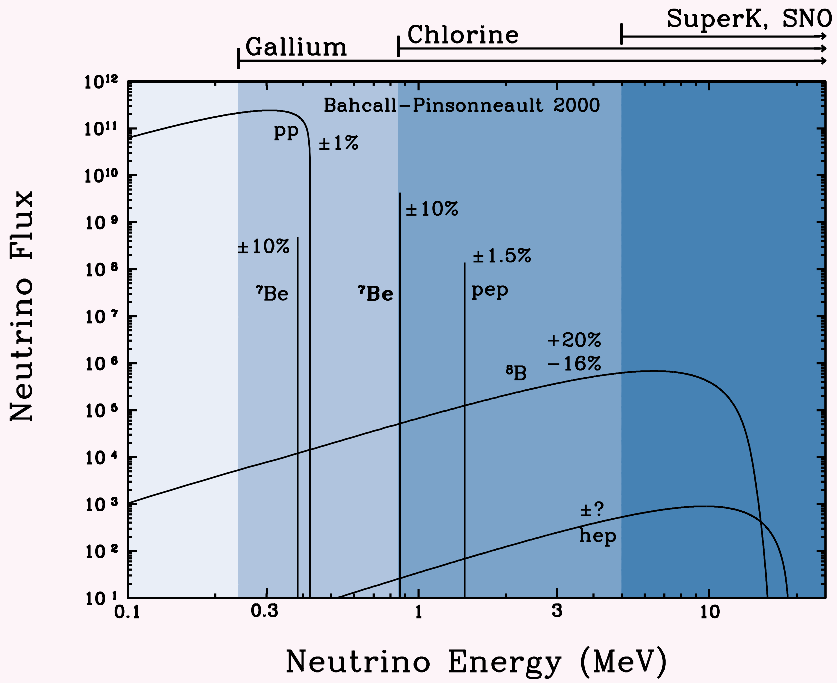

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$\Rightarrow$ ν can cross resonance condition in its way out of the Sun

For $\theta \ll \frac{\pi}{4}$: In vacuum $\nu_e = \cos\theta\,\nu_1 + \sin\theta\,\nu_2$ is mostly ν_1

In Sun core $\nu_e = \cos\theta_{m,0}\,\nu_1 + \sin\theta_{m,0}\,\nu_2$ is mostly ν_2

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If $\frac{(\Delta m^2/\text{eV}^2)\sin^2 2\theta}{(E/\text{MeV})\cos 2\theta} \gg 3\times 10^{-9}$

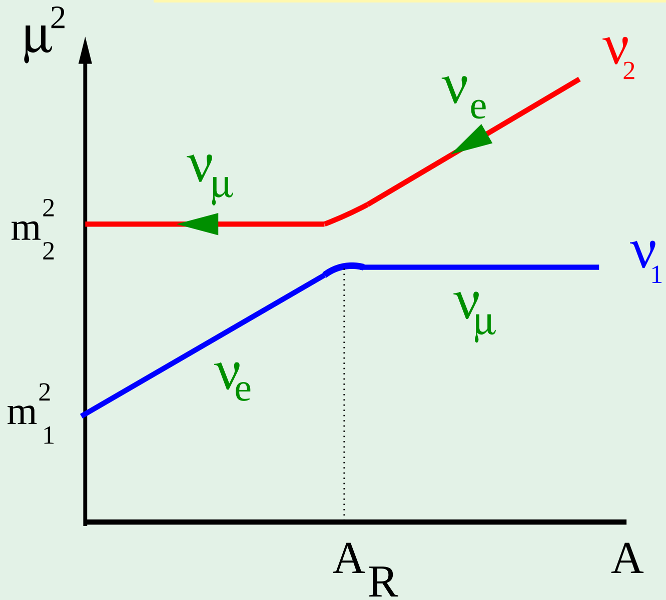
$\Rightarrow$ Adiabatic transition

$*$ ν is mostly ν_2 before and after resonance

$*$ $\theta_m \downarrow$ *dramatically* at resonance

$\Rightarrow \nu_e$ component $\downarrow \Rightarrow P_{ee} \downarrow$

This is the MSW effect



$$P_{ee} = \frac{1}{2}\left[1 + \cos 2\theta_{m,0}\cos 2\theta\right] \simeq \sin^2\theta$$

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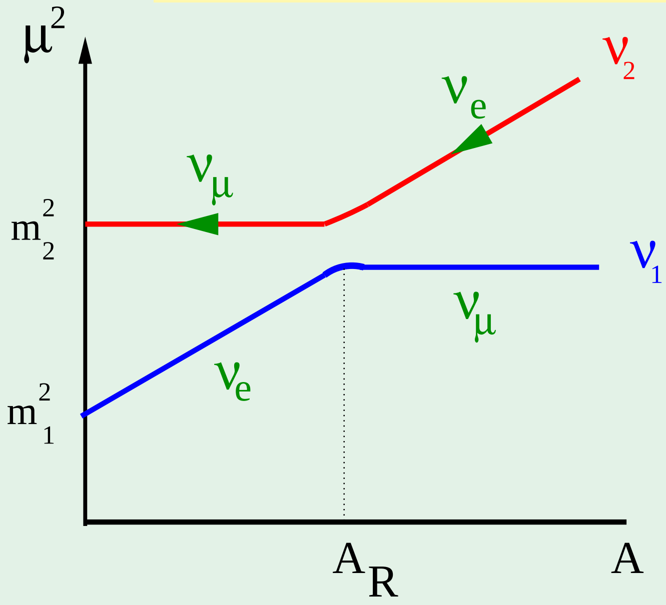

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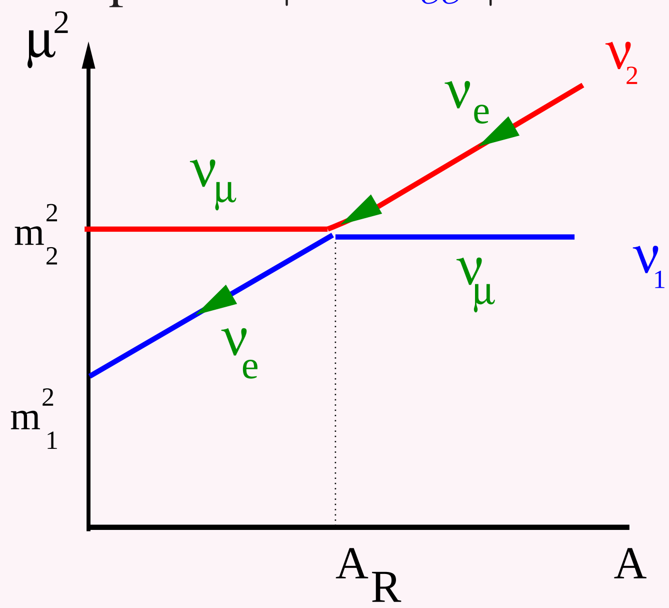
$\Rightarrow$ Non-Adiabatic transition

$*$ ν is mostly ν_2 till the resonance

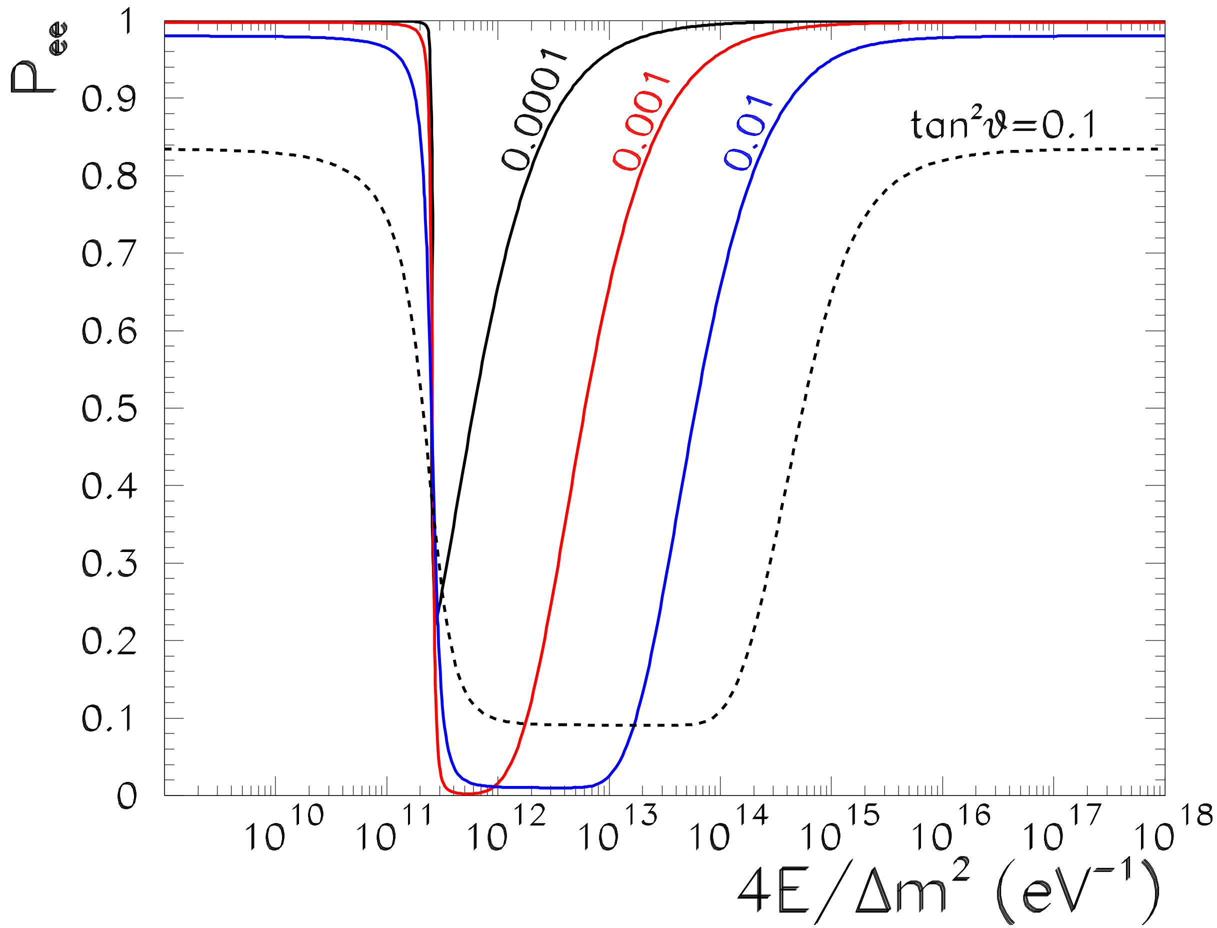
$*$ At resonance the state can jump into ν_1 (with probability P_{LZ})

$\Rightarrow \nu_e$ component $\uparrow\ \Rightarrow P_{ee}\uparrow$



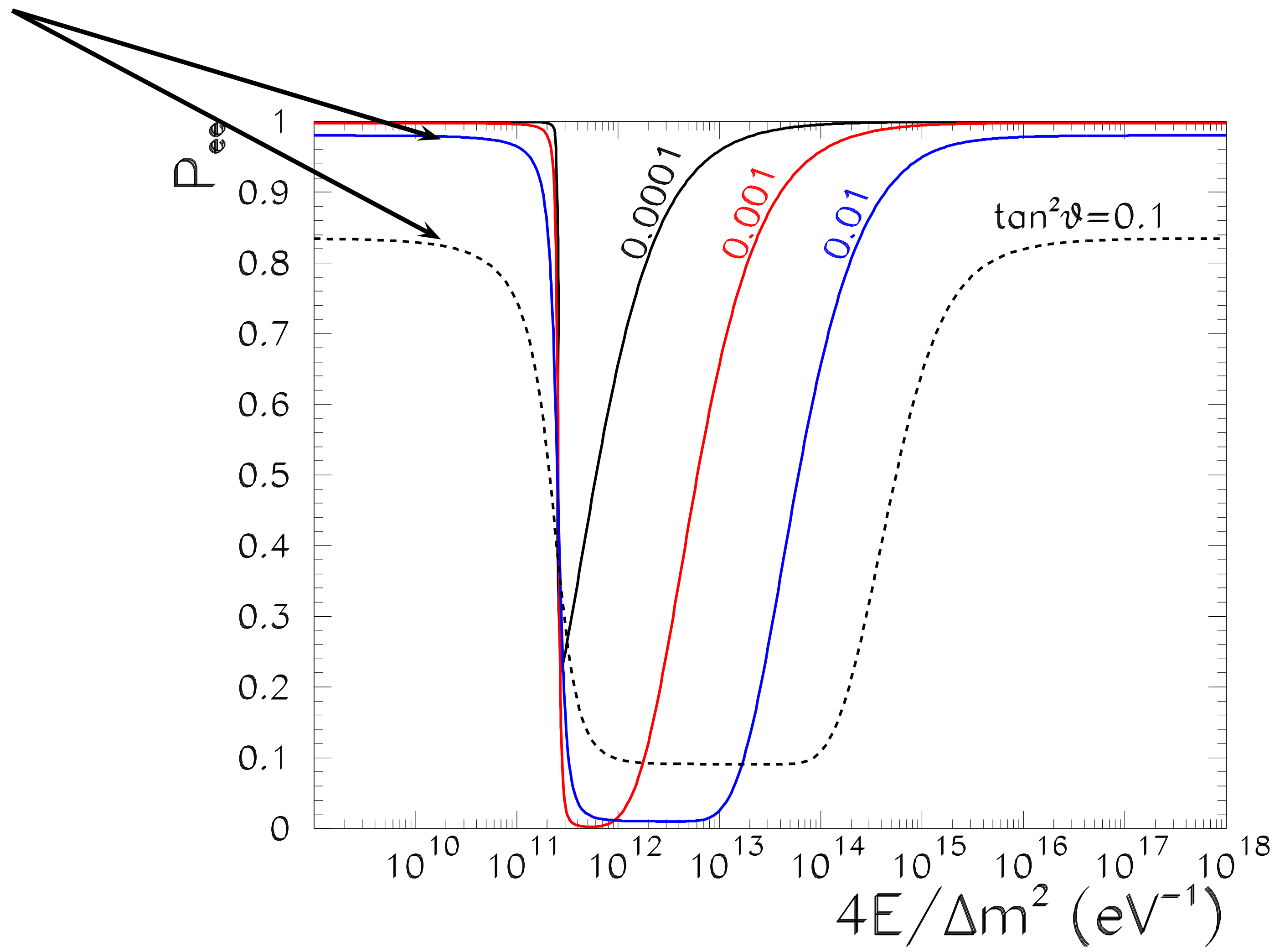
$$P_{ee} = \frac{1}{2}\left[1 + (1-2P_{LZ})\cos 2\theta_{m,0}\cos 2\theta\right]$$

Neutrinos in The Sun : MSW Effect



Neutrinos in The Sun : MSW Effect

ν does not cross resonance: $$P_{ee} = 1 - \frac{1}{2}\sin^2 2\theta > \frac{1}{2}$$



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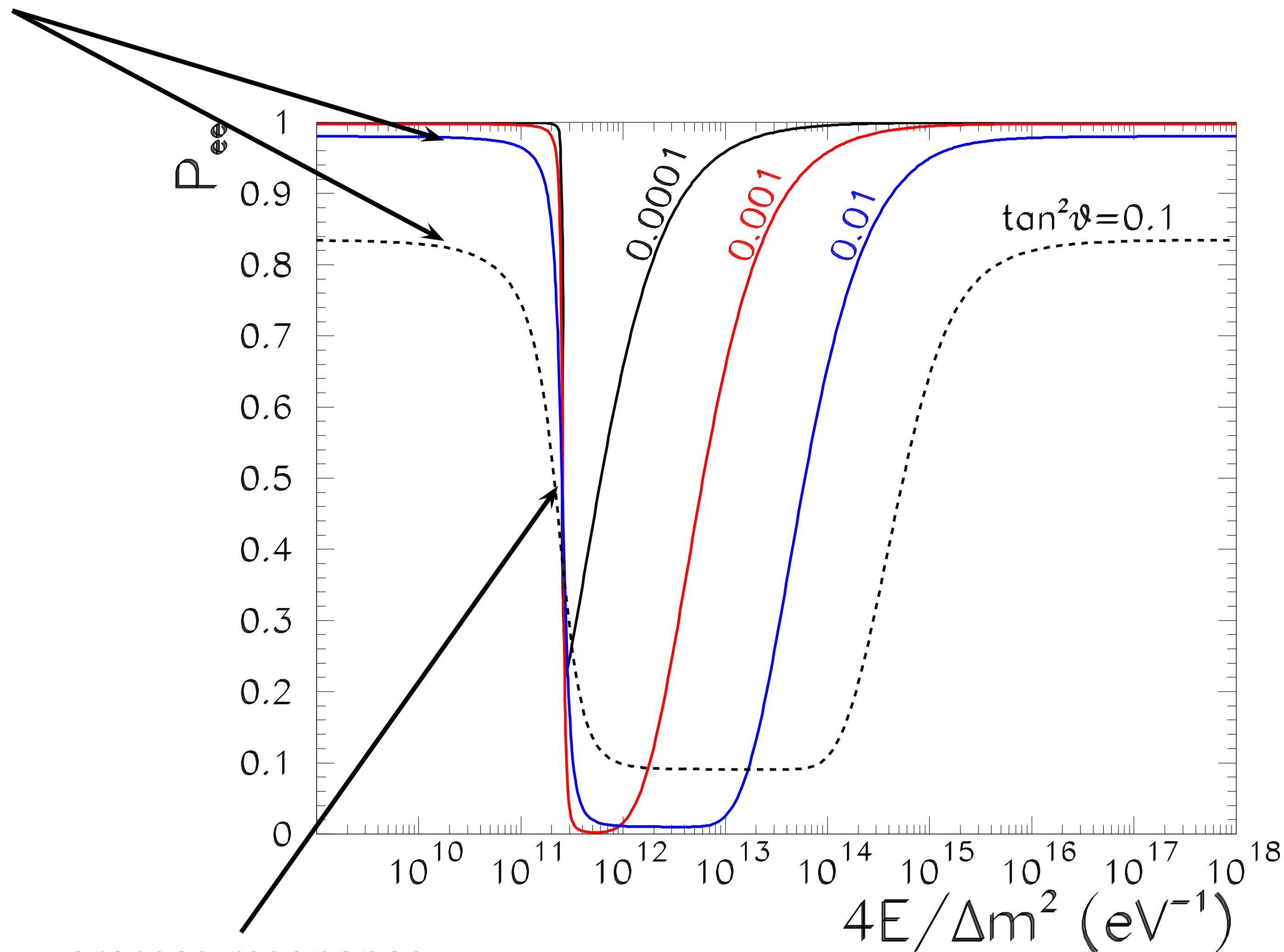

ν crosses resonance

MSW effect

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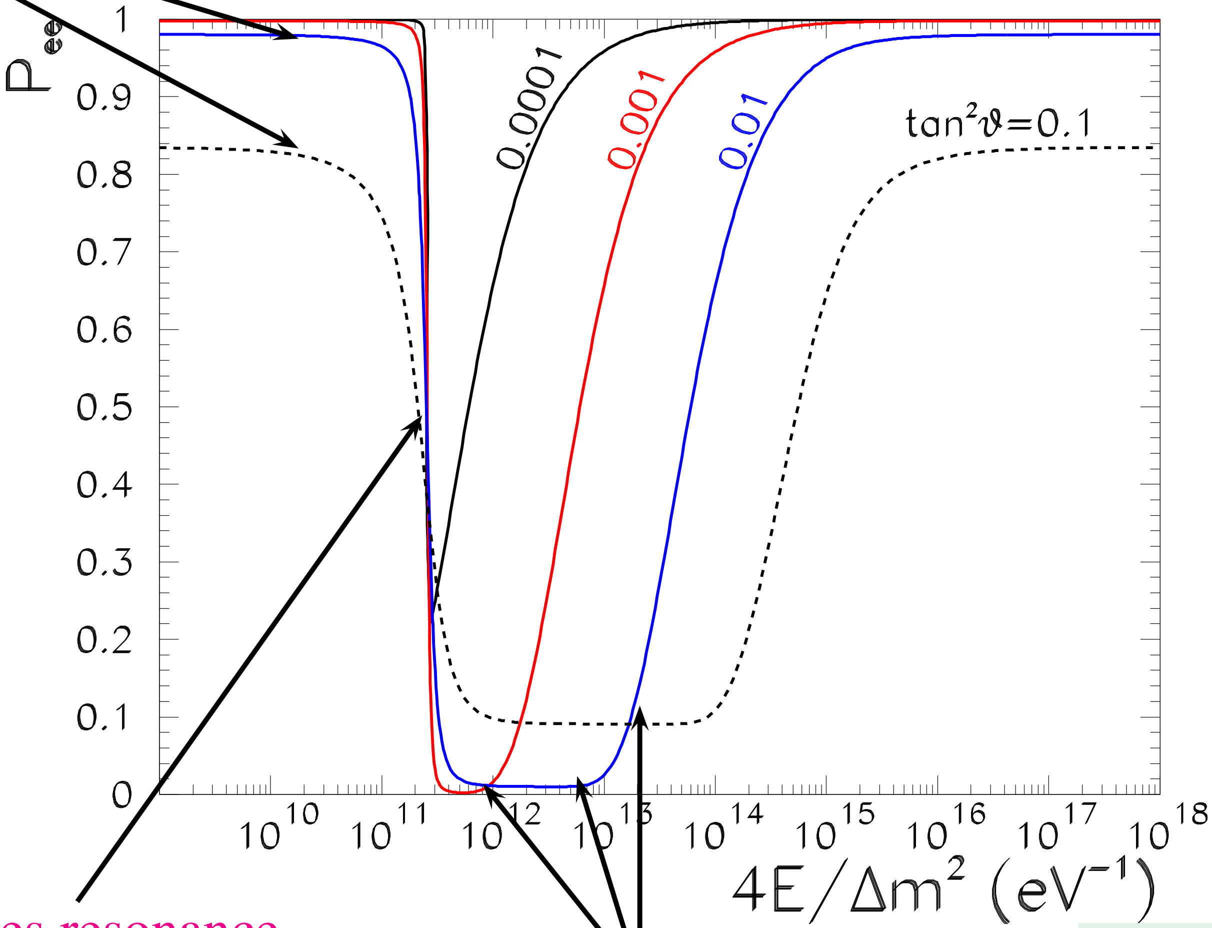

ν crosses resonance

MSW effect

Adiabatic MSW transition $P_{ee} = \sin^2\theta < \frac{1}{2}$

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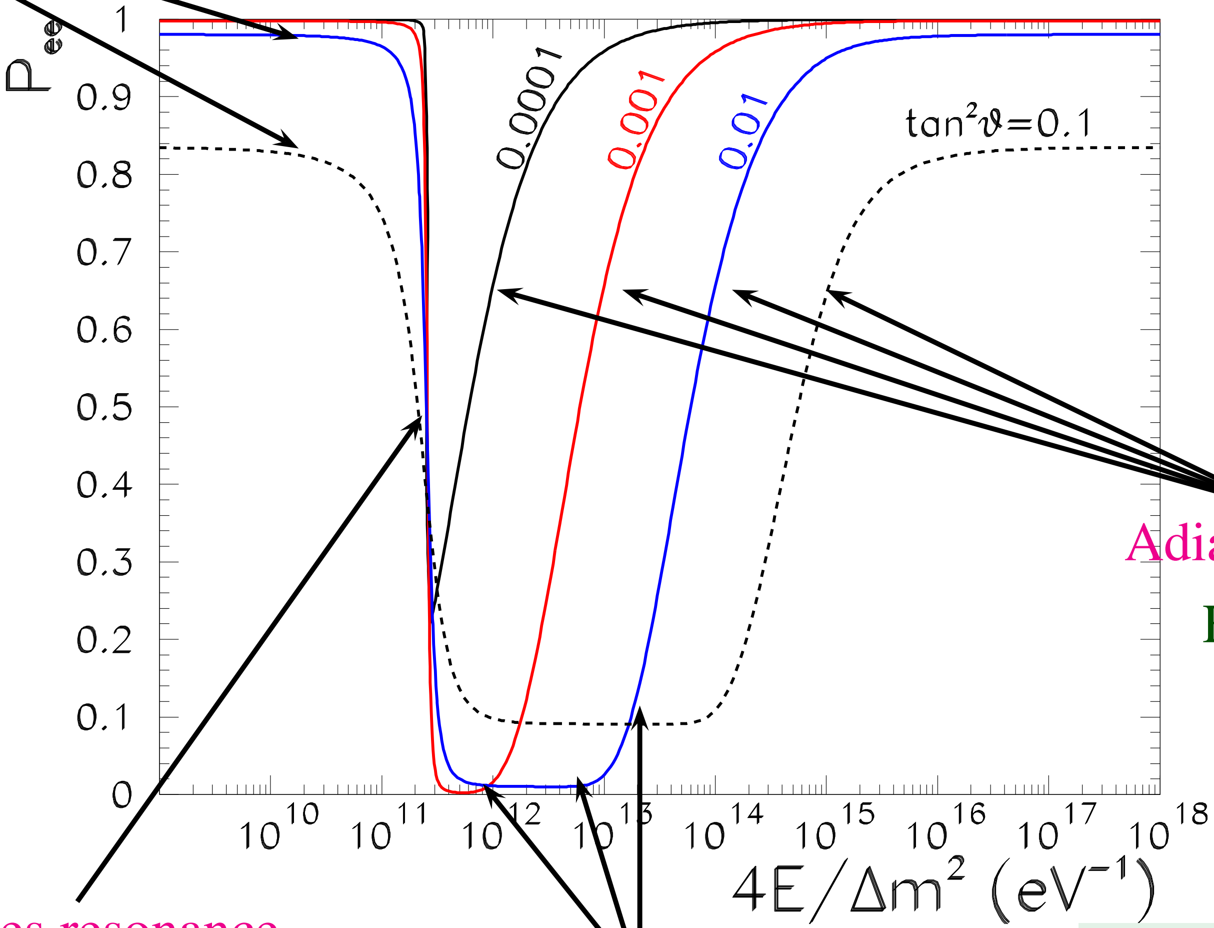


