A TOP-DOWN DE SITTER MAXIMUM IN M-THEORY FROM CASIMIR ENERGIES

Miguel Montero
IFT Madrid

PASCOS 2025,

Durham University, July 21st, 2025

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putting everything together, i.e. getting the exact fields & couplings of the SM+ cosmo history.

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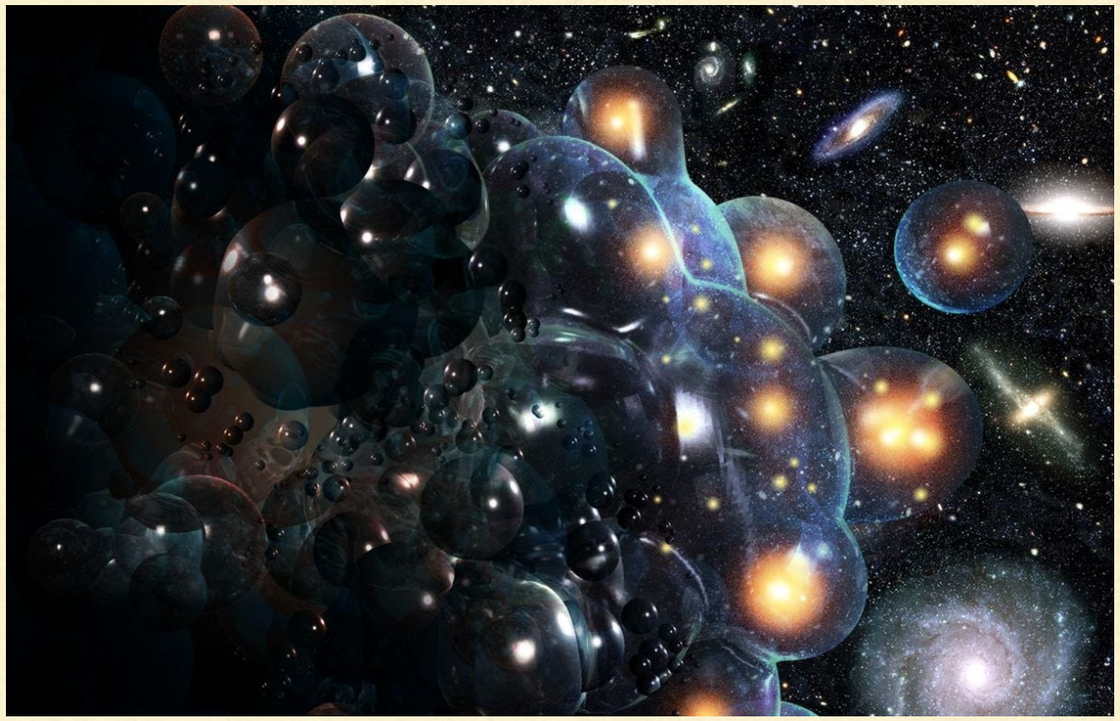

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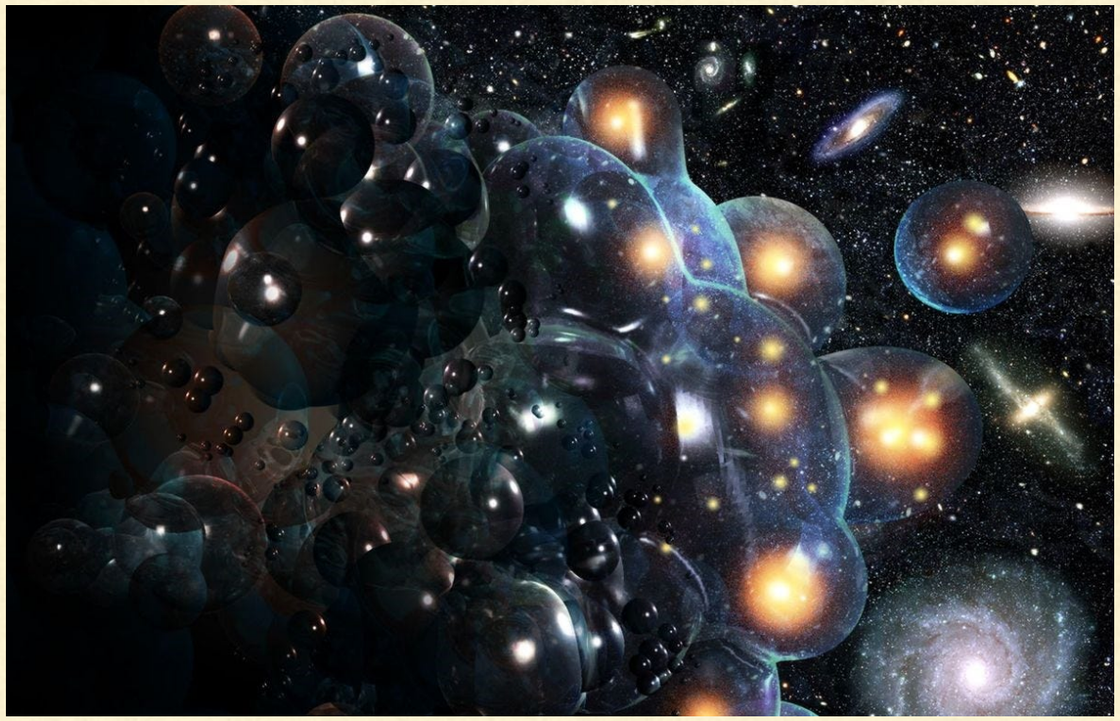

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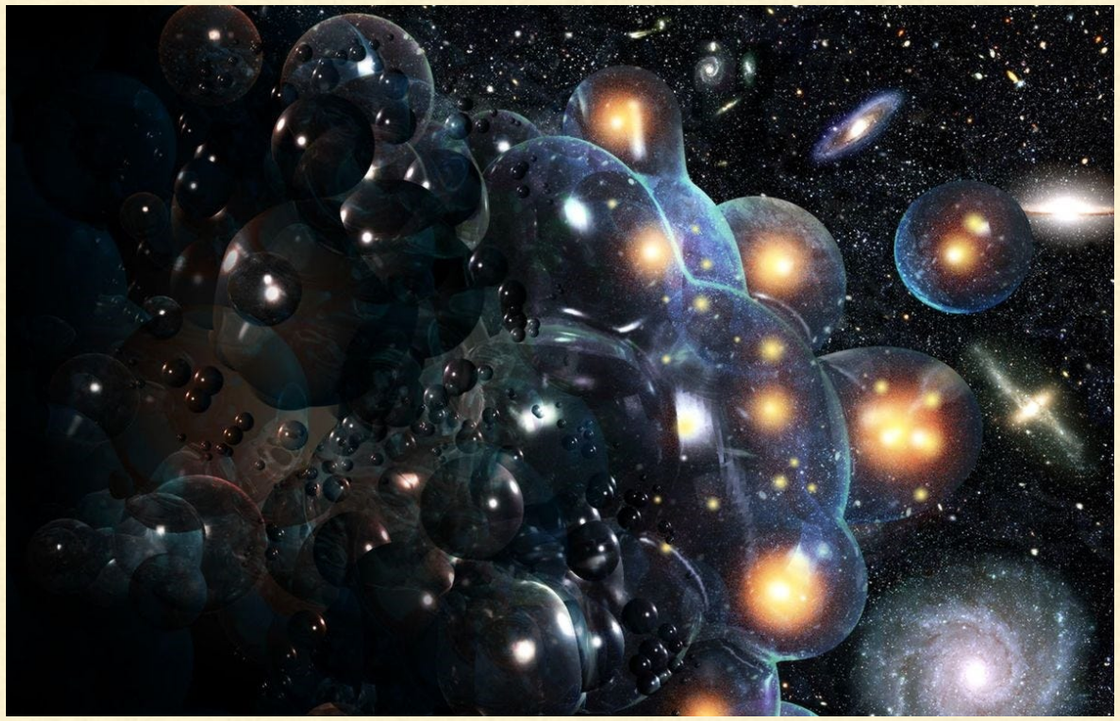

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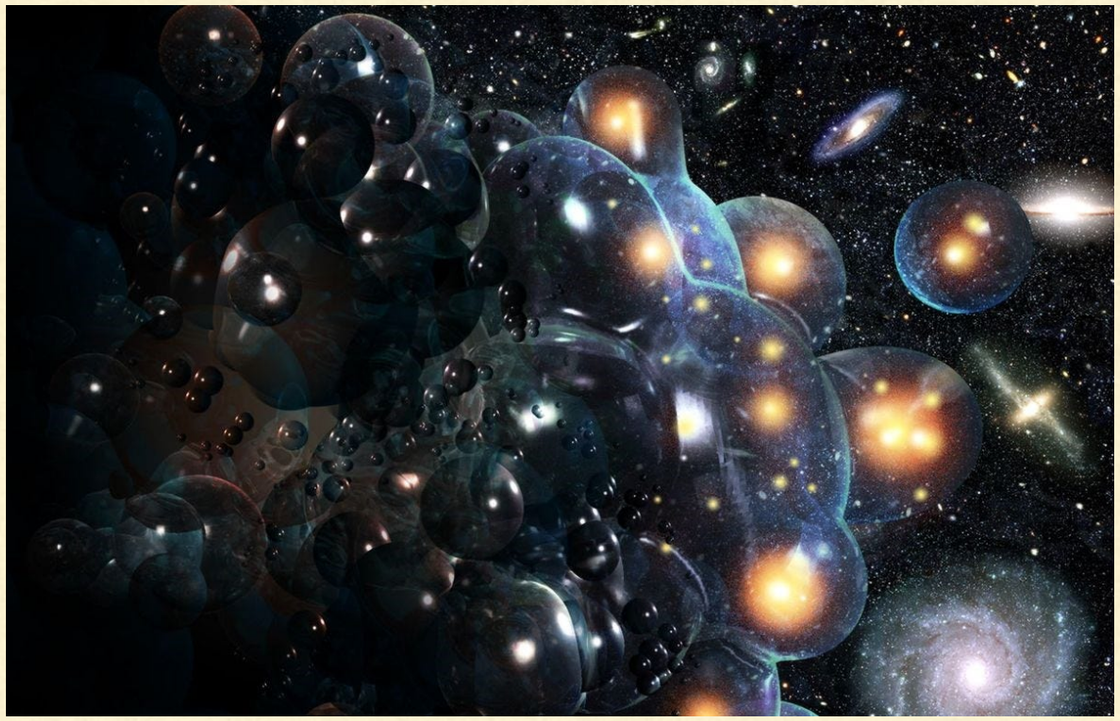

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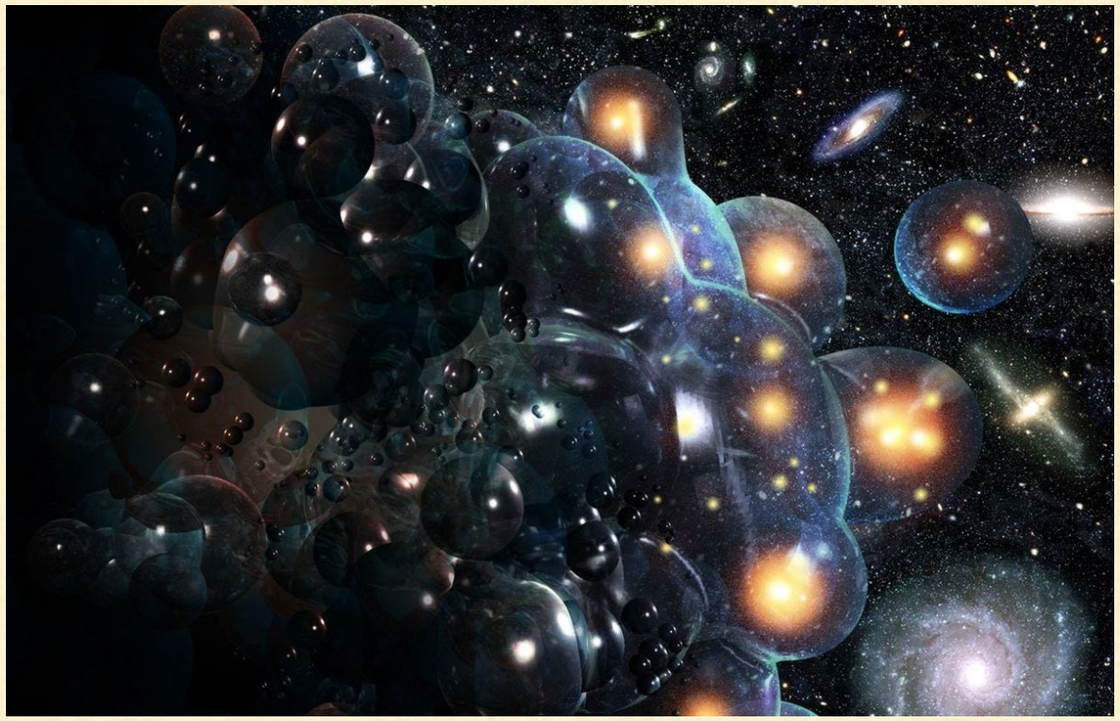

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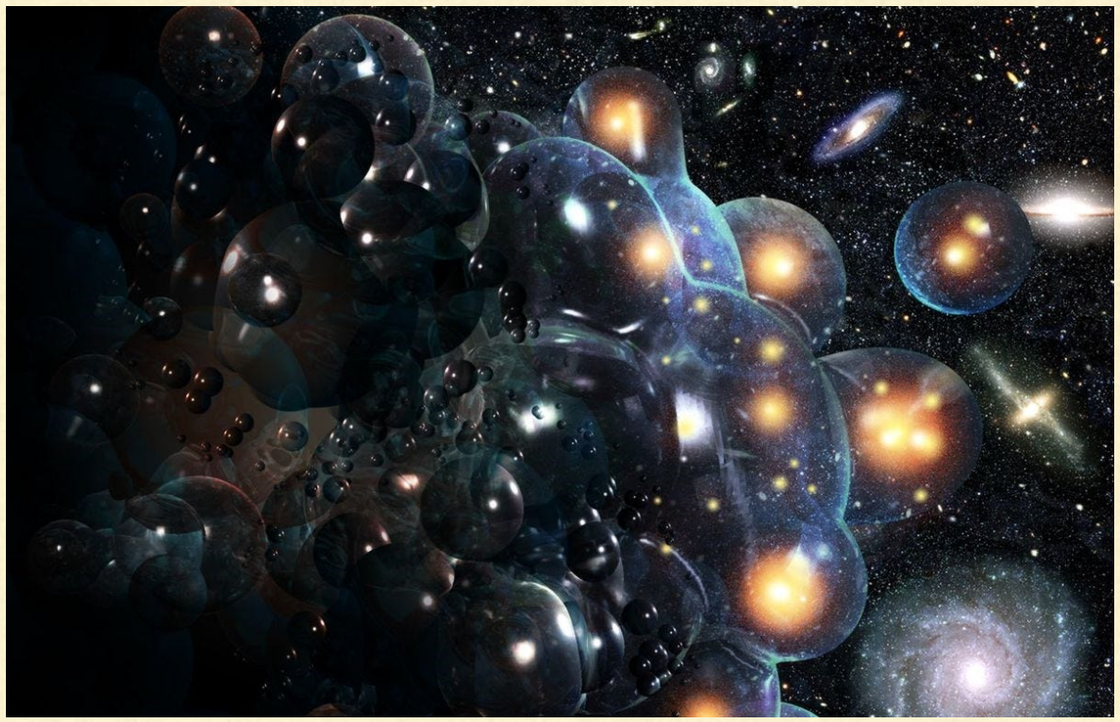

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So the quest **continues**!

