Quasi-Dirac neutrino oscillations

Student Talks UK HEP Forum 2024

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8th November 2024

Neutrinos in the Standard Model

Neutrinos in the Standard Model

In the Standard Model, fermions behave according to the Dirac equation,

mass

$$(i\gamma^\mu\partial_\mu - m)(\psi_L^D + \psi_R^D) = 0$$

4 components: $\psi_L, \psi_R, \psi_L^C, \psi_R^C$

left-chiral part

right-chiral part

But for a massless fermion, $m = 0$, and

$$i\gamma^\mu\partial_\mu\psi_L = 0 \qquad\qquad i\gamma^\mu\partial_\mu\psi_L = 0$$

ψ_L and ψ_R are thus independent in this case. We can eliminate two degrees of freedom, giving us a left-chiral Weyl field,

$$i\gamma^\mu\partial_\mu\psi_L^W = 0$$

2 components: ψ_L, ψ_R^C

Neutrinos are strictly LH within the SM so are described by Weyl spinors

$\Longrightarrow$ Neutrinos are thus massless within the SM

$\Longrightarrow$ But neutrino oscillations tell us that they must be massive!!

Massive neutrinos

Introducing sterile neutrinos

So, in order to generate neutrino masses, we need to add a right-handed neutrino.

In the SM, the W boson only interacts with left-chiral particles, such as ν_L and ν_R^C. As those particles interact with the SM, we refer to them as active neutrinos, ν

This means that right-chiral neutrinos, ν_L^C and ν_R do not interact with any of the forces within the SM, and are thus sterile neutrinos, N.

Dirac vs. Majorana

Option 1: Dirac $\nu_L \neq \nu_R^C$

Add a RH neutrino field, $\nu^D = \nu_L + \nu_R$,

$$-\frac{\lambda_\nu v}{\sqrt{2}}(\overline{\nu_L}\nu_R + \overline{\nu_R}\nu_L) \xrightarrow[\text{Higgs mechanism}]{} -m_D(\overline{\nu_L}\nu_R + \overline{\nu_R}\nu_L)$$

$$\Longrightarrow \boxed{\text{4 chiral fields : } \nu_L, \nu_L^C, \nu_R, \nu_R^C}$$

$\Longrightarrow$ As $m_D \ll 1\text{eV}$, this implies the need for unnatural Yukawa couplings $\lambda_\nu \approx 10^{-12}$

Option 2: Majorana $\nu_L = \nu_R^C$

Neutrinos could also be their own antiparticle: $\underbrace{\nu^M = (\nu^M)^C}_{\text{Majorana condition}}$. This implies,

$$\nu_R = (\nu_L)^C \longrightarrow -m_L(\overline{\nu_L}(\nu_L)^C + (\overline{\nu_L})^C\nu_L)$$

$$\Longrightarrow \boxed{\text{2 chiral fields : } \nu_L, \nu_L^C}$$

$\Longrightarrow$ Lepton Number is violated

Dirac-Majorana masses

Rewrite the mass Lagrangian assuming left and right-handed neutrinos

$$\begin{aligned}\mathcal{L}_M &= \mathcal{L}_D + \mathcal{L}_M \\ &= \left(-\frac{1}{2}m_L(\overline{\nu}_L(\nu_L)^C + (\overline{\nu}_L)^C\nu_L) - \frac{1}{2}m_R(\overline{\nu}_R(\nu_R)^C + (\overline{\nu}_R)^C\nu_R)\right) - \left(m_D\overline{\nu}_L\nu_R\right) \\ &= -\overline{\nu^{DM}}\,\mathcal{M}\,(\nu^{DM})^C\end{aligned}$$

$$\boxed{\mathcal{M} = \begin{pmatrix} m_L & m_D \\ m_D & m_R \end{pmatrix}}$$

$$\boxed{\nu^{DM} = \begin{pmatrix} \nu_L^{DM} = \frac{\nu_L+(\nu_L)^C}{\sqrt{2}} \\ \nu_R^{DM} = \frac{\nu_R+(\nu_R)^C}{\sqrt{2}} \end{pmatrix}}$$

Eigenvalues give the physical masses

Dirac-Majorana masses

$$\mathcal{M} = \begin{pmatrix} m_L & m_D \\ m_D & m_R \end{pmatrix} \underset{\text{diagonalise}}{\Longrightarrow} \quad m_{1,2} = \frac{1}{2}\left(m_L + m_R \pm \sqrt{(m_L - m_R)^2 + 4m_D^2}\right)$$