Adiabacity breaking

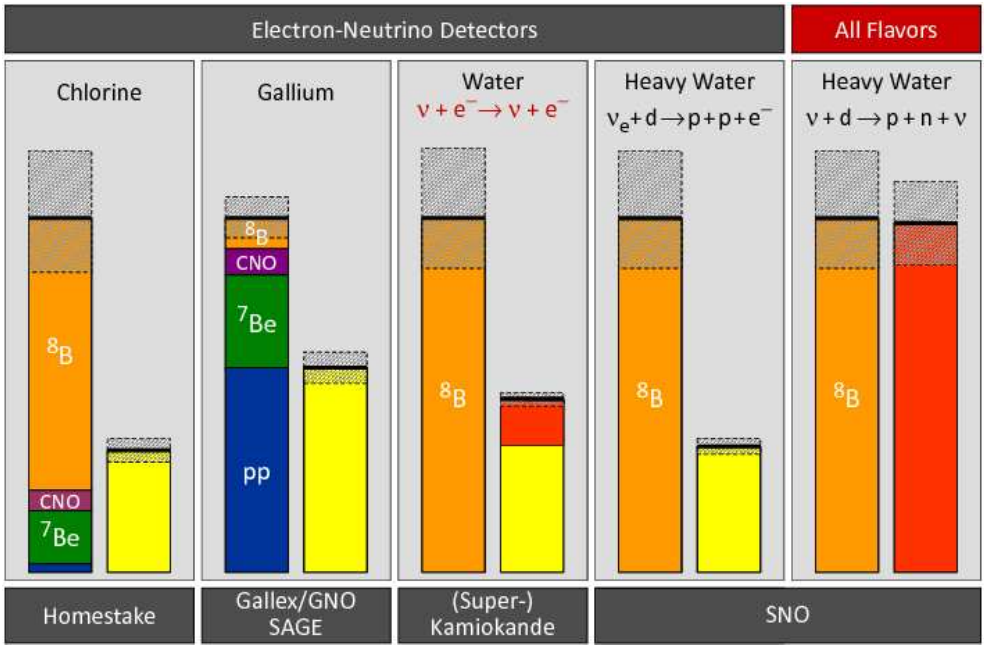
Effect of P_{LZ}

ν crosses resonance

MSW effect

Adiabatic MSW transition $P_{ee} = \sin^2\theta < \frac{1}{2}$

Solar Neutrinos: Results



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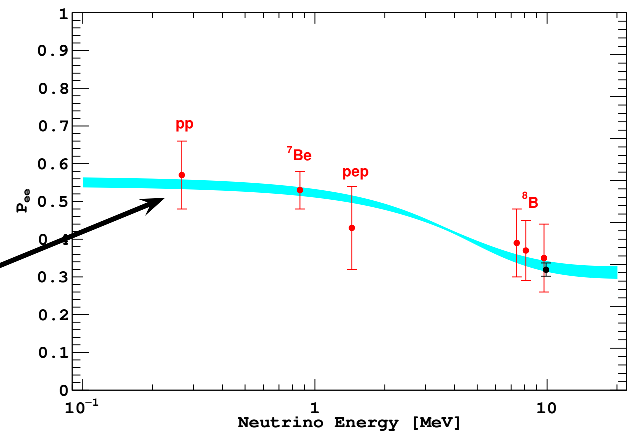
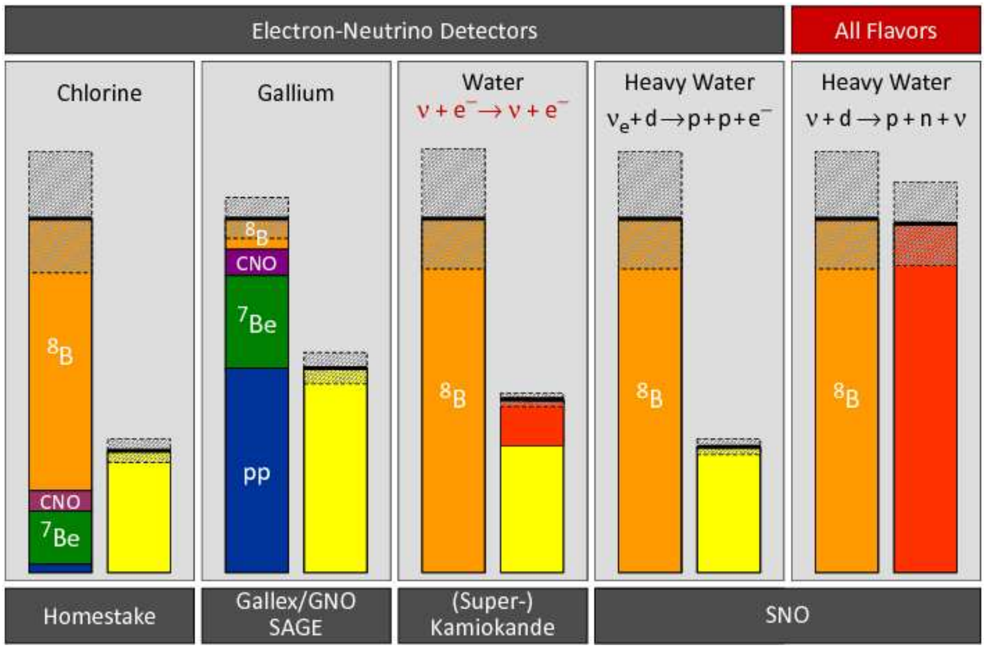
Best explained by MSW $\nu_e \to \nu_{\mu,\tau}$



P_{ee} for $\Delta m^2_{21} = (7.41^{+0.21}_{-0.20}) \times 10^{-5}$ eV^2 and $\theta_{12} = 33.41^{\circ}{}^{+0.78}_{-0.75}$

Solar Neutrinos: Results



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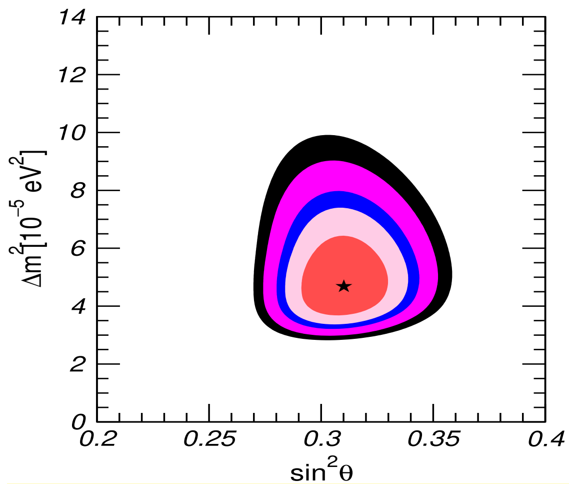

$\Delta m^2 \sim 5 \times 10^{-5}\ \text{eV}^2,\ \theta \sim \frac{\pi}{6}$

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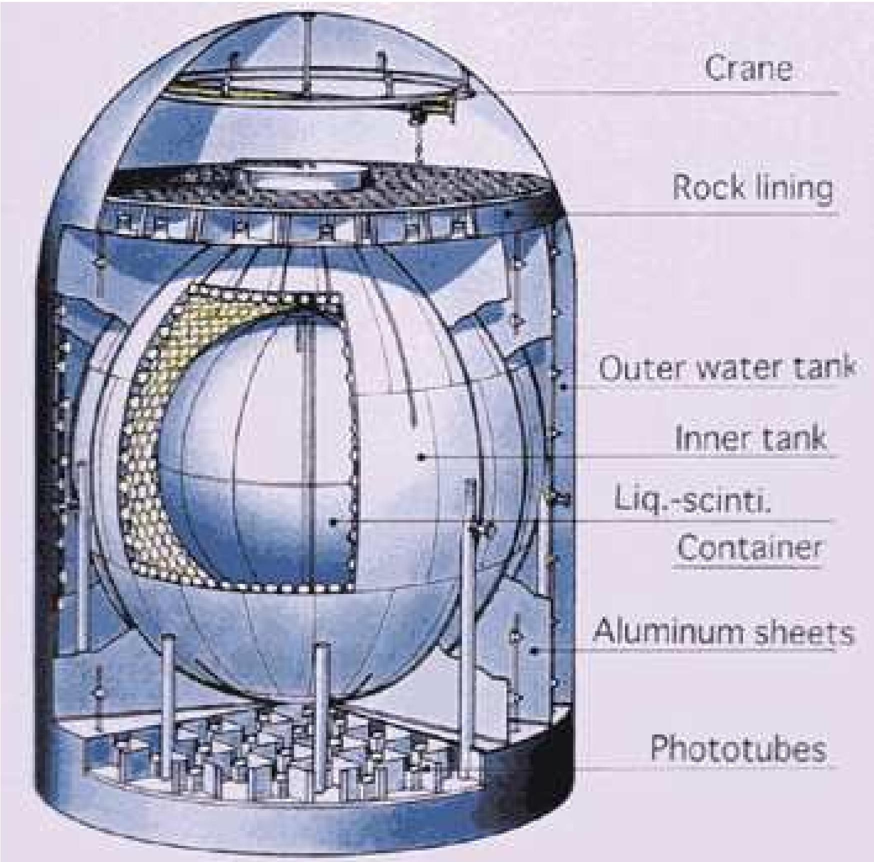
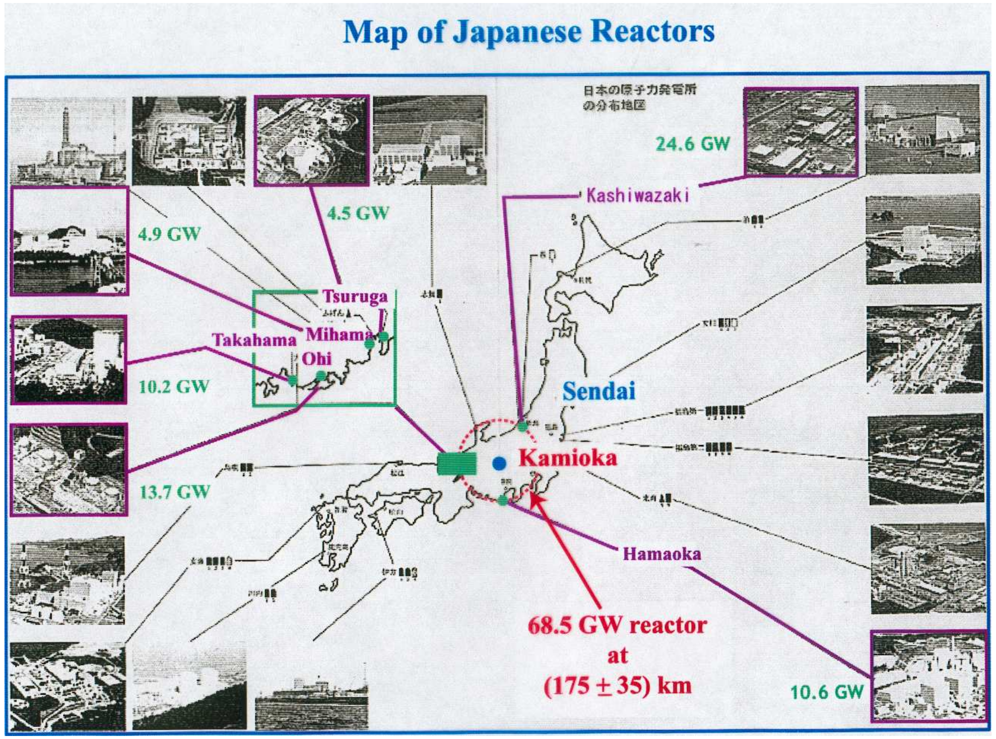
Terrestrial test: KamLAND

KamLAND: Detector of $\bar{\nu}_e$ produced in nuclear reactors in Japan at an average distance of 180 Km





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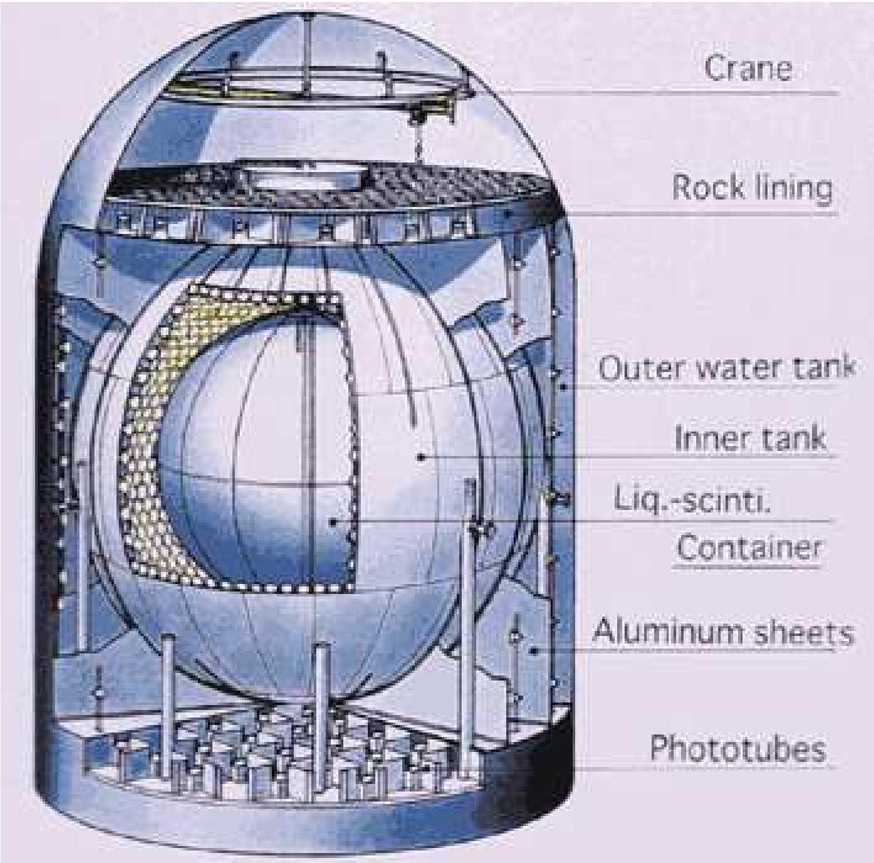
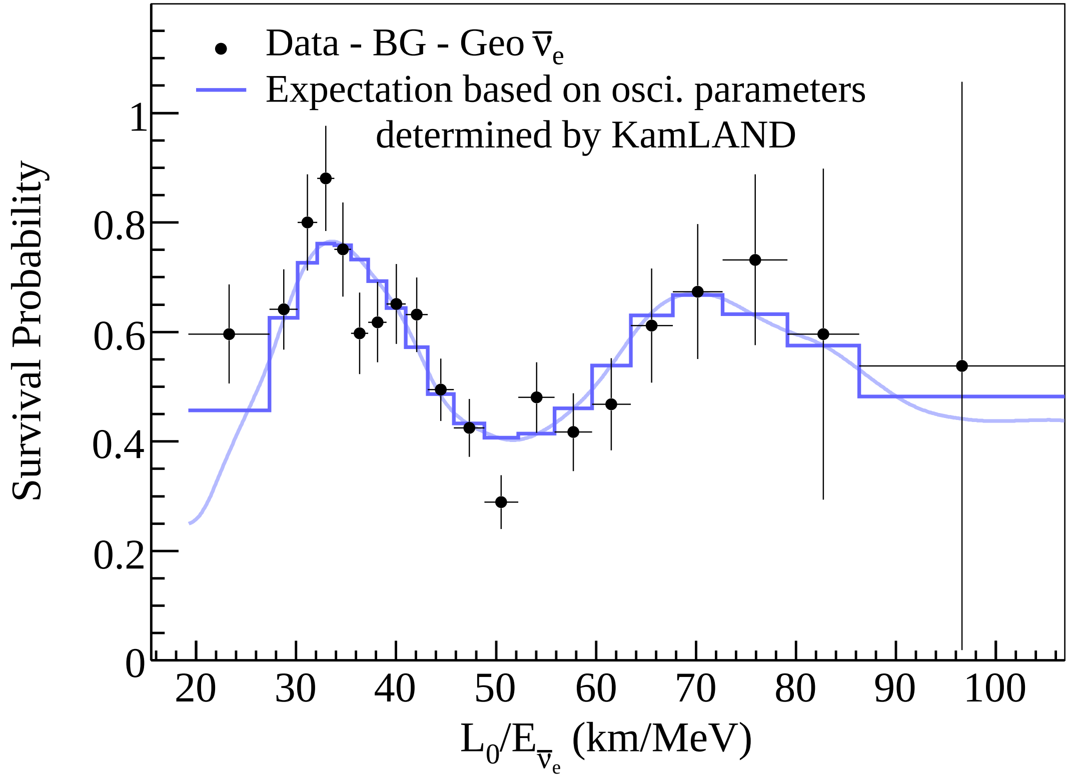


Results of KamLAND compared with P_{ee} for

$\theta = 35^\circ$ y $\Delta m^2 = 7.5 \times 10^{-5}$ (eV/c^2)^2



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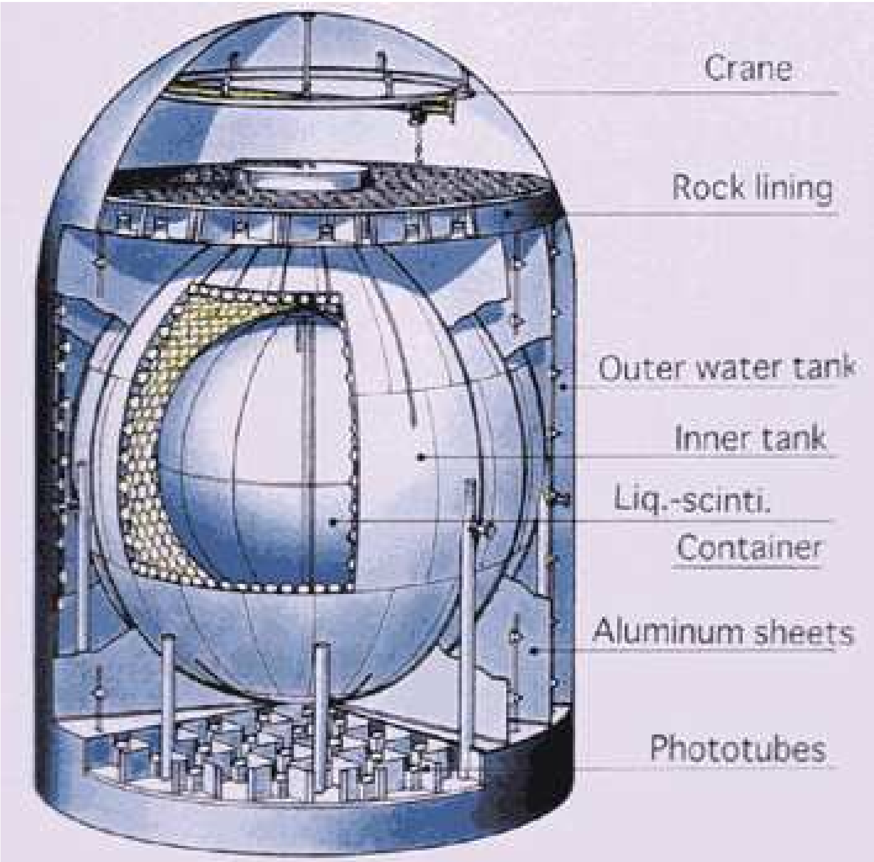
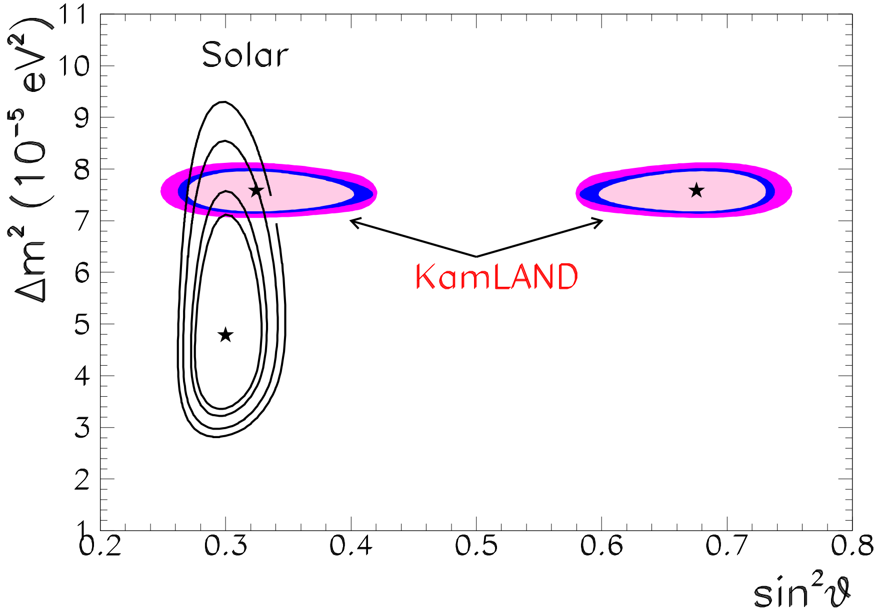
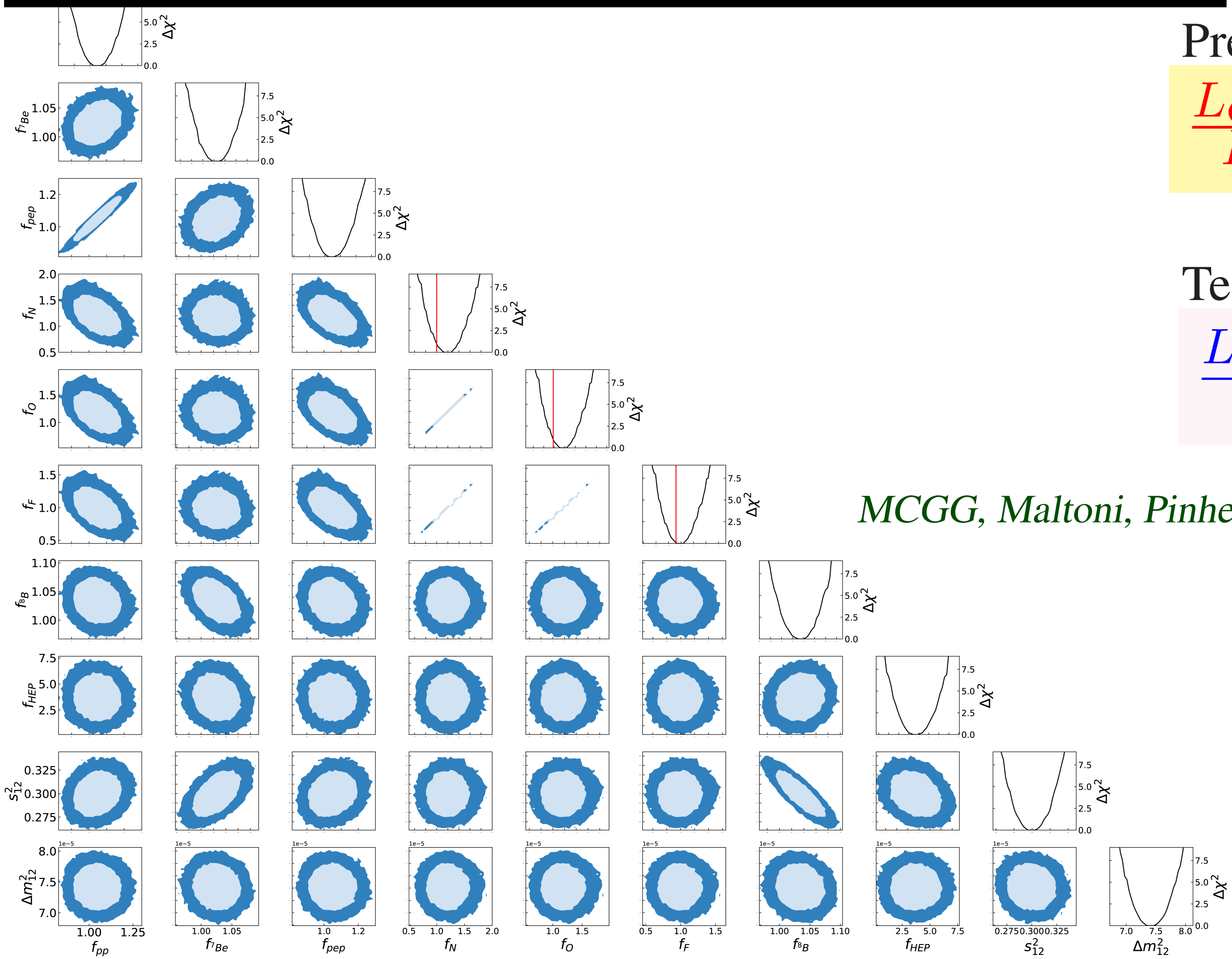




Byproduct: Testing How the Sun Shines with $\nu' s$

Fitting together Δm^2, θ and normalization of ν-producing reactions: $f_i = \frac{\Phi_i}{\Phi_i^{SSM}}$

$\Rightarrow$ Constraint on solar energy produced by nuclear reactions



Present limit on CNO:

$$\frac{L_{\rm CNO}}{L_\odot} < (0.75 \pm 0.3)\,\%\ (3\sigma)$$

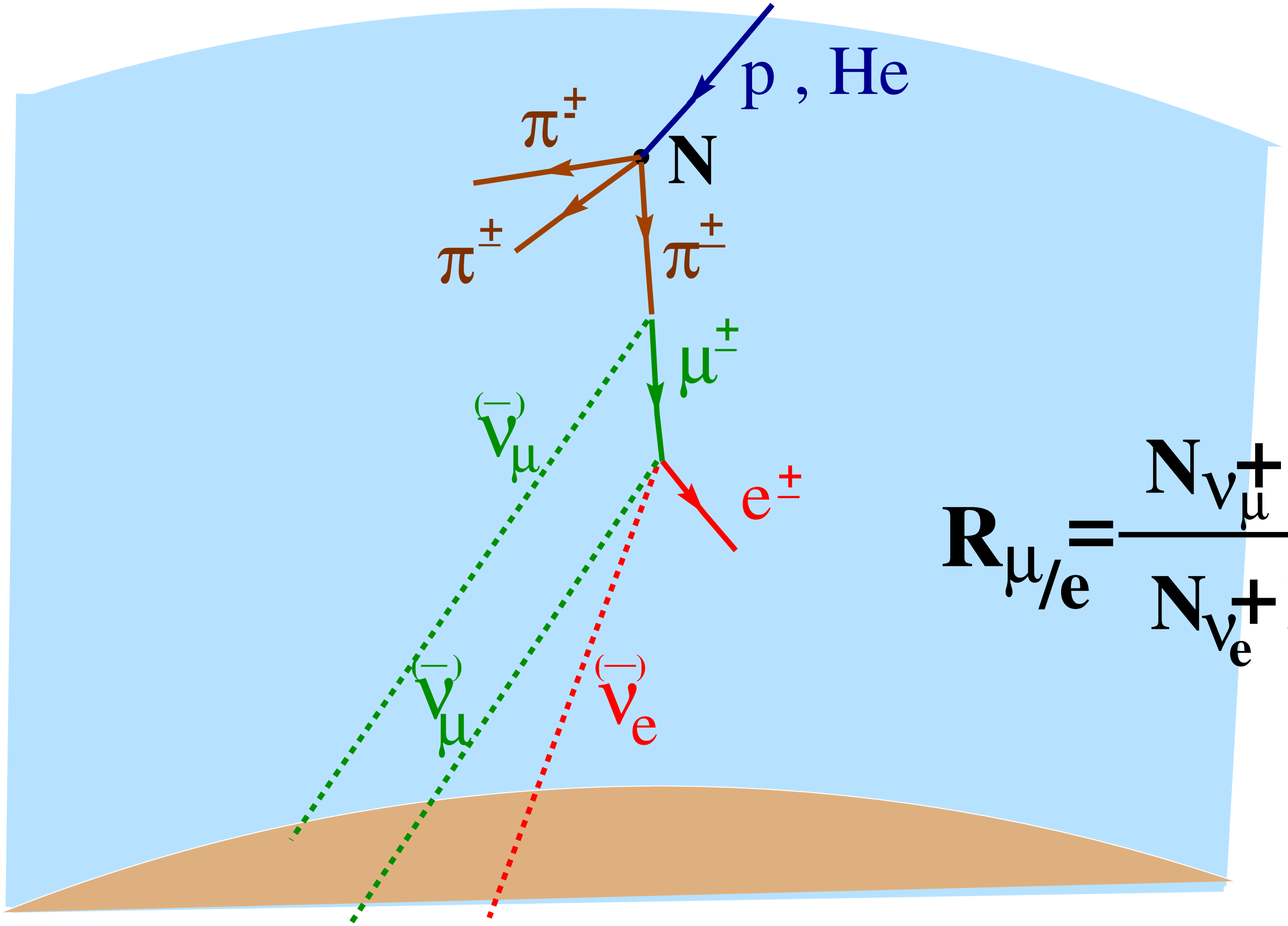
Test of Lum Constraint:

$$\frac{L_\odot(\nu - \text{inferred})}{L_\odot} = 1.04 \pm 0.06$$

MCGG, Maltoni, Pinheiro, Serenelli 2311.16226

Atmospheric Neutrinos

Atmospheric $\nu_{e,\mu}$ are produced by the interaction of cosmic rays (p, He ...) with the atmosphere