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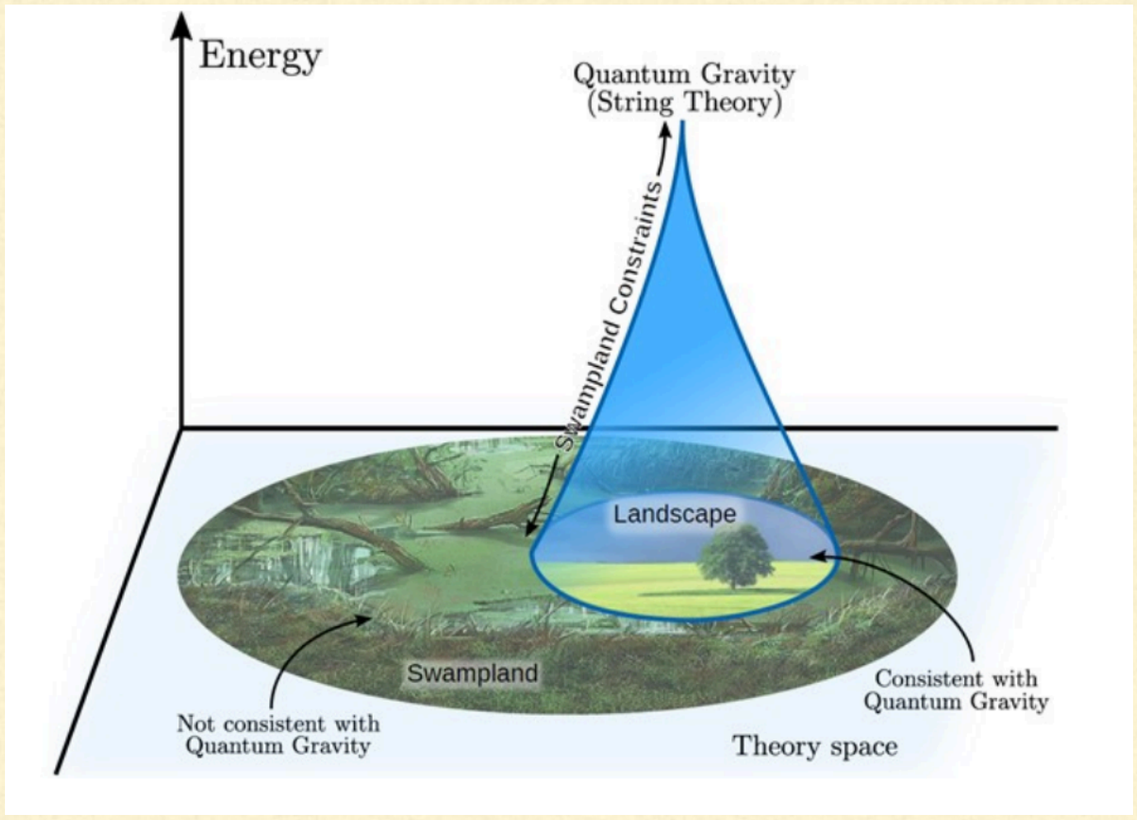
Others try to identify **universal features** that may be used to make a prediction even without the exact solution.



(SWAMPLAND PROGRAM)

[Review: 2102.01111 by van Beest, Calderon-Infante, Mirfendereski, Valenzuela]

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so these models are all missing a crucial ingredient, even when one ignores everything else (SM, etc).

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Some conjectures that long-lived dS does not exist in ST!

[Obied-Ooguri-Vafa-Spodyneiko '18, ...]

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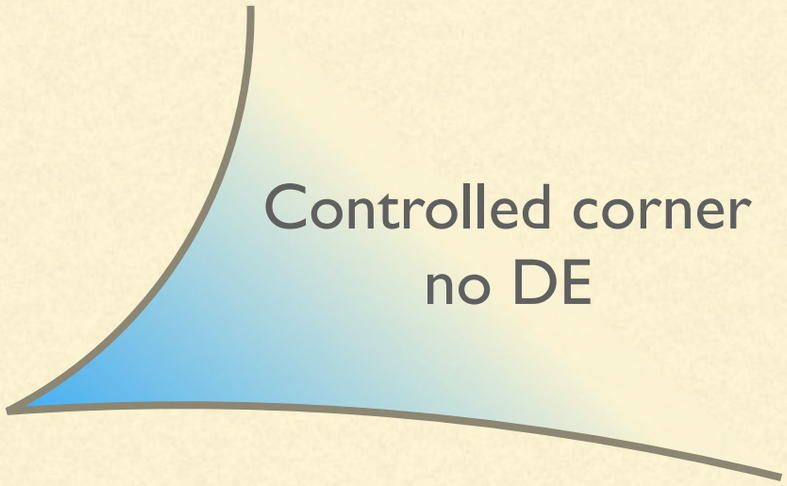

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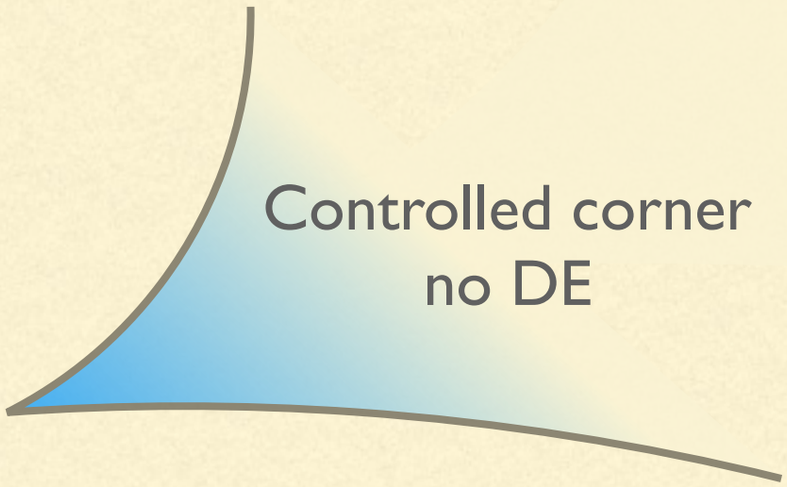

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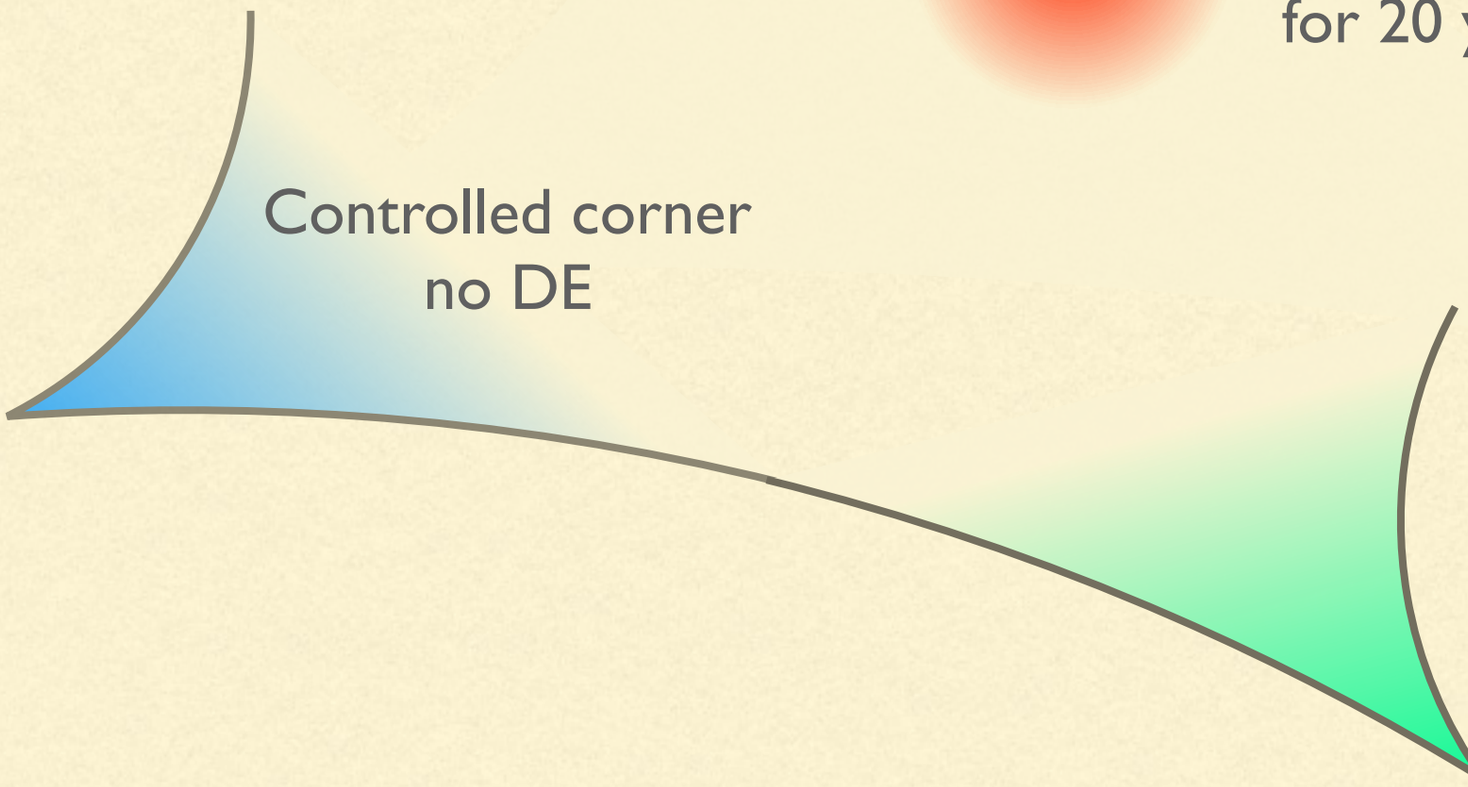

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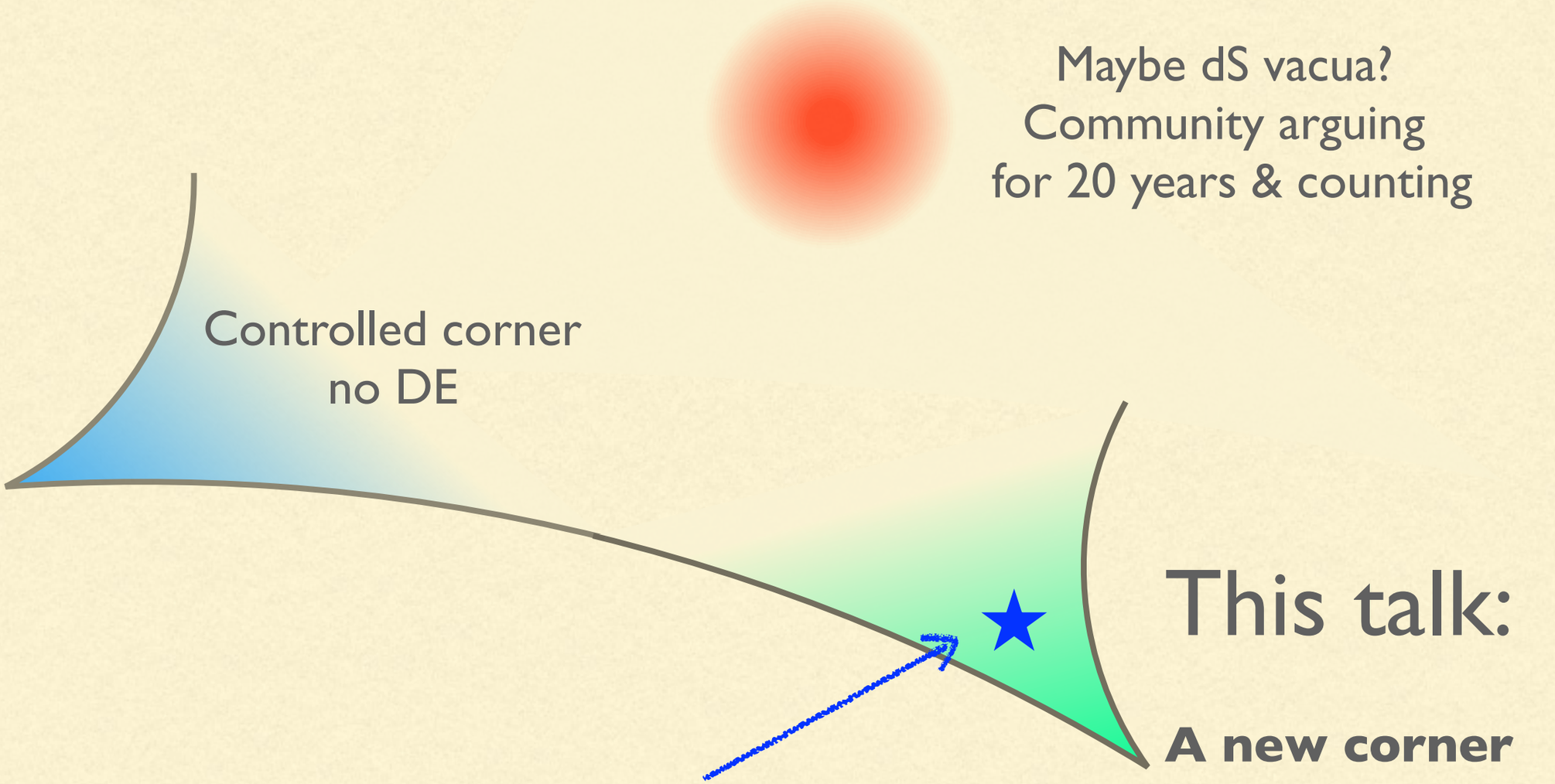
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Controlled corner
no DE
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A simpler, more controlled model
of **dynamical** (i.e. time-dependent)
Dark Energy

