Higgs and ALP-Higgs near-criticality at future colliders

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Asymmetry
Essential Asymmetries of Nature

Motivation

Higgs Potential: $V_{\text{eff}}(H) = -m_{\text{eff}}^2 |H|^2 + \lambda_{\text{eff}} |H|^4$

Hierarchy problem ...**If** there is a scale Λ_{UV}^2

$$m_{\text{eff}}^2 \ll \Lambda_{\text{UV}}^2$$

Why is the Higgs so light?

Tuning* problem

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Traditional path: **Naturalness ≡ symmetry justification**

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$$m_{\text{eff}}^2 \ll \Lambda_{\text{UV}}^2$$

Traditional path: **Naturalness ≡ symmetry justification**

This has (yet) led nowhere —> near-criticality alternative?

Motivation: The two parameters of the Higgs potential take near critical values

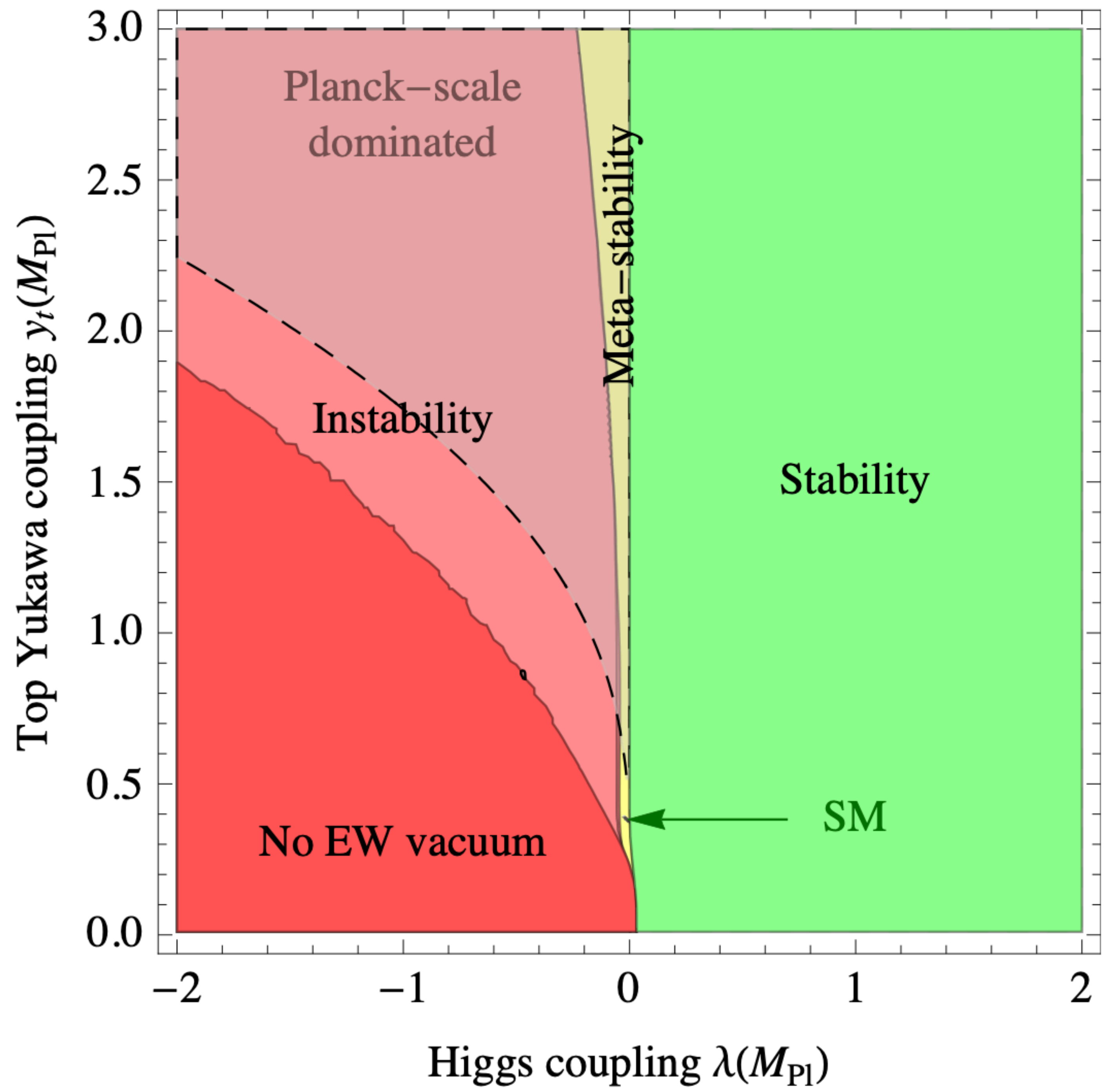
Higgs Potential: $V_{\text{eff}}(H) = -m_{\text{eff}}^2 |H|^2 + \lambda_{\text{eff}} |H|^4$

Metastability of the EW vacuum

Hierarchy problem

$m_{\text{eff}}^2 \ll \Lambda_{\text{UV}}^2$

near-criticality alternative?



Motivation: The two parameters of the Higgs potential take near critical values

SM phase diagram

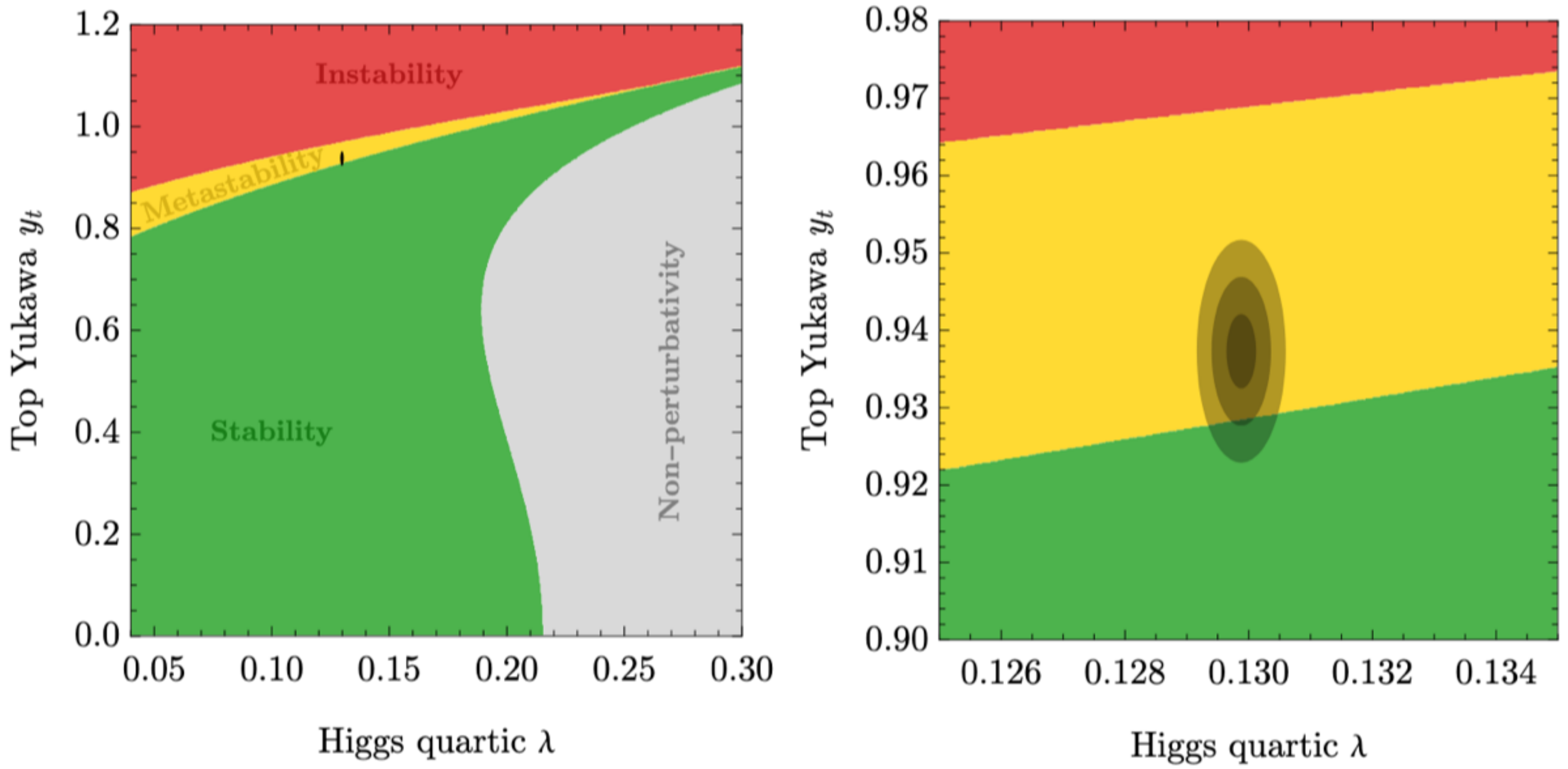


Figure 1: *Standard Model phase diagram in the plane spanned by the top Yukawa coupling and Higgs quartic coupling, renormalised at the top mass scale. The measured SM values are shown with a 3-σ ellipse on the left and with 1-, 2-, and 3-σ contours on the right. The uncertainties are given in Eq.* (D.2) *and include the experimental uncertainty only. The SM values for the top Yukawa and gauge couplings are given in Eq.* (D.2).

"stable vacuum"=never decays

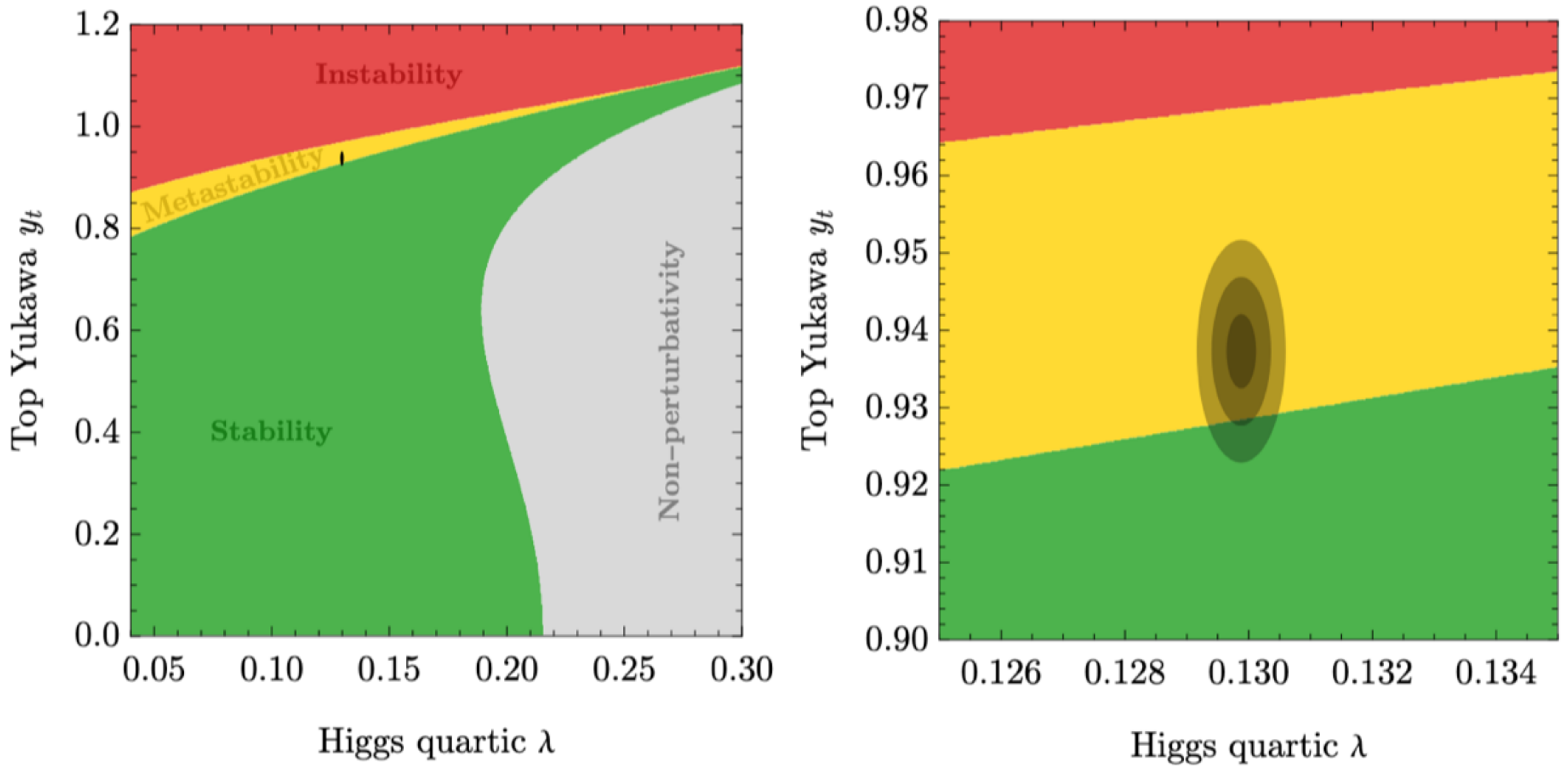
"metastable vacuum"="decays, but with a lifetime longer than the current age of the Universe"

"unstable vacuum"="should have decayed already"

Recent updated drawing by Detering-You

Motivation: The two parameters of the Higgs potential take near critical values

SM phase diagram



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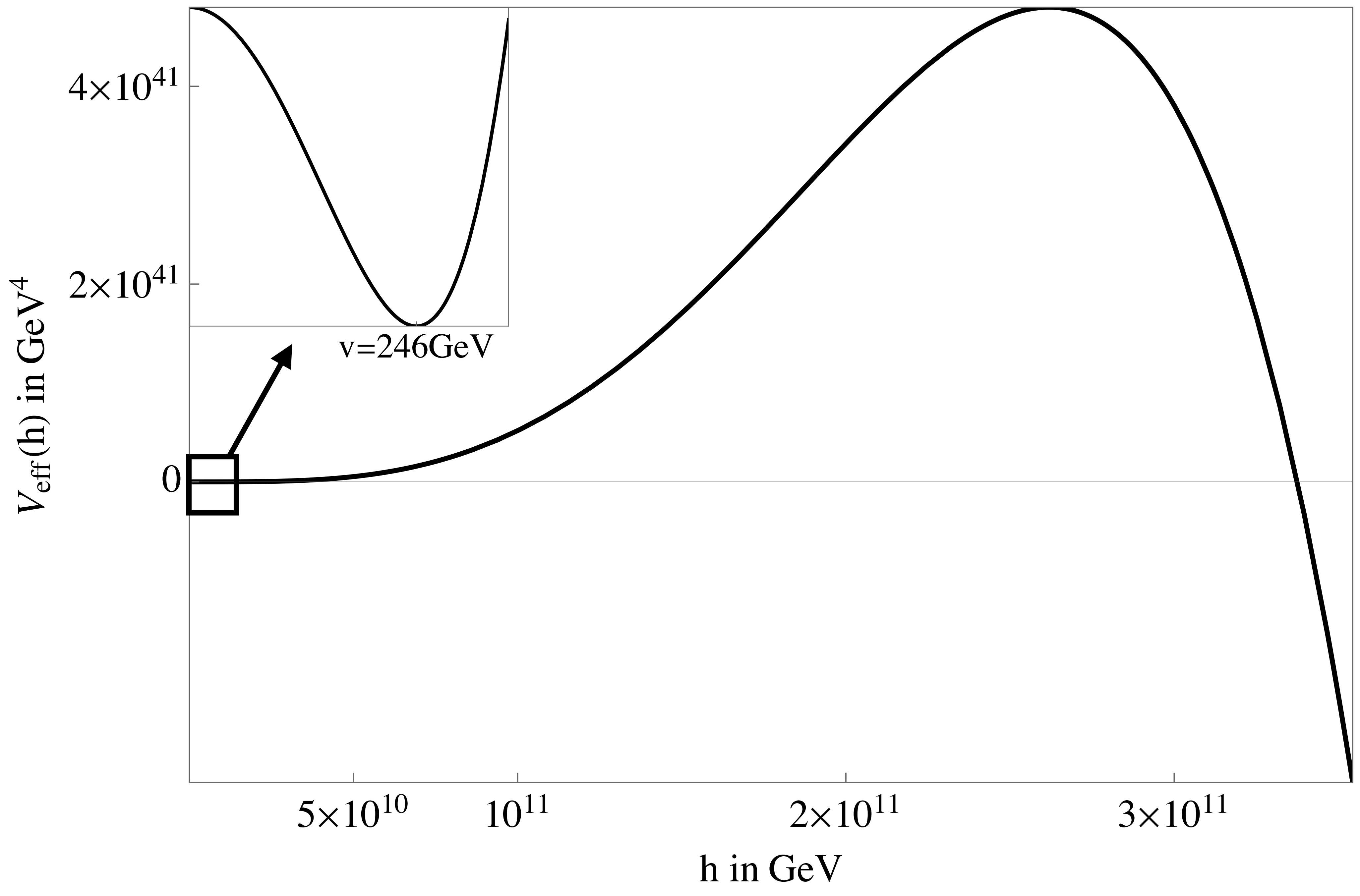
$$V_{\rm eff}(H) = -\,m_{\rm eff}^2|H|^2 + \lambda_{\rm eff}|H|^4$$



$$\lambda_{\rm eff}^{\rm LO} \sim \lambda_H + \frac{1}{4\pi^2}(-y_t^4 + g_2^4 + (g_1^2 + g_2^2)^2)$$

Near-criticality in the SM

Higgs Potential: $V_{\rm eff}(H) = -m_{\rm eff}^2 |H|^2 + \lambda_{\rm eff} |H|^4$

$m_{\rm eff}^2$: close to transition — “SSB”↔”no SSB”

$\lambda_{\rm eff}$: close to transition — “stable”↔”unstable”

“Critical values”

“**Quantum** phase transitions”

Dynamical explanation?

Near-criticality in the SM

Near-criticality alternative ≡ assumption

that some unknown underlying dynamics forces the two parameters of the SM potential to take near-critical values

The golden question: which dynamics ??? 🤔 💁 🙀

* Idea of underlying dynamics : self-organised criticality

PerBak, ChaoTang, and KurtWiesenfeld, `87

is a property of some dynamical systems that have a critical point as an attractor —> macroscopic behaviour alike to phase transitions

Sometimes based on cosmological dynamics:

G. F. Giudice, M. McCullough, and T. You, "Self-organised localisation," arXiv:2105.08617

J. Khoury, "Accessibility Measure for Eternal Inflation: Dynamical Criticality and Higgs Metastability¨ arXiv:1912.06706

J. Khoury and O. Parrikar, "Search Optimization, Funnel Topography, and Dynamical Criticality on the String Landscape, arXiv:1907.07603

G. Kartvelishvili, J. Khoury, and A. Sharma, "The Self-Organized Critical Multiverse, arXiv:2003.12594

J. Khoury and S. S. C. Wong, "Early-time measure in eternal inflation", arXiv:2106.12590,

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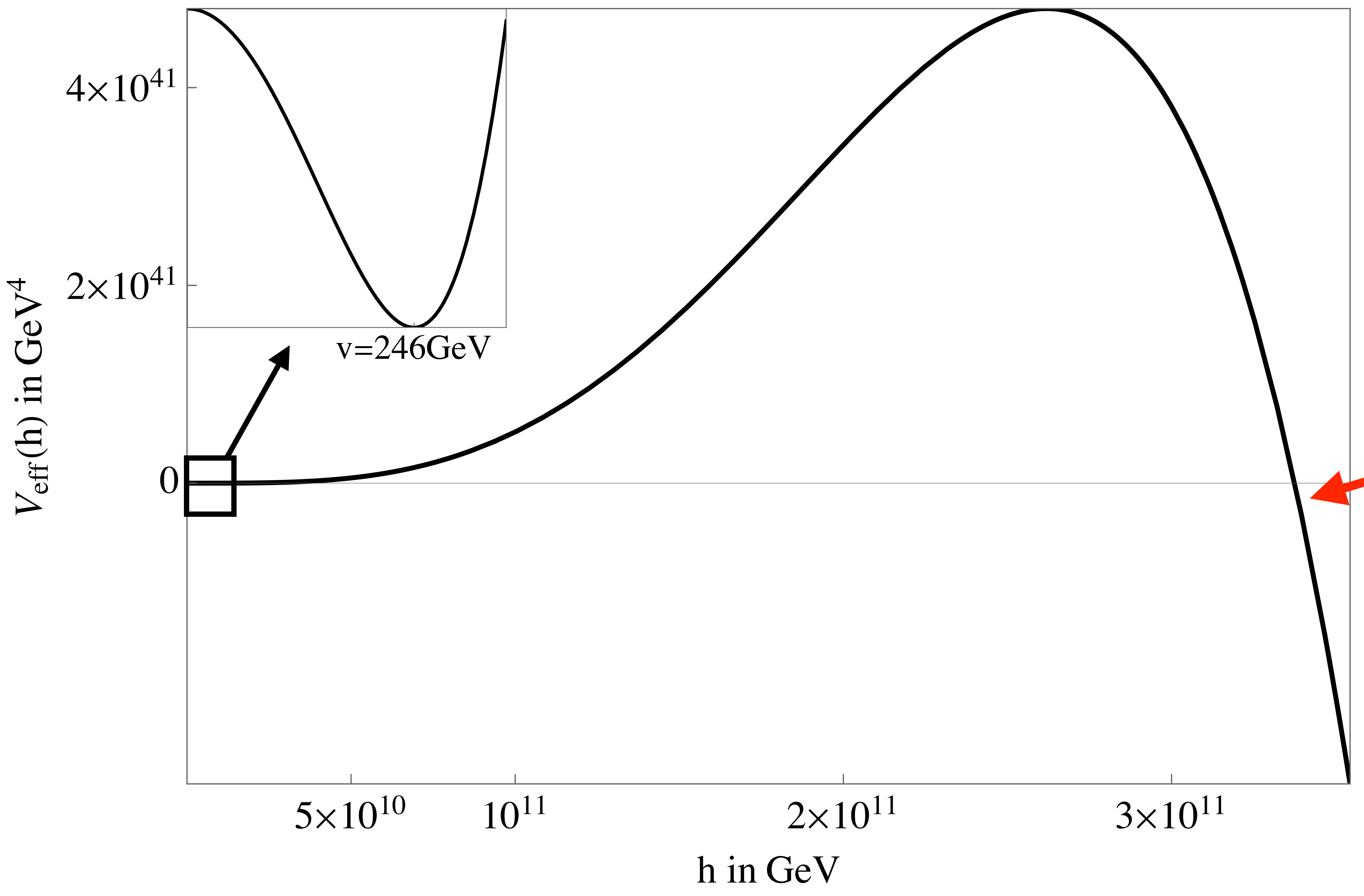
Even ignoring the dynamics, the hypothesis has testable consequences

—> Metastability bounds on the Higgs mass

There is an upper bound to the Higgs mass in the SM

Higgs Potential: $V_{\text{eff}}(H) = -m^2_{\text{eff}}|H|^2 + \lambda_{\text{eff}}|H|^4$

$$\lambda^{\text{LO}}_{\text{eff}} \sim \lambda_H + \frac{1}{4\pi^2}(-y_t^4 + g_2^4 + (g_1^2 + g_2^2)^2)$$



μ_I is the instability scale:

$$\lambda(\mu_I) = 0$$

There is an upper bound to the Higgs mass in the SM

SM Higgs Potential: $$V_{\rm eff}(H) = -\,m_{\rm eff}^2 |H|^2 + \lambda_{\rm eff} |H|^4$$

$$\frac{\partial V_{\rm eff}(H)}{\partial |H|} = |H| \left[-2m_{\rm eff}^2 - \beta_m^2 + |H|^2 \left\{ 4\lambda_{\rm eff} + \beta_\lambda \right\} \right] = 0$$

Staying at leading-log, and expanding around the instability scale μ_I

$$\lambda_{\rm eff} \simeq \lambda_{\rm eff}(\mu_I) + \beta_\lambda \ln \frac{\mu}{\mu_I} \quad \longrightarrow \quad m_{\rm eff}^2 \simeq \mu^2 \left(\ln \frac{\mu}{\mu_I} + \frac{1}{4} \right) \beta_{\lambda|\mu_I}$$

$\lambda_{\rm eff}(\mu_I) = 0$

This equation only has solutions with an IR minimum if

$$m_{\rm eff}^2 \leq m_{\rm crit}^2 \equiv -\,\beta_{\lambda|\mu_I} e^{-3/2} \mu_I^2 = |\beta_{\lambda|\mu_I}| \, e^{-3/2} \mu_I^2$$

$$m_h^2 \leq |\beta_{\lambda|\mu_I}| \, e^{-3/2} \, \mu_I^2$$

D. Buttazzo et al. 1307.3536
Khoury, Steingasser, 2108.09315
Detering, You ,2412.03542

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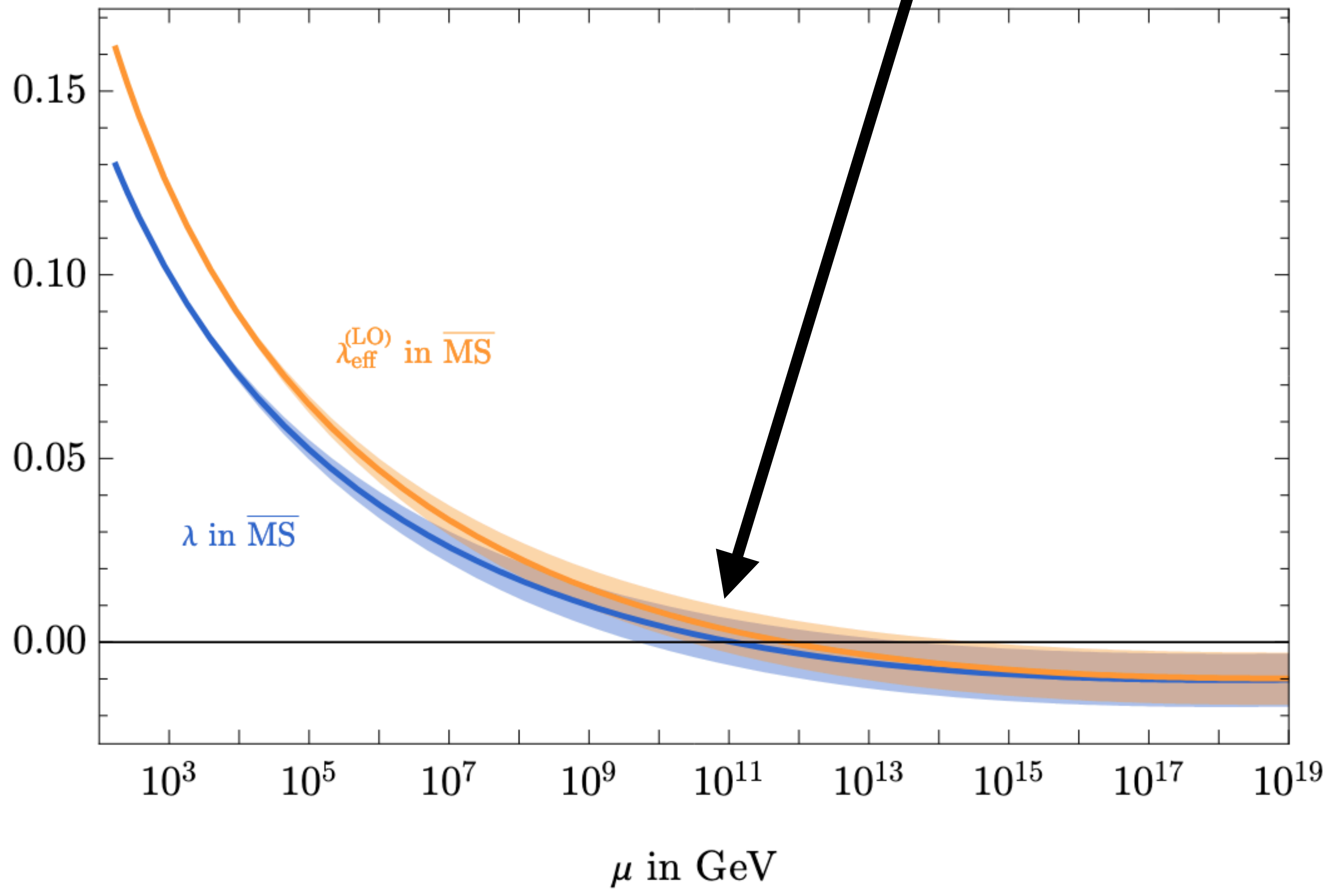
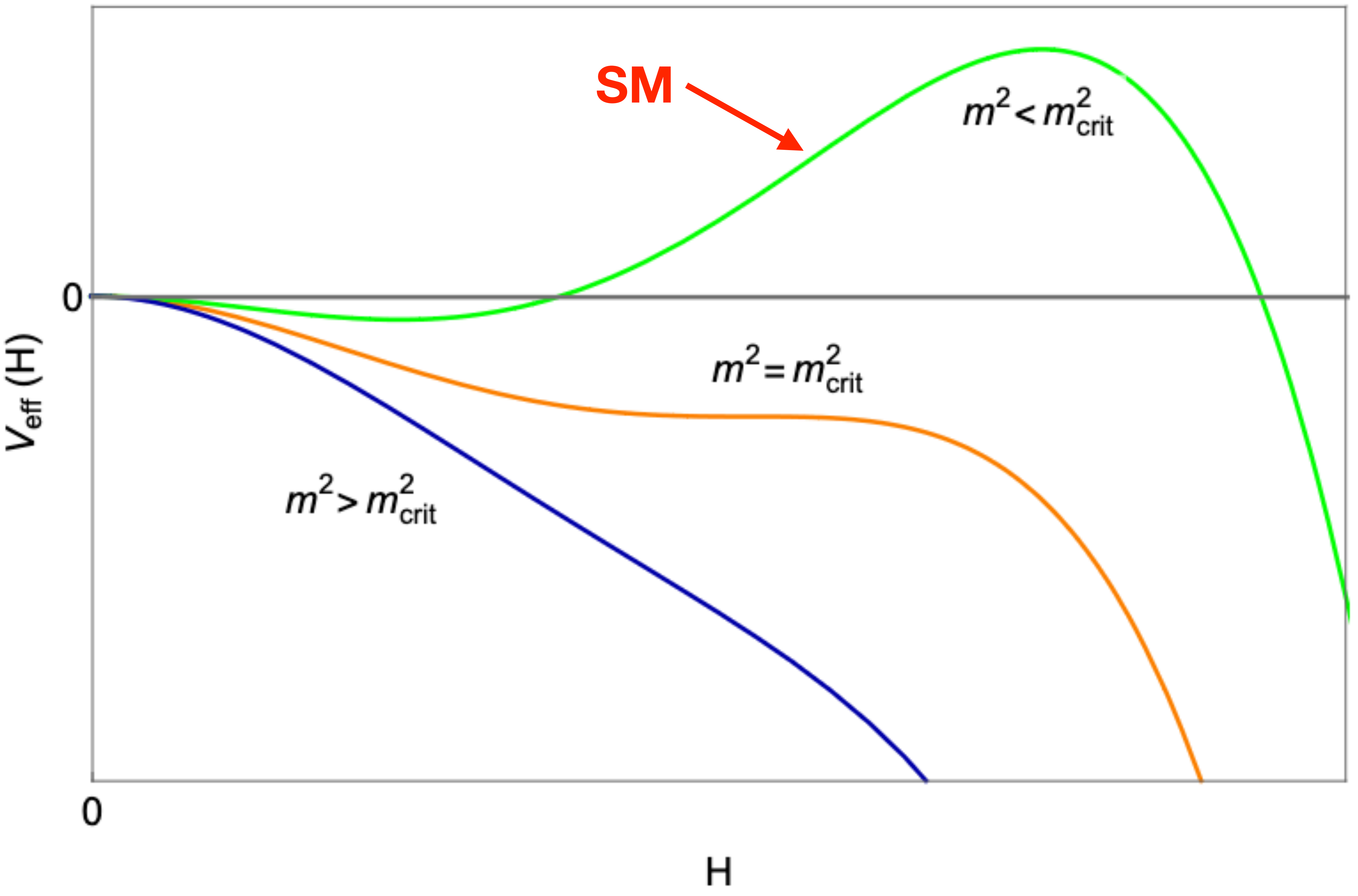
[1307.3536]
(D. Buttazzo et al)

$$m_h^2 \lesssim e^{-3/2} \, |\beta_\lambda| \, \mu_I^2$$





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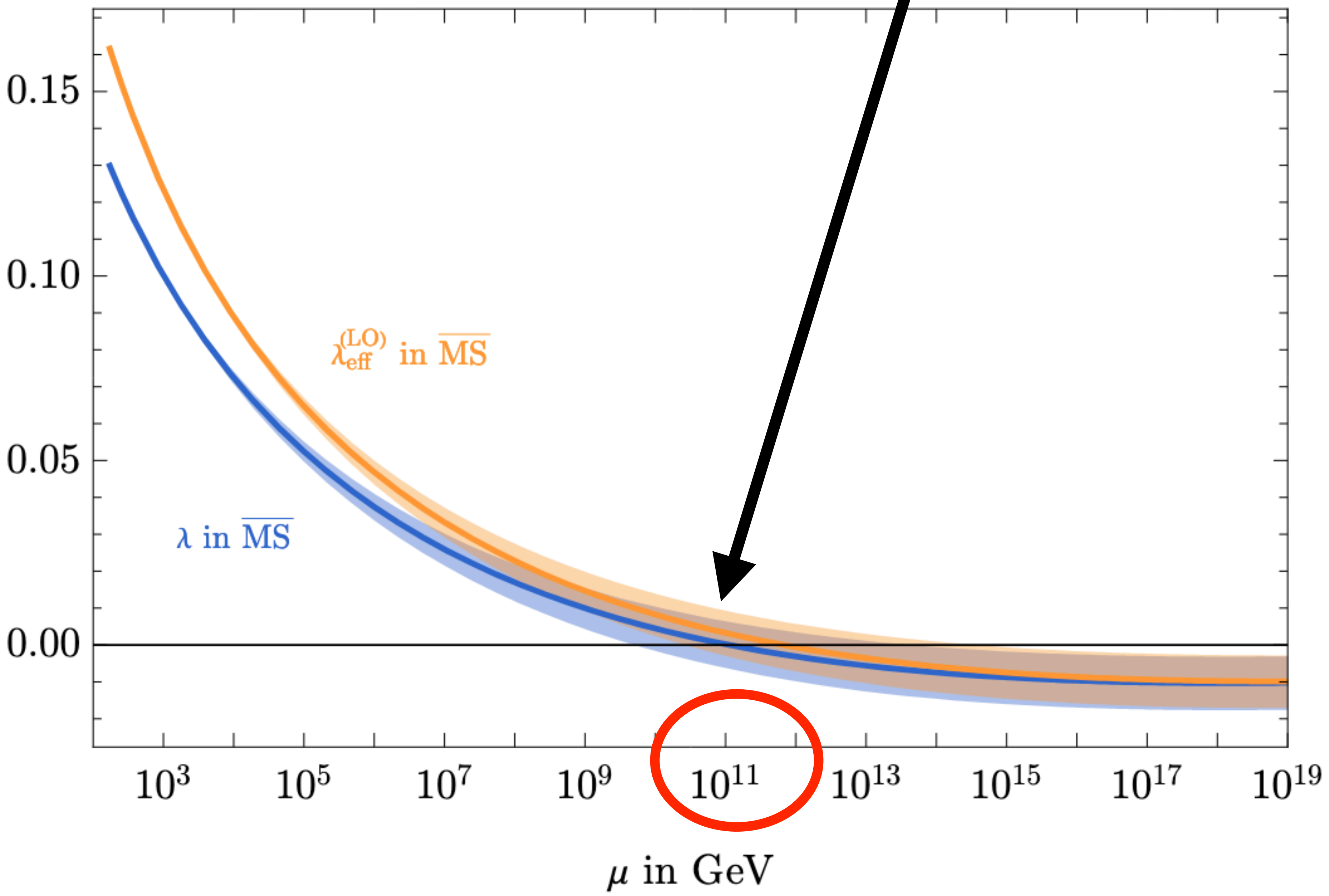

