Muon cLFV + Kaon motivation

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Motivation

- Origin and stability of the electroweak scale
- Strong CP problem
- Neutrino masses
- Structure of fermion generations and masses
- Dark matter
- Origin of the matter/ antimatter asymmetry
- Dark energy = cosmological constant?
- Cosmological history at $T \gtrsim \mathrm{MeV}$
- Quantum gravity. Inflation. Supersymmetry.

Talk by: Ed Hardy

- Structure of fermion generations and masses
- For non ultra heavy NP: link
  - Seesaw mechanism
  - Lepto- / Baryogenesis
  - Test ν interactions
  - New sources of CP violation
  - Decay into light new particles

Charged (μ) Lepton Flavour Violation

- Dirac masses: $\mathcal{B}(\mu \to e\gamma) \simeq (G_F \Delta m_\nu^2)^2 \sim 10^{-50}, \ldots$
- Majorana masses: (Dim 5)^2 $\to (H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{L}\gamma^\mu L)$

	Current	Future
$\mu \to e\,\gamma$	$< 4.2 \times 10^{-13}$	$\sim 10^{-14}$ (MEG II)
$\mu \to e\,e\,e$	$< 1.0 \times 10^{-12}$	$\sim 10^{-16}$ (Mu3e)
μ A $\to$ e A	$< 7 \times 10^{-13}$	$\sim 10^{-16}$ (COMET, Mu2e)

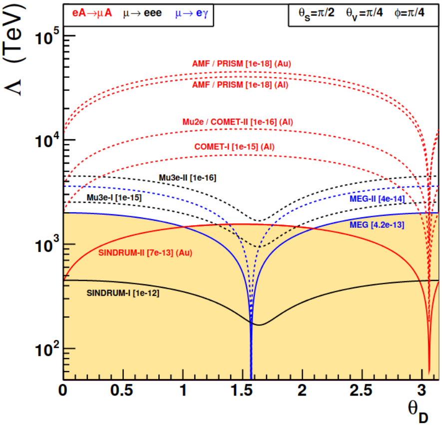
- Observation of LFV: Clear signal of new physics
- Could test mechanism of neutrino masses
- Review: Kuno, Okada hep-ph/9909265

Weak Effective Theory Constraints

$$\delta\mathcal{L} = \frac{1}{\Lambda_{LFV}^2}\Big[C_D(m_\mu\bar{e}\sigma^{\alpha\beta}P_R\mu)F_{\alpha\beta} + C_S(\bar{e}P_R\mu)(\bar{e}P_Re) + C_{VR}(\bar{e}\gamma^\alpha P_L\mu)(\bar{e}\gamma_\alpha P_Re)$$
$$+ C_{VL}(\bar{e}\gamma^\alpha P_L\mu)(\bar{e}\gamma_\alpha P_Le) + C_{Alight}\mathcal{O}_{Alight} + C_{Aheavy\perp}\mathcal{O}_{Aheavy\perp}\Big]$$

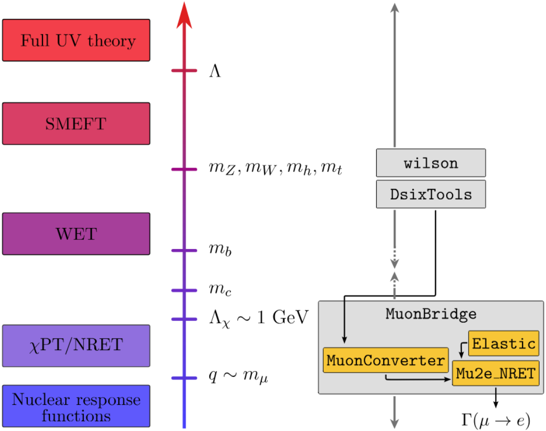
- Complementary information
- Including RGE & Parametrisation 2204.00564

