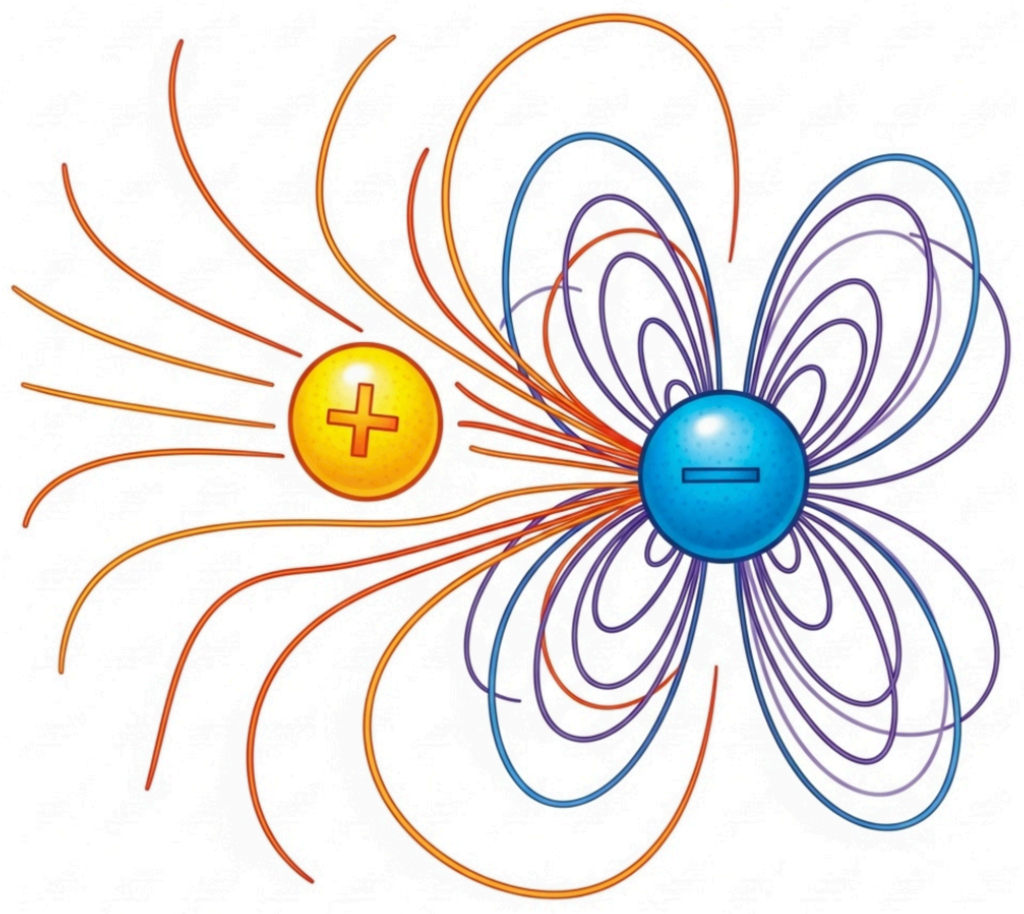
The Classical Equations of Quantized Gauge Theories