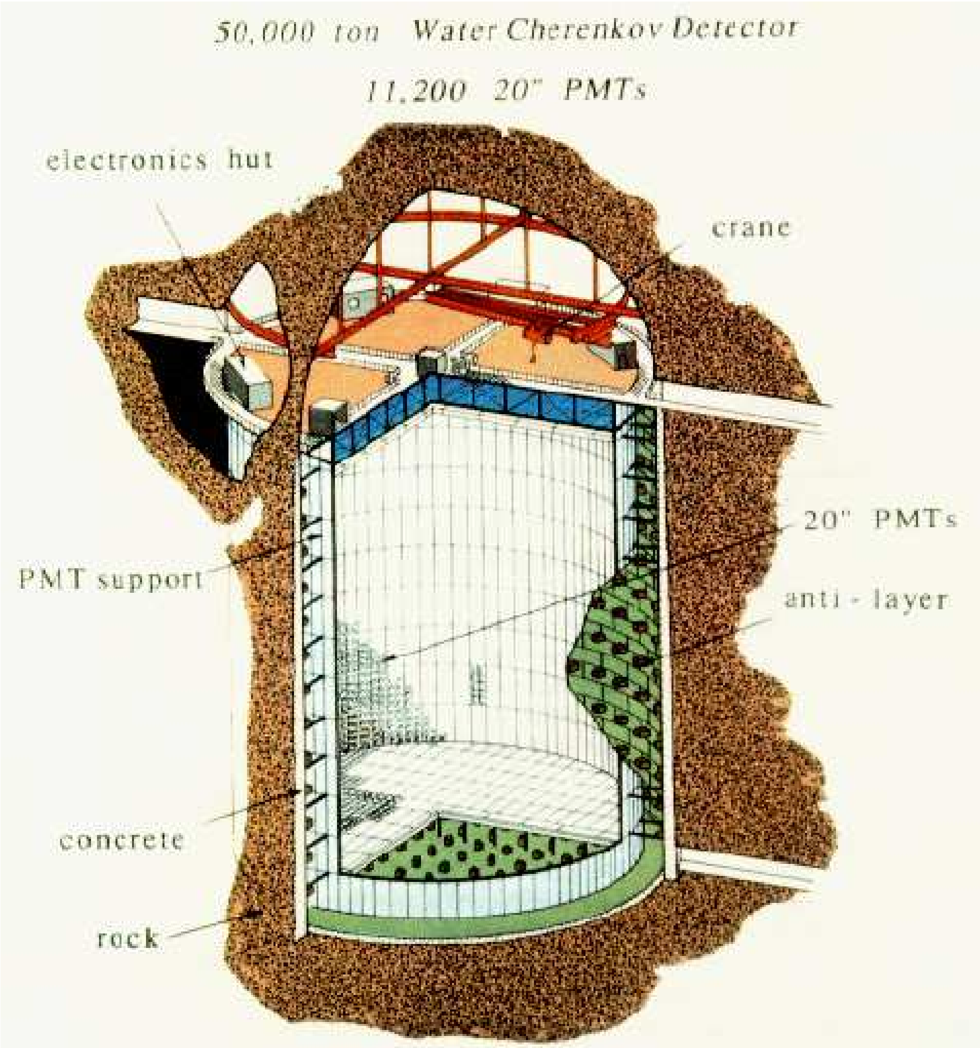


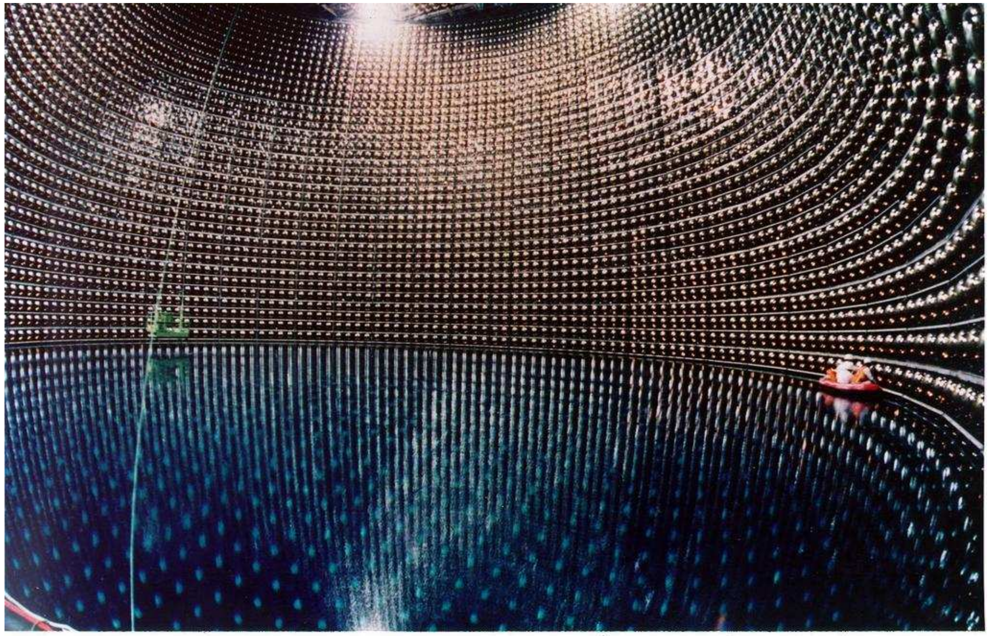
$$R_{\mu/e} = \frac{N_{\nu_\mu} + N_{\bar{\nu}_\mu}}{N_{\nu_e} + N_{\bar{\nu}_e}} \sim 2$$

Detection of Atmpospheric Neutrinos: SuperKamiokande

Located in the Kamiokande mine in the center of Japan at $\sim$ 1Km deep

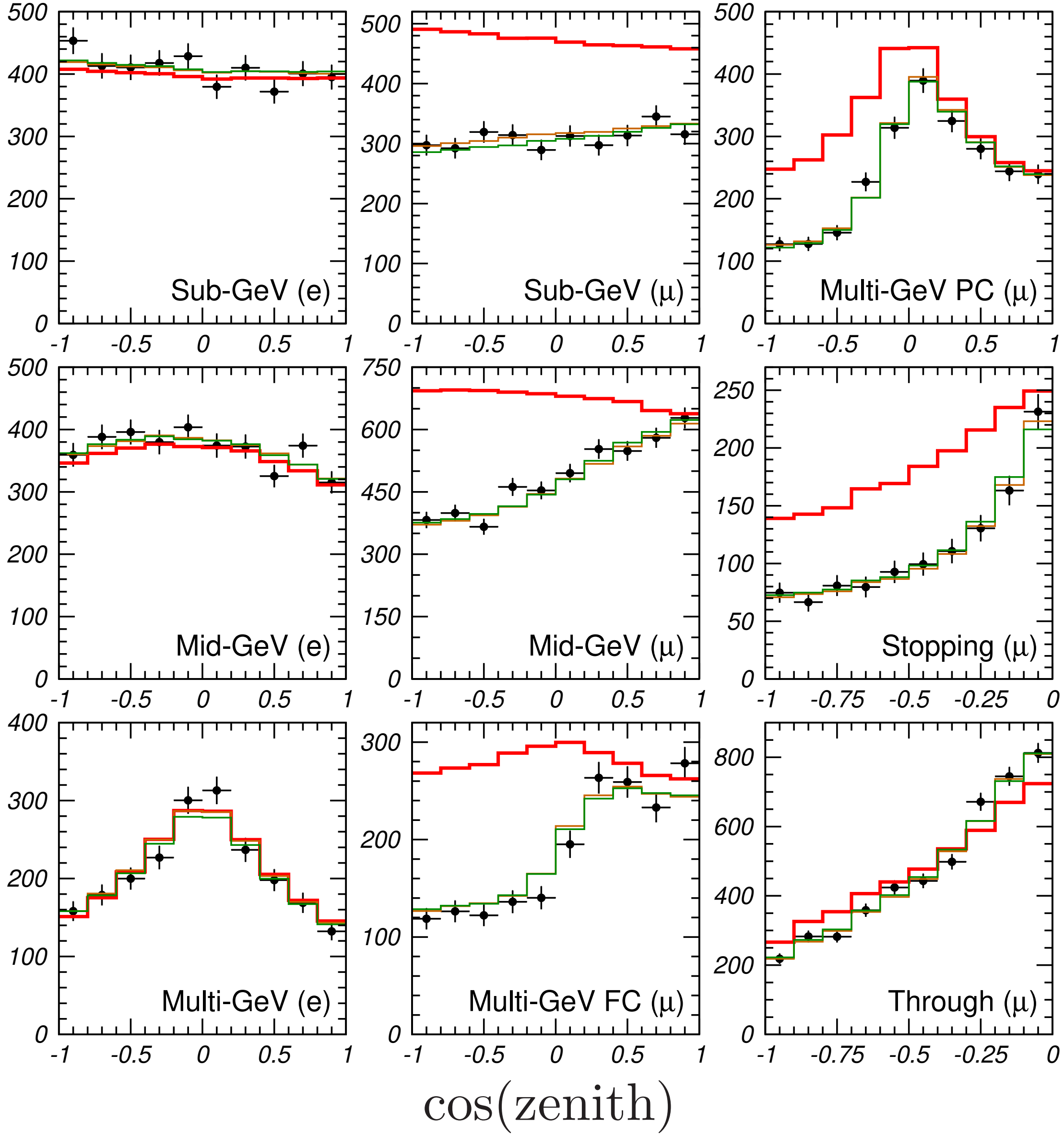
50 Kton of water surounded by $\sim$ 12000 photomultipliers

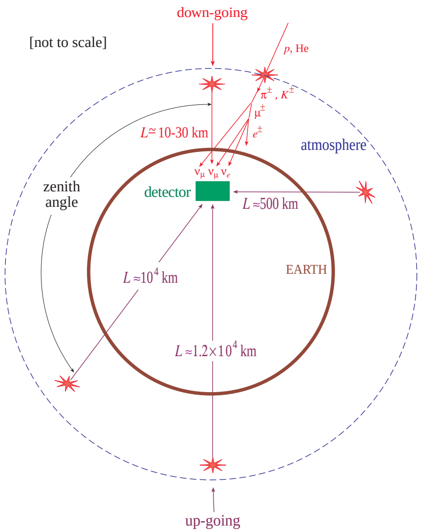


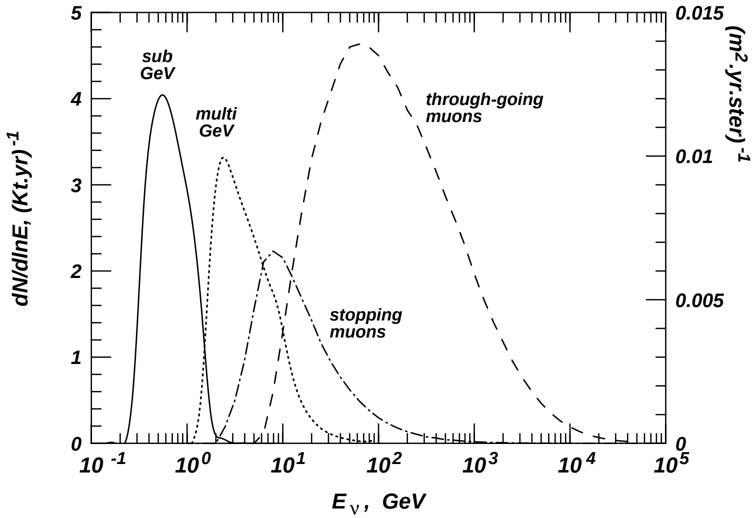


Atmospheric Neutrinos: Results

- SKI+II+III+IV data:







Atmospheric Neutrinos: Results

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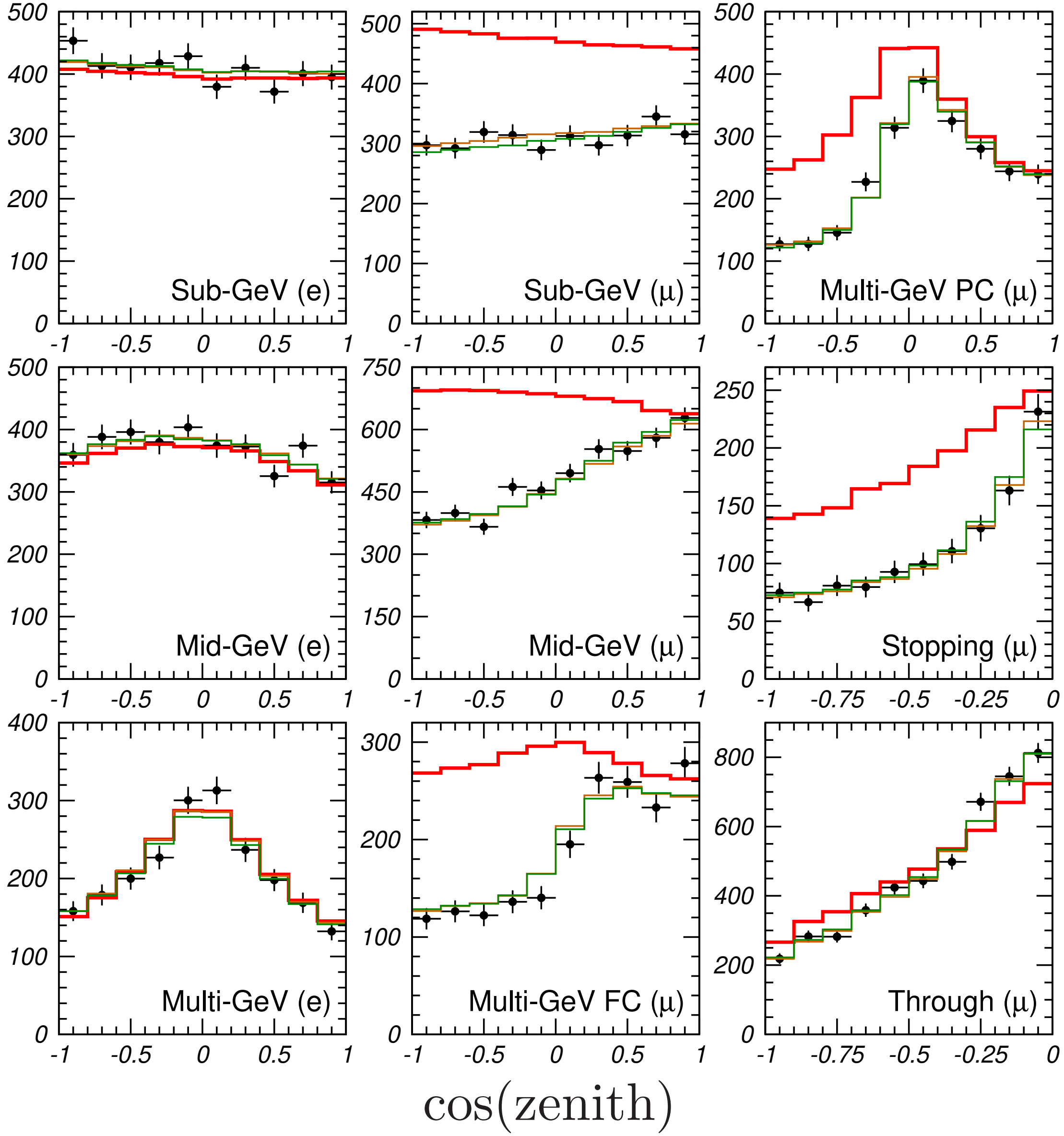
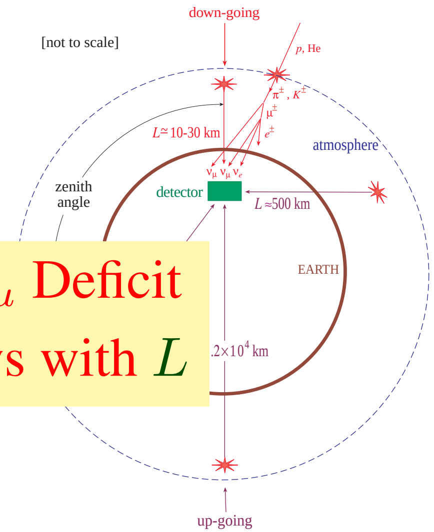
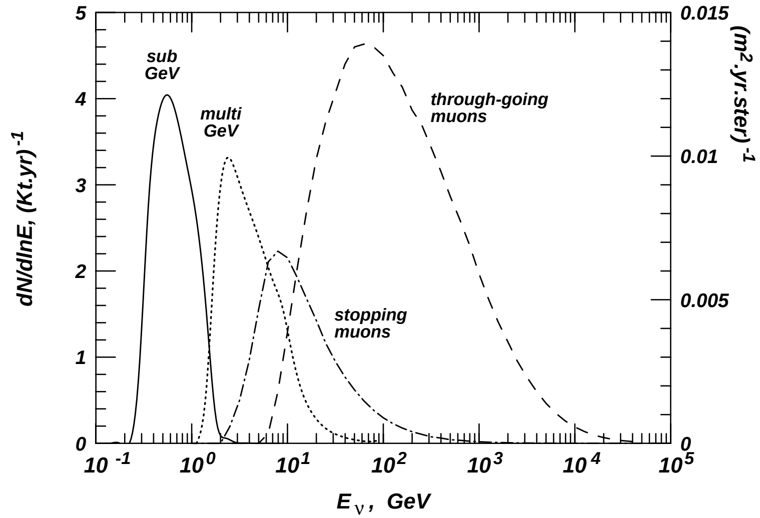
ν_e in agreement with SM





$\rightarrow \nu_\mu$ Deficit grows with L



$\rightarrow \nu_\mu$ Deficit decreases with E

Atmospheric Neutrinos: Results

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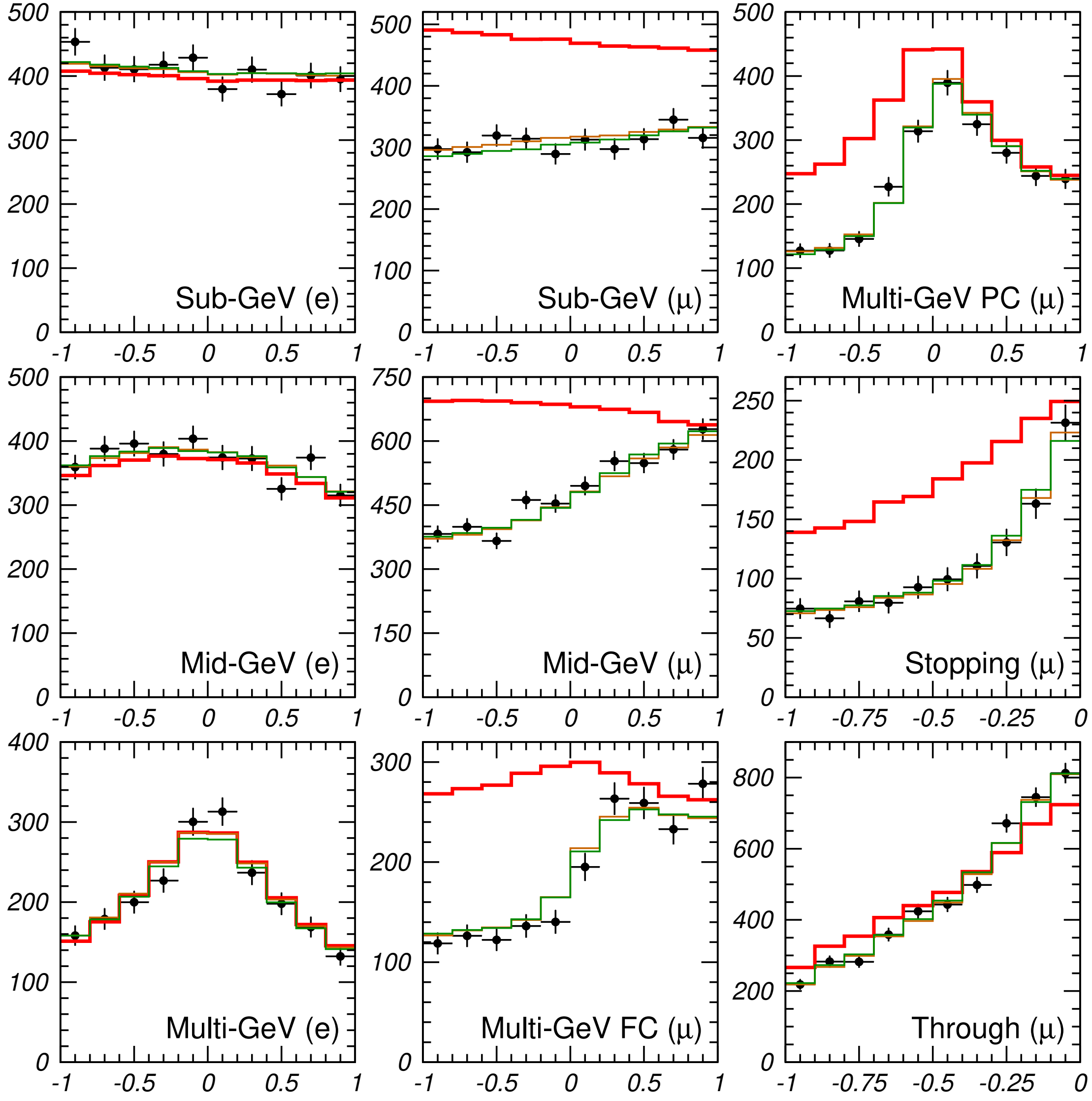
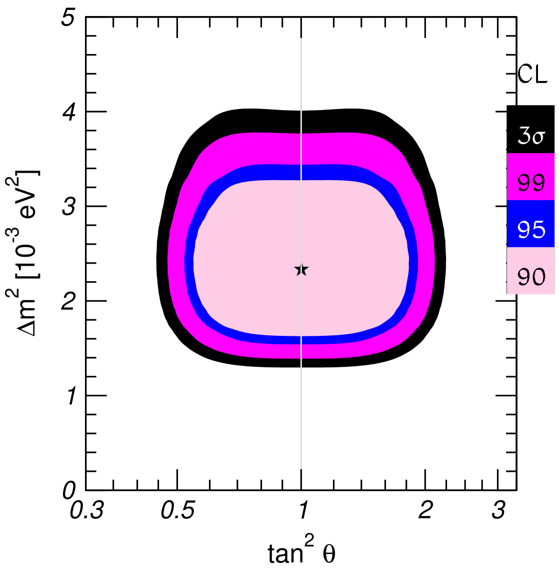


cos(zenith)

Best explained by $\nu_\mu \to \nu_\tau$



$$\Delta m^2 \sim 2.4 \times 10^{-3}\ \text{eV}^2$$
$$\tan^2\theta \sim 1 \;\Rightarrow\; \theta \sim \frac{\pi}{4}$$

Alternative Oscillation Mechanisms

- Oscillations are due to:
  - Misalignment between CC-int and propagation states: Mixing $\Rightarrow$ Amplitude
  - Difference phases of propagation states $\Rightarrow$ Wavelength. For Δm^2-OSC $\lambda = \dfrac{4\pi E}{\Delta m^2}$

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- ν masses are not the only mechanism for oscillations

Violation of Equivalence Principle (VEP): Gasperini 88, Halprin,Leung 01

Non universal coupling of neutrinos $\gamma_1 \neq \gamma_2$ *to gravitational potential* ϕ

$$\lambda = \frac{\pi}{E|\phi|\delta\gamma}$$

Violation of Lorentz Invariance (VLI): Coleman, Glashow 97

Non universal asymptotic velocity of neutrinos $c_1 \neq c_2 \Rightarrow E_i = \frac{m_i^2}{2p} + c_i p$

$$\lambda = \frac{2\pi}{E\Delta c}$$

Interactions with space-time torsion: Sabbata, Gasperini 81

Non universal couplings of neutrinos $k_1 \neq k_2$ *to torsion strength* Q

$$\lambda = \frac{2\pi}{Q\Delta k}$$

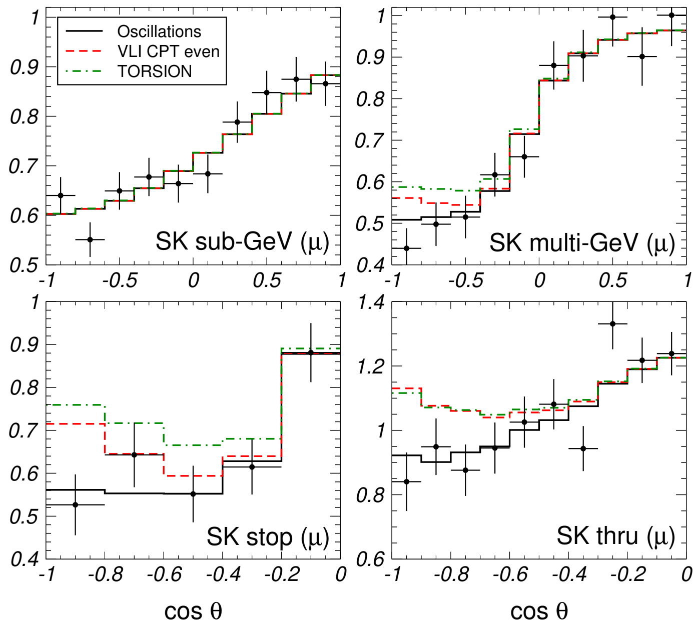
Violation of Lorentz Invariance (VLI) Colladay, Kostelecky 97; Coleman, Glashow 99

due to CPT violating terms: $\bar{\nu}_L^\alpha b_\mu^{\alpha\beta}\gamma_\mu \nu_L^\beta \Rightarrow E_i = \frac{m_i^2}{2p} \pm b_i$

$$\lambda = \pm\frac{2\pi}{\Delta b}$$

Alternative Mechanisms vs ATM ν's

- Strongly constrained with ATM data (MCG-G, M. Maltoni PRD 04,07)