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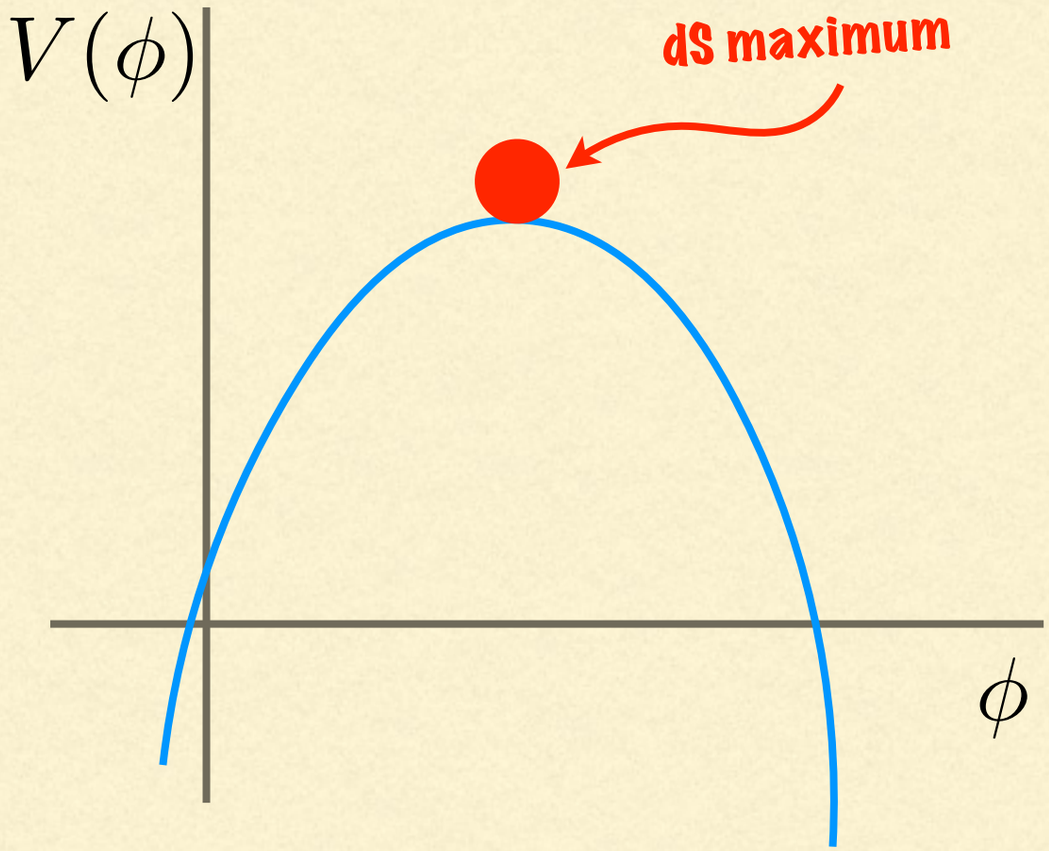

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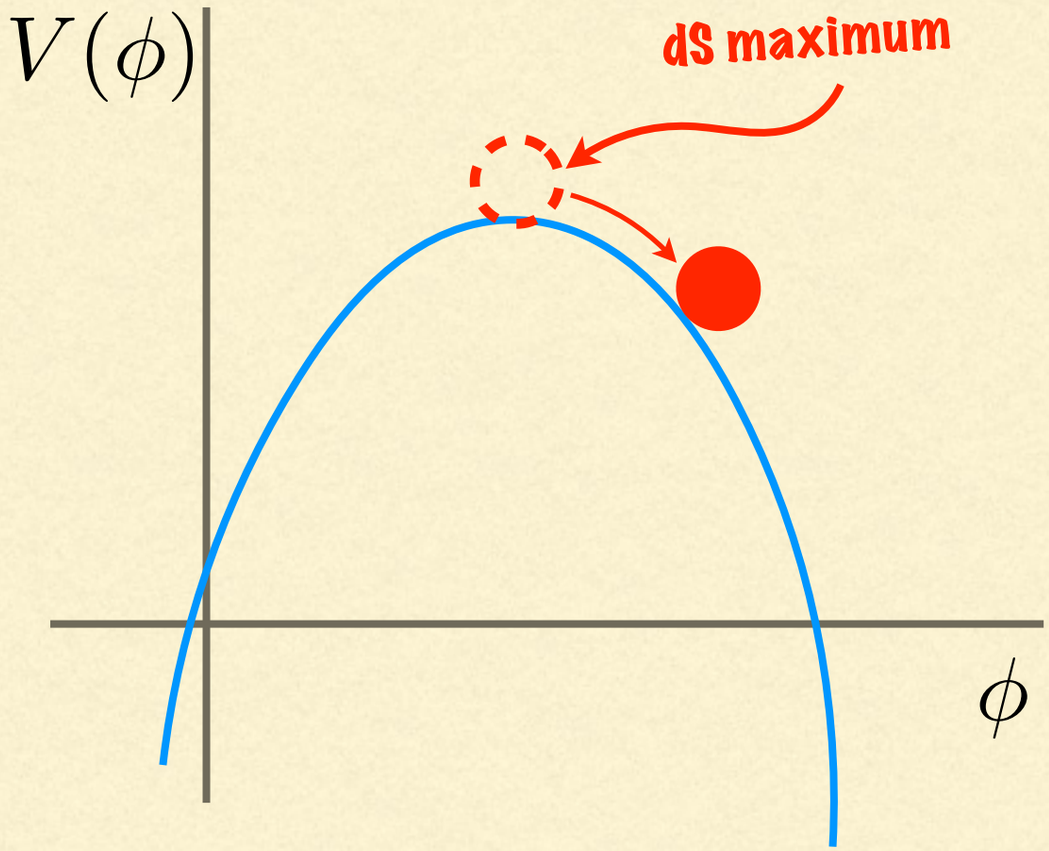

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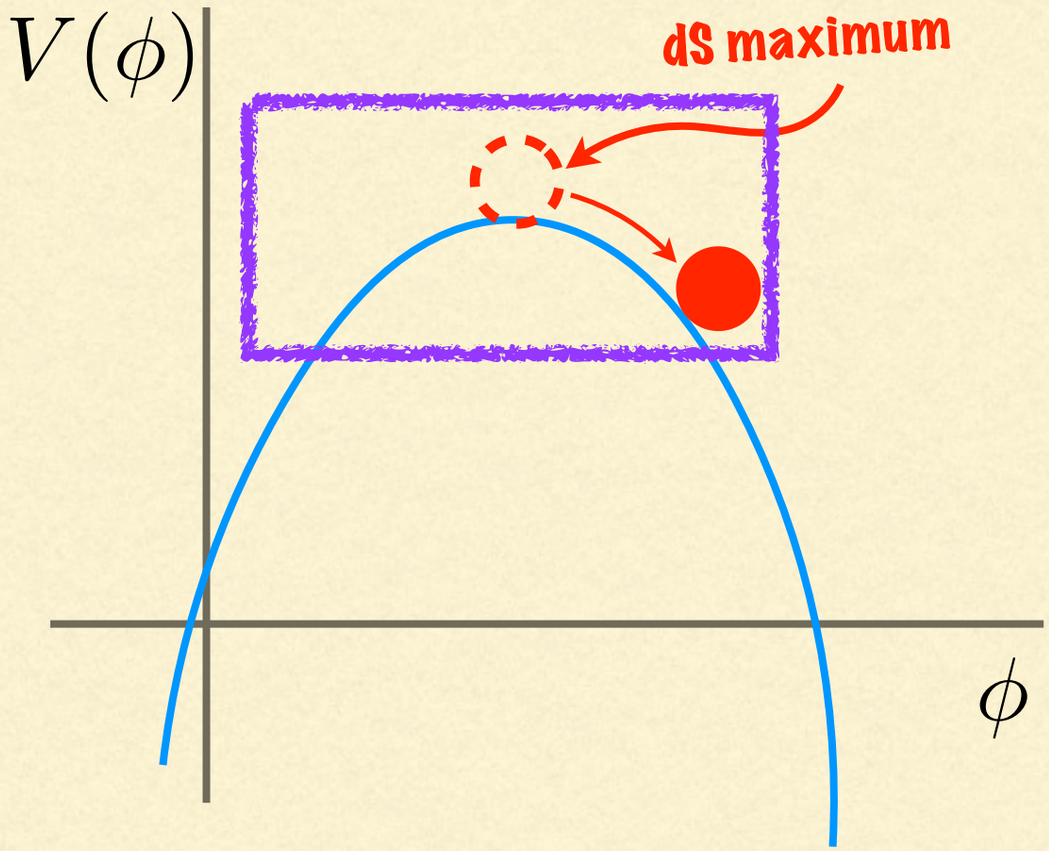

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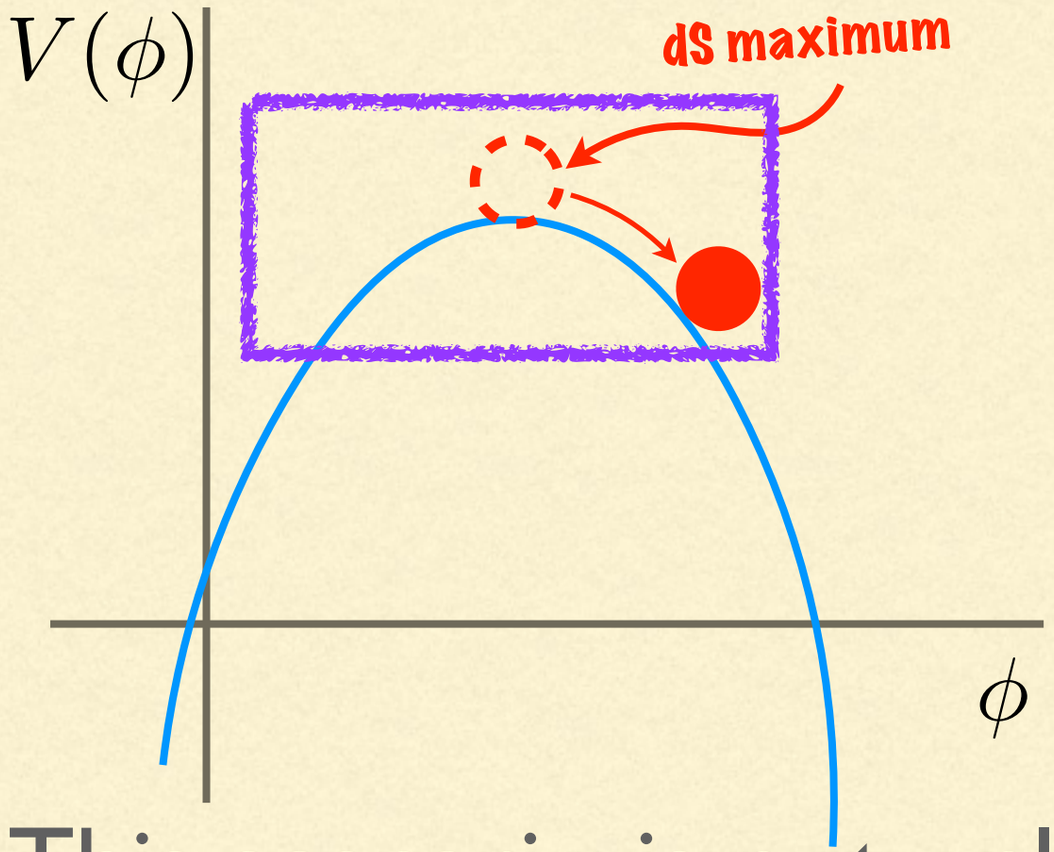

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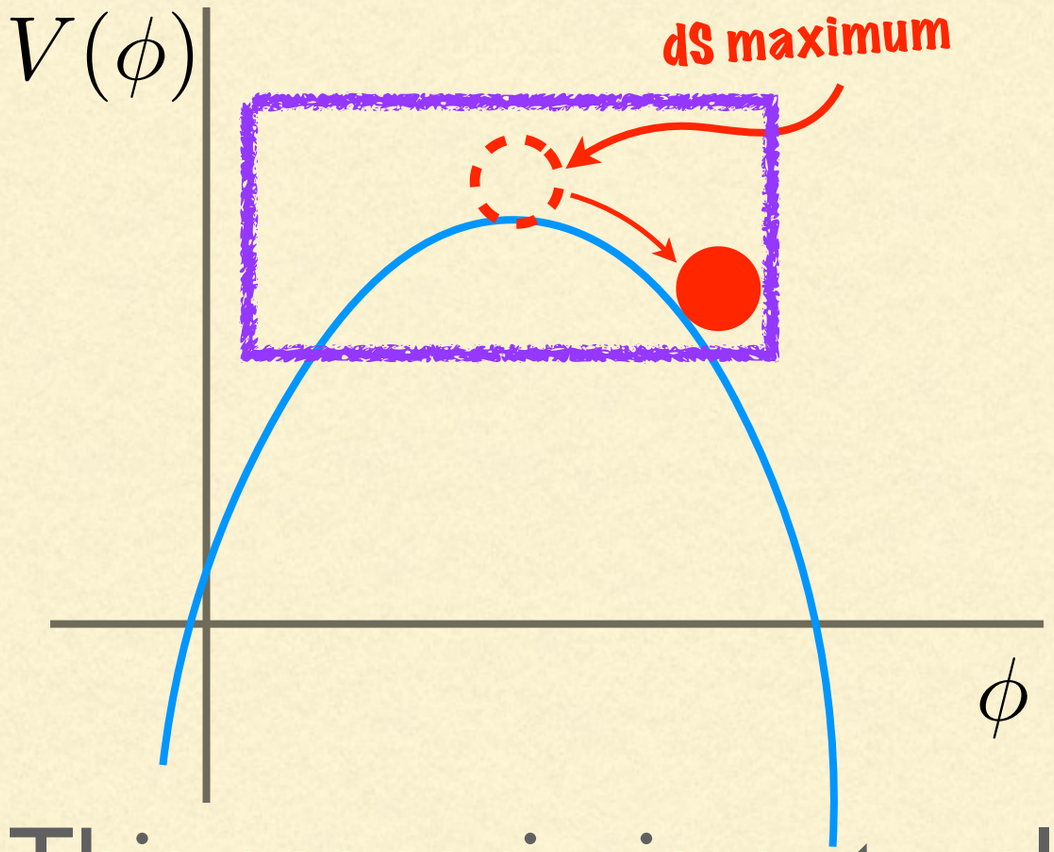

