Infrared surprises in quantum gravity

2403.13053, 2412.16149, 2412.16142
& work in progress

with Sangmin Choi & Alok Laddha

to appear & work in progress

with Sangmin Choi & Ameya Kadhe

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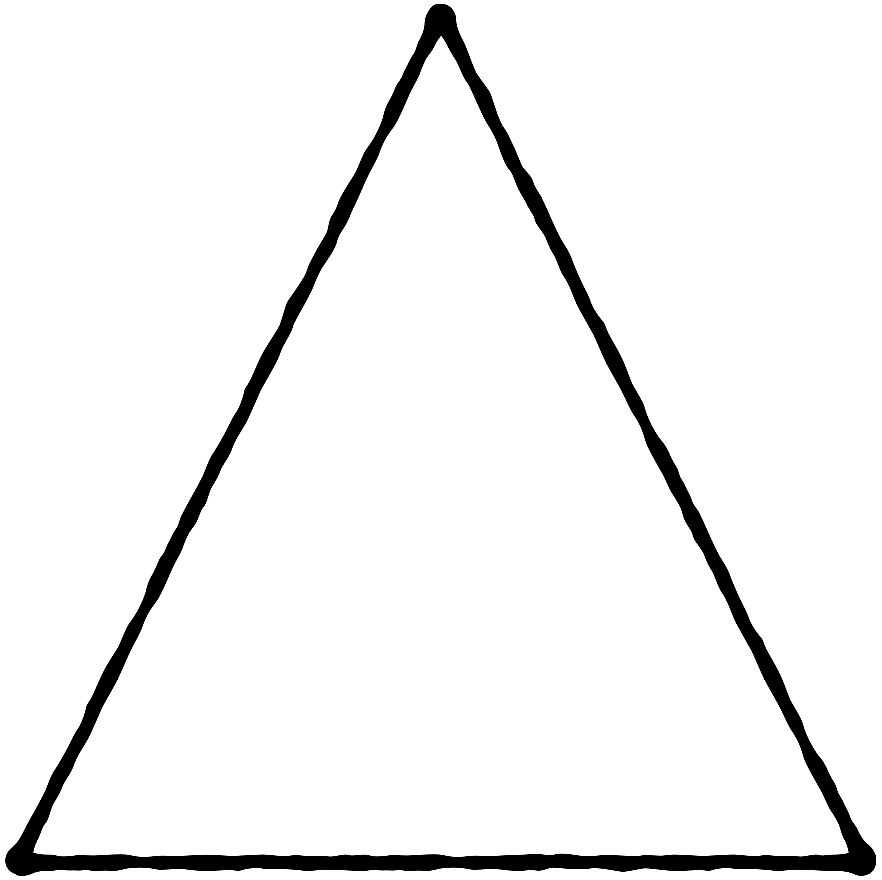
SIMONS FOUNDATION

SIMONS COLLABORATION ON CELESTIAL HOLOGRAPHY

Universality in the IR

Classical and quantum gravitational scattering in 4D exhibits universal behavior:

approximating
our world by
$\Lambda \approx 0$



asymptotic symmetry

soft theorem

low-energy

memory effect

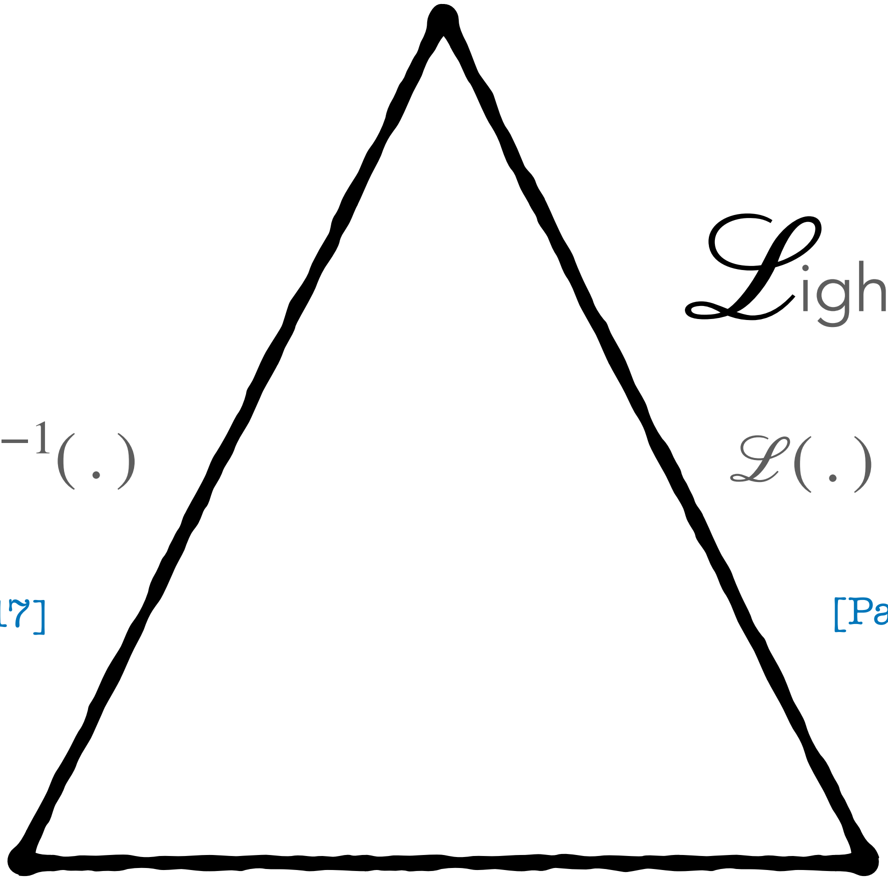
large-distance

3 bases for the IR

boost weight

Δ

asymptotic symmetry



$\mathscr{M}$ellin

$$\mathscr{M}(\,.\,) = \int_0^\infty d\omega \omega^{\Delta-1}(\,.\,)$$

[Pasterski,Shao,Strominger'17]

$\mathscr{L}$ight-ray

$$\mathscr{L}(\,.\,) = \int_{-\infty}^{+\infty} du u^{-\Delta}(\,.\,)$$

[Pasterski,AP,Trevisani'21]

soft theorem

energy

ω

$\mathscr{F}$ourier

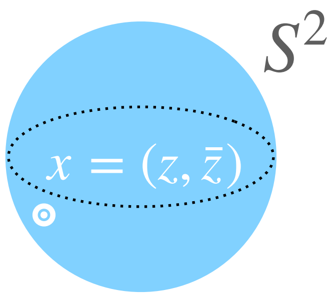
$$\mathscr{F}(\,.\,) = \int_{-\infty}^{+\infty} du e^{i\omega u}(\,.\,)$$

memory effect

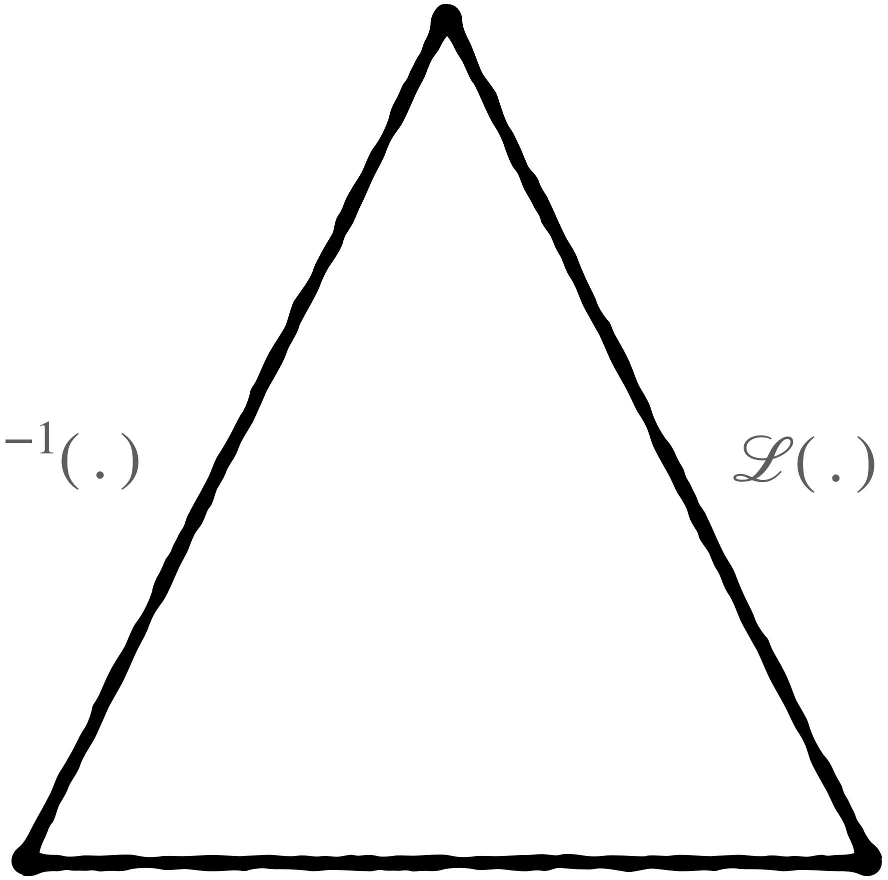
null time

u

IR universality is 2D



graviton: directed towards point $x = (z, \bar{z})$ on the celestial sphere



asymptotic symmetry → generated by 2D conserved operators $\mathcal{O}_\Delta$

$$\mathcal{M}(\,.\,) = \int_0^\infty d\omega \omega^{\Delta-1}(\,.\,)$$

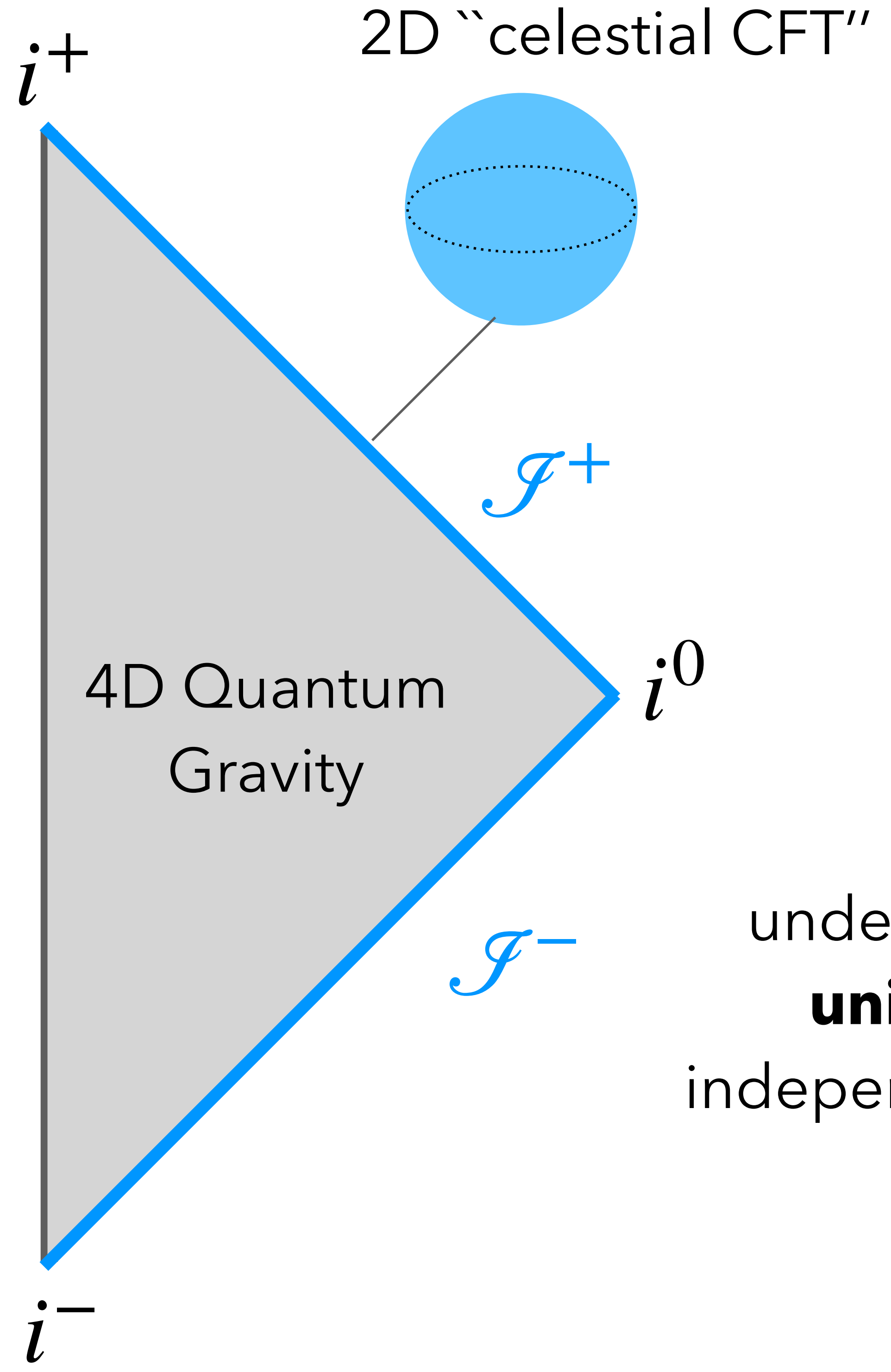
$$\mathcal{L}(\,.\,) = \int_{-\infty}^{+\infty} du u^{-\Delta}(\,.\,)$$

soft theorem

memory effect

2D Hologram for 4D Quantum Gravity ?

(at least for the IR sector)



Celestial Holography

bottom-up

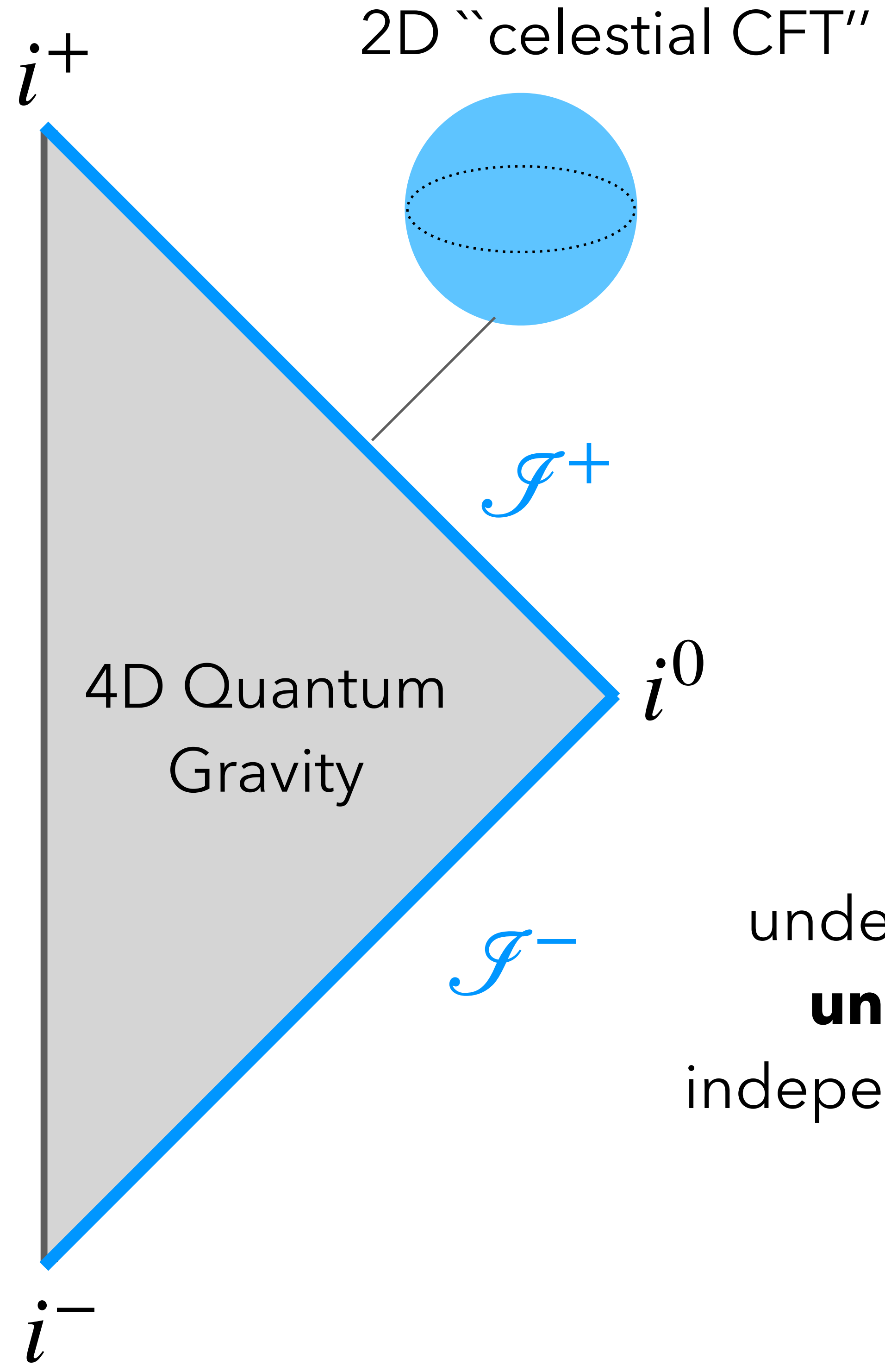
understand long-distance
universal properties
independent of short distance
microphysics

top-down

explore toy models to
construct dual pairs

2D Hologram for 4D Quantum Gravity ?