At 90% CL:

$$\frac{|\Delta c|}{c} \leq 1.2 \times 10^{-24}$$

$$|\phi\,\Delta\gamma| \leq 5.9 \times 10^{-25}$$

$$|Q\,\Delta k| \leq 4.8 \times 10^{-23}\ \text{GeV}$$

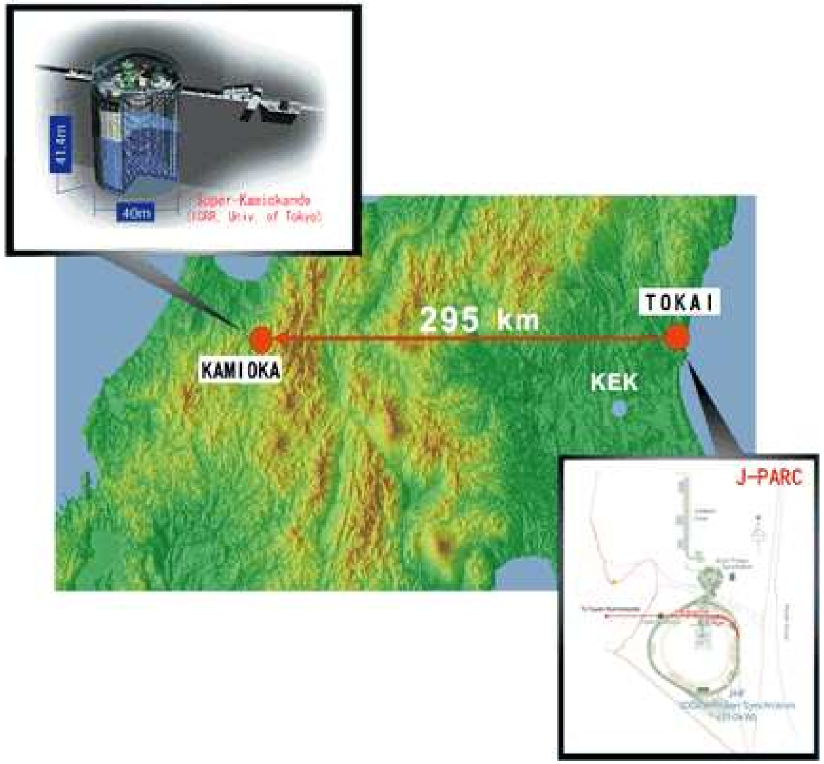
$$|\Delta b| \leq 3.0 \times 10^{-23}\ \text{GeV}$$

Long Baseline Accelerator ν Experiments

T2K:

ν_μ produced in Tokai (Japan)

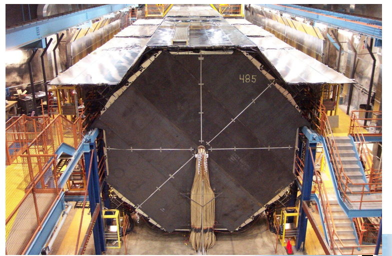
detected in SK at $\sim$ 250 Km



MINOS, NOνA

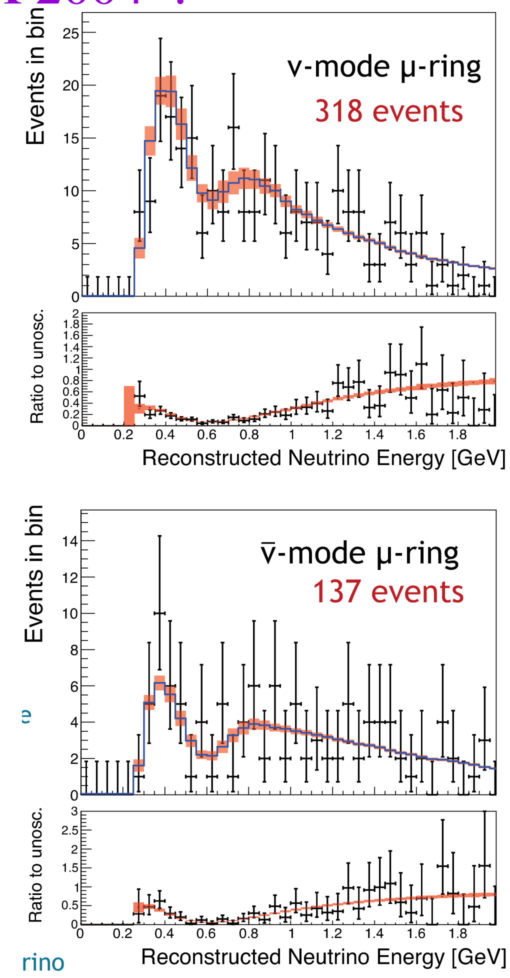
ν_μ produced en Fermilab (Illinois)

detected in Minnesota at $\sim$ 800 Km

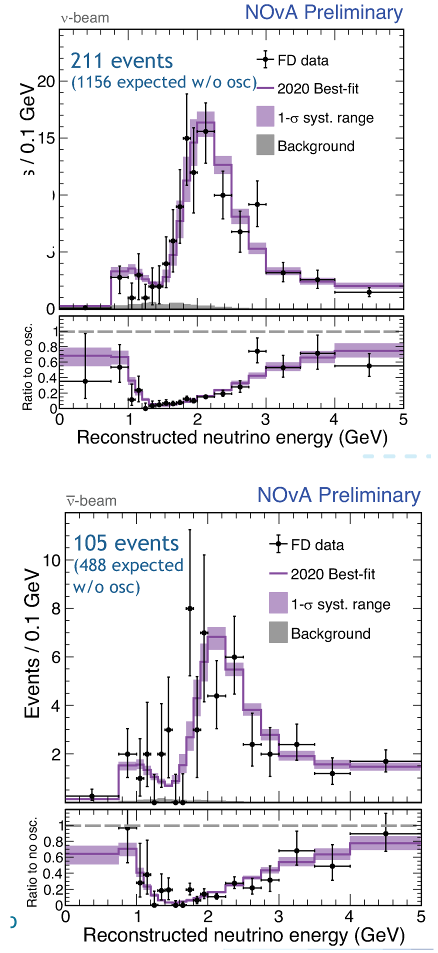


Long Baseline Experiments: ν_μ Disappearance

K2K/T2K 2004–:



NOνA: 2015–



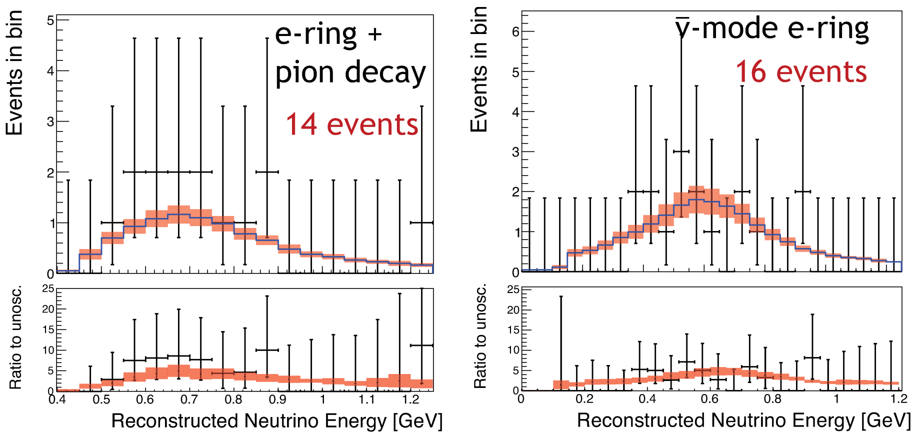
ν_μ oscillations with $\Delta m^2 \sim 2.5 \times 10^{-3}$ eV^2 and mixing compatible with $\frac{\pi}{4}$

Long Baseline Experiments: ν_e Appearance

- Observation of $\nu_\mu \to \nu_e$ transitions with $E/L \sim 10^{-3}$ eV^2

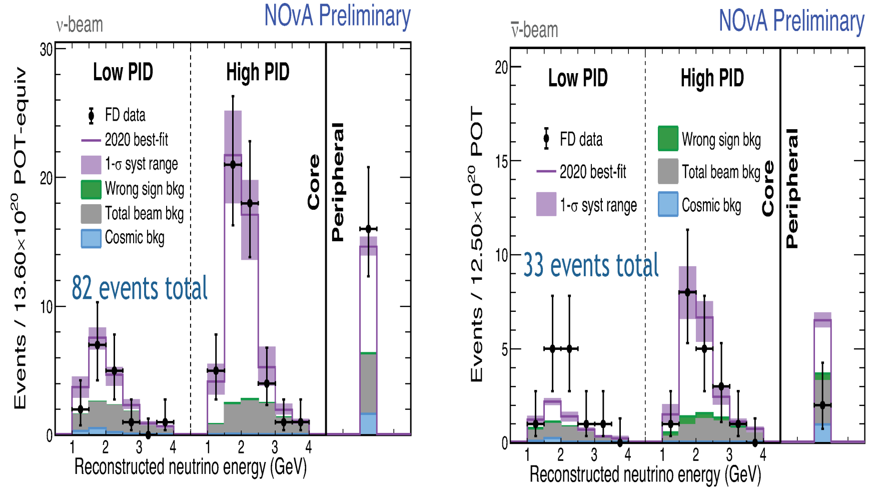
T2K

$\nu_\mu \to \nu_e$ $\quad$ $\bar\nu_\mu \to \bar\nu_e$



NOνA

$\nu_\mu \to \nu_e$ $\quad$ $\bar\nu_\mu \to \bar\nu_e$

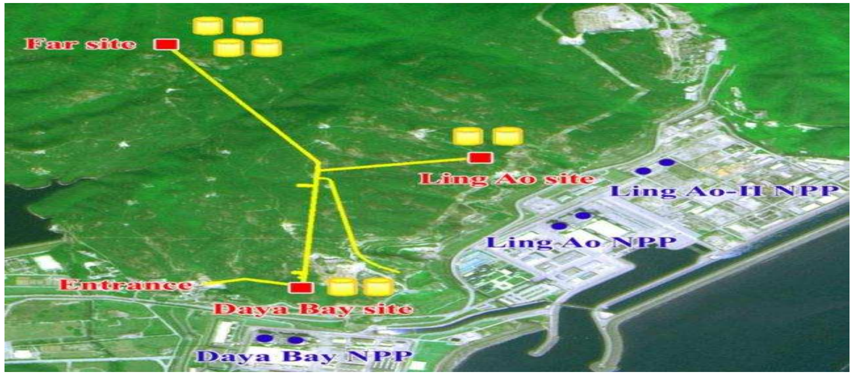


- Test of $P(\nu_\mu \to \nu_e)$ vs $P(\bar\nu_\mu \to \bar\nu_e) \Rightarrow$ Leptonic CP violation

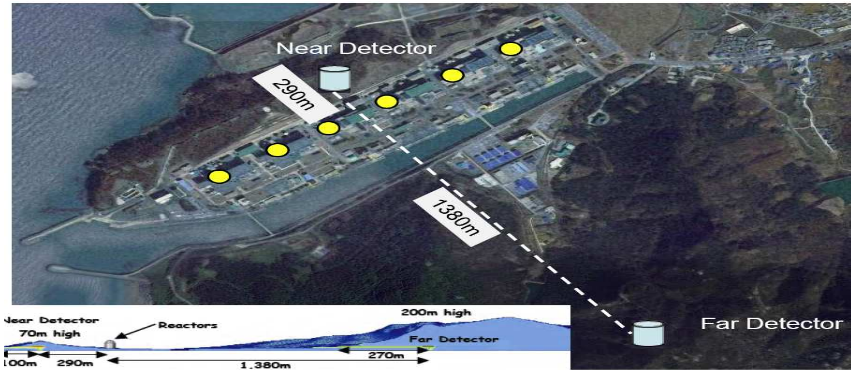
Medium Baseline Reactor Experiments

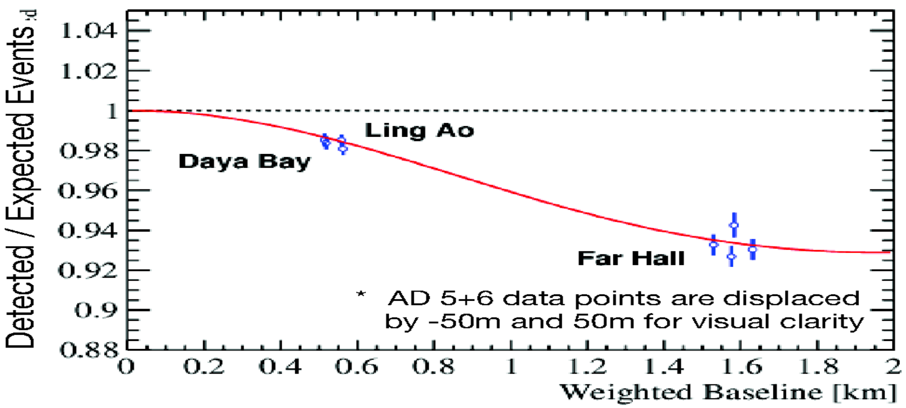
- Searches for $\bar{\nu}_e \to \bar{\nu}_e$ disapperance at $L \sim$ Km $(E/L \sim 10^{-3}\ \text{eV}^2)$
- Relative measurement: near and far detectors

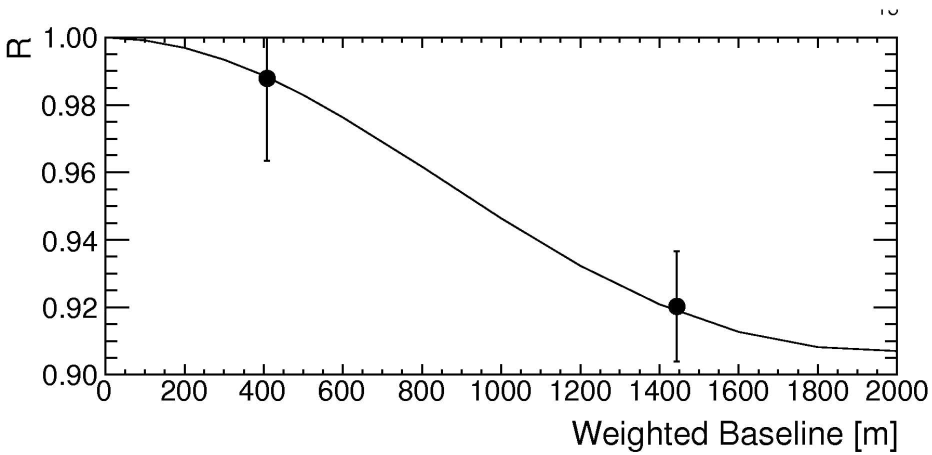
Daya-Bay



Reno







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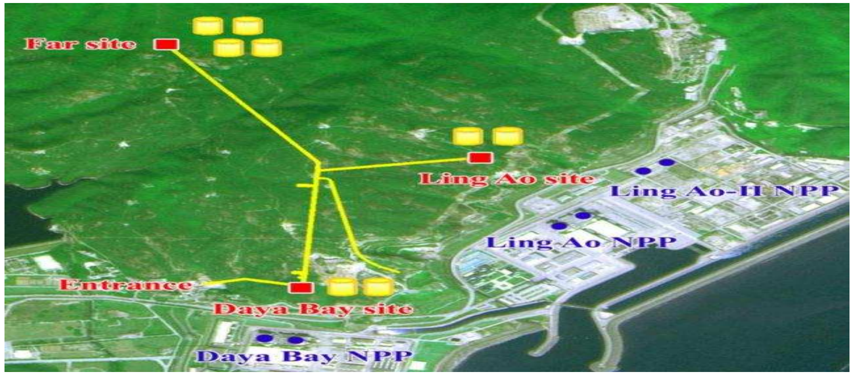
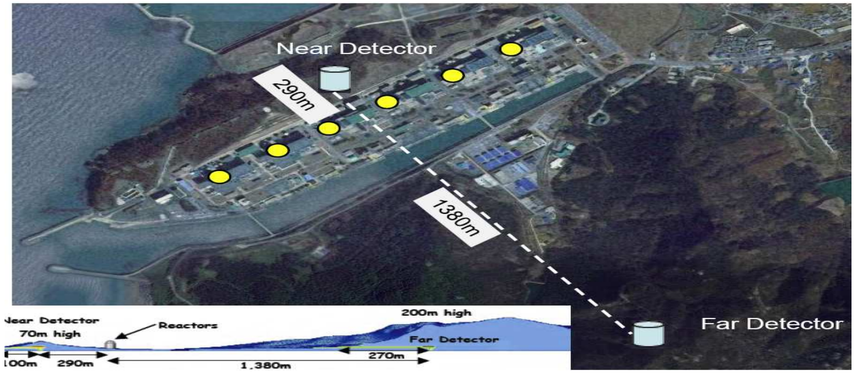
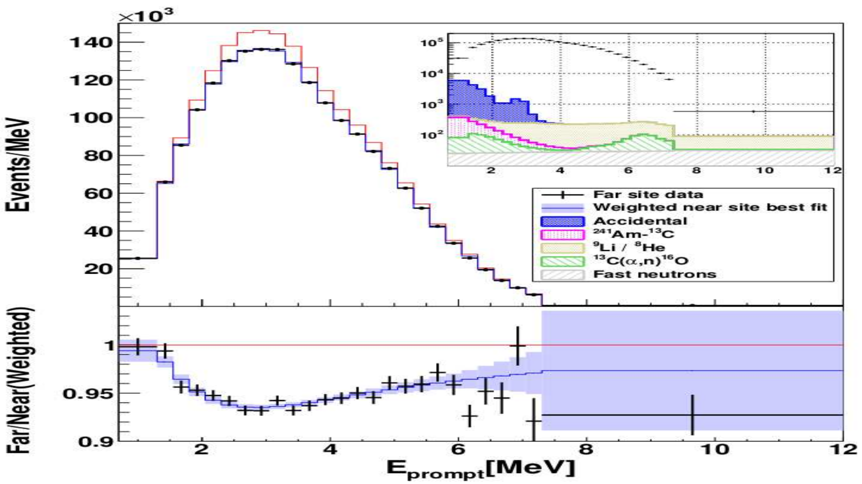
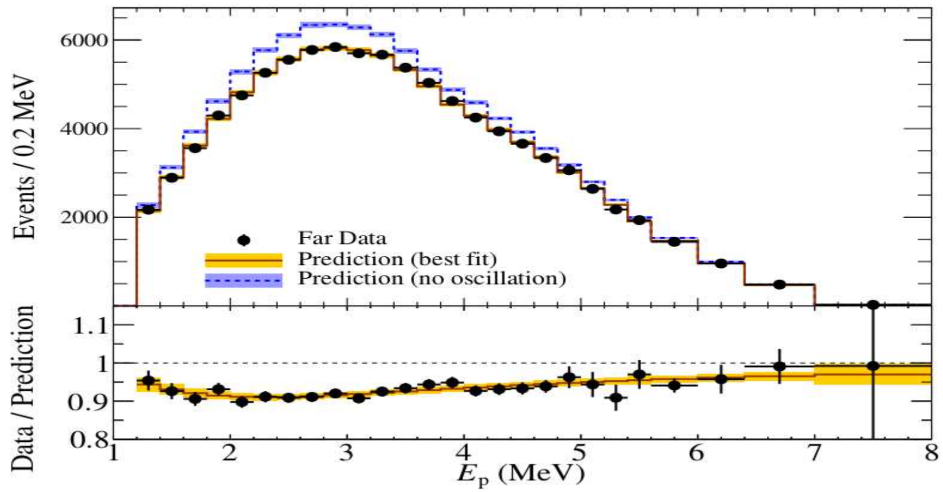


Reno







Medium Baseline Reactor Experiments

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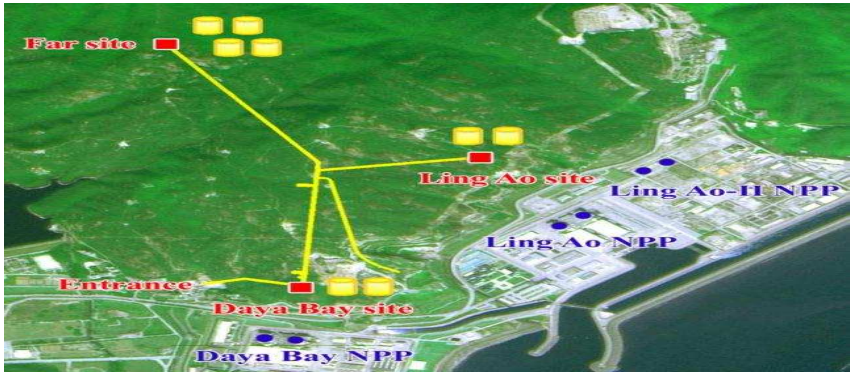
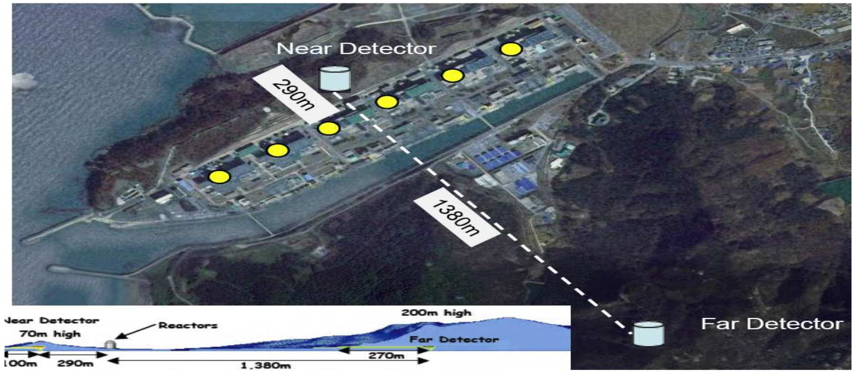
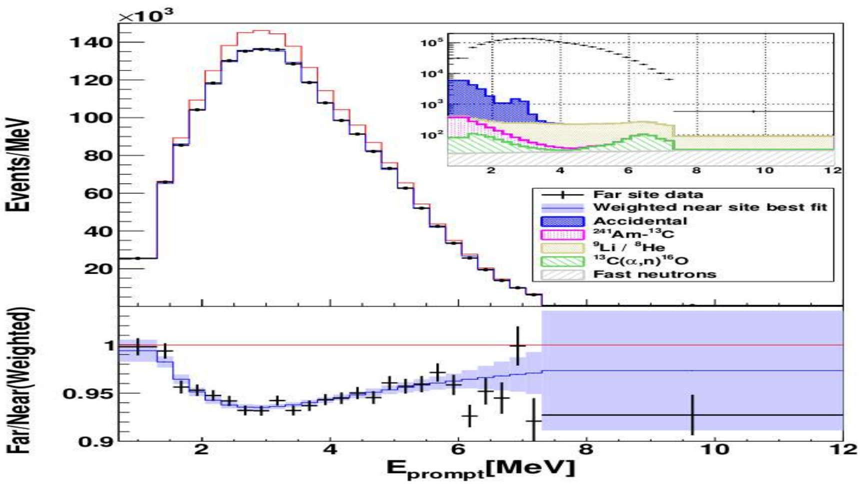
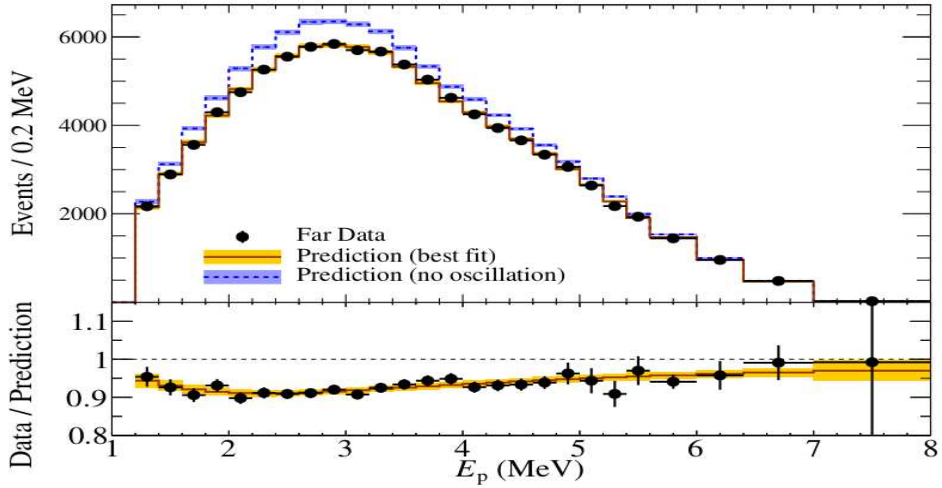


Reno



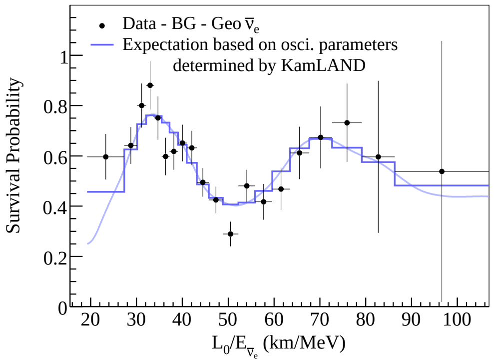




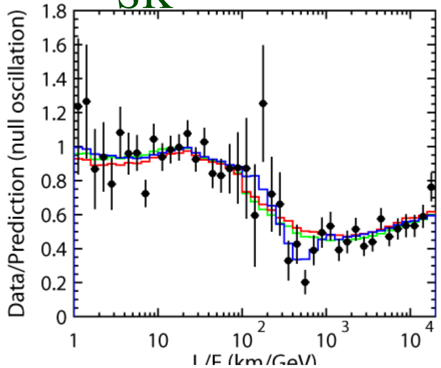
Described with $\Delta m^2 \sim 2.5 \times 10^{-3}\text{eV}^2$ (as ν_μ ATM and LBL acc but for ν_e) and $\theta \sim 9^\circ$