$\vec{C}\cdot\vec{e}_D$	$\lvert\vec{e}_D\rvert\cos\theta_D$
$\vec{C}\cdot\vec{e}_S$	$\lvert\vec{e}_S\rvert\sin\theta_D\cos\theta_S$
$\vec{C}\cdot\vec{e}_{VL}$	$\lvert\vec{e}_{VL}\rvert\sin\theta_D\sin\theta$
$\vec{C}\cdot\vec{e}_{VR}$	$\lvert\vec{e}_{VR}\rvert\sin\theta_D\sin\theta$
$\vec{C}\cdot\vec{e}_{Alight}$	$\lvert\vec{e}_{Alight}\rvert\sin\theta_D$ si
$\vec{C}\cdot\vec{e}_{Aheavy\perp}$	$\lvert\vec{e}_{Aheavy\perp}\rvert\sin\theta_D$



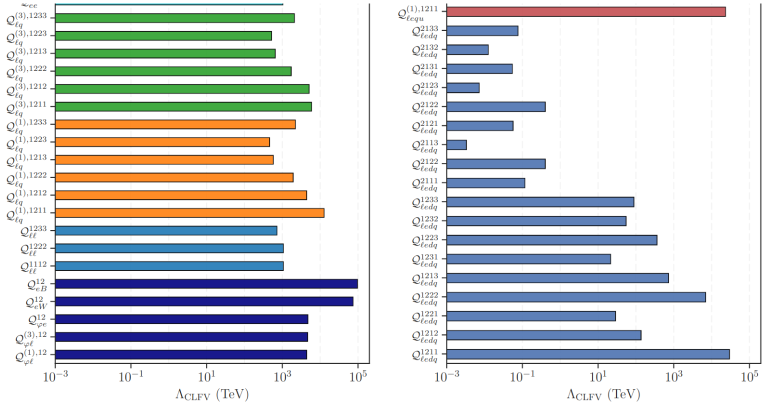
EFT for muon conversion

- μ A $\to$ e A similar to WIMP. Tower of EFTs and Matching: 2406.13818



SMEFT constraints

- Constraints from $\mathcal{B}(\mu + Al \to e + AL) < 10^{-17}$ (2406.13818)
- e.g. on $\mathrm{O_{eB}} = (\bar{L}\sigma^{\mu\nu}e)HB_{\mu\nu}$



Beyond $\mu \to$ e EFT

- Distinguish TeV scale models,
  - E.g. Type-I and Type-II seesaw. 2308.16897
- τ LFV: $\mu \to \tau \to e$ physics 2202.09246
- NSIs for $q^2 \ll M_W^2$: $\mathcal{L} \supset -2\sqrt{G_F}\varepsilon^{\rho\sigma}_{f,(L)}(\overline{\nu}_\rho \gamma_\alpha \nu_\sigma)(\bar{f}\gamma^\alpha (P_L) f)$
  - In SMEFT: NSI typically generate LFV 1807.04283

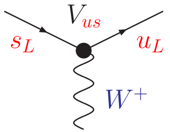
f	$C_{V,LR}^{\mu\, eff}$	$\varepsilon^{\mu\, e}_{fR}(1l)$	$\varepsilon^{\mu\, e}_{fR}(2l)$
e	$< 9.3 \cdot 10^{-7}$	$< 5 \cdot 10^{-5}$	$9 \cdot 10^{-3}$
u	$< 5.4 \cdot 10^{-8}$	$< 3 \cdot 10^{-6}$	$5 \cdot 10^{-4}$
d	$< 6.3 \cdot 10^{-8}$	$< 3 \cdot 10^{-6}$	$6 \cdot 10^{-4}$

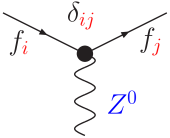
μ Lepton Flavour Violation

- No SM background
- LFV Measurement: Clear signal of new physics
  - Could give handle ν-mass generation, lepton number violation
- Limits give complementary information to collider- and ν-experiments

FCNCs in the Quark Sector

Mass ≠ flavour eigenstates





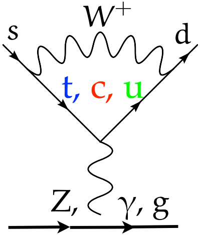
SM: Only charged currents change the flavour ($\propto V_{us}$)

SM: Neutral currents do not change the flavour (i=j) at tree-level

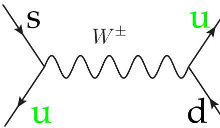
$$V_{\text{CKM}} = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho+i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

- SM: Yukawas only source of flavour & CP violation
- CKM parametrises CP & flavour violation
- First row from tree-level semi-leptonic decays