(at least for the IR sector)



Celestial Holography

bottom-up

understand long-distance **universal properties** independent of short distance microphysics

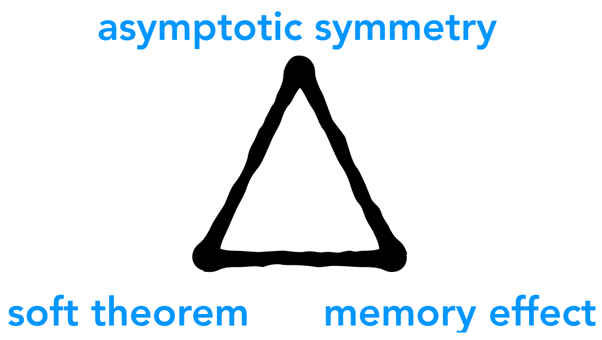
top-down

explore toy models to **construct dual pairs**

Plan: infrared surprises

& their connection to ``celestial CFT"

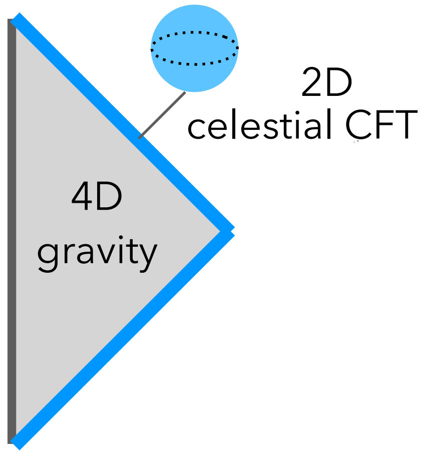
Infrared triangles


asymptotic symmetry

soft theorem

memory effect

Towers of ∞ symmetries


2D
celestial CFT

4D
gravity

4D = 2D

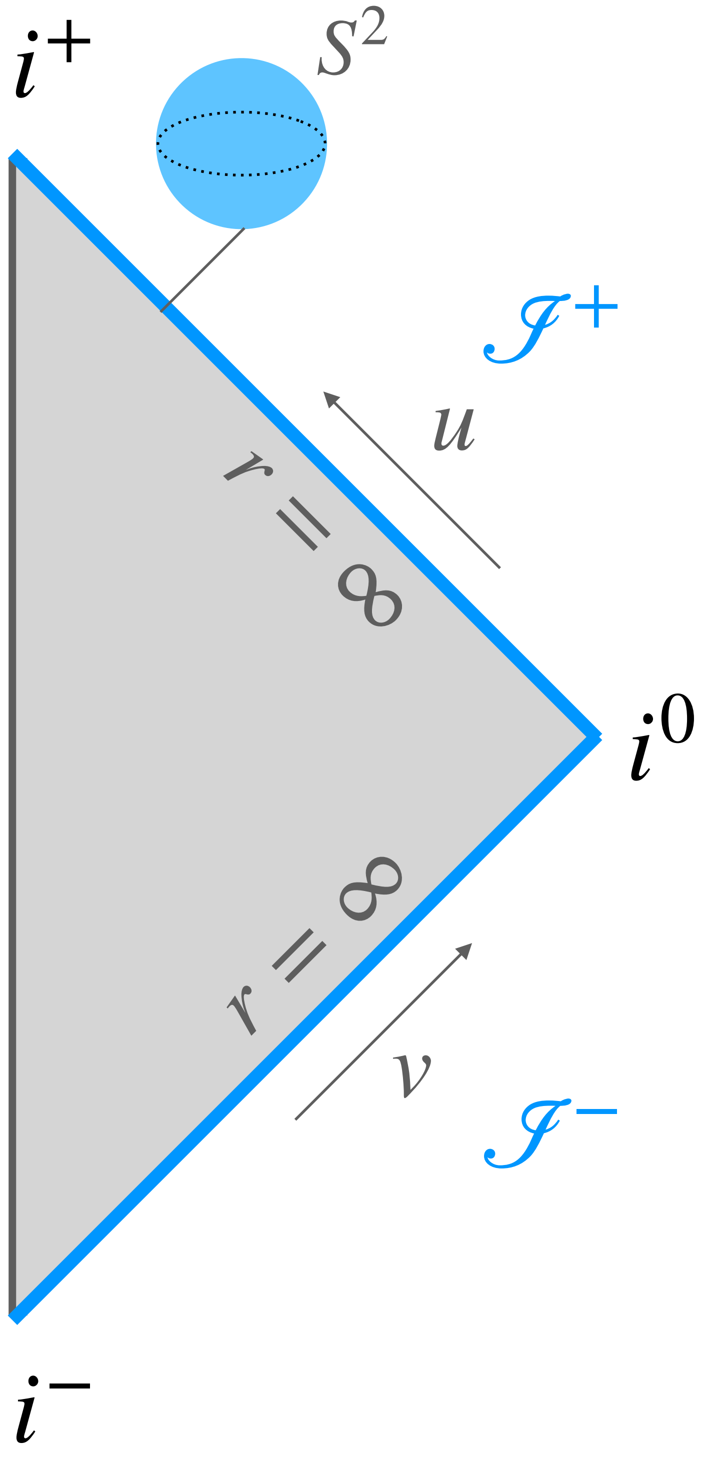
Long-range effects →

New conservation laws in Einstein gravity!

[Choi,Laddha,AP'24]

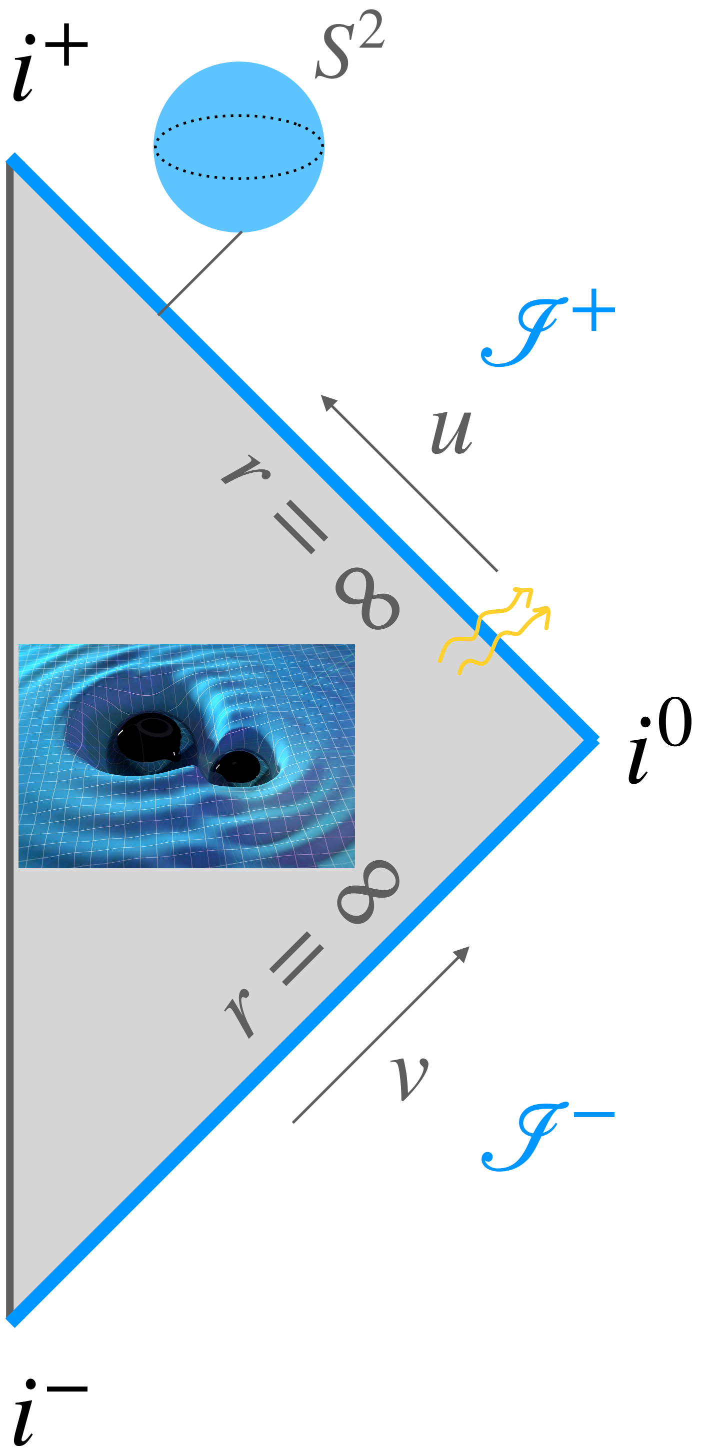
Infrared triangles

Asymptotic symmetries



"Large" gauge/diffeo transformations that preserve the boundary conditions of the fields, i.e. their large-distance fall-offs.

Asymptotic symmetries



"Large" gauge/diffeo transformations that preserve the boundary conditions of the fields, i.e. their large-distance fall-offs.

gravity:

mass

$$ds^2 = -(1+\ldots)du^2 - (2+\ldots)dudr$$

angular momentum

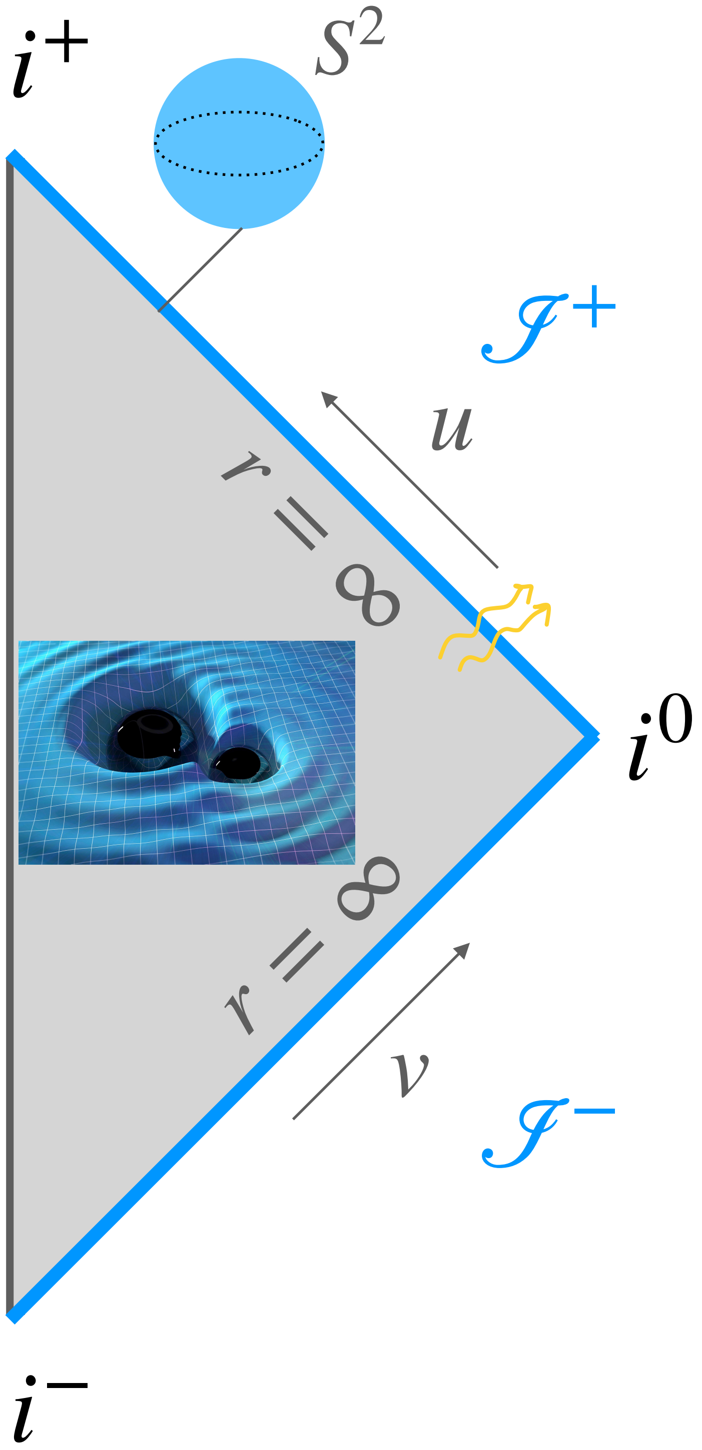
$$+(\ldots)dudx^A$$

$$+(r^2\gamma_{AB} + r\,C_{AB} + \ldots)dx^A dx^B$$

shear: gravitational waves

⇒ Bondi news $N_{AB} = \partial_u C_{AB}$

Asymptotic symmetries



"Large" gauge/diffeo transformations that preserve the boundary conditions of the fields, i.e. their large-distance fall-offs.

gravity:

Find ξ such that $\mathscr{L}_\xi g_{\mu\nu} \approx "0"$ as $r \to \infty$.

$$\updownarrow$$

$$O(1/r^{\#})$$

Unlike gauge redundancies, asymptotic symmetries act non-trivially on the physical data → non-zero charges.

IR triangle

[He,Lysov,Mitra,Strominger'14]

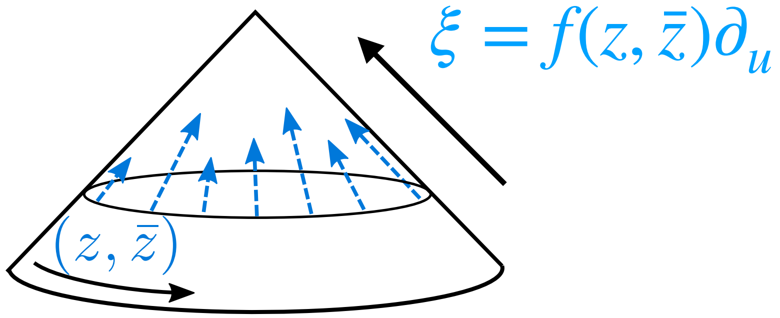
[Strominger,Zhiboedov'14]

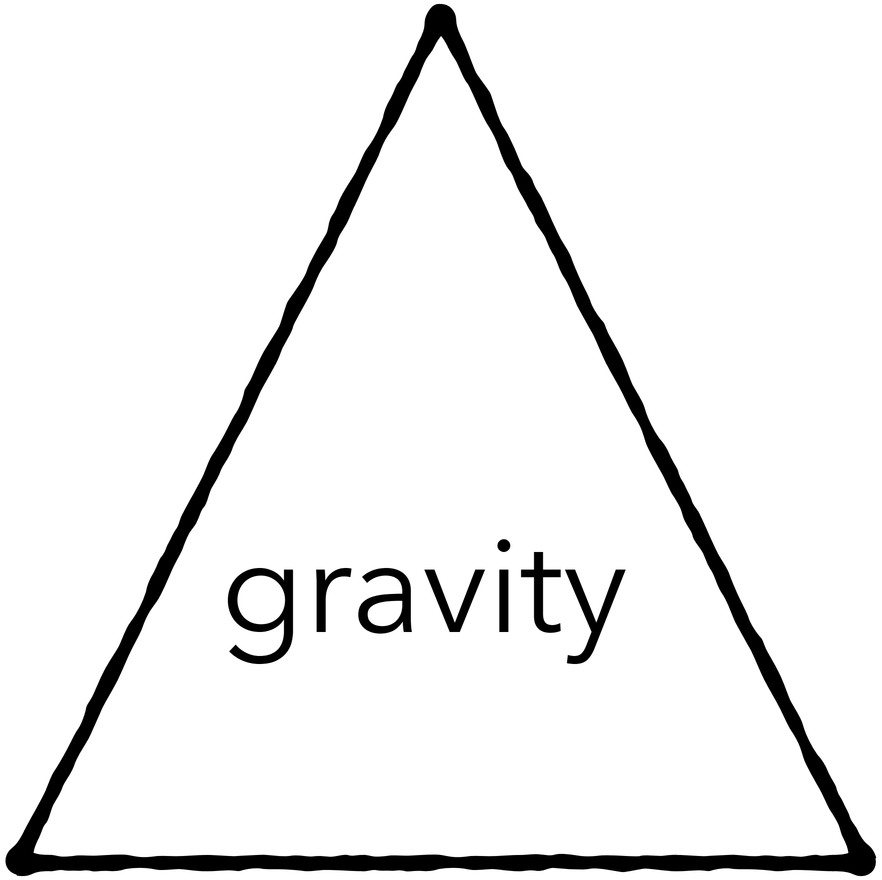
The symmetries of asymptotically flat space are not just Poincaré but an infinite extension!