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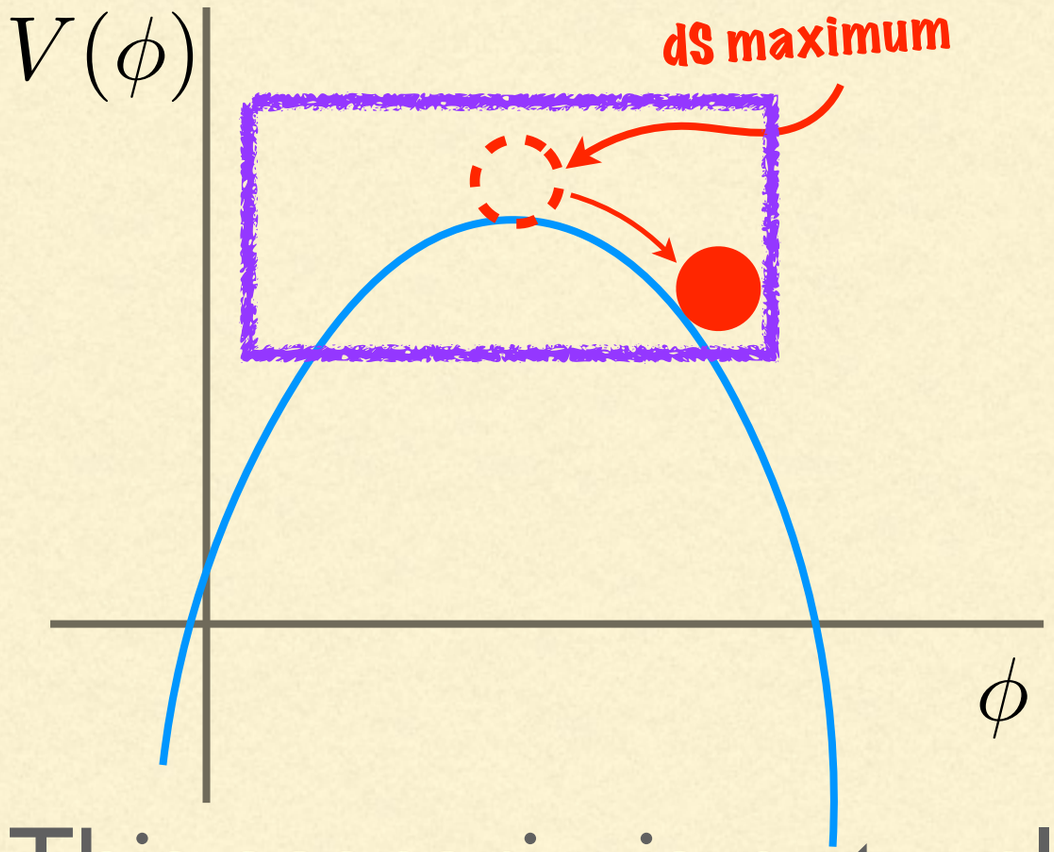

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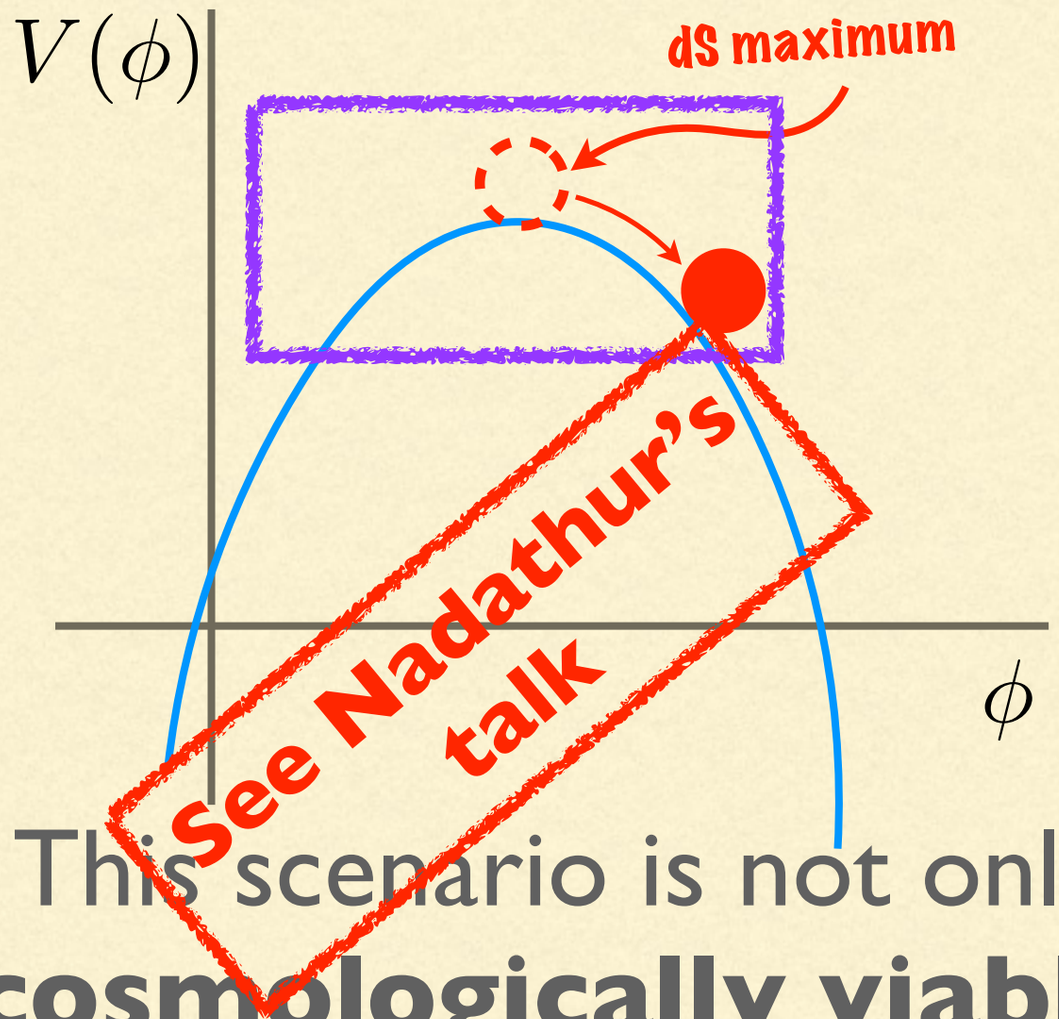

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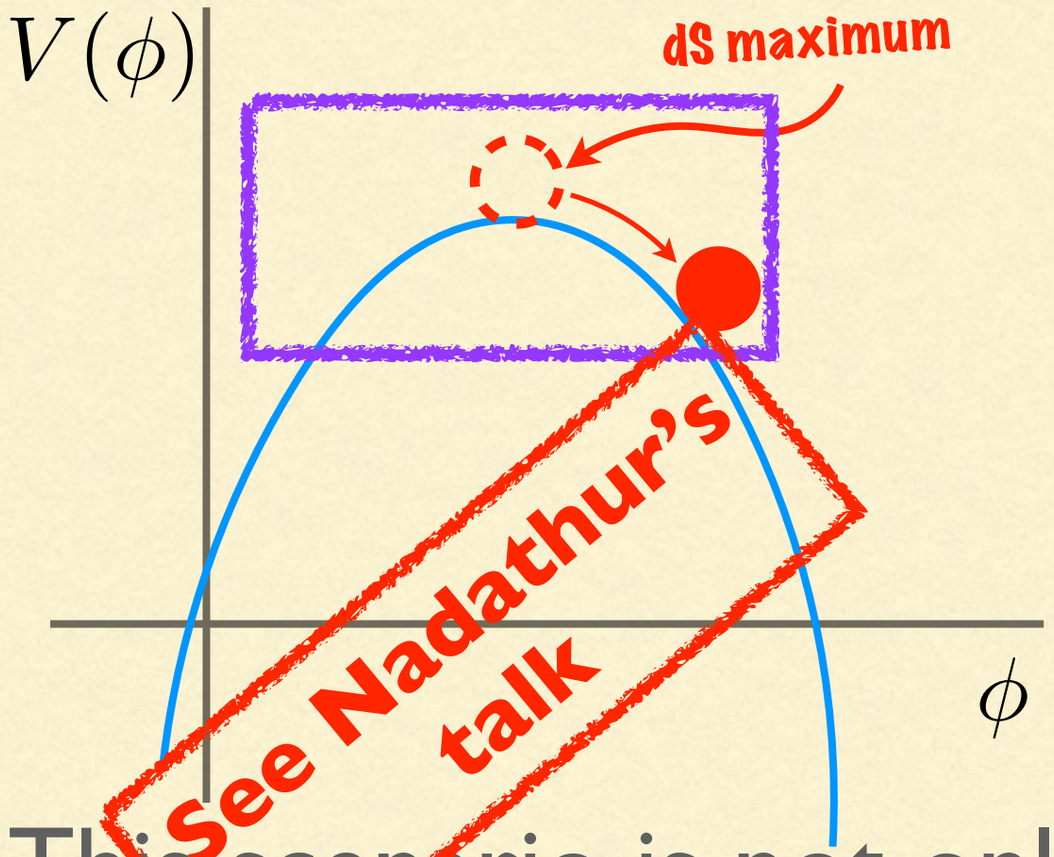

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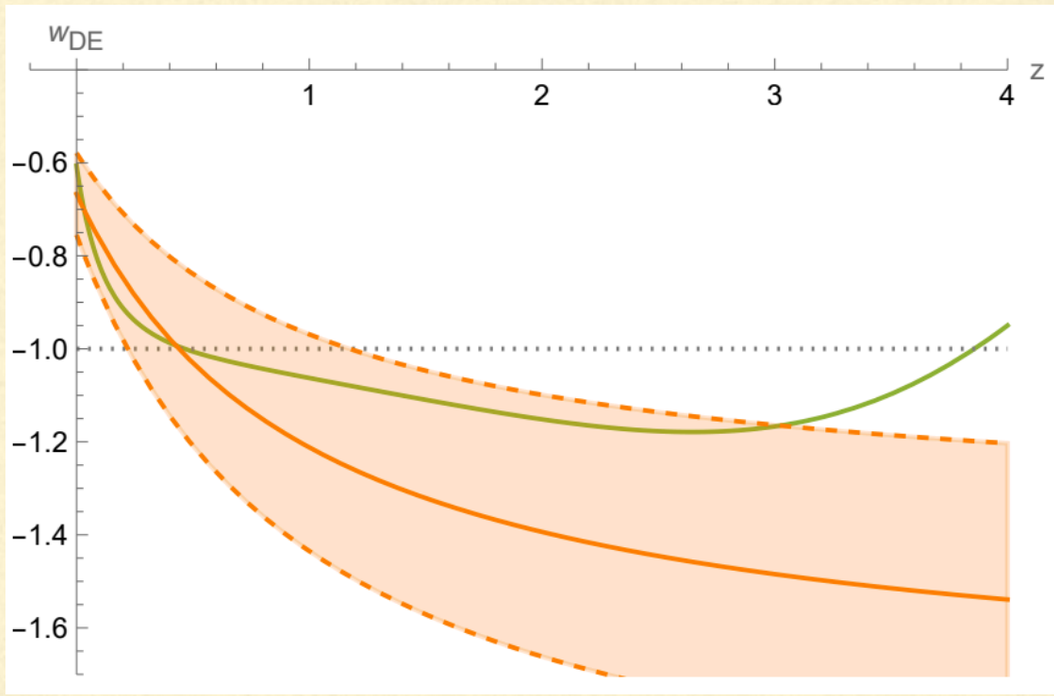

(Figure taken from **2505.10410, "Phantom matters"** by D. Andriot)

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Fully explicit; proof of concept of dS maxima in ST

Techniques generalize and can be used beyond ST

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If you are hiring next fall, **keep him in mind**

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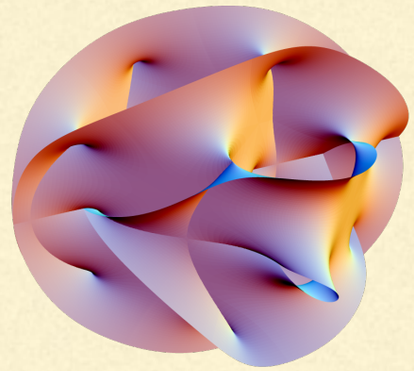

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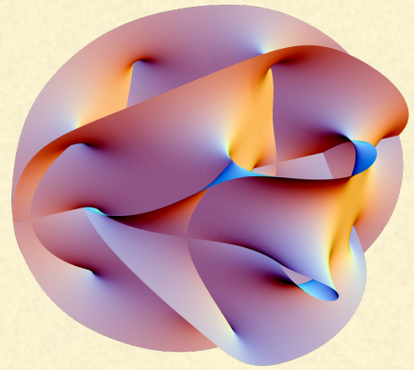

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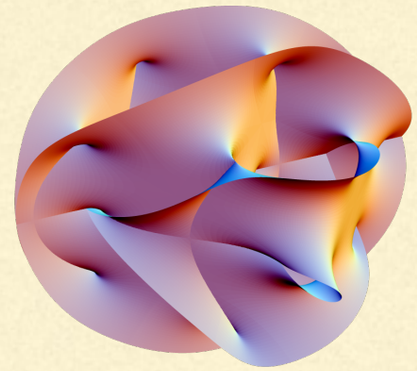

Primary goal:
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To do this: Look for dS maxima free of all the **complications** that appear in the dS constructions

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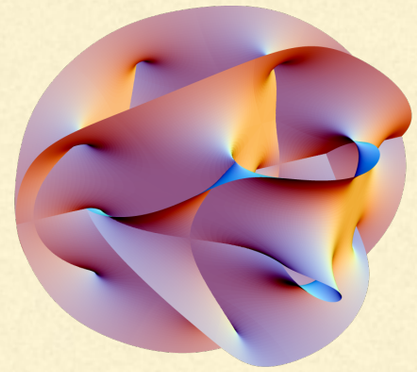