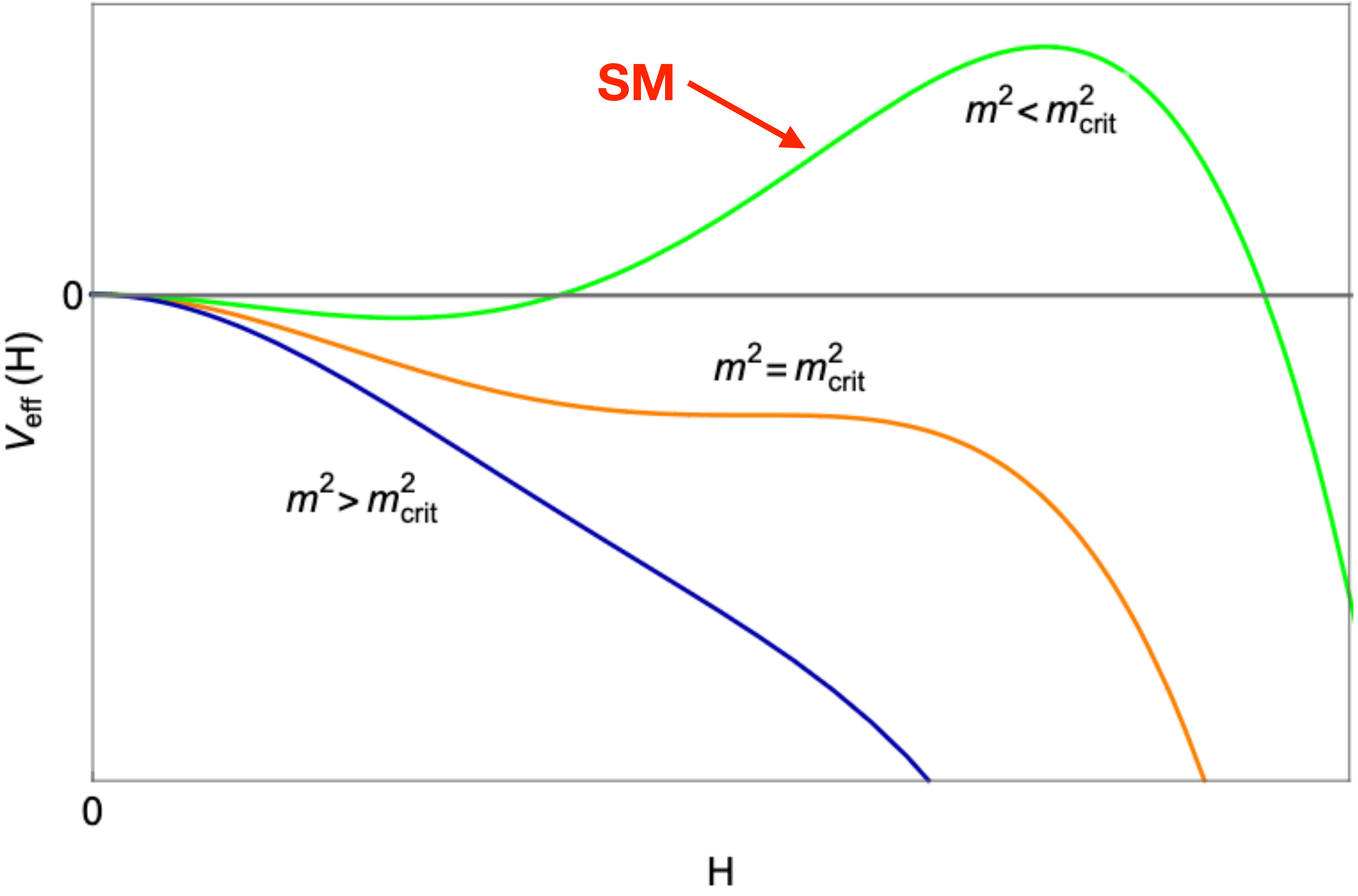

$$\mu_I = 10^{11.8^{+2.7}_{-1.4}}\ {\rm GeV}$$

RG at NNLO, Detering-You 2024

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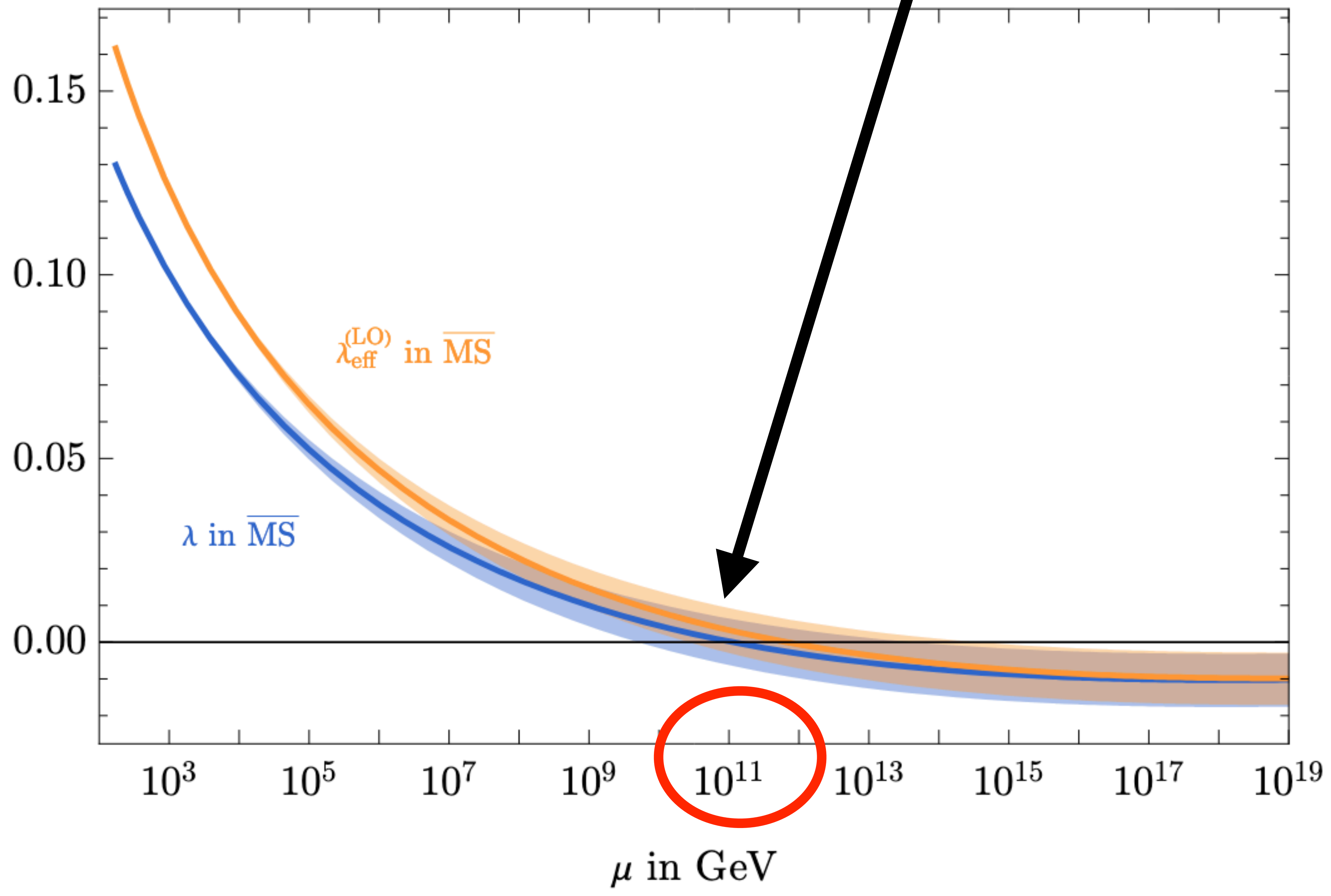
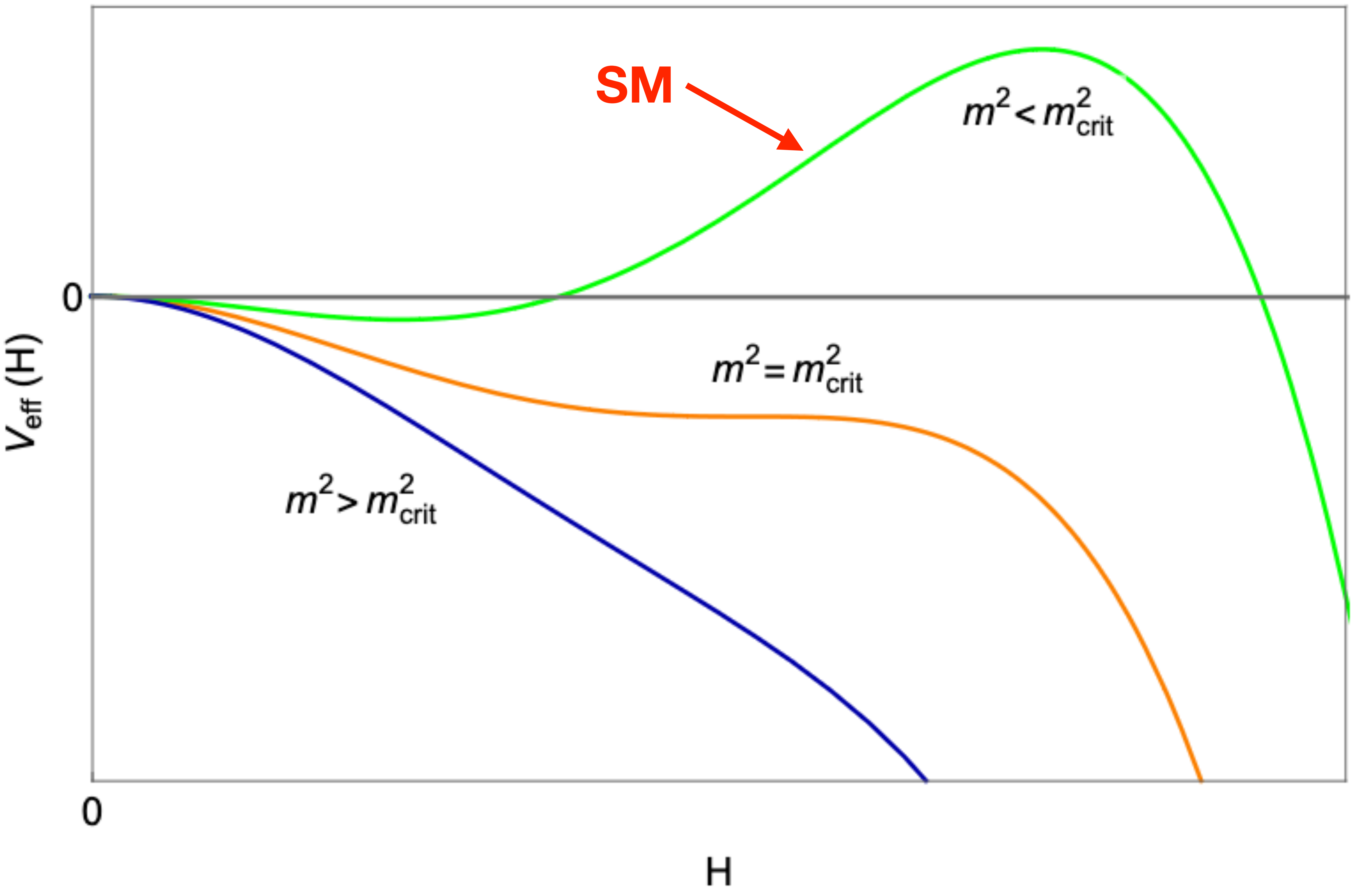
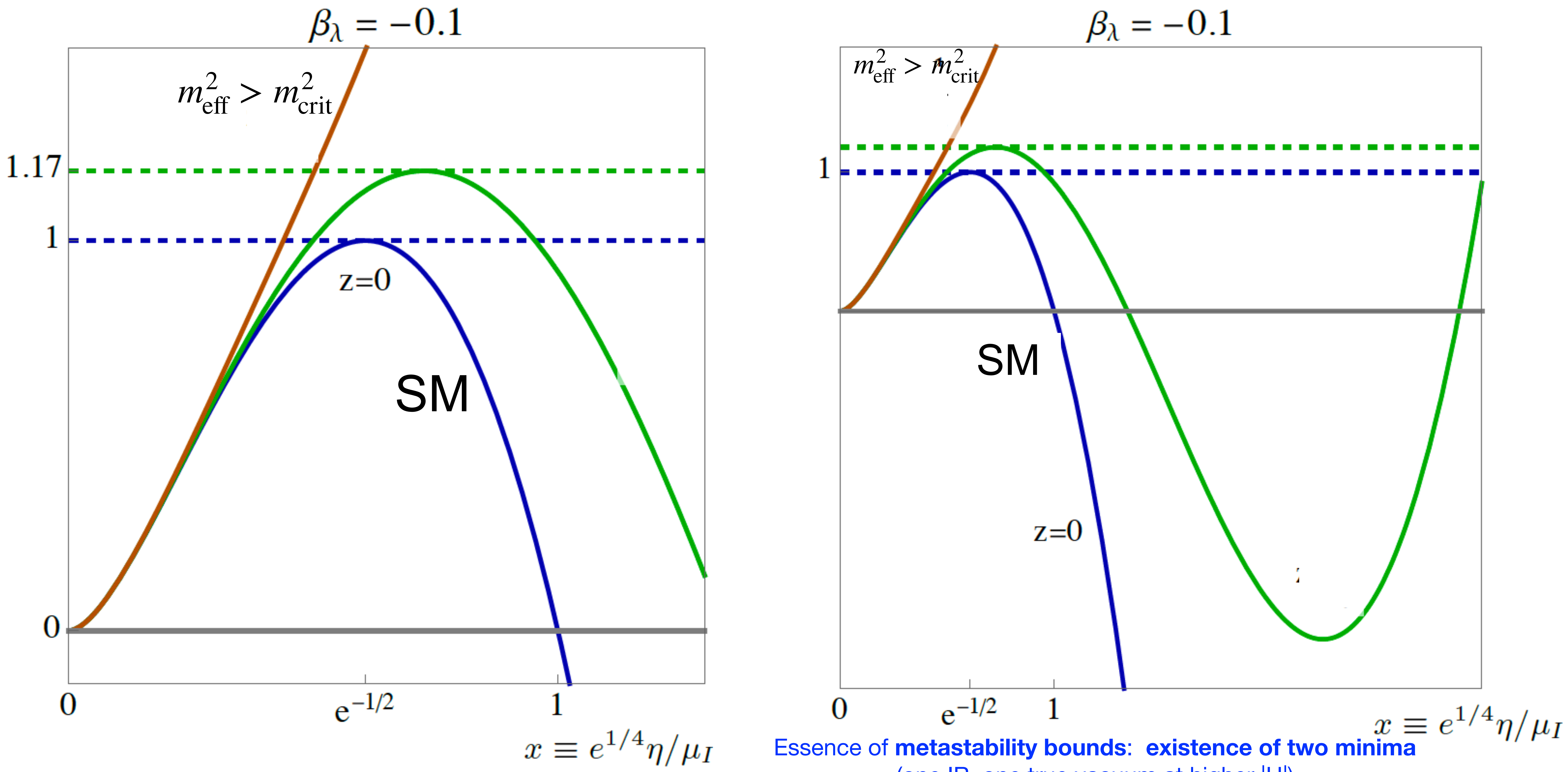




In the SM $m^2_{\rm eff} \sim \left[10^2 \text{ GeV}\right]^2 \ll \left[10^{10} \text{ GeV}\right]^2$

Metastability bound - motivation —> BSM

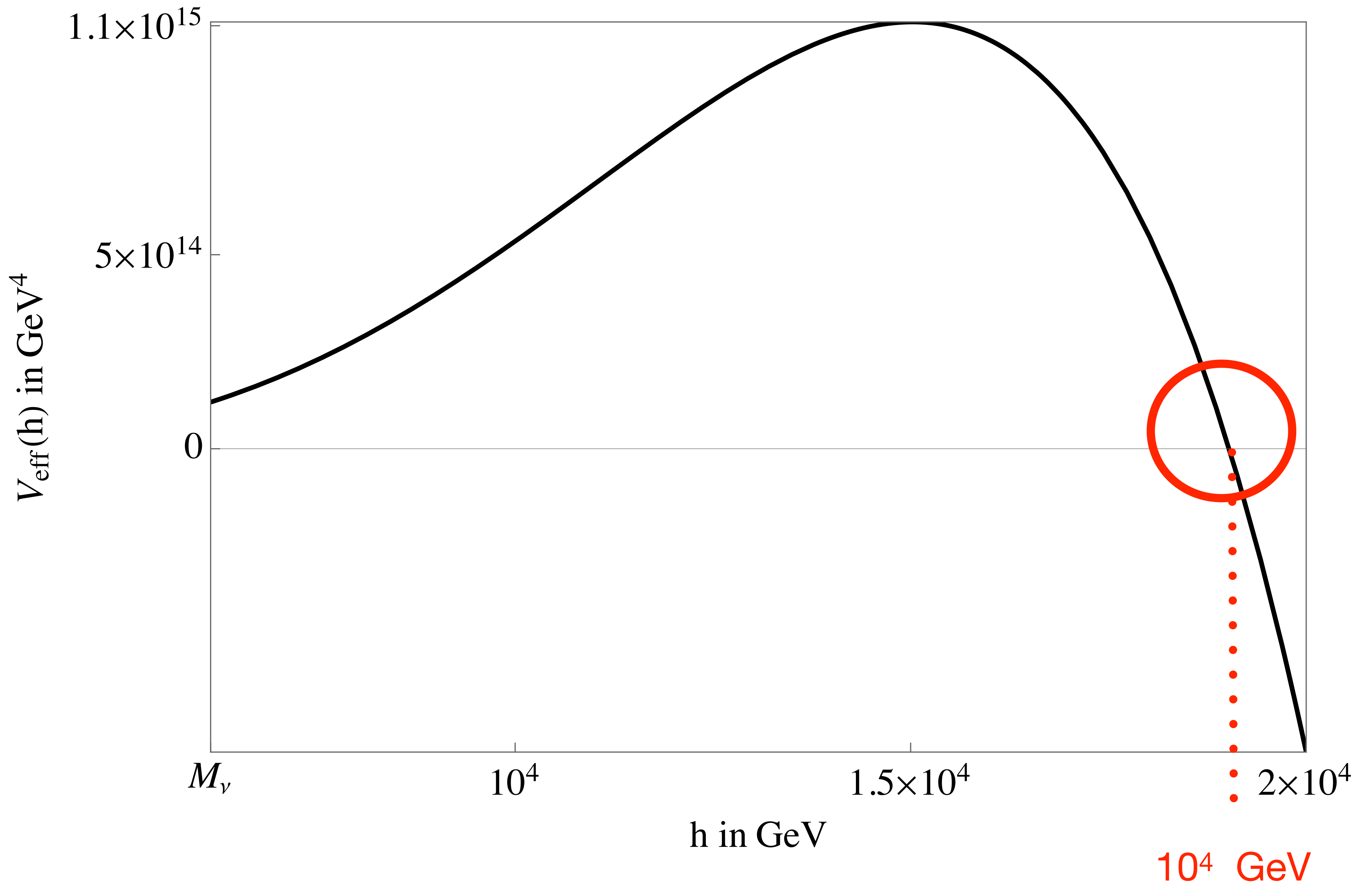


Essence of **metastability bounds**: **existence of two minima**
(one IR, one true vacuum at higher |H|)
and a maximum in-between
—> Higgs mass bound

Can you lower the instability scale? —> BSM

Higgs Potential: $V_{\text{eff}}(H) = -\frac{1}{2}m_{\text{eff}}^2|H|^2 + \frac{1}{4}\lambda_{\text{eff}}|H|^4 + \ldots$

$m_h^2 \lesssim |\beta_\lambda|\mu_I^2 + \ldots$



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$$m_h^2 \lesssim |\beta_\lambda(\mu_I)|\mu_I^2 \ll \Lambda_{UV}^2$$

lowered by BSM physics?

VLFs
[2105.08617]
[2502.07876]
[2408.10297]

Sparticles
[2502.07876]

ALPs
[2412.03542]
[2506.06426]

"UV-complete model"

V. Enguita, B. Gavela, T. Steingasser

Majoron
[2503.03825]

Can you lower the instability scale? —> BSM

Victor Enguita

Thomas Steingasser

“UV-complete model”

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Majoron

[2503.03825]

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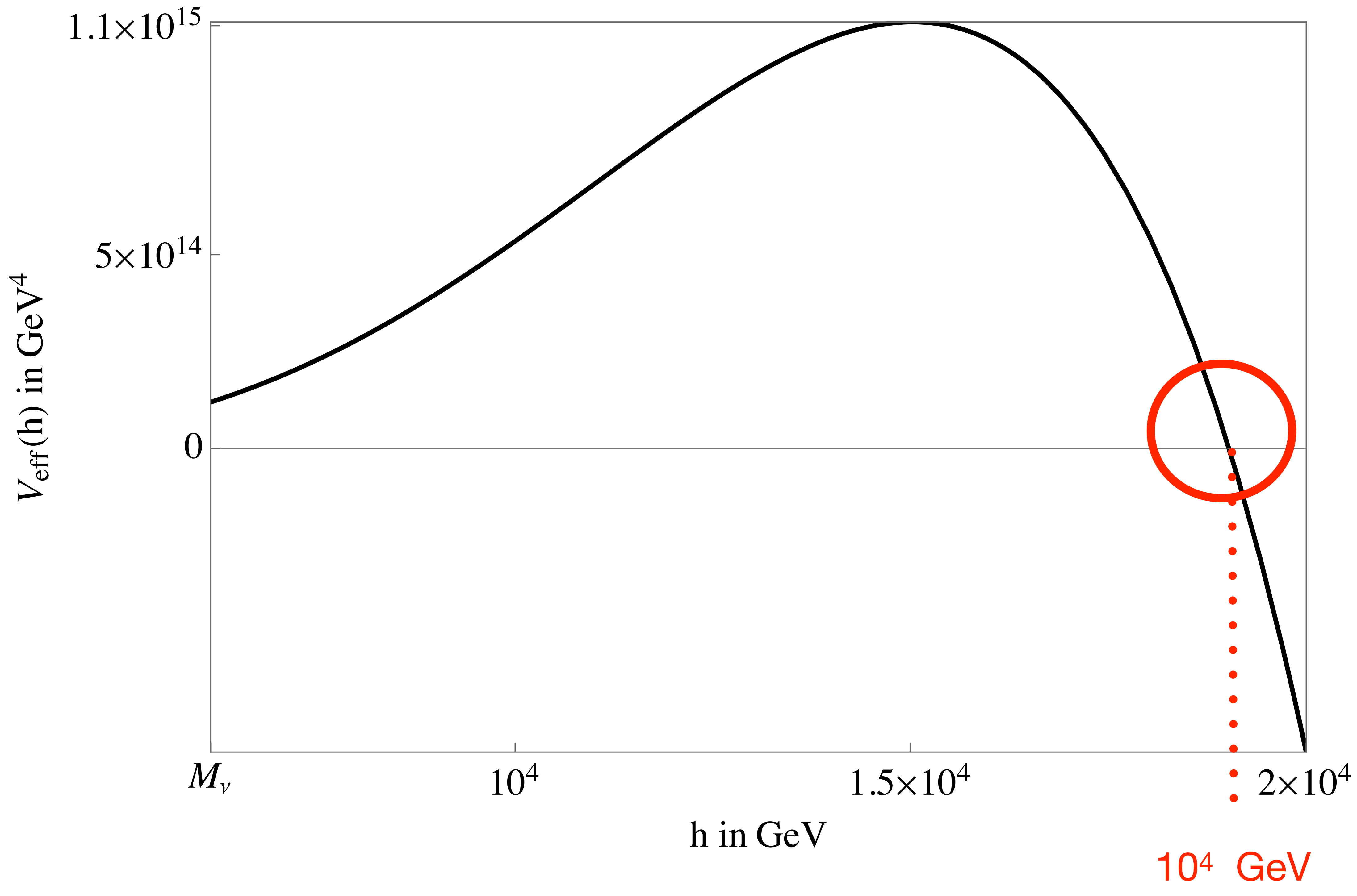
Higgs Potential: $V_{\text{eff}}(H) = -\frac{1}{2}m_{\text{eff}}^2|H|^2 + \frac{1}{4}\lambda_{\text{eff}}|H|^4$ +Type-I seesaw

Very easy: e.g. heavy neutrinos



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$$\mathcal{L}_{N_R} = \overline{N_R}\partial\!\!\!/ N_R - \bar{\ell}_L\,\mathbf{Y}_\nu\,\tilde{H}\,N_R - \frac{1}{2}\overline{N_R^c}\,\mathbf{Y}_R\,M_N\,N_R + \mathrm{h.c.}$$

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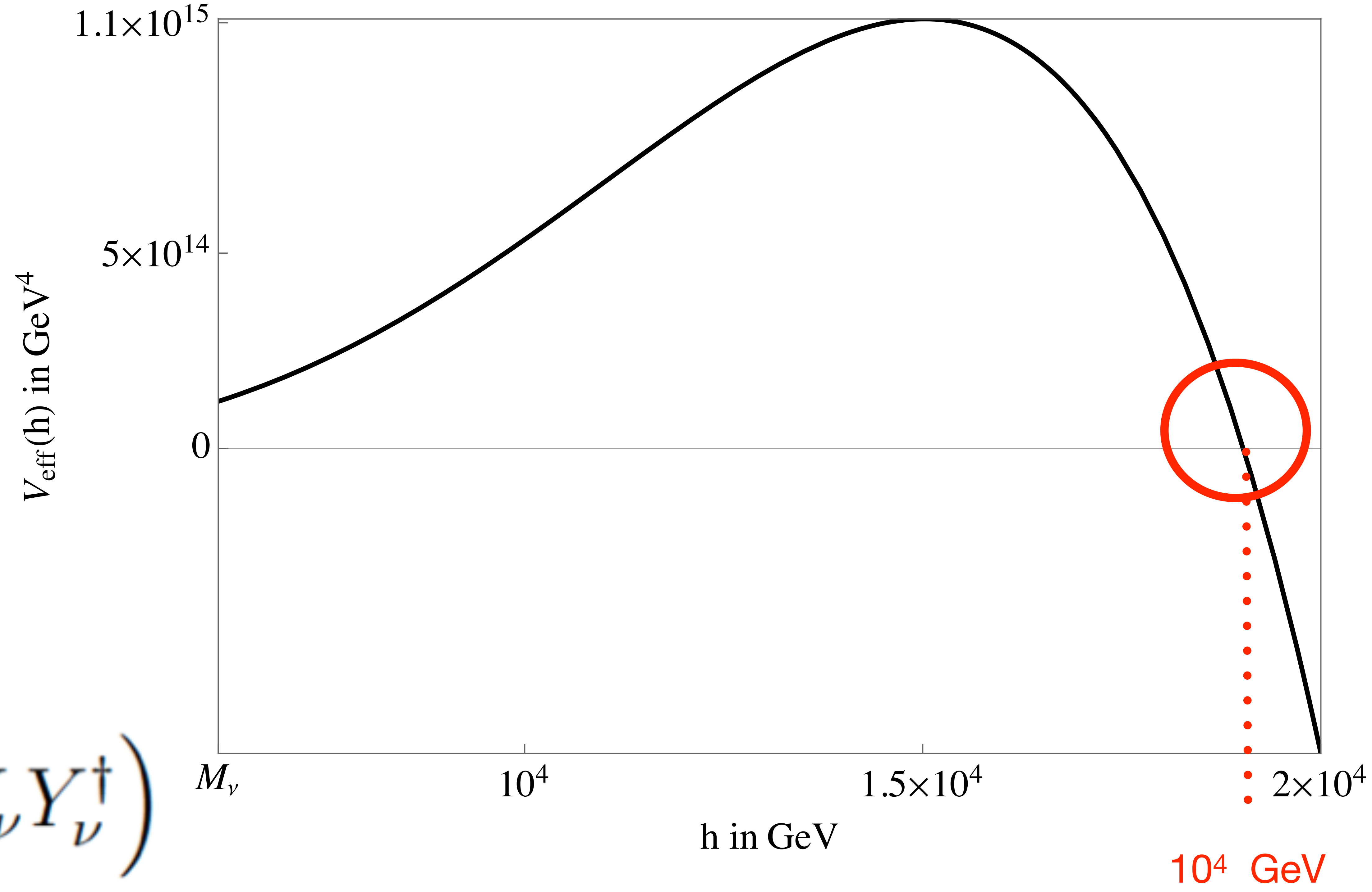

$$\delta\,\beta_{\lambda_H}^{(1)} = -4\lambda_H \mathrm{Tr}\left(Y_\nu Y_\nu^\dagger\right) - 2\mathrm{Tr}\left(Y_\nu Y_\nu^\dagger Y_\nu Y_\nu^\dagger\right)$$