- We have observed with high (or good) precision:
  - $*$ Atmospheric ν_μ & $\bar\nu_\mu$ disappear most likely to ν_τ (**SK**,MINOS, **ICECUBE**) $\frac{\Delta m^2}{\text{eV}^2} \sim 2\,10^{-3}$
  - $*$ Accel. ν_μ & $\bar\nu_\mu$ disappear at $L \sim 300/800$ Km (K2K, **T2K**, MINOS, **NOνA**) $\theta \sim 45°$
  - $*$ Some accel ν_μ & $\bar\nu_\mu$ appear as ν_e & $\bar\nu_e$ at $L \sim 300/800$ Km (**T2K**, MINOS,**NOνA**) $\theta \sim 8°$
  - $*$ Solar ν_e convert to ν_μ/ν_τ (Cl, Ga, **SK, SNO, Borexino**) $\frac{\Delta m^2}{\text{eV}^2} \sim 10^{-5}, \theta \sim 30°$
  - $*$ Reactor $\overline{\nu}_e$ disappear at $L \sim 200$ Km (**KamLAND**)
  - $*$ Reactor $\overline{\nu}_e$ disappear at $L \sim 1$ Km (D-Chooz, **Daya Bay**, Reno) $\frac{\Delta m^2}{\text{eV}^2} \sim 2\,10^{-3}, \theta \sim 8°$
- Confirmed:
  - – Vacuum oscillation L/E pattern with 2 frequencies

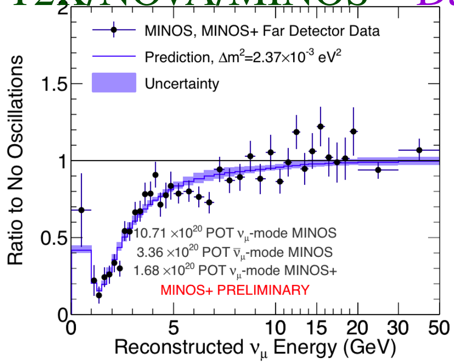
KamLAND



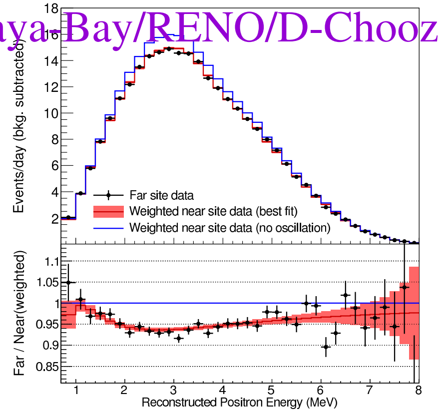
SK



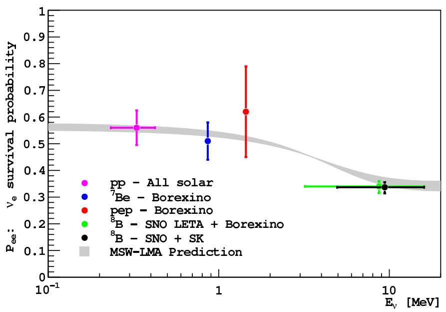
T2K/NOνA/MINOS



Daya Bay/RENO/D-Chooz



– MSW conversion in Sun



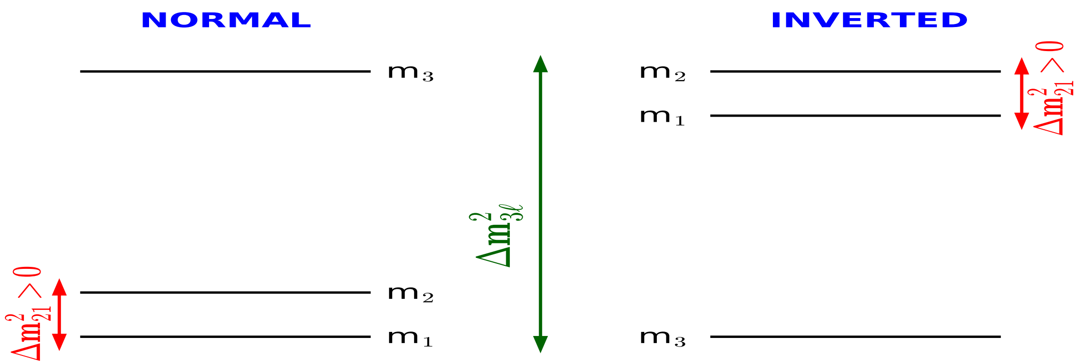
3ν Flavour Parameters

Concha Gonzalez-Garcia

- For **for 3 ν's** : 3 Mixing angles + 1 Dirac Phase + 2 Majorana Phases

$$U_{\rm LEP} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{i\delta_{\rm cp}} \\ 0 & 1 & 0 \\ -s_{13} e^{-i\delta_{\rm cp}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{21} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{i\eta_1} & 0 & 0 \\ 0 & e^{i\eta_2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- Convention: $0 \leq \theta_{ij} \leq 90^\circ \quad 0 \leq \delta \leq 360^\circ \quad \Rightarrow$ 2 Orderings



$\Delta m^2_{21} > 0$ by convention

$$\Delta m^2_{3\ell} = \begin{cases} \Delta m^2_{31} > 0 \text{ for NO} \\ \Delta m^2_{32} < 0 \text{ for IO} \end{cases}$$

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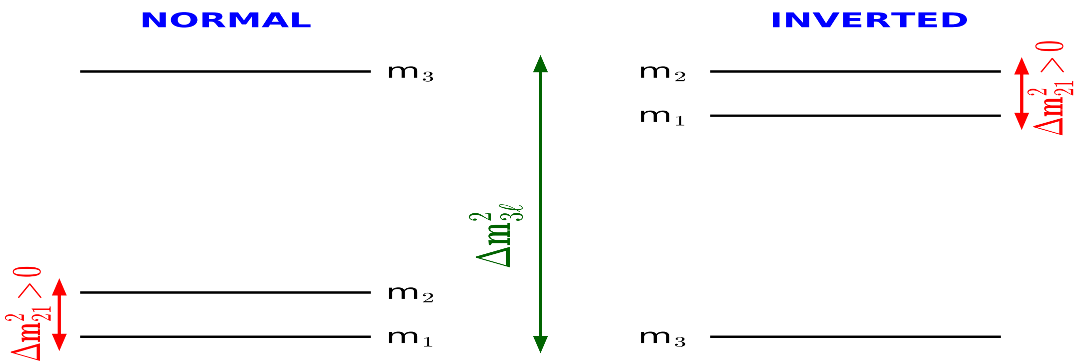

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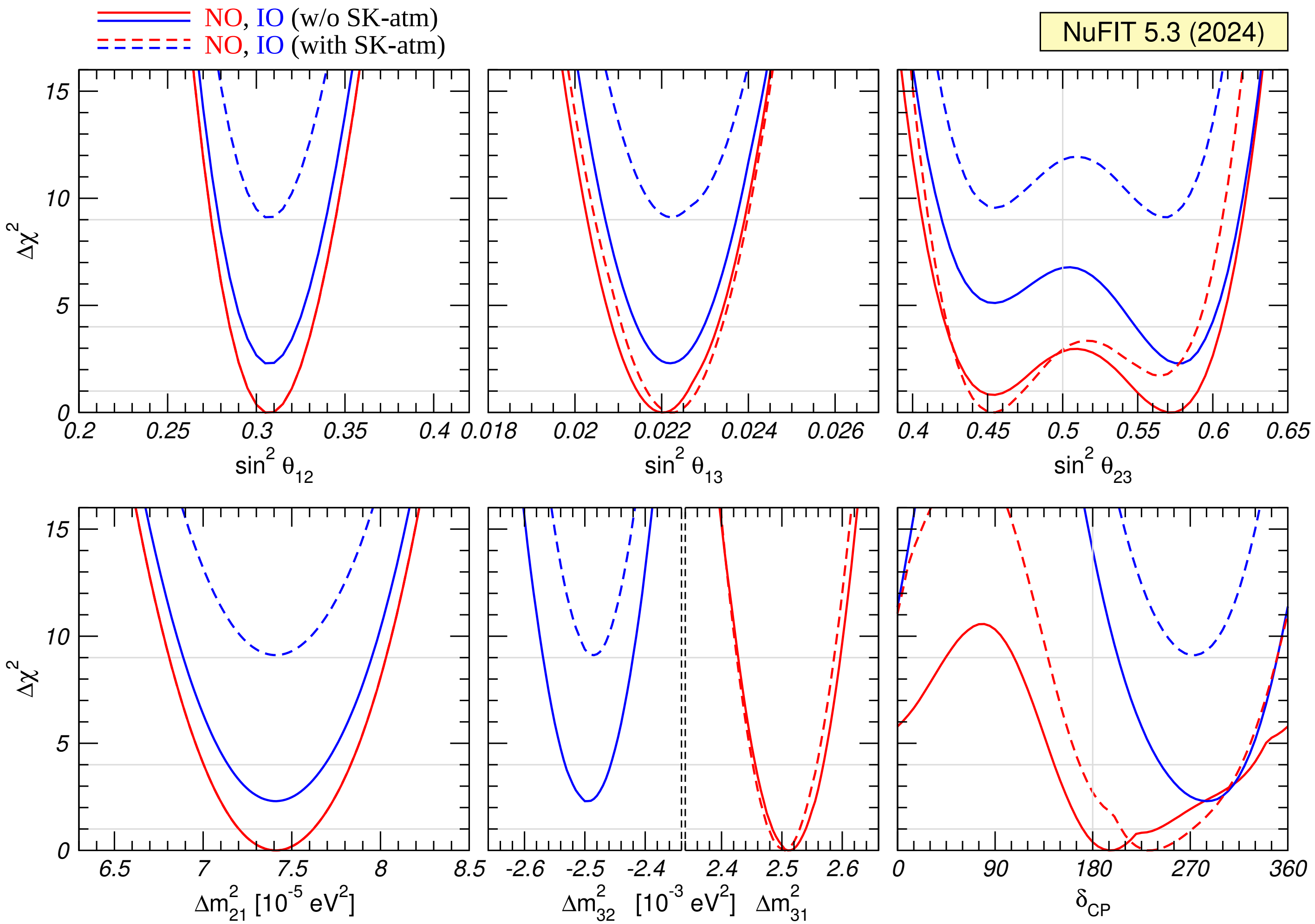
Experiment	Dominant	Important	Additional
Solar Experiments	θ_{12}	Δm^2_{21}	θ_{13}
Reactor LBL (KamLAND)	Δm^2_{21}	θ_{12}	θ_{13}
Reactor MBL (Daya Bay, Reno, D-Chooz)	θ_{13}, $\Delta m^2_{3\ell}$		
Atmospheric Experiments (SK,IC)	θ_{23}	$\Delta m^2_{3\ell}$	θ_{13} ,$\delta_{\rm cp}$
Acc LBL ν_μ Disapp (Minos,T2K,NOvA)	$\Delta m^2_{3\ell}$. θ_{23}		
Acc LBL ν_e App (Minos,T2K,NOvA)	$\delta_{\rm cp}$		θ_{13}

Summary: Global 3 ν Flavour Parameters

Global 6-parameter fit http://www.nu-fit.org

Esteban, Gonzalez-Garcia, Maltoni, Schwetz, Zhou, JHEP'20 [2007.14792]

(Good agreement with other groups': Capozzi,et al, 2107.00532; Salas etal 2006.11237)



SK-atm $\equiv \chi^2$ table from SK1-5

Summary: Global 3 ν Flavour Parameters

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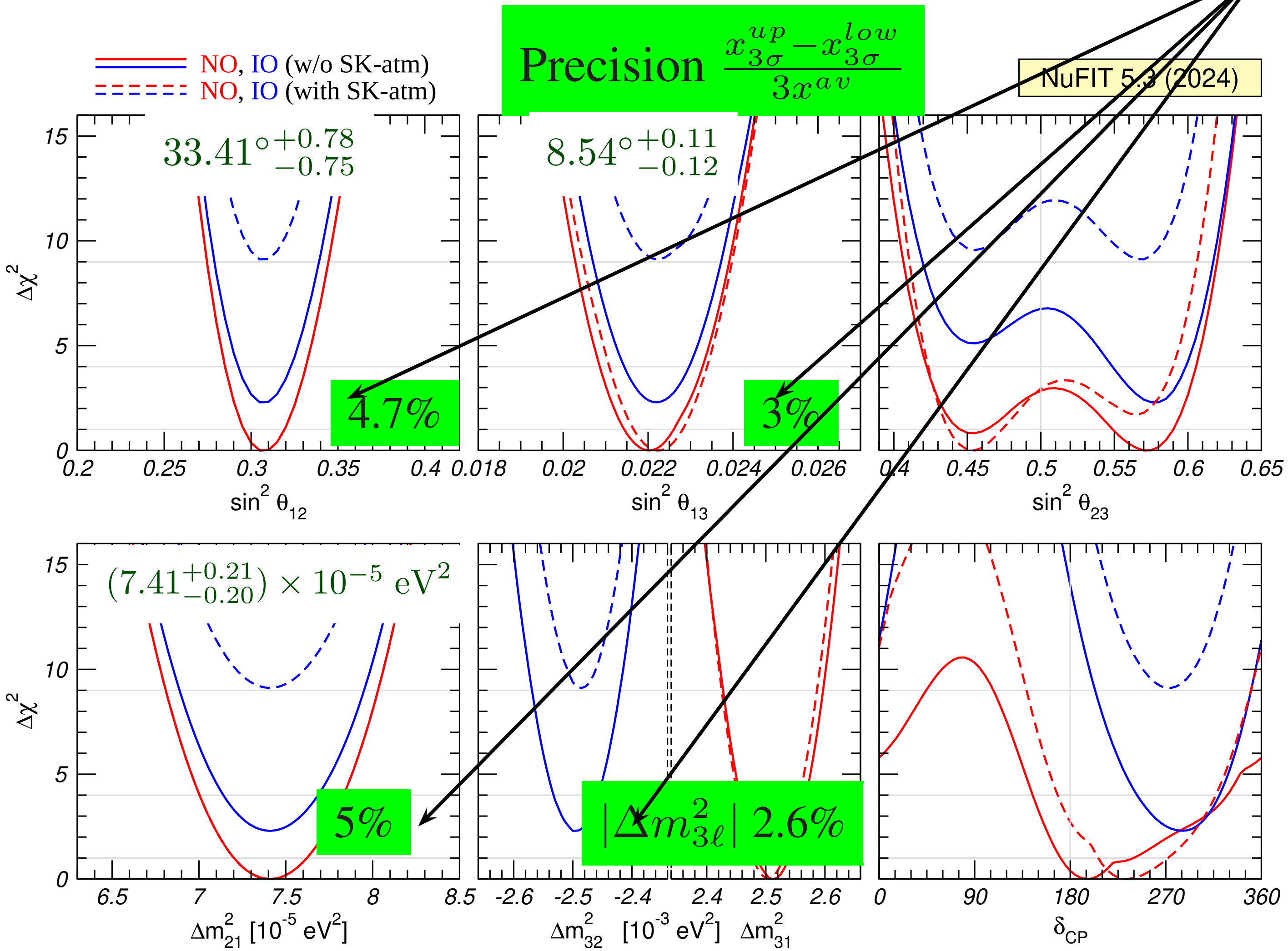
Esteban, Gonzalez-Garcia, Maltoni, Schwetz, Zhou, JHEP'20 [2007.14792]

- 4 well-known parameters: θ_{12}, θ_{13}, Δm^2_{21}, $|\Delta m^2_{3\ell}|$



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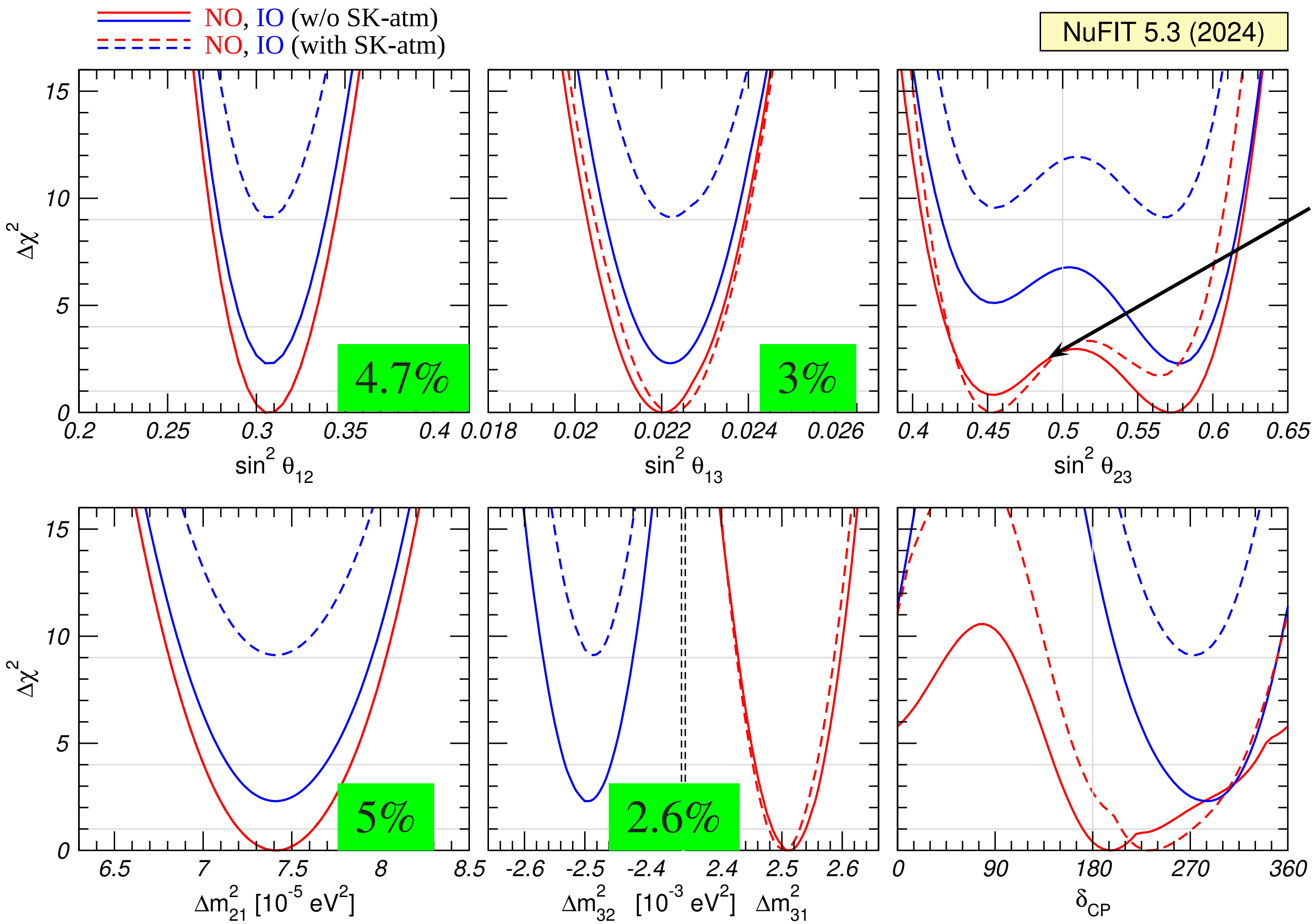


- 4 well-known parameters:
  $\theta_{12}, \theta_{13}, \Delta m^2_{21}, |\Delta m^2_{3\ell}|$
  Δm^2_{21} Solar vs KLAND
  Tension Resolved
- θ_{23}: Least known angle
  Maximal? Octant?
  non-robust wrt ATM

Flavour Parameters: Mixing Matrix

- We have the three leptonic mixing angles determined (at $\pm 3\sigma/6$)

$$|U|_{3\sigma} = \begin{pmatrix} 0.80 \to 0.85 & 0.51 \to 0.56 & 0.14 \to 0.16 \\ 0.23 \to 0.51 & 0.46 \to 0.69 & 0.63 \to 0.78 \\ 0.26 \to 0.53 & 0.47 \to 0.70 & 0.61 \to 0.76 \end{pmatrix}$$

- Good progress but still precision very far from:

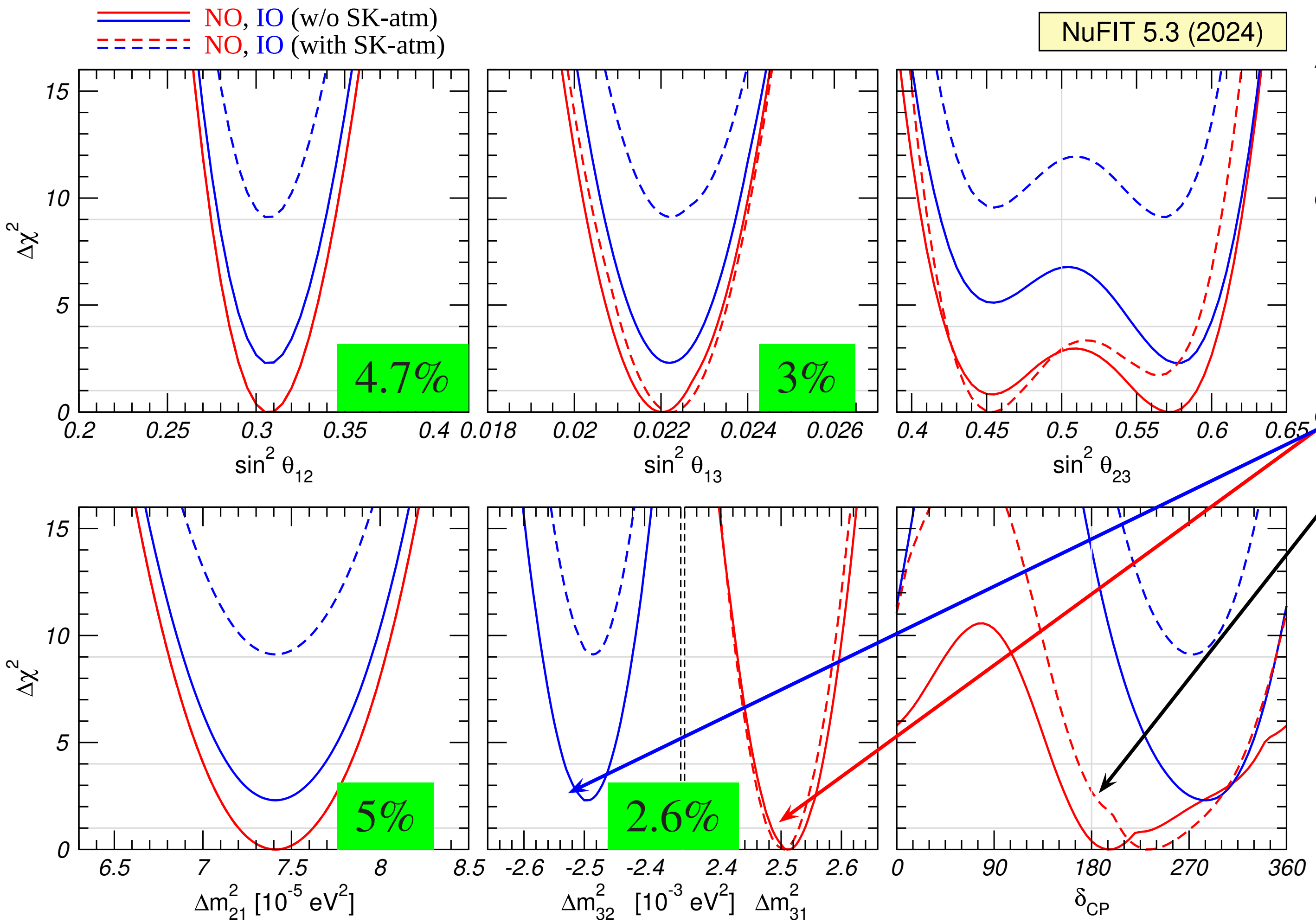
$$|V|_{\rm CKM} = \begin{pmatrix} 0.97427 \pm 0.00015 & 0.22534 \pm 0.0065 & (3.51 \pm 0.15) \times 10^{-3} \\ 0.2252 \pm 0.00065 & 0.97344 \pm 0.00016 & (41.2^{+1.1}_{-5}) \times 10^{-3} \\ (8.67^{+0.29}_{-0.31}) \times 10^{-3} & (40.4^{+1.1}_{-0.5}) \times 10^{-3} & 0.999146^{+0.000021}_{-0.000046} \end{pmatrix}$$

- But clearly very different flavour mixing of leptons vs quarks $\equiv$ *Flavour Puzzle*

Summary: Global 3 ν Flavour Parameters

Global 6-parameter fit http://www.nu-fit.org

Esteban, Gonzalez-Garcia, Maltoni, Schwetz, Zhou, JHEP'20 [2007.14792]



- 4 well-known parameters: $\theta_{12}, \theta_{13}, \Delta m^2_{21}, |\Delta m^2_{3\ell}|$
  Δm^2_{21} Solar vs KLAND Tension Resolved
- θ_{23}: Least known angle
  Maximal? Octant?
  non-robust wrt ATM
- Ordering NO or IO?
  CPV?:

Matter effects in LBL

- At LBL: $\sqrt{2}G_F N_e \equiv V_{\oplus,\text{CRUST}} \sim 5\times 10^{-14}\,\text{eV}$ $\sim$ constant at ν trajectory
- Most relevant for $\nu_\mu \to \nu_e$

$$
\begin{aligned}
P_{\mu e(\bar\mu\bar e)} \quad \simeq \quad & s_{23}^2 \sin^2 2\theta_{13} \left(\frac{\Delta_{31}}{\Delta_{31}\mp V_\oplus}\right)^2 \sin^2\left(\frac{(\Delta_{31}\mp V_\oplus)\,L}{2}\right) \\
+ \quad & \tilde J \,\frac{\Delta_{21}}{V_\oplus}\,\frac{\Delta_{31}}{\Delta_{31}\mp V_\oplus}\,\sin\left(\frac{V_\oplus L}{2}\right)\sin\left(\frac{(\Delta_{31}\mp V_\oplus)L}{2}\right)\cos\delta\,\cos\left(\frac{\Delta_{31}\,L}{2}\right) \\
\pm \quad & \tilde J \,\frac{\Delta_{21}}{V_\oplus}\,\frac{\Delta_{31}}{\Delta_{31}\mp V_\oplus}\,\sin\left(\frac{V_\oplus L}{2}\right)\sin\left(\frac{(\Delta_{31}\mp V_\oplus)L}{2}\right)\sin\delta\,\sin\left(\frac{\Delta_{31}\,L}{2}\right) + \ldots
\end{aligned}
$$