Rare Kaon Decays: CKM Structure



Using the GIM mechanism, we can eliminate either $V_{cs}^* V_{cd}$ or $V_{us}^* V_{ud} \rightarrow - V_{cs}^* V_{cd} - V_{ts}^* V_{td}$



Z-Penguin and Boxes (high virtuality): power expansion in: $A_c - A_u \propto 0 + O(m_c^2/M_W^2)$

γ/g-Penguin (expand in mom.): $A_c - A_u \propto O(\mathrm{Log}(m_c^2/m_u^2))$

$$\mathrm{Im}V_{ts}^* V_{td} = -\mathrm{Im}V_{cs}^* V_{cd} = \mathcal{O}(\lambda^5) \qquad \mathrm{Im}V_{us}^* V_{ud} = 0$$

$$\mathrm{Re}V_{us}^* V_{ud} = -\mathrm{Re}V_{cs}^* V_{cd} = \mathcal{O}(\lambda^1) \qquad \mathrm{Re}V_{ts}^* V_{td} = \mathcal{O}(\lambda^5)$$

- $K \rightarrow \pi\bar{\nu}\nu$ (from Z & Boxes): Clean and suppressed
- $K \rightarrow (\pi)\mu\mu$: γ-Penguin pollution $\rightarrow$ isolate SD piece from mixing

Rare Kaon Decays SD contributions

- $K \to \pi\nu\bar{\nu}$ are theoretically super clean
- Use CP properties for neutral decays into leptons
  - K_S: CP even ; K_L CP odd
  - $K_S \to \mu\ \mu$ – extract CP violating part via mixing
  - $K_L \to \pi\ \ell\ \ell$ – indirect and direct CP violation interfere
    - indirect $K_L \to K_S \to \pi\ \ell\ \ell$

$K \to \mu^+\mu^-$

SD: $Y_t + Y_{NNLO}$ from SM $\mathcal{L}_{eff}^{d=6}$ gives $\ell = 0$. $[\mathcal{O}(\lambda_t, \frac{m_c^2}{M_W^2}\lambda_c)]$

LD: from γ-γ loop is CP conserving: $\mathcal{O}(\frac{\alpha_{QED}}{4\pi})$

Setting ϵ_K to zero, K_L and K_S are CP eigenstates:

K_i^{CP}	BR_{exp}	$(\mu^+\mu^-)^-_{\ell=0}$	$(\mu^+\mu^-)^+_{\ell=1}$
$K_L^{(-)}$	$6.84(11) \times 10^{-9}$	λ_t, $\frac{m_c^2}{M_W^2}\lambda_c$, γ-γ	(~~CP~~) $\simeq 0$
$K_S^{(+)}$	$< 2.1 \times 10^{-10}$	~~CP~~: Im λ_t	γ-γ

ℓ is not experimentally accessible, but interference term in time dependent decay sensitive to *SD* (D'Ambrosio, Kitahara '17)

$K(t) \to \mu^+\mu^-$

$$\frac{d\Gamma}{dt} \propto C_L e^{-\Gamma_L} + C_S e^{-\Gamma_S} + 2[C_{sin}\sin(\Delta M t) + C_{cos}\cos(\Delta M t)]e^{-\Gamma t}$$

$$C_{sin/cos} = Im/Re\left\{(A_0^S)^* A_0^L + (A_1^S)^* A_1^L\right\}$$

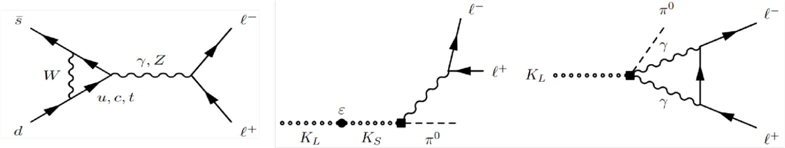
$$\frac{BR(K_S \to \mu^+\mu^-)^{(pert)}_{\ell=0}}{BR(K_L \to \mu^+\mu^-)} = \frac{\tau_S}{\tau_L}\frac{|A_0^S A_0^L|^2}{|A_0^L|^4} = \frac{\tau_S}{\tau_L}\frac{C_{int}^2}{C_L^2}$$

where we assumed $A_1^L = 0$, $\epsilon_K = 0$

- Measurement of $BR(K_S \to \mu^+\mu^-)_{\ell=0}$ (Dery et.al.'21)