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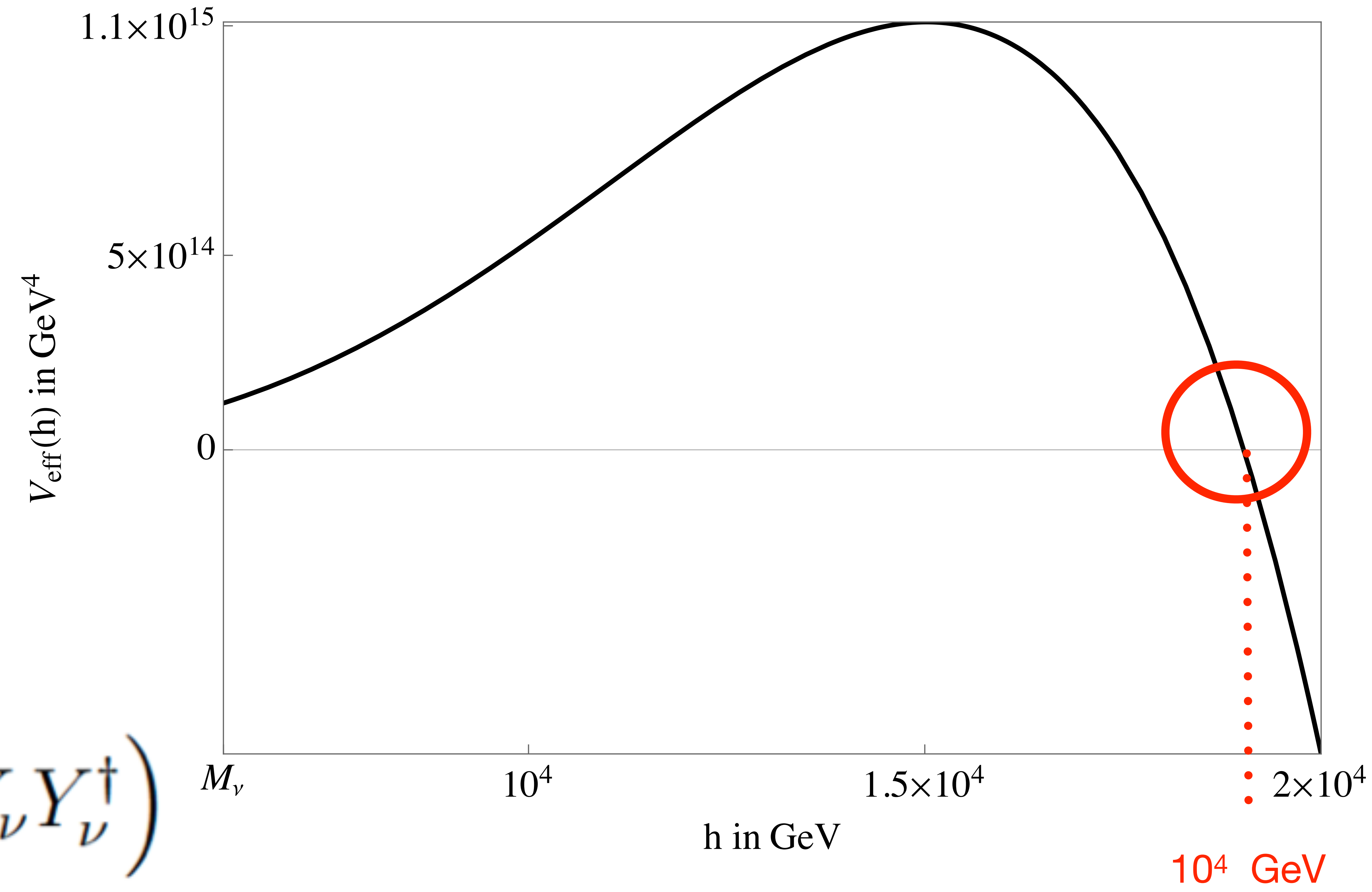

low-scale Type-I seesaw model

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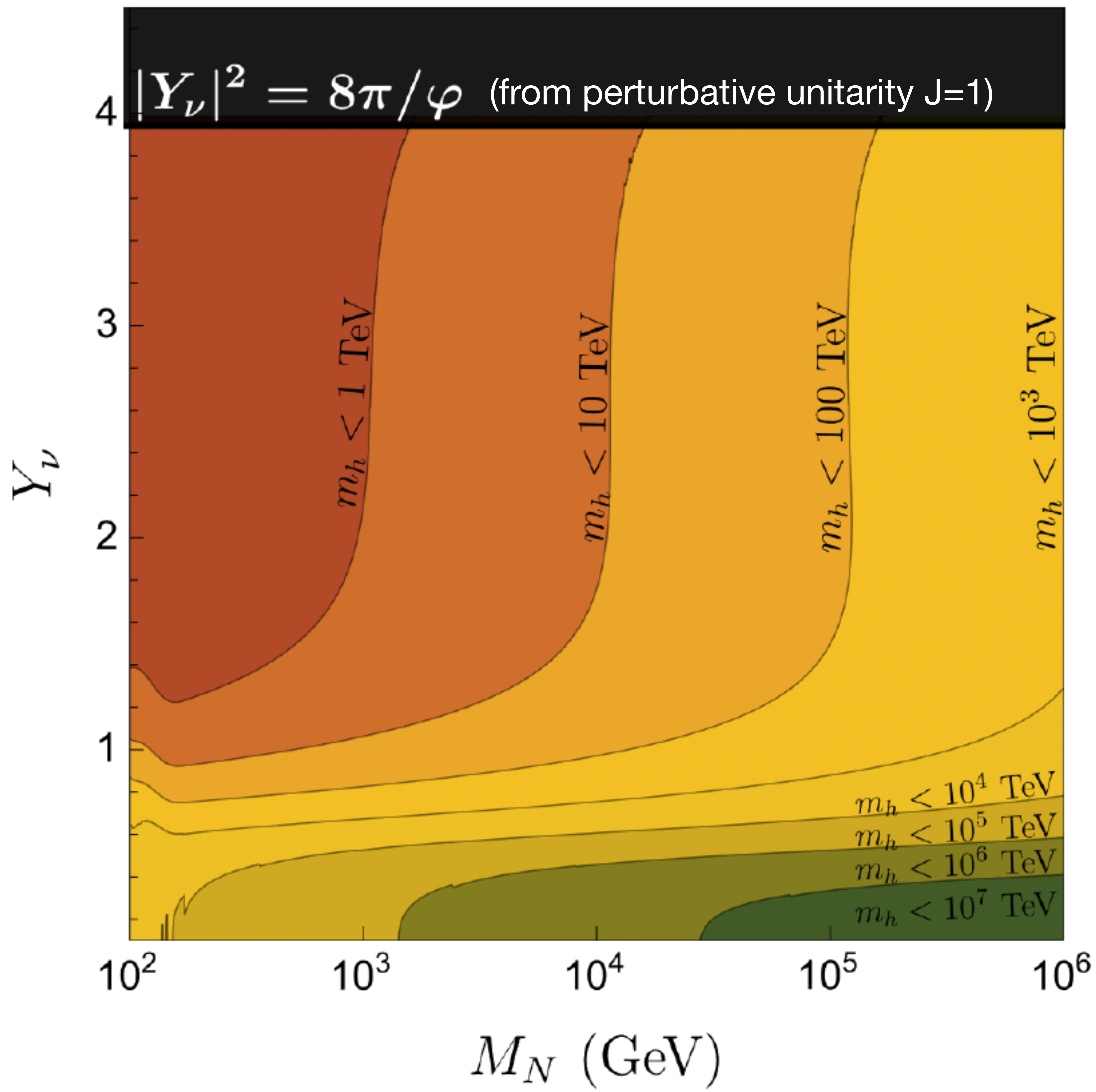
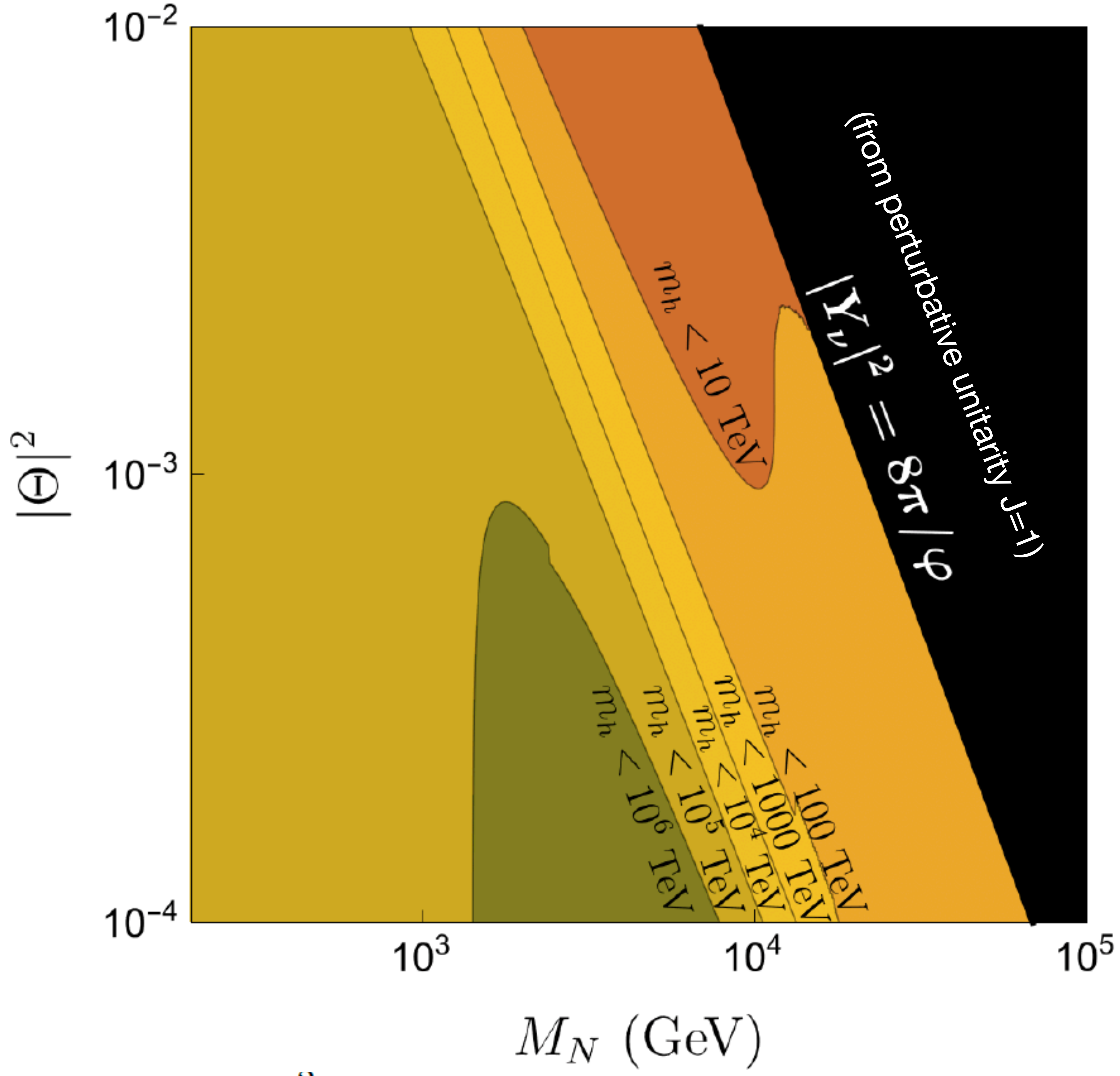
Metastability bound @FCC - Heavy sterile neutrinos

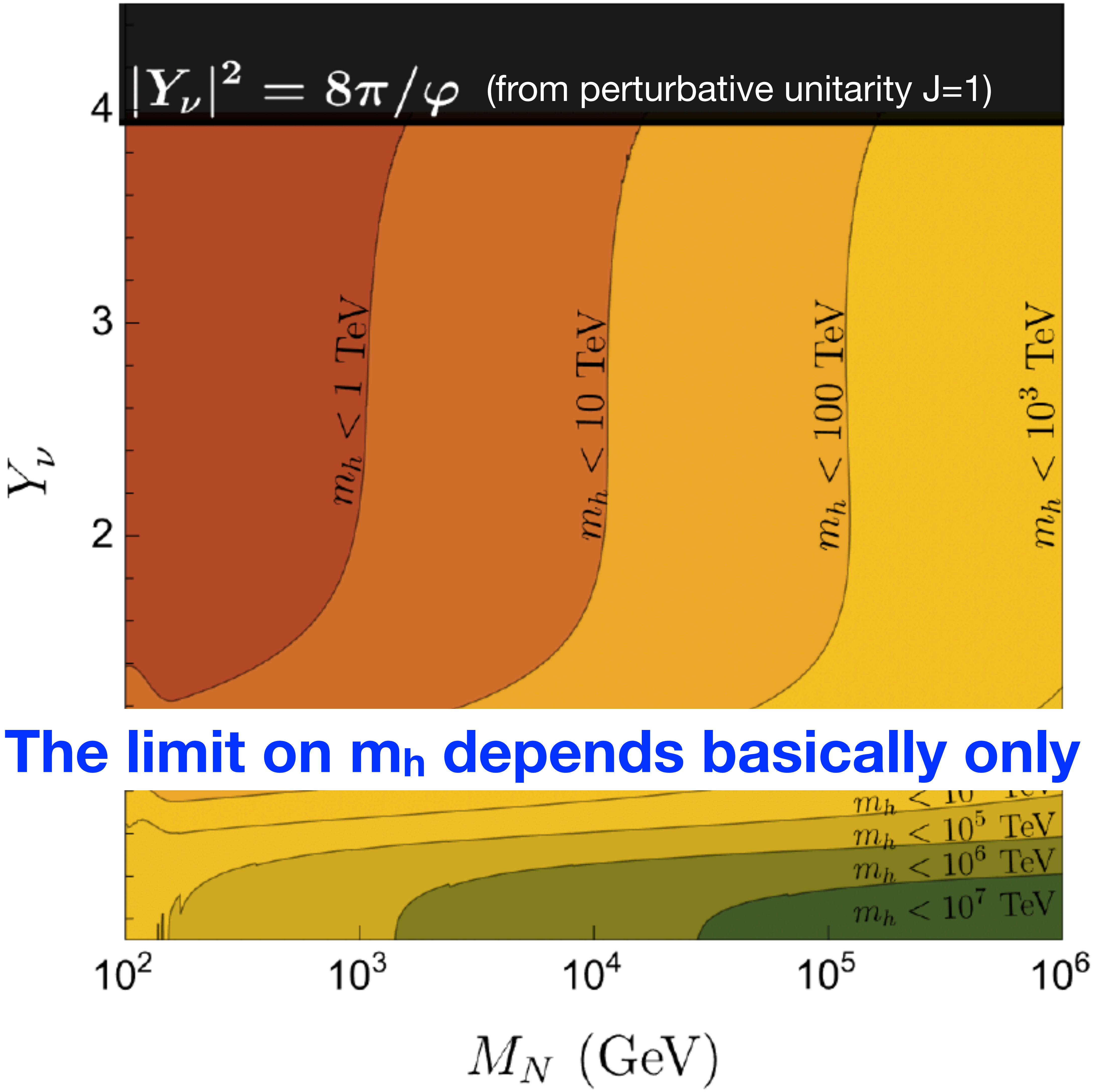


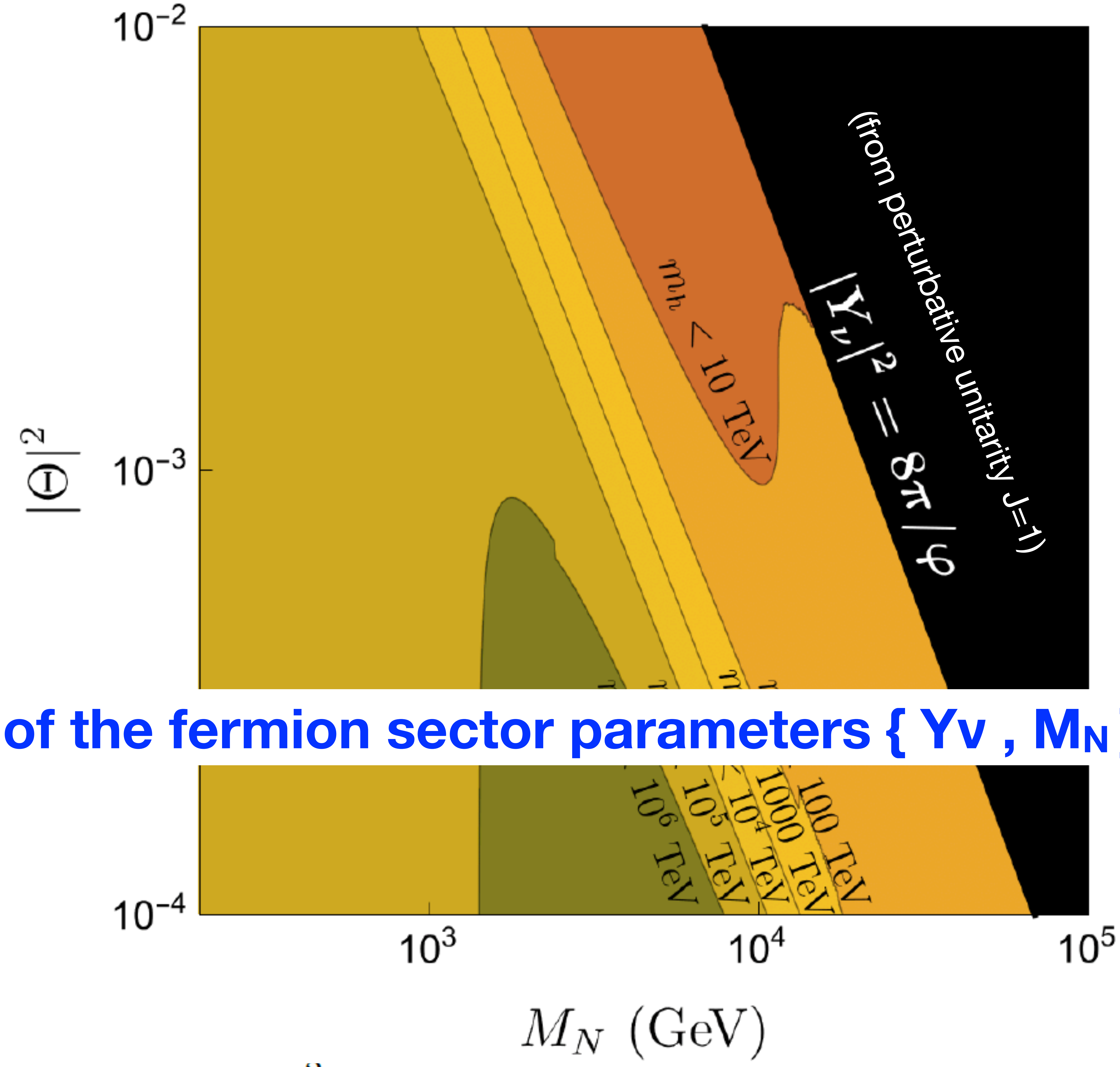


[2503.03825]
(V. Enguita, B. Gavela, T, Steingasser)

$$\Theta^a_\nu = \frac{Y^{*a2}_\nu}{\sqrt{2}} \frac{v}{M_N} \quad \text{and} \quad |\Theta_\nu|^2 \equiv \sum_a |\Theta^a|^2$$

Metastability bound @FCC - Heavy sterile neutrinos





The limit on m_h depends basically only of the fermion sector parameters { Y_ν , M_N }

$$|Y_\nu|^2 = 8\pi/\varphi$$ (from perturbative unitarity J=1)

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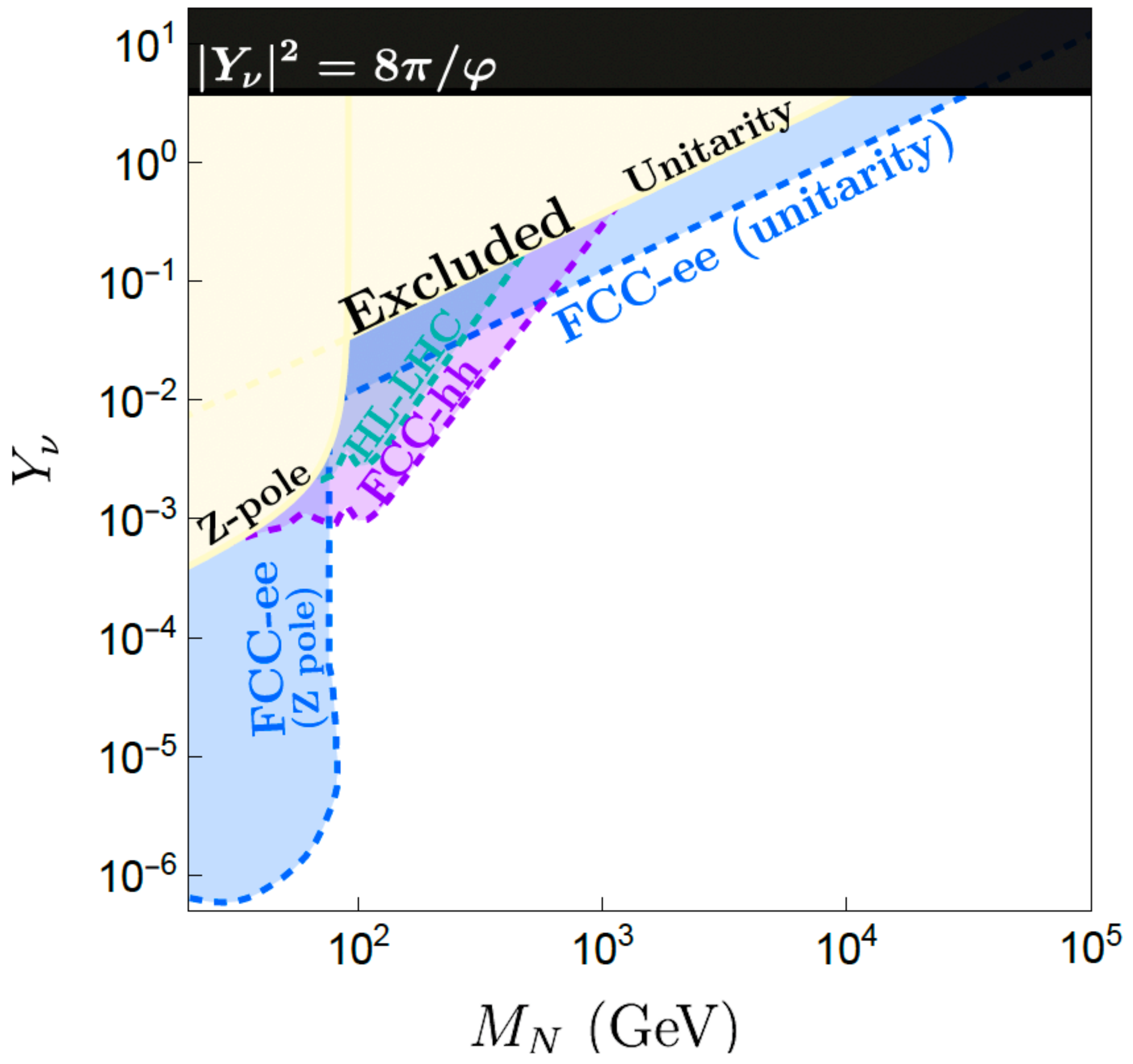

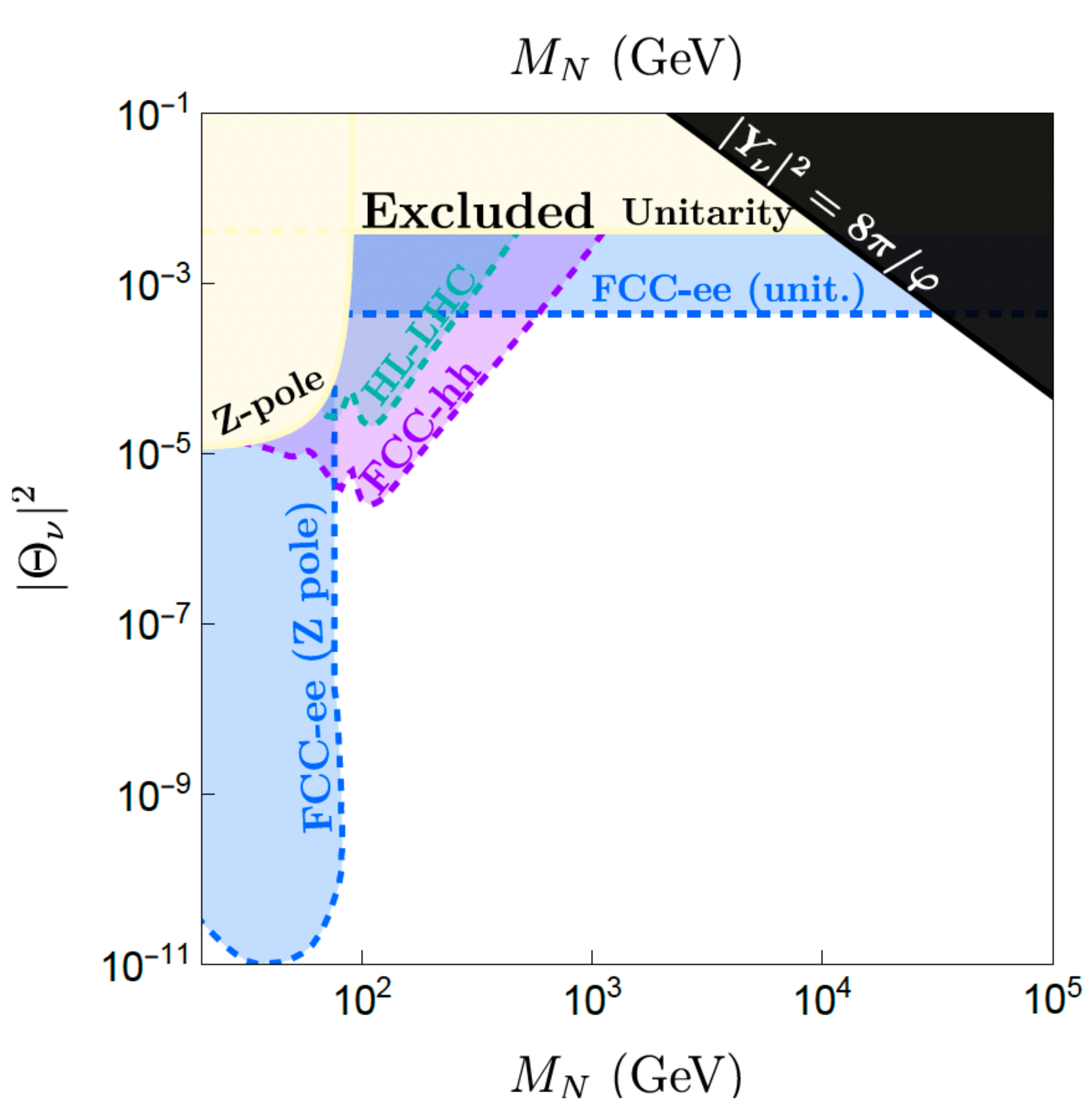

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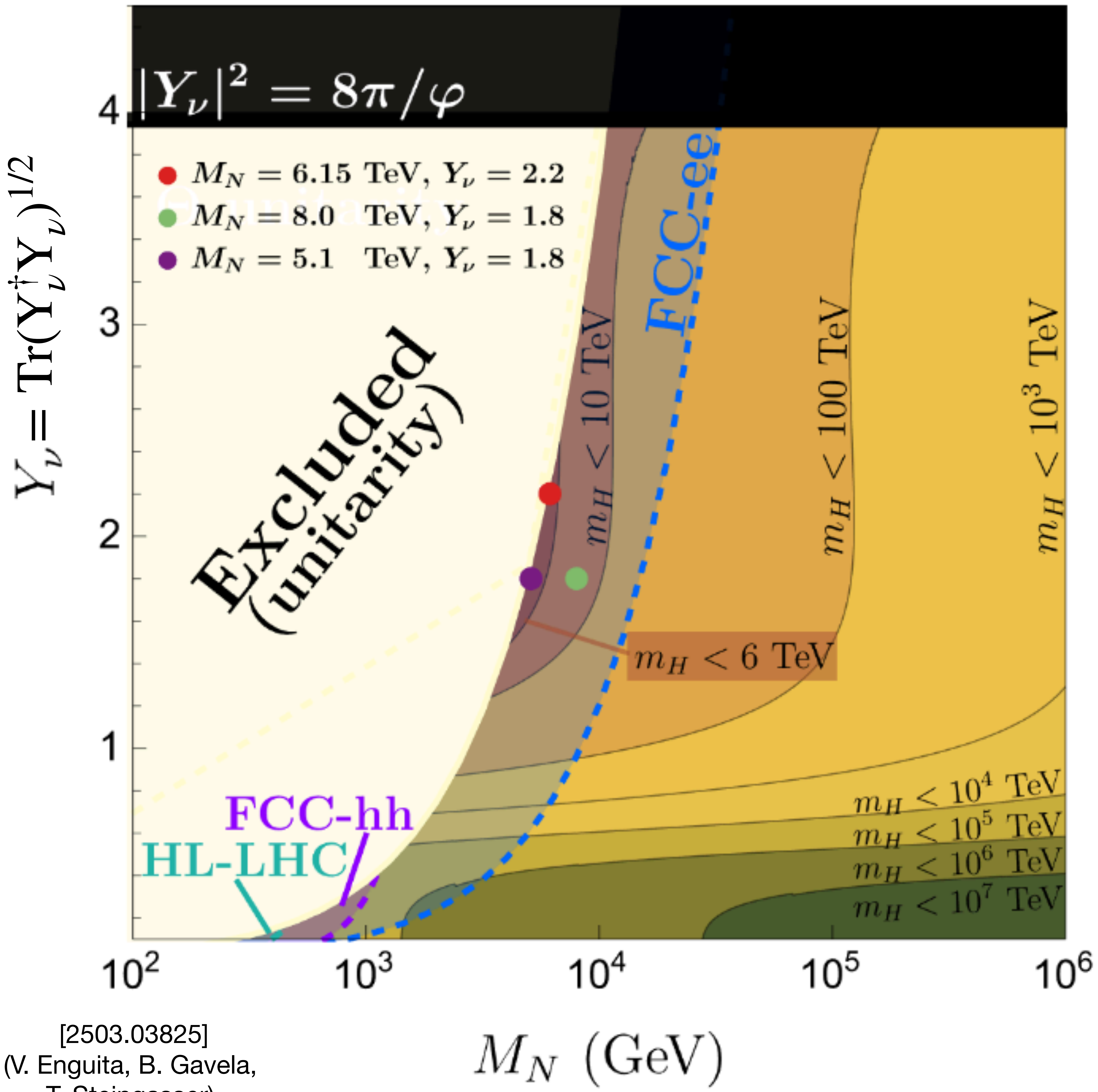
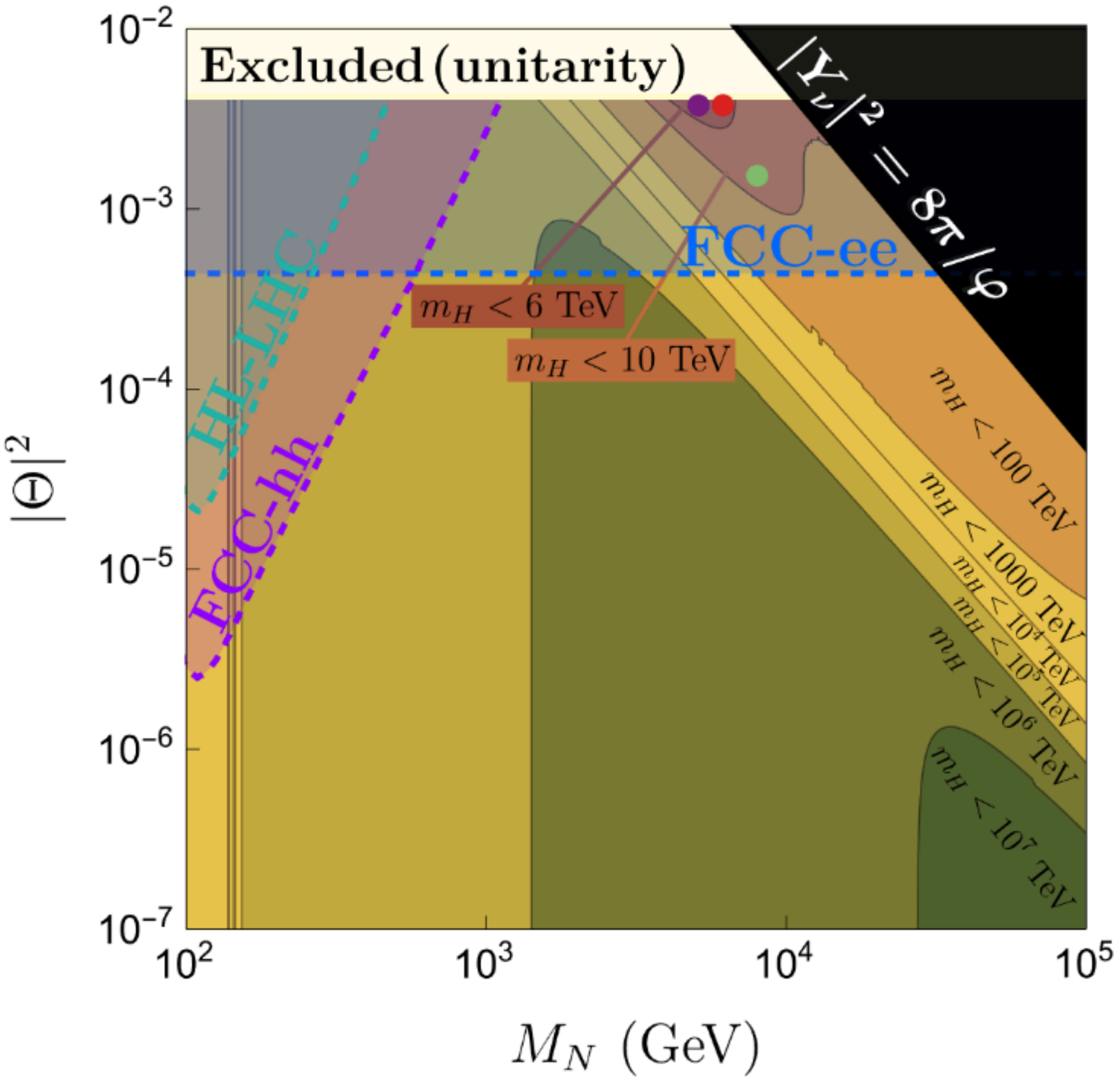
Metastability bound @FCC-ee Heavy sterile neutrinos