$\Delta_{ij} = \frac{\Delta m_{ij}^2}{2E_\nu}$

$\tilde J = c_{13}\sin^2 2\theta_{13}\sin^2 2\theta_{23}\sin^2 2\theta_{12}$

$\Rightarrow$ Sensitivity to θ_{13}, octant of θ_{23}, δ_{CP}, sign$\Delta m_{31}^2 \equiv$ Ordering

Matter effects in LBL

- Most relevant for $\nu_\mu \to \nu_e$

$$
\begin{aligned}
P_{\mu e(\bar\mu\bar e)} \simeq\ & s_{23}^2 \sin^2 2\theta_{13} \left(\frac{\Delta_{31}}{\Delta_{31} \mp V_\oplus}\right)^2 \sin^2\left(\frac{(\Delta_{31} \mp V_\oplus)\, L}{2}\right) \\
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\end{aligned}
$$

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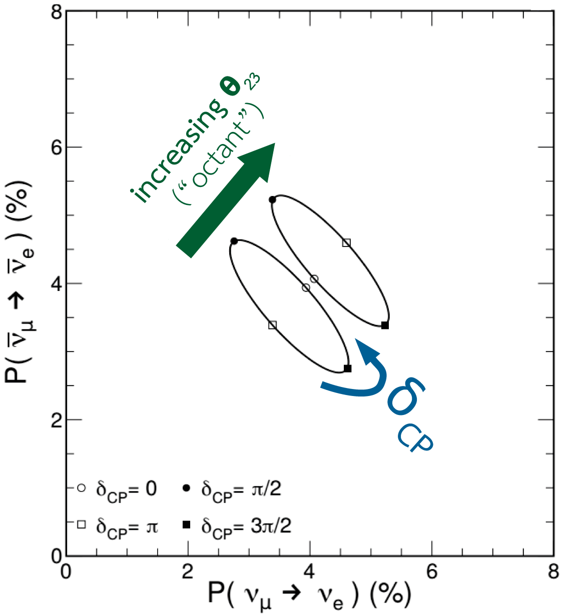
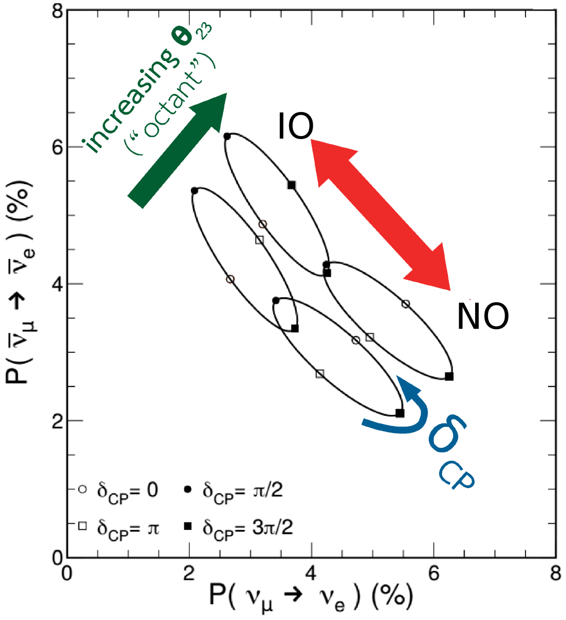




In plots: $\theta_{13} \sim 8^\circ$ fix

In plots: $\Delta_{31} L \sim \pi$ (osc max)

Left: $V_\oplus \ll \Delta_{31}$ (no matter)

Right: $V_\oplus L \sim 0.2$ (NOνA)

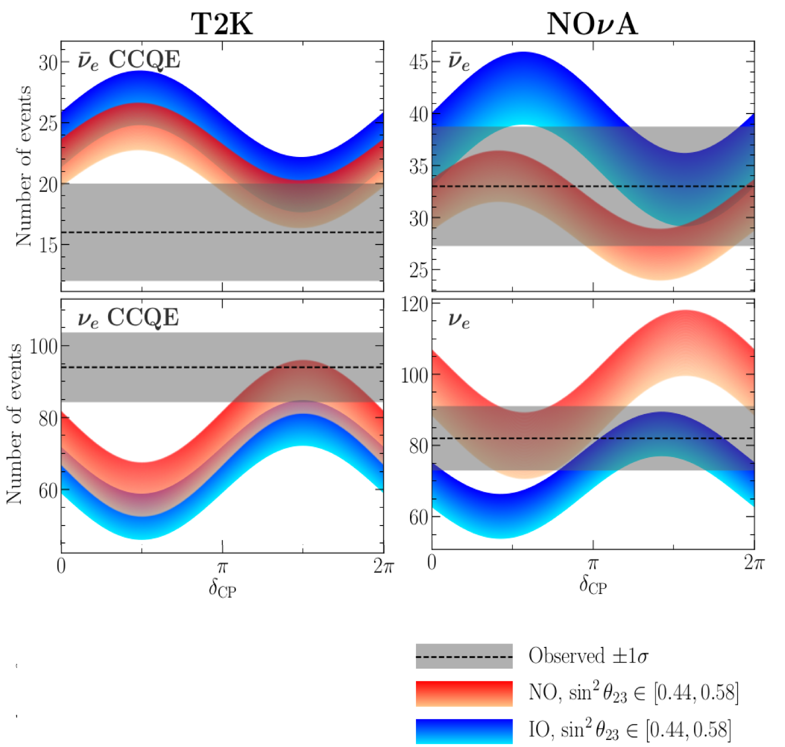
Plots taken from J. Wolcott 52nd FNAL users meeting talk

Ordering and CPV in LBL: ν_e appearace

- Dominant information from ν_e appearance in LBL

$$P_{\mu e} \simeq s_{23}^2 \sin^2 2\theta_{13} \left(\frac{\Delta_{31}}{B_\mp}\right)^2 \sin^2\left(\frac{B_\mp L}{2}\right) + \tilde{J}\, \frac{\Delta_{21}}{V_\oplus} \frac{\Delta_{31}}{B_\mp} \sin\left(\frac{V_\oplus L}{2}\right) \sin\left(\frac{B_\mp L}{2}\right) \cos\left(\frac{\Delta_{31} L}{2} \pm \delta_{CP}\right)$$

$$\Delta_{ij} = \frac{\Delta m^2_{ij}}{4E} \qquad B_\pm = \Delta_{31} \pm V_\oplus \qquad \tilde{J} = c_{13} \sin^2 2\theta_{13} \sin^2 2\theta_{23} \sin^2 2\theta_{12}$$



$\Rightarrow$ Each T2K and NOνA favour NO

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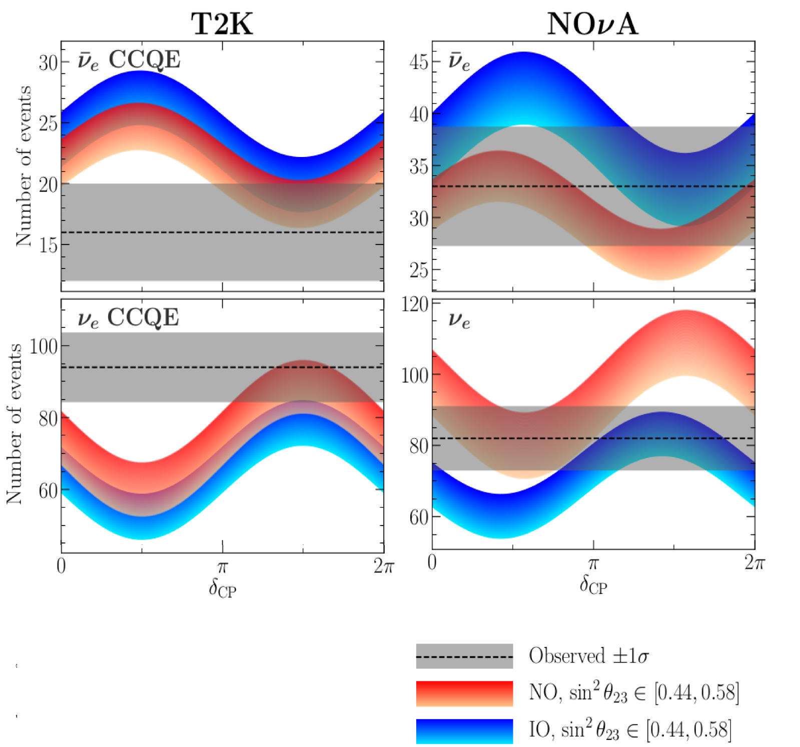
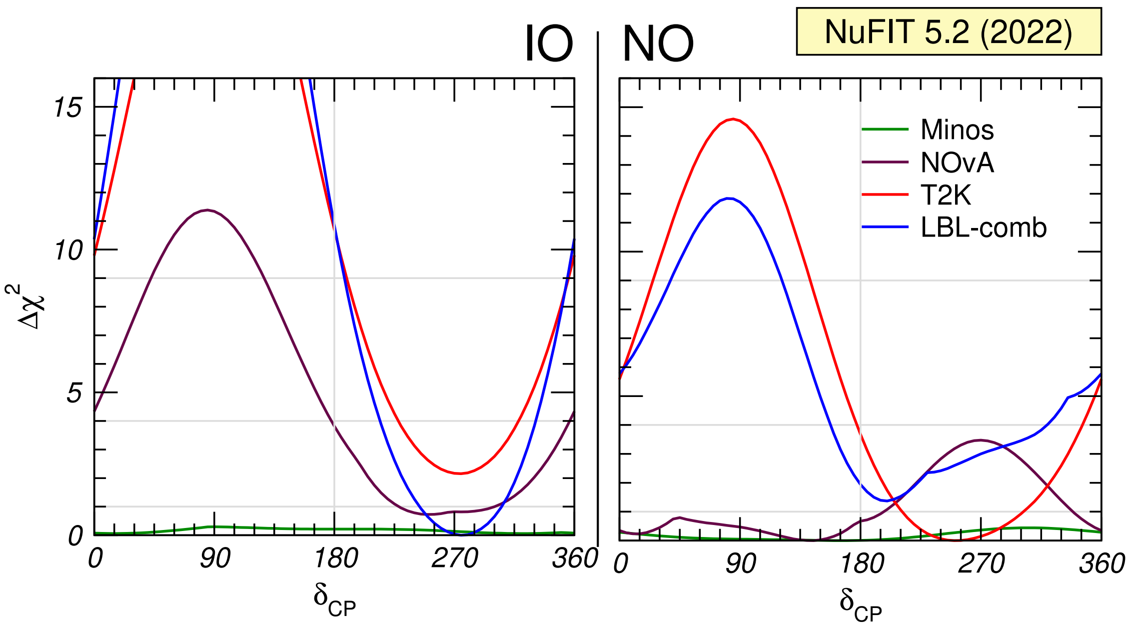
$$\Delta_{ij} = \frac{\Delta m_{ij}^2}{4E} \qquad B_\pm = \Delta_{31} \pm V_\oplus \qquad \tilde{J} = c_{13}\,\sin^2 2\theta_{13}\,\sin^2 2\theta_{23}\,\sin^2 2\theta_{12}$$



$\Rightarrow$ Each T2K and NOνA favour NO

But tension in favoured values of δ_{CP} in NO



$\Rightarrow$ IO best fit in LBL combination

Δm^2_{3l} in LBL & Reactors

- At LBL determined in ν_μ and $\bar\nu_\mu$ disappearance spectrum

$$\Delta m^2_{\mu\mu} \simeq \Delta m^2_{3l} + {}^{c^2_{12}\Delta m^2_{21} \ \text{NO}}_{s^2_{12}\Delta m^2_{21} \ \text{IO}} + \ldots$$

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Nunokawa,Parke,Zukanovich (2005)

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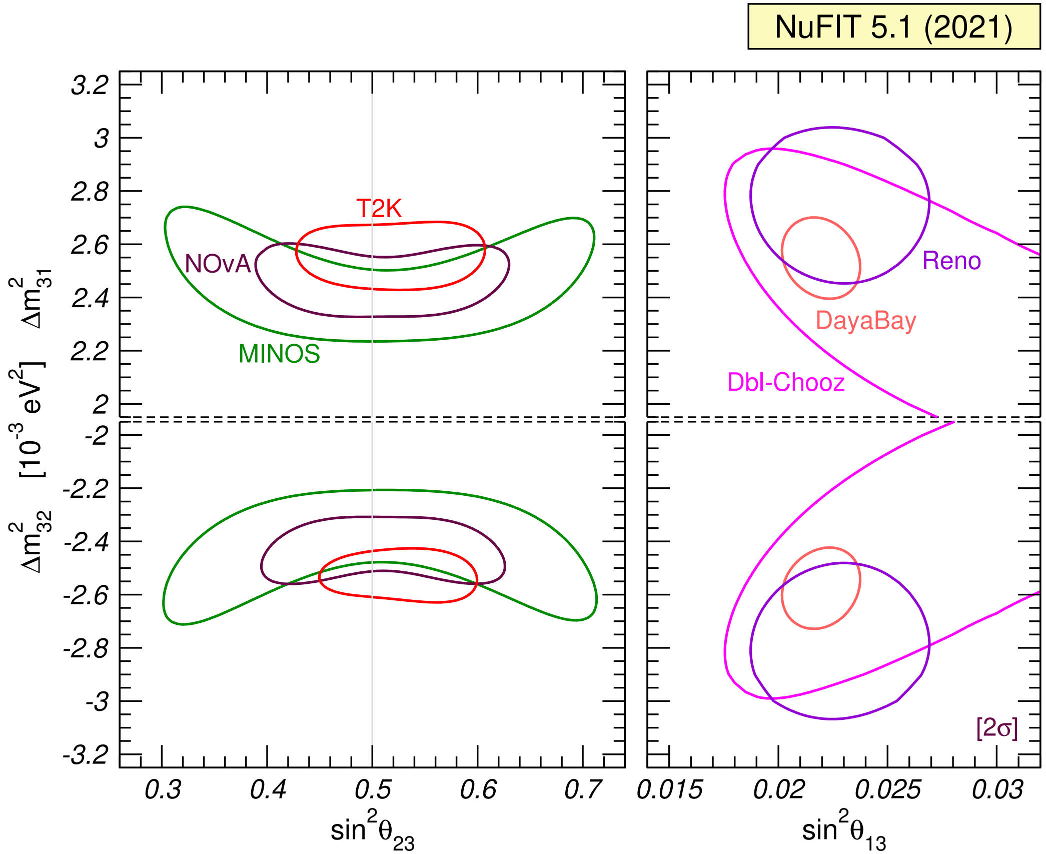

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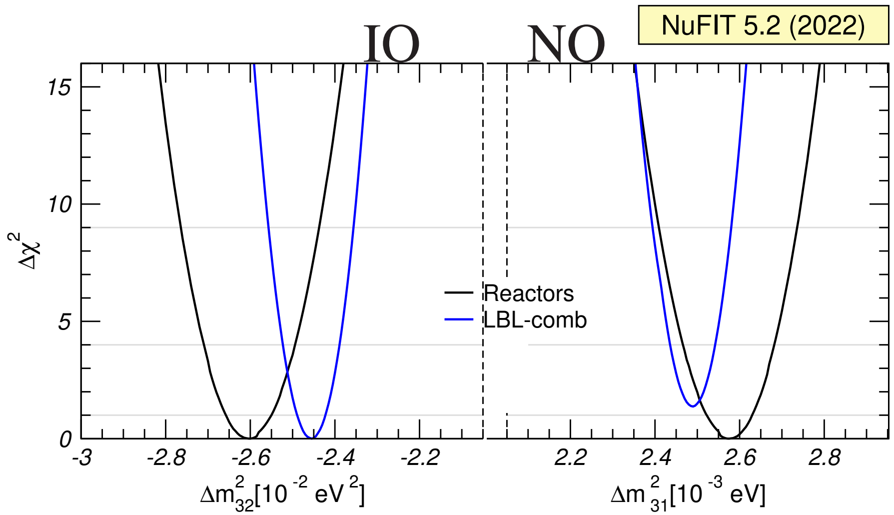

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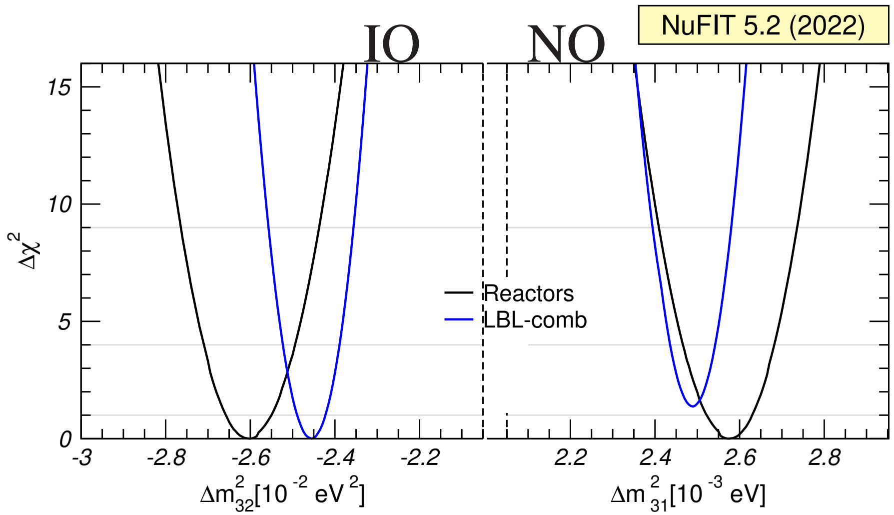

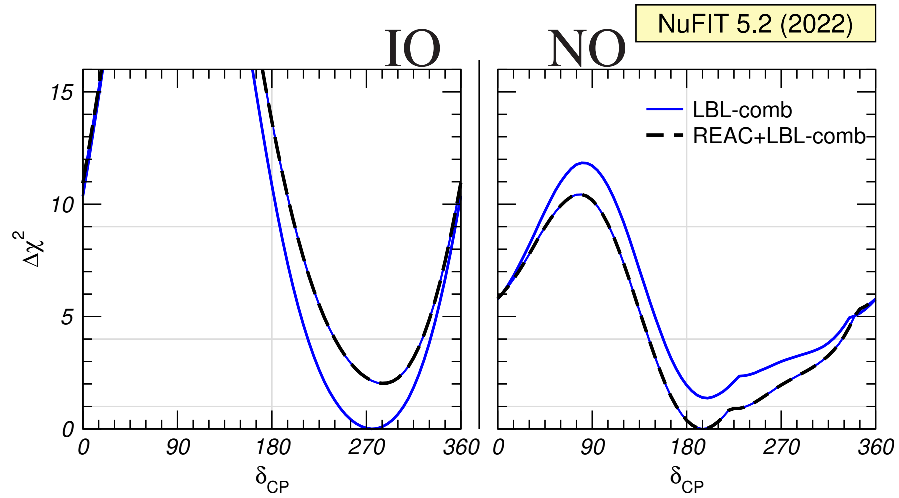

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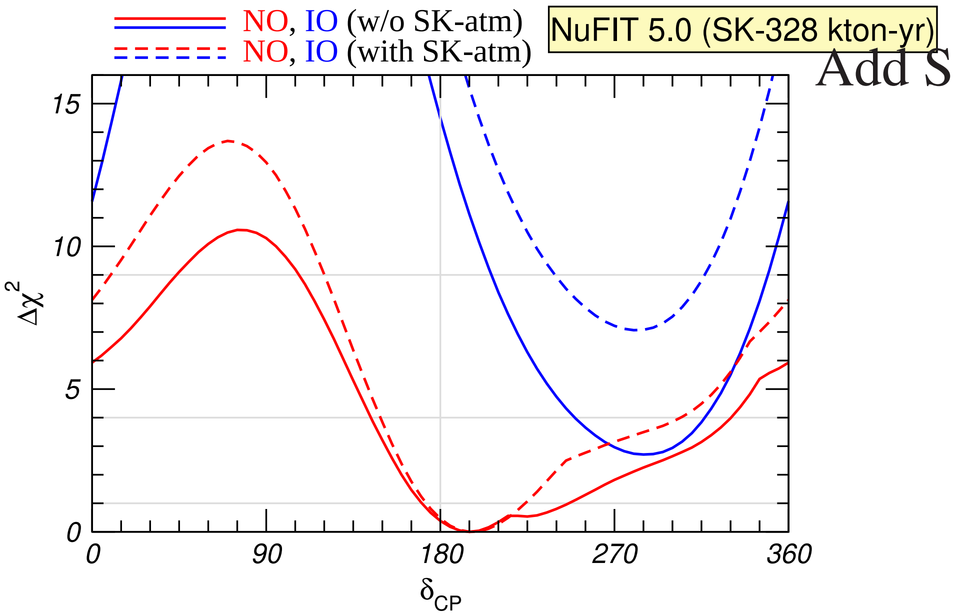
- LBL/Reactor complementarity in $\Delta m^2_{3\ell}$ $\Rightarrow$ NO best fit in LBL+Reactors
- in NO: b.f $\delta_{\rm CP} \sim 195^\circ$ $\Rightarrow$ CPC allowed at 0.6 σ
- in IO: b.f $\delta_{\rm CP} \sim 270^\circ$ $\Rightarrow$ CPC disfavoured at 3 σ

Ordering and CPV including SK-ATM

ATM results added to global fit using SK χ^2 tables

- NUFIT 5.0: included SK I-IV 328 kton-years table
- NUFIT 5.1 and 5.2: include SK I-IV 372.8 kton-years table
- NUFIT 5.3: include SK I-V 484 kton-years table



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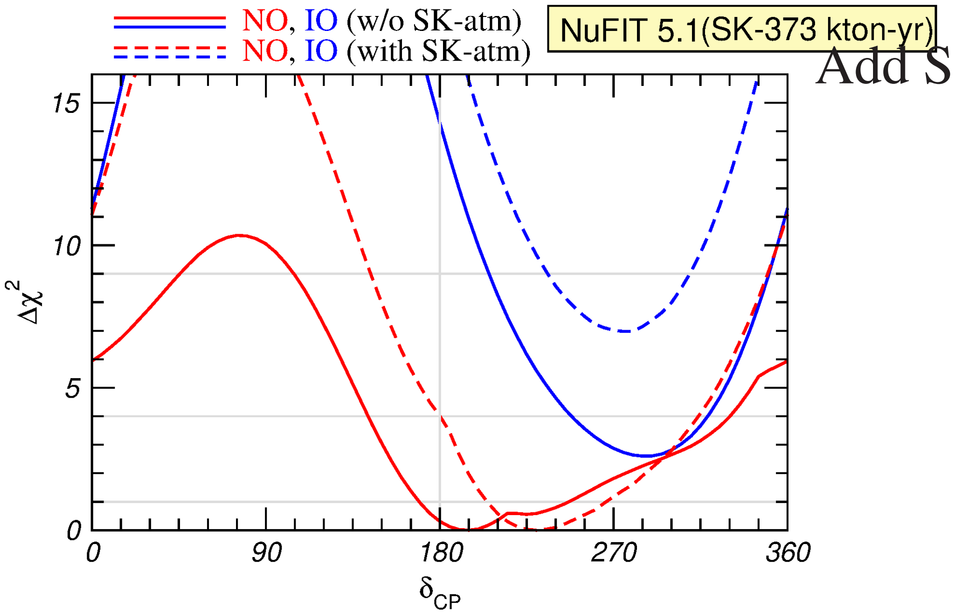

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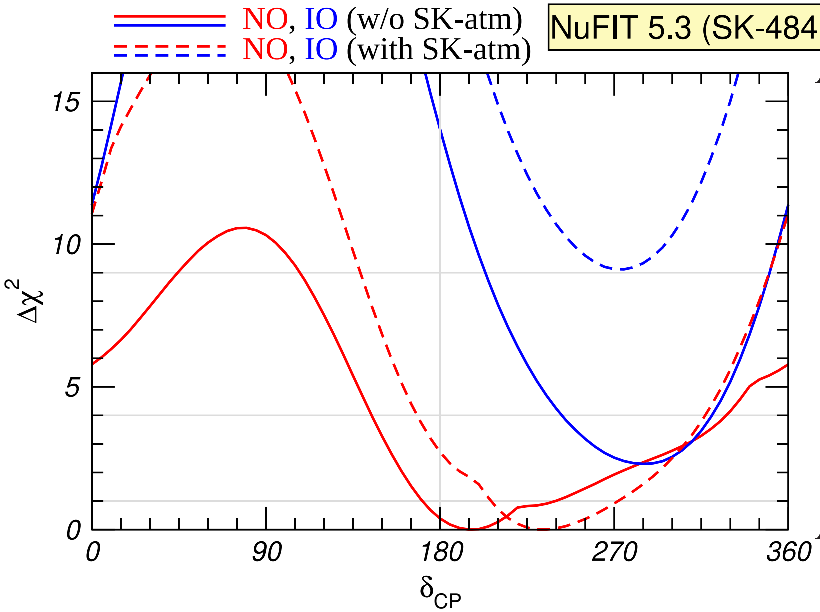

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$$\Delta\chi^2_{\rm NO-IO,with\ SK-atm484} = 9.0$$

Add 373 (484) table ⇒ slight increase of signif of CPV in NO:

w/o SK-Atm b.f $\delta_{\rm CP} = 197^\circ$ CPC at 0.6σ

with SK-Atm: b.f $\delta_{\rm CP} = 232^\circ$ CPC at $\sim 2\ \ (1.5)\sigma$

Near Future for CP and Ordering: Strategies

Lecture by N. McCauley

- $\nu/\bar{\nu}$ comparison with or without Earth matter effects in $\nu_\mu \to \nu_e$ & $\bar{\nu}_\mu \to \bar{\nu}_e$ at LBL: DUNE (wide band beam, L=1300 km), HK (narrow band beam, L=300 km)

$$P_{\mu e} \simeq s_{23}^2 \sin^2 2\theta_{13} \left(\frac{\Delta_{31}}{\Delta_{31} \pm V}\right)^2 \sin^2\left(\frac{\Delta_{31} \pm V\, L}{2}\right)$$
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$$J_{\rm CP}^{\max} = c_{13}^2 s_{13} c_{23} s_{23} c_{12} s_{12}$$

  – Challenge: Parameter degeneracies, Normalization uncertainty, E_ν reconstruction

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- Reactor experiment at $L \sim 50$ km (vacuum) able to observe the difference between oscillations with Δm^2_{31} and Δm^2_{32}: JUNO, RENO-50

$$P_{\nu_e,\nu_e} = 1 - c^4_{13} \sin^2 2\theta_{12} \sin^2\left(\frac{\Delta m^2_{21} L}{4E}\right) - \sin^2 2\theta_{13} \left[c^2_{12} \sin^2\left(\frac{\Delta m^2_{31} L}{4E}\right) + s^2_{12} \sin^2\left(\frac{\Delta m^2_{32} L}{4E}\right)\right]$$

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  – Challenge: Energy resolution

- Earth matter effects in large statistics ATM ν_μ disapp : HK,INO,ORCA …
  – Challenge: ATM flux contains both ν_μ and $\bar\nu_\mu$, ATM flux uncertainties

Confirmed Low Energy Picture

- At least two neutrinos are massive $\Rightarrow$ There is BSM Physics
- Oscillations DO NOT determine the lightest mass

  Model independent probe of m_ν β decay: $\sum m_i^2 |U_{ei}|^2 \leq (0.8\text{ eV})^2$ (Katrin 21)
- Dirac or Majorana?: Best probe ν-less $\beta\beta$ decay *Lecture by C. Patrick*
- 3ν scenario: Robust determination of θ_{12}, θ_{13}, Δm^2_{21}, $|\Delta m^2_{3\ell}|$

  $U_{\rm LEP}$ very different from $U_{\rm CKM}$

  Mass ordering, θ_{23} Octant, CPV depend on subdominant 3ν-effects

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- What about a UV complete model?