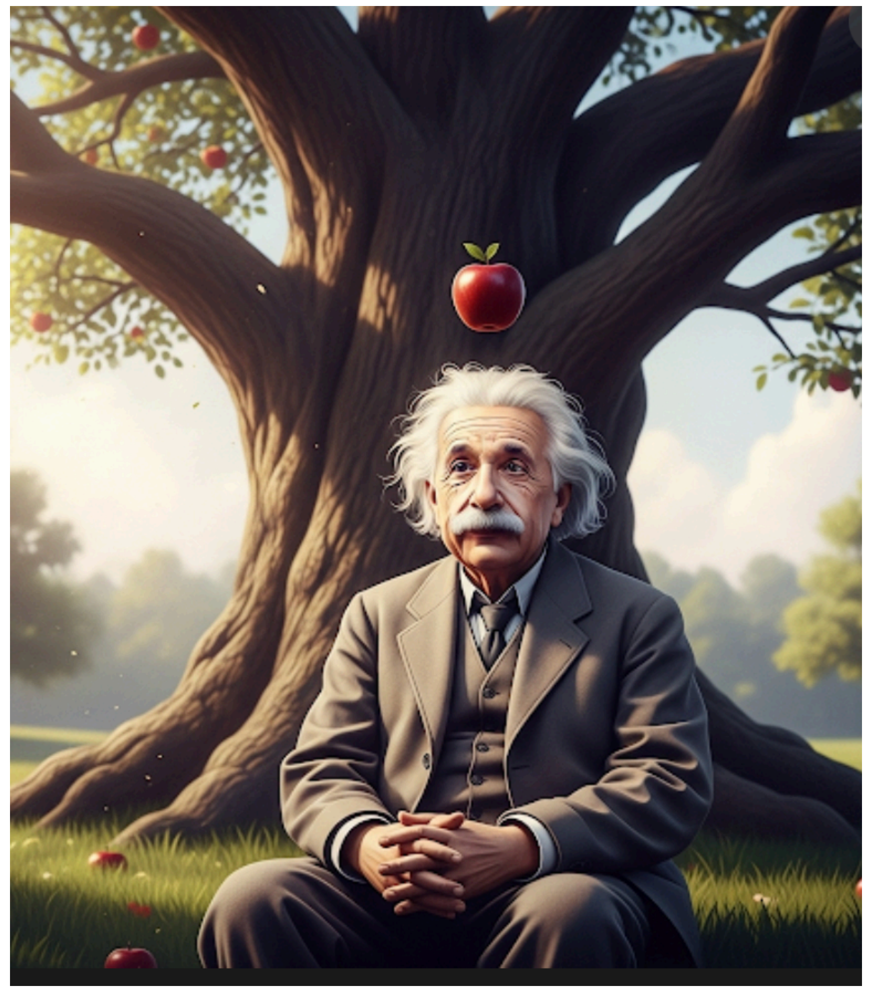
Surjeet Rajendran

Classical and Quantum Physics

$$\dot{p} = -\nabla V$$



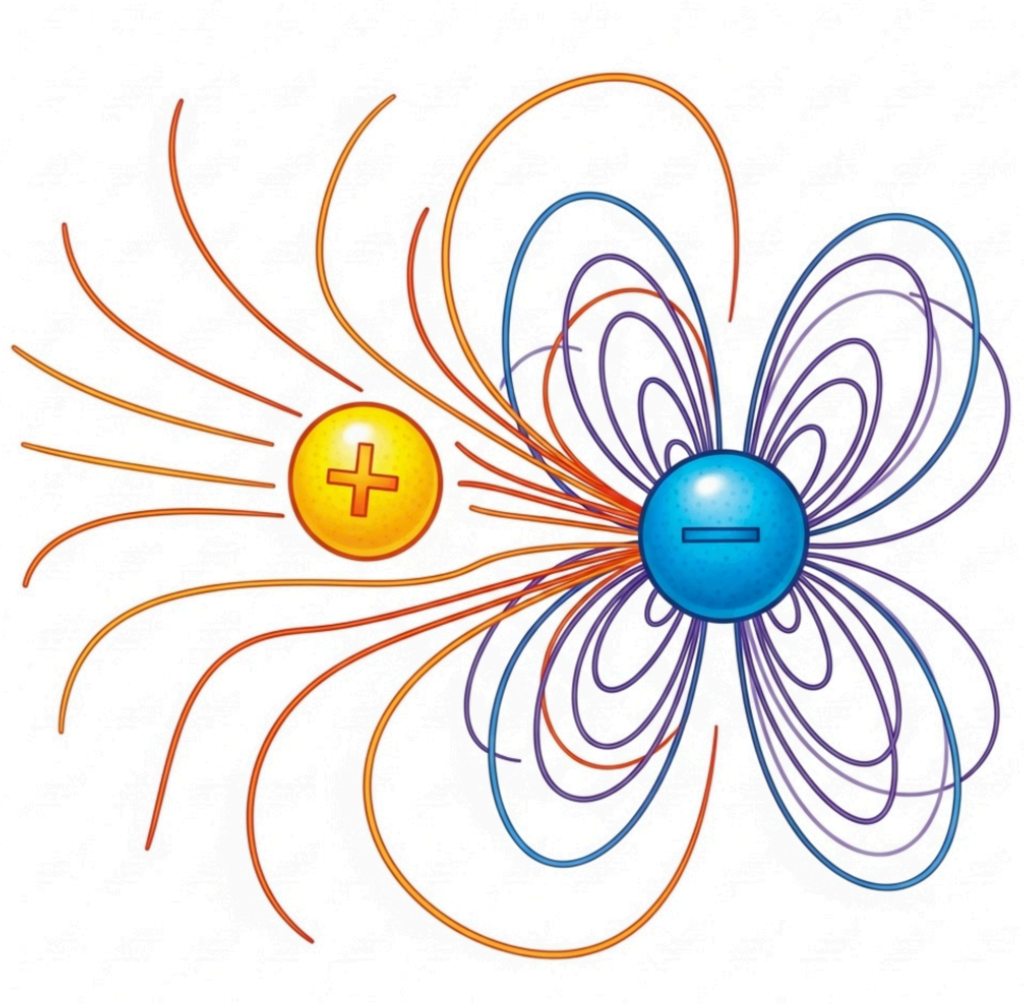
$$\partial_\mu F^{\mu\nu} = J^\nu$$



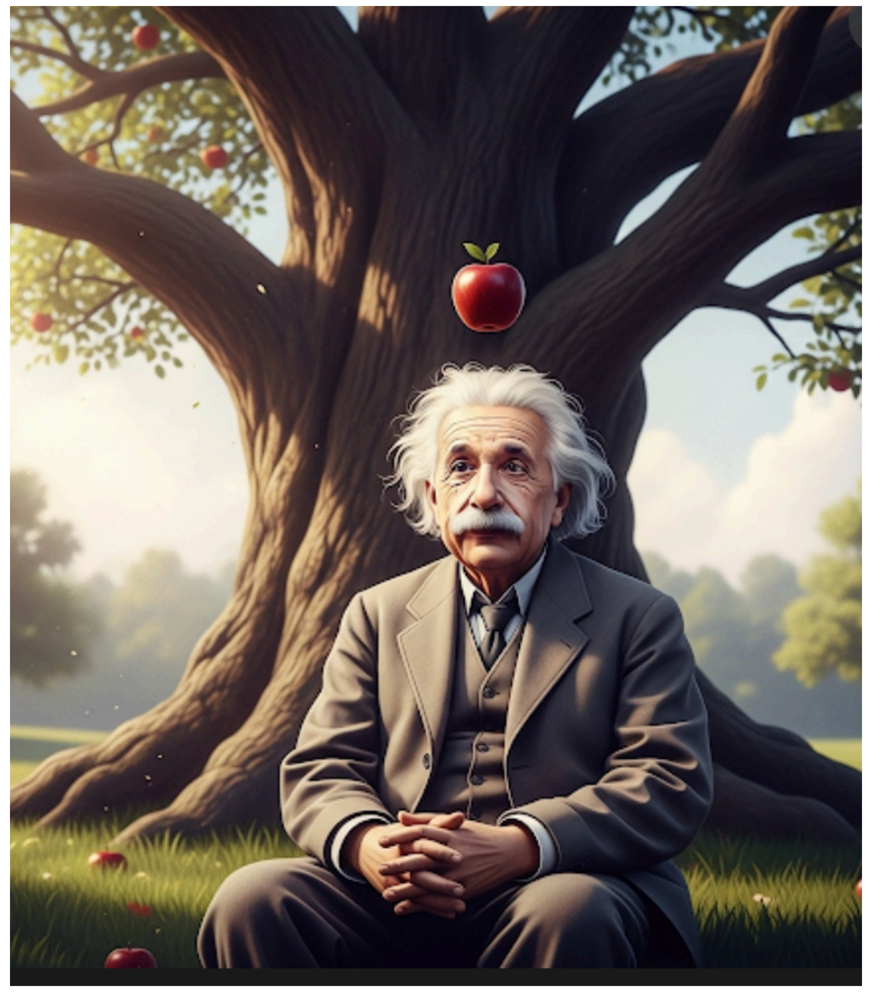
$$G_{\mu\nu} = T_{\mu\nu}$$

Classical and Quantum Physics

$$\dot{p} = -\nabla V$$



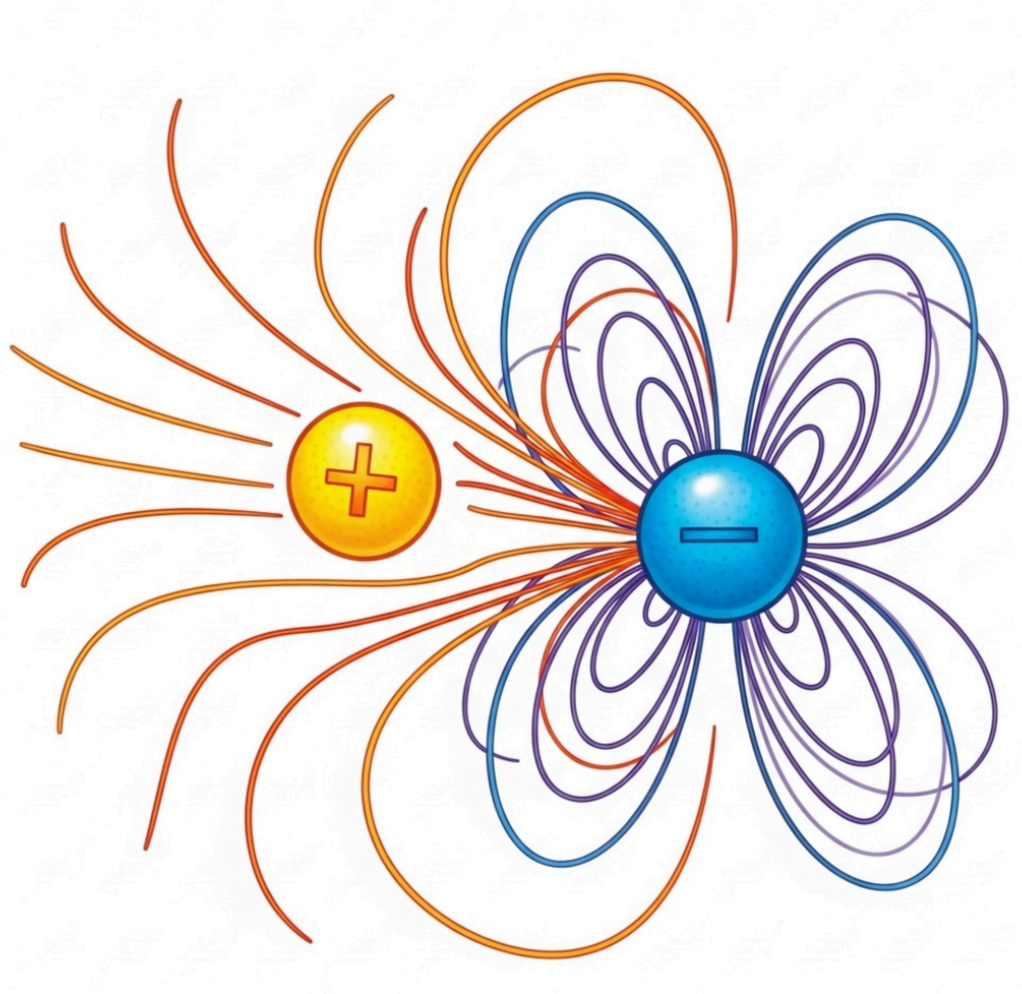
$$\partial_\mu F^{\mu\nu} = J^\nu$$



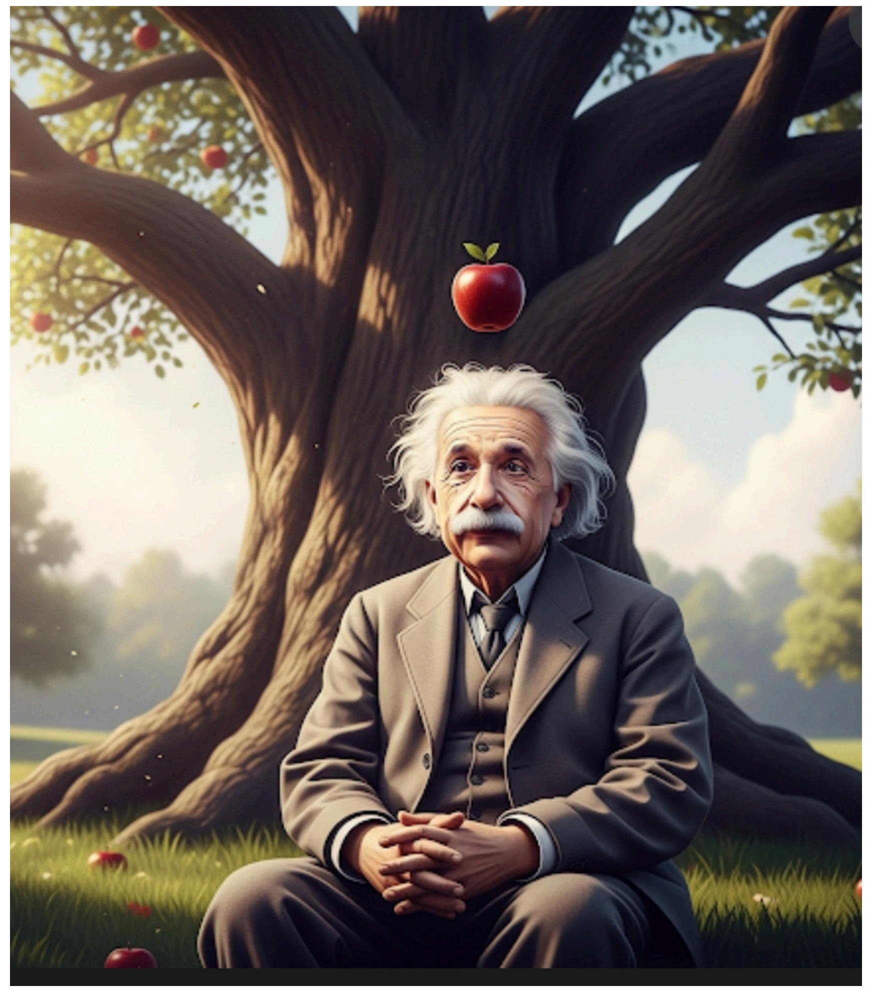
$$G_{\mu\nu} = T_{\mu\nu}$$

Classical and Quantum Physics

$$\dot{p} = -\nabla V$$



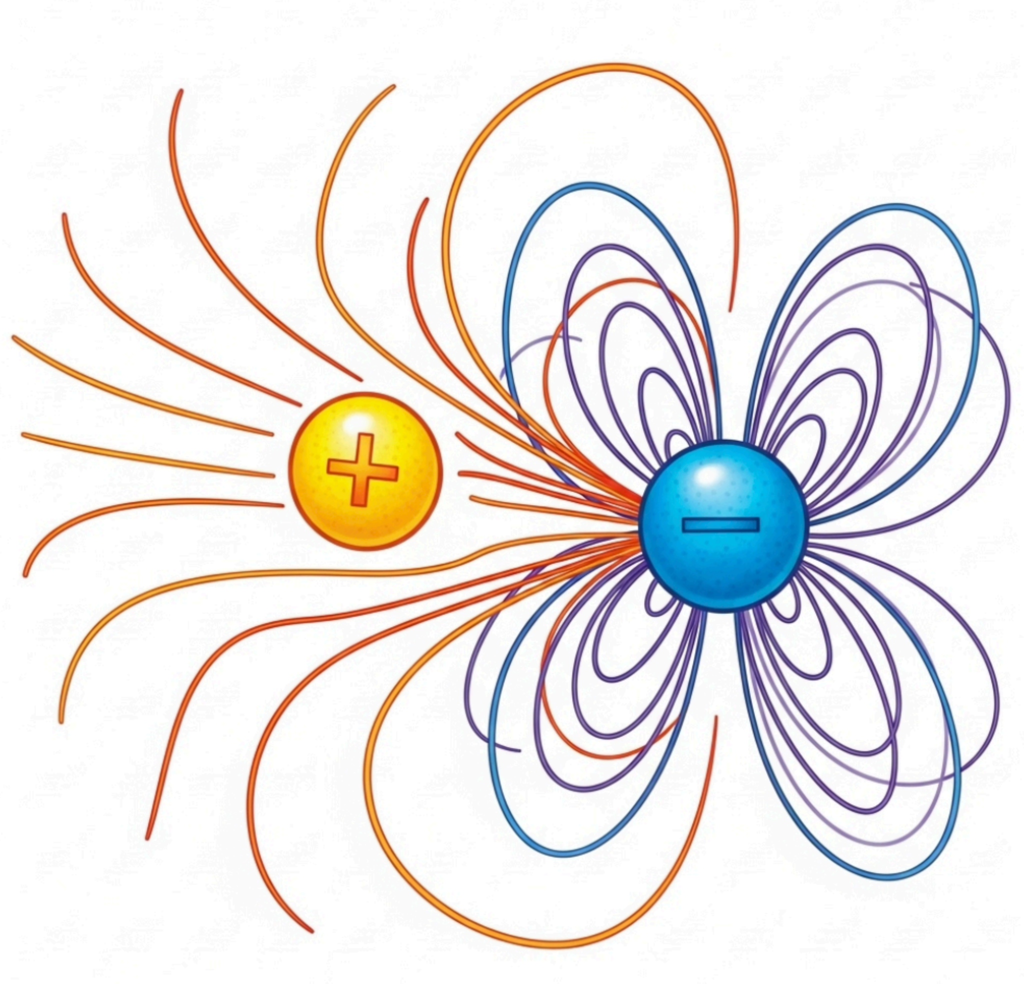
$$\partial_\mu F^{\mu\nu} = J^\nu$$

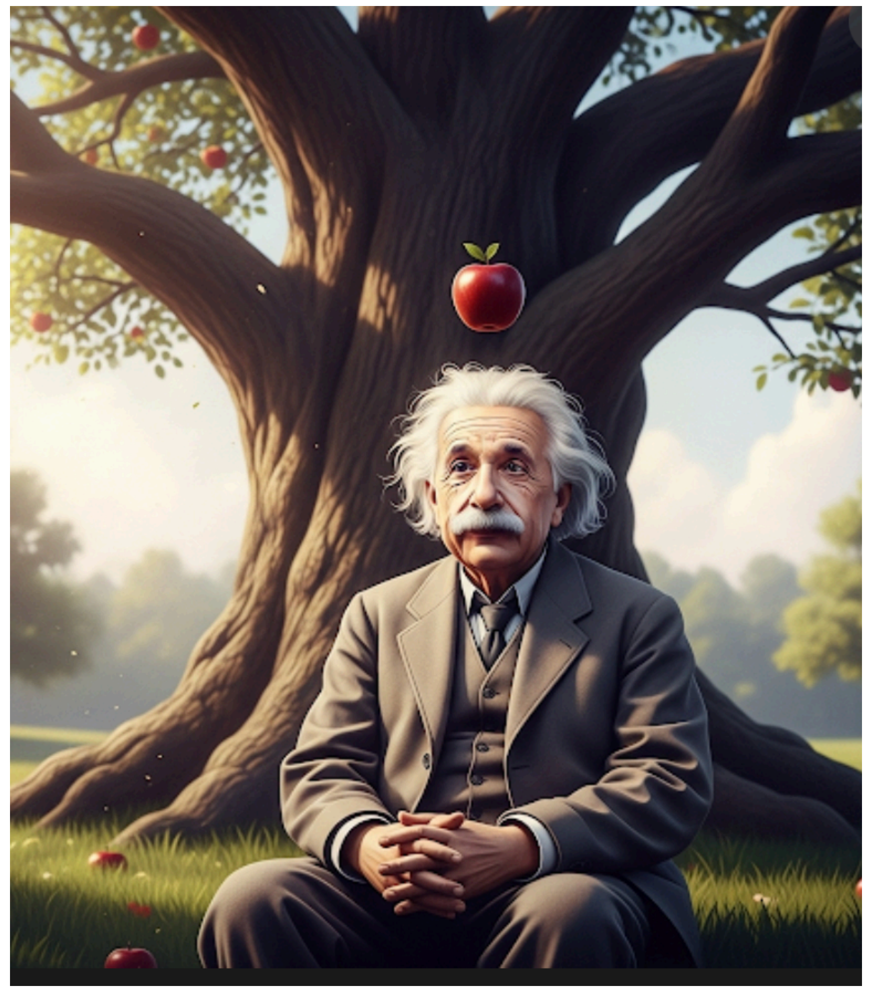


$$G_{\mu\nu} = T_{\mu\nu}$$

$$i\frac{\partial|\Psi\rangle}{\partial t} = H|\Psi\rangle$$ ✓

Classical and Quantum Physics





$$\langle \dot{p} \rangle = -\langle \nabla V \rangle$$

$$\langle \partial_\mu F^{\mu\nu} \rangle = \langle J^\nu \rangle$$

$$\langle G_{\mu\nu} \rangle = \langle T_{\mu\nu} \rangle$$

$$i\frac{\partial|\Psi\rangle}{\partial t} = H|\Psi\rangle \checkmark$$

$\Uparrow$

Ehrenfest's Theorem

Ehrenfest's Theorem

Given |Ψ(0)>, what is |Ψ(T)>?

Particle Mechanics

$$|\Psi(0)\rangle = \int dx\, f(x)\,|x\rangle$$

$$|x\rangle \to \int dy\, K(y,T;x,0)\,|y\rangle$$

$$|\Psi(T)\rangle = \int dx\, dy\, f(x)\, K(y,T;x,0)\,|y\rangle$$

$$K(y,T;x,0) = \langle y|\mathrm{e}^{-iHT}|x\rangle = \int_{\gamma(0)=x}^{\gamma(T)=y} D\gamma\, e^{i\int_0^T dt\, L(x,\dot{x})}$$

Ehrenfest's Theorem

Given |Ψ(0)>, what is |Ψ(T)>?

Particle Mechanics

$$|\Psi(0)\rangle = \int dx\, f(x)\, |x\rangle$$

$$|x\rangle \to \int dy\, K(y,T;x,0)\, |y\rangle$$

$$|\Psi(T)\rangle = \int dx\, dy\, f(x)\, K(y,T;x,0)\, |y\rangle$$

$$K(y,T;x,0) = \langle y|\mathrm{e}^{-iHT}|x\rangle = \int_{\gamma(0)=x}^{\gamma(T)=y} D\gamma\, e^{i\int_0^T dt\, L(x,\dot{x})}$$

Quantum Fields

$$|\Psi(0)\rangle = \int d\phi\, f(\phi)\, |\phi\rangle$$

$$|\Psi(T)\rangle = \int d\phi_f\, d\phi_i\, f(\phi_i)\, K(\phi_f,T;\phi_i,0)\, |\phi_f\rangle$$

$$K(\phi_f,T;\phi_i,0) = \int_{\gamma(0)=\phi_i}^{\gamma(T)=\phi_f} D\phi e^{iS[\phi]}$$

Ehrenfest's Theorem

Given |Ψ(0)>, what is |Ψ(T)>?

Particle Mechanics

$$|\Psi(0)\rangle = \int dx\, f(x)\, |x\rangle$$

$$|x\rangle \to \int dy\, K(y, T; x, 0)\, |y\rangle$$

$$|\Psi(T)\rangle = \int dx\, dy\, f(x)\, K(y, T; x, 0)\, |y\rangle$$

$$K(y, T; x, 0) = \langle y|\mathrm{e}^{-iHT}|x\rangle = \int_{\gamma(0)=x}^{\gamma(T)=y} D\gamma\, e^{i\int_0^T dt\, L(x,\dot{x})}$$

Quantum Fields

$$|\Psi(0)\rangle = \int d\phi\, f(\phi)\, |\phi\rangle$$

$$|\Psi(T)\rangle = \int d\phi_f\, d\phi_i\, f(\phi_i)\, K(\phi_f, T; \phi_i, 0)\, |\phi_f\rangle$$

$$K(\phi_f, T; \phi_i, 0) = \int_{\gamma(0)=\phi_i}^{\gamma(T)=\phi_f} D\phi e^{iS[\phi]}$$

$$\text{Change Variables: } \gamma \to \gamma + \delta\gamma \implies \delta K = 0 \implies \langle\Psi|\frac{\partial S}{\partial \gamma}|\Psi\rangle = 0$$

Ehrenfest's Theorem

Given |Ψ(0)>, what is |Ψ(T)>?

Particle Mechanics

$$|\Psi(0)\rangle = \int dx\, f(x)\,|x\rangle$$

$$|x\rangle \to \int dy\, K(y,T;x,0)\,|y\rangle$$

$$|\Psi(T)\rangle = \int dx\, dy\, f(x)\, K(y,T;x,0)\,|y\rangle$$

$$K(y,T;x,0) = \langle y|\mathrm{e}^{-iHT}|x\rangle = \int_{\gamma(0)=x}^{\gamma(T)=y} D\gamma\, e^{i\int_0^T dt\, L(x,\dot{x})}$$

Quantum Fields

$$|\Psi(0)\rangle = \int d\phi\, f(\phi)\,|\phi\rangle$$

$$|\Psi(T)\rangle = \int d\phi_f\, d\phi_i\, f(\phi_i)\, K(\phi_f,T;\phi_i,0)\,|\phi_f\rangle$$

$$K(\phi_f,T;\phi_i,0) = \int_{\gamma(0)=\phi_i}^{\gamma(T)=\phi_f} D\phi e^{iS[\phi]}$$

$$\text{Change Variables: } \gamma \to \gamma + \delta\gamma \implies \delta K = 0 \implies \langle\Psi|\frac{\partial S}{\partial\gamma}|\Psi\rangle = 0$$

Classical Physics is an identity of Quantum Mechanics

Ehrenfest's Theorem

Given |Ψ(0)>, what is |Ψ(T)>?