[2503.03825]
(V. Enguita, B. Gavela, T. Steingasser)

Metastability bounds - BSM features for fermion path

General: Smaller μ_I → Shorter lifetime

$\mu_I \sim \mathcal{O}(\mathrm{TeV})$ → lifetime < age of the universe

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Note that, in general:

* Fermions destabilise (Casas et al. 2000)

* Scalars stabilise (Elias-Miro et al. 2012)

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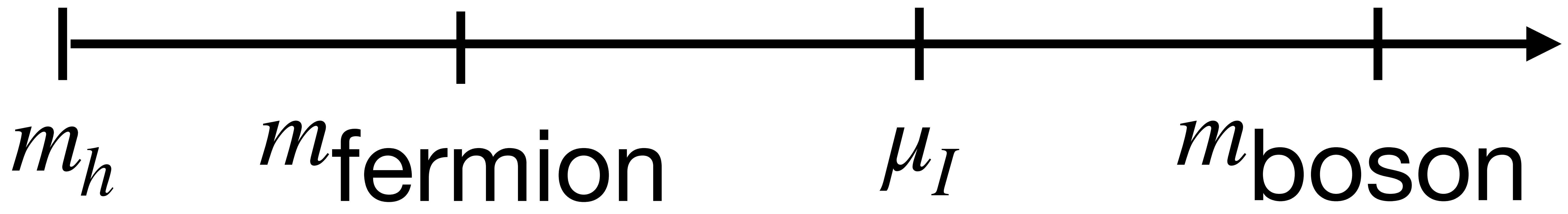
→ bosons to partially stabilize

→ 

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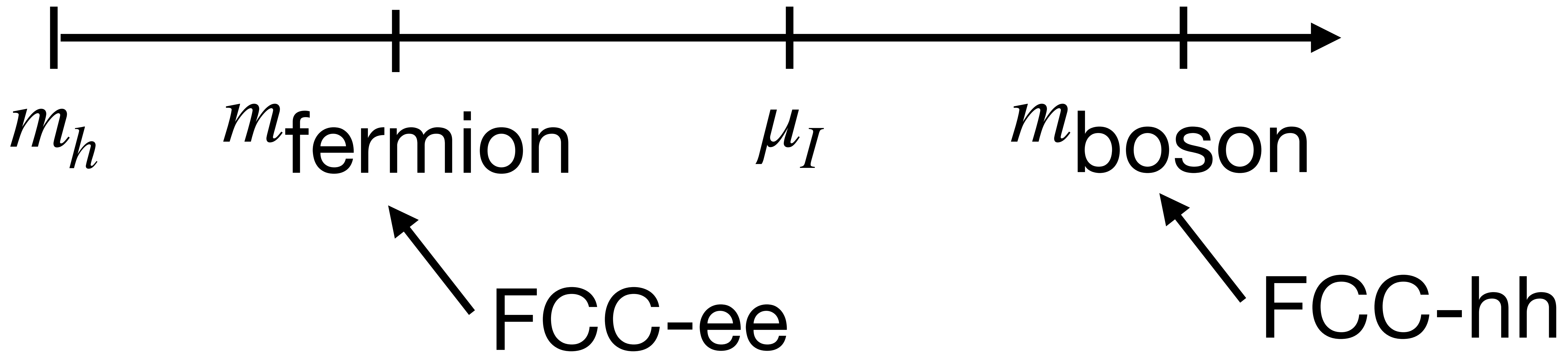


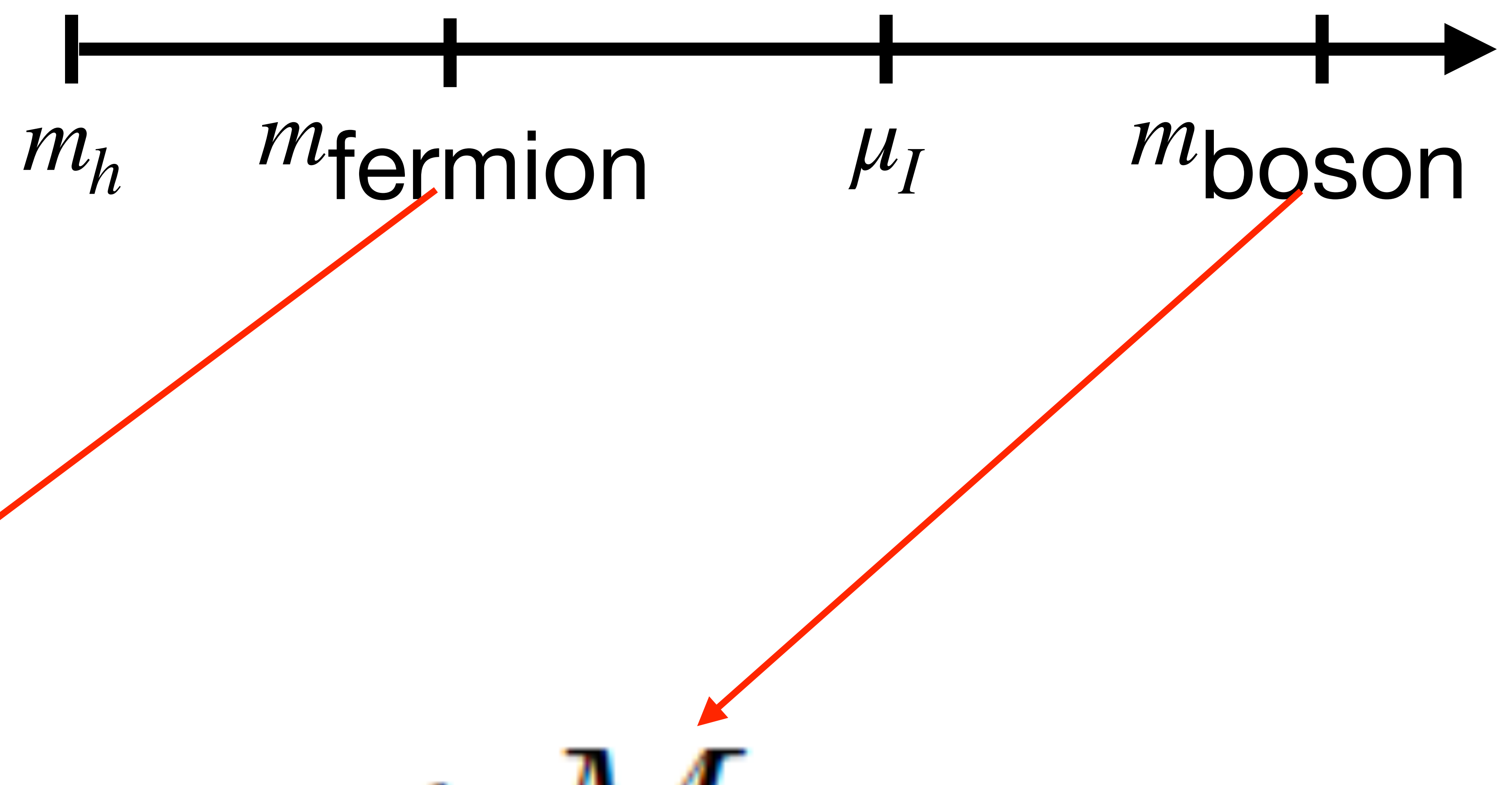
We contend that

[2503.03825]
(V. Enguita, B. Gavela, T. Steingasser)

Metastability bounds - BSM features for fermion path

Important subtleties:

- specific ordering of scales?
- lifetime of vacuum?
- why is running necessary?



$$m_N < \mu_I < M_s$$

Heavy sterile
neutrino masses

Heavy radial
Scalar mass

The Majoron model (Chikashige, R. N. Mohapatra, and R. D. Peccei) naturally implements this, with all scales close

[2503.03825]
(V. Enguita, B. Gavela, T, Steingasser)

Metastability bounds - Majoron model

$S \equiv |S| e^{i\frac{J}{f}}$

pGB: Majoron J

Majoron model: [2503.03825]
(V. Enguita, B. Gavela, T. Steingasser)

m_h — m_{fermion} — μ_I — m_{boson}

$$V = -m_H^2 |H|^2 + \lambda_H |H|^4 - m_S^2 |S|^2 + \lambda_S |S|^4 + \kappa |H|^2 |S|^2$$

$\rightarrow M_S = \sqrt{\lambda_S} \langle S \rangle$

Metastability bounds - Majoron model

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➡ Majoron model: [2503.03825]
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coupling to RHNs

➡ $M_S = \sqrt{\lambda_S} \langle S \rangle$

➡ $M_N \sim Y_R \langle S \rangle$

Metastability bounds - Majoron model

$S \equiv |S| e^{i\frac{J}{f}}$ — pGB: Majoron J

O #(TeV)

m_h — m_{fermion} — μ_I — m_{boson}

→ Majoron model: [2503.03825]
(V. Enguita, B. Gavela, T. Steingasser)

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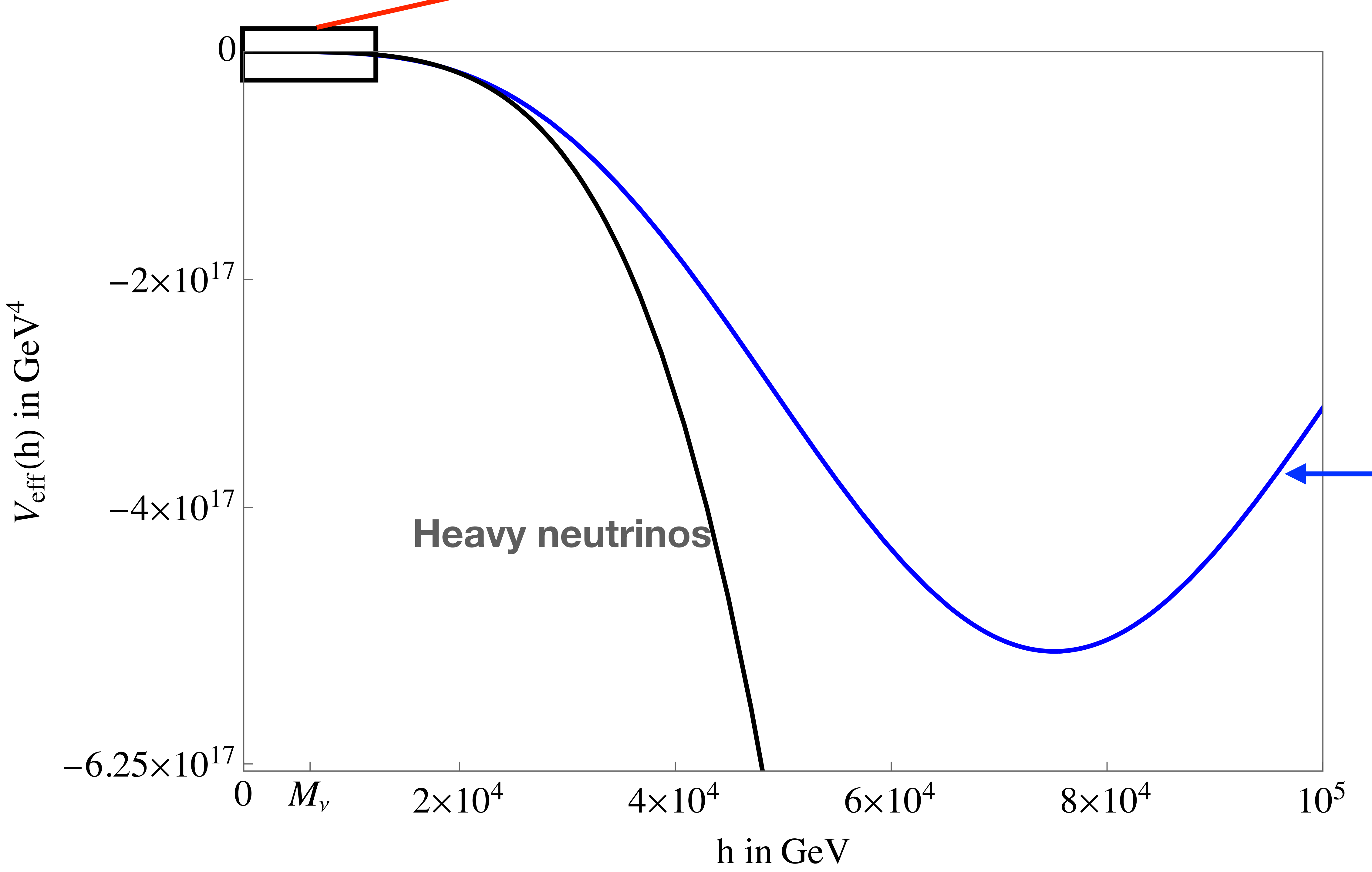
coupling to RHNs

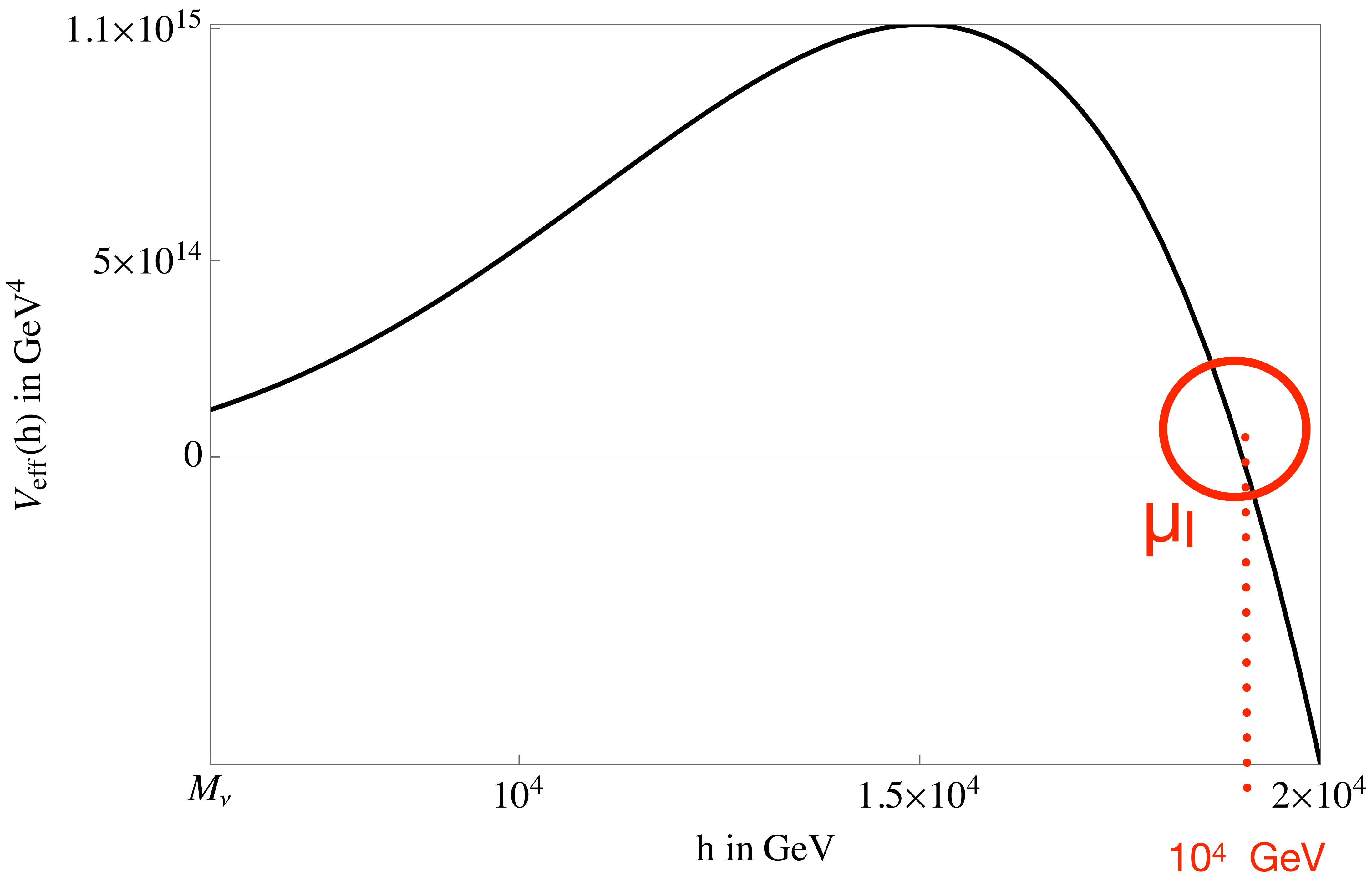
→ $M_S = \sqrt{\lambda_S}\langle S\rangle$

→ $M_N \sim Y_R\langle S\rangle$

Metastability bounds - BSM features

$$\beta_\lambda \to \beta_\lambda + \frac{1}{(4\pi)^2}\left(4\lambda|Y_\nu|^2 - 2|Y_\nu|^4\right) + \ldots$$





the heavy scalar S stabilises the vacuum

(Black line = only heavy neutrinos included in running)

Metastability bounds - BSM features

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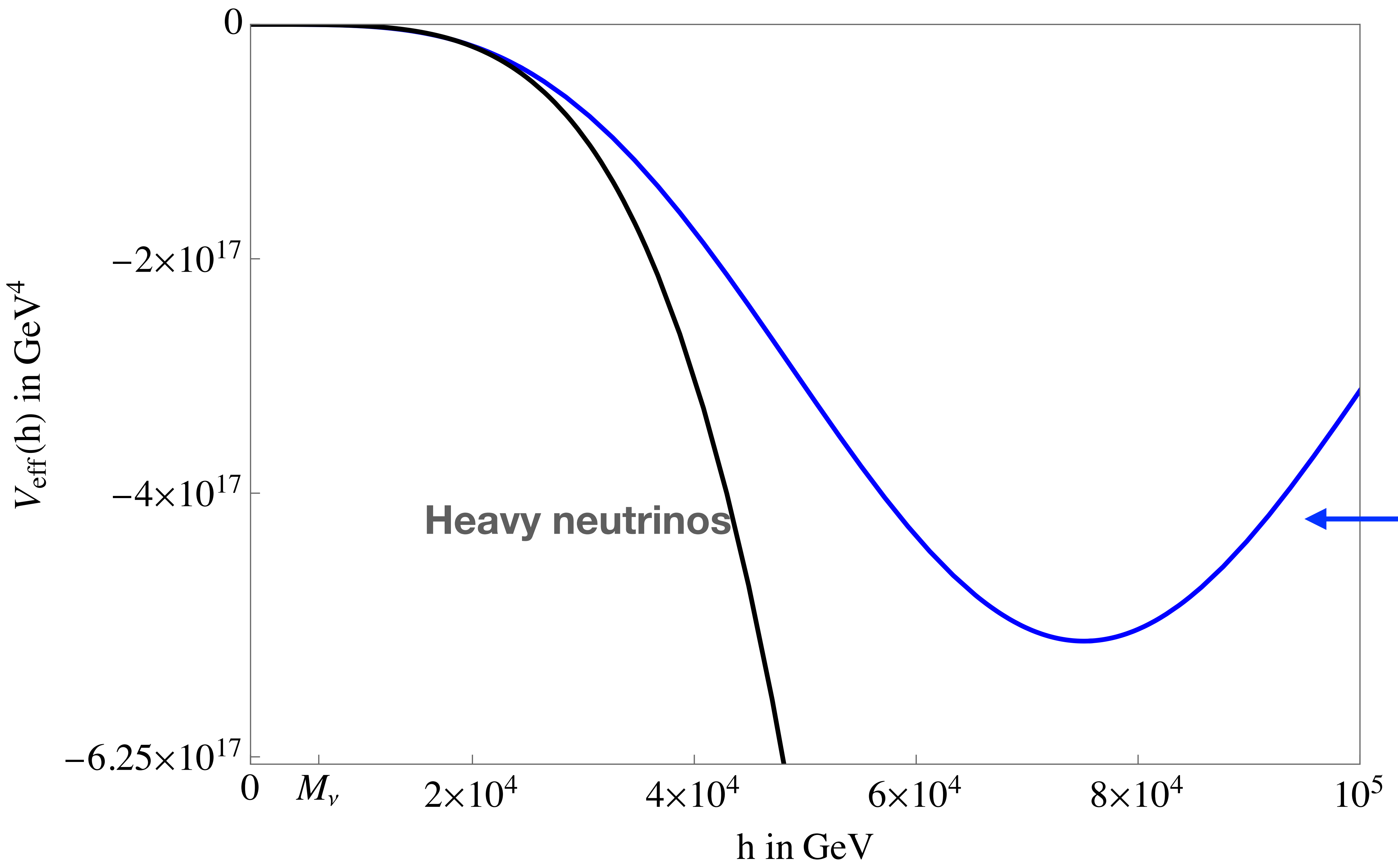

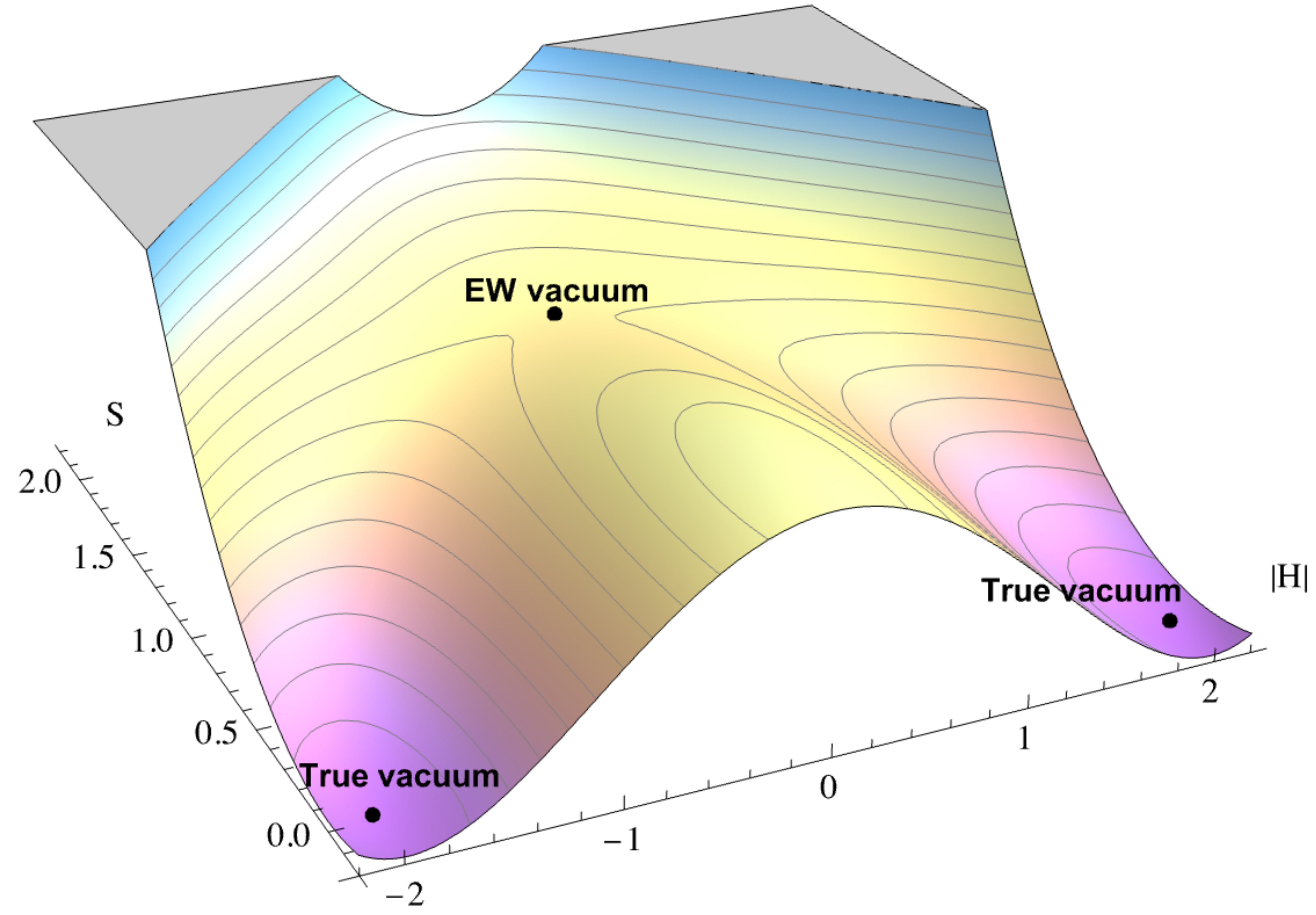

the heavy scalar S stabilises the vacuum

Metastability bounds - BSM features

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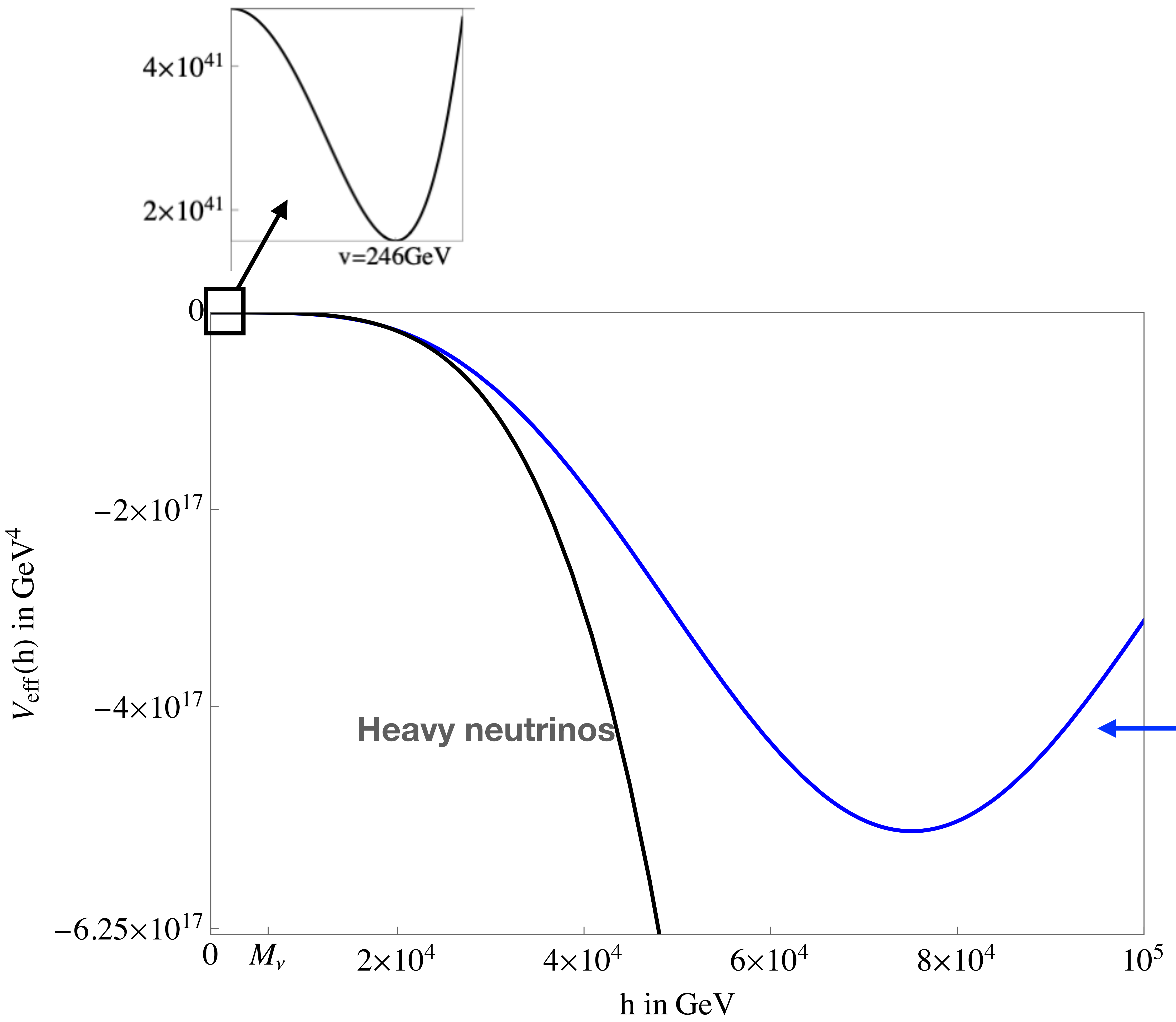