[Bondi,van der Burg,Metzner'62]
[Sachs'62]

BMS group

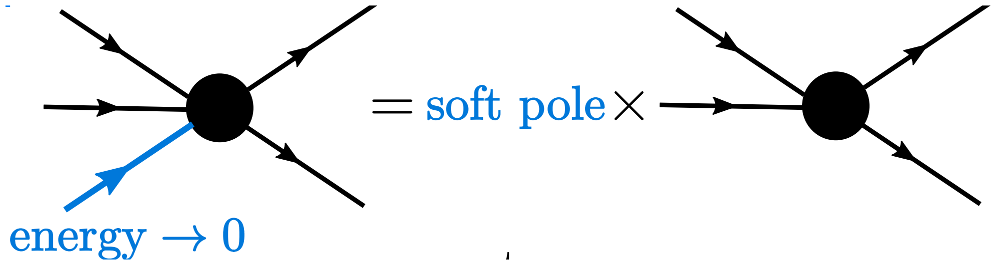
supertranslations





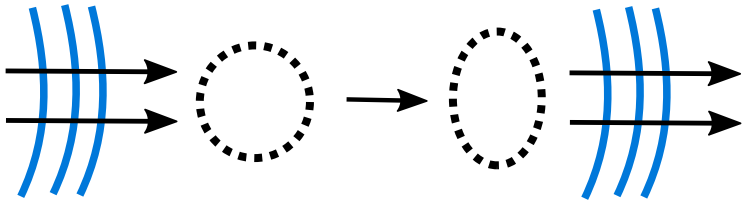
[Weinberg'65]

soft graviton theorem



[Zel'dovich,Polnarev'74 [Braginsky,Thorne'87]

displacement memory effect



IR triangle

on $\mathscr{I}^+$:

supertranslations

superrotations

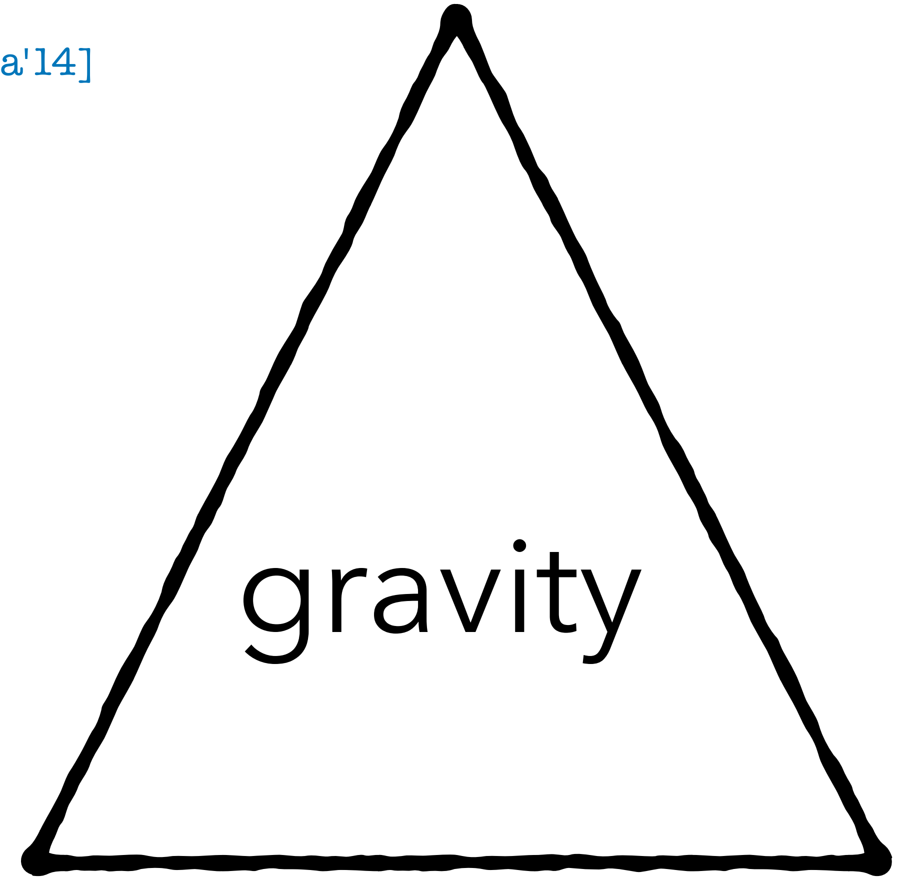
asymptotic symmetry

$\xi^u = f$ $\qquad$ $\xi^A = \frac{1}{r} D_A f$

$\xi^u = -\frac{1}{2} D_A Y^A$ $\qquad$ $\xi^A = Y^A$

[Barnich,Troessaert'11][Campiglia,Laddha'14]

extended / generalized
BMS group



gravity

soft theorem

ω^{-1} leading soft graviton

ω^{0} subleading soft graviton

[Cachazo,Strominger'14]

memory effect

displacement

spin

[Pasterski,Strominger,Zhiboedov'15]

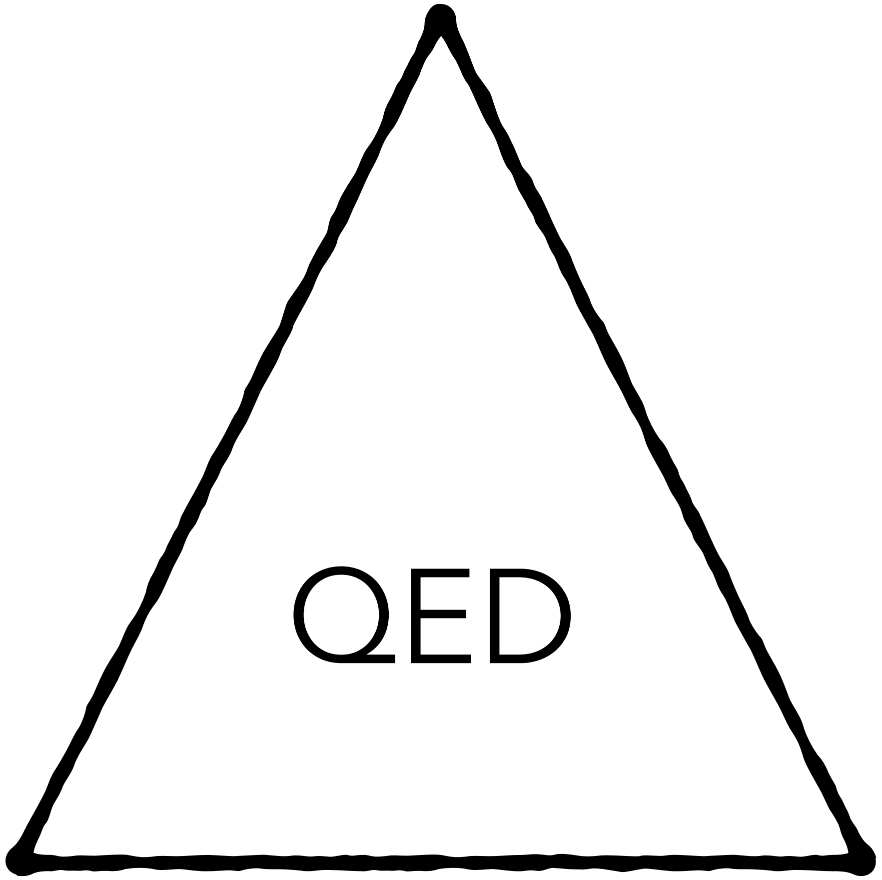
IR triangle

superphaserotation **asymptotic symmetry**

[He,Mitra,Porfyriadis,Strominger'14]

[Kapec,Pate,Strominger'15]

[Campiglia,Laddha'15]



soft theorem

memory effect

ω^{-1} leading soft photon

[Weinberg'65]

electromagnetic kick

[Bieri,Garfinkle'13]

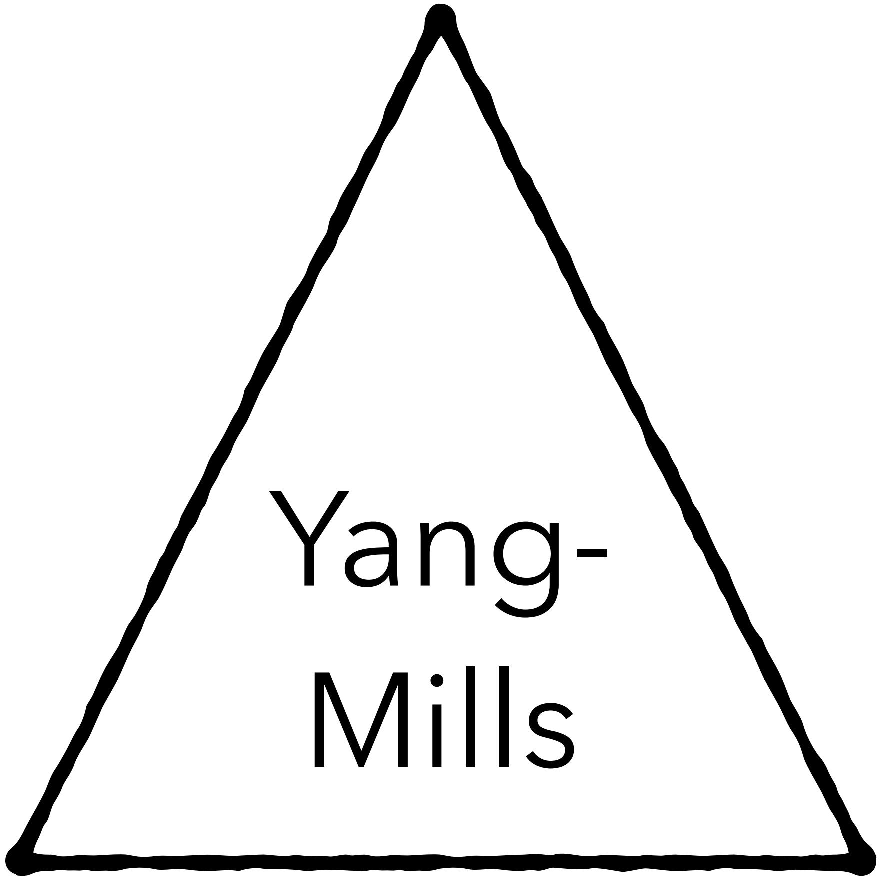
[Pasterski'15]

IR triangle

superphaserotation

asymptotic symmetry

[He,Mitra,Strominger'14]



Yang-Mills

soft theorem

memory effect

ω^{-1} leading soft gluon

[Berends,Giele'89]

color

[Pate,Raclariu,Strominger'17]

4D = 2D

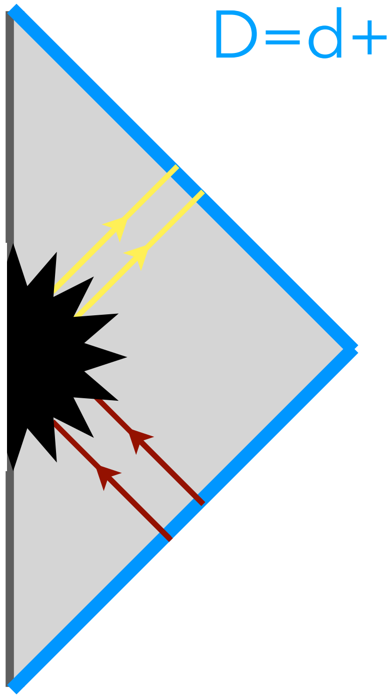
Symmetry & observables

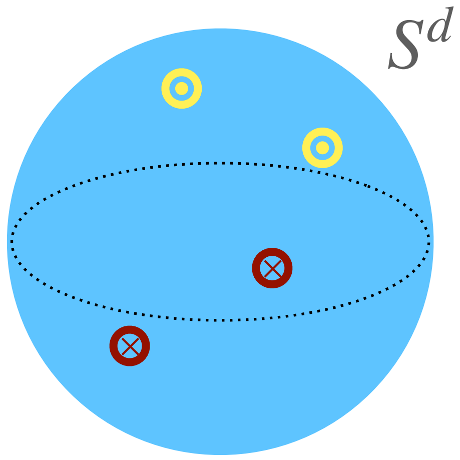
symmetry:

Lorentz group in
D=d+2 dimensions

$\cong$

Euclidean conformal
group in d dimensions

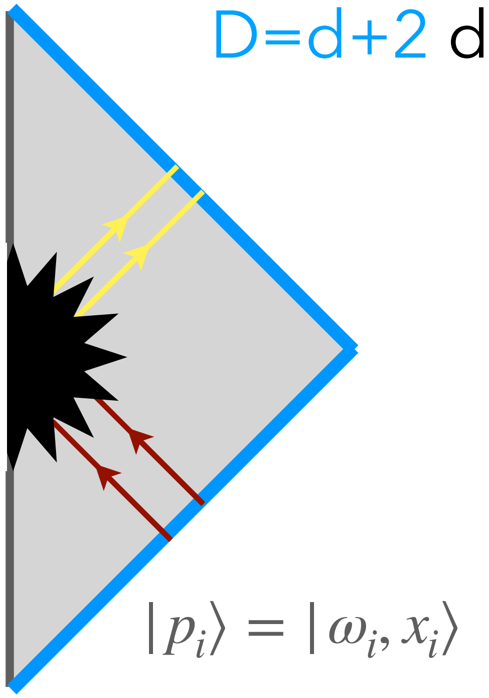




Symmetry & observables

symmetry:

Lorentz group in D=d+2 dimensions $\cong$ Euclidean conformal group in d dimensions

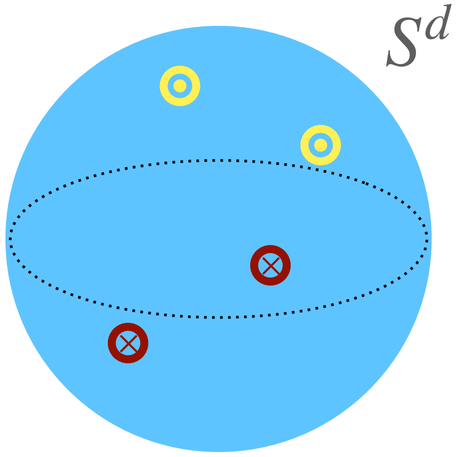


$|p_i\rangle = |\omega_i, x_i\rangle$

energy basis

basic observables in flat space:

S-matrix



Standard amplitudes

$$\langle p_n^{\text{out}} \ldots | \mathcal{S} | \ldots p_1^{\text{in}} \rangle$$

translation symmetry

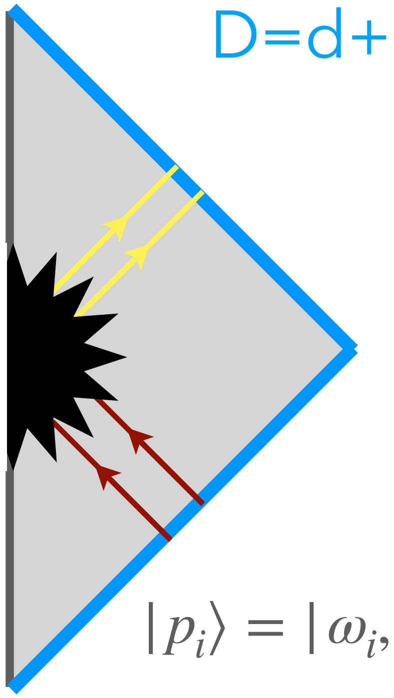
Symmetry & observables

symmetry:

Lorentz group in
D=d+2 dimensions

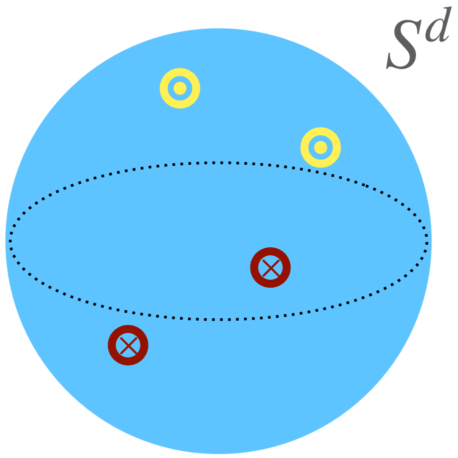
$\cong$

Euclidean conformal
group in d dimensions



basic observables in flat space:

S-matrix



$|p_i\rangle = |\omega_i, x_i\rangle$

$$\xrightarrow[\int_0^\infty d\omega\,\omega^{\Delta-1}]{\mathscr{M}\text{ellin}}$$

$|\Delta_i, x_i\rangle$

energy basis

boost-weight basis

Standard amplitudes

$$\langle p_n^{\text{out}} \ldots | \mathcal{S} | \ldots p_1^{\text{in}} \rangle$$

translation symmetry

Celestial amplitudes

$$\langle \mathcal{O}^-_{\Delta_1}(x_1) \ldots \mathcal{O}^+_{\Delta_n}(x_n) \rangle$$

Lorentz symmetry

↕

global conformal symmetry

[de Boer, Solodukhin'03] [Pasterski,Shao,Strominger'17] [Pasterski,Shao'17]

From global to local conformal

supertranslations,

superrotations, …

[Barnich,Troessaert'11] ↕

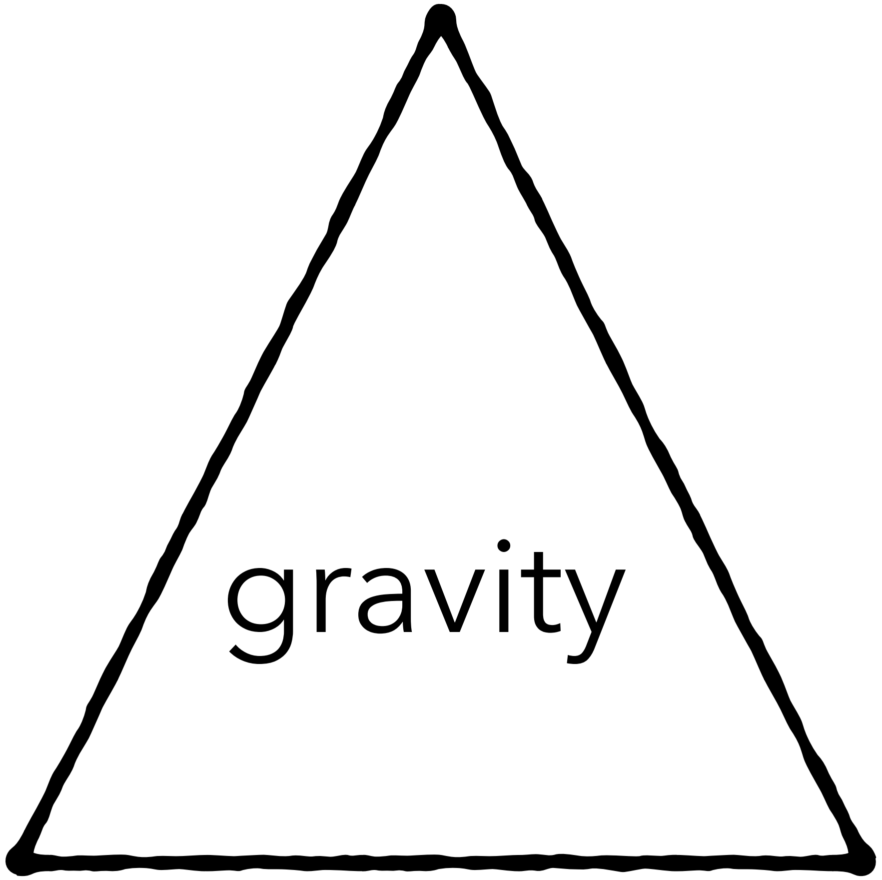
local conformal symmetry on S^2 !

asymptotic symmetry

on $\mathscr{I}^+$:

$\xi^u = f$ $\quad$ $\xi^A = \frac{1}{r} D_A f$

$\xi^u = -\frac{1}{2} D_A Y^A$ $\quad$ $\xi^A = Y^A$



gravity

soft theorem

ω^{-1} leading soft graviton,

ω^{0} subleading soft graviton, …

[Cachazo,Strominger'14]

memory effect

displacement,

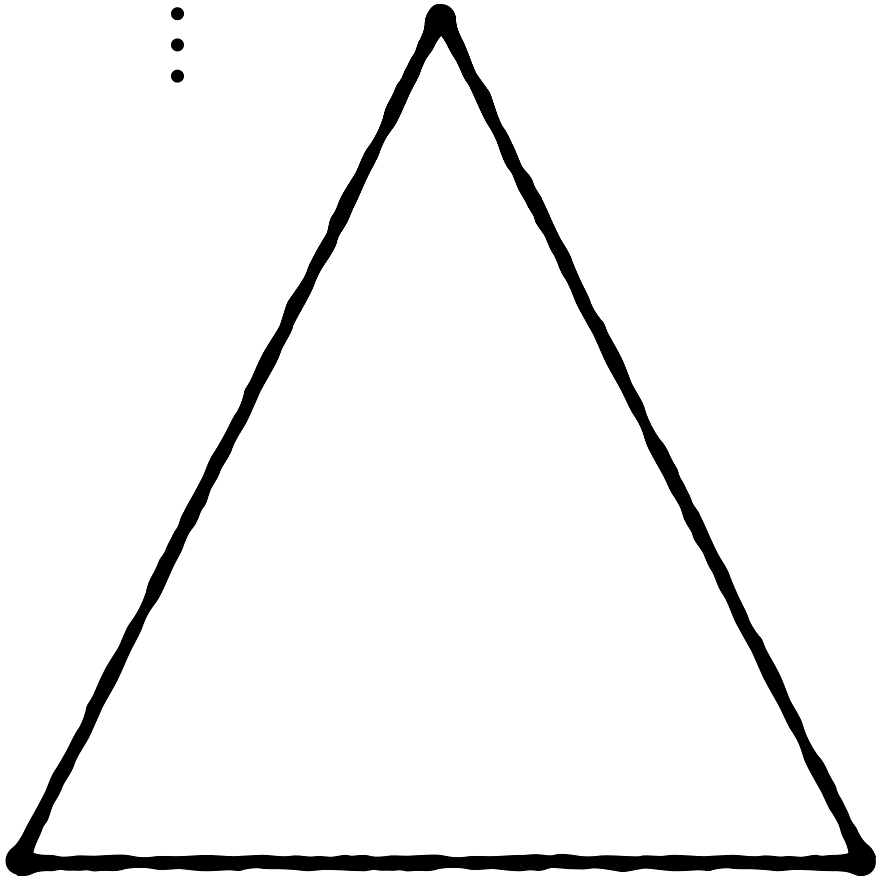
spin, …

[Pasterski,Strominger,Zhiboedov'15]

Tower of ∞ symmetries

∞ IR triangle

(for projected S-matrix)



asymptotic symmetry

⋮

[Hamada,Shiu'18]
[Li,Lin,Zhang'18]

soft theorem

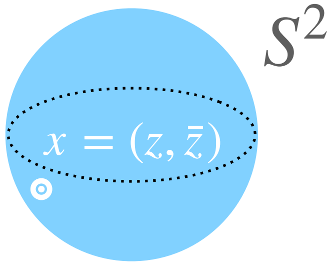
memory effect

ω^n ⋮

⋮

$n = -1, 0, 1, \ldots$

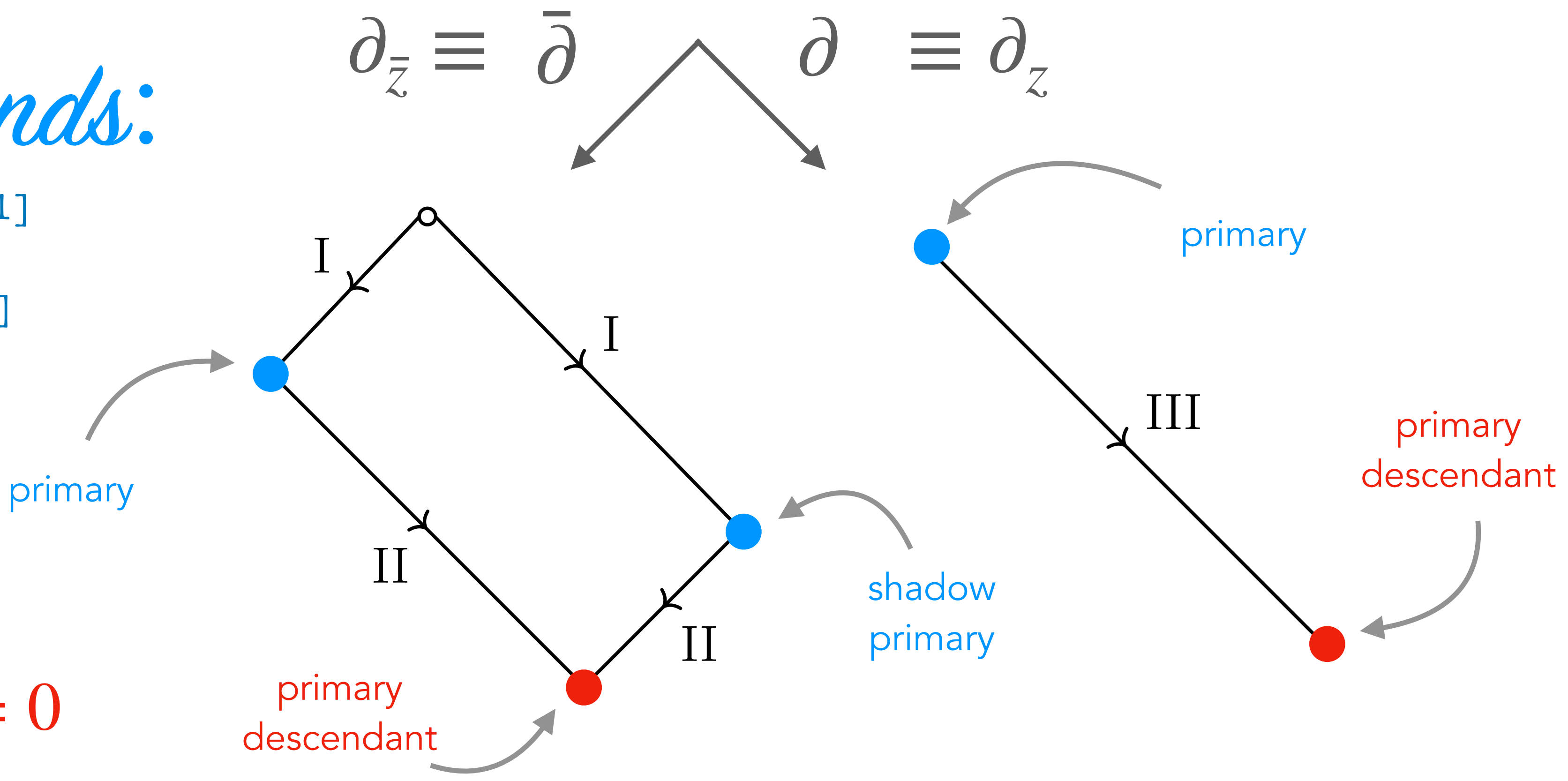
Celestial symmetries



celestial diamonds:

[Pasterski,AP,Trevisani'21]

also: [Banerjee,Pandey,Paul'19]



conservation equation $\bar{\partial}^{\#}\mathcal{O}(z,\bar{z}) = 0$

$\uparrow$

depends on types I, II, III: spin of descendant >,<,= spin of parent primary

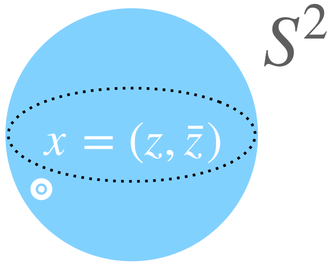
Conformally soft operator = primary

$$\downarrow \bar{\partial}^{\#}$$

primary descendant = conservation equation for **topological operator**

Noether current: $\mathcal{J} = \sum_{m=0}^{\#-1} (-1)^m \bar{\partial}^m \epsilon(z,\bar{z}) \bar{\partial}^{\#-m-1}\mathcal{O}(z,\bar{z})$

Celestial symmetries



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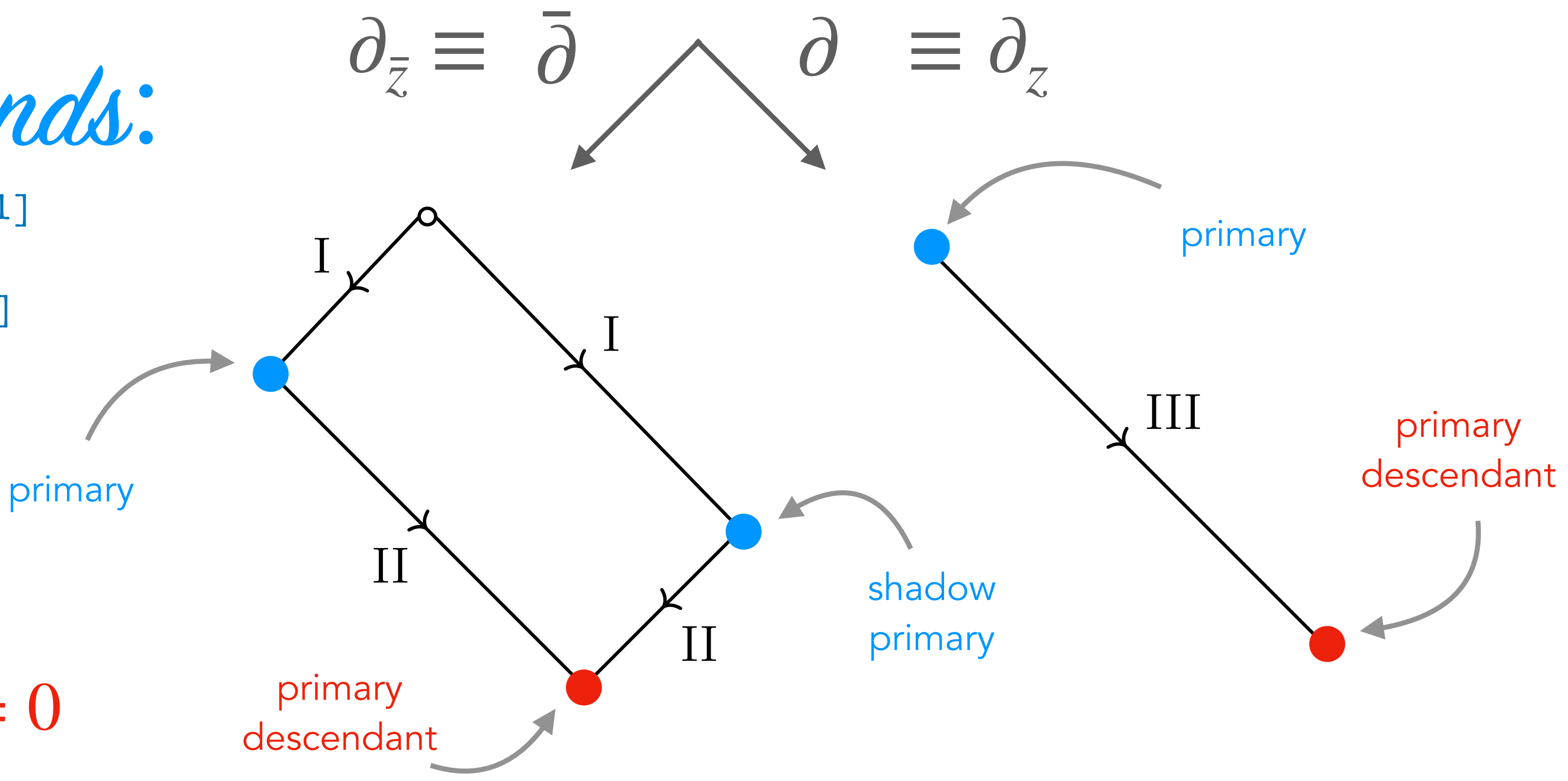


conservation equation $\bar{\partial}^{\#}\mathcal{O}(z, \bar{z}) = 0$

depends on types I, II, III: spin of descendant >,<,= spin of parent primary

Classification of all symmetries in celestial CFT$_{d\geq 2}$.

[Pasterski,AP,Trevisani'21] $d = 2$

[Pano,AP,Trevisani'23] $d > 2$

$$\mathcal{O}_\Delta : \quad \Delta = 1, 0, -1, \ldots$$

supertranslations

superrotation

∞ tower

Towers of ∞ symmetries

Define a **discrete family** of conformally soft positive-helicity gravitons

supertranslations

$$H^k = \lim_{\varepsilon \to 0} \varepsilon \mathcal{O}_{k+\epsilon,+2} \qquad k = 2,1,0,-1,-2,\ldots$$

superrotation

with weights $(h, \bar{h}) = \left(\frac{k+2}{2}, \frac{k-2}{2}\right)$

and a **consistently-truncated** antiholomorphic mode expansion

\`wedge condition' = conservation equation for topological operator

$$H^k(z, \bar{z}) = \sum_{n=\frac{k-2}{2}}^{\frac{2-k}{2}} \frac{H^k_n(z)}{\bar{z}^{n+\frac{k-2}{2}}}$$

$$w^p_n = \frac{1}{\kappa}(p-n-1)!(p+n-1)!H_n^{-2p+4} \qquad p, q = 1, \frac{3}{2}, 2, \frac{5}{2}, \ldots$$

$$[w^p_m, w^q_n] = [m(q-1) - n(p-1)]w^{p+q-2}_{m+n}$$

Arises already in Penrose's twistor construction!

This is the $w_{1+\infty}$ algebra.

[Guevara,Himwich,Pate,Strominger'21]

[Strominger'21]

2D soft actions

A toy model that captures the features of the diamond structure is the higher derivative Gaussian theory with action

[Pasterski,AP,Trevisani'21]

$$S = \int d^2z \left[\partial^k \mathcal{O}^s_{\Delta,J} \bar{\partial}^{\bar{k}} \mathcal{O}^s_{\Delta,J} + \partial^k \mathcal{O}^s_{\Delta,-J} \bar{\partial}^{\bar{k}} \mathcal{O}^s_{\Delta,-J}\right]$$

and with conservation equation

$$\Delta = 1 - \frac{k+\bar{k}}{2} \qquad J = \frac{k-\bar{k}}{2}$$

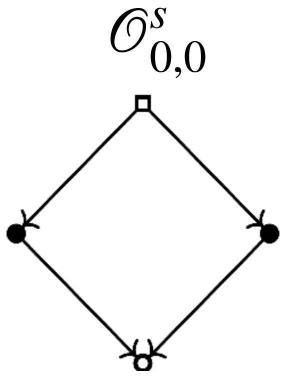
$$\partial_z^k \partial_{\bar{z}}^{\bar{k}} \mathcal{O}^s_{\Delta,J} = 0$$

$$k, \bar{k} \in \mathbb{Z}_>$$

QED

Simplest example:

free boson $(k = \bar{k} = 1)$



Yang-Mills

[Cheung,de La Fuente, Sundrum'15]

[Magnea'21] [Gonzáles,Rojas'21]

gravity

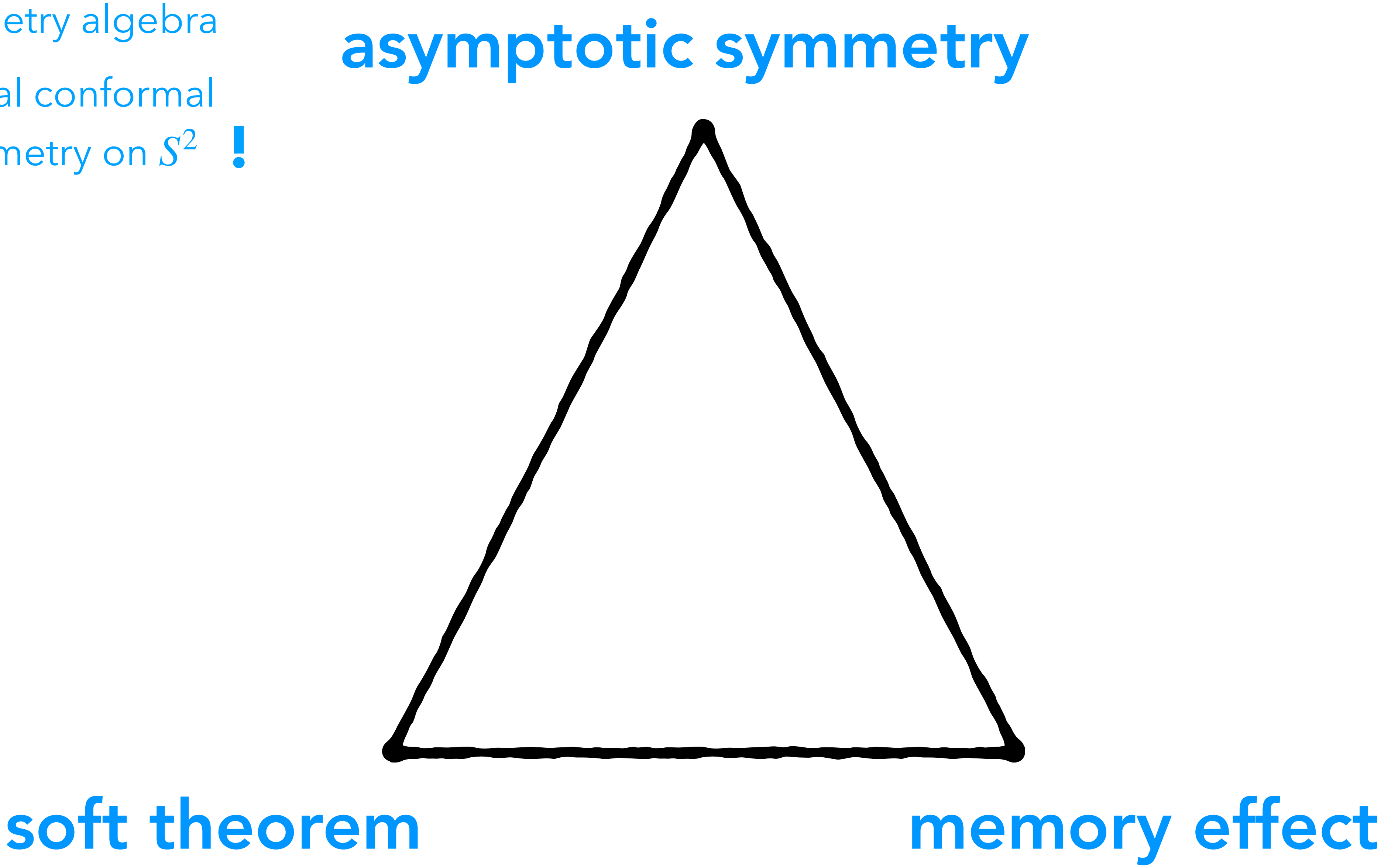
[Nguyen,Salzer'20] [Nguyen'21]