Particle Mechanics

$$|\Psi(0)\rangle = \int dx\, f(x)\, |x\rangle$$

$$|x\rangle \to \int dy\, K(y,T;x,0)\, |y\rangle$$

$$|\Psi(T)\rangle = \int dx\, dy\, f(x)\, K(y,T;x,0)\, |y\rangle$$

$$K(y,T;x,0) = \langle y|\mathrm{e}^{-iHT}|x\rangle = \int_{\gamma(0)=x}^{\gamma(T)=y} D\gamma\, e^{i\int_0^T dt\, L(x,\dot{x})}$$

Quantum Fields

$$|\Psi(0)\rangle = \int d\phi\, f(\phi)\, |\phi\rangle$$

$$|\Psi(T)\rangle = \int d\phi_f\, d\phi_i\, f(\phi_i)\, K(\phi_f,T;\phi_i,0)\, |\phi_f\rangle$$

$$K(\phi_f,T;\phi_i,0) = \int_{\gamma(0)=\phi_i}^{\gamma(T)=\phi_f} D\phi e^{iS[\phi]}$$

$$\text{Change Variables: } \gamma \to \gamma + \delta\gamma \implies \delta K = 0 \implies \langle\Psi|\frac{\partial S}{\partial\gamma}|\Psi\rangle = 0$$

Classical Physics is an identity of Quantum Mechanics

Guess quantum theory based on observed classical dynamics

The Classical Equations of Motion

Particle Mechanics

$$S \supset \int dt \left(\frac{1}{2} M(t) \dot{x}^2 - V(x) \right)$$

$$\text{Do Not } \frac{\partial S}{\partial M} = 0$$

Yukawa Theory

$$S \supset \int d^4x \, \lambda(x) \phi \bar{\Psi} \Psi$$

$$\text{Do Not } \frac{\partial S}{\partial \lambda} = 0$$

The Classical Equations of Motion

Particle Mechanics

$$S \supset \int dt \left(\frac{1}{2} M(t) \dot{x}^2 - V(x) \right)$$

$$\text{Do Not } \frac{\partial S}{\partial M} = 0$$

$$\pi_M = \frac{\partial \mathcal{L}}{\partial \dot{M}} = 0$$

Why?

Yukawa Theory

$$S \supset \int d^4x \, \lambda(x) \, \phi \bar{\Psi} \Psi$$

$$\text{Do Not } \frac{\partial S}{\partial \lambda} = 0$$

$$\pi_\lambda = \frac{\partial \mathcal{L}}{\partial \dot{\lambda}} = 0$$

The Classical Equations of Motion

Particle Mechanics

$$S \supset \int dt \left(\frac{1}{2} M(t)\, \dot{x}^2 - V(x) \right)$$

$$\text{Do Not } \frac{\partial S}{\partial M} = 0$$

$$\pi_M = \frac{\partial \mathcal{L}}{\partial \dot{M}} = 0$$

Yukawa Theory

$$S \supset \int d^4x\, \lambda(x)\, \phi \bar{\Psi} \Psi$$

$$\text{Do Not } \frac{\partial S}{\partial \lambda} = 0$$

$$\pi_\lambda = \frac{\partial \mathcal{L}}{\partial \dot{\lambda}} = 0$$

Why?

Not degrees of freedom, but parameters

Do not integrate over them in the path integral

The Classical Equations of Motion

Particle Mechanics

$$S \supset \int dt \left(\frac{1}{2} M(t)\, \dot{x}^2 - V(x) \right)$$

$$\text{Do Not } \frac{\partial S}{\partial M} = 0$$

$$\pi_M = \frac{\partial \mathcal{L}}{\partial \dot{M}} = 0$$

Yukawa Theory

$$S \supset \int d^4x\, \lambda(x)\, \phi \bar{\Psi} \Psi$$

$$\text{Do Not } \frac{\partial S}{\partial \lambda} = 0$$

$$\pi_\lambda = \frac{\partial \mathcal{L}}{\partial \dot{\lambda}} = 0$$

Why?

Not degrees of freedom, but parameters

Do not integrate over them in the path integral

Gauge Theories?

Classical Gauge Theories

Electromagnetism

$$S = \int d^4x \left(-F^2 + A_\mu J^\mu + \mathcal{L}_J\right)$$

Gravity

$$S = \int d^4x \sqrt{-g} \left(R + \mathcal{L}_M \left(\phi, \partial_\mu \phi\right)\right)$$

Classical Gauge Theories

Electromagnetism

$$S = \int d^4x \left(-F^2 + A_\mu J^\mu + \mathcal{L}_J\right)$$

Gravity

$$S = \int d^4x \sqrt{-g} \left(R + \mathcal{L}_M \left(\phi, \partial_\mu \phi\right)\right)$$

Dynamical Equations

$$\frac{\delta S}{\delta A^i} = 0 \implies \frac{\partial \vec{E}}{\partial t} = \nabla \times \vec{B} + \vec{J}$$

$$\frac{\partial S}{\partial g^{ij}} \implies G_{ij} = T_{ij}$$

Classical Gauge Theories

Electromagnetism

$$S = \int d^4x \left(-F^2 + A_\mu J^\mu + \mathcal{L}_J\right)$$

Gravity

$$S = \int d^4x \sqrt{-g} \left(R + \mathcal{L}_M \left(\phi, \partial_\mu \phi\right)\right)$$

Dynamical Equations

$$\frac{\delta S}{\delta A^i} = 0 \implies \frac{\partial \vec{E}}{\partial t} = \nabla \times \vec{B} + \vec{J}$$

$$\frac{\partial S}{\partial g^{ij}} \implies G_{ij} = T_{ij}$$

Constraint Equations

$$\frac{\delta S}{\delta A^0} = 0 \implies \vec{\nabla} \cdot \vec{E} = J^0$$

$$\frac{\partial S}{\partial g^{0\mu}} \implies G_{0\mu} = T_{0\mu}$$

Classical Gauge Theories

Electromagnetism

$$S = \int d^4x \left(-F^2 + A_\mu J^\mu + \mathcal{L}_J\right)$$

Gravity

$$S = \int d^4x \sqrt{-g}\left(R + \mathcal{L}_M\left(\phi, \partial_\mu \phi\right)\right)$$

Dynamical Equations

$$\frac{\delta S}{\delta A^i} = 0 \implies \frac{\partial \vec{E}}{\partial t} = \nabla \times \vec{B} + \vec{J}$$

$$\frac{\partial S}{\partial g^{ij}} \implies G_{ij} = T_{ij}$$

Constraint Equations

$$\frac{\delta S}{\delta A^0} = 0 \implies \vec{\nabla} \cdot \vec{E} = J^0$$

$$\frac{\partial S}{\partial g^{0\mu}} \implies G_{0\mu} = T_{0\mu}$$

But

$$\pi_{A_0} = \frac{\partial \mathcal{L}}{\partial \dot{A}_0} = 0$$

$$\pi_{0\mu} = \frac{\partial L}{\partial \dot{g}^{0\mu}} = 0$$

Classical Gauge Theories

Electromagnetism

$$S = \int d^4x \left(-F^2 + A_\mu J^\mu + \mathcal{L}_J\right)$$

Gravity

$$S = \int d^4x \sqrt{-g} \left(R + \mathcal{L}_M \left(\phi, \partial_\mu \phi\right)\right)$$

Dynamical Equations

$$\frac{\delta S}{\delta A^i} = 0 \implies \frac{\partial \vec{E}}{\partial t} = \nabla \times \vec{B} + \vec{J}$$

$$\frac{\partial S}{\partial g^{ij}} \implies G_{ij} = T_{ij}$$

Constraint Equations

$$\frac{\delta S}{\delta A^0} = 0 \implies \vec{\nabla} \cdot \vec{E} = J^0$$

$$\frac{\partial S}{\partial g^{0\mu}} \implies G_{0\mu} = T_{0\mu}$$

But

$$\pi_{A_0} = \frac{\partial \mathcal{L}}{\partial \dot{A}_0} = 0$$

$$\pi_{0\mu} = \frac{\partial L}{\partial \dot{g}^{0\mu}} = 0$$

Treat as parameters

Outline

1. Electromagnetism

2. Gravity

3. Conclusions

QED in the Weyl Gauge

$$H = \int d^3x \frac{1}{2}\left(E^2 + B^2\right) + A_0\left(\nabla . E - J^0\right) + \vec{A} . \vec{J} + H_J$$

QED in the Weyl Gauge

$$H = \int d^3x \frac{1}{2}\left(E^2 + B^2\right) + A_0\left(\nabla . E - J^0\right) + \vec{A}.\vec{J} + H_J$$

$$A_0 = 0$$

$$H = \int d^3x \frac{1}{2}\left(E^2 + B^2\right) + \vec{A}.\vec{J} + H_J \qquad G = \nabla . E - J^0$$

QED in the Weyl Gauge

$$H = \int d^3x \frac{1}{2}\left(E^2 + B^2\right) + A_0\left(\nabla . E - J^0\right) + \vec{A}.\vec{J} + H_J$$

$$A_0 = 0$$

$$H = \int d^3x \frac{1}{2}\left(E^2 + B^2\right) + \vec{A}.\vec{J} + H_J \qquad G = \nabla . E - J^0$$

$$[H, G] = 0$$

QED in the Weyl Gauge

$$H = \int d^3x \frac{1}{2}\left(E^2 + B^2\right) + A_0\left(\nabla.E - J^0\right) + \vec{A}.\vec{J} + H_J$$

$$A_0 = 0$$

$$H = \int d^3x \frac{1}{2}\left(E^2 + B^2\right) + \vec{A}.\vec{J} + H_J \qquad G = \nabla.E - J^0$$

$$[H, G] = 0$$

Fixing Gauge

$$|A\rangle \neq |A + \nabla\alpha\rangle$$

Dirac: Restrict Hilbert Space

$$|A\rangle \equiv |A + \nabla\alpha\rangle$$

How?

$$|A\rangle \equiv |A + \nabla\alpha\rangle$$

$$G = \nabla . E - J^0$$

Generates spatial gauge transformations

$$e^{i\int d^3x G\alpha}|A\rangle = |A + \nabla\alpha\rangle$$

How?

$$|A\rangle \equiv |A + \nabla\alpha\rangle$$

$$G = \nabla.E - J^0$$

Generates spatial gauge transformations

$$e^{i\int d^3x G\alpha}|A\rangle = |A + \nabla\alpha\rangle$$

Demand State that doesn't change under G

$$G|A_P\rangle = 0 \implies e^{i\int d^3x G\alpha}|A_P\rangle = |A_P\rangle$$

State obeys Gauss's law

$$[H, G] = 0 \implies \text{Remain in same eigenspace}$$

Hilbert Space Restriction

$$G = \nabla . E - J^0$$

$$G|A'_P\rangle = J_D\left(x\right)|A'_P\rangle \implies e^{i\int d^3xG\alpha}|A'_P\rangle = e^{i\int d^3x\alpha J_0}|A'_P\rangle$$

Overall phases do not matter in quantum mechanics

Hilbert Space Restriction

$$G = \nabla . E - J^0$$

$$G|A'_P\rangle = J_D(x)|A'_P\rangle \implies e^{i\int d^3x G\alpha}|A'_P\rangle = e^{i\int d^3x \alpha J_0}|A'_P\rangle$$

Overall phases do not matter in quantum mechanics

Dynamical Equation

$$i\frac{\partial|\Psi(t)\rangle}{\partial t} = H|\Psi(t)\rangle$$

Hilbert Space Restriction

$$G = \nabla . E - J^0$$

$$G|A'_P\rangle = J_D\left(x\right)|A'_P\rangle \implies e^{i\int d^3xG\alpha}|A'_P\rangle = e^{i\int d^3x\alpha J_0}|A'_P\rangle$$

Overall phases do not matter in quantum mechanics

Dynamical Equation

$$i\frac{\partial|\Psi\left(t\right)\rangle}{\partial t} = H|\Psi\left(t\right)\rangle$$

$$[H, G] = 0 \implies \text{Remain in same eigenspace}$$

State evolves, looks like a world with static background charge $J_D(x)$

Massless Photon

H invariant under A-> A + da

Hilbert Space Restriction

$$G = \nabla . E - J^0$$

$$G|A'_P\rangle = J_D(x)|A'_P\rangle \implies e^{i\int d^3x G\alpha}|A'_P\rangle = e^{i\int d^3x \alpha J_0}|A'_P\rangle$$

Overall phases do not matter in quantum mechanics

Dynamical Equation

$$i\frac{\partial|\Psi(t)\rangle}{\partial t} = H|\Psi(t)\rangle$$

$$[H, G] = 0 \implies \text{Remain in same eigenspace}$$

State evolves, looks like a world with static background charge $J_D(x)$

Massless Photon

H invariant under A-> A + da

No new degree of freedom - just a different state of electromagnetism

Classical Limit

Dynamical Equations

$$i\frac{d|\Psi\rangle}{dt} = H|\Psi\rangle \implies \langle\frac{\partial\vec{E}}{\partial t} - \nabla\times\vec{B} - \vec{J}\rangle = 0$$

Classical Limit

Dynamical Equations

$$i\frac{d|\Psi\rangle}{dt} = H|\Psi\rangle \implies \langle\frac{\partial\vec{E}}{\partial t} - \nabla\times\vec{B} - \vec{J}\rangle = 0$$

Constraint Equation

$$[H,G] = 0$$

$$\langle\vec{\nabla}.\left(\frac{\partial\vec{E}}{\partial t} - \nabla\times\vec{B} - \vec{J}\right)\rangle = 0 \implies \frac{d}{dt}\left(\langle\vec{\nabla}.\vec{E} - J^0\rangle\right) = 0$$

$$G|A_P\rangle = J_D\left(x\right)|A_P\rangle$$

$$\text{Initial }\nabla.E - J_0 = J_D\left(x\right) \implies \frac{d}{dt}\left(J_D\left(x\right)\right) = 0$$

No new degree of freedom - just a different state of electromagnetism

Operator Choice

Do we need to set $A_0 = 0$?