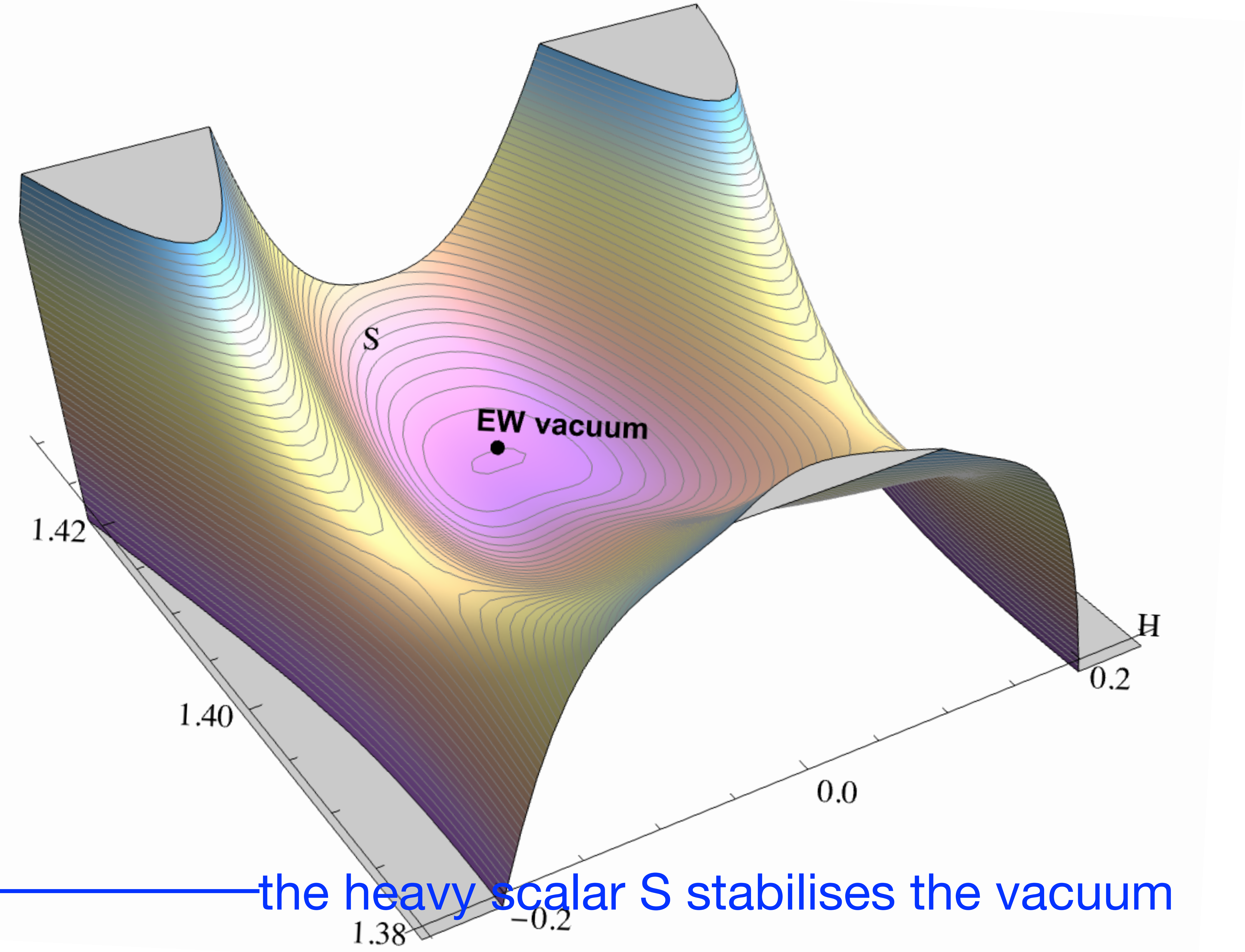

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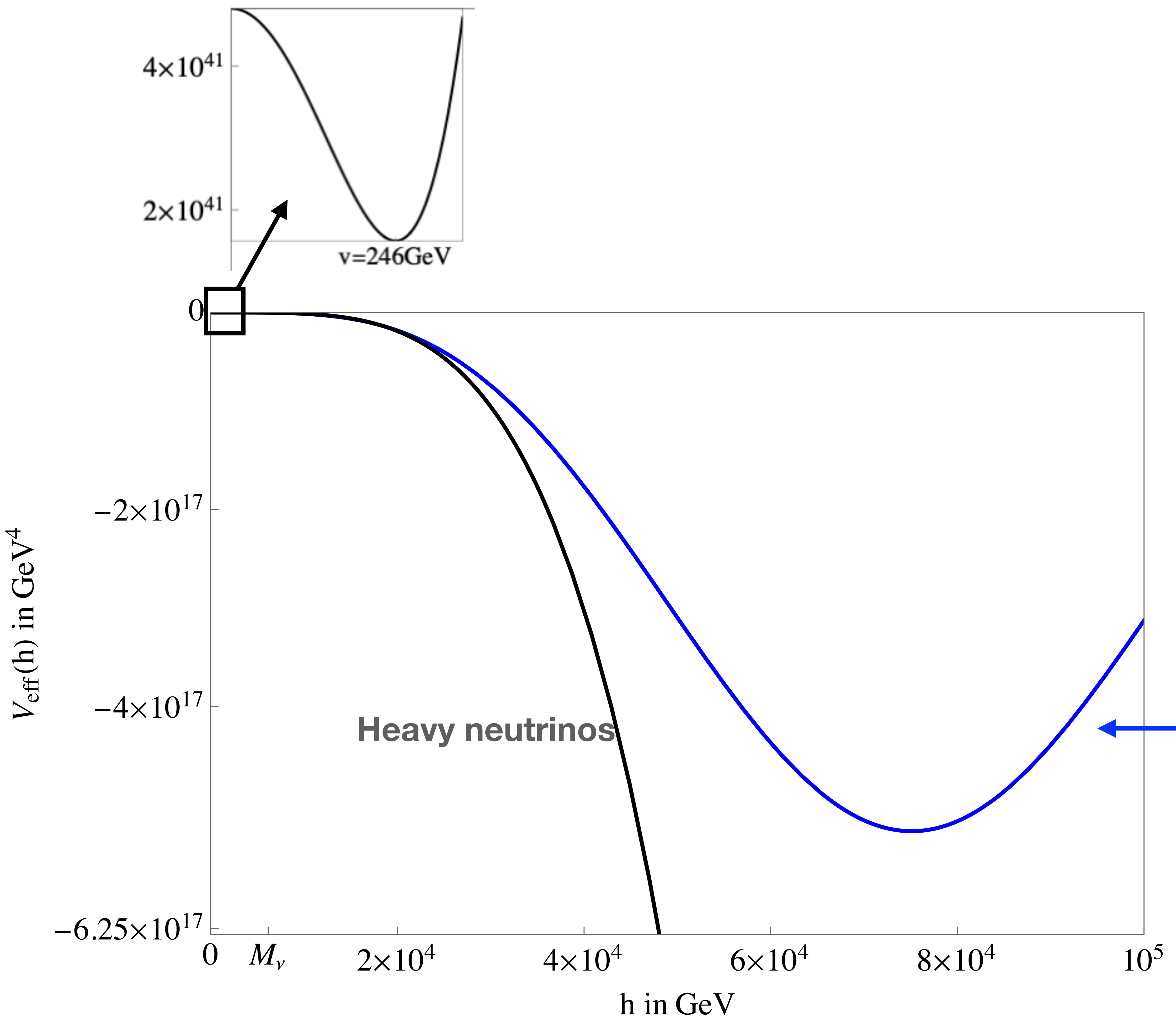

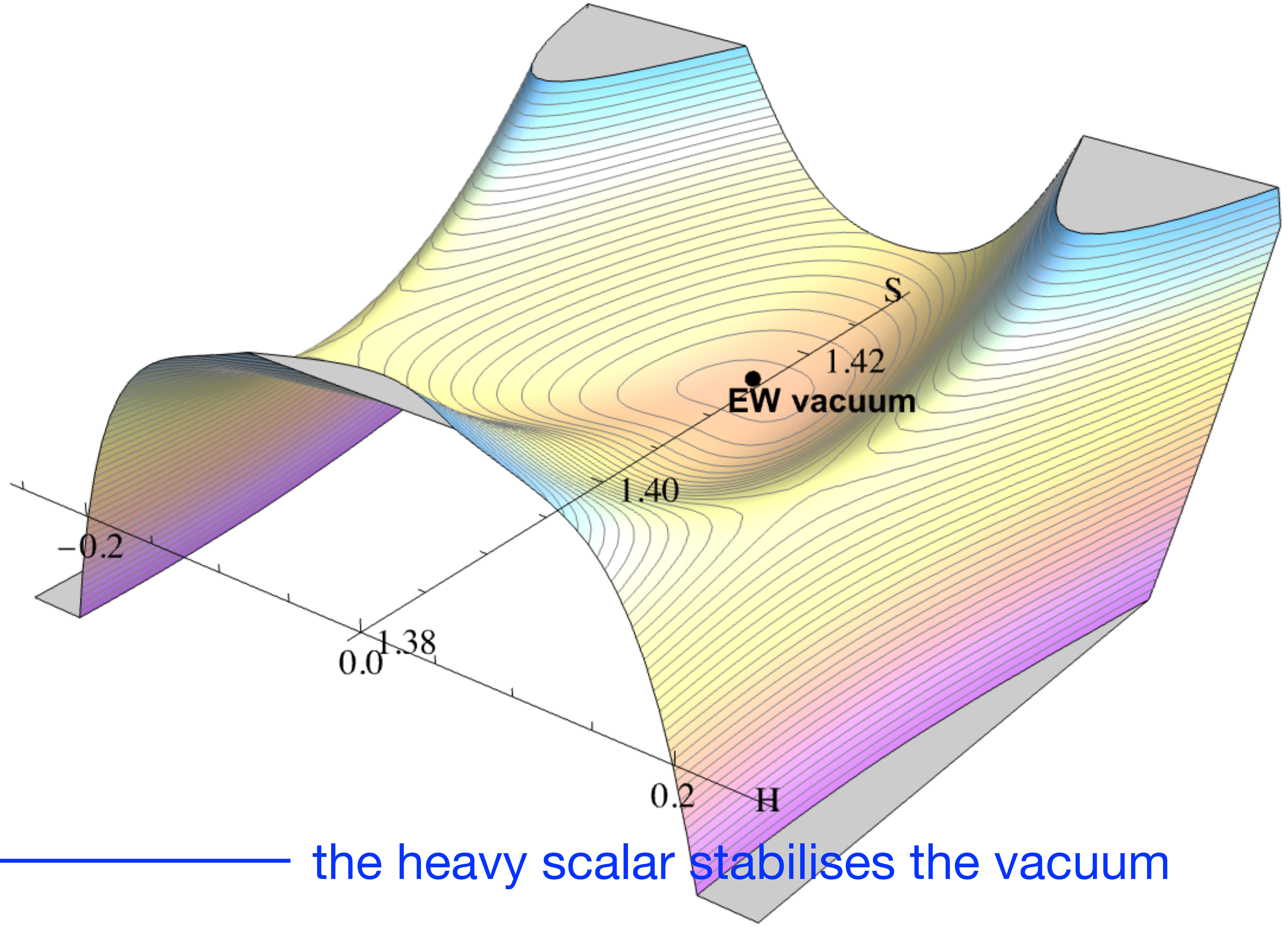

the heavy scalar stabilises the vacuum

Metastability bounds - BSM features

[2503.03825]
(V. Enguita, B. Gavela, T, Steingasser)

Metastability bounds - Scalar sector of Majoron model

$$V = -m_H^2|H|^2 + \lambda_H|H|^4 - m_S^2|S|^2 + \lambda_S|S|^4 + \kappa|H|^2|S|^2$$

↓ assuming SSB

running effects necessary!

↘ calculate tunneling rates!

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(V. Enguita, B. Gavela, T. Steingasser)

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Tree-level analysis:

$$P_i \equiv (<H>\ ,\ <S>)$$

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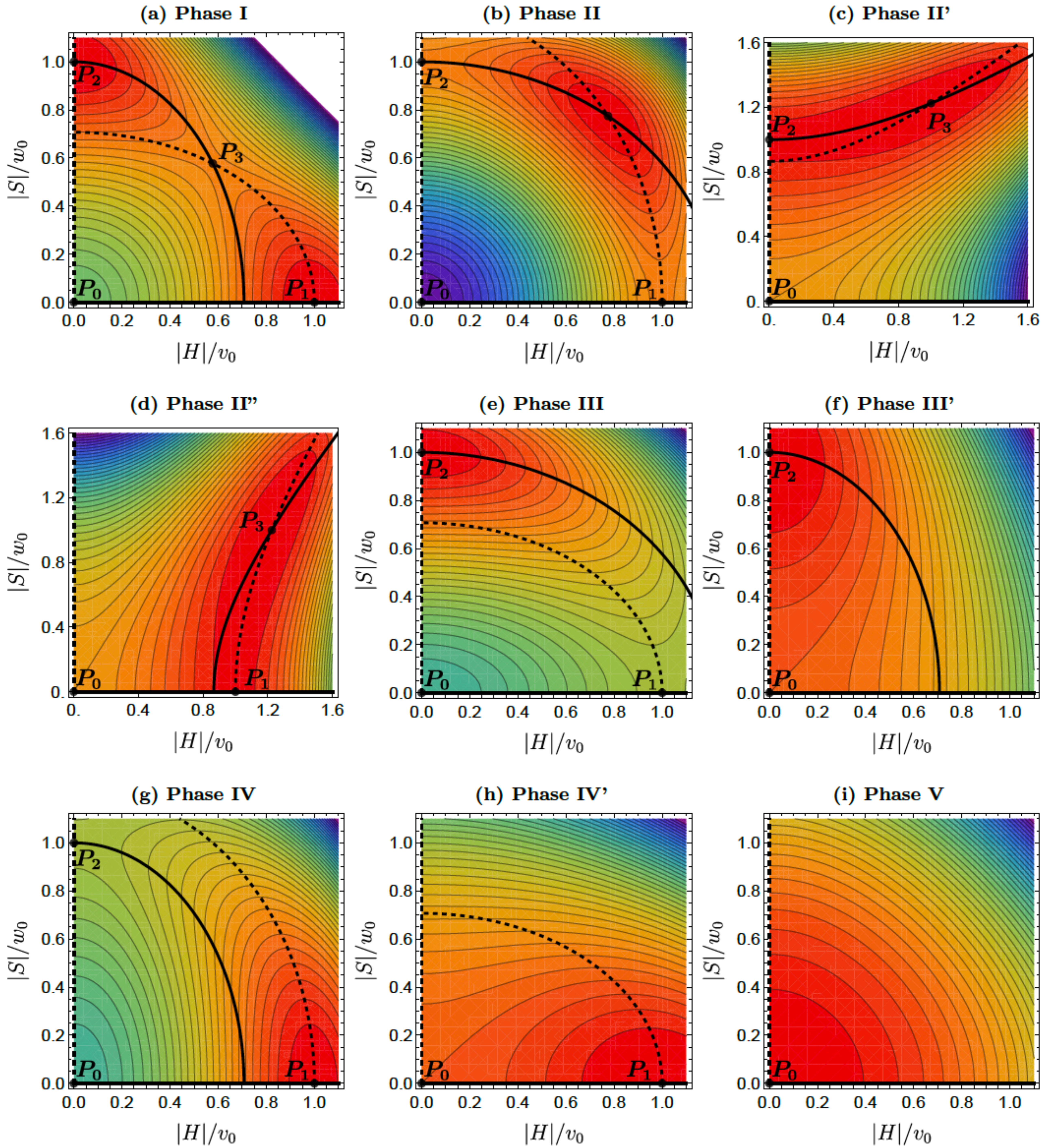

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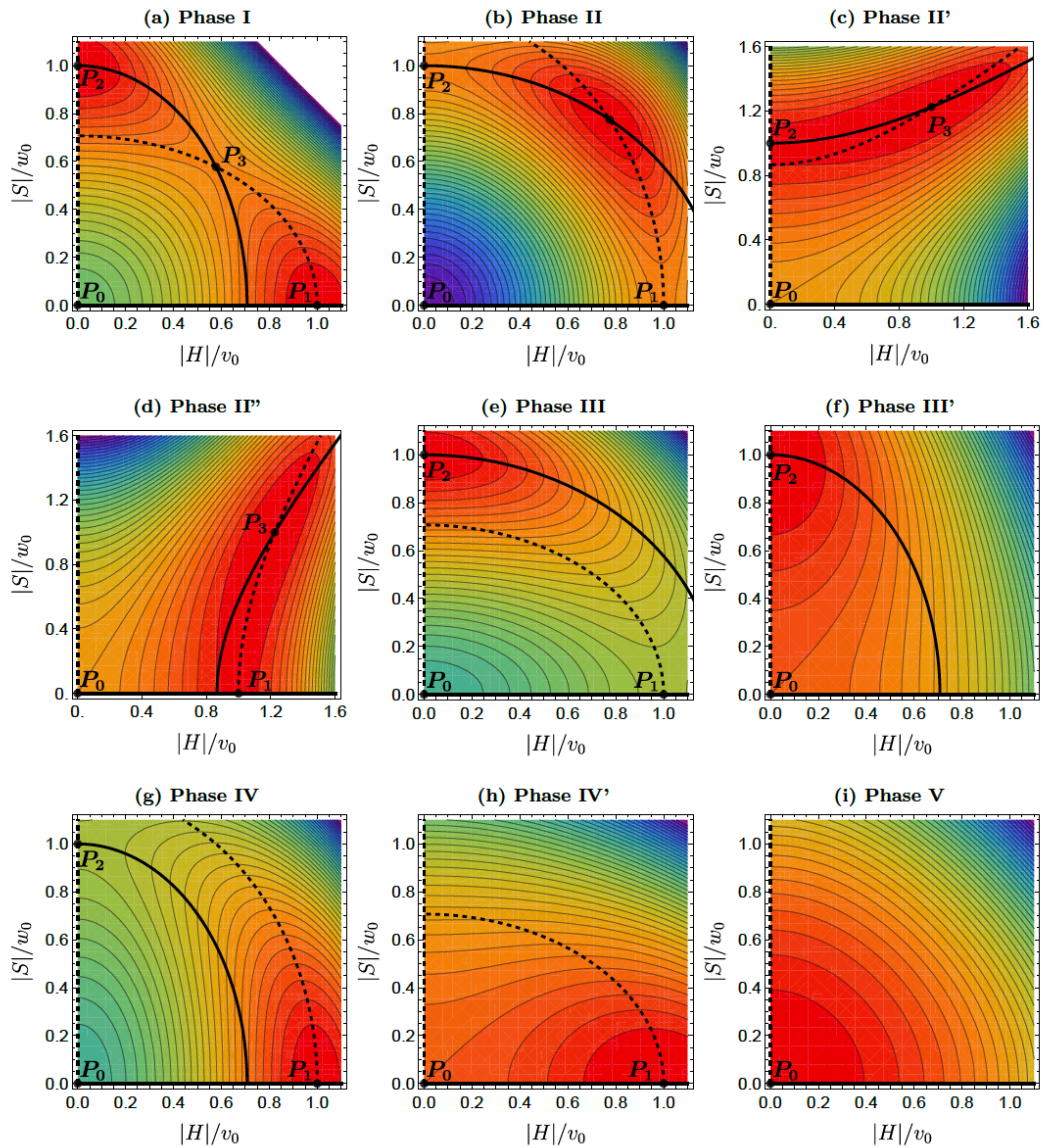

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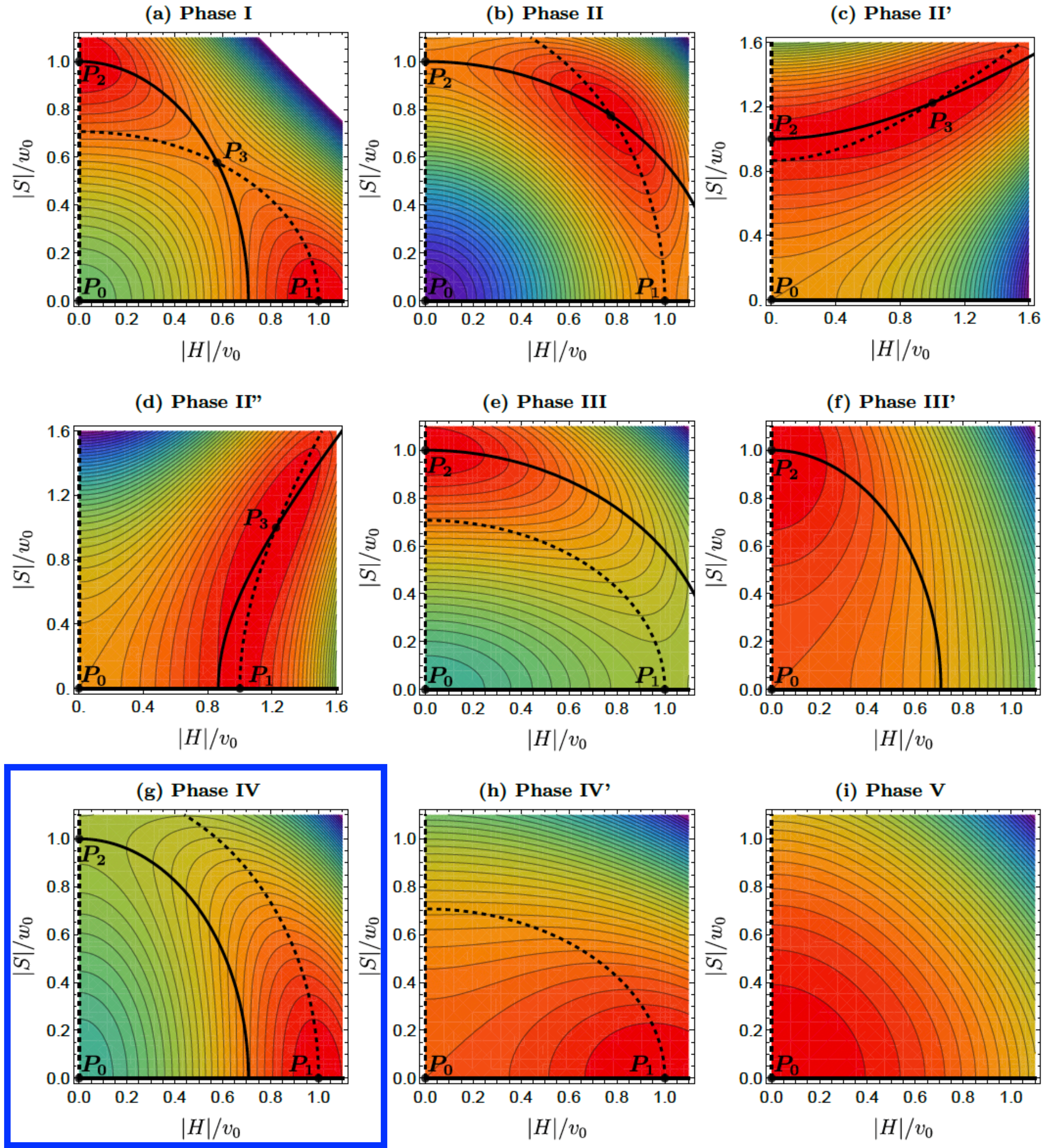

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Metastability bounds - Scalar sector of Majoron model

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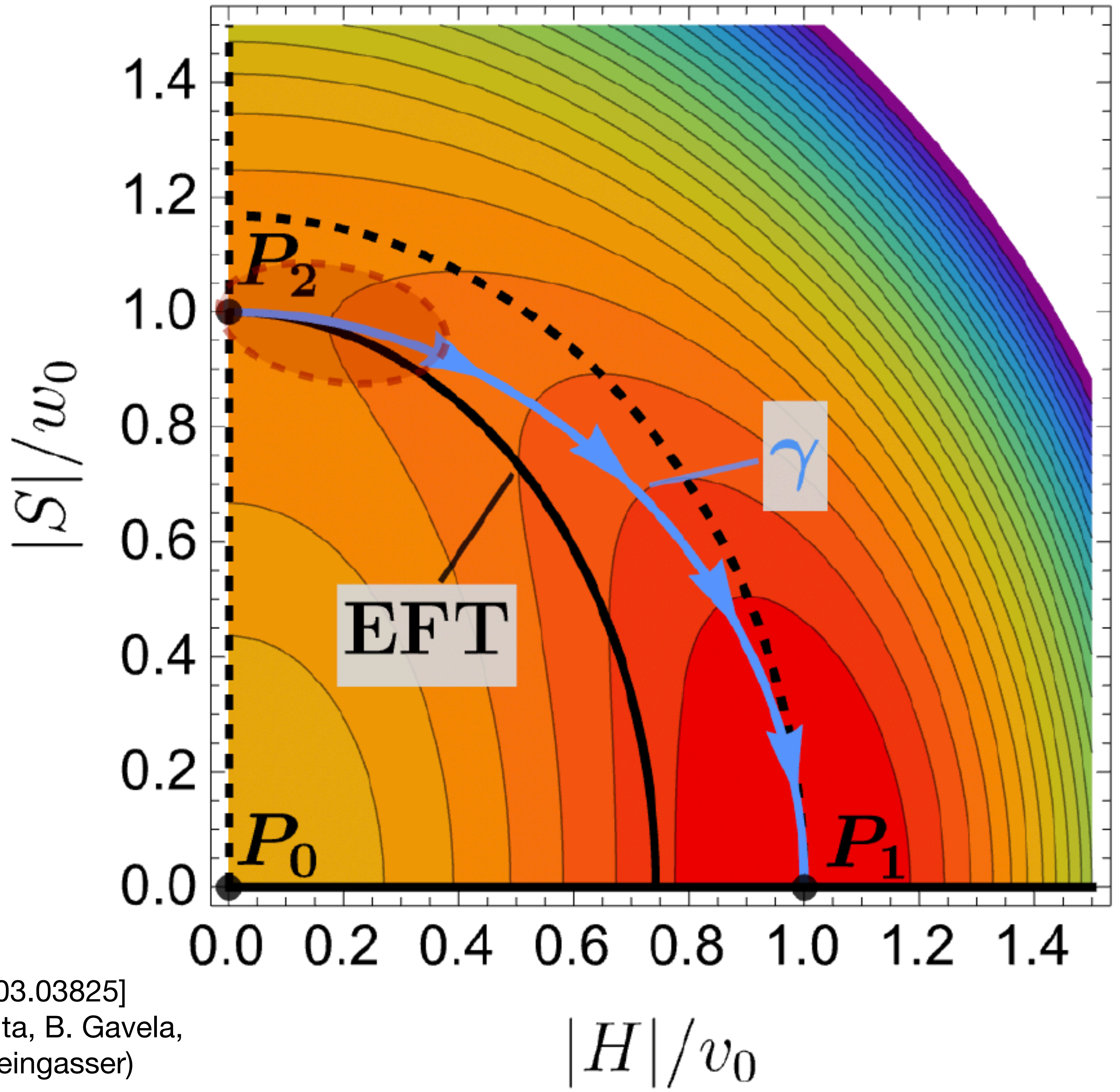

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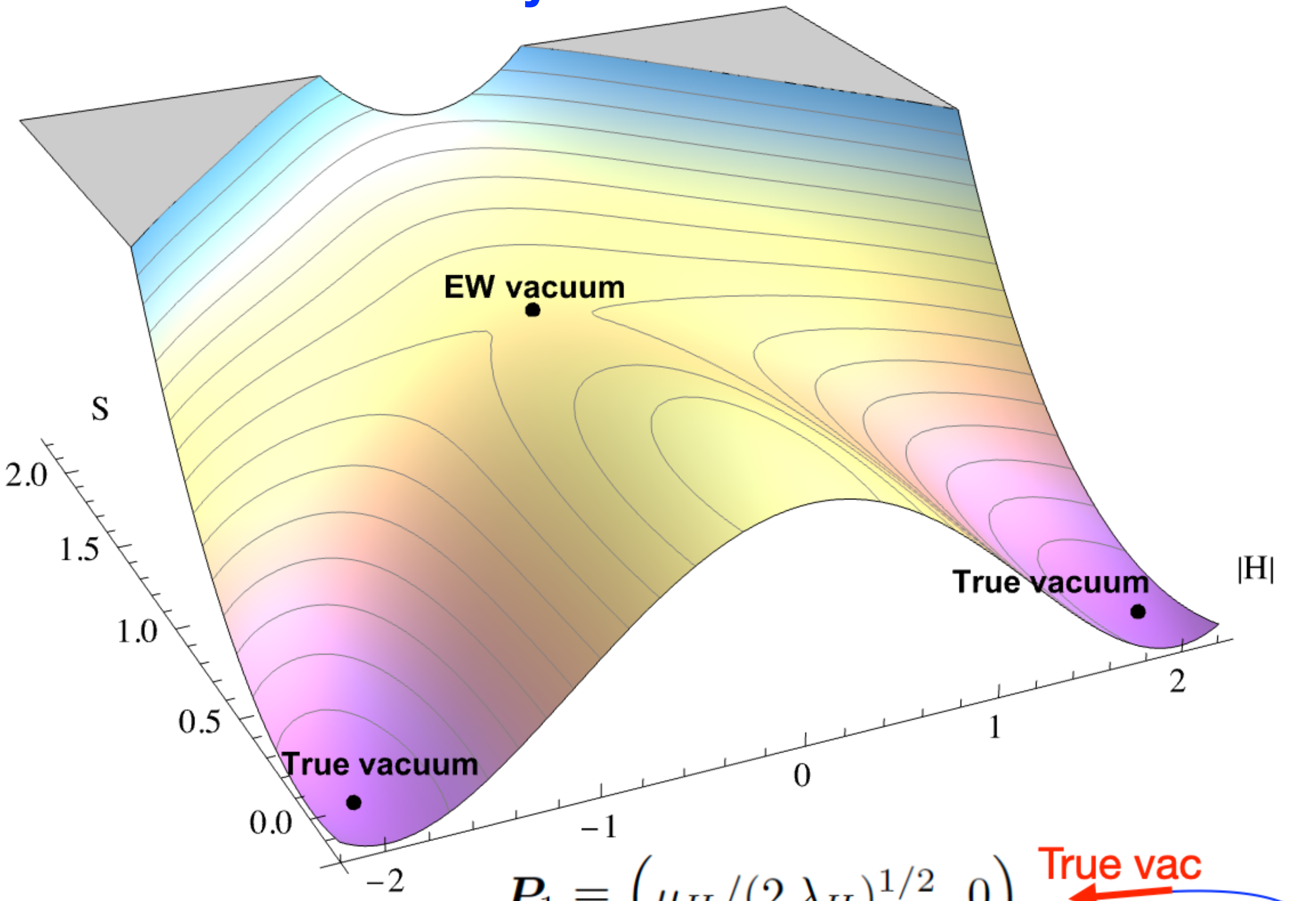