[Kalyanapuram'20+'21]

Long-range effects

IR triangle **@ tree !**

∞-dimensional
symmetry algebra
$\supset$ local conformal
symmetry on S^2 **!**



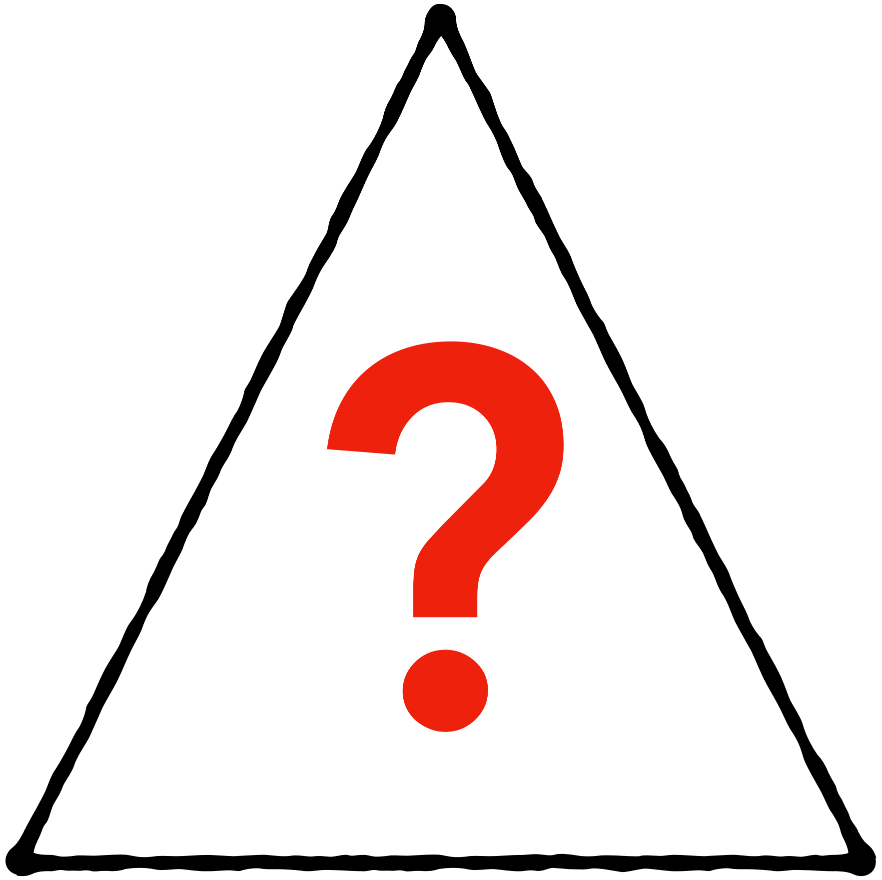
asymptotic symmetry

soft theorem

memory effect

IR triangle @ loop ?

∞-dimensional
symmetry algebra
⊃ local conformal
symmetry on S^2 ?



asymptotic symmetry

soft theorem

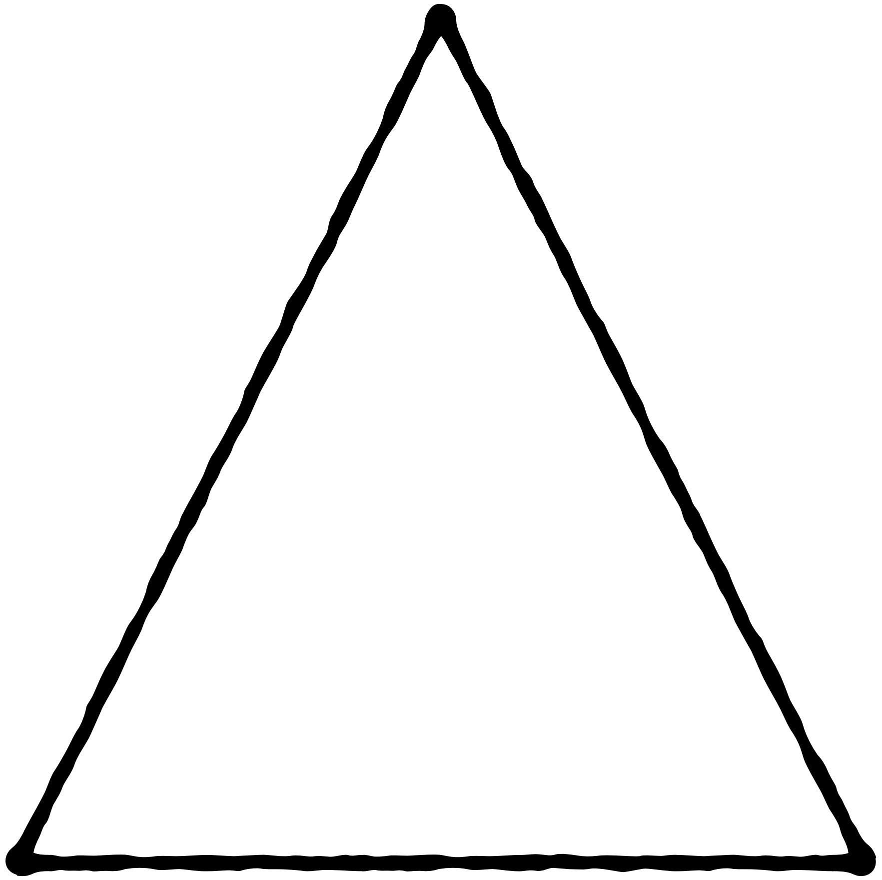
memory effect

IR triangle @ long-range IR effects

classical and quantum

∞-dimensional
symmetry algebra

$\supset$ local conformal
symmetry on S^2 ?



asymptotic ? symmetry

logarithmic soft theorem

tail memory effect

[Laddha,Sen'18]
[Sahoo,Sen'18]
[Saha,Sahoo,Sen'19]

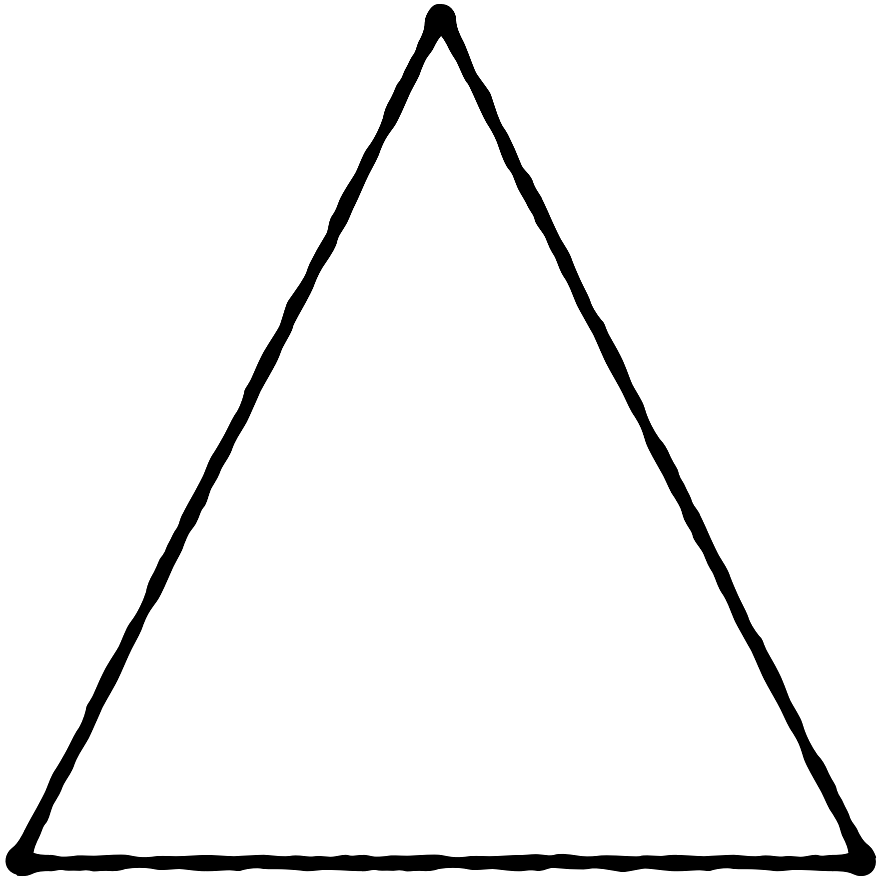
[Sahoo'20]
[Sahoo,Sen'21]
[Sahoo,Krishna'23]

IR triangle @ long-range IR effects

classical and quantum

∞-dimensional symmetry algebra

$\supset$ local conformal symmetry on S^2 ?



asymptotic ? symmetry

logs render ambiguous all subleading tree-level soft theorems

↑

logarithmic soft theorem

tail memory effect

ω^{-1} leading soft

$\log\omega$ subleading soft

$\vdots$

[Laddha,Sen'18]
[Sahoo,Sen'18]
[Saha,Sahoo,Sen'19]

[Sahoo'20]
[Sahoo,Sen'21]
[Sahoo,Krishna'23]

Power-law soft theorems

Tree-level amplitudes admit a soft expansion:

[Low'58] [Weinberg'65]
[Cachazo,Strominger'14]
[Hamada,Shiu'18] [Li,Lin,Zhang'18]

hard momenta

helicity

$$\mathcal{M}_{N+1}(p_1,\dots,p_N;(\omega,q,\ell)) = \sum_{n=-1}^{\infty} \omega^n S_n(\{p_i\},(q,\ell))\mathcal{M}_N(p_1,\dots,p_N) + \dots$$

soft momentum

$$p^\mu = \omega q^\mu(z,\bar{z})$$

$\supset$ non-universal *

$n=-1$	S_{-1}	Weinberg (leading) soft factor	tree exact & universal
$n=0$	S_0	subleading tree soft factor	
$n>0$	$S_{n>0}$	sub^{n+1} leading tree soft factors	

---

* see [Elvang,Jones,Naculich'16] for classification in EFT of massless particles

Soft graviton factor

[Weinberg'65]
[Cachazo,Strominger'14]

Leading soft factor $\sim \frac{1}{\omega}$:

$$S_{-1} = \frac{\kappa}{2} \sum_{i=1}^{N} \frac{\varepsilon_{\mu\nu} p_i^\mu p_i^\nu}{q \cdot p_i}$$

p_i^μ... hard momenta
$p^\mu = \omega q^\mu$... soft momentum
$\varepsilon^{\mu\nu}$...soft graviton polarization
$\kappa = \sqrt{32\pi G_N} \ldots$ coupling

Subleading soft factor $\sim \omega^0$:

$$S_0 = -\frac{i\kappa}{2} \sum_{i=1}^{N} \frac{\varepsilon_{\mu\nu} p_i^\mu q_\lambda J_i^{\lambda\nu}}{q \cdot p_i}$$

ambiguous if long-range IR effects

Logarithmic Soft Theorems

Long-range effects yield **novel soft theorems**:

[Sahoo,Sen'18]
[Saha,Sahoo,Sen'19]

hard momenta

helicity

$$\mathcal{M}_{N+1}(p_1,\ldots,p_N;(\omega,q,\ell)) = \sum_{n=-1}^{\infty} \omega^n(\ln\omega)^{n+1}\, S_n^{(\ln\omega)}(\{p_i\},(q,\ell))\mathcal{M}_N(p_1,\ldots,p_N) + \ldots$$

soft momentum

$$p^\mu = \omega q^\mu(z,\bar{z})$$

$\supset$ non-universal

$\sim \omega^n(\ln\omega)^{m\neq n+1}$

$n=-1$	$S_{-1}^{(\ln\omega)} \equiv S_{-1}$	Weinberg (leading) soft factor	tree exact & universal
$n=0$	$S_0^{(\ln\omega)} \neq S_0$	leading log soft factor	1-loop exact & universal
$n>0$	$S_{n>0}^{(\ln\omega)} \neq S_{n>0}$	sub^n leading log soft factor	(n+1)-loop exact & universal ?

Is the **universality** of the loop exact logarithmic soft theorems a consequence of **asymptotic symmetries of the S-matrix** ?

Soft graviton factor

[Weinberg'65]
[Sahoo,Sen'18]
[Saha,Sahoo,Sen'19]

Leading soft factor $\sim \frac{1}{\omega}$:

$$S_{-1} = \frac{\kappa}{2} \sum_{i=1}^{N} \frac{\varepsilon_{\mu\nu} p_i^\mu p_i^\nu}{q \cdot p_i}$$

p_i^μ... hard momenta
$p^\mu = \omega q^\mu$... soft momentum
$\varepsilon^{\mu\nu}$...soft graviton polarization

Subleading soft factor $\sim \ln \omega$:

$$S_0^{(\ln \omega)} = S_{0,\text{classical}}^{(\ln \omega)} + S_{0,\text{quantum}}^{(\ln \omega)}$$

classical:

late time gravitational radiation from particle acceleration via long-range gravitational interaction

$$S_{0,\text{classical}}^{(\ln \omega)} = \frac{i(\frac{\kappa}{2})^3}{8\pi} \sum_{i=1}^{N} \frac{\varepsilon_{\mu\nu} p_i^\nu q_\rho}{q \cdot p_i} \sum_{j \neq i, \eta_i \eta_j = 1} \frac{(p_i \cdot p_j)\left[p_i^\mu p_j^\rho - p_j^\mu p_i^\rho\right]\left[2(p_j \cdot p_j)^2 - 3p_i^2 p_j^2\right]}{\left[(p_i \cdot p_j)^2 - p_i^2 p_j^2\right]^{3/2}}$$

quantum:
(1-loop)

$\omega_{\text{soft}} \ll \omega_{\text{loop}} \ll \omega_{\text{hard}}$

$$S_{0,\text{quantum}}^{(\ln \omega)} = -\frac{(\frac{\kappa}{2})^3}{16\pi^2} \sum_{i=1}^{N} \frac{\varepsilon_{\mu\rho} p_i^\rho q_\nu}{q \cdot p_i} \left(p_i^\mu \partial_{p_i}^\nu - p_i^\nu \partial_{p_i}^\mu\right) \sum_{j \neq i} \frac{2(p_i \cdot p_j)^2 - p_i^2 p_j^2}{\sqrt{(p_i \cdot p_j)^2 - p_i^2 p_j^2)}} \ln\left(\frac{p_i \cdot p_j + \sqrt{(p_i \cdot p_j)^2 - p_i^2 p_j^2}}{p_i \cdot p_j - \sqrt{(p_i \cdot p_j)^2 - p_i^2 p_j^2}}\right)$$

Soft graviton factor

[Weinberg'65]
[Sahoo,Sen'18]
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quantum:
(1-loop)

$\omega_{\mathrm{soft}} \ll \omega_{\mathrm{loop}} \ll \omega_{\mathrm{hard}}$

$\omega_{\mathrm{IR}} \ll \omega_{\mathrm{loop}} \ll \omega_{\mathrm{soft}}$

$$S_{0,\mathrm{quantum}}^{(\ln \omega)} = -\frac{(\frac{\kappa}{2})^3}{16\pi^2} \sum_{i=1}^{N} \frac{\varepsilon_{\mu\rho} p_i^\rho q_\nu}{q \cdot p_i} \left(p_i^\mu \partial_{p_i}^\nu - p_i^\nu \partial_{p_i}^\mu\right) \sum_{j \neq i} \frac{2(p_i \cdot p_j)^2 - p_i^2 p_j^2}{\sqrt{(p_i \cdot p_j)^2 - p_i^2 p_j^2)}} \ln\left(\frac{p_i \cdot p_j + \sqrt{(p_i \cdot p_j)^2 - p_i^2 p_j^2}}{p_i \cdot p_j - \sqrt{(p_i \cdot p_j)^2 - p_i^2 p_j^2}}\right) \quad + \text{drag}$$

Soft graviton factor

[Weinberg'65]
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Conservation laws

To establish a symmetry interpretation for a soft theorem from *first principles*: for asymptotic symmetry transformations δ compute charges $Q^{\pm}$ from the symplectic structure

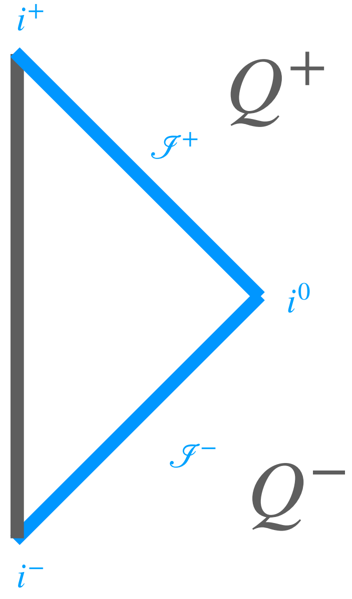
$$\Omega_{i^{\pm} \cup \mathscr{I}^{\pm}}(\delta, \delta') = \delta' Q^{\pm}$$

in the **covariant phase space formalism** and show that the **charge conservation law**