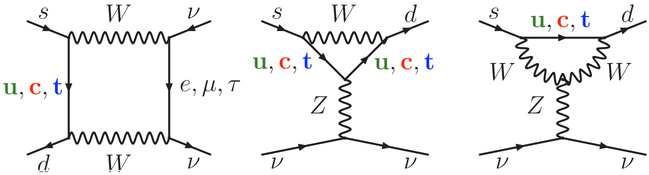
$$BR(K_S \to \mu^+\mu^-)^{(pert)}_{\ell=0} = 1.70(02)_{QCD/EW}(01)_{f_K}(19)_{param.} \times 10^{-13}$$

$K \to \pi\, \ell\, \ell$



- CP violation: direct, indirect & interference
  - $K_s \to \pi\, \ell\, \ell$ gives 1 γ contribution, can be extracted from data
  - Sign of interference
    - large N_c 2409.08568
    - future: Lattice 1608.07585, 1507.03094 – interesting interplay of perturbation theory 2112.11140.

$K \to \pi\bar{\nu}\nu$ at M_W



$$x_i = \frac{m_i^2}{M_W^2}$$

$$\sum_i V_{is}^* V_{id} F(x_i) = V_{ts}^* V_{td}(F(x_t) - F(x_u)) + V_{cs}^* V_{cd}(F(x_c) - F(x_u))$$

Quadratic GIM: $\lambda^5 \frac{m_t^2}{M_W^2}$ $\quad \lambda \frac{m_c^2}{M_W^2} \ln \frac{M_W}{m_c}$ $\quad \lambda \frac{\Lambda_{QCD}^2}{M_W^2}$

Matching (NLO +EW):

Operator Mixing (RGE)

ChiPT & Lattice

$$Q_\nu = (\bar{s}_L \gamma_\mu d_L)(\bar{\nu}_L \gamma^\mu \nu_L)$$

- Below the charm: Only Q_ν, ME from K_{l3}
- semi-leptonic $(\bar{s}\gamma_\mu u_L)(\bar{\nu}\gamma^\mu \ell_L)$ operator: χ PT gives small contribution (10% of charm contribution)

Leading Effective Hamiltonian for $\mu < m_c$

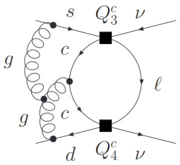
SM: $\nu\bar{\nu}$ are only invisibles $\Rightarrow$ no γ-Penguin $\Rightarrow$

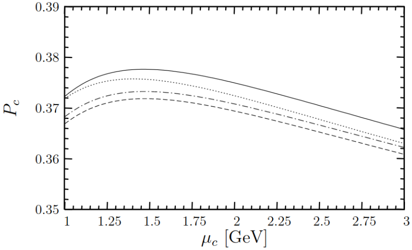
$$\mathcal{H}_{\text{eff}} = \frac{\sqrt{2}\alpha G_F}{\pi \sin^2\theta_w} \sum_{\ell=e,\mu,\tau} (\lambda_c X^\ell + \lambda_t X_t)(\bar{s}_L \gamma_\mu d_L)(\bar{\nu}_{\ell L}\gamma^\mu \nu_{\ell L}) + \text{h.c.}$$

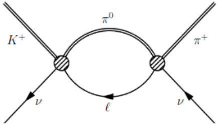
generated by highly virtual particles + tiny light quark contribution $\Rightarrow$ clean & CKM suppressed ($\lambda_i = V_{is}^* V_{id}$).

- X_t known at NLO QCD (Buchalla,Buras;Misiak,Urban'99) and two-loop EW (Brod et.al.'10): $X_t = 1.462 \pm 0.017_{\text{QCD}} \pm 0.002_{\text{EW}}$
- $P_c = \lambda^{-4}(\frac{2}{3}X^e + \frac{1}{3}X^\tau)$ at NNLO QCD (Buras et.al.'05) + NLO EW (Brod et.al.'08)

