



University
of Glasgow

USING FLAVOUR TO SIMPLIFY RUNNING IN THE LEFT

Ben Smith

(based on WIP w/ S. Renner, D. Sutherland)

15th July 2024, **UK HEP Forum, 2024**

THE LOW-ENERGY EFFECTIVE FIELD THEORY

Effective field theory valid below the electroweak scale.

$$\mathcal{L}_{LEFT} = \mathcal{L}_{QCD+QED} + \sum_{k,D>4} c_k^{(D)} \mathcal{O}_k^{(D)},$$

where $c_k^{(D)}$ have an implicit suppression of $\frac{1}{\Lambda_{EW}^{D-4}}$.

Running first calculated at one-loop in (Jenkins, Manohar, and Stoffer 2018)

Focus on vectorial operators

$$(\bar{\psi} \gamma_{\mu} P_{L/R} \psi) (\bar{\chi} \gamma^{\mu} P_{L/R} \chi) \quad \psi, \chi \in \{d, e, u\}$$

RUNNING AT ONE LOOP IN THE LEFT

$$(4\pi)^2 \frac{d}{dt} c_V(t) = \boxed{\gamma(t)} c_V(t) \quad (t \equiv \ln \mu)$$



DSixTools basis (Fuentes-Martin, Ruiz-Femenia, Vicente, and Virto [2021](#))

THE LEFT HAS A LARGE (BROKEN) FLAVOUR SYMMETRY

$$(U(3)_{d_L} \times U(3)_{d_R} \rtimes \mathbb{Z}_{2,d}) \times (U(3)_{e_L} \times U(3)_{e_R} \rtimes \mathbb{Z}_{2,e}) \\ \times (U(2)_{u_L} \times U(2)_{u_R} \rtimes \mathbb{Z}_{2,u}) \times U(3)_{\nu_L}$$

Kinetic terms invariant under $d_L^i \rightarrow U_{d_L}^{ij} d_L^j$, $d_R^i \rightarrow U_{d_R}^{ij} d_R^j$,
 $d_L \leftrightarrow d_R, \dots$

Masses and **other operators** break this – their components are *charged* under the flavour group.

$$\mathcal{L} = i\bar{d}_L^i D d_L^i + i\bar{d}_R^i D d_R^i + [\text{sim. for } u_L, u_R, e_L, e_R, \nu_L] \\ - [M^d]_{ij} \bar{d}_L^i d_R^j + \text{h.c.} + [\text{sim. for } M^u, M^e] \\ + c_{ijkl} (\bar{d}_L^i \gamma d_L^j) (\bar{e}_L^k \gamma e_L^l) + [\text{other ops}]$$

(Neglecting purely gluonic operators)

WE USE A SMALLER LARGE (BROKEN) FLAVOUR SYMMETRY

$$SU(3)_d \times SU(3)_e \times SU(2)_u \times \mathbb{Z}_2$$

Kinetic terms invariant under $d_L^i \rightarrow U_d^{ij} d_L^j$, $d_R^i \rightarrow U_d^{ij} d_R^j$, ..., $(d_L, e_L, u_L) \leftrightarrow (d_R, e_R, u_R)$.

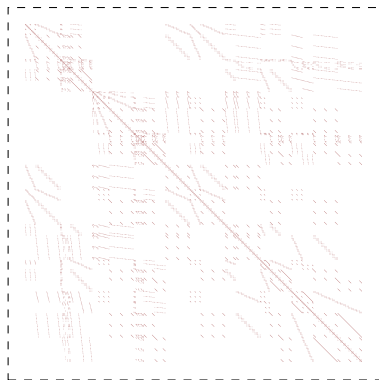
Masses and **other operators** break this – their components are *charged* under the flavour group.

$$\begin{aligned} \mathcal{L} = & i\bar{d}_L^i D d_L^i + i\bar{d}_R^i D d_R^i + [\text{sim. for } u_L, u_R, e_L, e_R, \nu_L] \\ & - [M^d]_{ij} \bar{d}_L^i d_R^j + \text{h.c.} + [\text{sim. for } M^u, M^e] \\ & + c_{ijkl} \left(\bar{d}_L^i \gamma d_L^j \right) \left(\bar{e}_L^k \gamma e_L^l \right) + [\text{other ops}] \end{aligned}$$