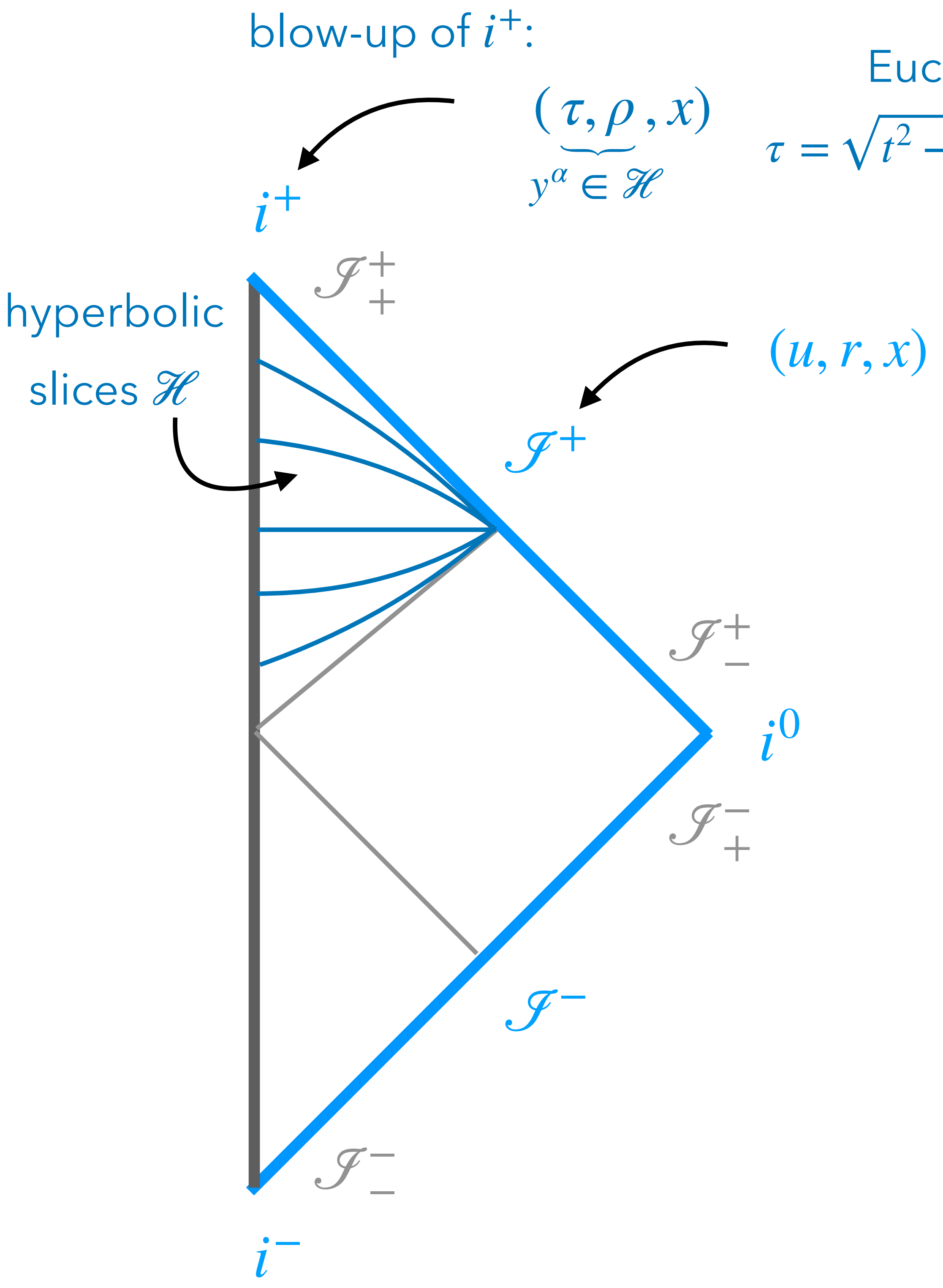
Upon identifying the fields and symmetry parameter antipodally:

$$Q^{+} = Q^{-}$$

corresponds to the **soft theorem**.



Structure at ∞



Euclidean AdS$_3$ coordinates

$\tau = \sqrt{t^2 - r^2}$ $\quad$ $\rho = \frac{r}{\sqrt{t^2 - r^2}}$

retarded Bondi coordinates

$u = t - r$

$$\Omega_{i^+ \cup \mathscr{I}^+}(\delta, \delta') = \delta' Q_+$$

limit $\tau \to \infty$ at fixed ρ $\quad$ limit $r \to \infty$ at fixed u

$$Q_{H,+} \quad + \quad Q_{S,+} \quad = \quad Q_+$$

hard charge $\quad$ soft charge

Structure at ∞

blow-up of i^+:

$$(\underbrace{\tau, \rho}_{y^\alpha \in \mathscr{H}}, x)$$

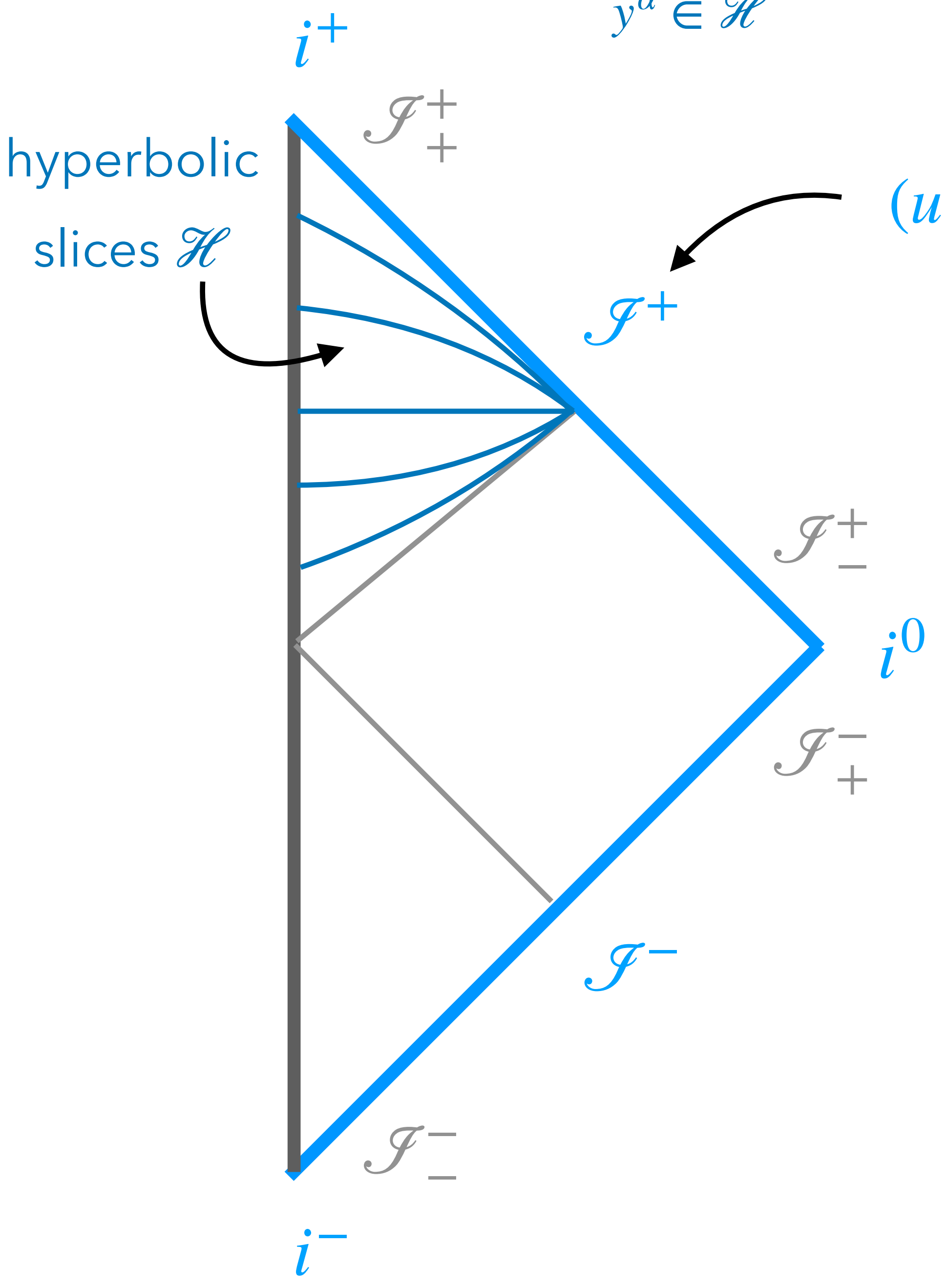
Euclidean AdS$_3$ coordinates

$$\tau = \sqrt{t^2 - r^2} \qquad \rho = \frac{r}{\sqrt{t^2 - r^2}}$$

$$(u, r, x)$$

retarded Bondi coordinates

$$u = t - r$$



$$\Omega_{i^+ \cup \mathscr{I}^+}(\delta, \delta') = \delta' Q_+$$

limit $\tau \to \infty$ at fixed ρ — limit $r \to \infty$ at fixed u

$$Q_{H,+} \quad + \quad Q_{S,+} \quad = \quad Q_+$$

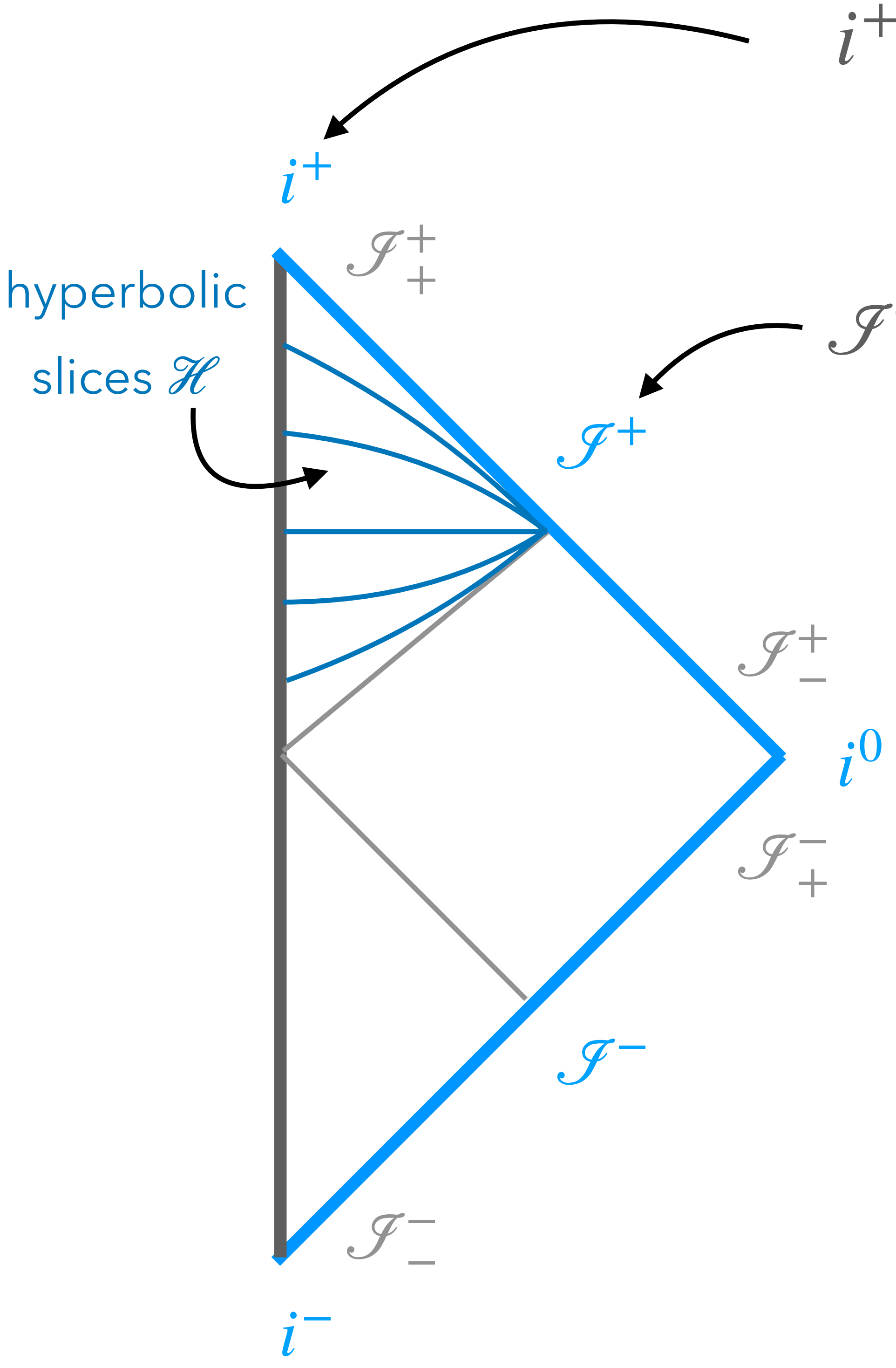
hard charge — soft charge

in soft theorem

action on hard matter — soft insertion

Symmetry at ∞

[Choi,Laddha,AP'24]



i^+ : $\boxed{\bar{Y}^\alpha(y)}$

$$\delta\varphi = \bar{Y}^\alpha \partial_\alpha \varphi$$

$\mathscr{I}^+$: $\boxed{Y^A(x)}$

$$\delta_Y \gamma_{AB} = 2D_{(A}Y_{B)} - D\cdot Y\,\gamma_{AB}$$

$$\delta_Y C_{AB} = \left[\mathscr{L}_Y - \tfrac{1}{2}D\cdot Y(1-u\partial_u)\right] C_{AB}$$
$$+u\left[D_{(A}(D^2+1)Y_{B)} - D_A D_B D\cdot Y - \tfrac{1}{2}\gamma_{AB}(D^2+4)D\cdot Y\right]$$

Superrotation vector field extends smoothly across $i^+ \cup \mathscr{I}^+$:

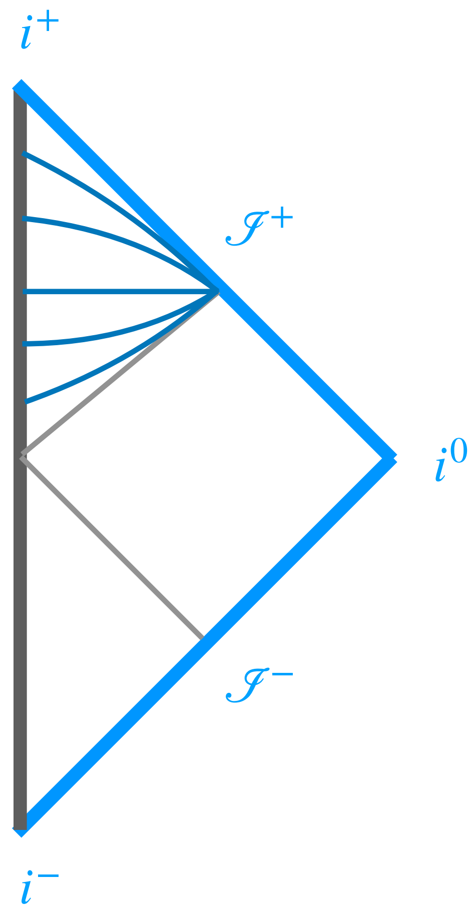
$$\boxed{\bar{Y}^\alpha(y)} = \int_{S^2} d^2x\, G^\alpha_A(y;x)\boxed{Y^A(x)}$$

vector Green's function

Effect of long-range interactions

[Choi,Laddha,AP'24]

Matter is not free at late times but gravitationally dressed by **logs**:



$$i^+ : \quad h_{\tau\tau}(\tau, y) \overset{\tau\to\infty}{=} \frac{1}{\tau}\overset{1}{h}_{\tau\tau}(y) + \ldots$$

'Coulombic' mode sourced by matter stress tensor

$$\varphi(\tau, y) = \frac{\sqrt{m}}{2(2\pi)^{3/2}} \sum_{n=0}^{\infty} \frac{e^{-im\tau}}{\tau^{\frac{3}{2}+n}} \left(\overset{\ln}{b}_n(y) \ln\tau + b_n(y) \right) + \text{c.c.} + \ldots$$

Late-time **tails** with observable consequences:

$$\mathscr{I}^+ : \ C_{AB}(u, x) \overset{u\to+\infty}{=} C^{(0),+}_{AB}(x) + \frac{1}{u} C^{(1),+}_{AB}(x) + \ldots$$

$$= \lim_{r\to\infty} \frac{1}{r} h_{AB}(u, r, x)$$

displacement memory

tail to the memory

Gravity: hard and soft charges

$$\begin{tikzcd}
\Omega_{i^\pm\cup\mathscr{I}^\pm} & = & \Omega_{i^\pm} \arrow[d] & + & \Omega_{\mathscr{I}^\pm} \arrow[d] \\
Q_\pm & = & Q_{H,\pm} & + & Q_{S,\pm}
\end{tikzcd}$$

hard charge soft charge

[Choi,Laddha,AP'24]

$$\begin{tikzcd}
\Omega_{i^\pm\cup\mathscr{I}^\pm}(\delta\,, & \delta_Y \arrow[d] &) = \delta Q_\pm \\
 & \textbf{superrotation} &
\end{tikzcd}$$

Gravity: hard and soft charges

[Choi,Laddha,AP'24]

diverges logarithmically as $\tau \to \infty, u \to \infty$ →

$$\Omega_{i^\pm \cup \mathscr{I}^\pm} = \Omega_{i^\pm} + \Omega_{\mathscr{I}^\pm}$$

↓ ↓

$$Q_\pm = Q_{H,\pm} + Q_{S,\pm}$$

hard charge soft charge

$$\Omega_{i^\pm \cup \mathscr{I}^\pm}(\delta\,,\delta_Y) = \delta Q_\pm$$

↓

superrotation

Gravity: hard and soft charges

[Choi,Laddha,AP'24]

diverges →

logarithmically as

$\tau \to \infty, u \to \infty$

↓

regulate via late-time

cutoff Λ^{-1}

$$\Omega_{i^\pm \cup \mathscr{I}^\pm} = \Omega_{i^\pm} + \Omega_{\mathscr{I}^\pm}$$

↓ ↓

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↓

superrotation

Gravity: hard and soft charges

[Choi,Laddha,AP'24]

$$\begin{tikzcd}
\Omega_{i^{\pm}\cup\mathscr{I}^{\pm}} \arrow[r, phantom, "="] & \Omega_{i^{\pm}} \arrow[d] \arrow[r, phantom, "+"] & \Omega_{\mathscr{I}^{\pm}} \arrow[d] \\
Q_{\pm} \arrow[r, phantom, "="] & Q_{H,\pm} \arrow[r, phantom, "+"] & Q_{S,\pm}
\end{tikzcd}$$

hard charge soft charge

$$\Omega_{i^{\pm}\cup\mathscr{I}^{\pm}}(\delta\,,\delta_Y) = \delta Q_{\pm}$$

$$\downarrow$$

superrotation

Regularized Noether charge:

$$\boxed{Q_{\pm}^{\Lambda} = \ln\Lambda^{-1}\left(Q_{H,\pm}^{(\ln)} + Q_{S,\pm}^{(\ln)}\right) + \left(Q_{H,\pm}^{(0)} + Q_{S,\pm}^{(0)}\right) + \ldots}$$