Higher order corrections for P_c







- GIM in the EFT: (Charm - Up) only $m_c^2 G_F^2 Q_\nu$ above m_c
  - $P_c = (0.2255/\lambda)^4 \times (0.3604 \pm 0.0087)$ @ NNLO+EW (Brod et.al.'21)
- Small corrections from higher dimensional operator + light quarks below m_c
  - χ PT matching logs: $\delta P_{c,u} = 0.04 \pm 0.02$ (Isidori et.al.'05)
  - Future Lattice: (Bai et.al.'18,2311.02923)

CKM parameters for $\mathcal{H}_{\text{eff}}$

- We need Imλ_t, Reλ_t and Reλ_c:
- Improvements 1911.06822 in $\epsilon_K \to$ change V_{cb}
- Use exact Wolfenstein parameterization (PDG)
  - $s_{12} = \lambda$, $s_{23} = A\lambda^2$, $s_{13}e^{i\delta} = A\lambda^3(\rho + i\eta)$:

$$\text{Im}\lambda_t = A^2\bar{\eta}\lambda^5 + \ldots, \text{Re}\lambda_c = -\lambda + \ldots$$

$$\text{Re}\lambda_t = A^2\lambda^5(\bar{\rho} - 1) + \ldots$$

	$\bar{\rho}$	$\bar{\eta}$	λ	A
PDG20	0.141(17)	0.357(11)	0.22650(48)	0.790(17)
PDG22	0.159(10)	0.348(10)	0.22500(67)	$0.826^{+0.018}_{-0.015}$
UTfit	0.159(10)	0.348(9)	0.22499(67)	0.833(11)

$K \to \pi\nu\bar{\nu}$ in the Standard Model

- 2105.02868 Standard Model Prediction

$$\mathrm{BR}(K^+ \to \pi^+\nu\bar{\nu}) = 7.73(16)_{SD}(25)_{LD}(54)_{para.} \times 10^{-11},$$
$$\mathrm{BR}(K_L \to \pi^0\nu\bar{\nu}) = 2.59(6)_{SD}(2)_{LD}(28)_{para.} \times 10^{-11}.$$

- 2311.02923 using UTfit CKM parameters:

$$\mathrm{BR}(K^+ \to \pi^+\nu\bar{\nu}) = 8.38(17)_{SD}(25)_{LD}(40)_{para.} \times 10^{-11},$$
$$\mathrm{BR}(K_L \to \pi^0\nu\bar{\nu}) = 2.87(7)_{SD}(2)_{LD(}23)_{para.} \times 10^{-11}.$$

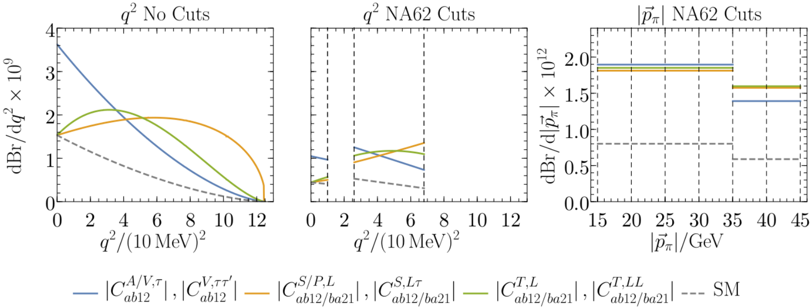
- Consistent with ϵ_K 'ratio' 2203.11960

$$\mathrm{BR}(K^+ \to \pi^+\nu\bar{\nu}) = 8.44(41) \times 10^{-11},$$

- NA62 collaboration (before 24.9.):
  $\mathrm{BR}(K^+ \to \pi^+\nu\bar{\nu}) = (10.6^{+4.0}_{-3.4}|_{\mathrm{stat}} \pm 0.9_{\mathrm{syst}}) \times 10^{-11}$
- JPARC-KOTO has $\mathrm{BR}(K_L \to \pi^0\nu\bar{\nu}) \leq 2.0 \times 10^{-9}$

Distributions for WET @ 90%CL

- Majorana & Dirac Wilson coefficients saturate 90% CL
  - left: q^2 distribution integrated over full $|p_\pi|$
  - middle: q^2 distribution for NA62 cuts
- right: $|p_\pi|$ distribution with q^2 region as NA62