Scenario 1: $m_L = m_D = 0 \longrightarrow m_1 = m_L, m_2 = 0$ **Majorana; Lepton number violated**

Scenario 2: $m_R \gg m_D \longrightarrow m_1 \approx m_R, m_2 \approx \frac{m_D^2}{m_R}$ **Type-I seesaw mechanism**

Scenario 3: $m_L = m_R = 0 \longrightarrow m_1 = m_2 = m_D$ **Dirac; lepton number conserved**

A fourth scenario: Quasi-Dirac neutrinos

In the third scenario, the eigenstates and masses are given by,

$$m_{1,2} \approx m_D$$

$$\nu_1 \approx \frac{1}{\sqrt{2}}[\nu + \nu^C] = \nu_1^C, \qquad \nu_2 \approx \frac{1}{\sqrt{2}}[-\nu + \nu^C] = \nu_2^C$$

$$\boxed{\nu = \nu_1 + \nu_2}$$

$\Longrightarrow$ in the limit that $m_L = m_R = 0$, we recover the Dirac scenario. We obtain two Majorana neutrinos which can be combined to form a single Dirac neutrino.

Dirac-Majorana masses

$$\mathcal{M} = \begin{pmatrix} m_L & m_D \\ m_D & m_R \end{pmatrix} \underset{\text{diagonalise}}{\Longrightarrow} \quad m_{1,2} = \frac{1}{2}\left(m_L + m_R \pm \sqrt{(m_L - m_R)^2 + 4m_D^2}\right)$$

Scenario 1: $m_L = m_D = 0 \longrightarrow m_1 = m_L, m_2 = 0$ **Majorana; Lepton number violated**

Scenario 2: $m_R \gg m_D \longrightarrow m_1 \approx m_R, m_2 \approx \frac{m_D^2}{m_R}$ **Type-I seesaw mechanism**

Scenario 3: $m_L = m_R = 0 \longrightarrow m_1 = m_2 = m_D$ **Dirac; lepton number conserved**

Scenario 4: $m_R \ll m_D \longrightarrow$???

A fourth scenario: Quasi-Dirac neutrinos

But, small deviations from the limit $m_L = m_R = 0$ lead to a quasi-Dirac scenario where lepton number is no longer exactly preserved. We define,

$$\Delta = \frac{m_L - m_R}{4m_D} \qquad \eta = \frac{m_L + m_R}{2m_D}$$

For $\eta \ll 1$ and $\Delta \ll 1$, we obtain

$$m_{1,2} \approx m_D(1 \pm \eta)$$

$$\nu_1 \approx \frac{1}{\sqrt{2}}[(1+\Delta)\nu + (1-\Delta)\nu^C], \qquad \nu_2 \approx \frac{1}{\sqrt{2}}[(-1+\Delta)\nu + (1+\Delta)\nu^C]$$

$\Longrightarrow$ if we have a small mass splitting due to deviations from the Dirac limit, we obtain two massive Majorana which don't exactly add up to one single Dirac neutrino. This induces high frequency oscillations as well as small sources of lepton number violation.

$\Longrightarrow$ Quasi-Dirac neutrinos

Motivation

Motivation

Why do we care?

- Soft probes for lepton number violation
- Would allow us to obtain neutrinos that behave almost like a Dirac particle without the need for very small Yukawa couplings

How can we probe them?

- We introduced two quasi-dirac neutrinos in addition to the three SM active neutrinos and studied their oscillations at Long and Short Baseline experiments

Results

Long Baseline experiments

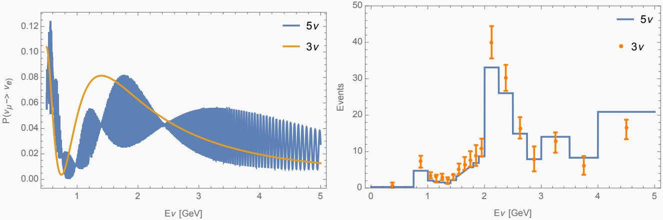


Figure 1: 3 and 5 neutrino oscillations at NOvA. The mass splitting induces rapid oscillations which influence $P(\nu_\mu \to \nu_e)$.