Gravity: hard and soft charges

[Choi,Laddha,AP'24]

$$\begin{tikzcd}[column sep=small] \Omega_{i^\pm\cup\mathscr{I}^\pm} = \Omega_{i^\pm} \arrow[d] & + & \Omega_{\mathscr{I}^\pm} \arrow[d] \\ Q_\pm = Q_{H,\pm} & + & Q_{S,\pm} \end{tikzcd}$$

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$$\boxed{Q^\Lambda_\pm = \ln\Lambda^{-1}\left(Q^{(\ln)}_{H,\pm} + Q^{(\ln)}_{S,\pm}\right) + \left(Q^{(0)}_{H,\pm} + Q^{(0)}_{S,\pm}\right) + \ldots}$$

Receives loop-corrections and is rendered ambiguous by logs.

$$Q^{(0)}_{H,+}[\bar{Y}] = \int_{i^+} d^3y\, \bar{Y}^\alpha \overset{3}{T}_{\tau\alpha}$$

free stress tensor

$$Q^{(0)}_{S,+}[Y] = -\frac{2}{\kappa}\int_{\mathscr{I}^+} du\, d^2x\, \underset{\uparrow}{D^3_z} Y^z\, \underbrace{u\partial_u}\, C^{zz} + \text{c.c.}$$

D_A ... covariant derivative on S^2

sub tree soft projector

$Q^{(0)}$ charge

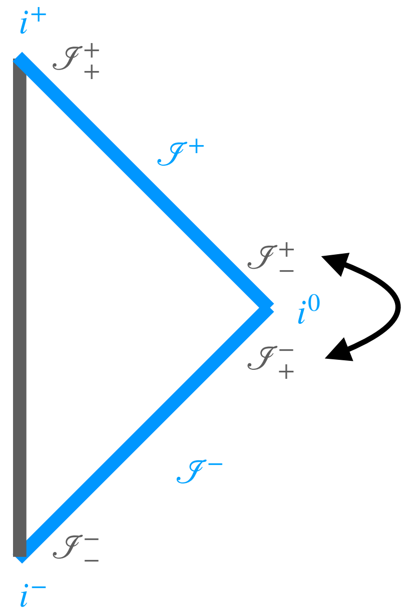
$$Q^{\Lambda} = \ln \Lambda^{-1} \left(Q_H^{(\ln)} + Q_S^{(\ln)} \right) + \underbrace{\left(Q_H^{(0)} + Q_S^{(0)} \right)}_{\equiv\, Q^{(0)}} + \dots$$

Conservation law:

[Campiglia,Laddha'15]

$$Q_+^{(0)} = Q_-^{(0)}$$

Upon identifying the fields and gauge parameter antipodally:



$Q^{(0)}$ charge

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Conservation law:

[Campiglia,Laddha'15]

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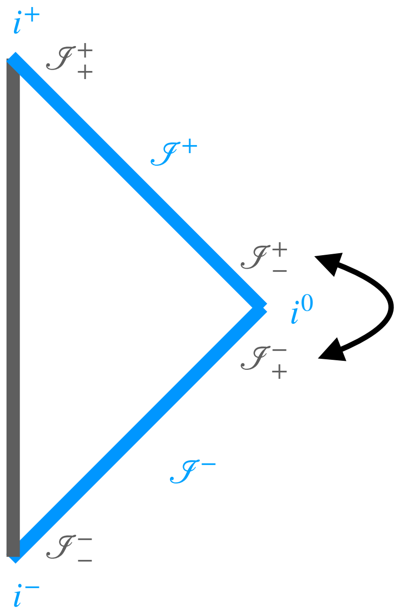
Upon identifying the fields and gauge parameter antipodally:

[Kapec,Lysov,Pasterski,Strominger'14]

judicious choice of **superrotation** $Y^A(x)$

$$\downarrow$$



Tree-level subleading soft **graviton** theorem:

$$\mathscr{M}_{N+1} = \left(\omega^{-1} S_{-1} + \omega^0 S_0 \right) \mathscr{M}_N + \ldots$$

tree-level soft expansion

[Cachazo,Strominger'14]

$Q^{(0)}$ charge

$$Q^{\Lambda} = \ln\Lambda^{-1}\left(Q_H^{(\ln)} + Q_S^{(\ln)}\right) + \underbrace{\left(Q_H^{(0)} + Q_S^{(0)}\right)}_{\equiv\, Q^{(0)}} + \dots$$

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$$\Big\downarrow$$

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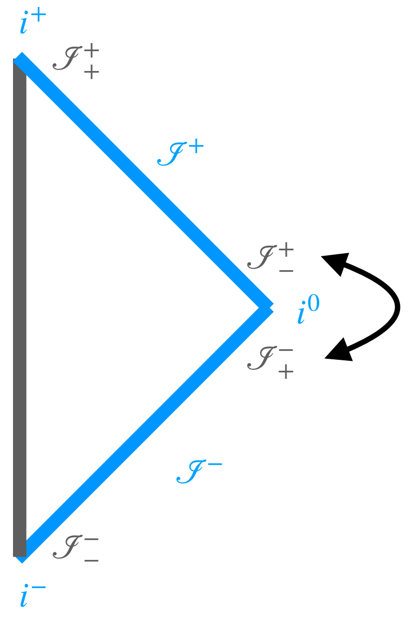

Tree-level subleading soft **graviton** theorem:

$$\mathscr{M}_{N+1} = \left(\omega^{-1}S_{-1} + \omega^{0}S_{0}\right)\mathscr{M}_N + \dots$$

tree-level soft expansion

[Cachazo,Strominger'14]

Recall: IR effects render subleading soft theorem at tree-level ambiguous.

Gravity: hard and soft charges

[Choi,Laddha,AP'24]

$$\begin{tikzcd}
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hard charge soft charge

$$\begin{tikzcd}
\Omega_{i^\pm \cup \mathscr{I}^\pm}(\delta\,, \delta_Y) = \delta Q_\pm \arrow[d] \\
\text{superrotation}
\end{tikzcd}$$

Regularized Noether charge:

$$\boxed{Q^\Lambda_\pm = \ln \Lambda^{-1} \left(Q^{(\ln)}_{H,\pm} + Q^{(\ln)}_{S,\pm} \right) + \left(Q^{(0)}_{H,\pm} + Q^{(0)}_{S,\pm} \right) + \ldots}$$

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Regularized Noether charge:

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$$Q_{H,+}^{(\ln)}[\bar{Y}] = \int_{i^+} d^3y\, \bar{Y}^\alpha\, \overset{3,\ln}{T}{}_{\tau\alpha}$$

leading interacting stress tensor

$$Q_{S,+}^{(\ln)}[Y] = -\frac{2}{\kappa}\int_{\mathscr{I}^+} du\, d^2x\, D_z^3 Y^z\, \underbrace{\partial_u(u^2\partial_u}C^{zz}) + \text{c.c.}$$

log soft projector

This is **exact** in the **gravitational coupling κ** !

$Q^{(\ln)}$ charge

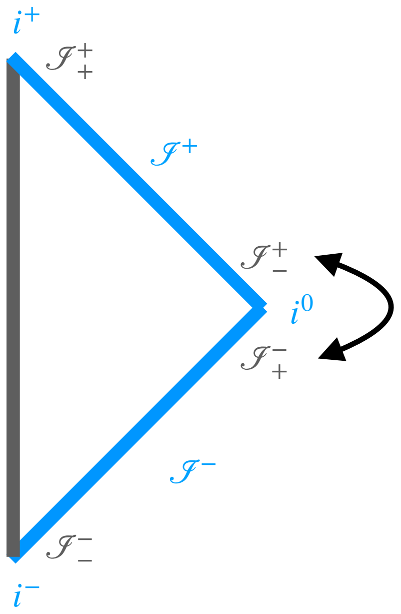
$$Q^{\Lambda} = \ln \Lambda^{-1} \underbrace{\left(Q_H^{(\ln)} + Q_S^{(\ln)}\right)}_{\equiv\, Q^{(\ln)}} + \left(Q_H^{(0)} + Q_S^{(0)}\right) + \dots$$

Conservation law:

[Choi,Laddha,AP'24]

$$Q_+^{(\ln)} = Q_-^{(\ln)}$$

Upon identifying the fields and gauge parameter antipodally:



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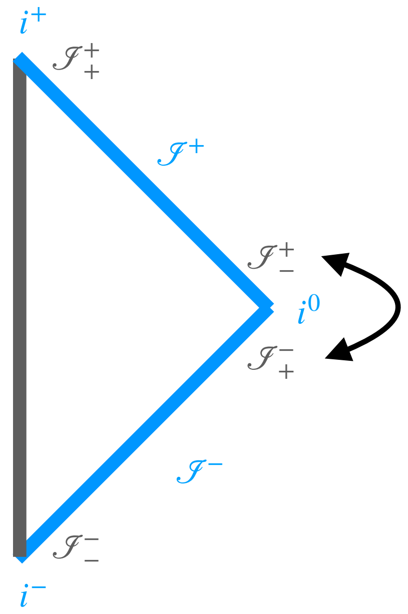


judicious choice of **superrotation** $Y^A(x)$

$$\downarrow$$

Logarithmic soft **graviton** theorem:

$$\mathscr{M}_{N+1} = \left(\omega^{-1} S_{-1} + \omega^{0} \ln \omega\, S_0^{(\ln \omega)} \right) \mathscr{M}_N + \dots$$

log soft expansion

[Sahoo,Sen'18] [Saha,Sahoo,Sen'19]

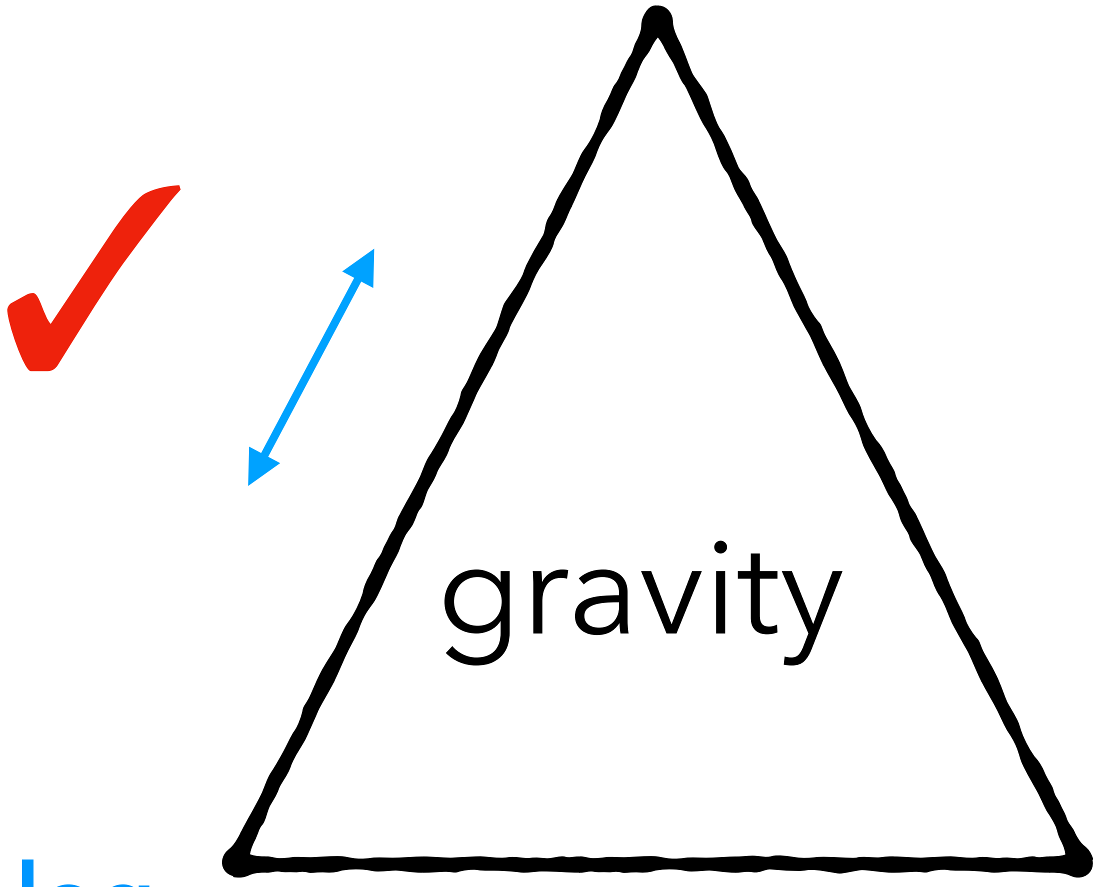
This establishes the symmetry interpretation of the classical logarithmic soft graviton theorem.* [Choi,Laddha,AP'24]

* drag term ≃ phase

Classical superrotation IR triangle

[Choi,Laddha,AP'24]

superrotation

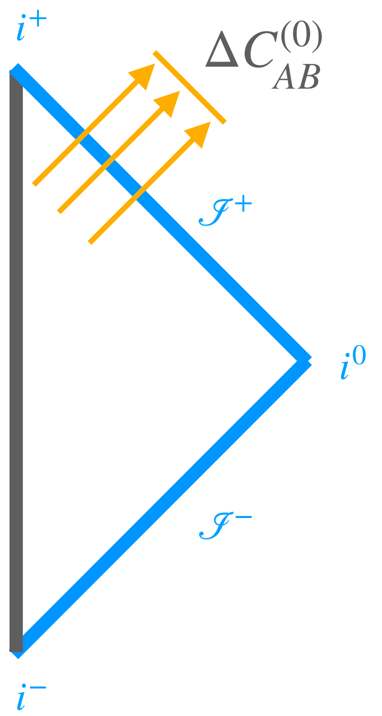


gravity

classical log
soft theorem

[Laddha,Sen'18]
[Sahoo,Sen'18]
[Saha,Sahoo,Sen'19]

Memory and its tail



Shear @ late times: $C_{AB}(u,x) \overset{u\to+\infty}{=} C^{(0),+}_{AB}(x) + \frac{1}{u}C^{(1),+}_{AB}(x) + \dots$

linear **displacement** **memory**
sourced by matter field:

$$\Delta C^{(0)}_{AB} = C^{(0),+}_{AB} - C^{(0),-}_{AB}$$

$$C^{(0),\pm}_{AB} = -\frac{\kappa^2}{8\pi}\int_{i^\pm} d^3y \frac{(\partial_A q\cdot\mathscr{Y})(\partial_B q\cdot\mathscr{Y}) + \frac{1}{2}\gamma_{AB}}{q\cdot\mathscr{Y}}\,\overset{3}{T}{}^{\text{matt}}_{\tau\tau}$$

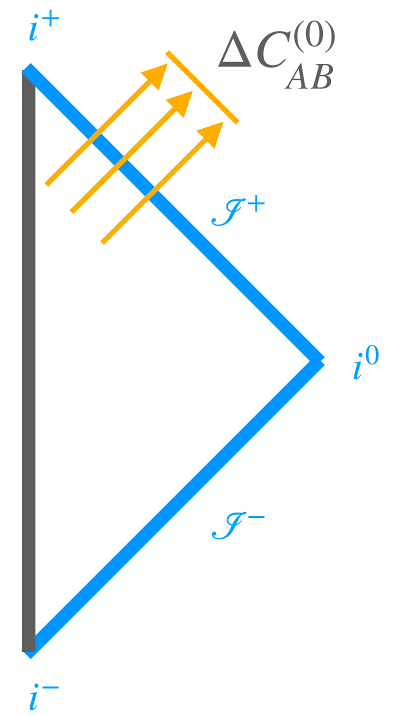
free

matter stress tensor

$x^\mu = \tau\mathscr{Y}(y)$

unit vector in Minkowski

Memory and its tail



Shear @ late times: $$C_{AB}(u,x) \overset{u\to+\infty}{=} C^{(0),+}_{AB}(x) + \frac{1}{u} C^{(1),+}_{AB}(x) + \dots$$

encodes gravitational waves

linear displacement memory

sourced by matter field: $$\Delta C^{(0)}_{AB} = C^{(0),+}_{AB} - C^{(0),-}_{AB}$$

$$C^{(0),\pm}_{AB} = -\frac{\kappa^2}{8\pi}\int_{i^\pm} d^3y \frac{(\partial_A q\cdot\mathscr{Y})(\partial_B q\cdot\mathscr{Y}) + \frac{1}{2}\gamma_{AB}}{q\cdot\mathscr{Y}} \overset{3}{T}{}^{\text{matt}}_{\tau\tau}$$

free

matter stress tensor

$$x^\mu = \tau\mathscr{Y}(y)$$

unit vector in Minkowski

tail to the memory

sourced by matter field: $$\Delta C^{(1)}_{AB} = C^{(1),+}_{AB} - C^{(1),-}_{AB}$$