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Maximize control

To do this: Look for dS maxima free of all the **complications** that appear in the dS constructions

- Branes

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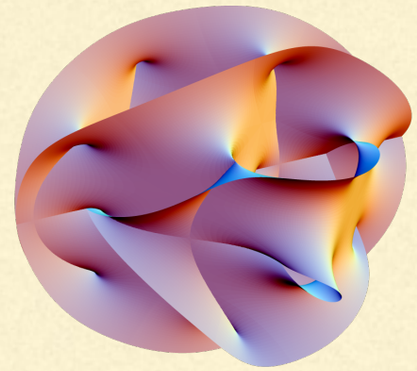

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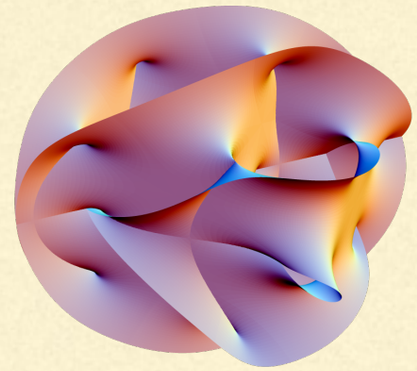

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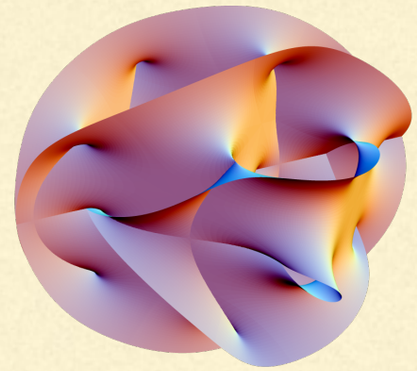

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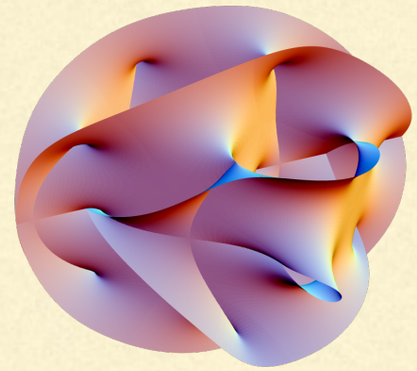

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~~Branes~~ ~~Orientifolds~~ Higher derivatives

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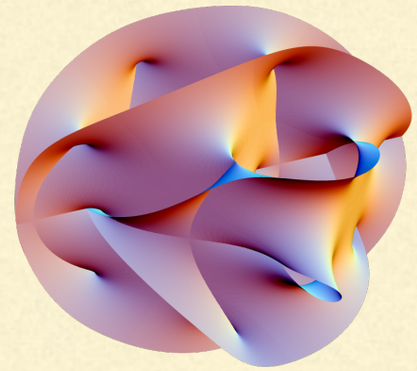

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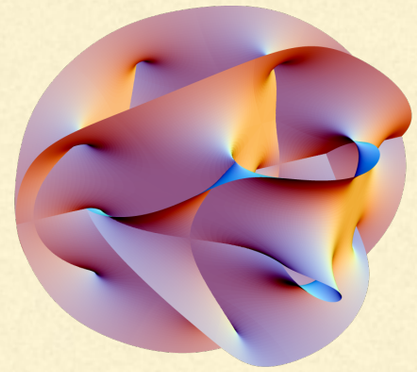

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Warping/curvature/ throats

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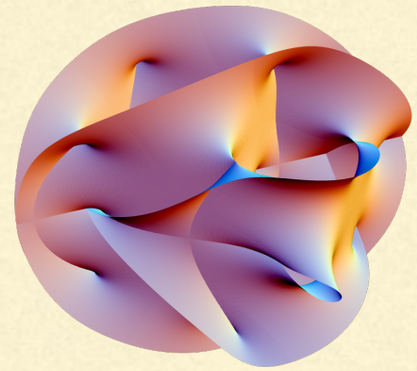

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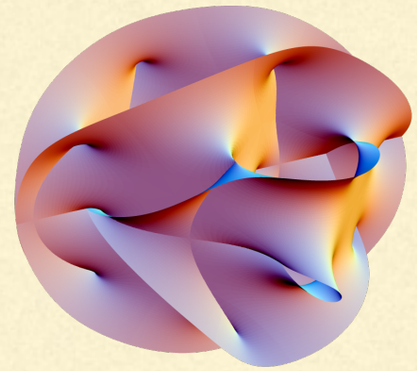

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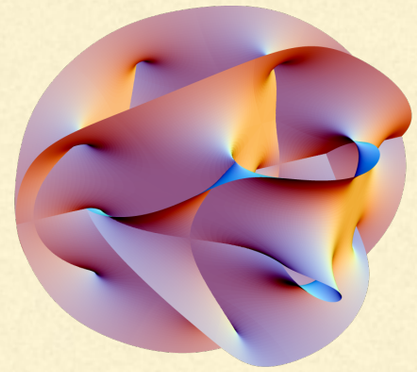

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Fluxes

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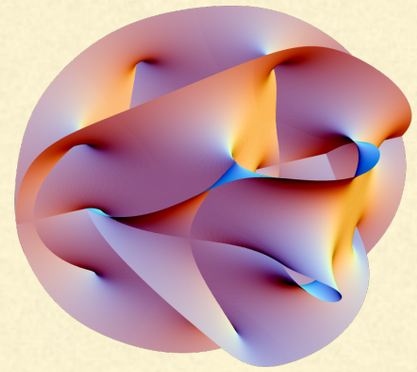

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Fluxes

Casimir energies

Following the idea of

[De Luca-Silverstein-Torroba '21]

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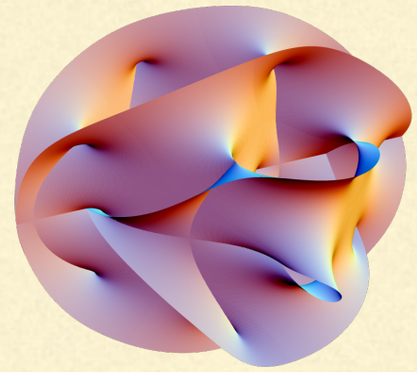

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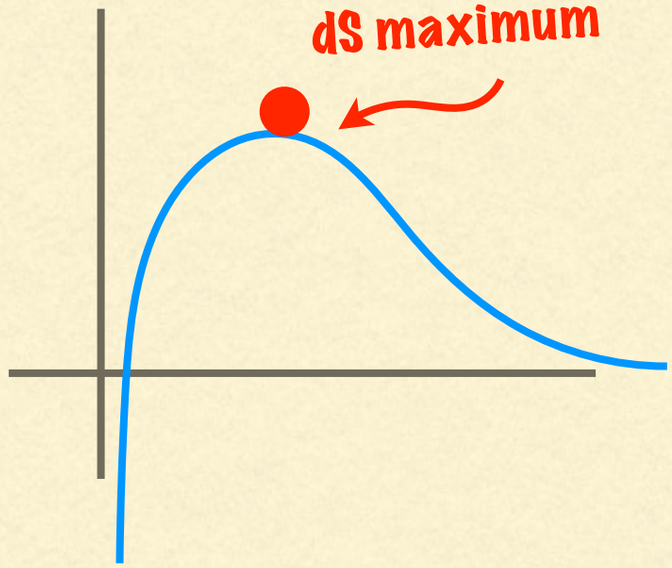

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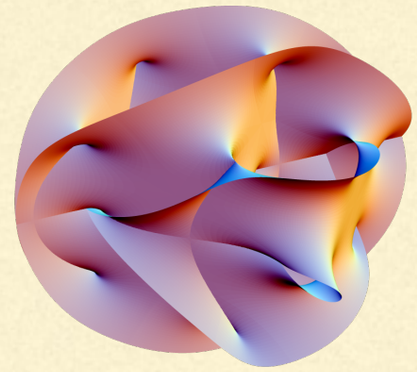

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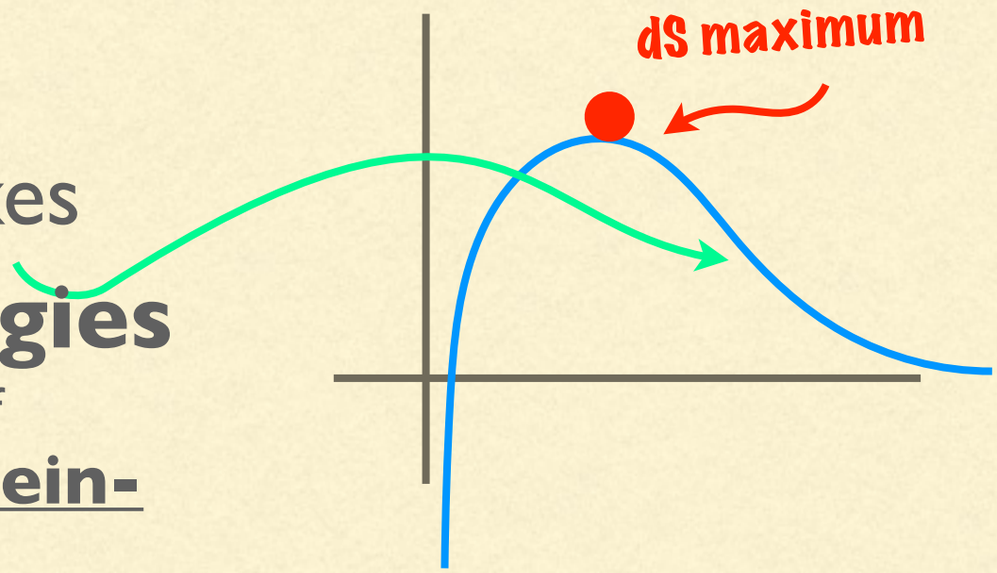

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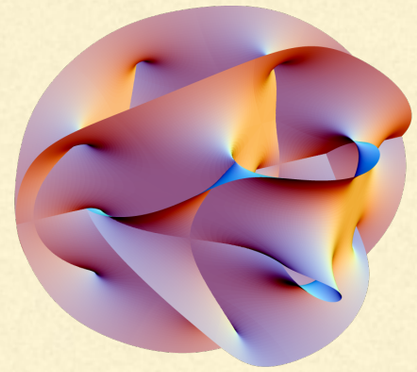

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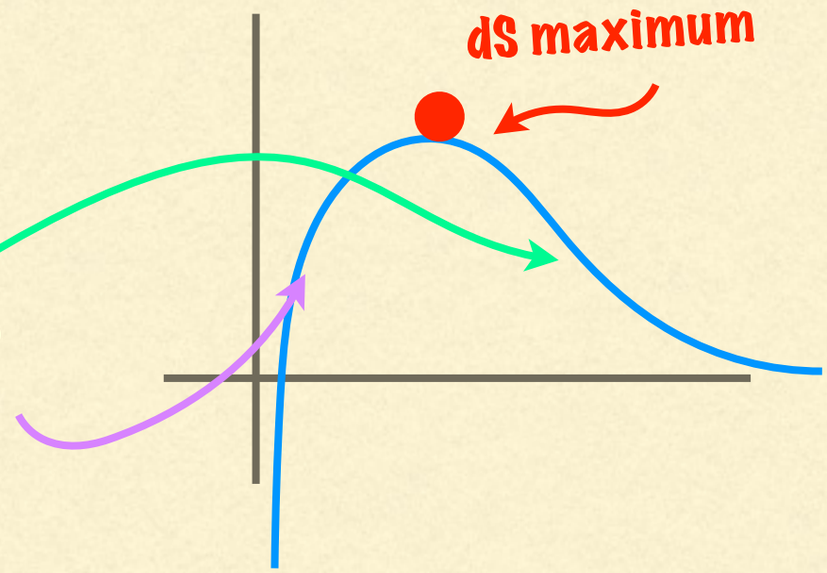

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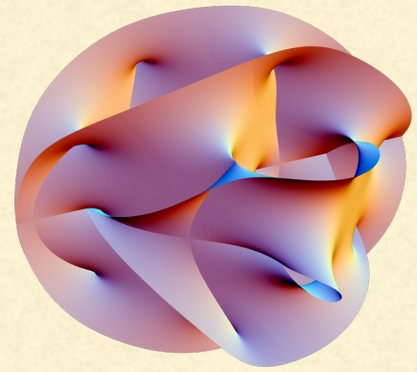

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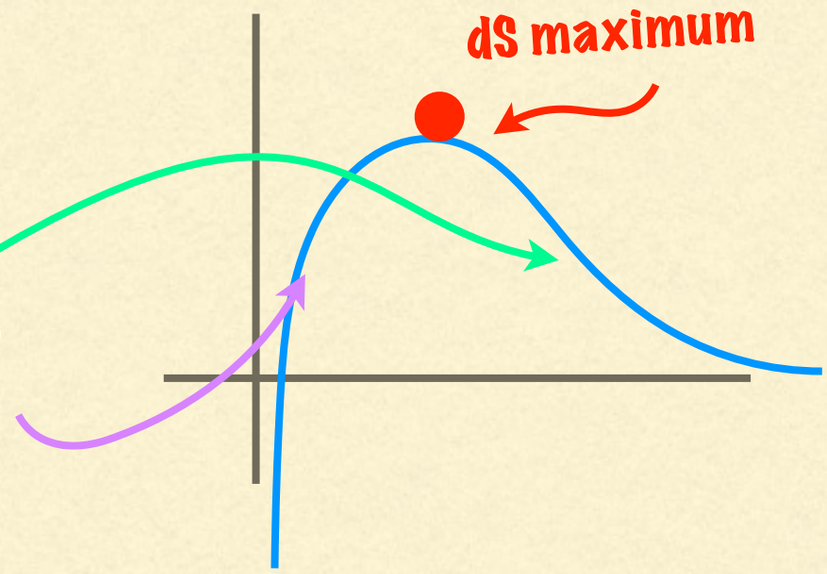

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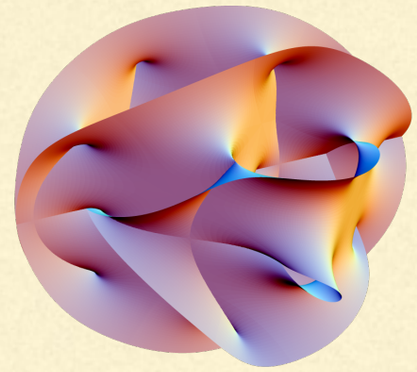