Operator Choice

Do we need to set $A_0 = 0$?

Choose A_0 to be any c-number function $\beta(t, x)$

$$H = \int d^3x \frac{1}{2}\left(E^2 + B^2\right) + A_0\left(\nabla . E - J^0\right) + \vec{A}.\vec{J} + H_J$$

Operator Choice

Do we need to set $A_0 = 0$?

Choose A_0 to be any c-number function $\beta(t, x)$

$$H = \int d^3x \frac{1}{2}\left(E^2 + B^2\right) + A_0\left(\nabla . E - J^0\right) + \vec{A}.\vec{J} + H_J$$

Does the physics depend on $\beta(t, x)$?

$$A_0 \rightarrow A_0 + \partial_t \alpha = 0, \vec{A} \rightarrow \vec{A} + \nabla \alpha$$

Maps to Weyl gauge Hamiltonian

Operator Choice

Do we need to set $A_0 = 0$?

Choose A_0 to be any c-number function $\beta(t, x)$

$$H = \int d^3x \frac{1}{2}\left(E^2 + B^2\right) + A_0\left(\nabla . E - J^0\right) + \vec{A}.\vec{J} + H_J$$

Does the physics depend on $\beta(t, x)$?

$$A_0 \rightarrow A_0 + \partial_t \alpha = 0, \vec{A} \rightarrow \vec{A} + \nabla \alpha$$

Maps to Weyl gauge Hamiltonian

More general choices of A_0 also possible - doesn't matter in linear quantum mechanics

Lagrangian

Classical Mechanics: L, H are classical functions related by Legendre Transformations

Quantum Mechanics: Hamiltonian defines differential equation, Lagrangian a solution

$$i\frac{\partial|\Psi(t)\rangle}{\partial t} = H|\Psi(t)\rangle$$

$$K(y,T;x,0) = \langle y|\mathrm{e}^{-iHT}|x\rangle = \int_{\gamma(0)=x}^{\gamma(T)=y} D\gamma\, e^{i\int_0^T dt\, L(x,\dot{x})}$$

Lagrangian

Classical Mechanics: L, H are classical functions related by Legendre Transformations

Quantum Mechanics: Hamiltonian defines differential equation, Lagrangian a solution

$$i\frac{\partial|\Psi(t)\rangle}{\partial t} = H|\Psi(t)\rangle \qquad K(y,T;x,0) = \langle y|\mathrm{e}^{-iHT}|x\rangle = \int_{\gamma(0)=x}^{\gamma(T)=y} D\gamma\, e^{i\int_0^T dt\, L(x,\dot{x})}$$

Solutions require boundary condition - Lagrangian can depend on it

$$G|A_P\rangle = J_D(x)\,|A_P\rangle$$

$$i\frac{\partial|A_p(t)\rangle}{\partial t} = H|A_p(t)\rangle$$

Lagrangian

Classical Mechanics: L, H are classical functions related by Legendre Transformations

Quantum Mechanics: Hamiltonian defines differential equation, Lagrangian a solution

$$i\frac{\partial|\Psi(t)\rangle}{\partial t} = H|\Psi(t)\rangle \qquad K(y,T;x,0) = \langle y|\mathrm{e}^{-iHT}|x\rangle = \int_{\gamma(0)=x}^{\gamma(T)=y} D\gamma\, e^{i\int_0^T dt\, L(x,\dot{x})}$$

Solutions require boundary condition - Lagrangian can depend on it

$$G|A_P\rangle = J_D(x)\,|A_P\rangle$$

$$i\frac{\partial|A_p(t)\rangle}{\partial t} = H|A_p(t)\rangle$$

Restrict evolution to just this sector

$$\mathcal{L} \supset A_\mu J_D^\mu$$

Lagrangian

Classical Mechanics: L, H are classical functions related by Legendre Transformations

Quantum Mechanics: Hamiltonian defines differential equation, Lagrangian a solution

$$i\frac{\partial|\Psi(t)\rangle}{\partial t} = H|\Psi(t)\rangle \qquad K(y,T;x,0) = \langle y|\mathrm{e}^{-iHT}|x\rangle = \int_{\gamma(0)=x}^{\gamma(T)=y} D\gamma\, e^{i\int_0^T dt\, L(x,\dot{x})}$$

Solutions require boundary condition - Lagrangian can depend on it

$$G|A_P\rangle = J_D(x)\,|A_P\rangle$$

$$i\frac{\partial|A_p(t)\rangle}{\partial t} = H|A_p(t)\rangle$$

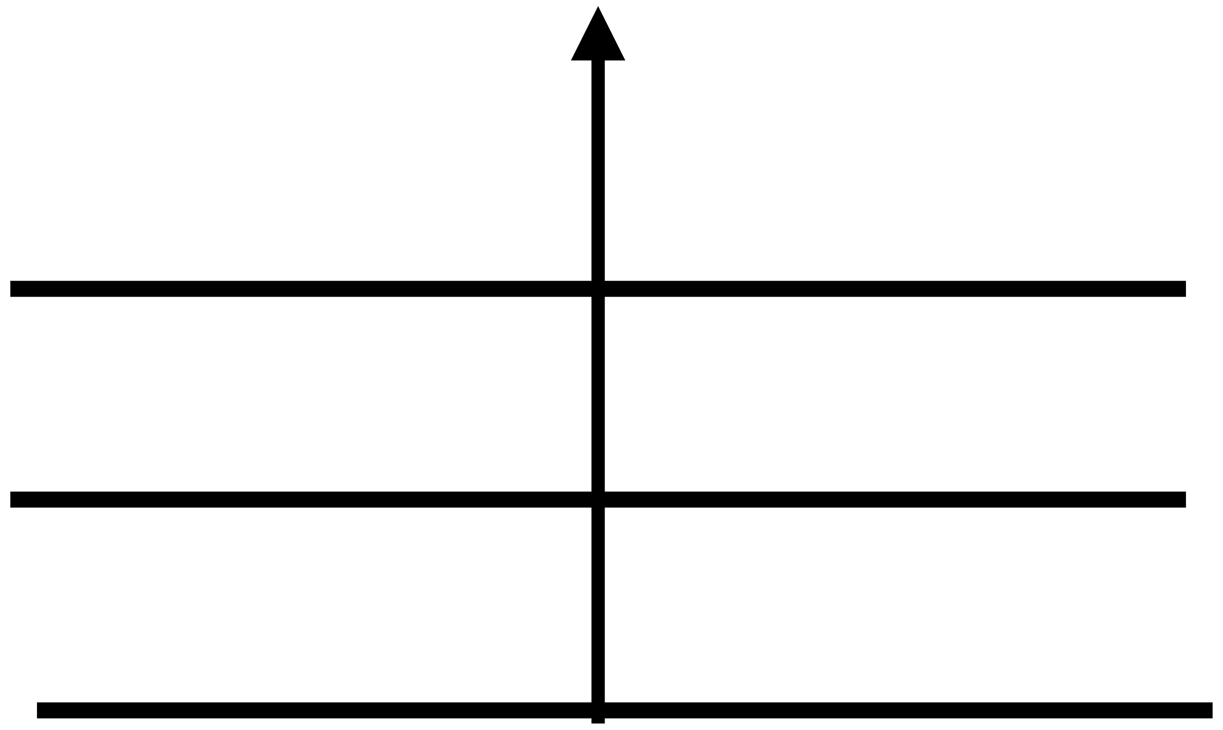
Restrict evolution to just this sector

$$\mathcal{L} \supset A_\mu J_D^\mu$$

Not a parameter, identifies sector of Hilbert space. Similar to θ

Gravitation

Quantum General Relativity

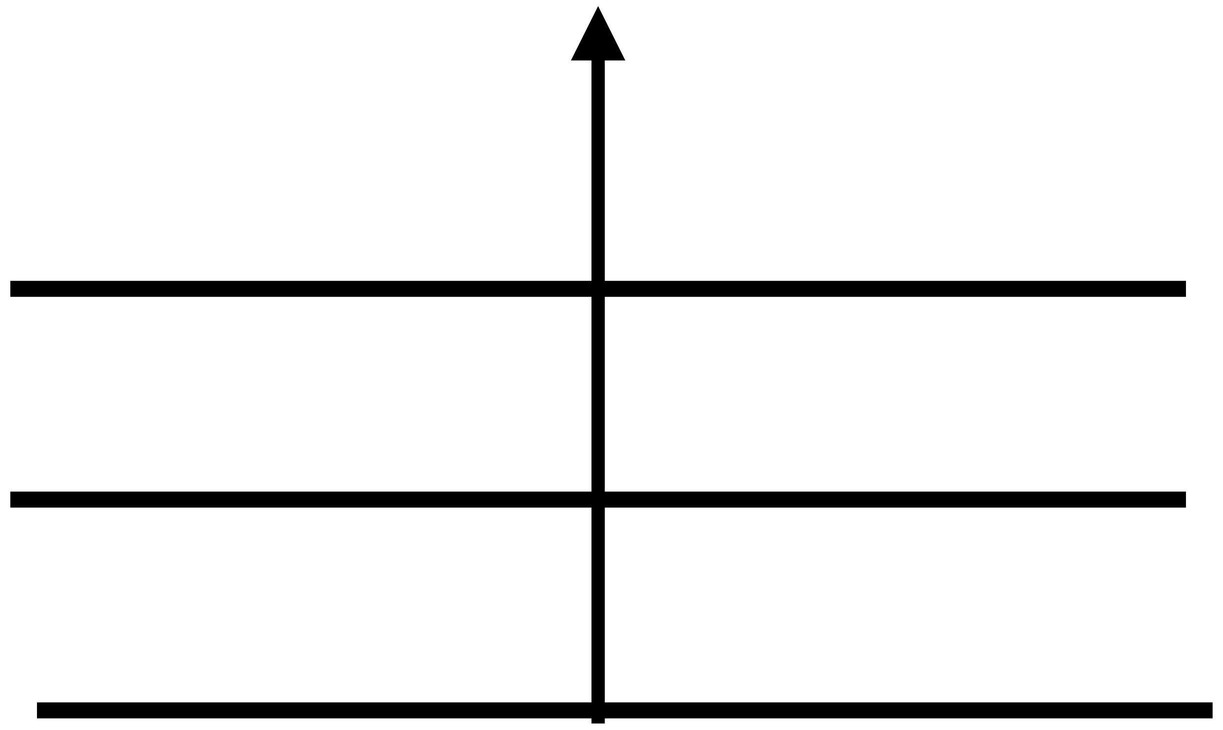


Manifold: R x Σ

$$g_{\mu\nu} = -N dt^2 + N_i dt dx^i + g_{ij} dx^i dx^j$$

$$\mathcal{L} = \sqrt{-g} R$$

Quantum General Relativity



Manifold: R x Σ

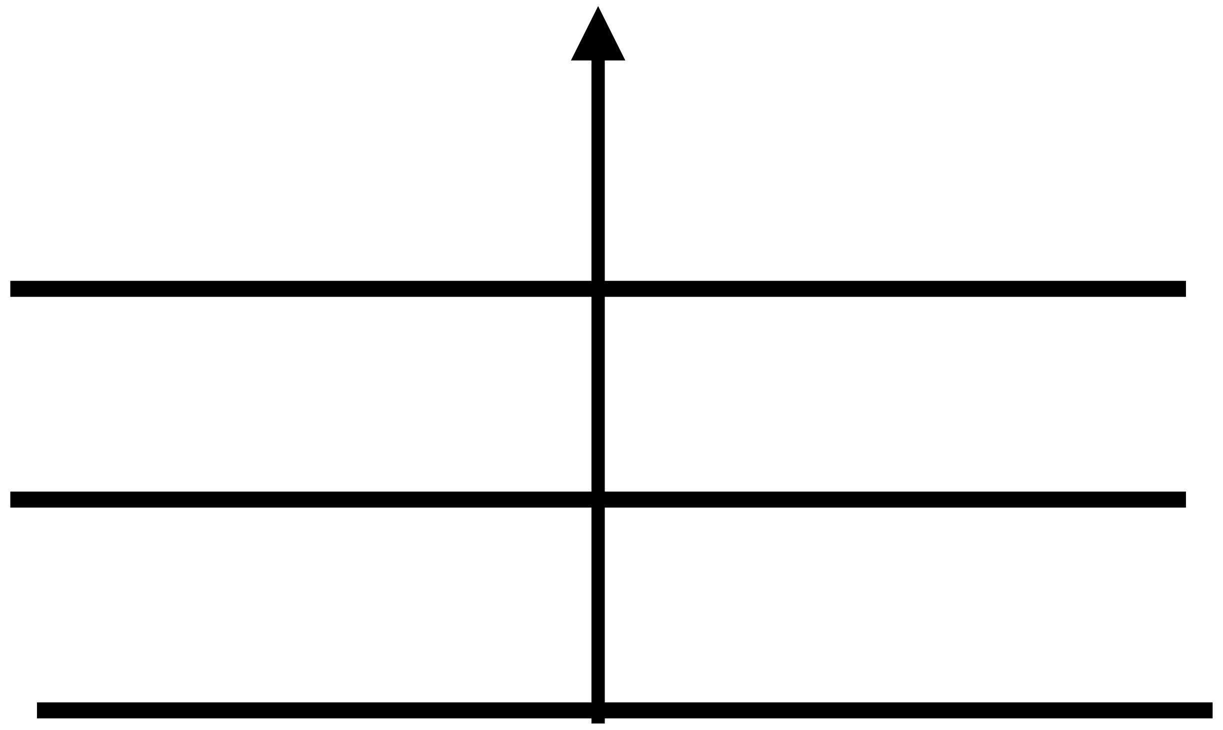
$$g_{\mu\nu} = -N dt^2 + N_i dt dx^i + g_{ij} dx^i dx^j$$

$$\mathcal{L} = \sqrt{-g} R$$

$$\pi_N = \frac{\partial L}{\partial \dot{N}} = 0 \qquad \pi_{N_i} = \frac{\partial L}{\partial \dot{N}_i} = 0$$

$$\pi_{ij} = \frac{\partial \mathcal{L}}{\partial \dot{g}_{ij}} \neq 0$$

Quantum General Relativity



Manifold: R x Σ

$$g_{\mu\nu} = -N dt^2 + N_i dt dx^i + g_{ij} dx^i dx^j$$

$$\mathcal{L} = \sqrt{-g} R$$

$$\pi_N = \frac{\partial L}{\partial \dot{N}} = 0 \qquad \pi_{N_i} = \frac{\partial L}{\partial \dot{N}_i} = 0$$

$$\pi_{ij} = \frac{\partial \mathcal{L}}{\partial \dot{g}_{ij}} \neq 0$$

g_{ij} are dynamical

What to do about N and N_i?