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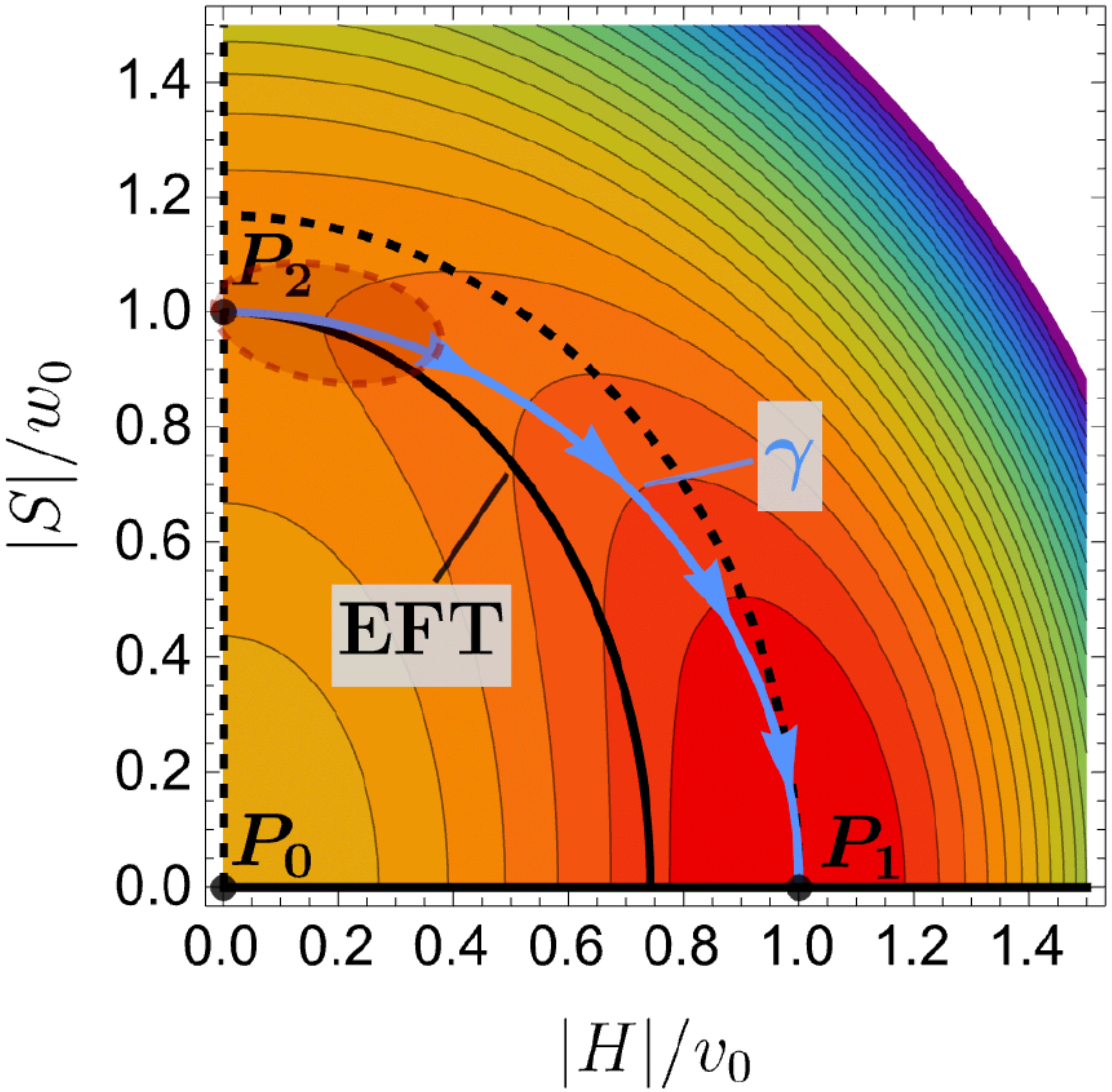
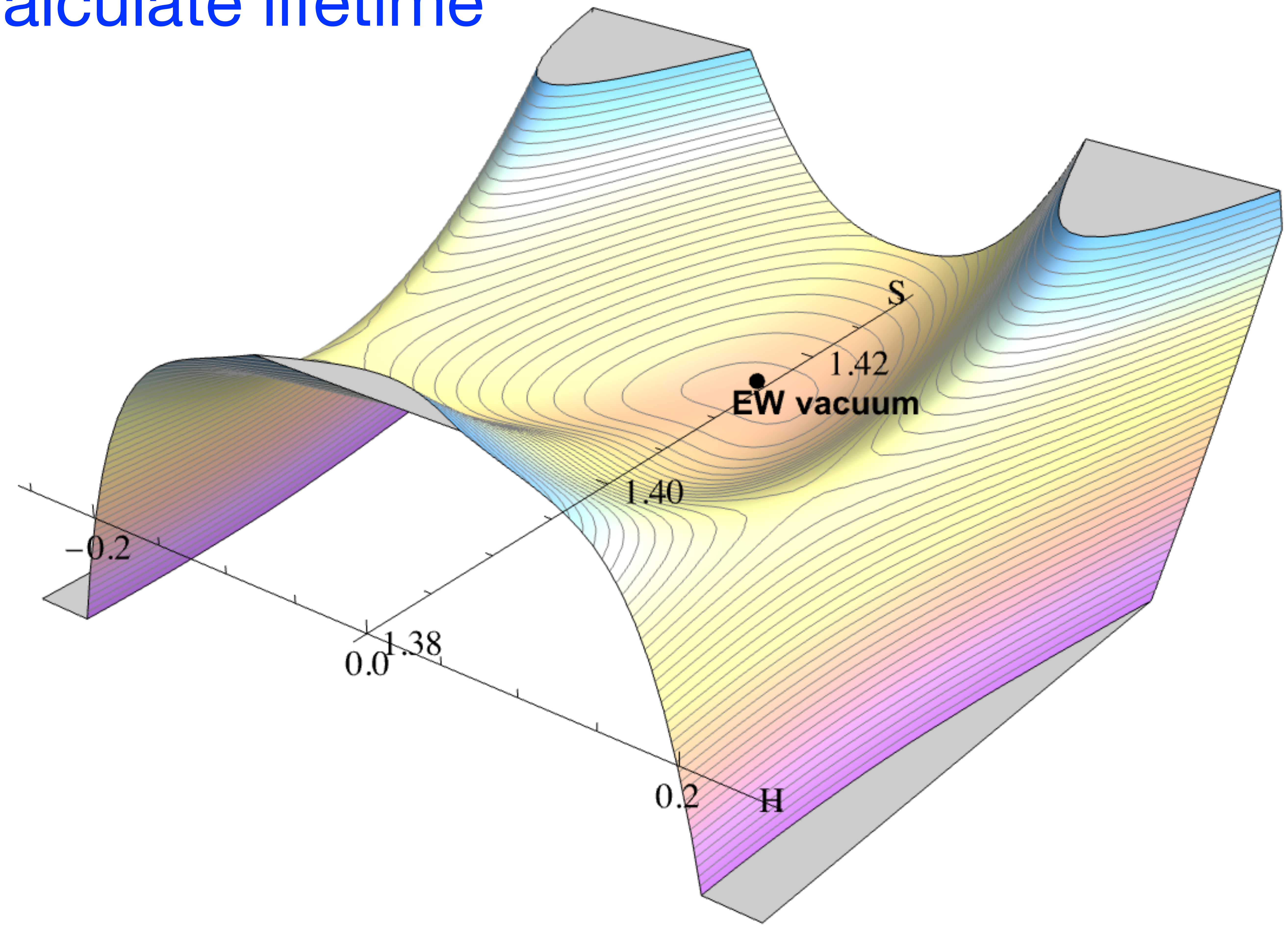


Calculate lifetime



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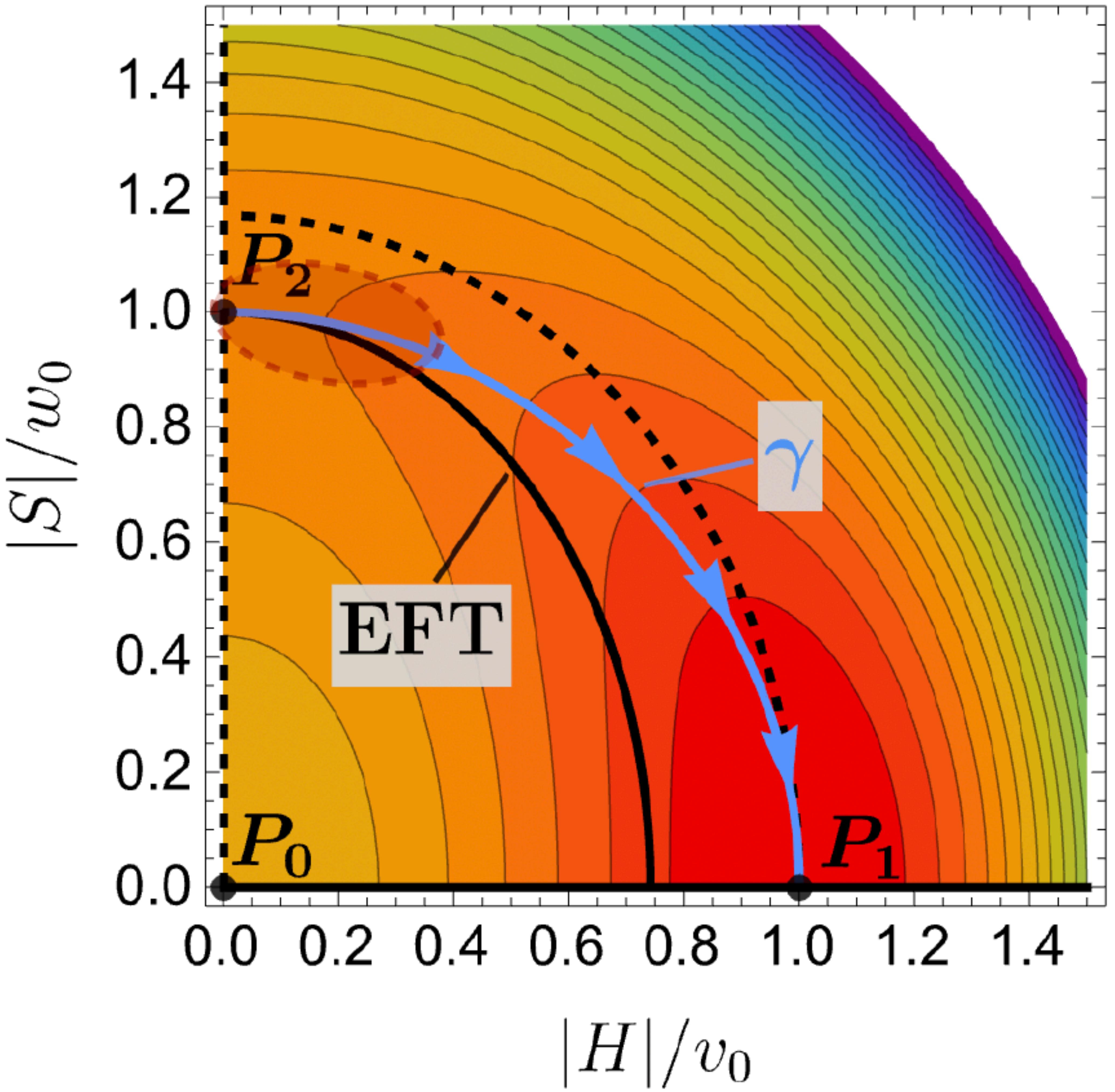

Calculate lifetime:

$$\mathcal{L}_\gamma = \frac{1}{2}\partial_\mu\eta\partial^\mu\eta - V_\gamma(\eta)$$

$$V_\gamma(\eta) = V(H_\gamma(\eta), S_\gamma(\eta))\,,$$

$$\frac{\mathrm{d}}{\mathrm{d}\eta}\begin{pmatrix} H_\gamma(\eta) \\ S_\gamma(\eta) \end{pmatrix} = \frac{-\nabla_{\{H,S\}}V(H_\gamma(\eta), S_\gamma(\eta))}{\sqrt{2}|\nabla_{\{H,S\}}V(H_\gamma(\eta), S_\gamma(\eta))|}$$

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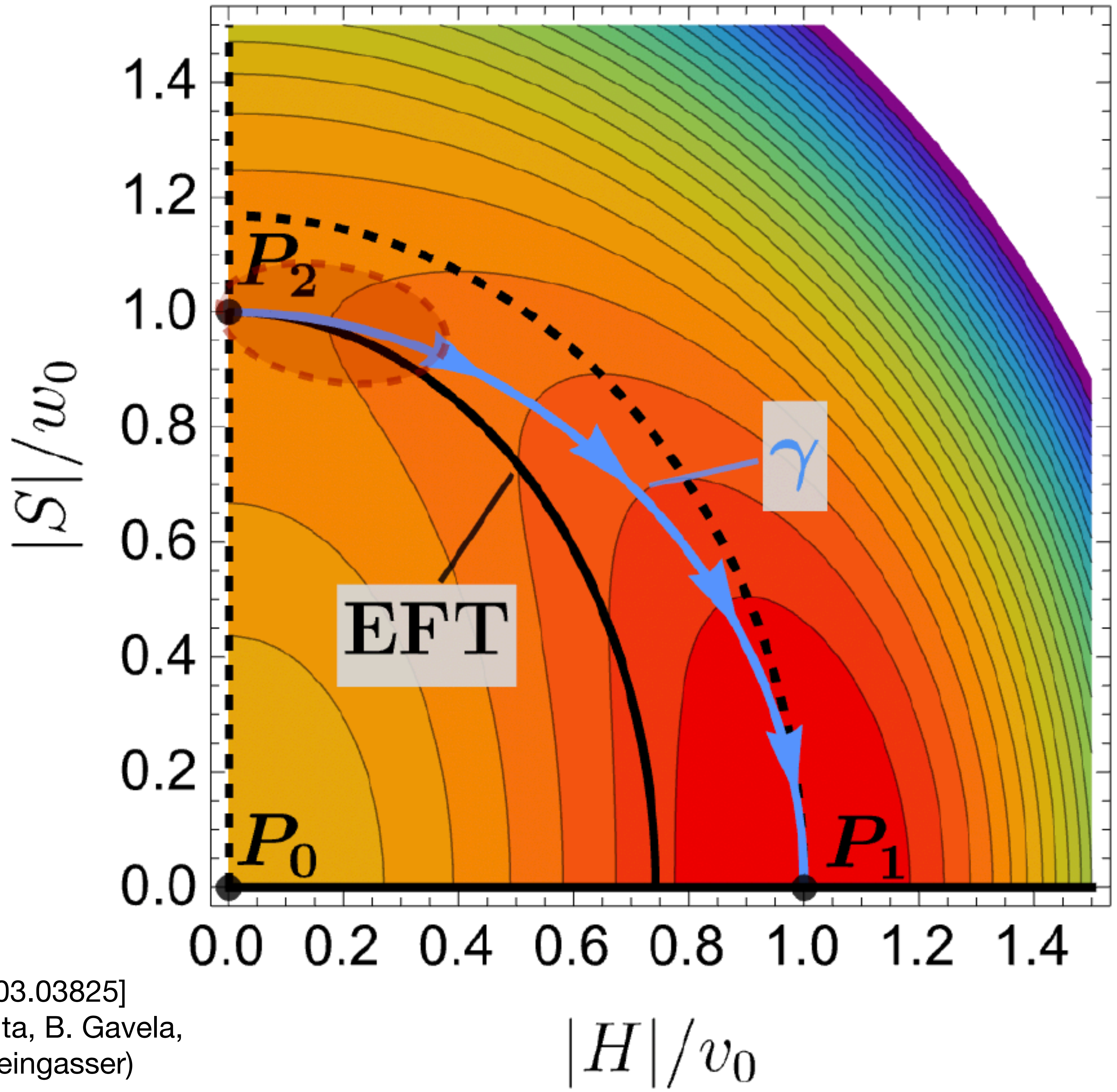

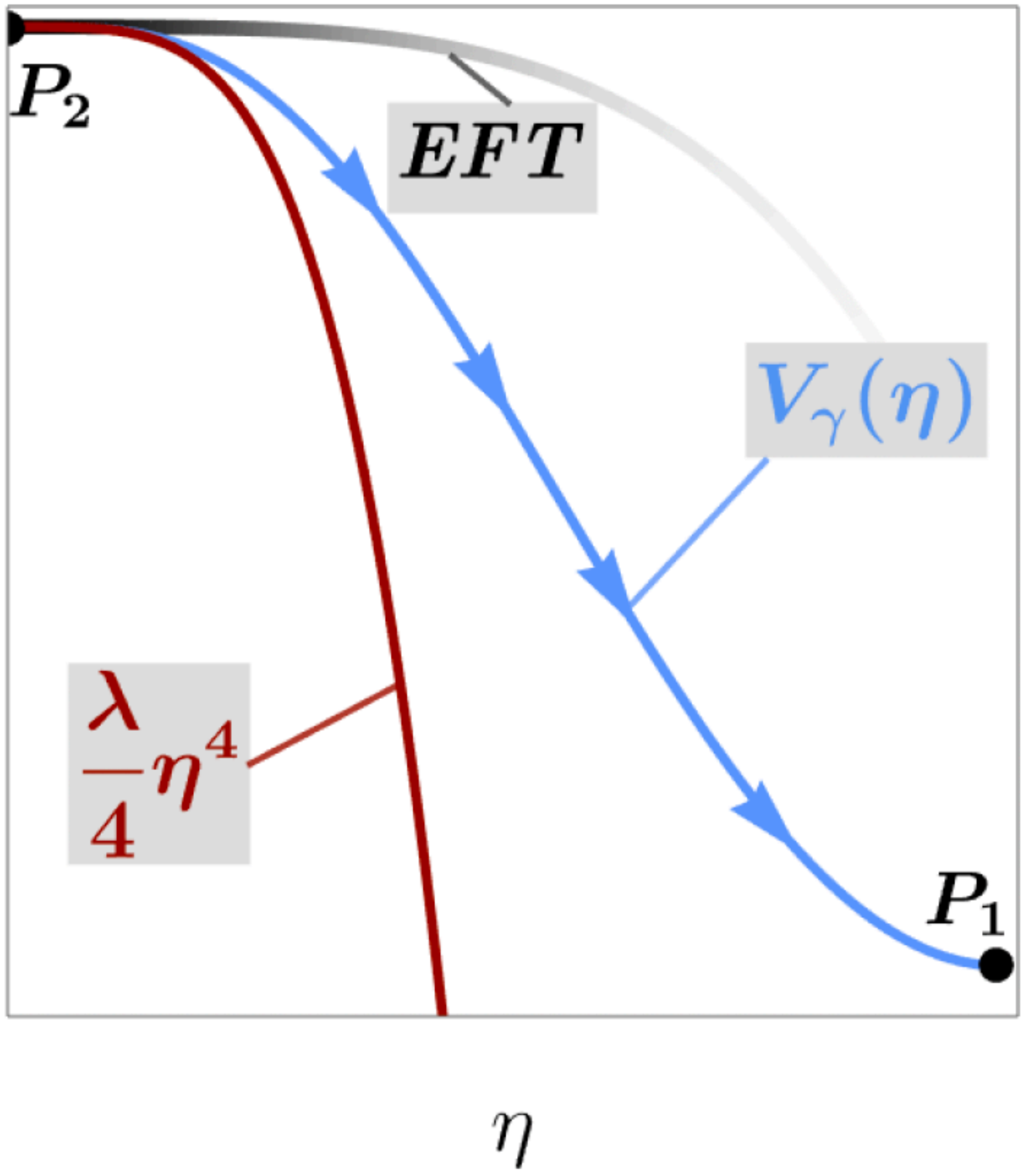

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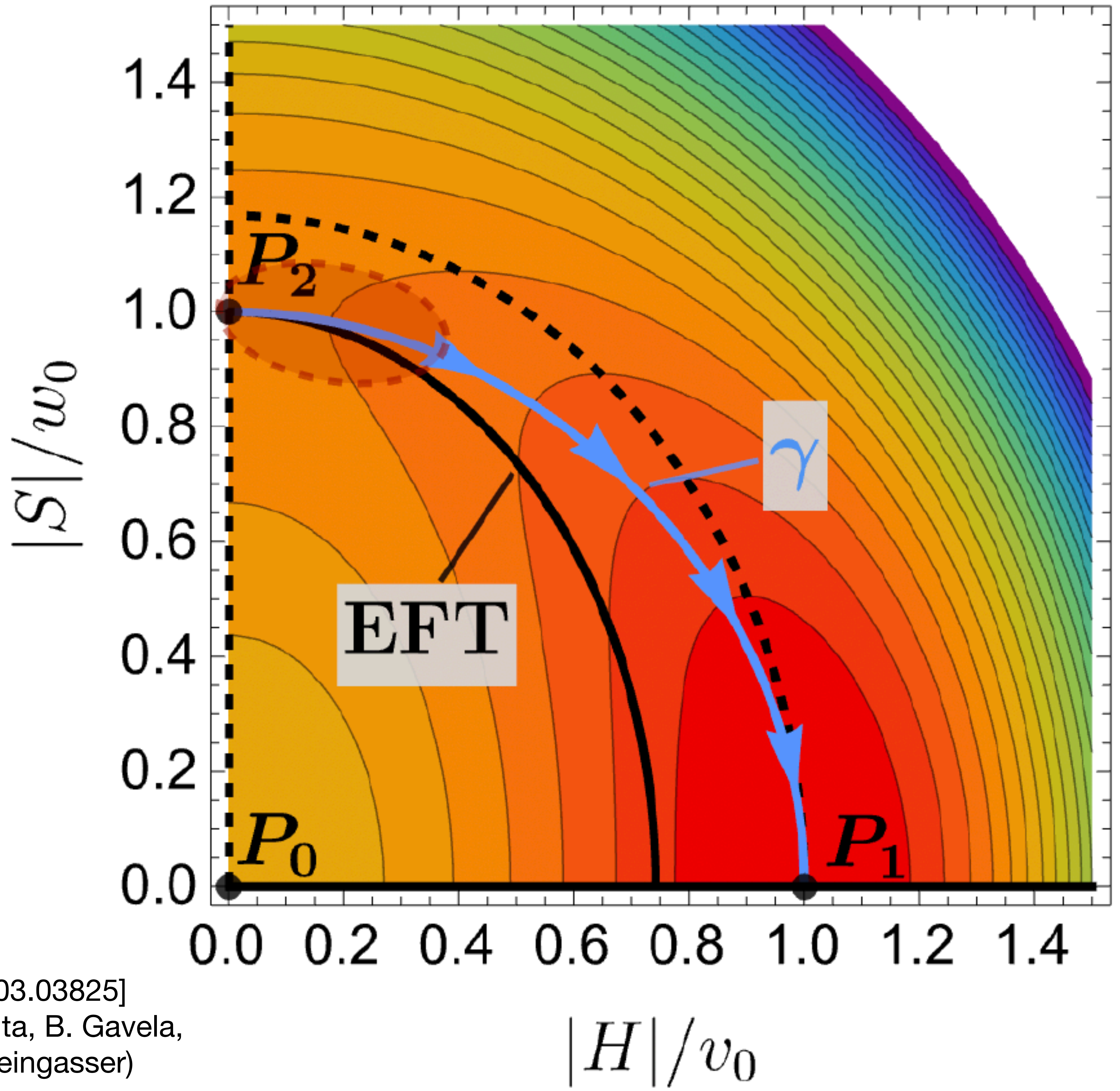

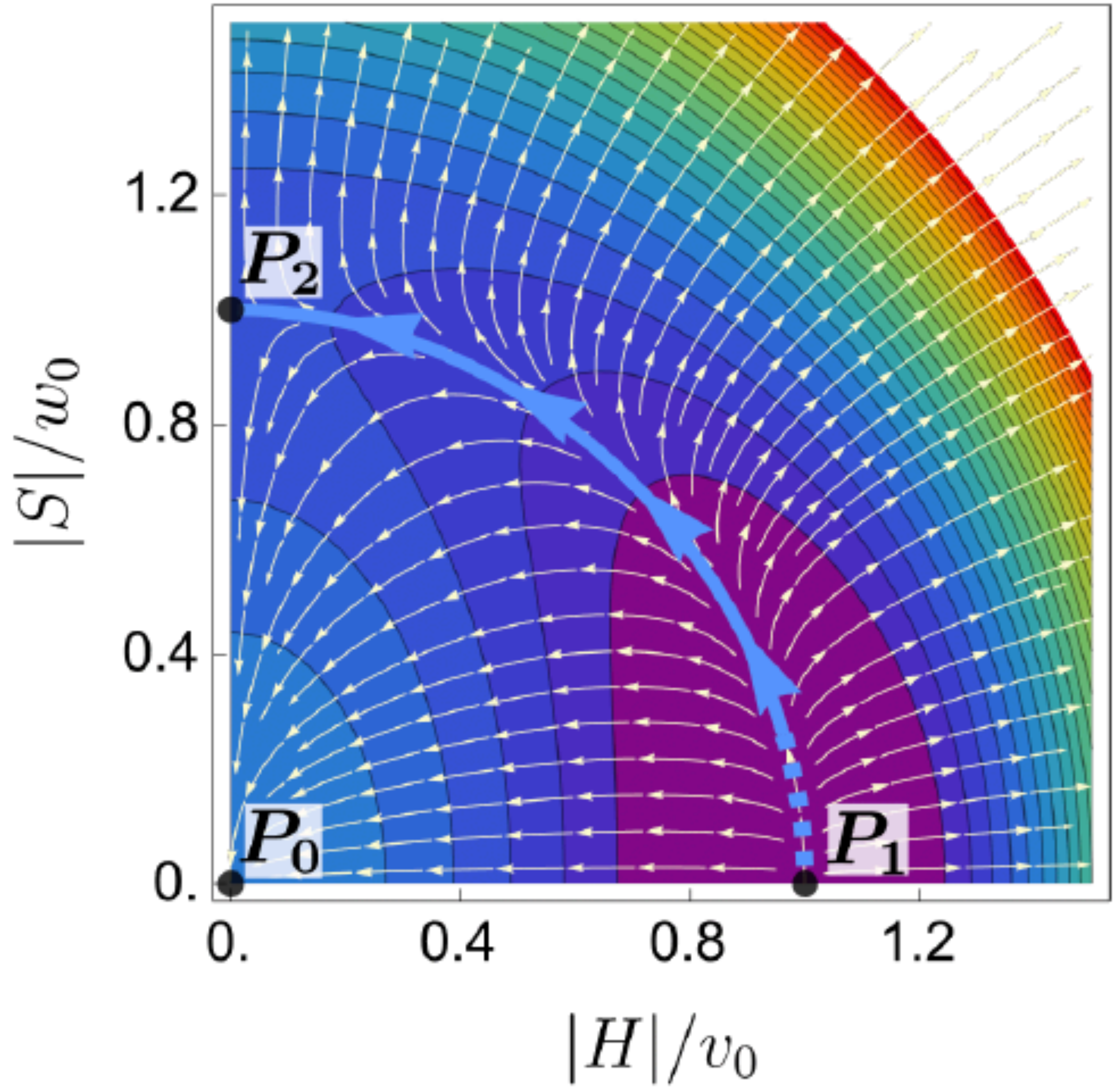

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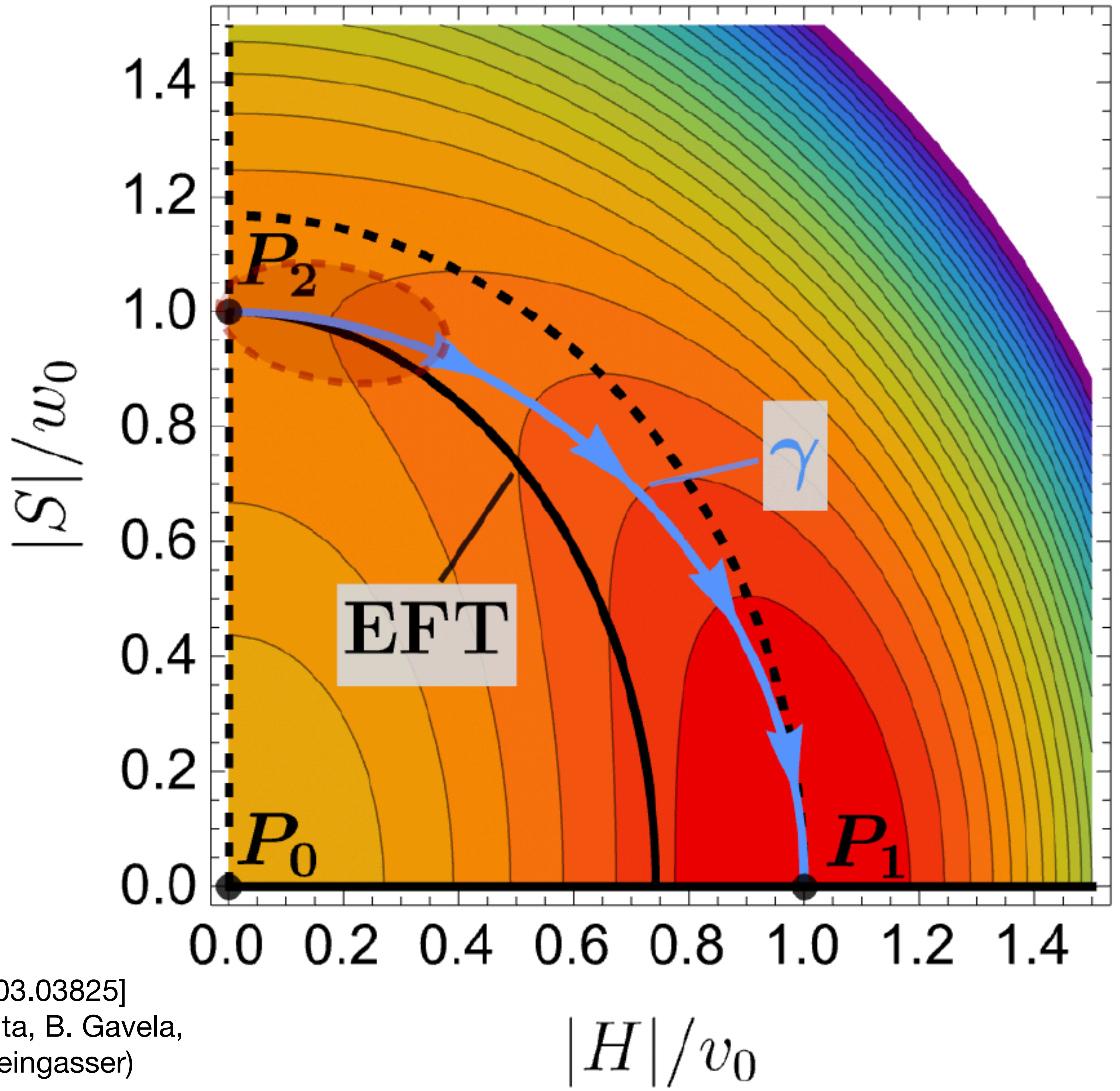