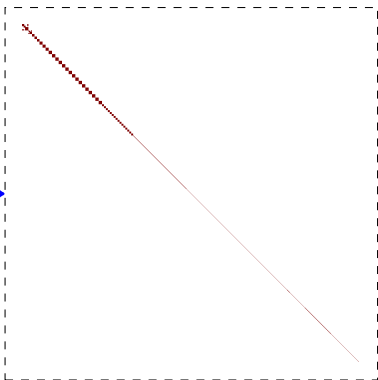
(Neglecting purely gluonic operators)

EFFECT ON ANOMALOUS DIMENSION MATRIX

$$(4\pi)^2 \frac{d}{dt} c_V(t) = \boxed{\gamma(t)} c_V(t) \quad (t \equiv \ln \mu)$$



DSixTools/San Diego basis



Flavour & parity basis

$$(4\pi)^2 \frac{d}{dt} c_V(t) = \gamma(t) c_V(t) \quad (t \equiv \ln \mu)$$

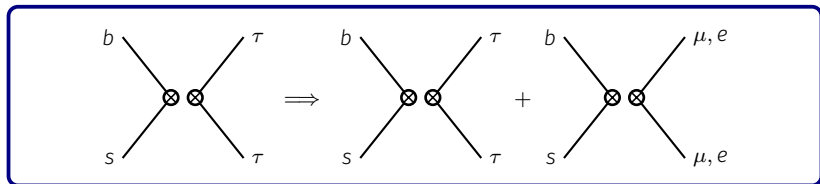
Solve for

$$c_V(t_b) = U(t_b, t_W) c_V(t_W)$$

$$(S^{-1}US)_{ij} = \left(\frac{m_b}{m_W} \right)^{\frac{\hat{\gamma}_i}{(4\pi)^2}} \delta_{ij}$$

No mixing, directions with +ve $\hat{\gamma}$ shrink, -ve $\hat{\gamma}$ grow.

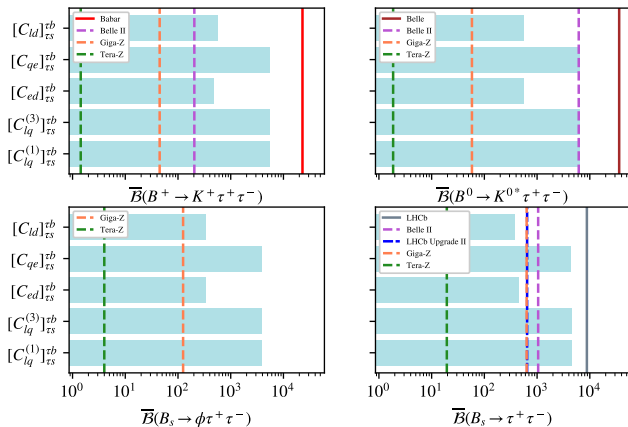
PHENO EXAMPLE



τ only at M_W scale $\implies \tau$, and some e and μ , at m_b scale

PHENO EXAMPLE

$bs\mu\mu$ (teal bars) can be better than current/projected $bS\tau\tau$ (solid/dashed lines)



Also (Cornella, Faroughy, Fuentes-Martin, Isidori, and Neubert [2021](#))

- Flavour and parity simplify running in the LEFT
- γ can be block diagonalised to all orders
- The map $M_W \rightarrow m_b$ is fully understandable in terms of eigenvalues and eigenvectors
- Many possible pheno applications!