Quantum General Relativity

$$H = \int d^3x \left(N\mathcal{H} + N^i\mathcal{H}_i\right)$$

Classical Hamiltonian Vanishes on Shell
Physics does not depend on N or N_i

Quantum General Relativity

$$H = \int d^3x \left(N\mathcal{H} + N^i\mathcal{H}_i \right)$$

Classical Hamiltonian Vanishes on Shell
Physics does not depend on N or N_i

Guesses

$$H = 0$$

Physics Does Not Exist

$$H|\Psi\rangle = 0, H_i|\Psi\rangle = \lambda|\Psi\rangle$$

State does not change in time or space

Quantum General Relativity

$$H = \int d^3x \left(N\mathcal{H} + N^i \mathcal{H}_i \right)$$

Classical Hamiltonian Vanishes on Shell
Physics does not depend on N or N_i

Guesses

$$H = 0$$

Physics Does Not Exist

$$H|\Psi\rangle = 0, H_i|\Psi\rangle = \lambda|\Psi\rangle$$

State does not change in time or space

Ehrenfest's Theorem

$$\langle H \rangle = 0?$$

General Relativity

$$g = -N^2 dt^2 + N_i dt dx^i + g_{ij} dx^i dx^j$$

$$\pi_N = \frac{\partial L}{\partial \dot{N}} = 0 \qquad \pi_{N_i} = \frac{\partial L}{\partial \dot{N}_i} = 0$$

General Relativity

$$g = -N^2 dt^2 + N_i dt dx^i + g_{ij} dx^i dx^j$$

$$\pi_N = \frac{\partial L}{\partial \dot{N}} = 0 \qquad \pi_{N_i} = \frac{\partial L}{\partial \dot{N}_i} = 0$$

Treat N, N_i as parameters. So pick them : N = 1, N_i =0

$$\mathcal{H} = \frac{1}{\sqrt{\gamma}}\left(\pi_{ij}\pi^{ij} - \frac{1}{2}\pi^2\right) - \sqrt{\gamma}\,{}^{(3)}R$$

General Relativity

$$g = -N^2 dt^2 + N_i dt dx^i + g_{ij} dx^i dx^j$$

$$\pi_N = \frac{\partial L}{\partial \dot{N}} = 0 \qquad \pi_{N_i} = \frac{\partial L}{\partial \dot{N}_i} = 0$$

Treat N, N_i as parameters. So pick them : N = 1, N_i =0

$$\mathcal{H} = \frac{1}{\sqrt{\gamma}}\left(\pi_{ij}\pi^{ij} - \frac{1}{2}\pi^2\right) - \sqrt{\gamma}^{(3)}R$$

Propose

$$i\frac{d|\Psi\rangle}{dt} = \int d^3x \mathcal{H}|\Psi\rangle$$

General Relativity

$$g = -N^2 dt^2 + N_i dt dx^i + g_{ij} dx^i dx^j$$

$$\pi_N = \frac{\partial L}{\partial \dot{N}} = 0 \qquad \pi_{N_i} = \frac{\partial L}{\partial \dot{N}_i} = 0$$

Treat N, N_i as parameters. So pick them : N = 1, N_i =0

$$\mathcal{H} = \frac{1}{\sqrt{\gamma}} \left(\pi_{ij} \pi^{ij} - \frac{1}{2} \pi^2 \right) - \sqrt{\gamma}^{(3)} R$$

Propose

$$i \frac{d|\Psi\rangle}{dt} = \int d^3 x \mathcal{H} |\Psi\rangle$$

Hamiltonian invariant under spatial co-ordinate transformations.
Theory of massless gravitons

Constraints

$$i\frac{\partial|\Psi(t)\rangle}{\partial t} = H|\Psi(t)\rangle \implies \frac{d\langle\Psi(t)|H|\Psi(t)\rangle}{dt} = 0$$

Initial State: $\langle\Psi(0)|H|\Psi(0)\rangle = 0 \implies \langle\Psi(t)|H|\Psi(t)\rangle = 0$

Constraints

$$i\frac{\partial|\Psi(t)\rangle}{\partial t} = H|\Psi(t)\rangle \implies \frac{d\langle\Psi(t)|H|\Psi(t)\rangle}{dt} = 0$$

Initial State: $\langle\Psi(0)|H|\Psi(0)\rangle = 0 \implies \langle\Psi(t)|H|\Psi(t)\rangle = 0$

Classical GR: $G_{ij} = T_{ij}$ **With initial:** $G_{0\mu}(0) = T_{0\mu}(0)$

Constraints

$$i\frac{\partial|\Psi(t)\rangle}{\partial t} = H|\Psi(t)\rangle \implies \frac{d\langle\Psi(t)|H|\Psi(t)\rangle}{dt} = 0$$

Initial State: $\langle\Psi(0)|H|\Psi(0)\rangle = 0 \implies \langle\Psi(t)|H|\Psi(t)\rangle = 0$

Classical GR: $G_{ij} = T_{ij}$ **With initial:** $G_{0\mu}(0) = T_{0\mu}(0)$

What if $\langle\Psi(0)|H|\Psi(0)\rangle = J(x) \neq 0$?

Constraints

$$i\frac{\partial|\Psi(t)\rangle}{\partial t} = H|\Psi(t)\rangle \implies \frac{d\langle\Psi(t)|H|\Psi(t)\rangle}{dt} = 0$$

Initial State: $\langle\Psi(0)|H|\Psi(0)\rangle = 0 \implies \langle\Psi(t)|H|\Psi(t)\rangle = 0$

Classical GR: $G_{ij} = T_{ij}$ **With initial:** $G_{0\mu}(0) = T_{0\mu}(0)$

What if $\langle\Psi(0)|H|\Psi(0)\rangle = J(x) \neq 0$?

Similar to Electromagnetism, can show:

$$G_{\mu\nu} = T_{\mu\nu} + T^D_{\mu\nu}$$

$$T^D_{ij} = 0, T^D_{0\mu} \neq 0$$

Evolves like cold dark matter - no new particle, just a state of gravity

Conclusions

Quantizing Gauge Theories

$$S_{EM} = \int dt \mathcal{L}\left(\vec{A}, \dot{\vec{A}}, A_0\right) \qquad S_{GR} = \int dt\, \mathcal{L}\left(g_{ij}, \dot{g}_{ij}, g_{0\mu}\right)$$

Analogy with Yukawa: Treat A_0, $g_{0\mu}$ as some unknown but fixed operators

Quantizing Gauge Theories

$$S_{EM} = \int dt \mathcal{L}\left(\vec{A}, \dot{\vec{A}}, A_0\right) \qquad S_{GR} = \int dt\, \mathcal{L}\left(g_{ij}, \dot{g}_{ij}, g_{0\mu}\right)$$

Analogy with Yukawa: Treat A_0, $g_{0\mu}$ as some unknown but fixed operators

$$\text{Not d.o.f.} \implies \frac{\partial S}{\partial A_0}|\Psi\rangle \neq 0, \frac{\partial S}{\partial g_{0\mu}}|\Psi\rangle \neq 0$$

Quantizing Gauge Theories

$$S_{EM} = \int dt \mathcal{L}\left(\vec{A}, \dot{\vec{A}}, A_0\right) \qquad S_{GR} = \int dt\, \mathcal{L}\left(g_{ij}, \dot{g}_{ij}, g_{0\mu}\right)$$

Analogy with Yukawa: Treat A_0, $g_{0\mu}$ as some unknown but fixed operators

$$\text{Not d.o.f.} \implies \frac{\partial S}{\partial A_0}|\Psi\rangle \neq 0, \frac{\partial S}{\partial g_{0\mu}}|\Psi\rangle \neq 0$$

Can show that for these states, still get massless photon and graviton - the only physical aspect of gauge invariance we actually care about

Quantizing Gauge Theories

$$S_{EM} = \int dt \mathcal{L}\left(\vec{A}, \dot{\vec{A}}, A_0\right) \qquad S_{GR} = \int dt\, \mathcal{L}\left(g_{ij}, \dot{g}_{ij}, g_{0\mu}\right)$$

Analogy with Yukawa: Treat A_0, $g_{0\mu}$ as some unknown but fixed operators

$$\text{Not d.o.f.} \implies \frac{\partial S}{\partial A_0}|\Psi\rangle \neq 0, \frac{\partial S}{\partial g_{0\mu}}|\Psi\rangle \neq 0$$

Can show that for these states, still get massless photon and graviton - the only physical aspect of gauge invariance we actually care about

Further, by suitable redefinitions, can show that these choices only influence physics at the level of violations of constraints with "dark" backgrounds

Quantizing Gauge Theories

$$S_{EM} = \int dt \mathcal{L}\left(\vec{A}, \dot{\vec{A}}, A_0\right) \qquad S_{GR} = \int dt\, \mathcal{L}\left(g_{ij}, \dot{g_{ij}}, g_{0\mu}\right)$$

Analogy with Yukawa: Treat A_0, $g_{0\mu}$ as some unknown but fixed operators

$$\text{Not d.o.f.} \implies \frac{\partial S}{\partial A_0}|\Psi\rangle \neq 0, \frac{\partial S}{\partial g_{0\mu}}|\Psi\rangle \neq 0$$

Can show that for these states, still get massless photon and graviton - the only physical aspect of gauge invariance we actually care about

Further, by suitable redefinitions, can show that these choices only influence physics at the level of violations of constraints with "dark" backgrounds

"Dark Matter/Charge" : New cosmological observables

Backup

Boundary Operator

$$i\frac{d|\Psi\rangle}{dt} = (H_0 + H_B)|\Psi\rangle$$

$$H_0|\Psi\rangle = 0$$

$$\frac{d\langle\Psi|O|\Psi\rangle}{dt} = \langle\Psi|[H_0 + H_B, O]|\Psi\rangle = \underbrace{\langle\Psi|[H_0, O]|\Psi\rangle}_{0} + \langle\Psi|[H_B, O]|\Psi\rangle$$

Local QFT - O needs to commute with H_B

In any case, H_B only knows about ADM mass - not totality of local dynamics

State Over All Time

$$|\Psi\rangle = \int dt|\Phi\left(t\right)\rangle$$

$$|\Phi\left(t\right)\rangle = \sum_{j} \alpha_j e^{-iE_j t}|j\rangle$$

$$|\Psi\rangle = \sum_{j} \int dt \alpha_j e^{-iE_j t}|j\rangle = \sum_{j} \alpha_j \delta\left(E_j\right)|j\rangle$$

$$|\Phi\left(t\right)\rangle = \sum_{j} \alpha_j e^{-iE_j t}|j\rangle = e^{-iE_k t}\sum_{j} \alpha_j e^{i\left(E_k - E_j\right)t}|j\rangle \equiv \sum_{j} \alpha_j e^{i\left(E_k - E_j\right)t}|j\rangle$$

$$|\Psi\rangle = \sum_{j} \int dt \alpha_j e^{i\left(E_k - E_j\right)t}|j\rangle = \sum_{j} \alpha_j \delta\left(E_k - E_j\right)|j\rangle$$

Homogeneous Universe

Study homogeneous space-times in General Relativity

$$g_{\mu\nu} = -N\left(t\right)^2 dt^2 + a\left(t\right)^2 \left(dx^2 + dy^2 + dz^2\right)$$

Couple this to homogenous sources of matter, for e.g. rolling scalar field φ

$$S = \int dt \mathcal{L}\left(a\left(t\right), \dot{a}\left(t\right), N\left(t\right), \phi\left(t\right), \dot{\phi}\left(t\right)\right)$$

Homogeneous Universe

Study homogeneous space-times in General Relativity

$$g_{\mu\nu} = -N(t)^2 dt^2 + a(t)^2 \left(dx^2 + dy^2 + dz^2\right)$$

Couple this to homogenous sources of matter, for e.g. rolling scalar field φ

$$S = \int dt \mathcal{L}\left(a(t), \dot{a}(t), N(t), \phi(t), \dot{\phi}(t)\right)$$

Physical Degrees of freedom: a(t), φ(t)
Gauge Freedom: N(t)

Homogeneous Universe

Study homogeneous space-times in General Relativity

$$g_{\mu\nu} = -N(t)^2\, dt^2 + a(t)^2 \left(dx^2 + dy^2 + dz^2\right)$$

Couple this to homogenous sources of matter, for e.g. rolling scalar field φ

$$S = \int dt \mathcal{L}\left(a(t), \dot{a}(t), N(t), \phi(t), \dot{\phi}(t)\right)$$

Physical Degrees of freedom: a(t), φ(t)
Gauge Freedom: N(t)

Classical Equations?

2 FRW + 1 Scalar Field

Homogeneous Universe

Study homogeneous space-times in General Relativity

$$g_{\mu\nu} = -N\left(t\right)^2 dt^2 + a\left(t\right)^2 \left(dx^2 + dy^2 + dz^2\right)$$

Couple this to homogenous sources of matter, for e.g. rolling scalar field φ

$$S = \int dt \mathcal{L}\left(a\left(t\right), \dot{a}\left(t\right), N\left(t\right), \phi\left(t\right), \dot{\phi}\left(t\right)\right)$$

Physical Degrees of freedom: a(t), φ(t)
Gauge Freedom: N(t)

Classical Equations?