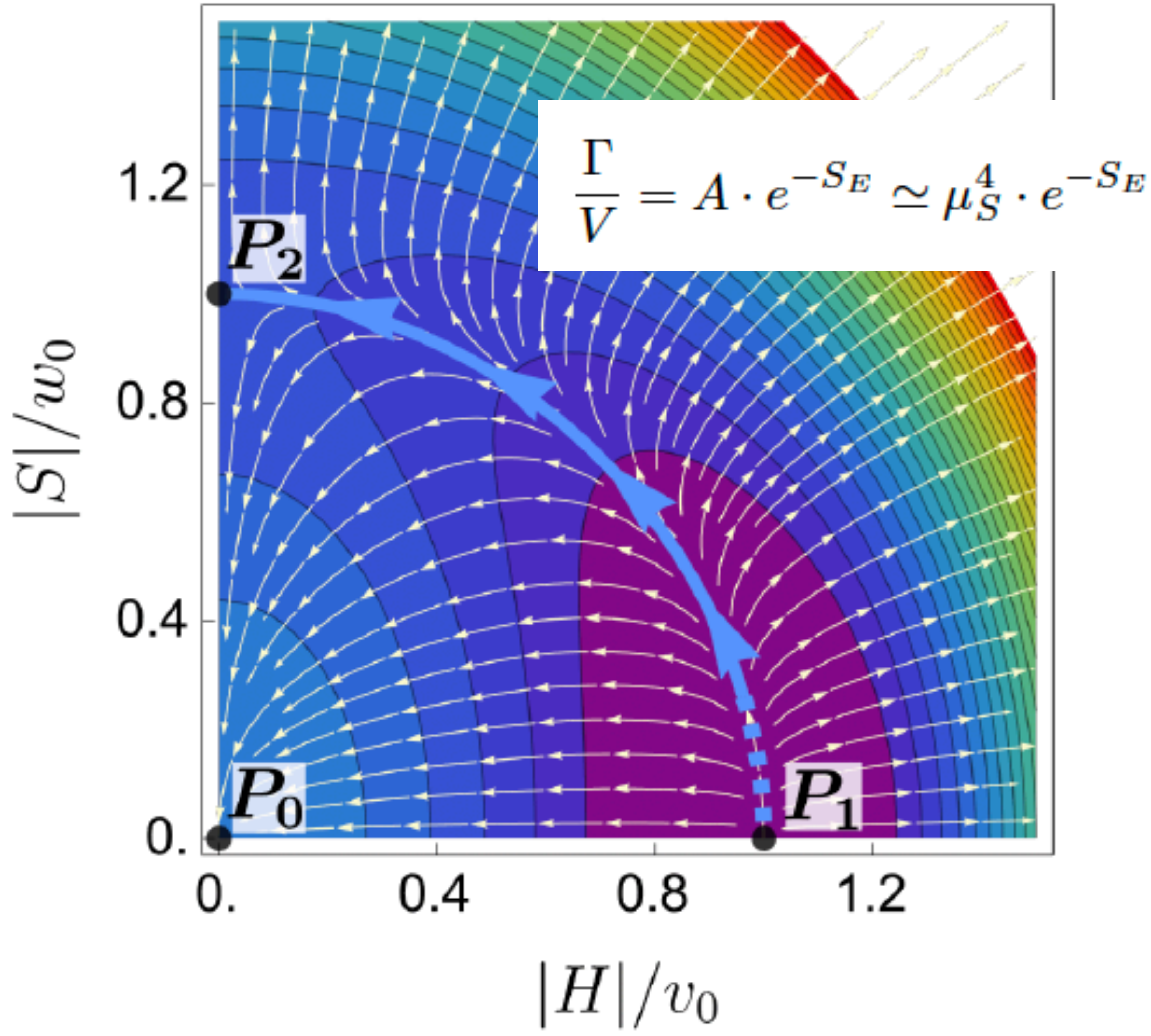

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$$m_N < \mu_I < M_s$$

Heavy sterile
neutrino masses

Heavy radial
Scalar mass

[2503.03825]
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Heavy sterile neutrino masses

Heavy radial Scalar mass

—> for FCC energies, you can integrate out S

We worked in the ``Majoron scheme” with physical inputs:

$$\{\underbrace{\alpha_{\rm em}, G_F, m_Z, m_h}_{\text{Z-scheme}}, \underbrace{M_N, M_s, \kappa, Y_\nu, \lambda_S}_{\text{Majoron model}}\}$$

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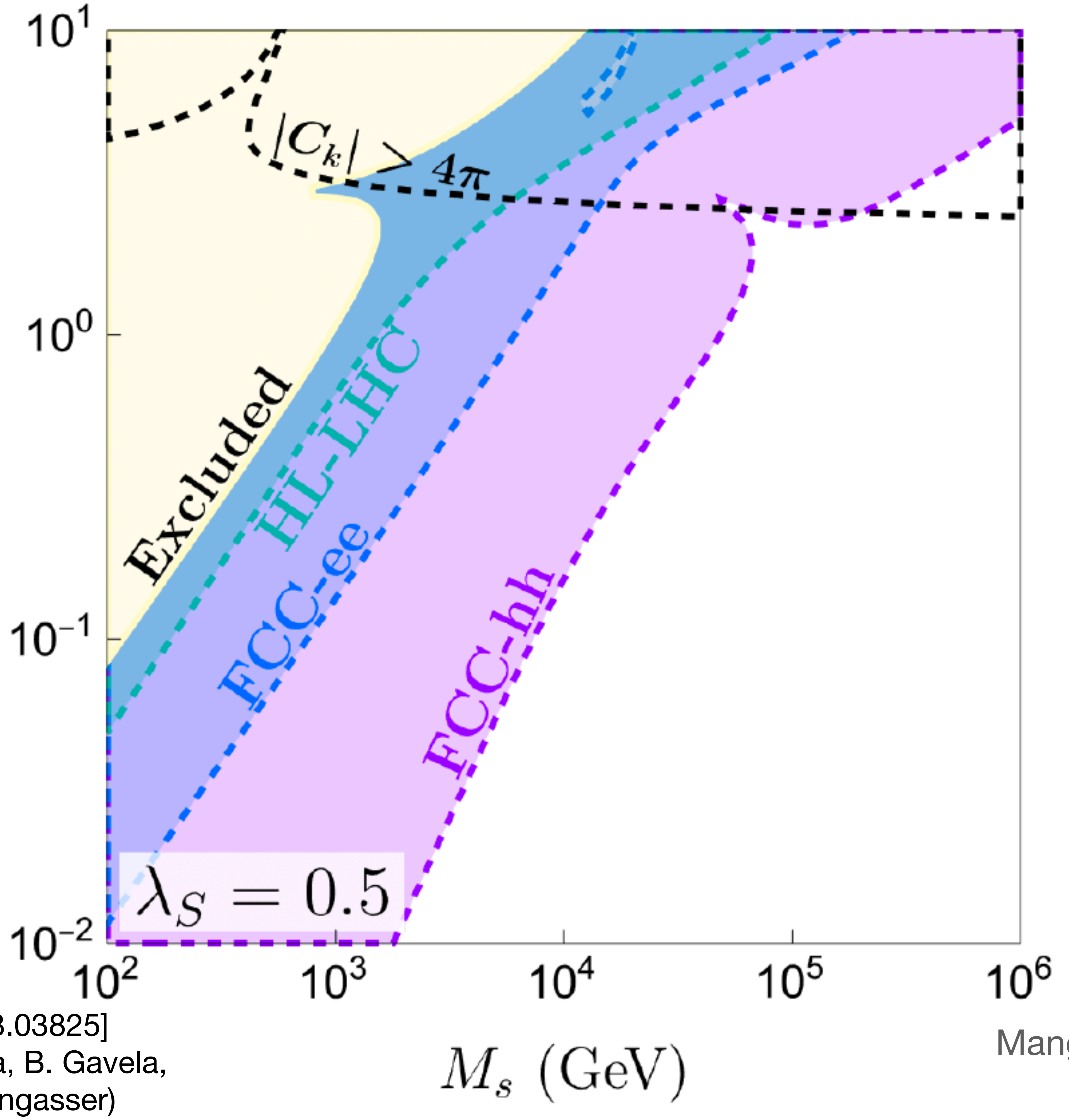
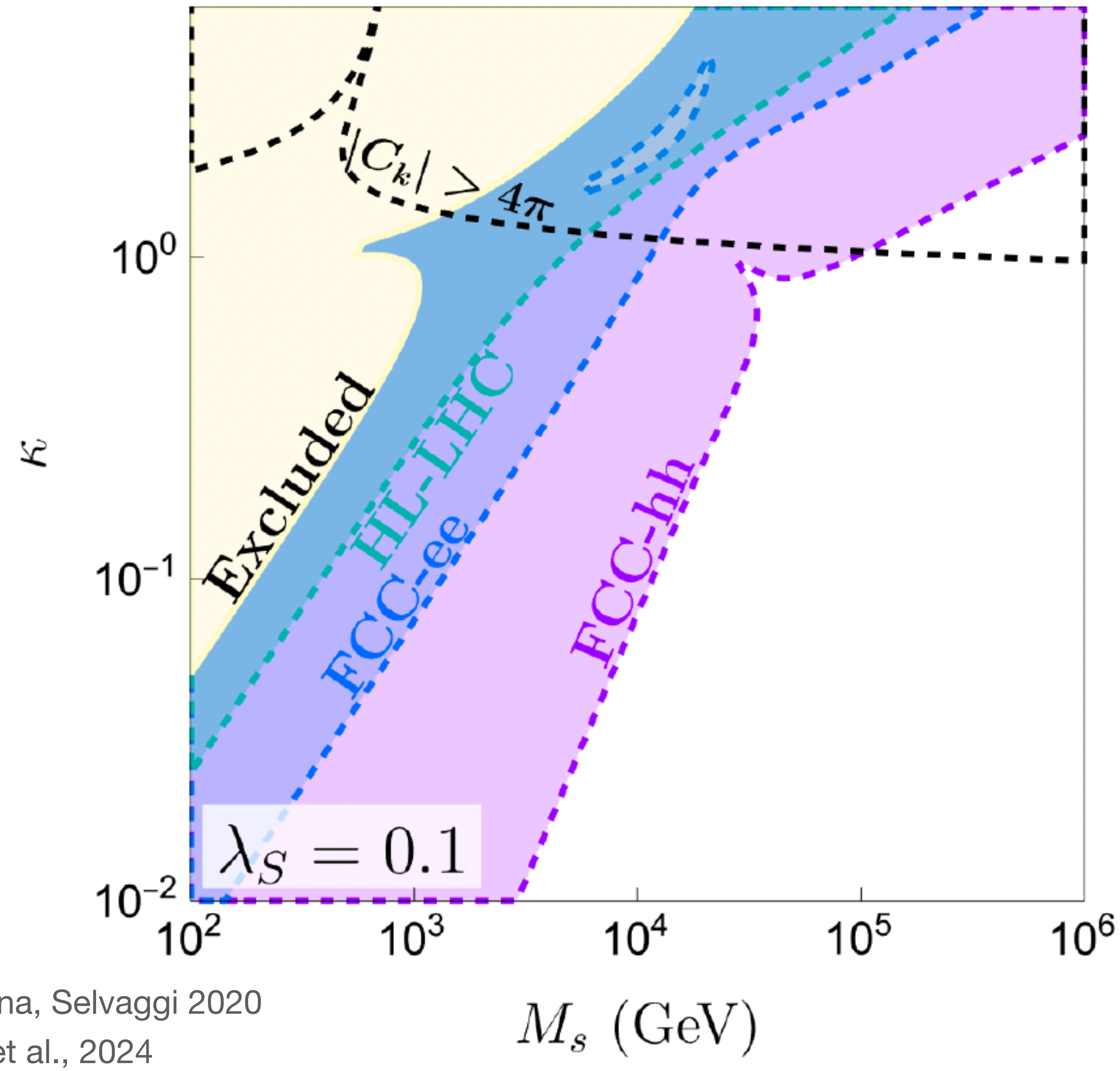
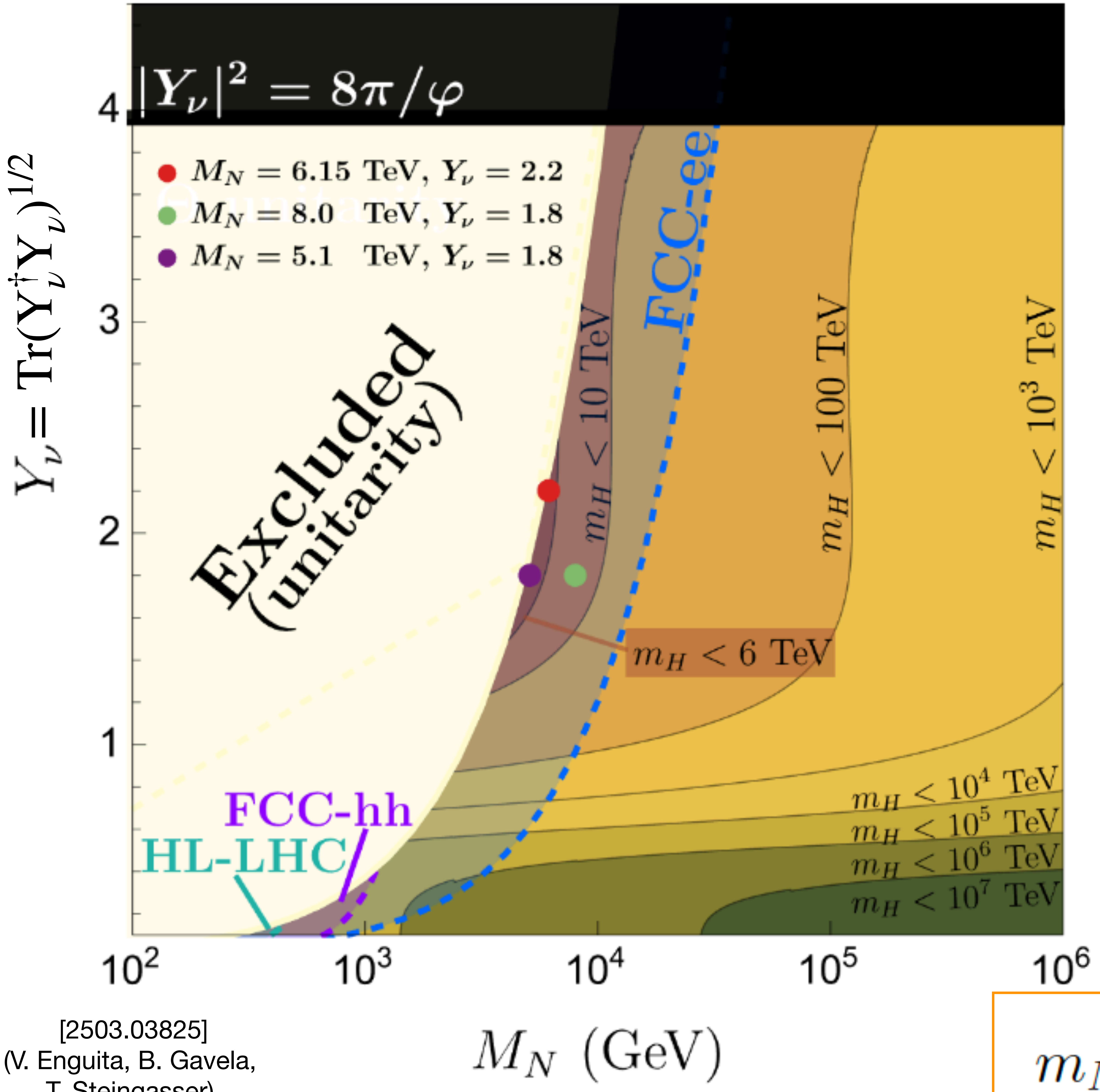


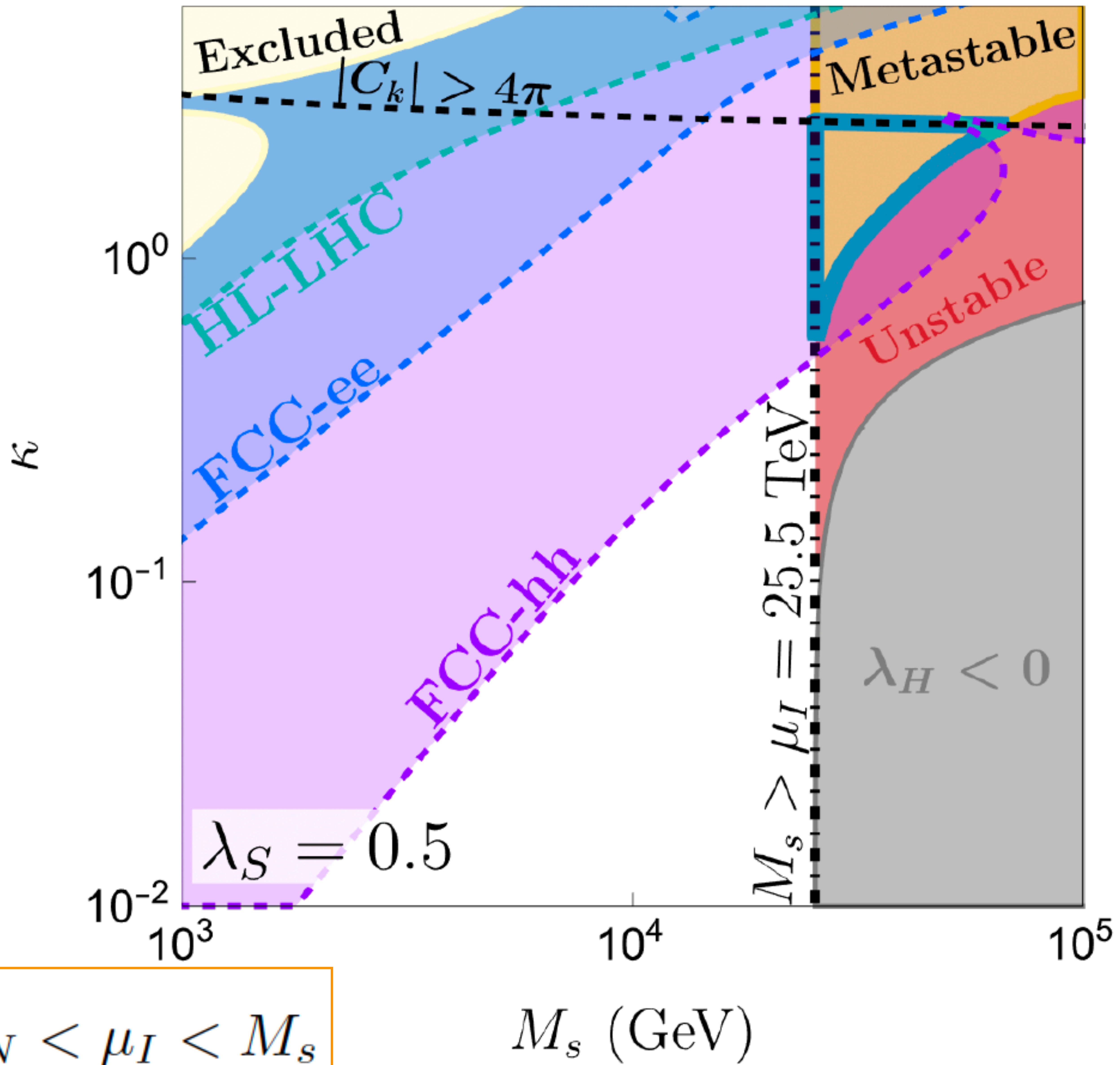


[2503.03825]
(V. Enguita, B. Gavela, T. Steingasser)

Mangano, Ortona, Selvaggi 2020
Celada et al., 2024

Low-scale Majoron @FCC-ee and @FCC-hh





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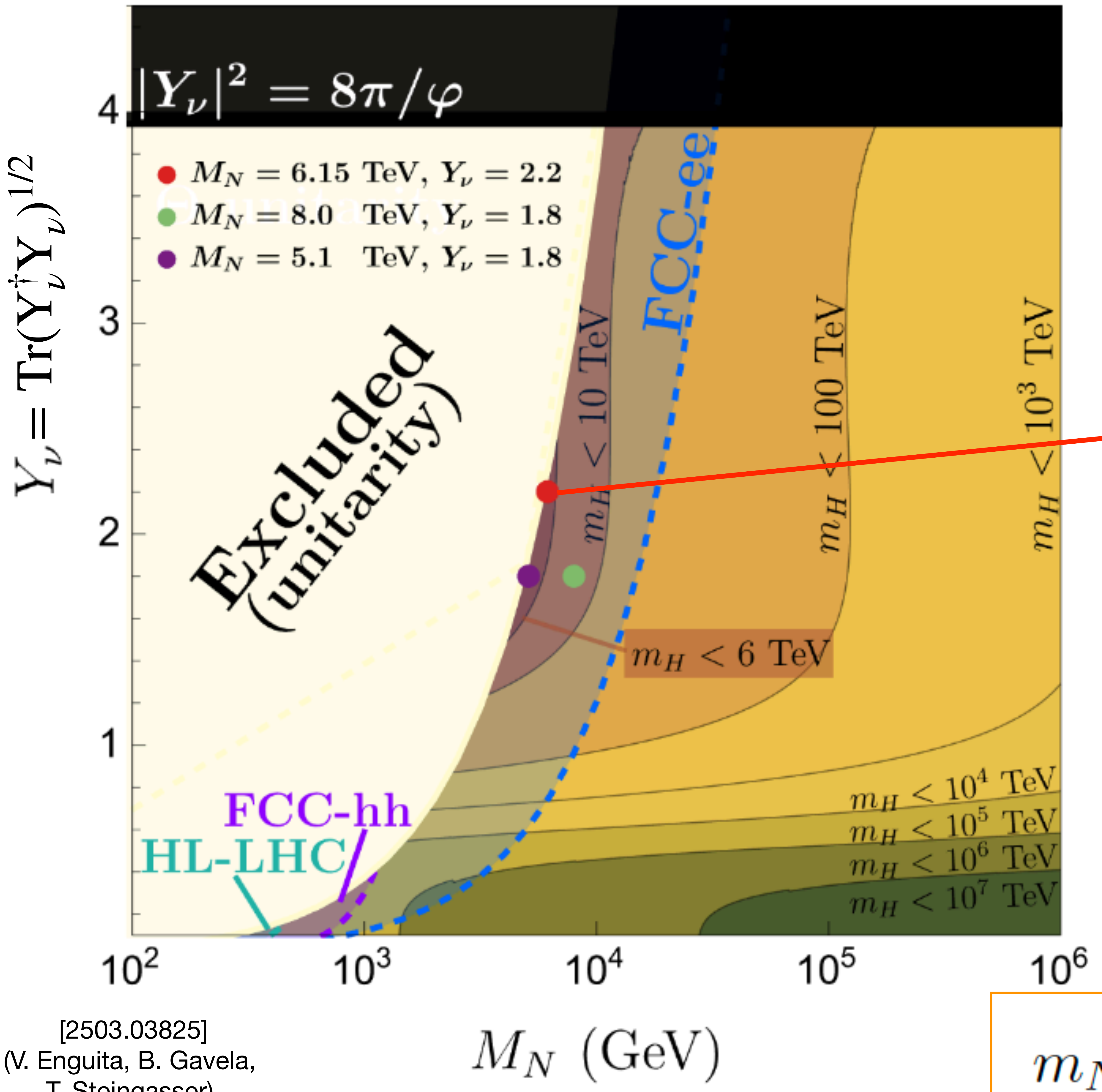
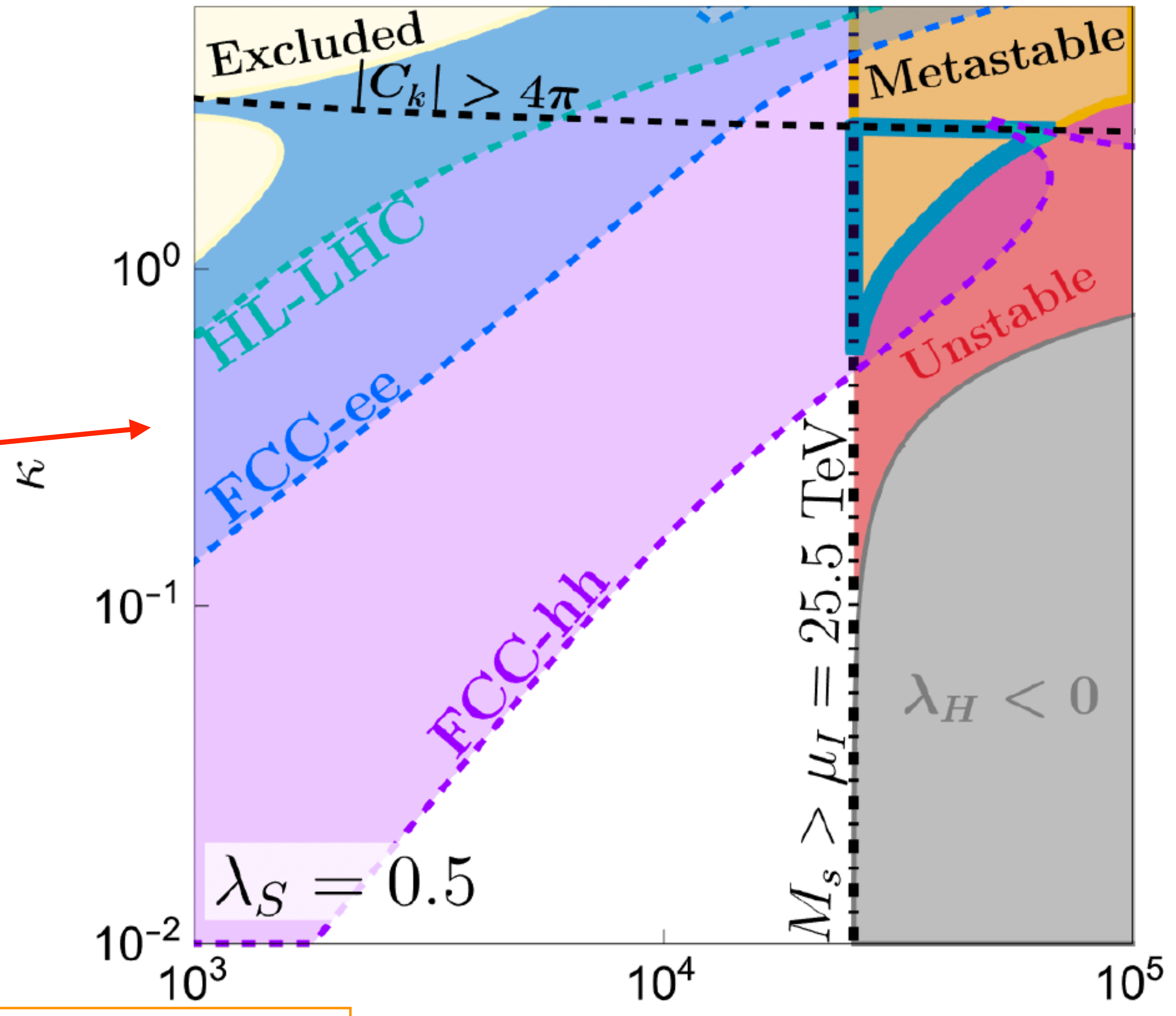


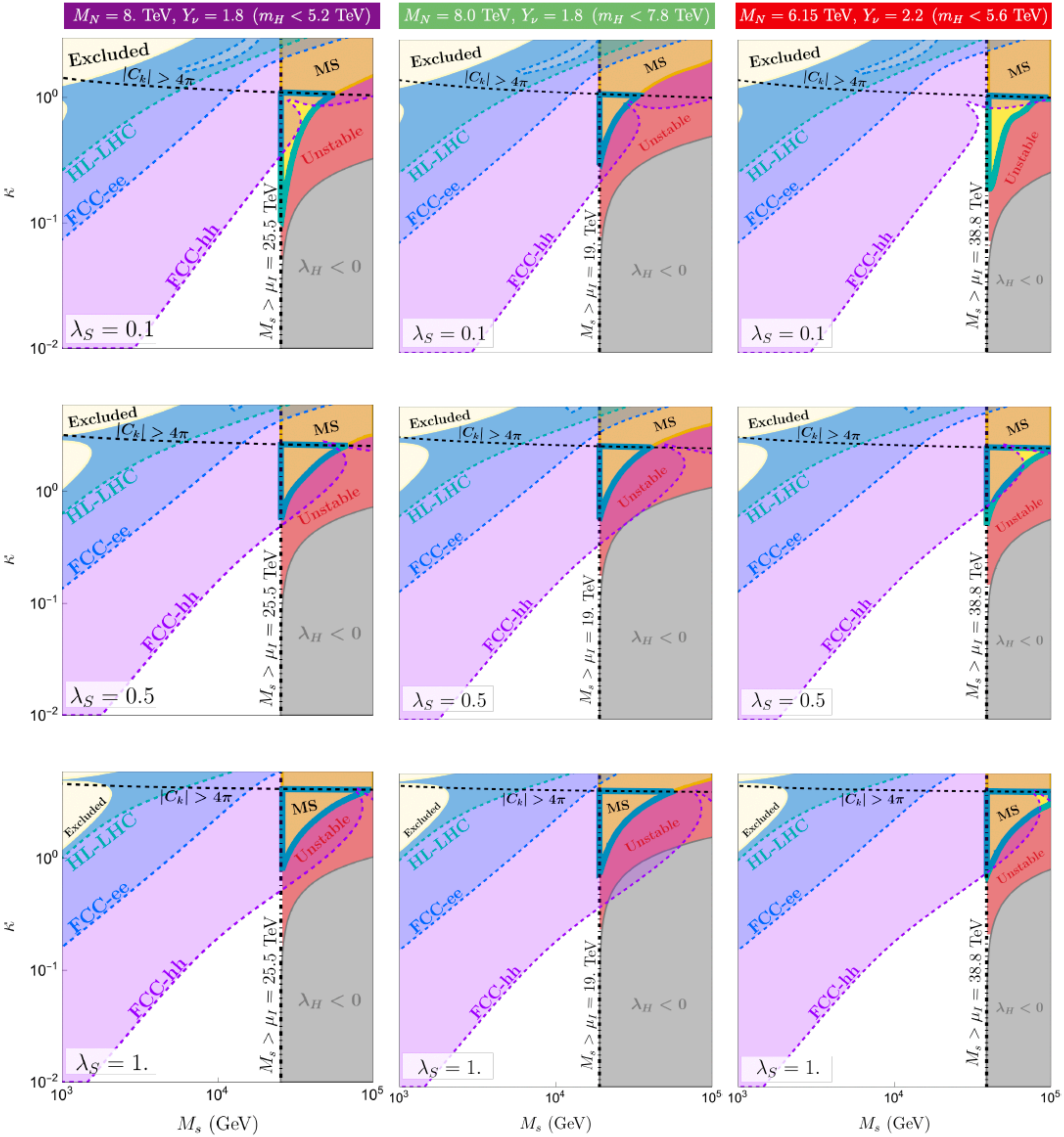


$m_N < \mu_I < M_s$

[2503.03825]
(V. Enguita, B. Gavela, T. Steingasser)

Metastability bound @FCC - hh Majoron scalar sector

$M_N = 8.\ \text{TeV},\ Y_\nu = 1.8\ \ (m_H < 5.2\ \text{TeV})$



[2503.03825]
(V. Enguita, B. Gavela,
T. Steingasser)

Summary of Majoron

Metastability bound: $m_h^2 \lesssim |\beta_\lambda(\mu_I)|\,\mu_I^2 \ll \Lambda_{UV}^2$

$\uparrow$

lowered by BSM physics (heavy sterile neutrinos)

$\longrightarrow$ check vacuum stability $\Rightarrow$ additional constraints

Majoron model:
- Heavy neutrinos in FCC-ee
- Scalar plausibly in FCC-hh

Summary of Majoron model

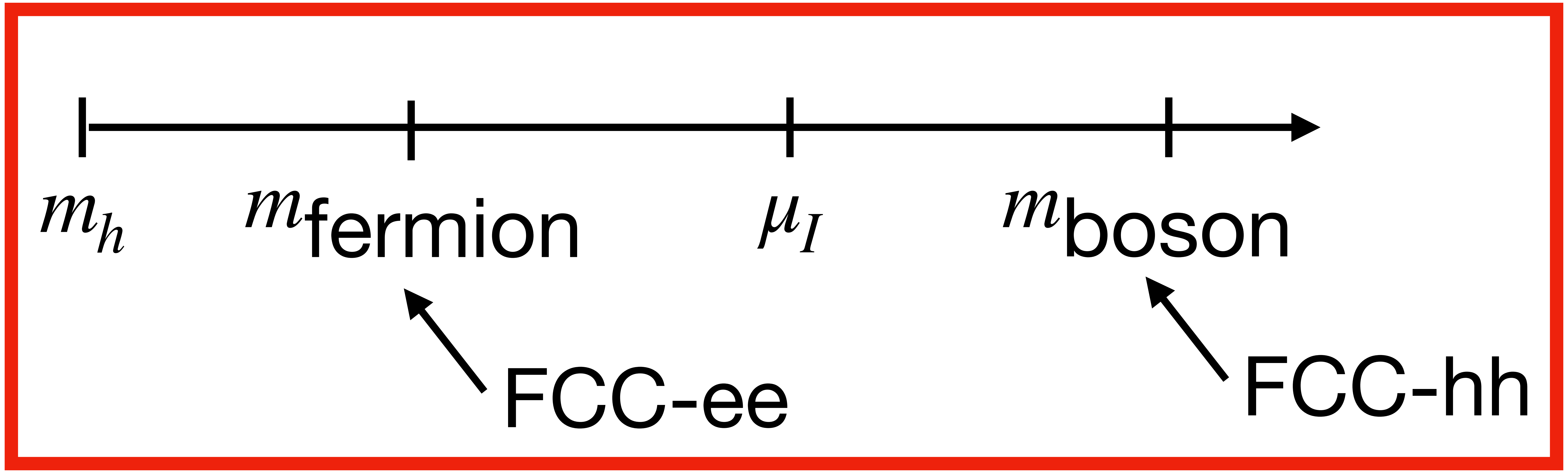
Metastability bound: $m_h^2 \lesssim |\beta_\lambda(\mu_I)|\,\mu_I^2 \ll \Lambda_{UV}^2$