Short Baseline experiments

In SBL, oscillations due to atmospheric and solar mass splittings are neglected, $m_1 = m_2 = m_3$ such that

$$P(\nu_\mu \to \nu_e) = 4\left|U_{\mu 4}^2\right|\left|U_{e4}^2\right| \sin^2 x_{41} + 4\left|U_{\mu 5}^2\right|\left|U_{e5}^2\right| \sin^2 x_{51} \\ + 8|U_{\mu 5}||U_{e5}||U_{\mu 4}||U_{e4}| \sin x_{41} \sin x_{51} \cos(x_{54} - \phi_{45}),$$

i.e the oscillations only depend on the quasi-dirac neutrinos.

Questions?

Backup slides

$3+2$ neutrino model

We consider a five-flavour neutrino model, and introduce two sterile neutrinos ν_{s1}, ν_{s2} in addition to the SM 3 active ones.

To switch between the weak and mass eigenbasis, we again need a unitary matrix U.

Because we have 5 eigenstates here,

$$U = R(\theta_{45}, \phi_{45})R(\theta_{35}, \phi_{35})...R(\theta_{23}, \phi_{23})R(\theta_{13}, \phi_{13})R(\theta_{12}, \phi_{12}) \tag{1}$$

$R(\theta_{ab}, \phi_{ab})$ is the matrix of complex rotations in the $a-b$ plane.

... and from this we can determine oscillations at neutrino experiments

Short Baseline experiments

In Short Baseline experiments, oscillations due to atmospheric and solar mass splittings are neglected, such that we can set $m_1 = m_2 = m_3$. In this case,

$$P(\nu_\mu \to \nu_e) = 4\left|U_{\mu 4}^2\right|\left|U_{e4}^2\right| \sin^2 x_{41} + 4\left|U_{\mu 5}^2\right|\left|U_{e5}^2\right| \sin^2 x_{51} \tag{2}$$

$$+ 8|U_{\mu 5}||U_{e5}||U_{\mu 4}||U_{e4}| \sin x_{41} \sin x_{51} \cos(x_{54} - \phi_{45}), \tag{3}$$

where

$$\phi_{45} = arg(U_{\mu 5}^* U_{e5} U_{\mu 4} U_{e4}^*) \qquad \text{and} \qquad x_{ij} = \frac{\Delta m_{ij}^2 L}{4E}, \tag{4}$$

$\Longrightarrow$ so for SB experiments, the oscillation is only dependent on the active/sterile mixing parameters.

Simulating the model at NOvA

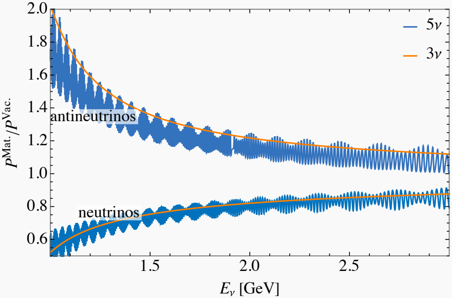
The MSW Effect

In the NOvA detector, neutrinos interact with matter as they propagate.

We assume that only e^- are present in the detector: they induce a CC potential V_{CC} which ν_e is subject to and a NC potential V_{NC} which all the active neutrinos are subject to.

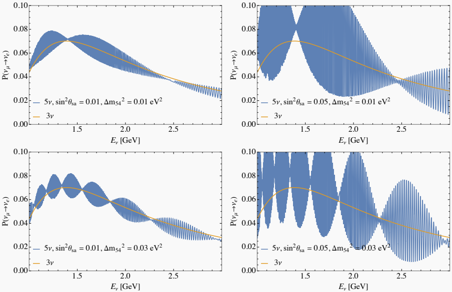
The Hamiltonian becomes

$$H' = H_0 + V = \frac{1}{2E}(U\boldsymbol{M}^2U^\dagger + \boldsymbol{V}), \quad \text{where} \quad \boldsymbol{V} = diag(2EV_{CC}, 0, 0, -2EV_{NC}, -2EV_{NC})$$



Simulating the model at NOvA

Oscillation Probability $P(\nu_\mu \to \nu_e)$



- $P(\nu_\mu \to \nu_e)$ fluctuates considerably more than in the 3ν case. This is because of the small Δm_{45}^2 which results in the superposition of two waves of similar wavelengths and frequencies.
- Larger values of Δm_{45}^2 induce a higher amplitude and wavepackets with a higher frequency.
- Larger values of $\sin^2(\theta_{sa})$ give rise to a large envelope and a higher average amplitude.
- As θ_{sa} and $\Delta m_{54}^2 \to 0$, we approach the 3ν model.