2 FRW + 1 Scalar Field

Gauge Symmetry: Time Reparameterization invariance

Classical Equations

$$S = \int dt \mathcal{L}\left(a\left(t\right), \dot{a}\left(t\right), N\left(t\right), \phi\left(t\right), \dot{\phi}\left(t\right)\right)$$

$$\frac{\partial S}{\partial a} = 0 \implies \partial_t \left(\frac{\partial \mathcal{L}}{\partial \dot{a}}\right) - \frac{\partial \mathcal{L}}{\partial a} = 0 \text{ i.e. } \left(\frac{\ddot{a}}{a} + \cdots = 0\right)$$

2nd FRW Equation

$$\frac{\partial S}{\partial \phi} = 0 \implies \partial_t \left(\frac{\partial \mathcal{L}}{\partial \dot{\phi}}\right) - \frac{\partial \mathcal{L}}{\partial \phi} = 0 \text{ i.e. } \left(\ddot{\phi} + \cdots = 0\right)$$

Klein Gordon

$$\frac{\partial S}{\partial N} = 0 \implies \frac{\partial \mathcal{L}}{\partial N} = 0 \text{ i.e. } \left(\left(\frac{\dot{a}}{a}\right)^2 + \cdots = 0\right)$$

1st FRW Equation (Hubble)

Classical Equations

$$S = \int dt \mathcal{L}\left(a(t), \dot{a}(t), N(t), \phi(t), \dot{\phi}(t)\right)$$

$$\frac{\partial S}{\partial a} = 0 \implies \partial_t \left(\frac{\partial \mathcal{L}}{\partial \dot{a}}\right) - \frac{\partial \mathcal{L}}{\partial a} = 0 \text{ i.e. } \left(\frac{\ddot{a}}{a} + \cdots = 0\right)$$

2nd FRW Equation

$$\frac{\partial S}{\partial \phi} = 0 \implies \partial_t \left(\frac{\partial \mathcal{L}}{\partial \dot{\phi}}\right) - \frac{\partial \mathcal{L}}{\partial \phi} = 0 \text{ i.e. } \left(\ddot{\phi} + \cdots = 0\right)$$

Klein Gordon

$$\frac{\partial S}{\partial N} = 0 \implies \frac{\partial \mathcal{L}}{\partial N} = 0 \text{ i.e. } \left(\left(\frac{\dot{a}}{a}\right)^2 + \cdots = 0\right)$$

1st FRW Equation (Hubble)

Classical Solutions are over-constrained (3 equations for 2 variables)

Solve: Klein Gordon + 2nd FRW with boundary condition from 1st FRW

Quantum Theory

$$g_{\mu\nu} = -N(t)^2 dt^2 + a(t)^2 \left(dx^2 + dy^2 + dz^2\right)$$

Physical Degrees of freedom: a(t), φ(t)
Gauge Freedom: N(t)

$$S = \int dt \mathcal{L}\left(a(t), \dot{a}(t), N(t), \phi(t), \dot{\phi}(t)\right)$$

Quantum Theory

$$g_{\mu\nu} = -N(t)^2 dt^2 + a(t)^2 \left(dx^2 + dy^2 + dz^2\right)$$

Physical Degrees of freedom: a(t), φ(t)
Gauge Freedom: N(t)

$$S = \int dt \mathcal{L}\left(a(t), \dot{a}(t), N(t), \phi(t), \dot{\phi}(t)\right)$$

$$\Pi_a = \frac{\partial \mathcal{L}}{\partial \dot{a}}, \Pi_\phi = \frac{\partial \mathcal{L}}{\partial \dot{\phi}}$$

Quantize These

Quantum Theory

$$g_{\mu\nu} = -N(t)^2 dt^2 + a(t)^2 \left(dx^2 + dy^2 + dz^2\right)$$

Physical Degrees of freedom: a(t), φ(t)
Gauge Freedom: N(t)

$$S = \int dt \mathcal{L}\left(a(t), \dot{a}(t), N(t), \phi(t), \dot{\phi}(t)\right)$$

$$\Pi_a = \frac{\partial \mathcal{L}}{\partial \dot{a}}, \Pi_\phi = \frac{\partial \mathcal{L}}{\partial \dot{\phi}}$$

Quantize These

$$\Pi_N = \frac{\partial \mathcal{L}}{\partial \dot{N}} = 0$$

What to do with N?

Hamiltonian Construction

$$S = \int dt \mathcal{L}\left(a\left(t\right), \dot{a}\left(t\right), N\left(t\right), \phi\left(t\right), \dot{\phi}\left(t\right)\right)$$

Construct Canonical Hamiltonian from this Lagrangian

$$H_N = N\, H_0\left(a, \Pi_a, \phi, \Pi_\phi\right)$$

Hamiltonian Construction

$$S = \int dt \mathcal{L}\left(a\left(t\right), \dot{a}\left(t\right), N\left(t\right), \phi\left(t\right), \dot{\phi}\left(t\right)\right)$$

Construct Canonical Hamiltonian from this Lagrangian

$$H_N = N\, H_0\left(a, \Pi_a, \phi, \Pi_\phi\right)$$

What is N? Different values of N yield different Hamiltonians
Different physics? Gauge symmetry?

N is like A_0 - just pick some c-number function
But still, different choices of N yield different Hamiltonians!

Hamiltonian Construction

$$S = \int dt \mathcal{L}\left(a\left(t\right), \dot{a}\left(t\right), N\left(t\right), \phi\left(t\right), \dot{\phi}\left(t\right)\right)$$

Construct Canonical Hamiltonian from this Lagrangian

$$H_N = N\, H_0\left(a, \Pi_a, \phi, \Pi_\phi\right)$$

What is N? Different values of N yield different Hamiltonians
Different physics? Gauge symmetry?

N is like A_0 - just pick some c-number function
But still, different choices of N yield different Hamiltonians!

Know: N doesn't affect physics

Schrodinger Equation

$$i\frac{\partial|\chi\left(t\right)\rangle}{\partial t}=NH_0|\chi\left(t\right)\rangle$$

N is a c-number, different choices of N yield different Hamiltonians

Schrodinger Equation

$$i\frac{\partial|\chi(t)\rangle}{\partial t} = NH_0|\chi(t)\rangle$$

N is a c-number, different choices of N yield different Hamiltonians

$$i\frac{1}{N}\frac{\partial|\chi(t)\rangle}{\partial t} = H_0|\chi(t)\rangle \qquad d\tilde{t} = N dt$$

Schrodinger Equation

$$i\frac{\partial|\chi(t)\rangle}{\partial t} = N H_0|\chi(t)\rangle$$

N is a c-number, different choices of N yield different Hamiltonians

$$i\frac{1}{N}\frac{\partial|\chi(t)\rangle}{\partial t} = H_0|\chi(t)\rangle \qquad d\tilde{t} = N dt$$

Different choices of N correspond to different choices of time co-ordinate.

What are the classical equations of motion?

Path Integral

$$S = \int dt \mathcal{L}\left(a\left(t\right), \dot{a}\left(t\right), N\left(t\right), \phi\left(t\right), \dot{\phi}\left(t\right)\right)$$

Physical Degrees of freedom: a(t), φ(t), Gauge Freedom: N(t)

To get finite path integral, need to fix N(t)

Path Integral

$$S = \int dt \mathcal{L}\left(a\left(t\right), \dot{a}\left(t\right), N\left(t\right), \phi\left(t\right), \dot{\phi}\left(t\right)\right)$$

Physical Degrees of freedom: a(t), φ(t), Gauge Freedom: N(t)

To get finite path integral, need to fix N(t)

$$\dot{N} = 0, \text{ enforce via Lagrange multiplier}$$

$$\langle \phi_f, a_f | T\left(t_2; t_1\right) | \phi_i, a_i \rangle = \int_{\phi(t_1)=\phi_i, a(t_1)=a_i, N(t_1)=N_1}^{\phi(t_2)=\phi_f, a(t_2)=a_f, N(t_2)=N_2} D\phi Da DN D\lambda e^{i\int_{t=t_1}^{t=t_2}\left(\mathcal{L}-\lambda\dot{N}\right)}$$

Path Integral

$$S = \int dt \mathcal{L}\left(a\left(t\right), \dot{a}\left(t\right), N\left(t\right), \phi\left(t\right), \dot{\phi}\left(t\right)\right)$$

Physical Degrees of freedom: a(t), φ(t), Gauge Freedom: N(t)

To get finite path integral, need to fix N(t)

$$\dot{N} = 0, \text{ enforce via Lagrange multiplier}$$

$$\langle \phi_f, a_f | T\left(t_2; t_1\right) | \phi_i, a_i \rangle = \int_{\phi(t_1)=\phi_i, a(t_1)=a_i, N(t_1)=N_1}^{\phi(t_2)=\phi_f, a(t_2)=a_f, N(t_2)=N_2} D\phi Da DN D\lambda e^{i\int_{t=t_1}^{t=t_2}\left(\mathcal{L}-\lambda\dot{N}\right)}$$

$$\dot{N} = 0 \implies N\left(t_2\right) = N_0, N\left(t_1\right) = N_0$$

Path Integral

$$\langle \phi_f, a_f | T(t_2; t_1) | \phi_i, a_i \rangle = \int_{\phi(t_1)=\phi_i, a(t_1)=a_i}^{\phi(t_2)=\phi_f, a(t_2)=a_f} D\phi D a e^{i \int_{t=t_1}^{t=t_2} \left(\tilde{\mathcal{L}}(N_0)\right)}$$

Can show that path integral over a(t), φ(t) are finite

$$\langle \phi_f, a_f | T(t_2; t_1) | \phi_i, a_i \rangle = e^{iP(\phi_f, a_f, t_2; \phi_i, a_i, t_1; N_0)}$$

Path Integral

$$\langle \phi_f, a_f | T(t_2; t_1) | \phi_i, a_i \rangle = \int_{\phi(t_1)=\phi_i, a(t_1)=a_i}^{\phi(t_2)=\phi_f, a(t_2)=a_f} D\phi D a e^{i \int_{t=t_1}^{t=t_2} \left(\tilde{\mathcal{L}}(N_0)\right)}$$

Can show that path integral over a(t), φ(t) are finite

$$\langle \phi_f, a_f | T(t_2; t_1) | \phi_i, a_i \rangle = e^{iP(\phi_f, a_f, t_2; \phi_i, a_i, t_1; N_0)}$$

Path Integral explicitly depends upon random choice N_0

But, redefine time to eliminate physical effects

Path Integral

$$\langle \phi_f, a_f | T\,(t_2; t_1)\,|\phi_i, a_i\rangle = \int_{\phi(t_1)=\phi_i, a(t_1)=a_i}^{\phi(t_2)=\phi_f, a(t_2)=a_f} D\phi D a e^{i\int_{t=t_1}^{t=t_2}\left(\tilde{\mathcal{L}}(N_0)\right)}$$

Can show that path integral over a(t), φ(t) are finite

$$\langle \phi_f, a_f | T\,(t_2; t_1)\,|\phi_i, a_i\rangle = e^{iP(\phi_f, a_f, t_2; \phi_i, a_i, t_1; N_0)}$$

Path Integral explicitly depends upon random choice N_0

But, redefine time to eliminate physical effects

Action invariant under time reparameterization - fix end points, can pick arbitrary time parameterization in the middle

Schwinger Dyson Equations

$$\langle \phi_f, a_f | T\left(t_2; t_1\right) | \phi_i, a_i \rangle = \int_{\phi(t_1)=\phi_i, a(t_1)=a_i, N(t_1)=N_1}^{\phi(t_2)=\phi_f, a(t_2)=a_f, N(t_2)=N_2} D\phi Da DN D\lambda e^{i \int_{t=t_1}^{t=t_2} \left(\mathcal{L} - \lambda \dot{N}\right)}$$

$$\phi \to \phi + \delta\phi, a \to a + \delta a, N \to N + \delta N$$

Schwinger Dyson Equations

$$\langle \phi_f, a_f | T(t_2; t_1) | \phi_i, a_i \rangle = \int_{\phi(t_1)=\phi_i, a(t_1)=a_i, N(t_1)=N_1}^{\phi(t_2)=\phi_f, a(t_2)=a_f, N(t_2)=N_2} D\phi Da DN D\lambda e^{i \int_{t=t_1}^{t=t_2} \left(\mathcal{L} - \lambda \dot{N}\right)}$$

$$\phi \rightarrow \phi + \delta\phi, a \rightarrow a + \delta a, N \rightarrow N + \delta N$$

$$\langle \Psi | \partial_t \left(\frac{\partial \mathcal{L}}{\partial \dot{a}} \right) - \frac{\partial \mathcal{L}}{\partial a} | \Psi \rangle = 0$$

2nd FRW Equation

Heisenberg Picture

$$\langle \Psi | \partial_t \left(\frac{\partial \mathcal{L}}{\partial \dot{\phi}} \right) - \frac{\partial \mathcal{L}}{\partial \phi} | \Psi \rangle = 0$$

Klein Gordon

Schwinger Dyson Equations

$$\langle \phi_f, a_f | T(t_2; t_1) | \phi_i, a_i \rangle = \int_{\phi(t_1)=\phi_i, a(t_1)=a_i, N(t_1)=N_1}^{\phi(t_2)=\phi_f, a(t_2)=a_f, N(t_2)=N_2} D\phi Da DN D\lambda e^{i\int_{t=t_1}^{t=t_2}(\mathcal{L}-\lambda\dot{N})}$$

$$\phi \to \phi + \delta\phi, a \to a + \delta a, N \to N + \delta N$$

$$\langle \Psi | \partial_t \left(\frac{\partial \mathcal{L}}{\partial \dot{a}} \right) - \frac{\partial \mathcal{L}}{\partial a} | \Psi \rangle = 0$$

2nd FRW Equation

Heisenberg Picture

$$\langle \Psi | \partial_t \left(\frac{\partial \mathcal{L}}{\partial \dot{\phi}} \right) - \frac{\partial \mathcal{L}}{\partial \phi} | \Psi \rangle = 0$$

Klein Gordon

$$\langle \Psi | \frac{\partial \lambda}{\partial t} - \frac{\partial \mathcal{L}}{\partial N} | \Psi \rangle = 0$$

Tells you how λ evolves in the path integral - not 1st FRW Equation?????