Sensitivity NA62 and HIKE like

- General quark-ν-interactions 2312.06494:

$$O^{V,L}_{abij} = \frac{1}{2}(\overline{\nu}_{Ma}\gamma_\mu \nu_{Mb})(\overline{d}_i \gamma^\mu P_L d_j) , \ldots$$

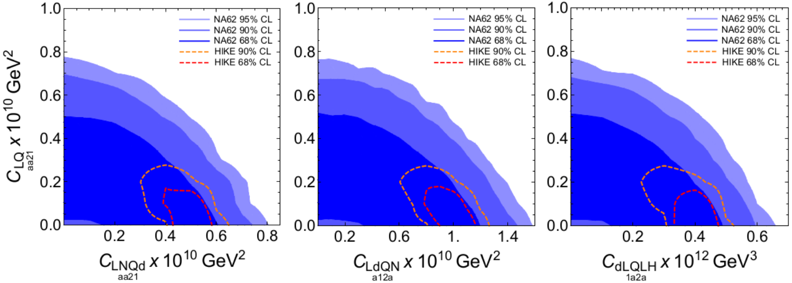
ν-Majorana EFT		**current**	**future**	**future[q^2 bins]**
$1/\sqrt{\lvert C^{V,L/R}_{ab12}\rvert}$	$a \neq b$	$7.6 \cdot 10^1$ TeV	$1.2 \cdot 10^2$ TeV	$1.2 \cdot 10^2$ TeV
$1/\sqrt{\lvert C^{A,L/R}_{ab12}\rvert}$	$\sum a = b$	$1.2 \cdot 10^2$ TeV	$2.8 \cdot 10^2$ TeV	$2.6 \cdot 10^2$ TeV
	$a = b$	$8.1 \cdot 10^1$ TeV	$1.7 \cdot 10^2$ TeV	$1.6 \cdot 10^2$ TeV
	$a \neq b$	$7.6 \cdot 10^1$ TeV	$1.2 \cdot 10^2$ TeV	$1.2 \cdot 10^2$ TeV
$1/\sqrt{\lvert C^{S,L}_{ab12/21}\rvert}$	$\sum a = b$	$1.6 \cdot 10^2$ TeV	$2.4 \cdot 10^2$ TeV	$2.5 \cdot 10^2$ TeV
	$a = b$	$1.2 \cdot 10^2$ TeV	$1.9 \cdot 10^2$ TeV	$1.9 \cdot 10^2$ TeV
	$a \neq b$	$1.4 \cdot 10^2$ TeV	$2.2 \cdot 10^2$ TeV	$2.3 \cdot 10^2$ TeV
$1/\sqrt{\lvert C^{P,L}_{ab12/21}\rvert}$	$\sum a = b$	$1.6 \cdot 10^2$ TeV	$2.4 \cdot 10^2$ TeV	$2.5 \cdot 10^2$ TeV
	$a = b$	$1.2 \cdot 10^2$ TeV	$1.9 \cdot 10^2$ TeV	$1.9 \cdot 10^2$ TeV
	$a \neq b$	$1.4 \cdot 10^2$ TeV	$2.2 \cdot 10^2$ TeV	$2.3 \cdot 10^2$ TeV
$1/\sqrt{\lvert C^{T,L}_{ab12/21}\rvert}$	$a \neq b$	$3.0 \cdot 10^1$ TeV	$4.7 \cdot 10^1$ TeV	$4.7 \cdot 10^1$ TeV

(ν) SMEFT

- $\mathcal{L}_{\nu\text{SMEFT}} \supset \mathcal{L}_{\text{SM}} + i\overline{N}\partial\!\!\!/ N + \left\{-\frac{1}{2} m_N \overline{N} N^c - Y\left(\overline{N} L\right)\epsilon H + \text{h.c.}\right\}$
- SMEFT has no extra sterile Weyl fermions N
- ν SMEFT with Lepton Number Conservation (LNC): $m_N = 0$
- scalar and tensor operators at d=6 (LNC)
  - $\mathcal{Q}^{(6)}_{LNQd} = \epsilon_{ij}(\overline{L}^i N)(\overline{Q}^j d)$ and $\mathcal{Q}^{(6)}_{LdQN} = \epsilon_{ij}(\overline{L}^i d)(\overline{Q}^j N)$
- scalar and (tensor) operator at d=7 in SMEFT (LNV)
  - $\mathcal{Q}^{(7)}_{dLQLH} = \epsilon_{ij}\epsilon_{mn}(\overline{d} L^i)(\overline{Q^c}^j L^m) H^n$