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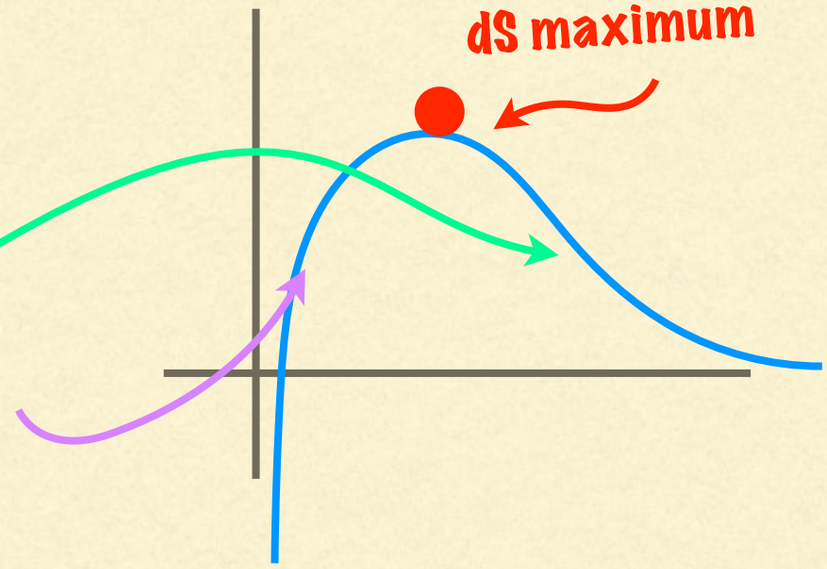

and for our compactification manifold: Calabi-Yau $R_{ab} = 0$

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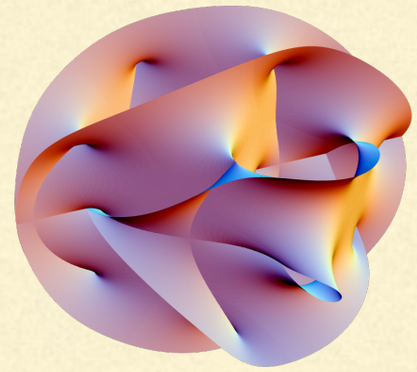

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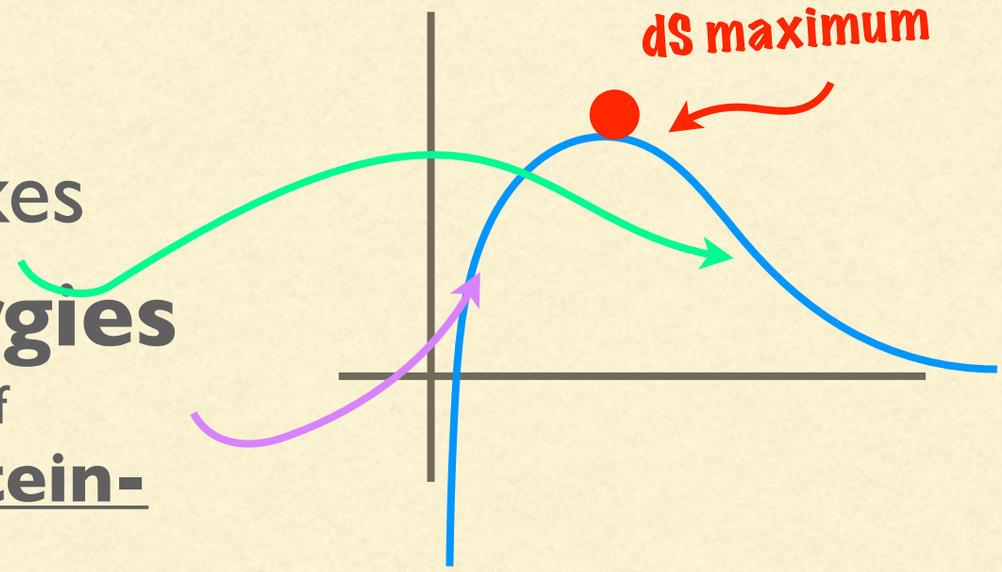

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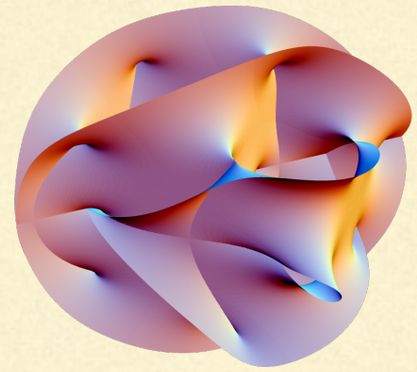

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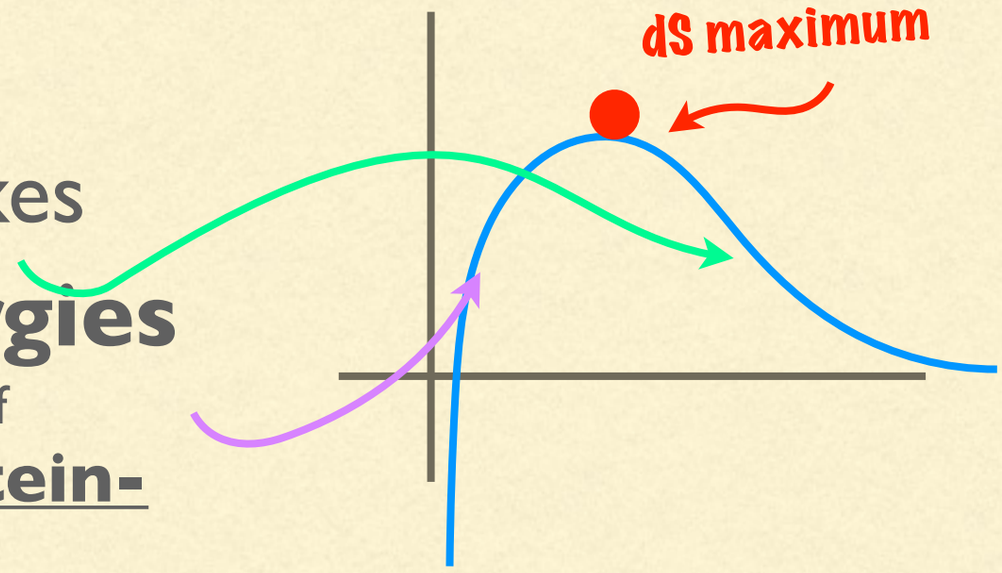

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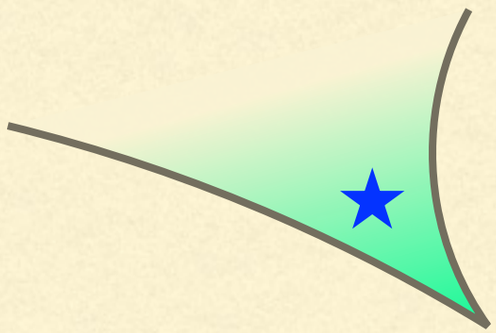
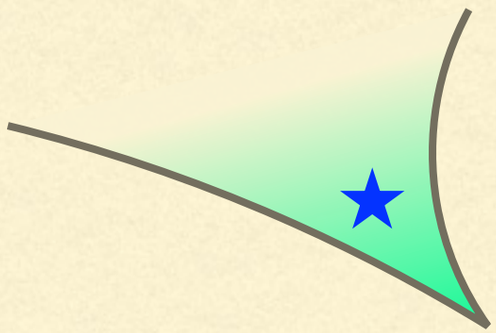
Riemann-flat-manifold (RFM) $R_{abcd} = 0$



New corner:
Riemann-flat manifolds



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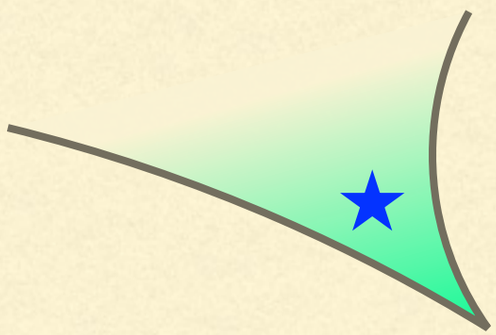

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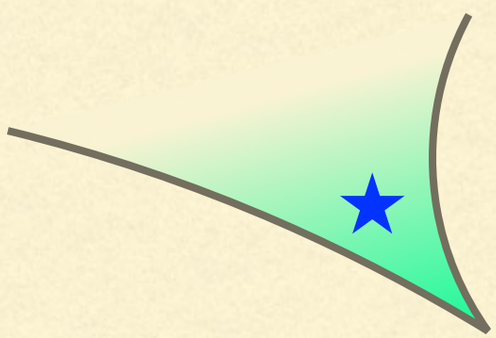
★ Break SUSY @ compactification scale



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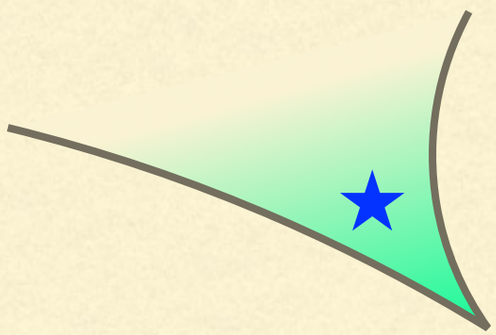

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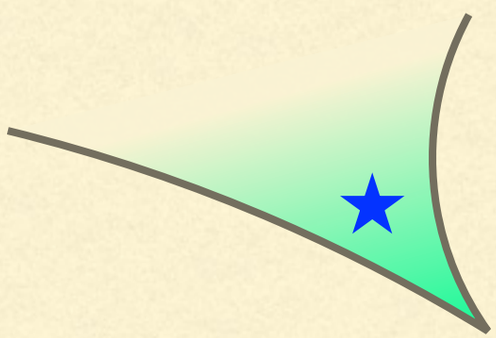

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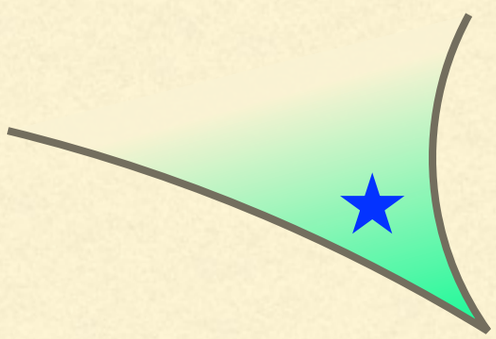

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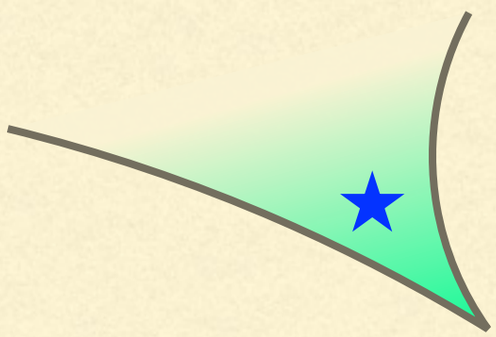
Most general RFM:

$$\frac{T^n}{\Gamma}$$

$T^n \longrightarrow$ Torus

$\Gamma \longrightarrow$ Crystallographic group of affine isometries of T^n,

$$\forall g \in \Gamma \qquad g = \mathbf{D}_g \vec{x} + \vec{b}_g$$



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Important: g's have
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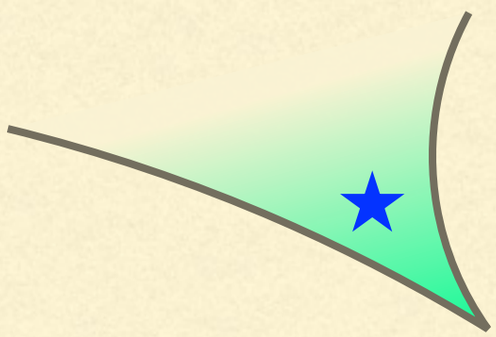
(i.e. manifold, not orbifold!)



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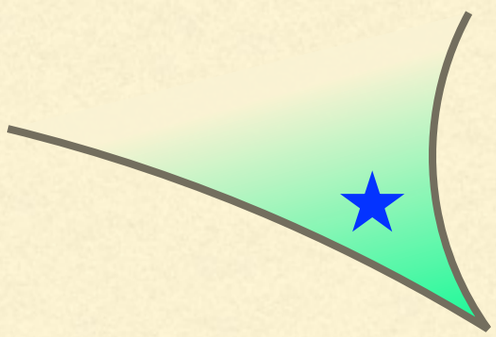
★ **Finite** # of RFM's in any dimension, classified until n<6
[Acharya, Fraiman, MM, Parra de Freitas, WIP]



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$\Gamma \longrightarrow$ Crystallographic group of affine isometries of T^n,

$$\forall g \in \Gamma \quad g = \mathbf{D}_g \vec{x} + \vec{b}_g$$

Important: g's have **no fixed points**

(i.e. manifold, not orbifold!)

★ **Finite** # of RFM's in any dimension, classified until n<6

[Acharya, Fraiman, MM, Parra de Freitas, WIP]

★ Cohomology & other technicalities easy to determine

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★ Having explicit 11d T means we can study higher-dimensional **backreaction** (not just a 5d EFT approach)

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i.e. Casimir energy **lumps** or **clusters** around certain regions of the internal space.