Consequence of Schrodinger Equation

$$i\frac{\partial|\chi(t)\rangle}{\partial t} = NH_0|\chi(t)\rangle \implies \frac{d\langle\chi(t)|H_0|\chi(t)\rangle}{dt} = 0$$

Identity, similar to Ehrenfest and Schwinger-Dyson

Consequence of Schrodinger Equation

$$i\frac{\partial|\chi(t)\rangle}{\partial t} = N H_0|\chi(t)\rangle \implies \frac{d\langle\chi(t)|H_0|\chi(t)\rangle}{dt} = 0$$

Identity, similar to Ehrenfest and Schwinger-Dyson

Can Show: $$\langle\chi(t)|H_0|\chi(t)\rangle = \langle\chi(t)|a^3\frac{\partial\mathcal{L}}{\partial N}|\chi(t)\rangle$$

Consequence of Schrodinger Equation

$$i\frac{\partial|\chi(t)\rangle}{\partial t} = NH_0|\chi(t)\rangle \implies \frac{d\langle\chi(t)|H_0|\chi(t)\rangle}{dt} = 0$$

Identity, similar to Ehrenfest and Schwinger-Dyson

Can Show: $\langle\chi(t)|H_0|\chi(t)\rangle = \langle\chi(t)|a^3\frac{\partial\mathcal{L}}{\partial N}|\chi(t)\rangle$

Thus: $\dfrac{d\langle\chi|a^3\frac{\partial\mathcal{L}}{\partial N}|\chi\rangle}{dt} = 0$

This is almost the 1st FRW equation - but not quite.
1st FRW equation only satisfied up to overall constant

Initial State

$$a = A + A^\dagger, \Pi_a = i\left(A - A^\dagger\right) \qquad \phi = B + B^\dagger, \Pi_\phi = i\left(B - B^\dagger\right)$$

Create quantum states of a, φ

$$|\chi\rangle = f\left(A, A^\dagger, B, B^\dagger\right)|0>$$

Initial State

$$a = A + A^\dagger, \Pi_a = i\left(A - A^\dagger\right) \qquad \phi = B + B^\dagger, \Pi_\phi = i\left(B - B^\dagger\right)$$

Create quantum states of a, φ

$$|\chi\rangle = f\left(A, A^\dagger, B, B^\dagger\right)|0>$$

$$\text{Choose } \langle\chi|a^3\frac{\partial\mathcal{L}}{\partial N}|\chi\rangle = 0 \implies \text{1st FRW holds } \left(\frac{d\langle\chi|a^3\frac{\partial\mathcal{L}}{\partial N}|\chi\rangle}{dt} = 0\right)$$

Initial State

$$a = A + A^\dagger, \Pi_a = i\left(A - A^\dagger\right) \qquad \phi = B + B^\dagger, \Pi_\phi = i\left(B - B^\dagger\right)$$

Create quantum states of a, φ

$$|\chi\rangle = f\left(A, A^\dagger, B, B^\dagger\right)|0>$$

$$\text{Choose } \langle\chi|a^3\frac{\partial\mathcal{L}}{\partial N}|\chi\rangle = 0 \implies \text{1st FRW holds } \left(\frac{d\langle\chi|a^3\frac{\partial\mathcal{L}}{\partial N}|\chi\rangle}{dt} = 0\right)$$

$$\text{Can } \langle\chi|a^3\frac{\partial\mathcal{L}}{\partial N}|\chi\rangle \neq 0?$$

Consequence of Schrodinger Equation

$$\text{Choose } \langle\chi| a^3 \frac{\partial \mathcal{L}}{\partial N} |\chi\rangle = 0 \implies \text{1st FRW holds}$$

$$\text{Can } \langle\chi| a^3 \frac{\partial \mathcal{L}}{\partial N} |\chi\rangle \neq 0?$$

In classical physics, we demand 2 FRW + 1 KG equation to hold - so we restrict initial conditions to obey 1st FRW

Consequence of Schrodinger Equation

$$\text{Choose } \langle\chi|a^3\frac{\partial\mathcal{L}}{\partial N}|\chi\rangle = 0 \implies \text{1st FRW holds}$$

$$\text{Can } \langle\chi|a^3\frac{\partial\mathcal{L}}{\partial N}|\chi\rangle \neq 0?$$

In classical physics, we demand 2 FRW + 1 KG equation to hold - so we restrict initial conditions to obey 1st FRW

Quantum Dynamics

$$i\frac{\partial|\chi(t)\rangle}{\partial t} = NH_0|\chi(t)\rangle$$

Consequence of Schrodinger Equation

$$\text{Choose } \langle\chi|a^3\frac{\partial\mathcal{L}}{\partial N}|\chi\rangle = 0 \implies \text{1st FRW holds}$$

$$\text{Can } \langle\chi|a^3\frac{\partial\mathcal{L}}{\partial N}|\chi\rangle \neq 0?$$

In classical physics, we demand 2 FRW + 1 KG equation to hold - so we restrict initial conditions to obey 1st FRW

Quantum Dynamics

$$i\frac{\partial|\chi(t)\rangle}{\partial t} = NH_0|\chi(t)\rangle$$

First order ODE - no issue with time evolving

$$\langle\chi|a^3\frac{\partial\mathcal{L}}{\partial N}|\chi\rangle \neq 0$$

Violating 1st FRW

$$\langle\chi|a^3\frac{\partial\mathcal{L}}{\partial N}|\chi\rangle = c$$

Quantum Dynamics: $$i\frac{\partial|\chi(t)\rangle}{\partial t} = NH_0|\chi(t)\rangle$$

Violating 1st FRW

$$\langle\chi|a^3\frac{\partial\mathcal{L}}{\partial N}|\chi\rangle = c$$

Quantum Dynamics: $$i\frac{\partial|\chi(t)\rangle}{\partial t} = NH_0|\chi(t)\rangle$$

Implied Classical Dynamics

$$\langle\chi|\partial_t\left(\frac{\partial\mathcal{L}}{\partial\dot{\phi}}\right) - \frac{\partial\mathcal{L}}{\partial\phi}|\chi\rangle = 0$$

Klein Gordon

$$\langle\chi|\partial_t\left(\frac{\partial\mathcal{L}}{\partial\dot{a}}\right) - \frac{\partial\mathcal{L}}{\partial a}|\chi\rangle = 0$$

2nd FRW Equation

$$\langle\chi|\frac{\partial\mathcal{L}}{\partial N}|\chi\rangle = \langle\chi|\frac{c}{a^3}|\chi\rangle$$

1st FRW but with "Dark" Matter

Quantum "Dark" Matter

$$\langle \chi | a^3 \frac{\partial \mathcal{L}}{\partial N} | \chi \rangle = c$$

$$a = A + A^\dagger, \Pi_a = i\left(A - A^\dagger\right) \qquad \phi = B + B^\dagger, \Pi_\phi = i\left(B - B^\dagger\right)$$

Create quantum states of a, φ
Quantum Dynamics: Just 1 first order ODE (Schrodinger)
No reason to constrain initial state!

Failure manifests classically as "dark" matter - though no real particle excitation there. Conservation implies super-selection sector.

Can be positive or negative!

Wheeler deWitt

Gauge Invariance

Dirac: $|A\rangle \equiv |A + \nabla\alpha\rangle$

Restrict to states that are invariant under gauge transformations

G generates the gauge transformation

$$G|A_P\rangle = 0$$

Gauge Invariance

Dirac: $|A\rangle \equiv |A + \nabla\alpha\rangle$

Restrict to states that are invariant under gauge transformations

G generates the gauge transformation

$$G|A_P\rangle = 0$$

Gauge symmetry of General Relativity is space-time translations

H generates time translation

$$H|\Psi\rangle = 0$$

Gauge Invariance

Dirac: $$|A\rangle \equiv |A + \nabla\alpha\rangle$$

Restrict to states that are invariant under gauge transformations

G generates the gauge transformation

$$G|A_P\rangle = 0$$

Gauge symmetry of General Relativity is space-time translations

H generates time translation

$$H|\Psi\rangle = 0$$

Universe is in an energy eigenstate

Conclusions

Conclusions

$$S = \int dt \mathcal{L}\left(a\left(t\right), \dot{a}\left(t\right), N\left(t\right), \phi\left(t\right), \dot{\phi}\left(t\right)\right)$$

Opinion A

$$i\frac{\partial|\chi\left(t\right)\rangle}{\partial t} = NH_0|\chi\left(t\right)\rangle$$

Opinion B

$$H|\Psi\rangle = 0$$

Conclusions

$$S = \int dt \mathcal{L}\left(a\left(t\right), \dot{a}\left(t\right), N\left(t\right), \phi\left(t\right), \dot{\phi}\left(t\right)\right)$$

Opinion A

$$i\frac{\partial|\chi\left(t\right)\rangle}{\partial t} = NH_0|\chi\left(t\right)\rangle$$

Pick N
Redefine time to get physics independent of N

1st FRW Equation only true up to constant

Quantum "Dark" Matter

Opinion B

$$H|\Psi\rangle = 0$$

Obey's Dirac's criteria

Only static states…no time!

The Classical Equations of Motion

Yukawa Theory

$$S \supset \int d^4x \, \lambda(x) \, \phi \bar{\Psi} \Psi$$

$$\text{Do Not } \frac{\partial S}{\partial \lambda} = 0$$

$$\text{Do Not } \phi \bar{\Psi} \Psi |\chi\rangle = 0$$

Why?

$$\pi_\lambda = \frac{\partial \mathcal{L}}{\partial \dot{\lambda}} = 0$$

λ not d.o.f.

Gauge Theory

$$S = \int dt \mathcal{L} \left(a(t), \dot{a}(t), N(t), \phi(t), \dot{\phi}(t) \right)$$

$$\pi_N = \frac{\partial \mathcal{L}}{\partial \dot{N}} = 0$$

N not d.o.f.

But:

$$\frac{\partial S}{\partial N} = 0$$

$$\frac{\partial S}{\partial N} |\chi\rangle = 0$$

????????

Quantizing Gauge Theories

$$S_{MS} = \int dt\, \mathcal{L}\left(a\left(t\right), \dot{a}\left(t\right), N\left(t\right)\right) \qquad S_{EM} = \int dt \mathcal{L}\left(\vec{A}, \dot{\vec{A}}, A_0\right) \qquad S_{GR} = \int dt\, \mathcal{L}\left(g_{ij}, \dot{g_{ij}}, g_{0\mu}\right)$$

Analogy with Yukawa: Treat N, A_0, $g_{0\mu}$ as some unknown but fixed operators

Quantizing Gauge Theories

$$S_{MS} = \int dt\, \mathcal{L}\left(a\left(t\right), \dot{a}\left(t\right), N\left(t\right)\right) \qquad S_{EM} = \int dt \mathcal{L}\left(\vec{A}, \dot{\vec{A}}, A_0\right) \qquad S_{GR} = \int dt\, \mathcal{L}\left(g_{ij}, \dot{g_{ij}}, g_{0\mu}\right)$$

Analogy with Yukawa: Treat N, A_0, $g_{0\mu}$ as some unknown but fixed operators

$$\text{Not d.o.f.} \implies \frac{\partial S}{\partial N}|\chi\rangle \neq 0, \frac{\partial S}{\partial A_0}|\chi\rangle \neq 0, \frac{\partial S}{\partial g_{0\mu}}|\chi\rangle \neq 0$$

Quantizing Gauge Theories

$$S_{MS} = \int dt\, \mathcal{L}\left(a\left(t\right), \dot{a}\left(t\right), N\left(t\right)\right) \qquad S_{EM} = \int dt \mathcal{L}\left(\vec{A}, \dot{\vec{A}}, A_0\right) \qquad S_{GR} = \int dt\, \mathcal{L}\left(g_{ij}, \dot{g_{ij}}, g_{0\mu}\right)$$

Analogy with Yukawa: Treat N, A_0, $g_{0\mu}$ as some unknown but fixed operators

$$\text{Not d.o.f.} \implies \frac{\partial S}{\partial N}|\chi\rangle \neq 0, \frac{\partial S}{\partial A_0}|\chi\rangle \neq 0, \frac{\partial S}{\partial g_{0\mu}}|\chi\rangle \neq 0$$

Can show that for these states, still get massless photon and graviton - the only physical aspect of gauge invariance we actually care about

Quantizing Gauge Theories

$$S_{MS} = \int dt\, \mathcal{L}\left(a\left(t\right), \dot{a}\left(t\right), N\left(t\right)\right) \qquad S_{EM} = \int dt \mathcal{L}\left(\vec{A}, \dot{\vec{A}}, A_0\right) \qquad S_{GR} = \int dt\, \mathcal{L}\left(g_{ij}, \dot{g_{ij}}, g_{0\mu}\right)$$

Analogy with Yukawa: Treat N, A_0, $g_{0\mu}$ as some unknown but fixed operators

$$\text{Not d.o.f.} \implies \frac{\partial S}{\partial N}|\chi\rangle \neq 0, \frac{\partial S}{\partial A_0}|\chi\rangle \neq 0, \frac{\partial S}{\partial g_{0\mu}}|\chi\rangle \neq 0$$

Can show that for these states, still get massless photon and graviton - the only physical aspect of gauge invariance we actually care about

Further, by suitable redefinitions, can show that these choices only influence physics at the level of violations of constraints with “dark” backgrounds

General Relativity

$$g_{\mu\nu} = g_{0\mu} dt dx^{\mu} + g_{ij} dx^{i} dx^{j}$$

$g_{0\mu}$ do not have conjugate momenta - fixed c-number functions

General Relativity

$$g_{\mu\nu} = g_{0\mu} dt dx^{\mu} + g_{ij} dx^i dx^j$$

$g_{0\mu}$ do not have conjugate momenta - fixed c-number functions

Quantum Mechanics Guarantees:

$G_{ij} = T_{ij}$

General Relativity

$$g_{\mu\nu} = g_{0\mu} dt dx^{\mu} + g_{ij} dx^{i} dx^{j}$$

$g_{0\mu}$ do not have conjugate momenta - fixed c-number functions

Quantum Mechanics Guarantees:

$G_{ij} = T_{ij}$

Classically:

$G_{\mu\nu} = T_{\mu\nu} + H_{\mu\nu}$

With: $H_{\mu\nu} = H_{0\mu}$

$$\nabla^{\mu} H_{\mu\nu} = 0$$

General Relativity

$$g_{\mu\nu} = g_{0\mu} dt dx^{\mu} + g_{ij} dx^i dx^j$$

$g_{0\mu}$ do not have conjugate momenta - fixed c-number functions

Quantum Mechanics Guarantees:

$G_{ij} = T_{ij}$

Classically:

$G_{\mu\nu} = T_{\mu\nu} + H_{\mu\nu}$

With: $H_{\mu\nu} = H_{0\mu}$

$$\nabla^{\mu} H_{\mu\nu} = 0$$

New cosmological observables

Quantum Gauge Theories

Classical Field Theory (e.g. Electromagnetism)

$$\frac{\delta S}{\delta A^\mu} = 0$$

Quantum Field Theory

$$Z = \int DA^\mu e^{iS[A]} \implies \delta Z = \int DA^\mu \left(\frac{\delta S}{\delta A^\mu} \right) \delta A^\mu = 0$$

Quantum Gauge Theories

Classical Field Theory
(e.g. Electromagnetism)

$$\frac{\delta S}{\delta A^\mu} = 0$$

Quantum Field Theory

$$Z = \int DA^\mu e^{iS[A]} \implies \delta Z = \int DA^\mu \left(\frac{\delta S}{\delta A^\mu}\right) \delta A^\mu = 0$$

Need to Gauge Fix to define Path Integral

$$Z = \int DA^\mu e^{iS_{gf}[A]} \implies \delta Z = \int DA^\mu \left(\frac{\delta S_{gf}}{\delta A^\mu}\right) \delta A^\mu = 0$$

Quantum Gauge Theories

Classical Field Theory
(e.g. Electromagnetism)

$$\frac{\delta S}{\delta A^\mu} = 0$$

Not the Same ????????