BACKUP SLIDES

RUNNING AT ONE LOOP IN THE LEFT

Schematically (Jenkins, Manohar, and Stoffer 2018)

$$\text{masses} \rightarrow (4\pi)^2 \dot{M} = (e^2 + g^2)M + (e + g)dM^2 + c_S M^3 + c_V M^3 + d^2 M^3,$$

$$\text{QED} \rightarrow (4\pi)^2 \dot{e} = e^3 + e^2 dM + d^2 M^2,$$

$$\text{QCD} \rightarrow (4\pi)^2 \dot{g} = g^3 + g^2 dM + d^2 M^2,$$

$$\text{dipoles} \rightarrow (4\pi)^2 \dot{d} = (e^2 + g^2 + eg)d + eM(c_S + c_T) + (e + g)d^2 M,$$

$$4f \text{ scalar} \rightarrow (4\pi)^2 \dot{c}_S = (e^2 + g^2)(c_S + c_T) + (e^2 + g^2 + eg)d^2,$$

$$4f \text{ tensor} \rightarrow (4\pi)^2 \dot{c}_T = (e^2 + g^2)(c_S + c_T),$$

$$4f \text{ vector} \rightarrow (4\pi)^2 \dot{c}_V = (e^2 + g^2)c_V + (e^2 + g^2 + eg)d^2,$$

Neglect operators in grey at $O(0.1\%)$ accuracy.

$\{c_S, c_T\}$ and c_V do not mix due to helicity selection rules.

(Cheung and Shen 2015)

$$\gamma(t) = e^2(t)\hat{\gamma}_e + g^2(t)\hat{\gamma}_g.$$

$$\begin{aligned} \ln U(t_b, t_W) &= \frac{1}{(4\pi)^2} \int \gamma(t_1) + \frac{1}{2(4\pi)^4} \int_{t_1 > t_2} [\gamma(t_1), \gamma(t_2)] \\ &\quad + \frac{1}{6(4\pi)^6} \int_{t_1 > t_2 > t_3} ([\gamma(t_1), [\gamma(t_2), \gamma(t_3)]] + [\gamma(t_3), [\gamma(t_2), \gamma(t_1)]]) + \dots, \\ &= -\hat{\gamma}_e \times 1.803 \times 10^{-3} - \hat{\gamma}_g \times 3.783 \times 10^{-2} - \frac{1}{2}[\hat{\gamma}_e, \hat{\gamma}_g] \times 7.379 \times 10^{-6} + \dots \end{aligned}$$

Neglecting higher order terms matches fully numerical solution to $O(0.0001\%)$ accuracy for lepton universal $b \rightarrow s$ block.



Cheung, Clifford and Chia-Hsien Shen (2015). "Nonrenormalization Theorems without Supersymmetry". In: *Phys. Rev. Lett.* 115.7, p. 071601. DOI: [10.1103/PhysRevLett.115.071601](https://doi.org/10.1103/PhysRevLett.115.071601). arXiv: [1505.01844](https://arxiv.org/abs/1505.01844) [[hep-ph](#)].



Cornella, Claudia et al. (2021). "Reading the footprints of the B-meson flavor anomalies". In: *JHEP* 08, p. 050. DOI: [10.1007/JHEP08\(2021\)050](https://doi.org/10.1007/JHEP08(2021)050). arXiv: [2103.16558](https://arxiv.org/abs/2103.16558) [[hep-ph](#)].



Fuentes-Martin, Javier et al. (2021). "DsixTools 2.0: The Effective Field Theory Toolkit". In: *Eur. Phys. J. C* 81.2, p. 167. DOI: [10.1140/epjc/s10052-020-08778-y](https://doi.org/10.1140/epjc/s10052-020-08778-y). arXiv: [2010.16341](https://arxiv.org/abs/2010.16341) [[hep-ph](#)].



Jenkins, Elizabeth E., Aneesh V. Manohar, and Peter Stoffer (2018). "Low-Energy Effective Field Theory below the Electroweak Scale: Anomalous Dimensions". In: *JHEP* 01. [Erratum: *JHEP* 12, 042 (2023)], p. 084. DOI: [10.1007/JHEP01\(2018\)084](https://doi.org/10.1007/JHEP01(2018)084). arXiv: [1711.05270](https://arxiv.org/abs/1711.05270) [[hep-ph](#)].