long-range interaction

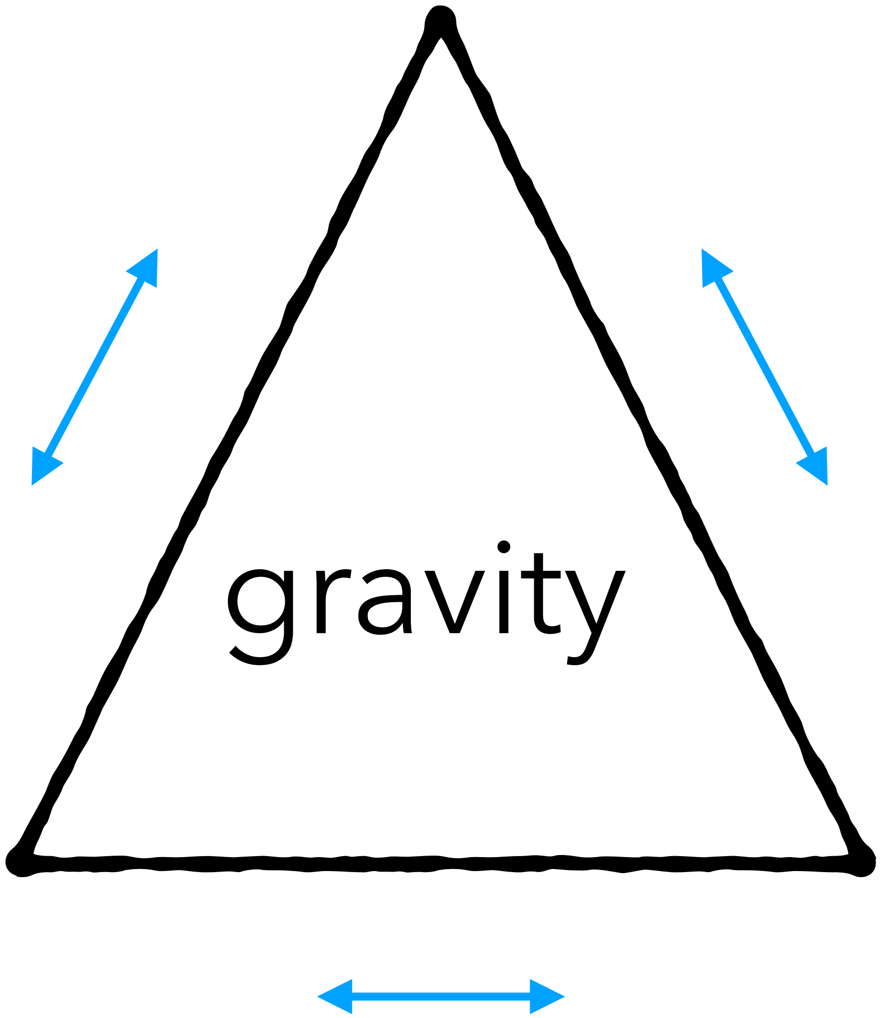
$$C^{(1),\pm}_{AB} = -\frac{\kappa^2}{8\pi}(\partial_A q^\mu)(\partial_B q^\nu)\int_{i^\pm} d^3y \Big[\frac{(q\cdot\mathscr{Y})\mathscr{D}^\alpha(\mathscr{Y}_\mu\mathscr{Y}_\nu) - (\mathscr{Y}_\mu\mathscr{Y}_\nu + \frac{1}{2}\eta_{\mu\nu})\mathscr{D}^\alpha(q\cdot\mathscr{Y})}{q\cdot\mathscr{Y}} \overset{3,\ln}{T}{}^{\text{matt}}_{\tau\alpha}$$

matter stress tensor

$$-\Big((\mathscr{Y}_\mu\mathscr{Y}_\nu + \frac{1}{2}\eta_{\mu\nu})k^{\alpha\beta} + (\mathscr{D}^\alpha\mathscr{Y}_\mu)(\mathscr{D}^\beta\mathscr{Y}_\nu) - \frac{1}{2}\eta_{\mu\nu}(\mathscr{D}^\alpha\mathscr{Y}_\sigma)(\mathscr{D}^\beta\mathscr{Y}^\sigma)\Big)\overset{2}{T}{}^h_{\alpha\beta}\Big]$$

gravity stress tensor

Classical superrotation IR triangle



[Choi,Laddha,AP'24]

superrotation

gravity

classical log soft theorem

[Laddha,Sen'18]
[Sahoo,Sen'18]
[Saha,Sahoo,Sen'19]

tail to the memory effect

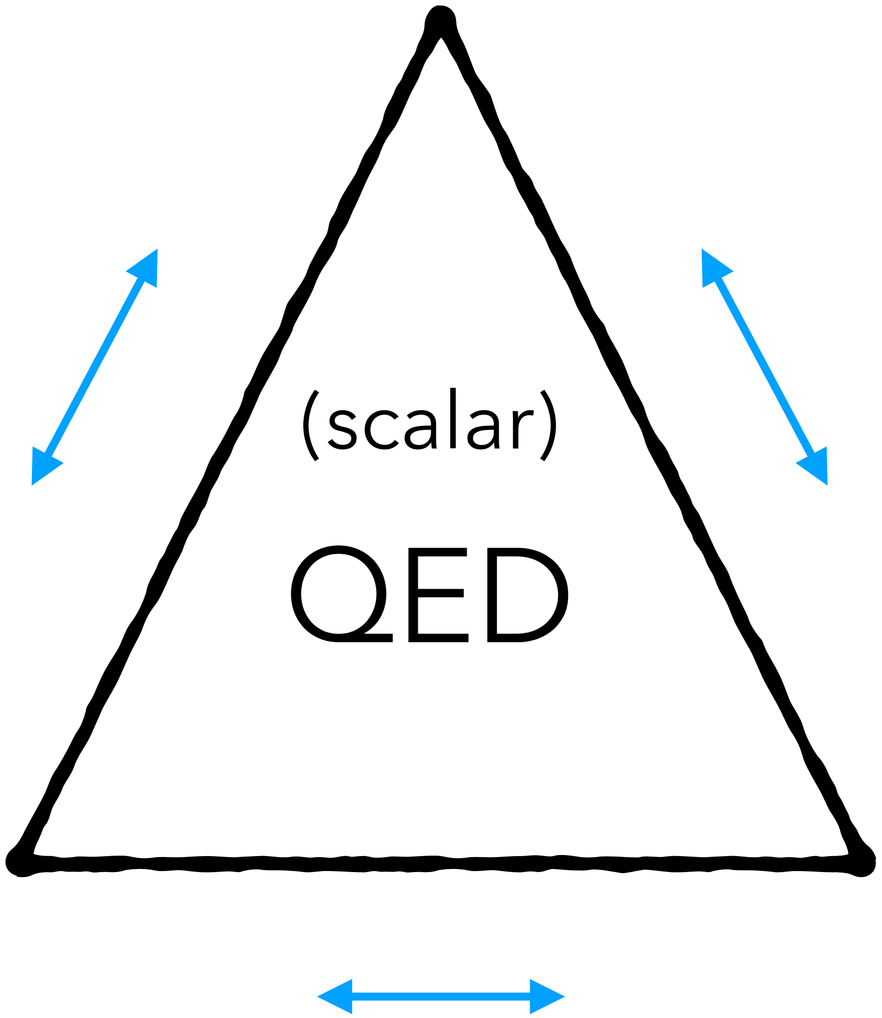
[Saha,Sahoo,Sen'19]

[Choi,Laddha,AP'24]

particles

fields ✓

Classical super*phase*rotation IR triangle



[Choi,Laddha,AP'24]

superphaserotation

(scalar)

QED

classical log soft theorem

[Laddha,Sen'18]
[Sahoo,Sen'18]
[Saha,Sahoo,Sen'19]

tail to the memory effect

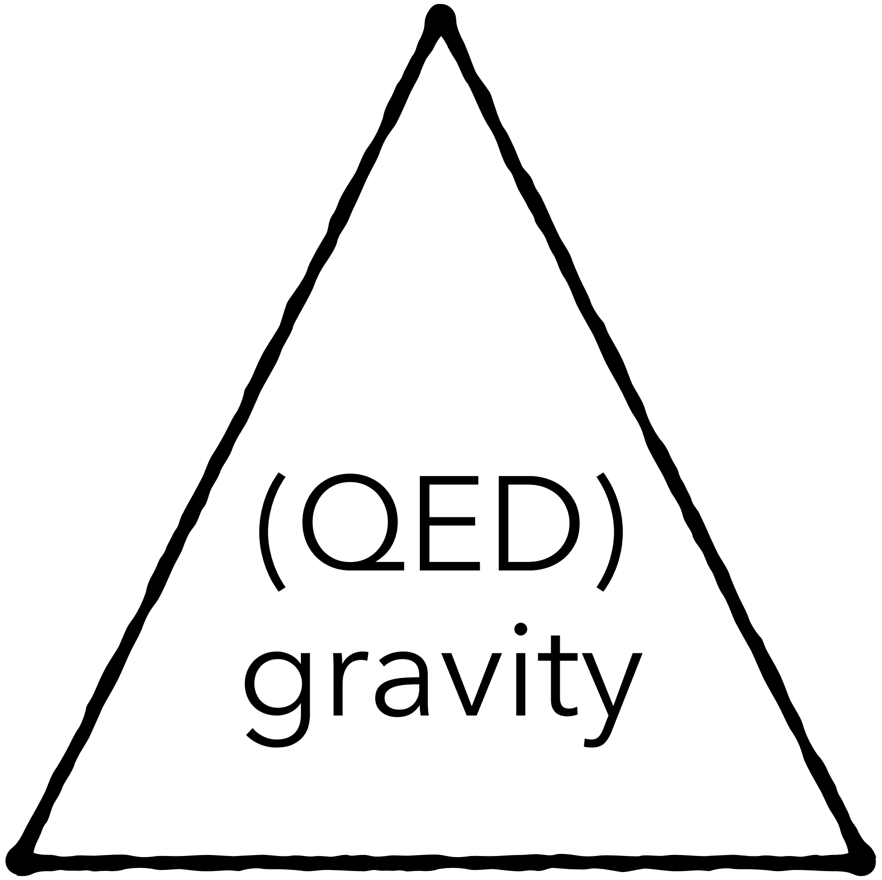
[Saha,Sahoo,Sen'19] particles

[Choi,Laddha,AP'24] fields

Classical IR triangle with massive matter

[Choi,Laddha,AP'24]

super(phase)rotation



(QED)
gravity

classical log
soft theorem

[Laddha,Sen'18]
[Sahoo,Sen'18]
[Saha,Sahoo,Sen'19]

tail to the
memory effect

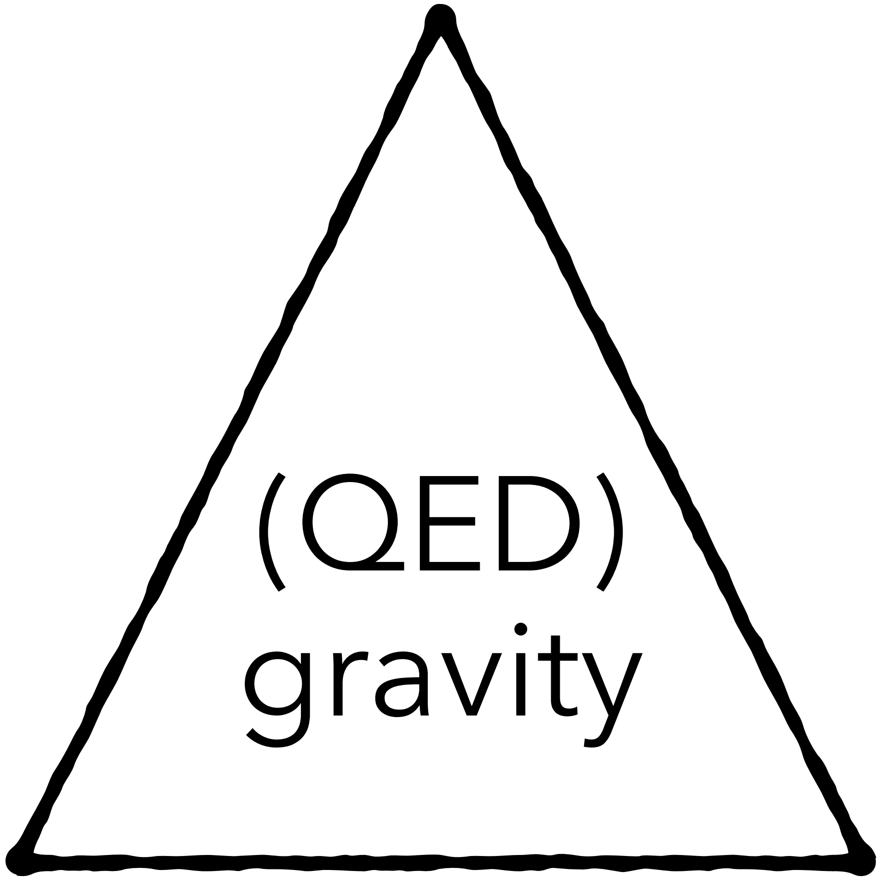
[Saha,Sahoo,Sen'19] particles

[Choi,Laddha,AP'24] fields

Classical IR triangle with massless matter

(more subtle)

[Choi,Kadhe,AP - to appear] [Choi,Kadhe,AP - work in progress]



super(phase)rotation

(QED)
gravity

classical log
soft theorem

tail to the
memory effect

New conservation laws

Noether charge from long-range IR effects:

$$Q^{\Lambda} = \ln \Lambda^{-1} \left(Q_H^{(\ln)} + Q_S^{(\ln)}\right) + \left(Q_H^{(0)} + Q_S^{(0)}\right) + \dots$$

physical IR scale

- super (phase) rotation symmetry ✓
- associated charge Q^{Λ} from first principles & exact in the coupling !
- regulator Λ^{-1} arises from the relevant infrared scale: large $|\tau|$ & $|u|$
- Λ can be removed in the end since $Q_{\pm}^{(\ln)}$ are finite
- log charge conservation law = log soft theorem ✓

Exact in **coupling** ! **Exact** in **coupling** !

Long-range interactions lead to new conservation laws!

Infrared surprises

Infrared triangles

Complete?

4D = 2D

What are the axioms of celestial CFT?

Exact celestial duals? [Costello,Paquette'22]

[Costello,Paquette,Sharma'22]

Towers of ∞ symmetries

Symmetries of what theories?

How powerful constraints?

Long-range effects

Beyond gravity & QED ? Beyond leading log ?

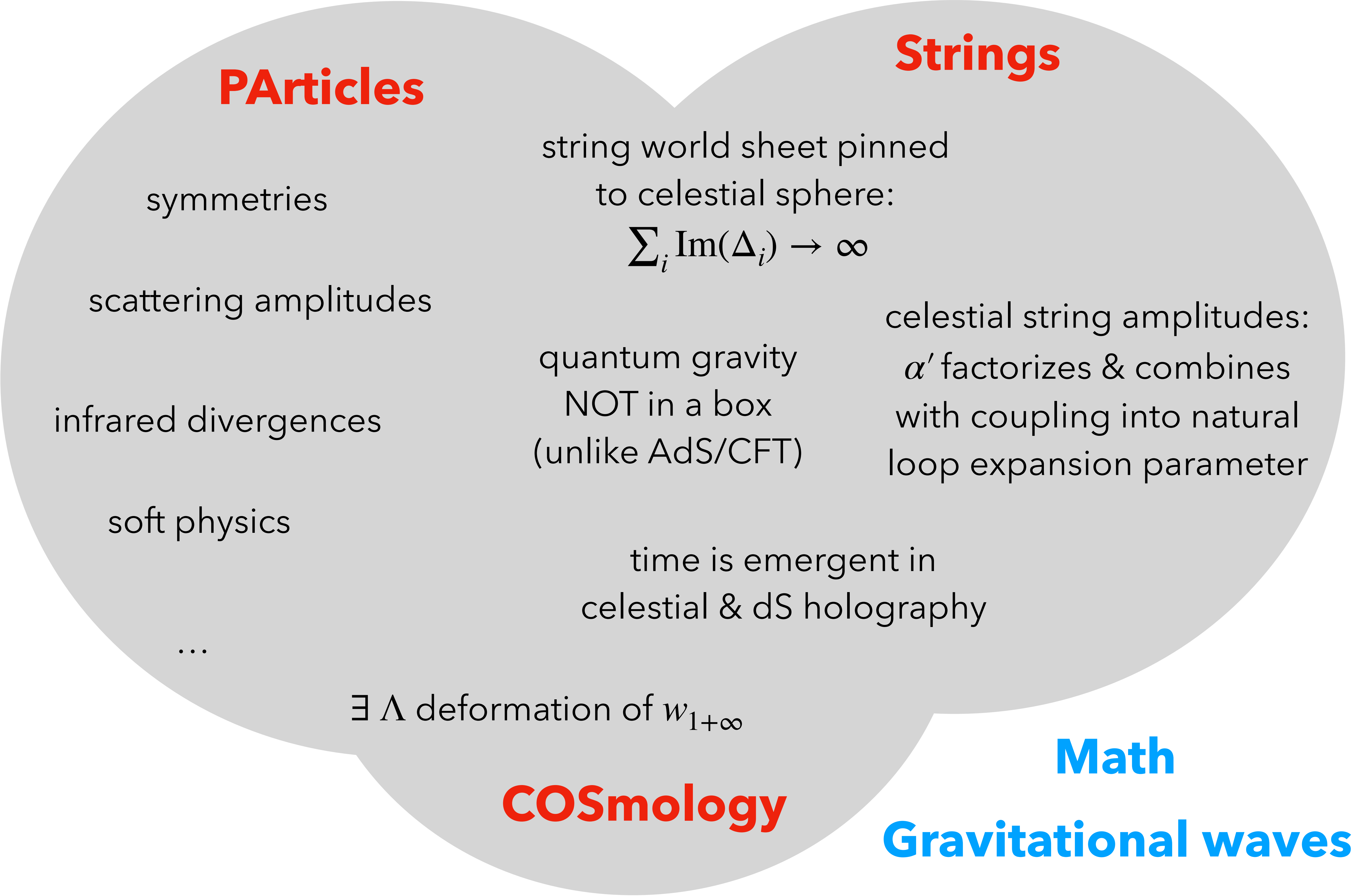
Quantum log soft factor?

see also [Donnay,Nguyen,Ruzziconi'22], [Agrawal,Donnay,Nguyen,Ruzziconi'23]

$\log(u)$? [Campiglia,Laddha'19]

logarithmic CFT? [Bissi,Donnay,Valsesia'24]

In the spirit of PASCOS



PArticles

Strings

symmetries

scattering amplitudes

infrared divergences

soft physics

…

string world sheet pinned
to celestial sphere:
$\sum_i \mathrm{Im}(\Delta_i) \to \infty$

celestial string amplitudes:
α' factorizes & combines
with coupling into natural
loop expansion parameter

quantum gravity
NOT in a box
(unlike AdS/CFT)

time is emergent in
celestial & dS holography

$\exists\ \Lambda$ deformation of $w_{1+\infty}$

COSmology

Math

Gravitational waves

Thank you!