Quantum Field Theory

$$Z = \int DA^\mu e^{iS[A]} \implies \delta Z = \int DA^\mu \left(\frac{\delta S}{\delta A^\mu}\right) \delta A^\mu = 0$$

Need to Gauge Fix to define Path Integral

$$Z = \int DA^\mu e^{iS_{gf}[A]} \implies \delta Z = \int DA^\mu \left(\frac{\delta S_{gf}}{\delta A^\mu}\right) \delta A^\mu = 0$$

Quantum Gauge Theories

$$i\frac{\partial|\chi\left(t\right)\rangle}{\partial t} = H|\chi\left(t\right)\rangle$$

Dynamical Equation

Quantum Gauge Theories

$$i\frac{\partial|\chi(t)\rangle}{\partial t} = H|\chi(t)\rangle$$

Dynamical Equation

Can only imply: $$\langle\frac{\delta S}{\delta A^i}\rangle = 0 \implies \langle\frac{\partial\vec{E}}{\partial t} - \nabla\times\vec{B} - \vec{J}\rangle = 0$$

Quantum Gauge Theories

$$i\frac{\partial|\chi(t)\rangle}{\partial t} = H|\chi(t)\rangle$$

Dynamical Equation

Can only imply: $$\langle\frac{\delta S}{\delta A^i}\rangle = 0 \implies \langle\frac{\partial\vec{E}}{\partial t} - \nabla\times\vec{B} - \vec{J}\rangle = 0$$

Constraint Equations?

Imposed by hand - does not follow from Schrodinger

$$\left(\vec{\nabla}\cdot\vec{E} - J^0\right)|\chi\rangle = 0$$

Quantum Gauge Theories

$$i\frac{\partial|\chi(t)\rangle}{\partial t} = H|\chi(t)\rangle$$

Dynamical Equation

Can only imply: $\langle\frac{\delta S}{\delta A^i}\rangle = 0 \implies \langle\frac{\partial \vec{E}}{\partial t} - \nabla \times \vec{B} - \vec{J}\rangle = 0$

Constraint Equations?

Imposed by hand - does not follow from Schrodinger

$$\left(\vec{\nabla} \cdot \vec{E} - J^0\right)|\chi\rangle = 0$$

First order ODE - can time evolve states that violate constraint

Does anything go wrong? i.e. massless photon?

Electromagnetism

$$A_\mu = A_0 dt + A_i dx^i$$

A_0 does not have a conjugate momentum - fixed c-number function

Electromagnetism

$$A_\mu = A_0 dt + A_i dx^i$$

A_0 does not have a conjugate momentum - fixed c-number function

Quantum Mechanics

(Gauss's Law) $\partial_\mu F^{\mu 0} = J^0$

$\partial_\mu F^{\mu i} = J^i$ **(Ampere's Law)**

Electromagnetism

$$A_\mu = A_0 dt + A_i dx^i$$

A_0 does not have a conjugate momentum - fixed c-number function

Quantum Mechanics

(Gauss's Law) $\partial_\mu F^{\mu 0} = J^0$ ✗

$\partial_\mu F^{\mu i} = J^i$ **(Ampere's Law)** ✓

$$\partial_0 \partial_\mu F^{\mu 0} = 0$$

Electromagnetism

$$A_\mu = A_0 dt + A_i dx^i$$

A_0 does not have a conjugate momentum - fixed c-number function

Quantum Mechanics

(Gauss's Law) $\partial_\mu F^{\mu 0} = J^0$

$\partial_\mu F^{\mu i} = J^i$ (Ampere's Law)

$$\partial_0 \partial_\mu F^{\mu 0} = 0$$

Classically

$$\partial_\mu F^{\mu\nu} = J^\nu + J_d^\mu$$

$$\text{With: } J_d^\mu = \left(J^0(x), 0, 0, 0\right) \implies \frac{d}{dt}\left(J_d^0\right) = 0$$

Electromagnetism

$$A_\mu = A_0 dt + A_i dx^i$$

A_0 does not have a conjugate momentum - fixed c-number function

Quantum Mechanics

(Gauss's Law) $\partial_\mu F^{\mu 0} = J^0$ ✗

$\partial_\mu F^{\mu i} = J^i$ **(Ampere's Law)** ✓

$$\partial_0 \partial_\mu F^{\mu 0} = 0$$

Classically

$$\partial_\mu F^{\mu\nu} = J^\nu + J_d^\mu$$

$$\text{With: } J_d^\mu = \left(J^0\left(x\right), 0, 0, 0\right) \implies \frac{d}{dt}\left(J_d^0\right) = 0$$

New cosmological observables

Backup

Quantizing Gauge Theories

Yukawa Theory

$$S \supset \int d^4x \, \lambda(x) \, \phi \bar{\Psi} \Psi$$

$$\text{Do Not } \frac{\partial S}{\partial \lambda} = 0$$

$$\text{Do Not } \phi \bar{\Psi} \Psi | \chi \rangle = 0$$

Quantizing Gauge Theories

Yukawa Theory

$$S \supset \int d^4x \, \lambda(x) \, \phi \bar{\Psi} \Psi$$

$$\text{Do Not } \frac{\partial S}{\partial \lambda} = 0$$

$$\text{Do Not } \phi \bar{\Psi} \Psi |\chi\rangle = 0$$

Why?

$$\pi_\lambda = \frac{\partial \mathcal{L}}{\partial \dot{\lambda}} = 0$$

λ not d.o.f.

Quantizing Gauge Theories

Yukawa Theory

$$S \supset \int d^4x \, \lambda(x) \, \phi \bar{\Psi} \Psi$$

$$\text{Do Not } \frac{\partial S}{\partial \lambda} = 0$$

$$\text{Do Not } \phi \bar{\Psi} \Psi |\chi\rangle = 0$$

Why?

$$\pi_\lambda = \frac{\partial \mathcal{L}}{\partial \dot{\lambda}} = 0$$

λ not d.o.f.

Gauge Theory

$$S = \int dt \mathcal{L} \left(a(t), \dot{a}(t), N(t), \phi(t), \dot{\phi}(t) \right)$$

$$\pi_N = \frac{\partial \mathcal{L}}{\partial \dot{N}} = 0$$

N not d.o.f.

Quantizing Gauge Theories

Yukawa Theory

$$S \supset \int d^4x \, \lambda(x) \, \phi \bar{\Psi} \Psi$$

$$\text{Do Not } \frac{\partial S}{\partial \lambda} = 0$$

$$\text{Do Not } \phi \bar{\Psi} \Psi |\chi\rangle = 0$$

Why?

$$\pi_\lambda = \frac{\partial \mathcal{L}}{\partial \dot{\lambda}} = 0$$

λ not d.o.f.

Gauge Theory

$$S = \int dt \mathcal{L} \left(a(t), \dot{a}(t), N(t), \phi(t), \dot{\phi}(t) \right)$$

$$\pi_N = \frac{\partial \mathcal{L}}{\partial \dot{N}} = 0$$

N not d.o.f.

But:

$$\frac{\partial S}{\partial N} = 0$$

$$\frac{\partial S}{\partial N} |\chi\rangle = 0$$

Quantizing Gauge Theories

Yukawa Theory

$$S \supset \int d^4x \, \lambda(x) \, \phi \bar{\Psi} \Psi$$

$$\text{Do Not } \frac{\partial S}{\partial \lambda} = 0$$

$$\text{Do Not } \phi \bar{\Psi} \Psi |\chi\rangle = 0$$

Why?

$$\pi_\lambda = \frac{\partial \mathcal{L}}{\partial \dot{\lambda}} = 0$$

λ not d.o.f.

Gauge Theory

$$S = \int dt \mathcal{L} \left(a(t), \dot{a}(t), N(t), \phi(t), \dot{\phi}(t) \right)$$

$$\pi_N = \frac{\partial \mathcal{L}}{\partial \dot{N}} = 0$$

N not d.o.f.

But:

$$\frac{\partial S}{\partial N} = 0$$

$$\frac{\partial S}{\partial N} |\chi\rangle = 0$$

????????

Classical and Quantum Gauge Theories

Classical Field Theory
(e.g. General Relativity)

$$\frac{\delta S}{\delta g^{\mu\nu}} = 0$$

Classical and Quantum Gauge Theories

Classical Field Theory (e.g. General Relativity)

$$\frac{\delta S}{\delta g^{\mu\nu}} = 0$$

Quantum Field Theory

$$Z = \int Dg^{\mu\nu} e^{iS[g]} \implies \delta Z = \int Dg^{\mu\nu} \left(\frac{\delta S}{\delta g^{\mu\nu}} \right) \delta g^{\mu\nu} = 0$$

Classical and Quantum Gauge Theories

Classical Field Theory
(e.g. General Relativity)

$$\frac{\delta S}{\delta g^{\mu\nu}} = 0$$

Quantum Field Theory

$$Z = \int Dg^{\mu\nu} e^{iS[g]} \implies \delta Z = \int Dg^{\mu\nu} \left(\frac{\delta S}{\delta g^{\mu\nu}} \right) \delta g^{\mu\nu} = 0$$

Need to Gauge Fix to define Path Integral

Classical and Quantum Gauge Theories

Classical Field Theory
(e.g. General Relativity)

$$\frac{\delta S}{\delta g^{\mu\nu}} = 0$$

Quantum Field Theory

$$Z = \int Dg^{\mu\nu} e^{iS[g]} \implies \delta Z = \int Dg^{\mu\nu} \left(\frac{\delta S}{\delta g^{\mu\nu}}\right) \delta g^{\mu\nu} = 0$$

Need to Gauge Fix to define Path Integral

$$Z = \int Dg^{\mu\nu} e^{iS_{gf}[g]} \implies \delta Z = \int Dg^{\mu\nu} \left(\frac{\delta S_{gf}}{\delta g^{\mu\nu}}\right) \delta g^{\mu\nu} = 0$$

Classical and Quantum Gauge Theories

Classical Field Theory
(e.g. General Relativity)

$$\frac{\delta S}{\delta g^{\mu\nu}} = 0$$

Not the Same ????????

Quantum Field Theory

$$Z = \int Dg^{\mu\nu} e^{iS[g]} \implies \delta Z = \int Dg^{\mu\nu} \left(\frac{\delta S}{\delta g^{\mu\nu}} \right) \delta g^{\mu\nu} = 0$$

✗

Need to Gauge Fix to define Path Integral

Quantum Supremacy

$$Z = \int Dg^{\mu\nu} e^{iS_{gf}[g]} \implies \delta Z = \int Dg^{\mu\nu} \left(\frac{\delta S_{gf}}{\delta g^{\mu\nu}} \right) \delta g^{\mu\nu} = 0$$

✓

Hamiltonian Construction

$$\Pi_a = \frac{\partial \mathcal{L}}{\partial \dot{a}}, \Pi_\phi = \frac{\partial \mathcal{L}}{\partial \dot{\phi}}$$

Quantize These

$$[a, \Pi_a] = [\phi, \Pi_\phi] = i$$

$$a = A + A^\dagger, \Pi_a = i\left(A - A^\dagger\right) \qquad \phi = B + B^\dagger, \Pi_\phi = i\left(B - B^\dagger\right)$$

Hamiltonian Construction

$$\Pi_a = \frac{\partial \mathcal{L}}{\partial \dot{a}}, \Pi_\phi = \frac{\partial \mathcal{L}}{\partial \dot{\phi}}$$

Quantize These

$$[a, \Pi_a] = [\phi, \Pi_\phi] = i$$

$$a = A + A^\dagger, \Pi_a = i\left(A - A^\dagger\right) \qquad \phi = B + B^\dagger, \Pi_\phi = i\left(B - B^\dagger\right)$$

Fully non-linear General Relativity - no “free” theory with “free” kinetic term

But, can still construct Hilbert space with Fock states of A, B - these are operator level statements independent of kinetic terms of the theory

$$A|0\rangle = 0, A^\dagger|0\rangle = |1\rangle \text{ etc.}$$

Electromagnetism

Outline

1. Electromagnetism

2. Mini Superspace

3. General Relativity

4. Cosmological Consequences

5. Conclusions

Do we need Hilbert space Restriction?

All we care about is we want a causal theory with a massless photon

$$A_0 = 0$$

$$H = \int d^3x \frac{1}{2}\left(E^2 + B^2\right) + \vec{A}.\vec{J} + H_J$$

Do we need Hilbert space Restriction?

All we care about is we want a causal theory with a massless photon

$$A_0 = 0$$

$$H = \int d^3x \frac{1}{2}\left(E^2 + B^2\right) + \vec{A}.\vec{J} + H_J$$

$$|\Psi(t)\rangle = e^{-iHt}|\Psi(0)\rangle = \left(1 + iHt + \frac{(iHt)^2}{2} + \dots\right)|\Psi(0)\rangle$$

Want to know if there is a photon mass operator at some order

Do we need Hilbert space Restriction?

All we care about is we want a causal theory with a massless photon

$$A_0 = 0$$

$$H = \int d^3x \frac{1}{2}\left(E^2 + B^2\right) + \vec{A}.\vec{J} + H_J$$

$$|\Psi(t)\rangle = e^{-iHt}|\Psi(0)\rangle = \left(1 + iHt + \frac{(iHt)^2}{2} + \ldots\right)|\Psi(0)\rangle$$

Want to know if there is a photon mass operator at some order

$$A \to A + \nabla\alpha \implies H \to H$$

Forbids photon mass: $A.A$

Do we need Hilbert space Restriction?

All we care about is we want a causal theory with a massless photon

$$A_0 = 0$$

$$H = \int d^3x \frac{1}{2}\left(E^2 + B^2\right) + \vec{A}.\vec{J} + H_J$$

$$|\Psi(t)\rangle = e^{-iHt}|\Psi(0)\rangle = \left(1 + iHt + \frac{(iHt)^2}{2} + \ldots\right)|\Psi(0)\rangle$$

Want to know if there is a photon mass operator at some order

$$A \to A + \nabla\alpha \implies H \to H$$

Forbids photon mass: $A.A$

Hilbert space restriction not needed - sufficient Hamiltonian symmetry

No Gauss Law since no restriction on initial state

Quantization of Electromagnetism

$$H = \int d^3x \frac{1}{2}\left(E^2 + B^2\right) + A_0\left(\nabla . E - J^0\right) + \vec{A}.\vec{J} + H_J$$

Opinion A (Dirac)

Gauge Invariance of States

$$|A\rangle \equiv |A + \nabla\alpha\rangle$$

$$G|A'_P\rangle = J_D\left(x\right)|A'_P\rangle$$

Opinion B (Team B)

Massless photon

Pick A_0 (e.g. any c-number function)

Requires Symmetry of Hamiltonian

$$A \rightarrow A + \nabla\alpha \implies H \rightarrow H$$

Quantization of Electromagnetism

$$H = \int d^3x \frac{1}{2}\left(E^2 + B^2\right) + A_0\left(\nabla.E - J^0\right) + \vec{A}.\vec{J} + H_J$$

Opinion A (Dirac)

Gauge Invariance of States

$$|A\rangle \equiv |A + \nabla\alpha\rangle$$

$$G|A'_P\rangle = J_D(x)\,|A'_P\rangle$$

Opinion B (Team B)

Massless photon

Pick A_0 (e.g. any c-number function)

Requires Symmetry of Hamiltonian

$$A \to A + \nabla\alpha \implies H \to H$$

Exist states of EM with Gauss law violation

Theory of massless photon, background charge - no new degrees of freedom