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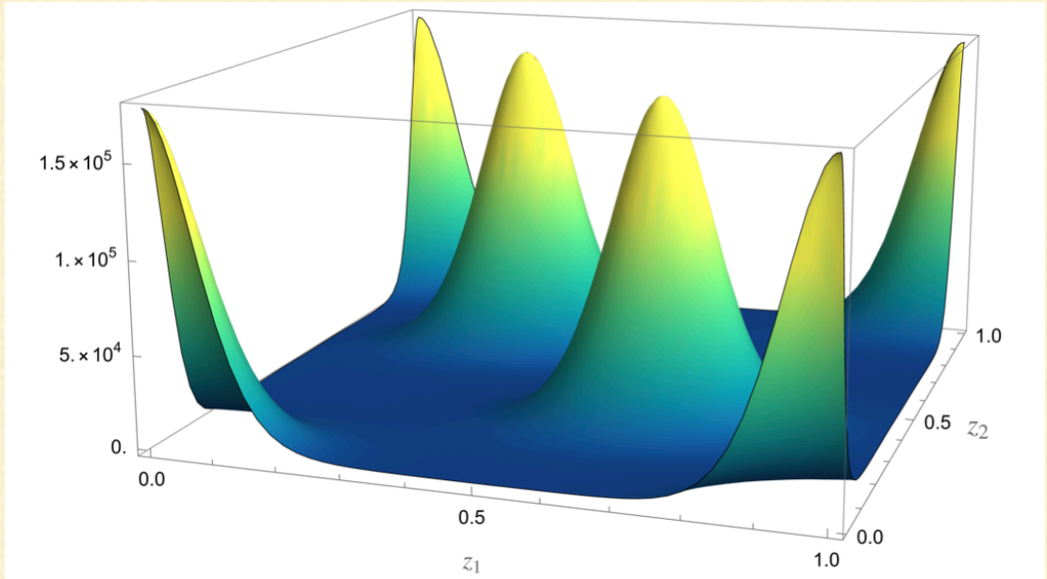

(b) $T^3/\mathbb{Z}_3$

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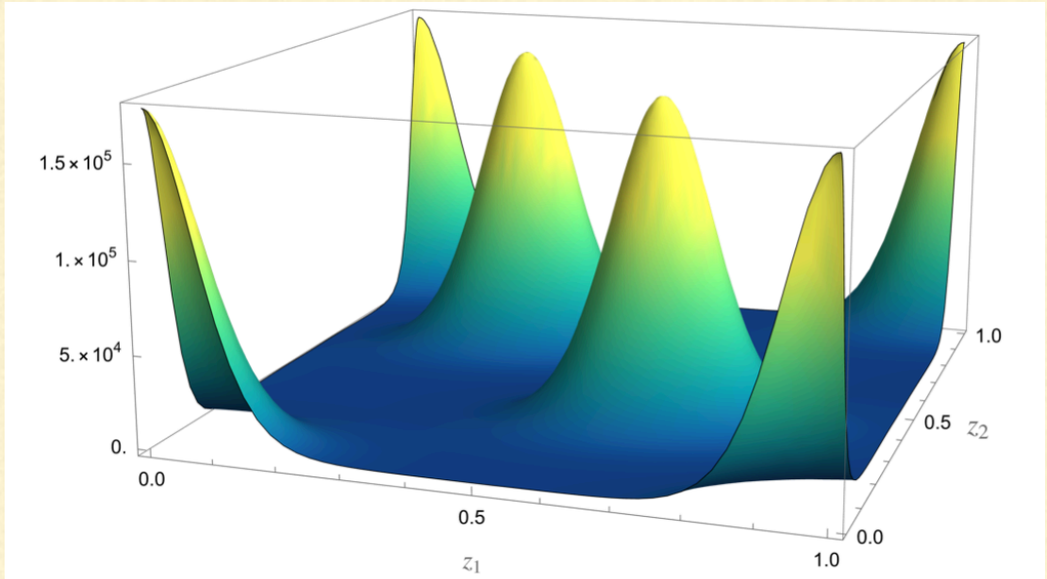

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Great for intuition, possibly hint of dualities…

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So they will yield **better** vacua, with smaller cosmological constant.

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I will spare you the details. After a very long search, we (i.e. Bruno) found **one.**

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We also add G4 flux on a particular 4-cycle

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Computing the traces for all M-theory fields:

k	0	1	2	3	4	5	6	7
$\mathrm{Tr}_{\mathbf{44}}(\mathbf{D}_\mathbf{g}^j)$	44	14	12	14	4	14	12	14
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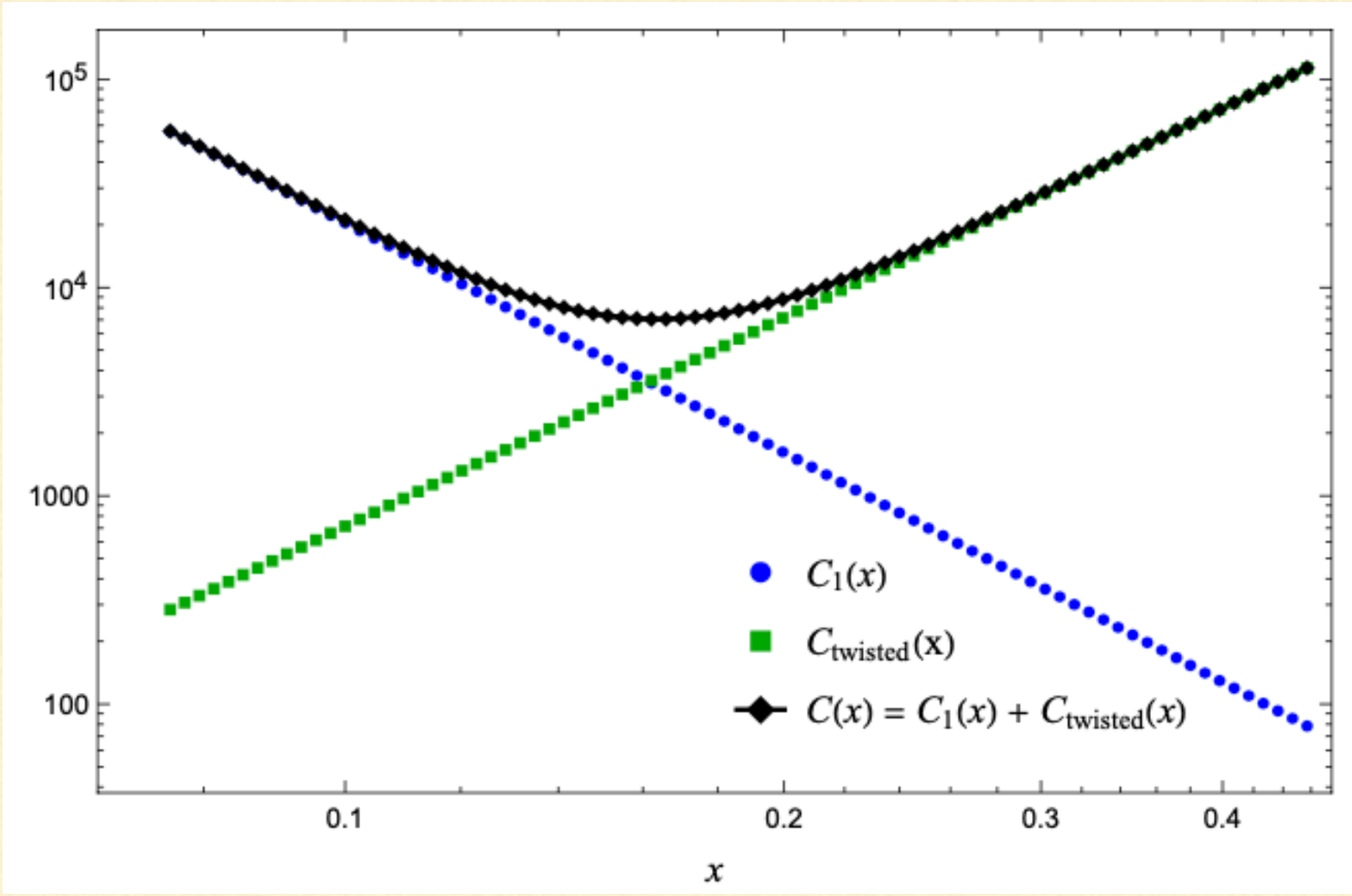
$$\approx 1.538 \times 10^6 \cdot x^{\frac{10}{3}} \,.$$

Huge enhancement of Casimir in twisted sector!

So, we get a saddle for the Casimir energy…



and therefore we can balance it against flux!

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with the following **properties**

5d vacuum energy	$V^{(5d)} \approx 3.68 \cdot 10^{-15} \ell_5^{-3}$
Hubble radius	$H_0^{-1} \sim 4.04 \cdot 10^7 \, \ell_5 = 2.45 \cdot 10^5 \, \ell_{11}$
Internal space volume	$\mathcal{V} = 4.50 \times 10^6 \, \ell_{11}^6 = 9.1 \cdot 10^{19} \, \ell_5^6$
Internal manifold radii	$\frac{1}{8} R_1 \sim 6.1 \, \ell_{11} = 1009 \, \ell_5,$ $\sqrt{2} \, R_2 \sim 13.2 \, \ell_{11} = 2176 \, \ell_5$
Scale separation	Yes: $\frac{1}{8} R_1 = 2.50 \cdot 10^{-5} \, H_0^{-1}$
Geometric moduli	Five, with masses $m_i^2 =$ $\{90.45 \, , \, 23.82 \, , \, -309.55 \, , \, -47.43 \, , \, -45.16\} \cdot H_0^2$
Axions	Five, with negligible masses
Vectors	Seven: two KK photons, five from C_3
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Light fermions	None
11d average energy density	$\rho_{11d} = 1.00 \cdot 10^{-10} \ell_{11}^{-11}$

Tiny vacuum energy!

Large hubble radius

Big volume

Tiny deviation from flat

M2,M5-brane instantons: negligible

5d vacuum energy	$V^{(5d)} \approx 3.68 \cdot 10^{-15} \ell_5^{-3}$
Hubble radius	$H_0^{-1} \sim 4.04 \cdot 10^7 \, \ell_5 = 2.45 \cdot 10^5 \, \ell_{11}$
Internal space volume	$\mathcal{V} = 4.50 \times 10^6 \, \ell_{11}^6 = 9.1 \cdot 10^{19} \, \ell_5^6$
Internal manifold radii	$\frac{1}{8} R_1 \sim 6.1 \, \ell_{11} = 1009 \, \ell_5,$ $\sqrt{2} \, R_2 \sim 13.2 \, \ell_{11} = 2176 \, \ell_5$
Scale separation	Yes: $\frac{1}{8} R_1 = 2.50 \cdot 10^{-5} \, H_0^{-1}$
Geometric moduli	Five, with masses $m_i^2 =$ $\{90.45 \, , \, 23.82 \, , \, -309.55 \, , \, -47.43 \, , \, -45.16\} \cdot H_0^2$
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Vectors	Seven: two KK photons, five from C_3
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Quasi-Planckian internal manifold...

$\ell_{KK}/\ell_{dS} \sim 40000$

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Similar suppression for loops:

$$\text{l-loop correction to C} \ \sim (\ell_{11}/R)^{9l}$$

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However, there is indeed a potentially **serious** issue…

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Same thing

We can **estimate** the effect of one such state in the Casimir formula

$$\delta V^{(5d)} = \frac{2^{\frac{3}{10}}}{R^{10}} \frac{R^6}{4} \left[\sum_{n=1}^{\infty} \frac{\left(2\mu^{11}\right) K_{\frac{11}{2}}\left(\mu n R_1\right)}{\left(2\pi\right)^{\frac{11}{2}} \left(\mu n R_1\right)^{\frac{11}{2}}} \right]$$

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I'm basically done, but for a couple small **surprises**…

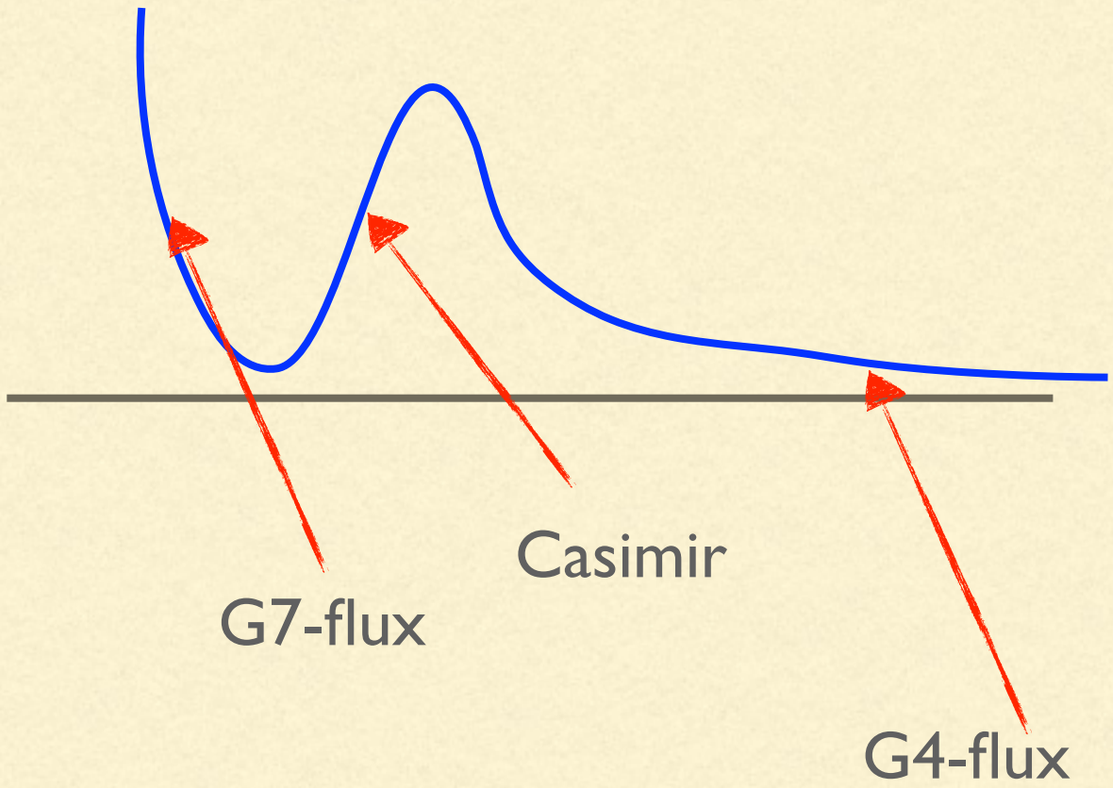
Surprise #1: A dS no-go from RFM's

[Bruno Bento, MM, to appear soon]

[Parameswaran, Serra '24]

Q: Can you modify your construction to get a dS minimum?

(i.e. copy **[De Luca-Silverstein-Torroba '21]** even more!)