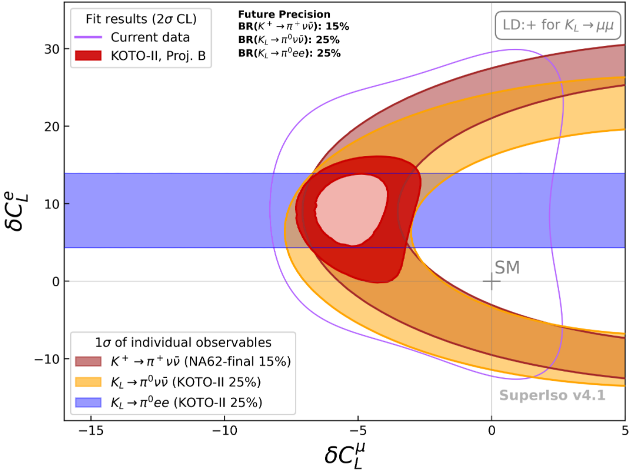
Can we disentangle Operators?

- Assume $C_{LNQd}^{(6)} = 0.5 \times 10^{-10}$ GeV^{-2}
- Try to fit events with SMEFT and ν SMEFT



- Does not allow for Vector explanation
- Can be explained with d=7, but at lower scale (10TeV) instead of (100TeV)

$$\mathcal{L}_{NP} = \delta C_L^\ell \sqrt{2} G_F V_{ts}^* V_{td} \frac{\alpha}{4\pi} (\bar{s}\gamma_\mu d_L) \left[(\bar{\ell}\gamma^\mu \ell_L) + (\bar{\nu}_\ell \gamma^\mu \nu_{\ell L}) \right] + \text{h.c.}$$

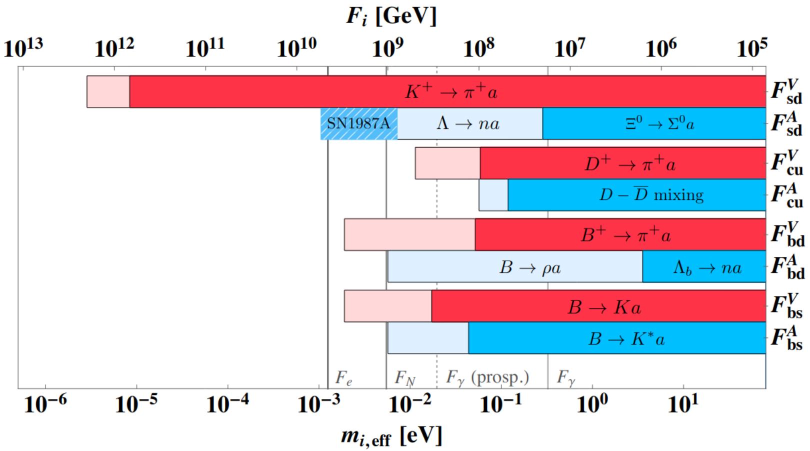


Cross check New Physics in $K \to \pi\bar{\nu}\nu$ with $K \to \pi e^+ e^-$

(2409.06545)

Axion s-d couplings

- $\mathcal{L}_{aff} = \frac{\partial_\mu a}{2f_a}\bar{f}_i\gamma^\mu(c^V_{ij} + c^A_{ij}\gamma_5)f_j$, with generic hermitian $c^{V,A}_{ij}$
- Tests generic $F^{V,A}_{ij} = 2f_a/c^{VA}_{ij}$ couplings 2002.04623



- Also $K_L \to \pi^0 a$ & $K_L \to \pi^0\pi^0 a$

Conclusion

- Flavour physics: sensitivities to High scales and light new physics
- EFT approach to generic new physics
- Precision prediction using perturbation theory
  - using interplay with lattice
- Lepton flavour physics $\leftrightarrow$ neutrino physics
- Strong Kaon program essential for high energy physics
  - Today: K $\rightarrow \pi\ \nu\ \nu$ is still a highlight observable
  - Outlook: Interplay with lattice will improve SM prediction & NP sensitivity and wider program at KOTO(2)