ν Mass from Non-Renormalizable Operator

If SM is an effective low energy theory, for $E \ll \Lambda_{\rm NP}$

– The same particle content as the SM and same pattern of symmetry breaking

– But there can be non-renormalizable (dim>4) operators

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \sum_n \frac{1}{\Lambda_{\rm NP}^{n-4}} \mathcal{O}_n$$

First NP effect $\Rightarrow$ dim=5 operator

There is only one!

$$\mathcal{L}_5 = \frac{Z^{\nu}_{ij}}{\Lambda_{\rm NP}} \left(\overline{L_{L,i}} \tilde{\phi}\right) \left(\tilde{\phi}^T L^C_{L,j}\right)$$

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– It is natural that ν mass is the first evidence of NP

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– $m_\nu > \sqrt{\Delta m^2_{\rm atm}} \sim 0.05$ eV for $Z^\nu \sim 1 \Longrightarrow \Lambda_{\rm NP} \sim 10^{15}$GeV $\Longrightarrow$ $\Lambda_{\rm NP} \sim$ GUT scale

$\Rightarrow$ Leptogenesis possible (*Tutorial by J. Turner*

If $Z^\nu \sim (Y_e)^2$ (or more complex NP sector) $\Rightarrow \Lambda_{\rm NP} \sim$ TeV scale $\Rightarrow$ Collider signals

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- Wh… …plete model?

ν masses are BSM physics effects to be put together with *all other NP effects*:
from charged LFV, Collider signals, Cosmo-astroparticle…to establish
the Next Standard Model

Back up Slides

The Other Flavours

ν coming out of a nuclear reactor is $\overline{\nu}_e$ because it is emitted together with an e^-

Question: Is it different from the muon type neutrino ν_μ that could be associated to the muon? Or is this difference a theoretical arbitrary convention?

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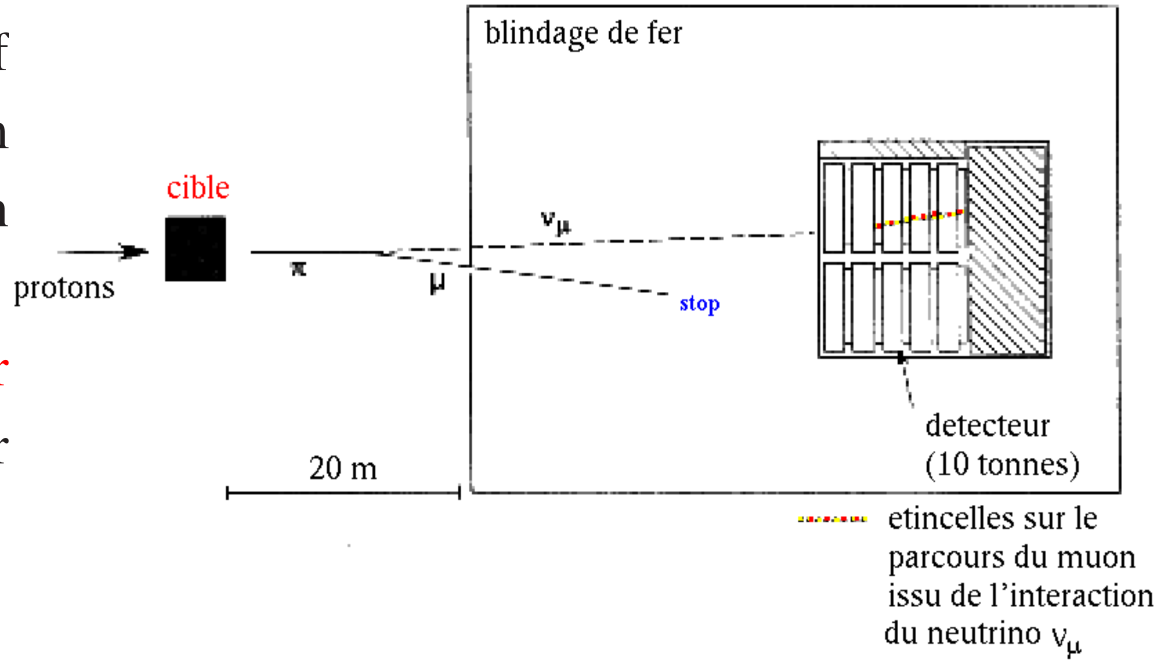

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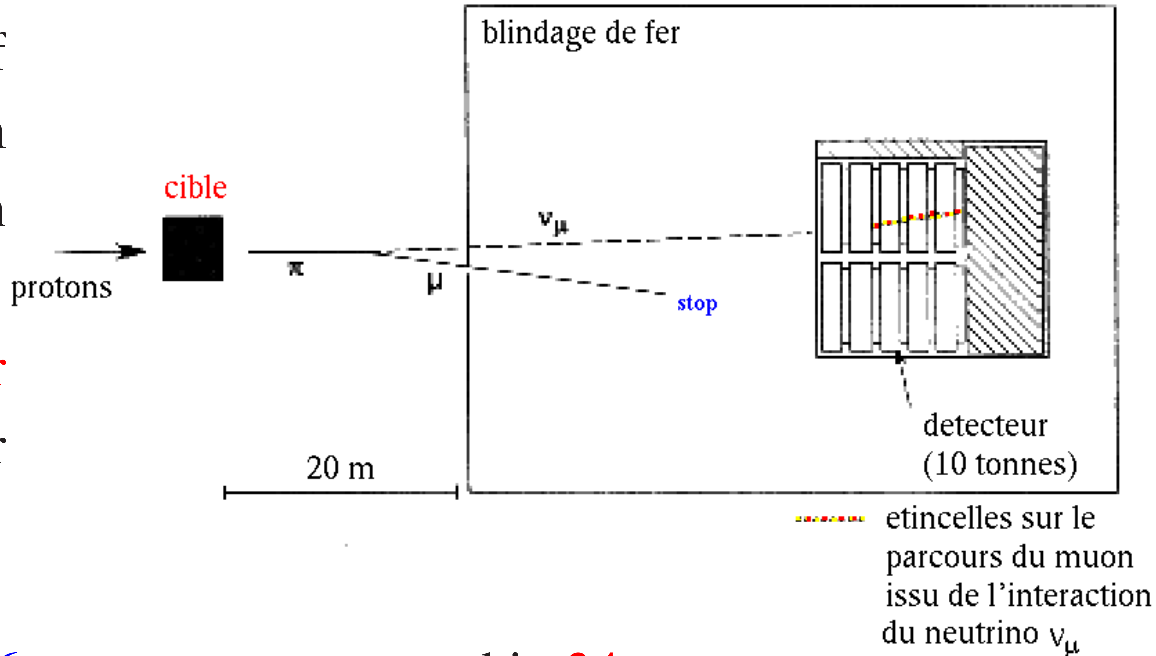

They observe 40 ν interactions: in 6 an e^- comes out and in 34 a μ^- comes out.

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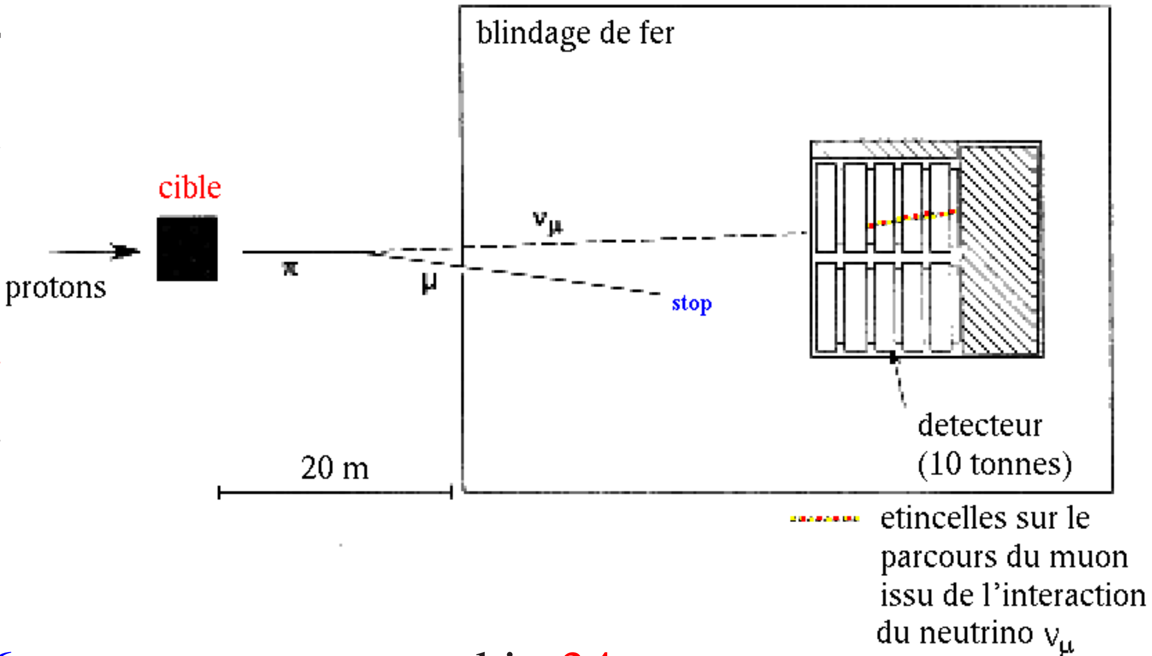

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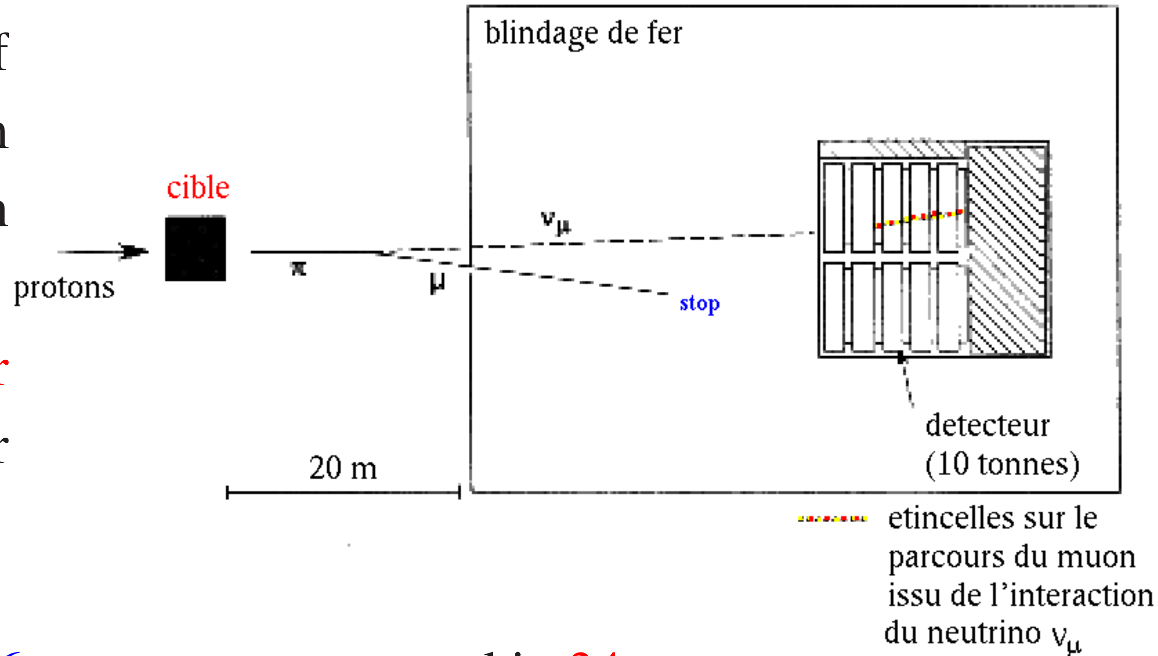

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In 1977 Martin Perl discovers the particle tau $\equiv$ the third lepton family.

The ν_τ was observed by DONUT experiment at FNAL in 1998 (*officially* in Dec. 2000).

Neutrino Helicity

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- The neutrino helicity was measured in 1957 in a experiment by Goldhaber et al.

- Using the electron capture reaction

$$e^- + {}^{152}Eu \to \nu + {}^{152}Sm^*$$
$$\to {}^{152}Sm + \gamma$$

with $J({}^{152}Eu) = J({}^{152}Sm) = 0$ and $L(e^-) = 0$

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- Nuclei are heavy $\Rightarrow \vec{p}({}^{152}Eu) \simeq \vec{p}({}^{152}Sm) \simeq \vec{p}({}^{152}Sm^*) = 0$

So momentum conservation $\Rightarrow \vec{p}(\nu) = -\vec{p}(\gamma) \Rightarrow$ ν helicity= γ helicity

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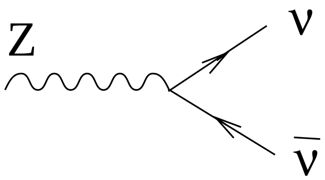
- Goldhaber et al found γ had negative helicity $\Rightarrow$ ν has helicity -1

Number of Neutrinos

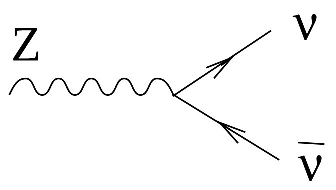
• The counting of light left-handed neutrinos is based on the family structure of the SM assuming a universal diagonal NC coupling:



$$j^{\mu}_{Z} = \sum_{\alpha} \bar{\nu}_{\alpha L} \gamma^{\mu} \nu_{\alpha L}$$

Number of Neutrinos

• The counting of light left-handed neutrinos is based on the family structure of the SM assuming a universal diagonal NC coupling:



$$j_Z^\mu = \sum_\alpha \bar{\nu}_{\alpha L} \gamma^\mu \nu_{\alpha L}$$

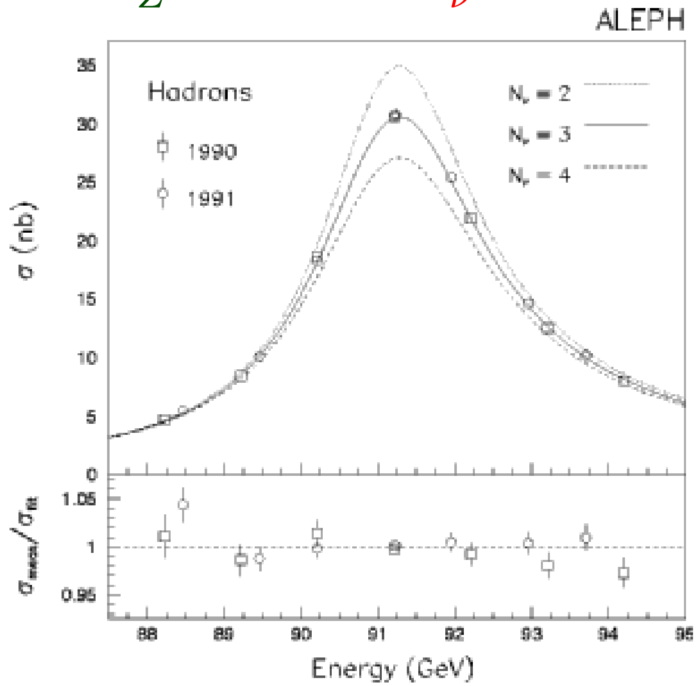
• For $m_{\nu_i} < m_Z/2$ one can use the total Z-width Γ_Z to extract N_ν

$$N_\nu = \frac{\Gamma_{\rm inv}}{\Gamma_\nu} \equiv \frac{1}{\Gamma_\nu}(\Gamma_Z - \Gamma_h - 3\Gamma_\ell) = \frac{\Gamma_\ell}{\Gamma_\nu}\left[\sqrt{\frac{12\pi R_{h\ell}}{\sigma_h^0 m_Z^2}} - R_{h\ell} - 3\right]$$

Γ_{inv}= the invisible width

Γ_h= the total hadronic width

Γ_l= width to charged lepton



Leads $N_\nu = 2.9840 \pm 0.0082$

- In terms of the instantaneous mass eigenstates in matter: $\begin{pmatrix} \nu_\alpha \\ \nu_\beta \end{pmatrix} = U[\theta_m(x)] \begin{pmatrix} \nu_1^m(x) \\ \nu_2^m(x) \end{pmatrix}$

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- For varying potential: $$\begin{pmatrix} \dot\nu_\alpha \\ \dot\nu_\beta \end{pmatrix} = \dot U[\theta_m(x)] \begin{pmatrix} \nu_1^m(x) \\ \nu_2^m(x) \end{pmatrix} + U[\theta_m(x)] \begin{pmatrix} \dot\nu_1^m(x) \\ \dot\nu_2^m(x) \end{pmatrix}$$

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⇒ the evolution equation in flavour basis (removing diagonal part)

$$i \begin{pmatrix} \dot\nu_\alpha \\ \dot\nu_\beta \end{pmatrix} = \frac{1}{4E} \begin{pmatrix} A - \Delta m^2 \cos 2\theta & \Delta m^2 \sin 2\theta \\ \Delta m^2 \sin 2\theta & -A + \Delta m^2 \cos 2\theta \end{pmatrix} \begin{pmatrix} \nu_\alpha \\ \nu_\beta \end{pmatrix}$$

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$$i \begin{pmatrix} \dot\nu_1^m \\ \dot\nu_2^m \end{pmatrix} = \frac{1}{4E} U^\dagger(\theta_m) \begin{pmatrix} A - \Delta m^2 \cos 2\theta & \Delta m^2 \sin 2\theta \\ \Delta m^2 \sin 2\theta & -A + \Delta m^2 \cos 2\theta \end{pmatrix} U(\theta_m) \begin{pmatrix} \nu_1^m \\ \nu_2^m \end{pmatrix} - i\, U^\dagger \dot U(\theta_m) \begin{pmatrix} \nu_1^m \\ \nu_2^m \end{pmatrix}$$

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$$\Rightarrow\; i \begin{pmatrix} \dot\nu_1^m \\ \dot\nu_2^m \end{pmatrix} = \frac{1}{4E} \begin{pmatrix} -\Delta\mu^2(x) & -4\,i\,E\,\dot\theta_m(x) \\ 4\,i\,E\,\dot\theta_m(x) & \Delta\mu^2(x) \end{pmatrix} \begin{pmatrix} \nu_1^m \\ \nu_2^m \end{pmatrix}$$

- Lets consider ν_e in a medium with e, p, and n. The low-energy Hamiltonian density:

$$H_W = \frac{G_F}{\sqrt{2}}[J^{(+)\alpha}(x)J^{(-)}_\alpha(x) + \frac{1}{4}J^{(N)\alpha}(x)J^{(N)}_\alpha(x)]$$

CC Int $\quad J^{(+)}_\alpha(x) = \overline{\nu_e}(x)\gamma_\alpha(1-\gamma_5)e(x) \qquad J^{(-)}_\alpha(x) = \overline{e}(x)\gamma_\alpha(1-\gamma_5)\nu_e(x)$

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$$+\overline{p}(x)[\gamma_\alpha(1-g_A^{(p)}\gamma_5) - 4s^2_W\gamma_\alpha]p(x) - \overline{n}(x)[\gamma_\alpha(1-g_A^{(n)}\gamma_5) - 4s^2_W\gamma_\alpha]n(x)$$

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• Example: The effect of CC with the e medium. The effective CC Hamiltonian density:

$$H_{CC}^{(e)} = \frac{G_F}{\sqrt{2}}\int d^3p_e f(E_e)\Big\langle \langle e(s,p_e)|\overline{e}\gamma^\alpha(1-\gamma_5)\nu_e\overline{\nu_e}\gamma_\alpha(1-\gamma_5)e|e(s,p_e)\rangle \Big\rangle$$

Fierz rearrange

$$= \frac{G_F}{\sqrt{2}}\overline{\nu_e}\gamma_\alpha(1-\gamma_5)\nu_e\int d^3p_e f(E_e)\Big\langle \langle e(s,p_e)|\overline{e}\gamma_\alpha(1-\gamma_5)e|e(s,p_e)\rangle \Big\rangle$$

$f(E_e)$ statistical energy distribution of e in *homogeneous and isotropic* medium.

$$\int d^3p_e f(E_e) = 1$$

$\Big\langle \ldots \Big\rangle \equiv$ summing over all e of momentum p_e.

coherence $\Rightarrow$ s, p_e same for initial and final e

- Expanding the electron fields e in plane waves (quantized in a volume $\mathcal{V}$)

$$\langle e(s,p_e)|\overline{e}\gamma_\alpha(1-\gamma_5)e|e(s,p_e)\rangle = \frac{1}{2E_e\,\mathcal{V}}\langle e(s,p_e)|\overline{u_s}(p_e)a_s^\dagger(p_e)\gamma_\alpha(1-\gamma_5)a_s(p_e)u_s(p_e)|e(s,p_e)\rangle$$

• Expanding the electron fields e in plane waves (quantized in a volume $\mathcal{V}$)

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• Since $a_s^\dagger(p_e)a_s(p_e) = \mathcal{N}_e^{(s)}(p_e)$ (number operator) and assuming that there are the same number of electrons with spin 1/2 and -1/2

$$\frac{1}{\mathcal{V}}\Big\langle\langle e(s,p_e)|a_s^\dagger(p_e)a_s(p_e)|e(s,p_e)\rangle\Big\rangle \equiv \sum_s N_e^s(p_e) = N_e(p_e)\frac{1}{2}\sum_s$$

where $N_e(p_e)$ number density of electrons with momentum p_e summed over helicities

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$$= \frac{N_e(p_e)}{4E_e}\sum_s Tr\left[\overline{u_s}(p_e)\gamma_\alpha(1-\gamma_5)u_s(p_e)\right] = \frac{N_e(p_e)}{4E_e}\sum_s Tr\left[u_s(p_e)\overline{u_s}(p_e)\gamma_\alpha(1-\gamma_5)\right]$$

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• For isotropic medium $\Rightarrow \int d^3p_e\vec{p_e}\,f(E_e)\,N_e(p_e)=0$

• By definition $\int d^3p_e\,f(E_e)\,N_e(p_e)=N_e$ electron number density

- The effective charged current Hamiltonian density due to electrons in matter is then:

$$H_{CC}^{(e)} = \frac{G_F N_e}{\sqrt{2}} \overline{\nu_e}(x)\gamma_0(1-\gamma_5)\nu_e(x)$$

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$$V_{CC} = \sqrt{2} G_F N_e$$

- for $\overline{\nu_e}$ the sign of V_{CC} is reversed

- Other potentials for ν_e $(\overline{\nu}_e)$ due to different particles in medium

medium	V_{CC}	V_{NC}
e^+ and e^-	$\pm\sqrt{2}G_F(N_e - N_{\bar{e}})$	$\mp\frac{G_F}{\sqrt{2}}(N_e - N_{\bar{e}})(1 - 4\sin^2\theta_W)$
p and $\bar{p}$	0	$\mp\frac{G_F}{\sqrt{2}}(N_p - N_{\bar{p}})(1 - 4\sin^2\theta_W)$
n and $\bar{n}$	0	$\mp\frac{G_F}{\sqrt{2}}(N_n - N_{\bar{n}})$
Neutral $(N_e = N_p)$	$\pm\sqrt{2}G_F N_e$	$\mp\frac{G_F}{\sqrt{2}}N_n$

For ν_μ and ν_τ: V_{NC} are the same as for ν_e BUT $V_{CC} = 0$ for any of these media

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- Estimating typical values:

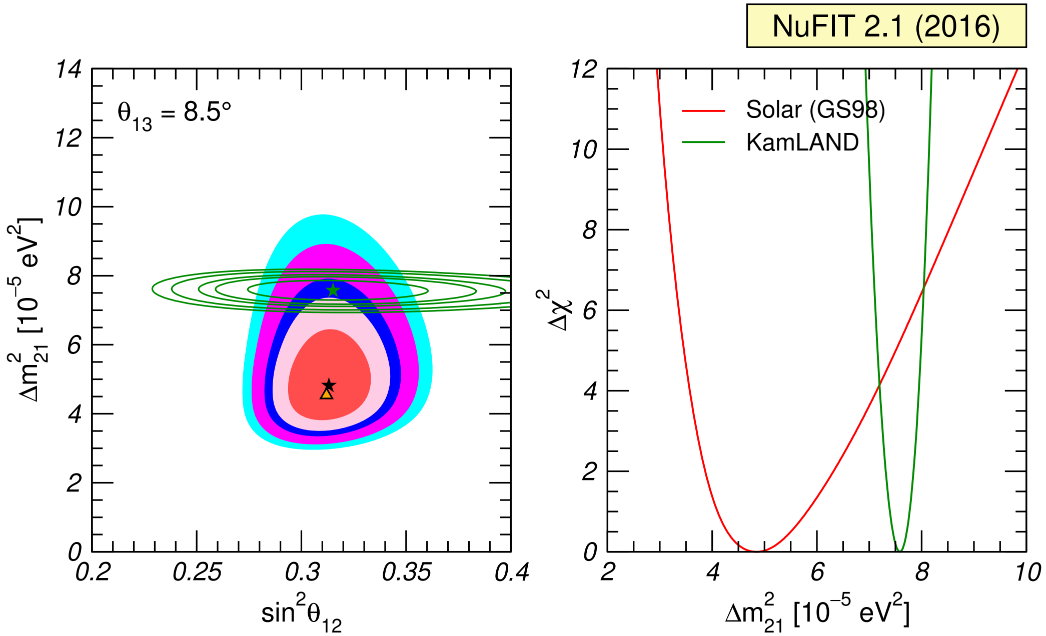
$$V_{CC} = \sqrt{2}G_F N_e \simeq 7.6\, Y_e \frac{\rho}{10^{14}\mathrm{g/cm^3}}\ \mathrm{eV}$$

$$Y_e = \frac{N_e}{N_p+N_n} \equiv \text{relative number density}$$
$$\rho \equiv \text{matter density}$$