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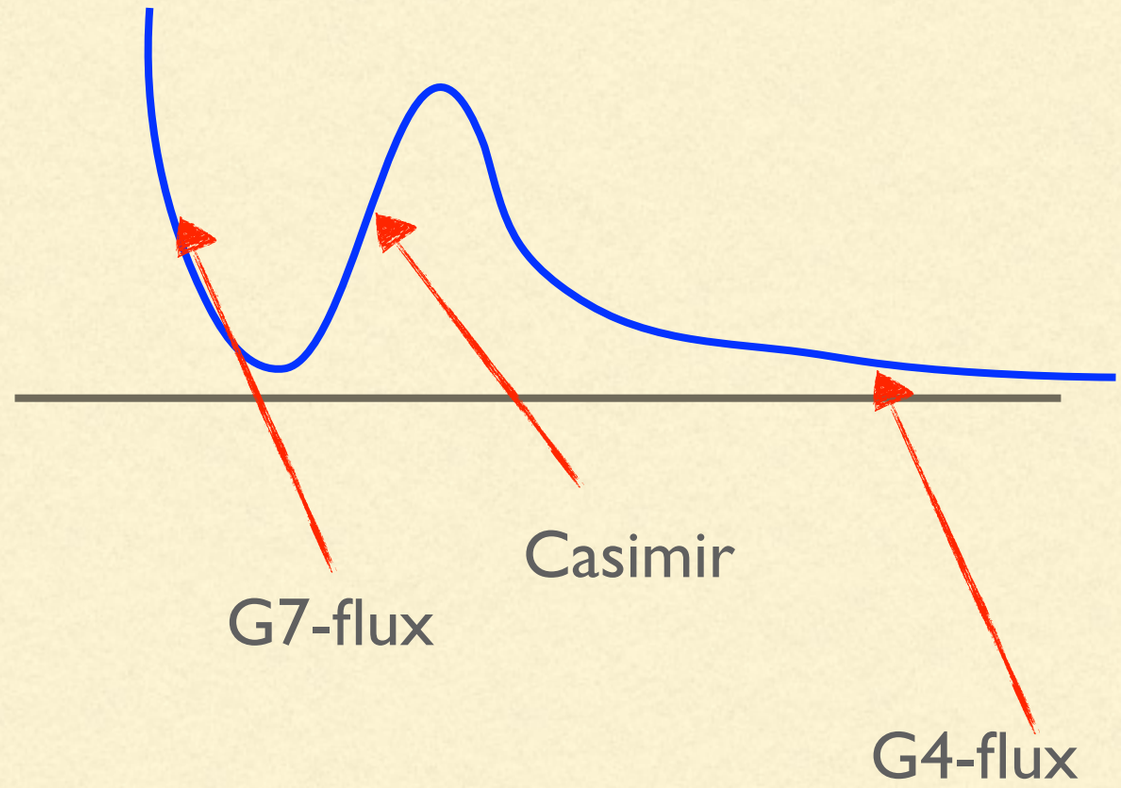

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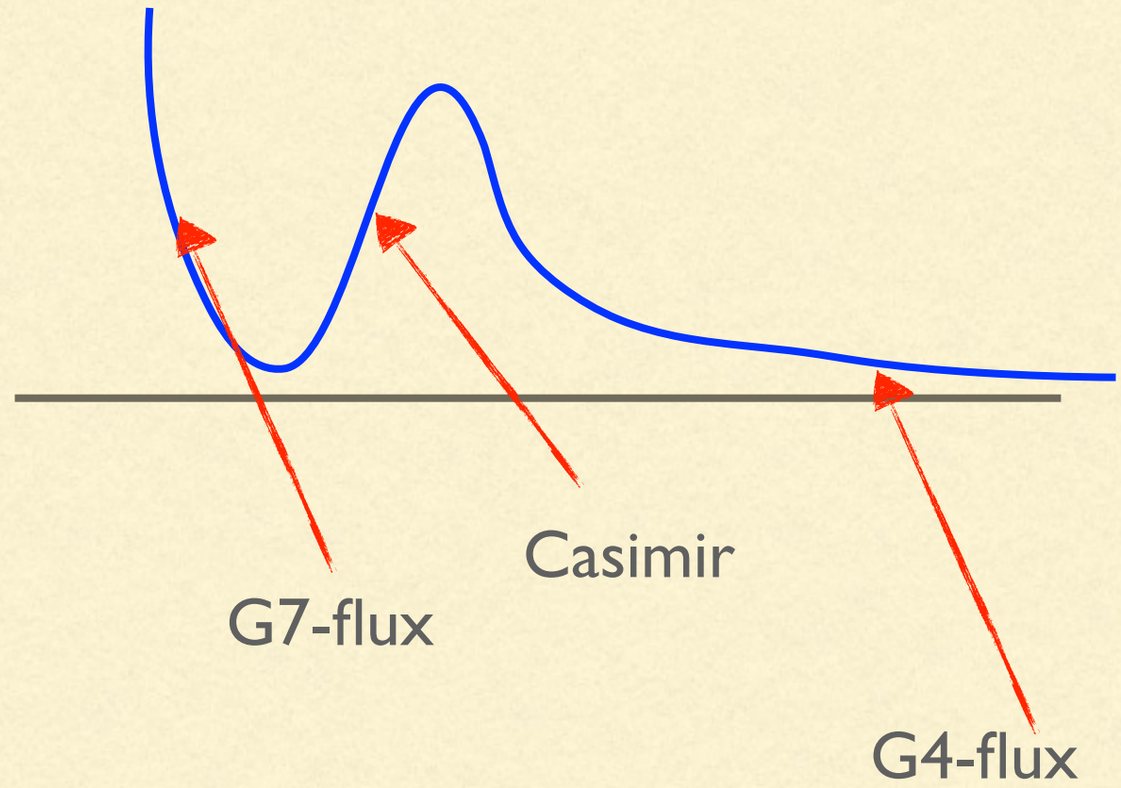

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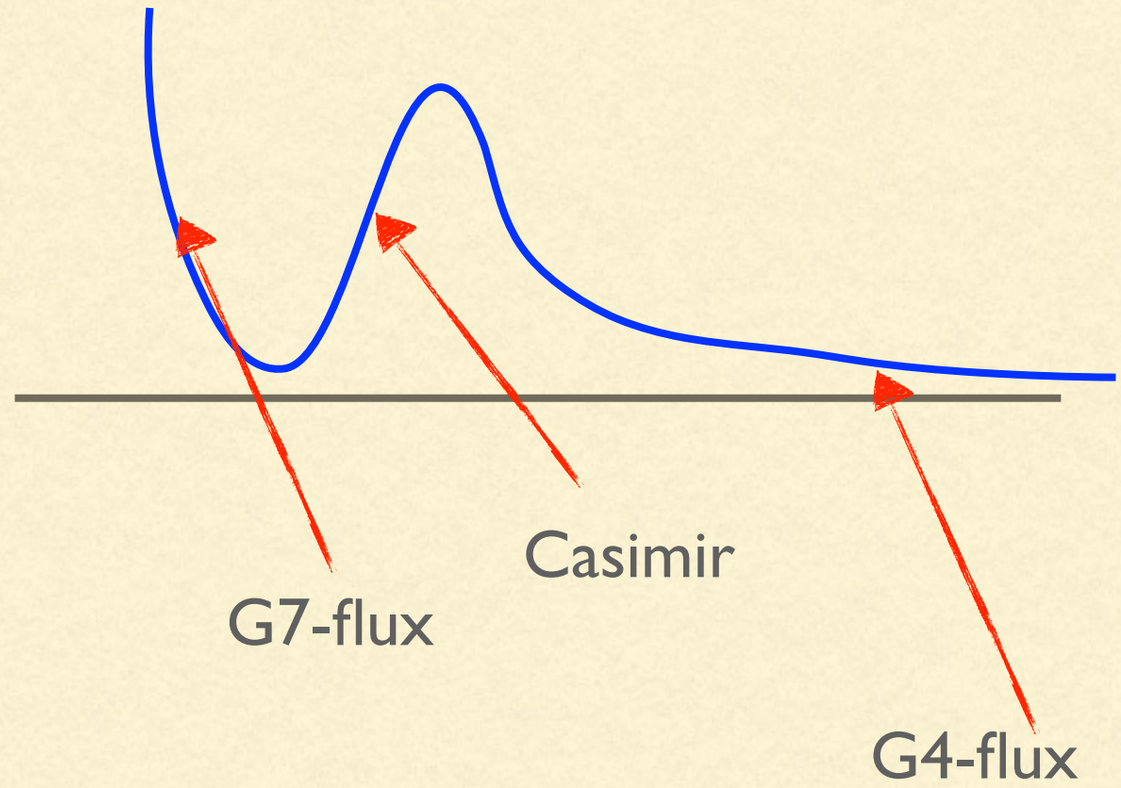

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Seems like RFM's are part of an **anti-dS conspiracy**

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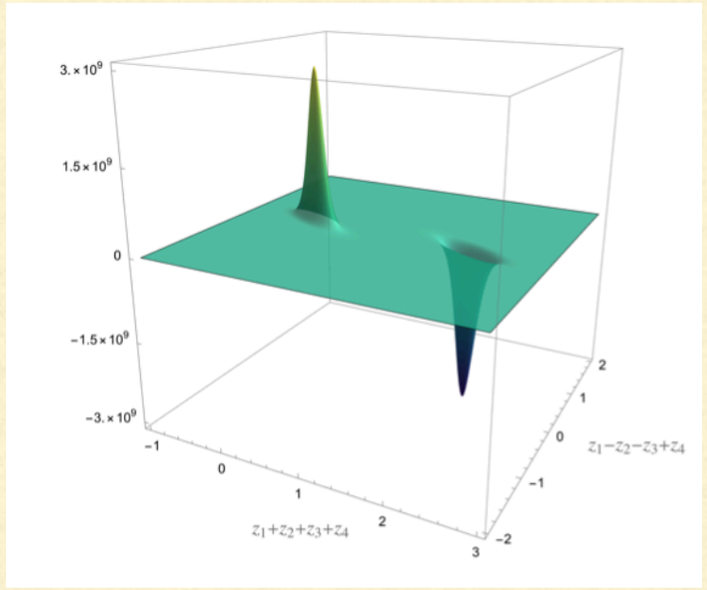

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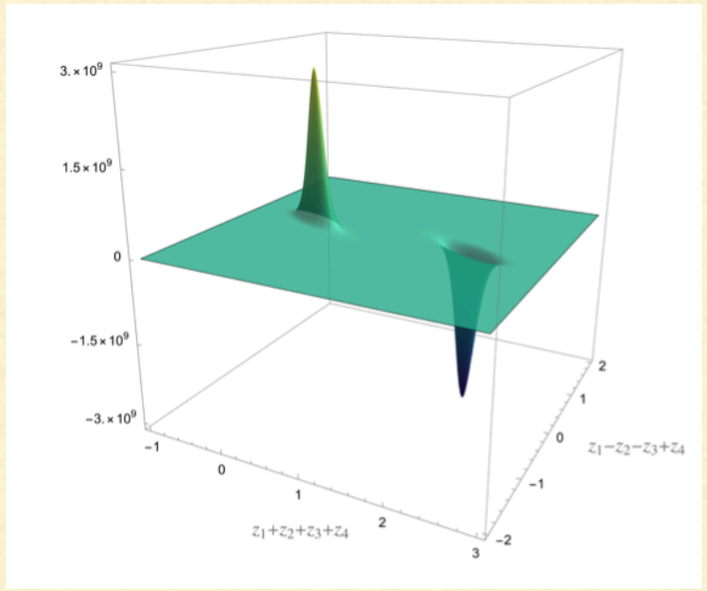

This is due to an anomalous symmetry in Casimir energy:

$$T_{00}(\vec{x}+\vec{a}) = -\rho(\vec{x}+\vec{a}),$$

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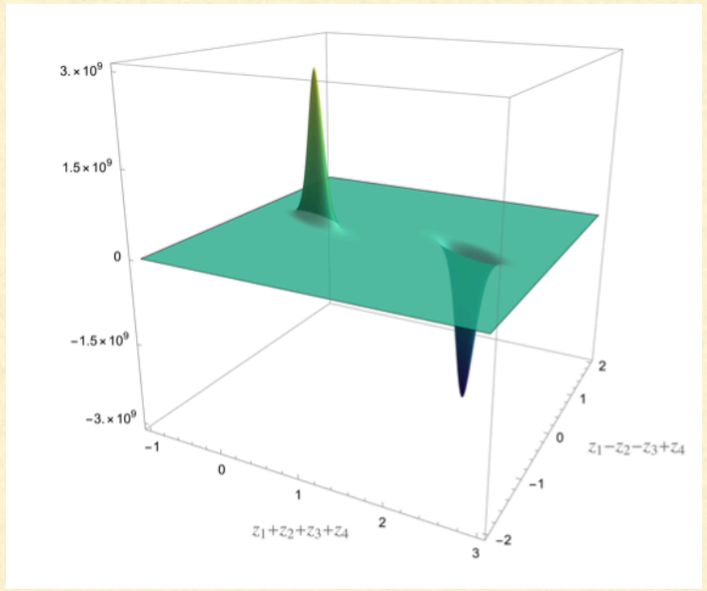

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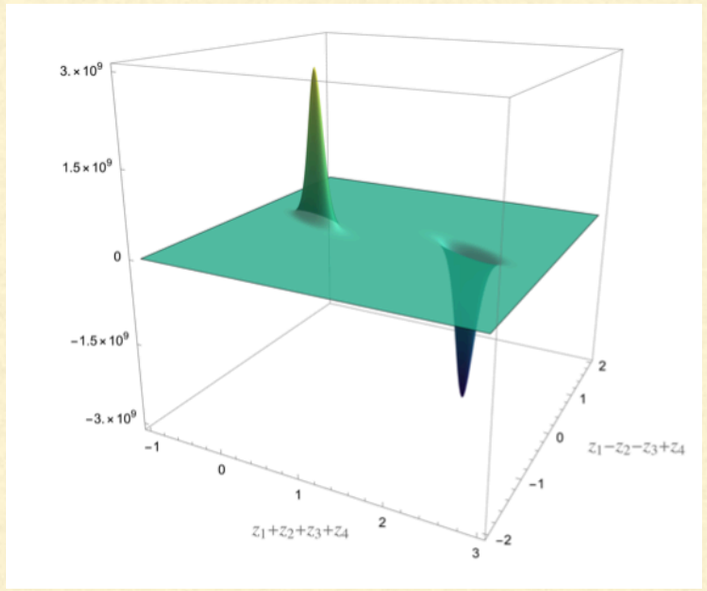

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Could have implications for **cosmological hierarchy?**

CONCLUSIONS

- **Riemann-flat manifolds** = nice, controllable, under explored, non-SUSY corner
- Explicit formula for Casimir energy in RFM, works also in EFT
- Constructed a fully explicit dS5 maximum in M-theory with Casimir, 10^{-15} vacuum energy and very small backreaction

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In summary:

join us in this exciting quest—the Earth is certainly not flat [159], but the extra dimensions just might be!

Thank you!