– At the solar core $\rho \sim 100$ g/cm^3 $\Rightarrow V \sim 10^{-12}$ eV

– At supernova $\rho \sim 10^{14}$ g/cm^3 $\Rightarrow V \sim$ eV

- Last decade: after including $\theta_{13} \simeq 9^\circ$ the comparison of KamLAND vs Solar



θ_{12} better than 1σ agreement

But $\sim 2\sigma$ tension on Δm^2_{12}

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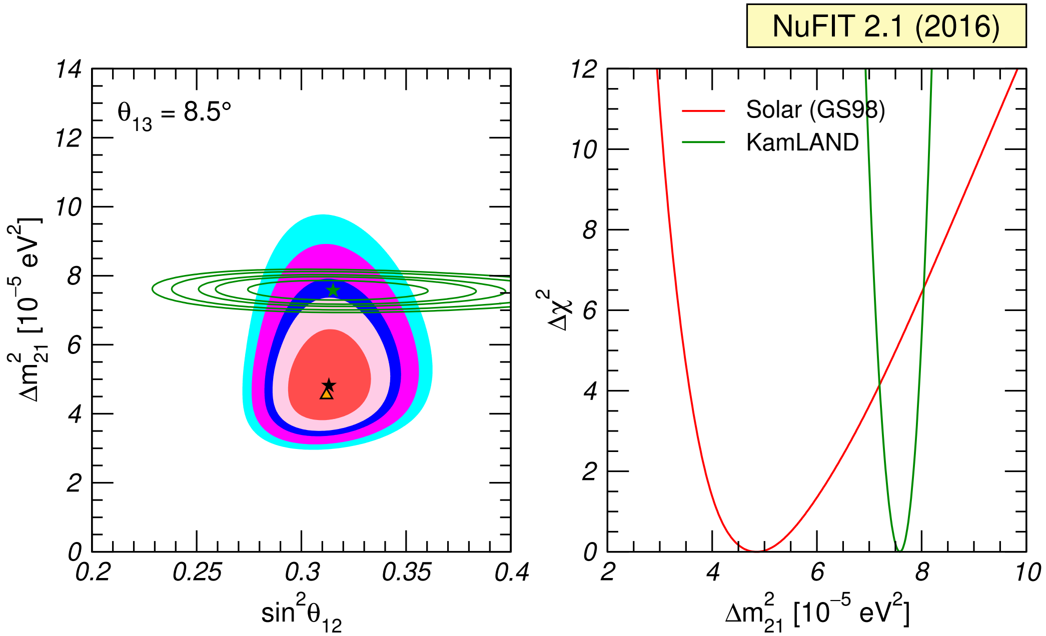

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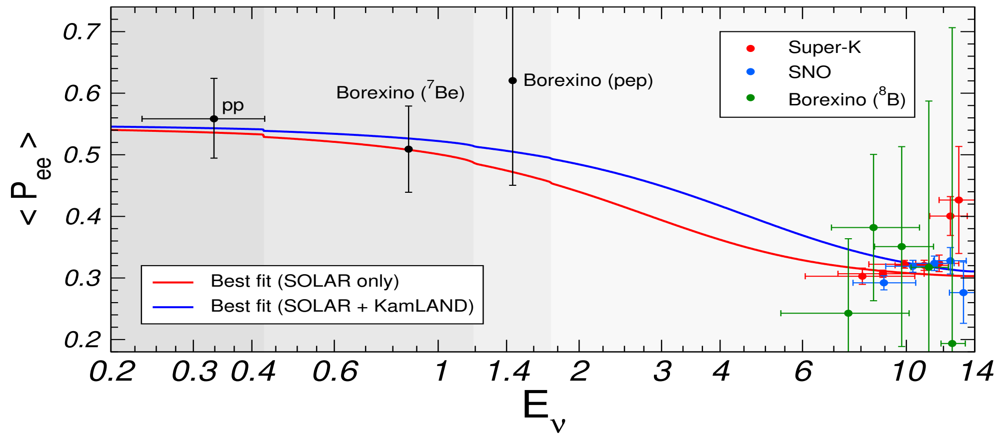
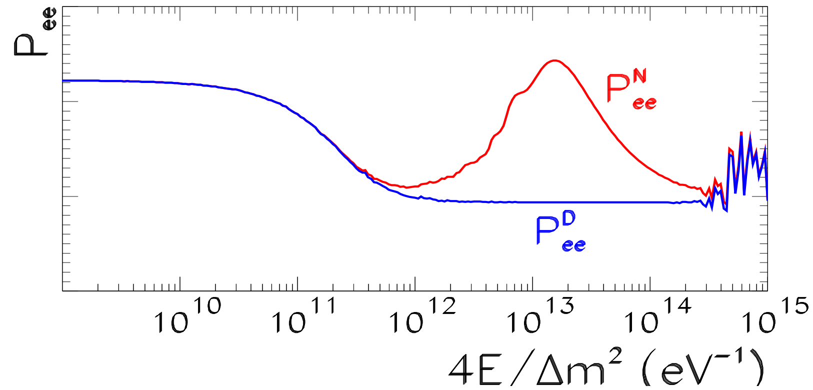
Smaller-than-expected MSW low-E turn-up in SK/SNO spectrum at global b.f.



"too large" of Day/Night at SK

$A_{\rm D/N,SK4-2055} = [-3.1 \pm 1.6({\rm stat.}) \pm 1.4({\rm sys.})]\%$



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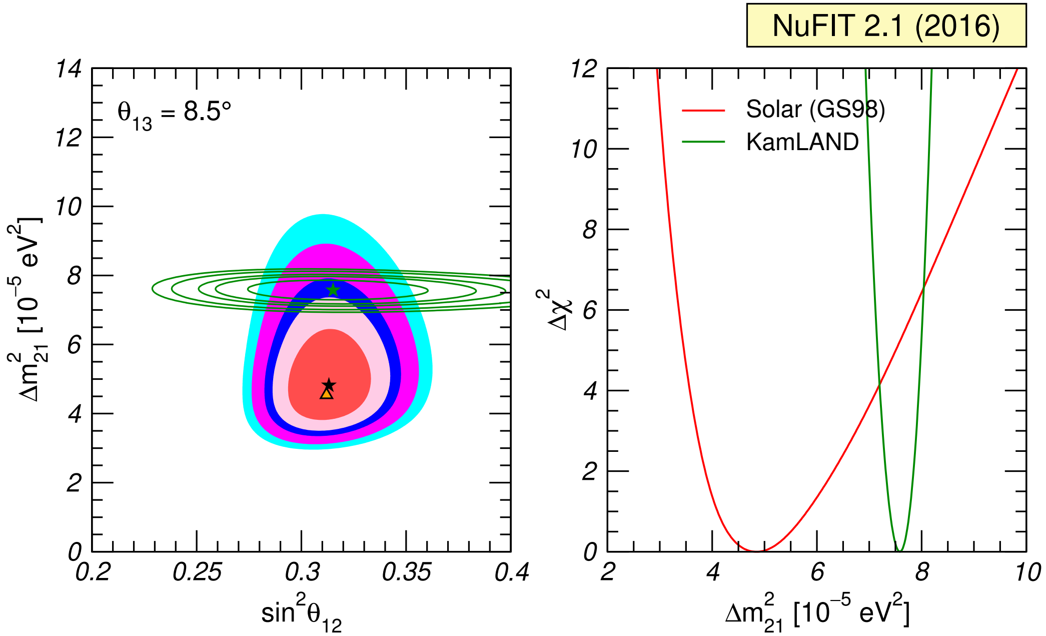

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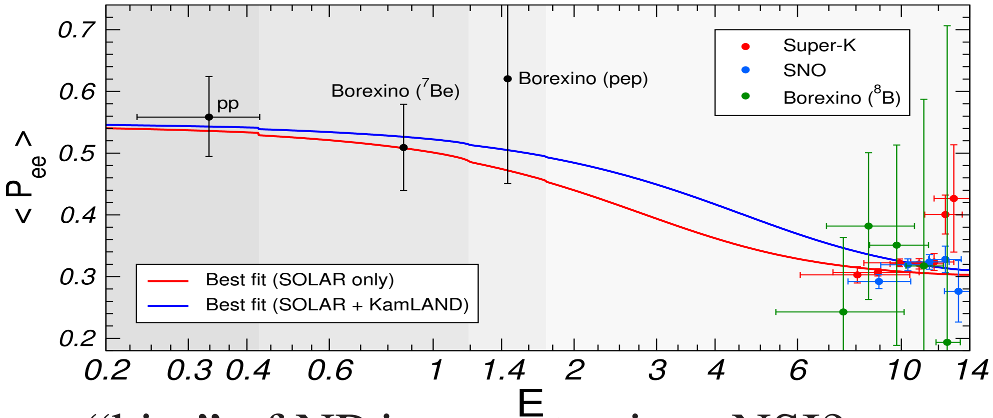
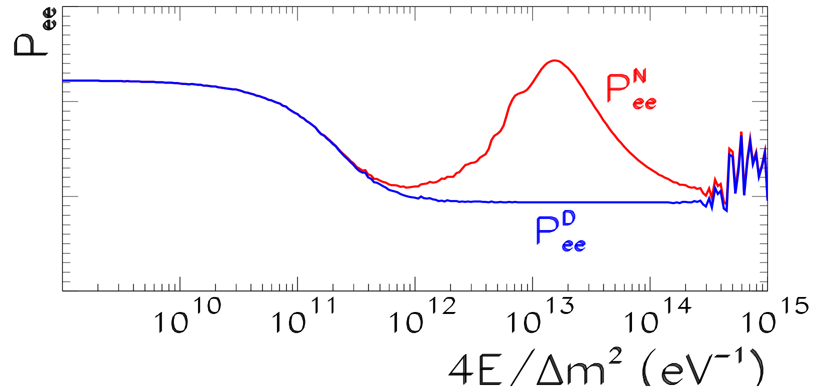
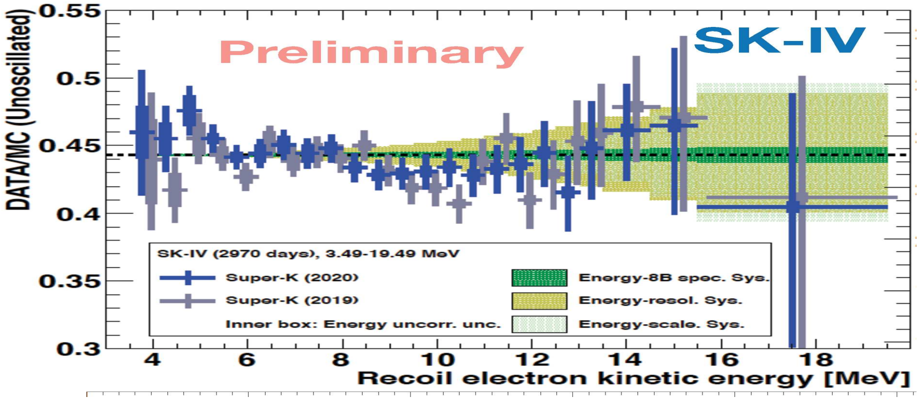




$\Rightarrow$ "hint" of NP in propagation: NSI?

• AFTER NU2020: With SK4 2970 days data

Slightly more pronounced low-E turn-up

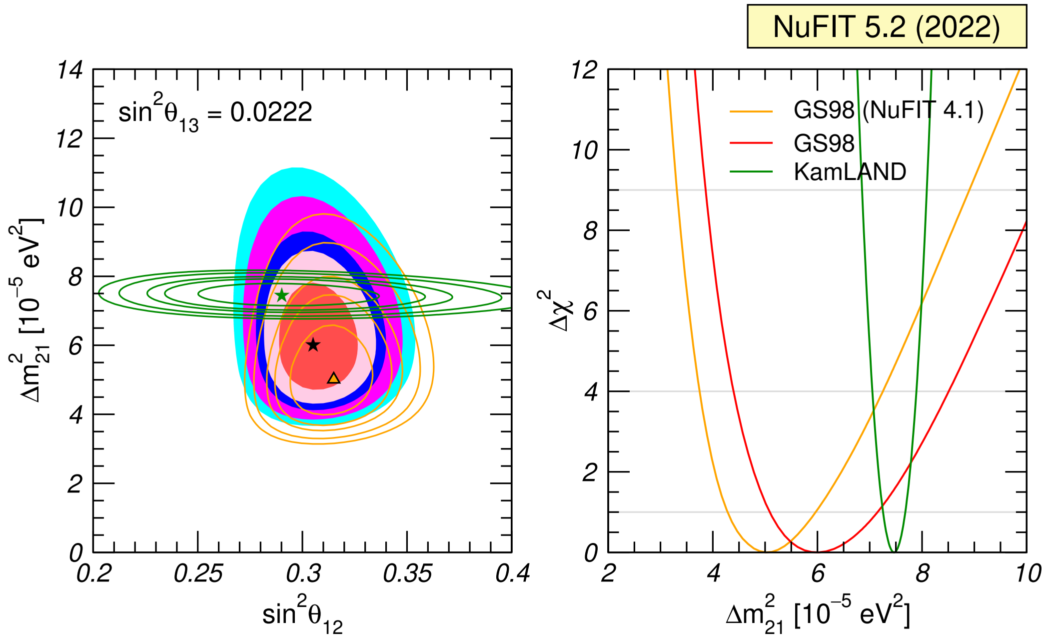


Smaller of Day/Night at

$$A_{\rm D/N,SK4-2055} = [-3.1 \pm 1.6({\rm stat.}) \pm 1.4({\rm sys.})]\%$$

$$A_{\rm D/N,SK4-2970} = [-2.1 \pm 1.1]\%$$

• In NuFIT 5.2



⇒ Agreement of Δm^2_{21} between solar and KamLAND at 1 σ

Leptonic CPV in 3ν Mixing: Jarlskog Invariant

- Leptonic $\not{C}$P $\Rightarrow P_{\nu_\alpha \to \nu_\beta} \neq P_{\bar{\nu}_\alpha \to \bar{\nu}_\beta}$
- In 3ν always

$$P_{\nu_\alpha \to \nu_\beta} - P_{\bar{\nu}_\alpha \to \bar{\nu}_\beta} \propto J \quad \text{with} \quad J = \text{Im}(U_{\alpha 1} U^*_{\alpha_2} U_{\beta 2} U^*_{\beta_1}) = J^{\max}_{\text{LEP,CP}} \sin \delta_{\text{CP}}$$

$$J^{\max}_{\text{LEP,CP}} = \frac{1}{8} c_{13} \sin^2 2\theta_{13} \sin^2 2\theta_{23} \sin^2 2\theta_{12}$$

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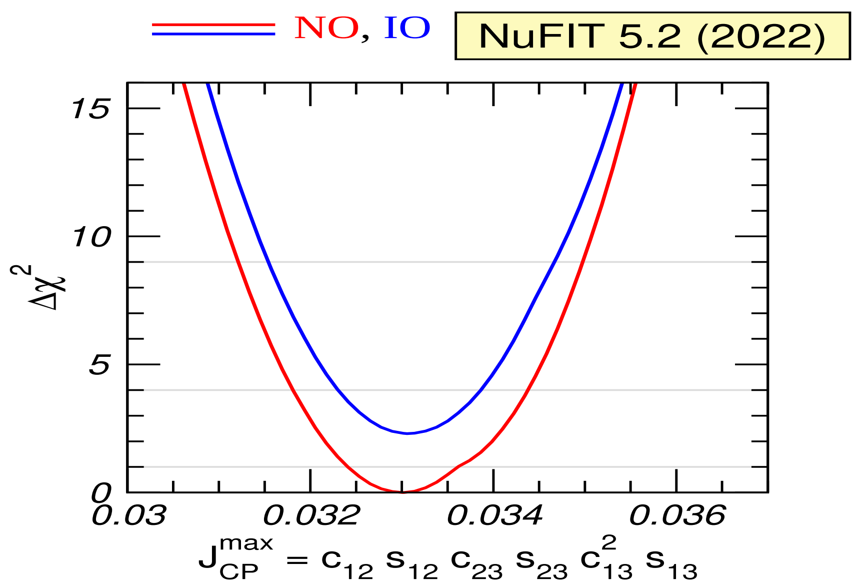
$$J^{\text{max}}_{\text{LEP,CP}} = \frac{1}{8} c_{13} \sin^2 2\theta_{13} \sin^2 2\theta_{23} \sin^2 2\theta_{12}$$

- Maximum Allowed Leptonic CPV:



$$J^{\text{max}}_{\text{LEP,CP}} = (3.29 \pm 0.07) \times 10^{-2}$$

to compare with

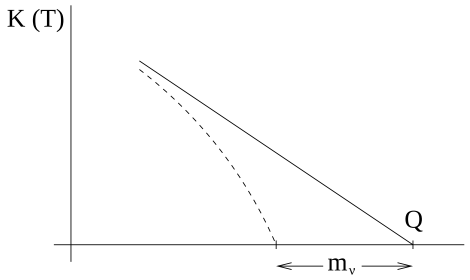
$$J_{\text{CKM,CP}} = (3.04 \pm 0.21) \times 10^{-5}$$

$\Rightarrow$ Leptonic CPV may be largest CPV in New Minimal SM

if $\sin\delta_{\text{CP}}$ not too small

Probes of Mass Scale in 3ν-mixing

Single β decay : Pure kinematics, Dirac or Majorana ν's, only model independent

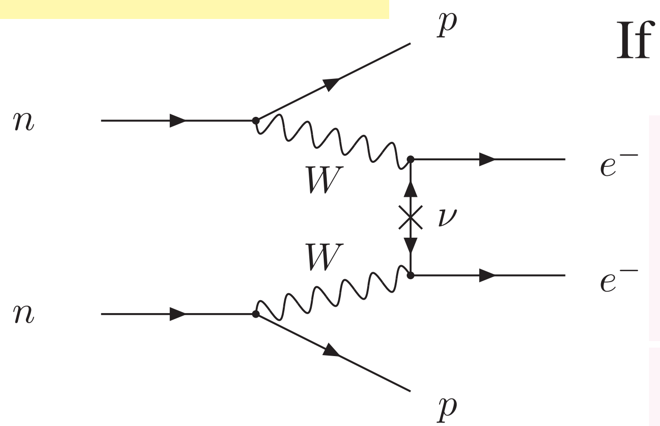


$$m^2_{\nu_e} = \sum m_j^2 |U_{ej}|^2 = \begin{cases} \text{NO}: \ m_\ell^2 + \Delta m^2_{21} c^2_{13} s^2_{12} + \Delta m^2_{31} s^2_{13} \\ \text{IO}: \ m_\ell^2 + \Delta m^2_{21} c^2_{13} s^2_{12} - \Delta m^2_{31} c^2_{13} \end{cases}$$

Present bound: $m_{\nu_e} \leq 0.8$ eV (90% CL KATRIN 2021)

Katrin (20XX) Sensitivity to $m_{\nu_e} \sim 0.2$ eV

ν-less Double-β decay: $\Leftrightarrow$ Majorana $\nu' s$



If m_ν only source of ΔL $\quad T^{0\nu}_{1/2} = \dfrac{m_e}{G_{0\nu}\, M^2_{\text{nucl}}\, m^2_{ee}}$

$$m_{ee} = |\sum U^2_{ej} m_j|$$

$$= \left|c^2_{13} c^2_{12} m_1\, e^{i\eta_1} + c^2_{13} s^2_{12} m_2\, e^{i\eta_2} + s^2_{13} m_3\, e^{-i\delta_{CP}}\right|$$

$$= f(m_\ell, \text{order}, \text{maj phases})$$

Present Bounds: $m_{ee} < 0.04$–0.2 eV

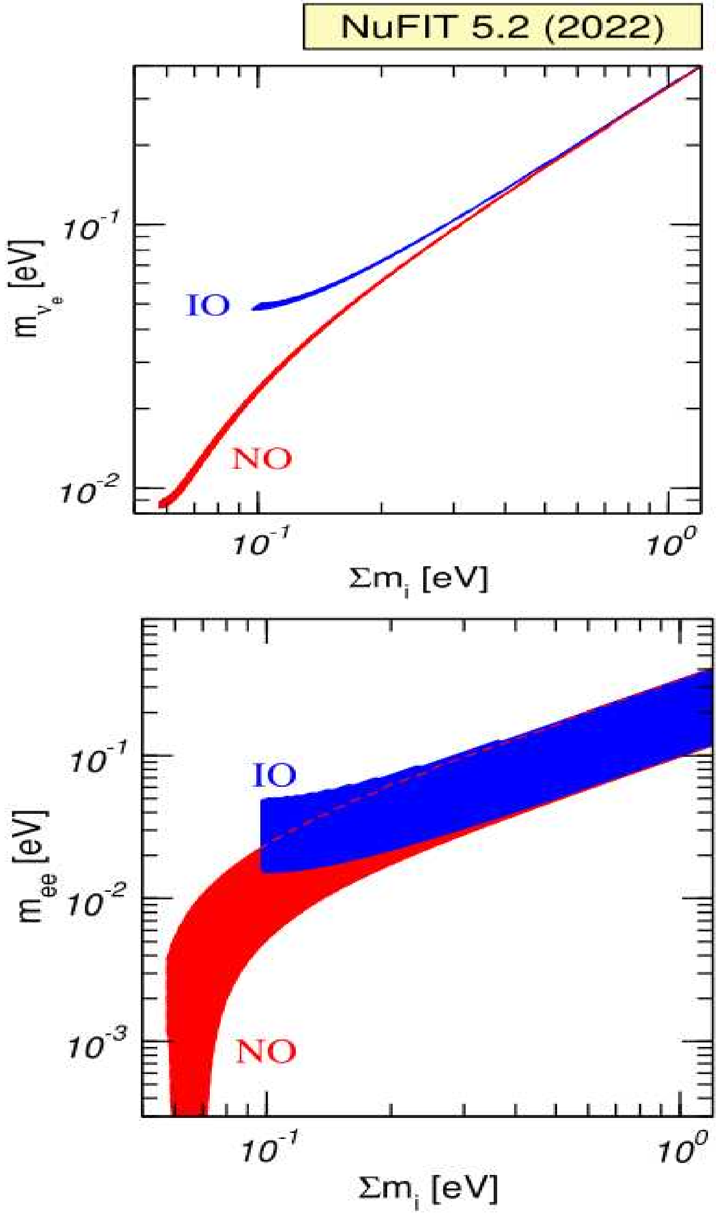
COSMO for Dirac or Majorana

m_ν affect growth of structures

$$\sum m_i = \begin{cases} \text{NO}: \ \sqrt{m_\ell^2} + \sqrt{\Delta m^2_{21} + m_\ell^2} + \sqrt{\Delta m^2_{31} + m_\ell^2} \\ \text{IO} \ \sqrt{m_\ell^2} + \sqrt{-\Delta m^2_{31} - \Delta m^2_{21} - m_\ell^2} + \sqrt{-\Delta m^2_{31} - m_\ell^2} \end{cases}$$

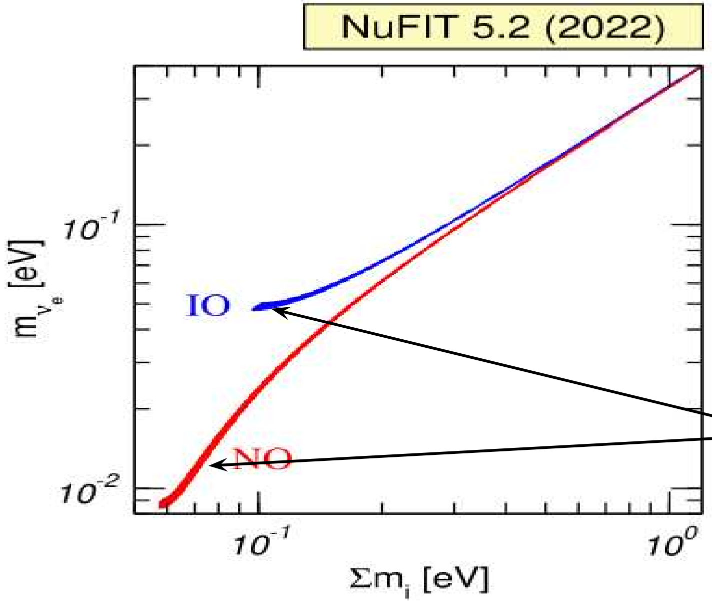
Neutrino Mass Scale: The Cosmo-Lab Connection

Global oscillation analysis ⇒ Correlations m_{ν_e}, m_{ee} and $\sum m_\nu$(Fogli *et al* (04))



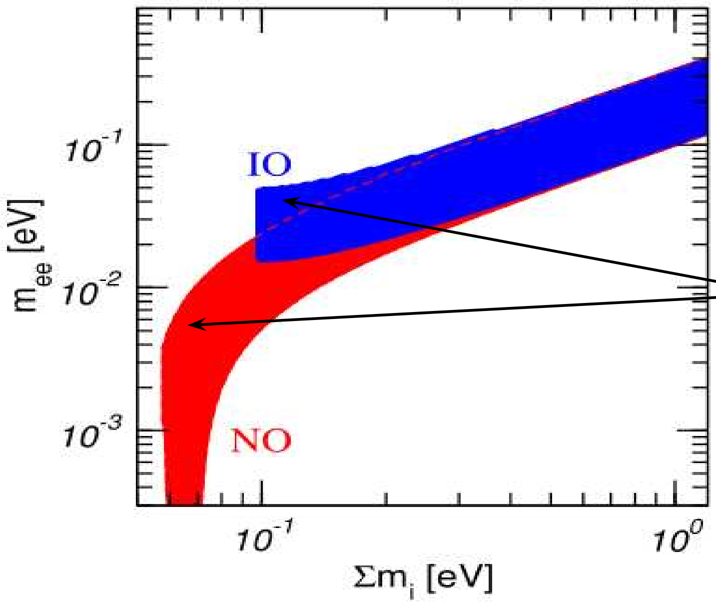
Neutrino Mass Scale: The Cosmo-Lab Connection

Global oscillation analysis $\Rightarrow$ Correlations m_{ν_e}, m_{ee} and $\sum m_\nu$(Fogli *et al* (04))



Width due to range in oscillation parameters very narrow

Lower bound on $\sum m_i$ depends on ordering



Wide band due to unknown Majorana phases $\Rightarrow$

Possible Det of Maj phases?