↑ lowered by BSM physics (heavy sterile neutrinos)

→ check vacuum stability → additional constraints

FCC-ee and FCC-hh are complementary to test Higgs criticality

→



We contend that

[2503.03825]
(V. Enguita, B. Gavela, T. Steingasser)

Can you lower the instability scale? —> BSM

* The ALP path : I

Metastability bounds - ALPs

An altogether different model : **SM+ ALPs: instability scale lowered by only scalars**

Detering-You 2024

Higgs-ALP potential:

$$V(H,S) = -\frac{1}{2}m_H^2 H^2 + \frac{1}{4}\lambda H^4 + m_a^2 f^2 \left(1 - \cos(\frac{a}{f})\right) - \frac{1}{2}Af(H^2 - v^2)\cos\left(\frac{a}{f} - \delta\right)$$

Jeong, Jun, Shin 2018
Harigaya, Wang, 2022

[2503.03825]
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$$m_h^2 \leq m_{\text{crit}}^2 = -\beta_\lambda(\mu_I) e^{-3/2}\mu_I^2 + \frac{\mathrm{v}^2 \mathrm{A}^2 \sin^2\delta}{m_a^2}$$

$$\lambda_{\text{eff}} \equiv \lambda_H - \frac{1}{2}\frac{A^2 \sin^2\delta}{m_a^2}$$

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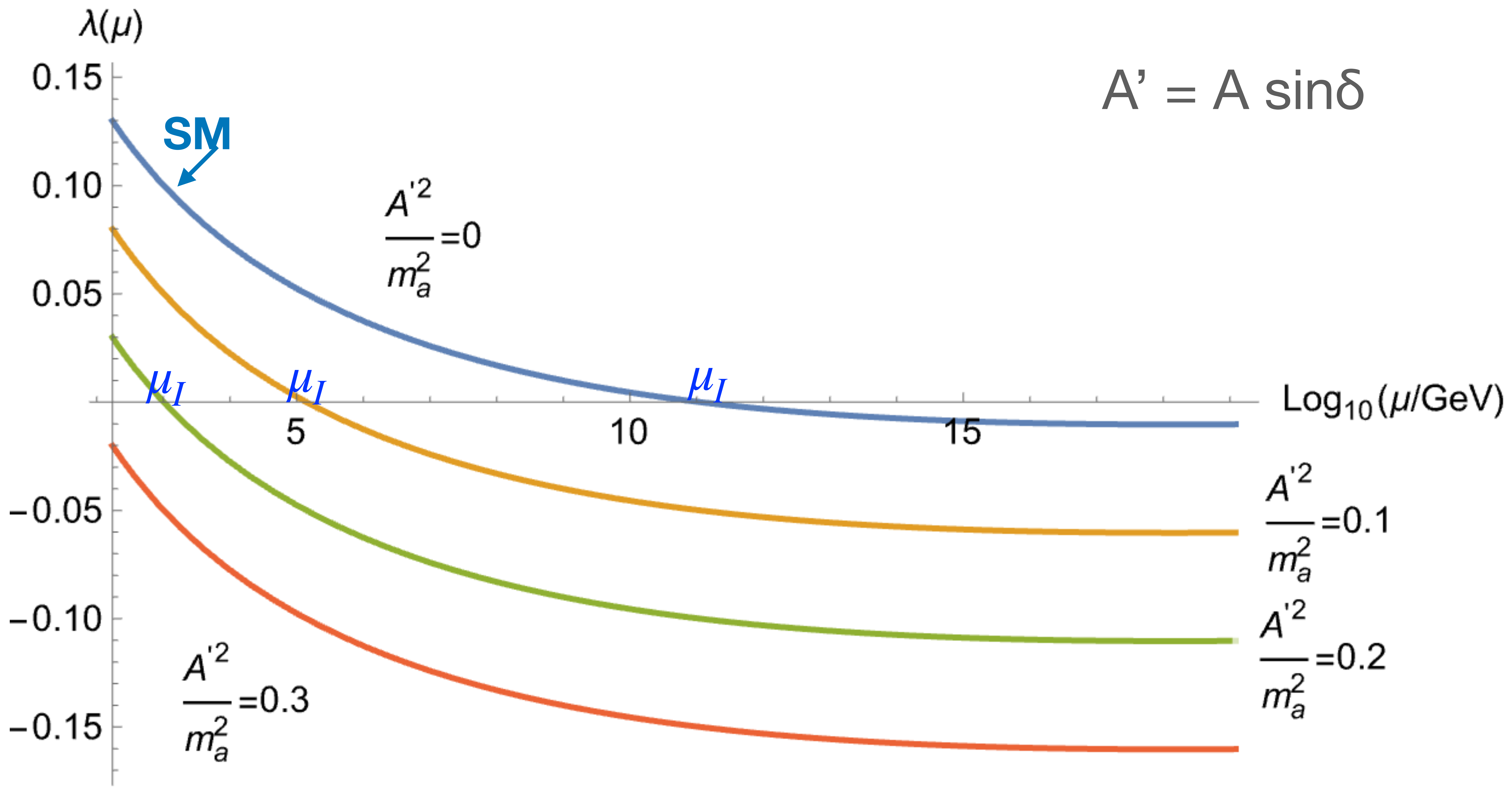
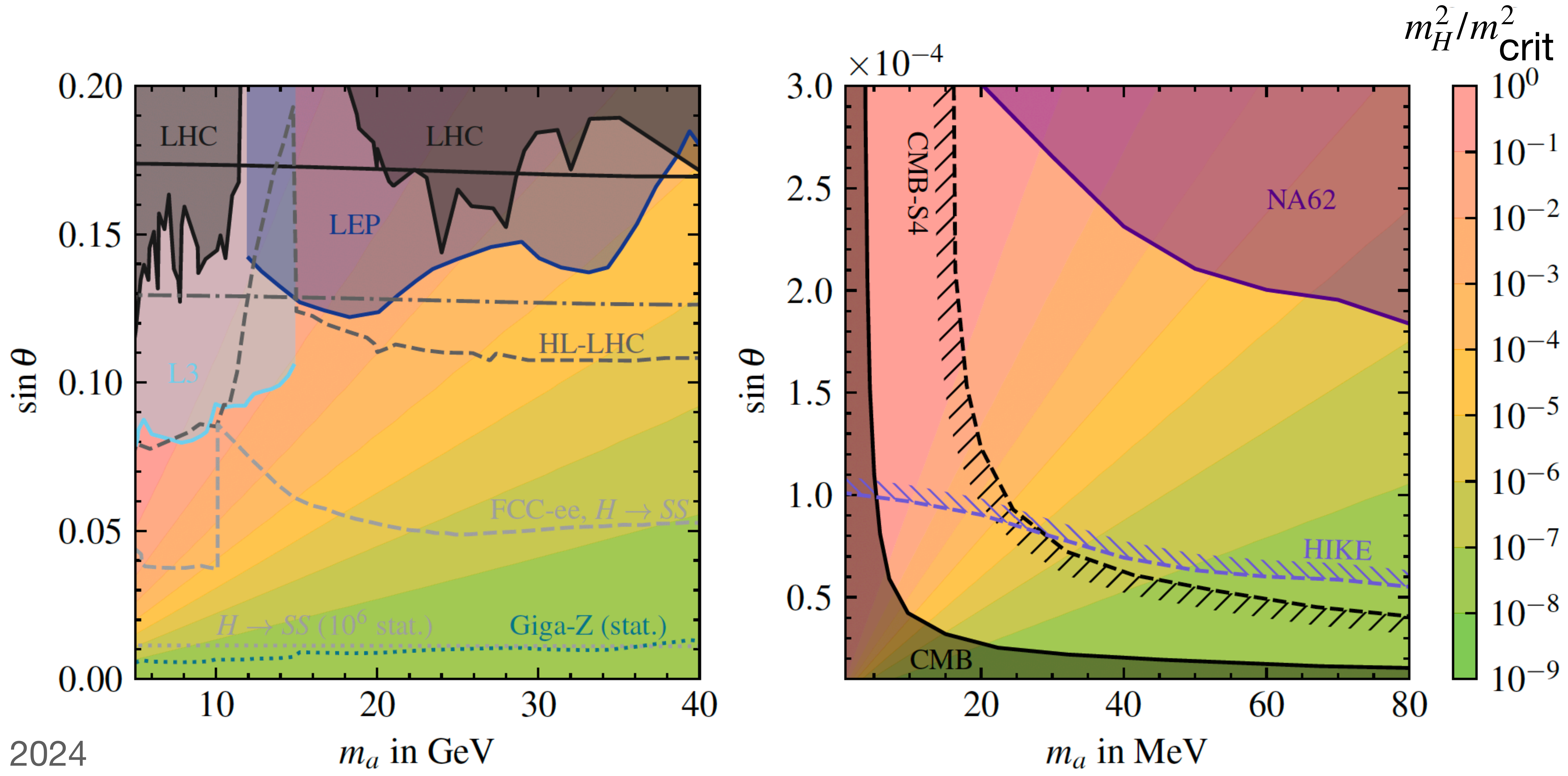
It is an important threshold correction

Metastability bounds - ALPs

A’ = A sinδ



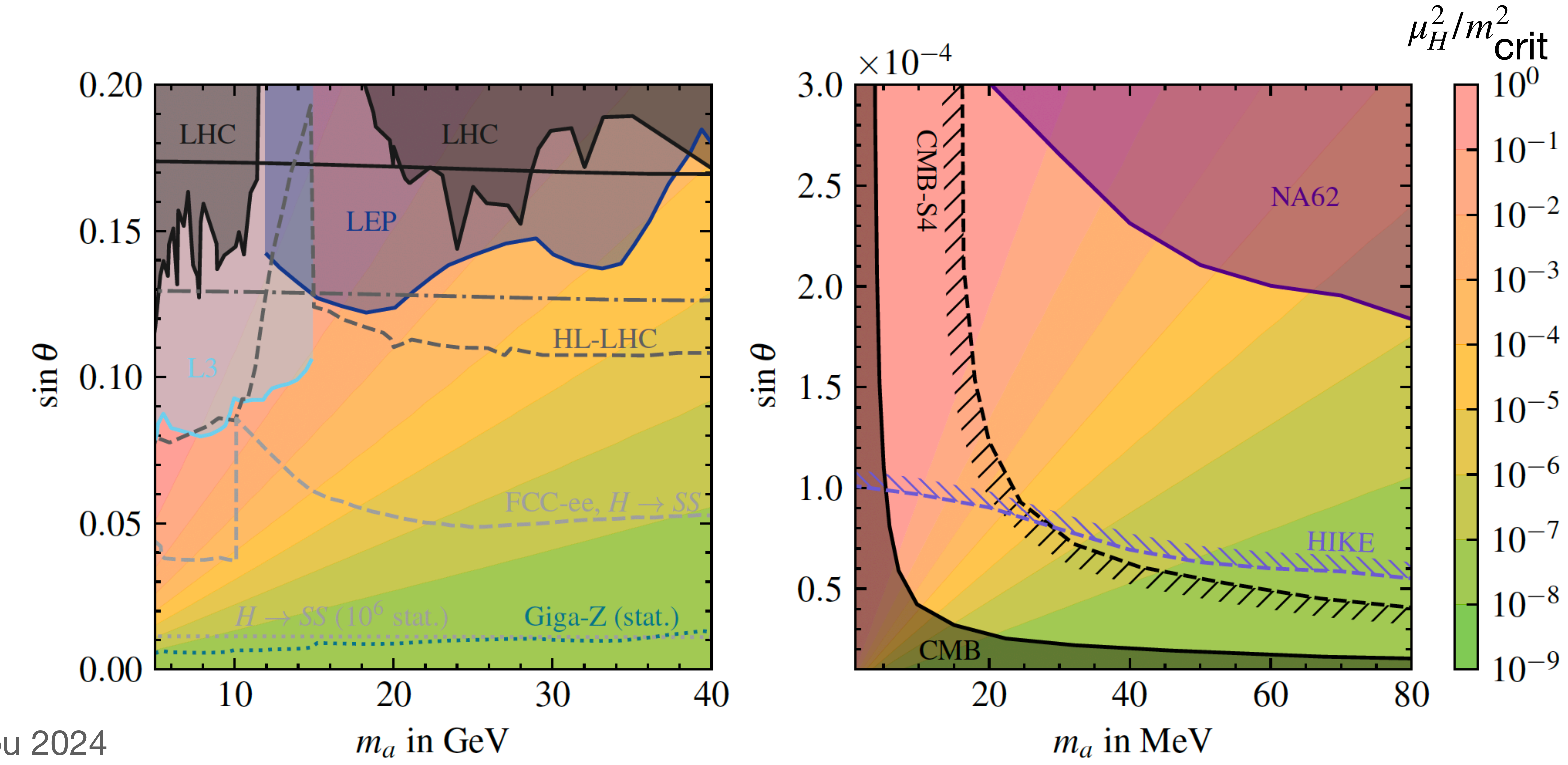
Metastability bounds - ALPs



Detering-You 2024

Figure 2: *Critical value of the Higgs mass parameter in the Axion-Higgs model. The contours of the remaining hierarchy between the observed Higgs mass parameter in the Axion-Higgs model and the metastability bound are shown from red (small hierarchy) to green (large hierarchy). Existing constraints on the parameter space are shaded and projected experimental sensitivities are indicated by dashed and dotted lines.* [9]

Metastability bounds - ALPs



Detering-You 2024

→ ALP mass in MeV - GeV range: $H \to aa, Zaa$ vertex, rare decays, CMB ...

Again the entire range to be covered at FCC-ee for GeV ALPs, and rare decays and/or CMB for MeV ALPs

Can you lower the instability scale? —> BSM

* The ALP path : II

Can you lower the instability scale? —> BSM

* Hold on! : in the Majoron model we also had an ALP that we disregarded = the Majoron pGB J

Can you lower the instability scale? —> BSM

*** Hold on! : in the Majoron model we also had an ALP that we disregarded = the Majoron pGB J**

$$S \equiv |S| e^{i\frac{J}{f}}$$

let us look at its possible potential

Metastability bounds - Majoron model with pGB J

$$V(H,S) = -\mu_H^2|H|^2 - \mu_S^2|S|^2 + \lambda_H|H|^4 + \lambda_S|S|^4 + \kappa\,|H|^2|S|^2 + V_{\not L}$$

$$S \equiv |S|\,e^{i\frac{J}{f}}$$

e.g. $$V_{\not L}(H,S,J) = \lambda_J\, e^{-i\delta}\, S^2\,|H|^2 + \text{h.c.} \quad = 2\lambda_J\,|H|^2\,|S|^2\,\cos\left(\frac{2J}{f} - \delta\right)$$

Frigerio, Hambye, Masso 2011

V. Enguita, B. Gavela,
T. Steingasser)

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$$V(H,S) = -\mu_H^2 |H|^2 - \mu_S^2 |S|^2 + \lambda_H |H|^4 + \lambda_S |S|^4 + \kappa\, |H|^2 |S|^2 \; + V_{\not{L}}$$

$$S \equiv |S|\, e^{i\frac{J}{f}}$$

e.g. $V_{\not{L}}(H,S,J) = \lambda_J\, e^{-i\delta}\, S^2\, |H|^2 \; + \mathrm{h.c.} \quad = 2\lambda_J\, |H|^2\, |S|^2\, \cos\left(\frac{2J}{f} - \delta\right)$

→ $m_J^2 = \lambda_J\, v^2$

$$\lambda \equiv \lambda_H - \frac{\kappa'^2}{4\lambda_S} = \lambda_H - \frac{(\kappa + 2\lambda_J)^2}{4\lambda_S}$$

Negligible correction for e.g. low m_J ; relevant for larger m_J ?

under construction

V. Enguita, B. Gavela, T. Steingasser)

Can you lower the instability scale? —> BSM

* The ALP path : III

Metastability and the ALP-SMEFT connection

*** The ALP path : III**

The ALP-SMEFT Interference and near-criticality

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Metastability and the ALP-SMEFT connection

$$\mathcal{L}_{\text{SM+ALP}}^{D\le 6} = c_{GG}\,\frac{a}{f}\,\frac{\alpha_s}{4\pi}\,G^A_{\mu\nu}\,\tilde{G}^{\mu\nu\,A}$$

$$+\, c_{WW}\,\frac{\alpha_L}{4\pi}\,\frac{a}{f}\,W^I_{\mu\nu}\,\tilde{W}^{\mu\nu\,I}$$

$$+\, c_{BB}\,\frac{\alpha_Y}{4\pi}\,\frac{a}{f}\,B_{\mu\nu}\,\tilde{B}^{\mu\nu}$$

$$+\,\frac{\partial^\mu a}{f}\sum_F \bar{\psi}_F\, c_F\,\gamma_\mu\,\psi_F$$

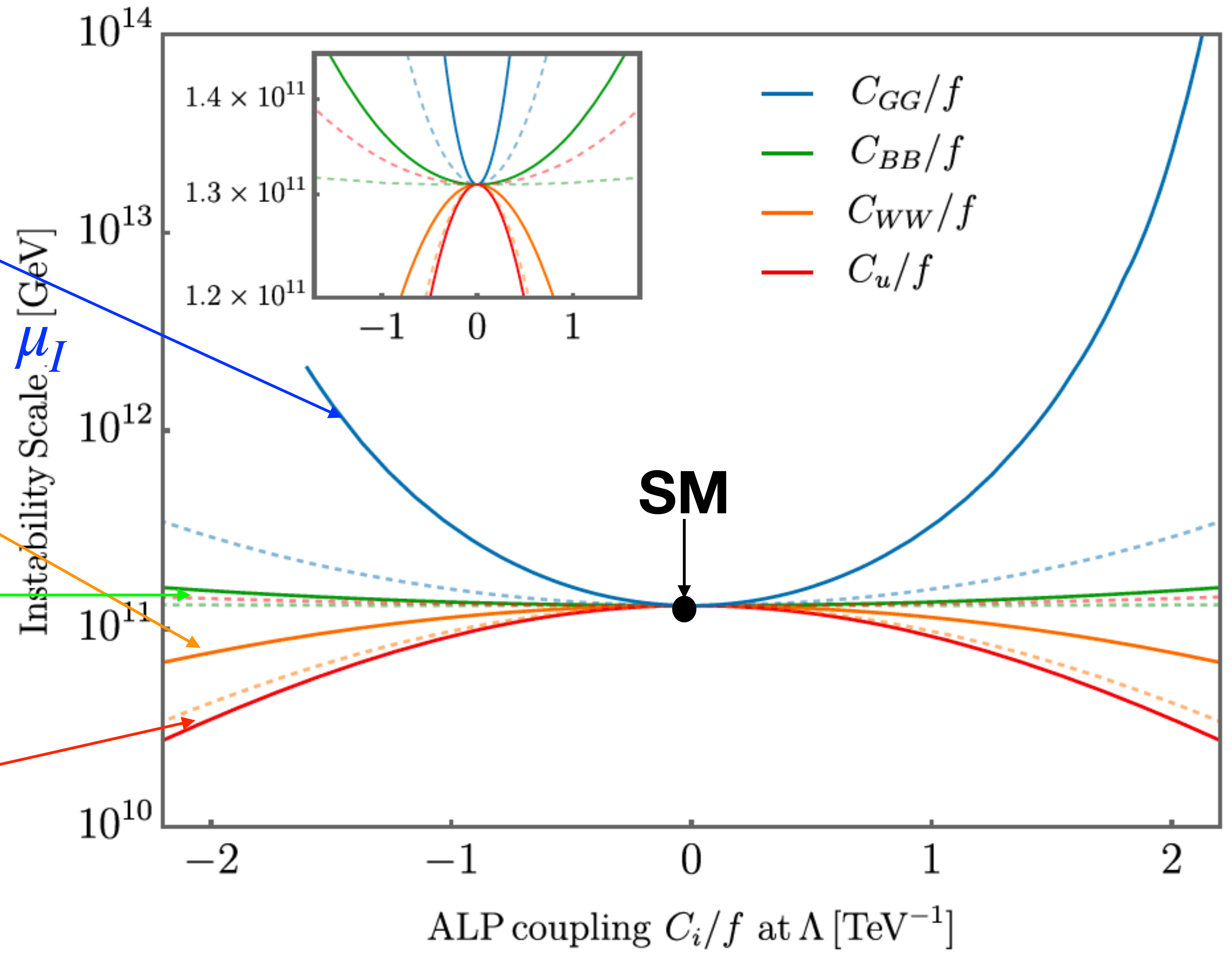

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Figure 5.1: Instability scale of the electroweak vacuum in the presence of nonzero ALP-SM couplings. The solid lines show the result for $m_a = 100$ GeV, while the dashed lines assume a vanishing ALP mass.

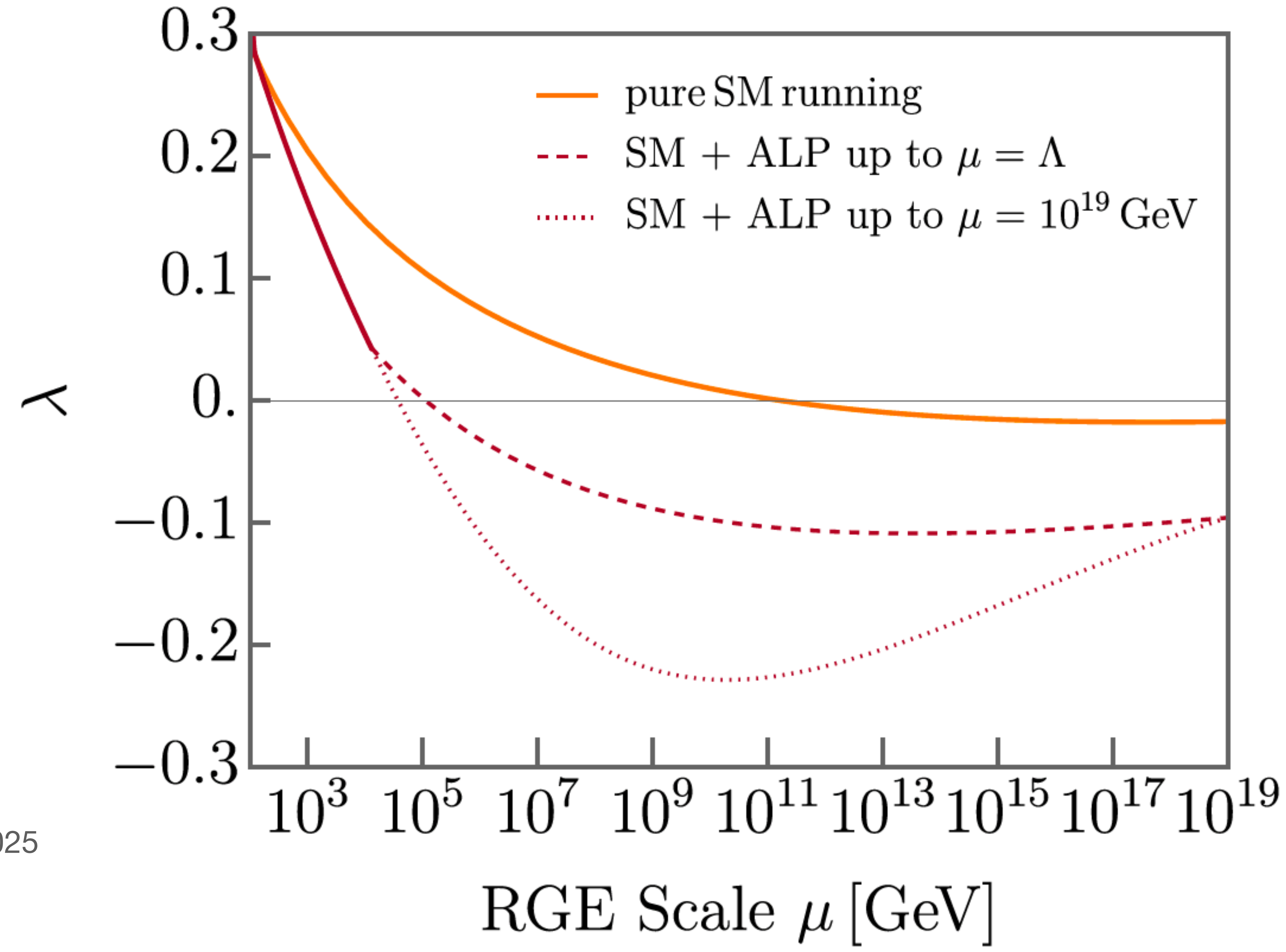

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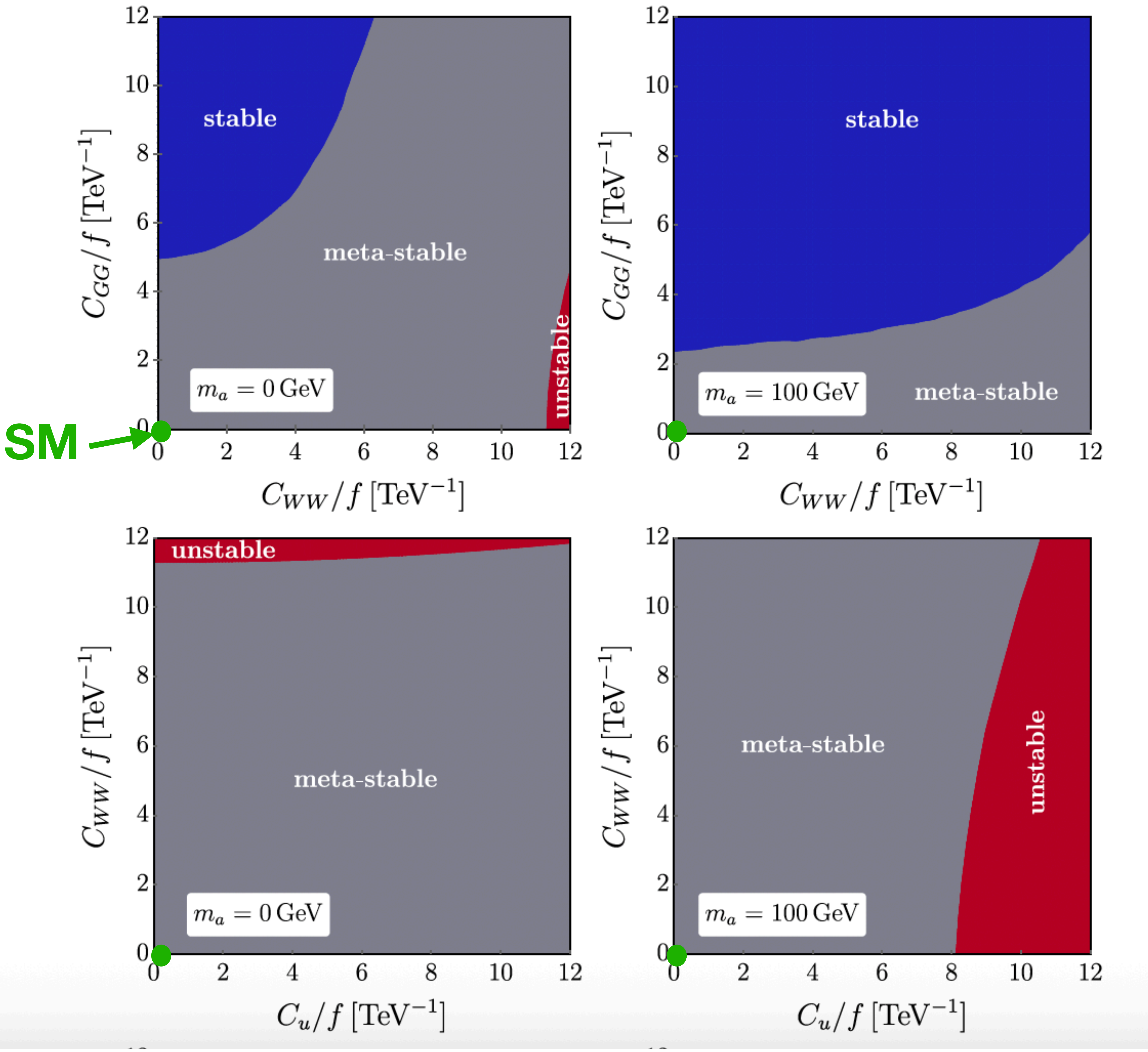
Figure 6.1: Scale evolution of λ for the case $C_{WW}/f = 12\,\text{TeV}^{-1}$, $m_a = 20\,\text{GeV}$. The red solid line shows the ALP + SM running until $\Lambda = 4\pi$ TeV. Above this scale, the dashed red line is obtained by taking only SM effects into account above this scale, while the brown dotted line employs ALP effects on the running up to the Planck scale. The orange line shows the pure SM case (no ALP) for comparison.

Metastability and the ALP-SMEFT connection



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Conclusions

If the low value of the Higgs mass is due to near-criticality

of the Higgs potential,

signals must be seen in the next round of experiments

???

Probably this idea is wrong,
But only those who wager can win

W. Pauli

Back up slides

Metastability bounds - Scalar sector of Majoron model

$$V = -m_H^2|H|^2 + \lambda_H|H|^4 - m_S^2|S|^2 + \lambda_S|S|^4 + \kappa|H|^2|S|^2$$

$$M_N < E < M_s$$

Tree-level analysis:

$$m_H^2 \to m_H^2 - \frac{\kappa}{2\lambda_S} \qquad \lambda_H^2 \to \lambda \equiv m_H^2 - \lambda_H \frac{\kappa^2}{2\lambda_S}$$

$$\mathcal{L}_{\text{eff}} \supset \frac{C_H}{\Lambda^2}\mathcal{O}_H + \frac{C_{H\Box}}{\Lambda^2}\mathcal{O}_{H\Box} + \frac{C_{HD}}{\Lambda^2}\mathcal{O}_{HD}$$

Metastability bounds - Scalar sector of Majoron model

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$$\mathcal{O}_H \equiv (H^\dagger H)^3, \quad \mathcal{O}_{H\Box} \equiv (H^\dagger H)\Box(H^\dagger H), \quad \mathcal{O}_{HD} \equiv (H^\dagger D^\mu H)^*(H^\dagger D_\mu H)$$

$$C_H^{(0)} = 0, \quad C_{H\Box}^{(0)} = -\frac{\kappa^2}{4\lambda_S}, \quad C_{HD}^{(0)} = 0,$$

$$g_{hhh} = 1 + \frac{3}{\sqrt{2}}\frac{1}{G_F M_s^2} C_{H\Box}^{(0)}$$

$$g_{hhhh} = 1 + \frac{50}{3\sqrt{2}}\frac{1}{G_F M_s^2} C_{H\Box}^{(0)}$$

Metastability bounds - Scalar sector of Majoron model

$$V = -m_H^2|H|^2 + \lambda_H|H|^4 - m_S^2|S|^2 + \lambda_S|S|^4 + \kappa|H|^2|S|^2$$

One-loop analysis:

Jiang, Craig, Li, Sutherland)

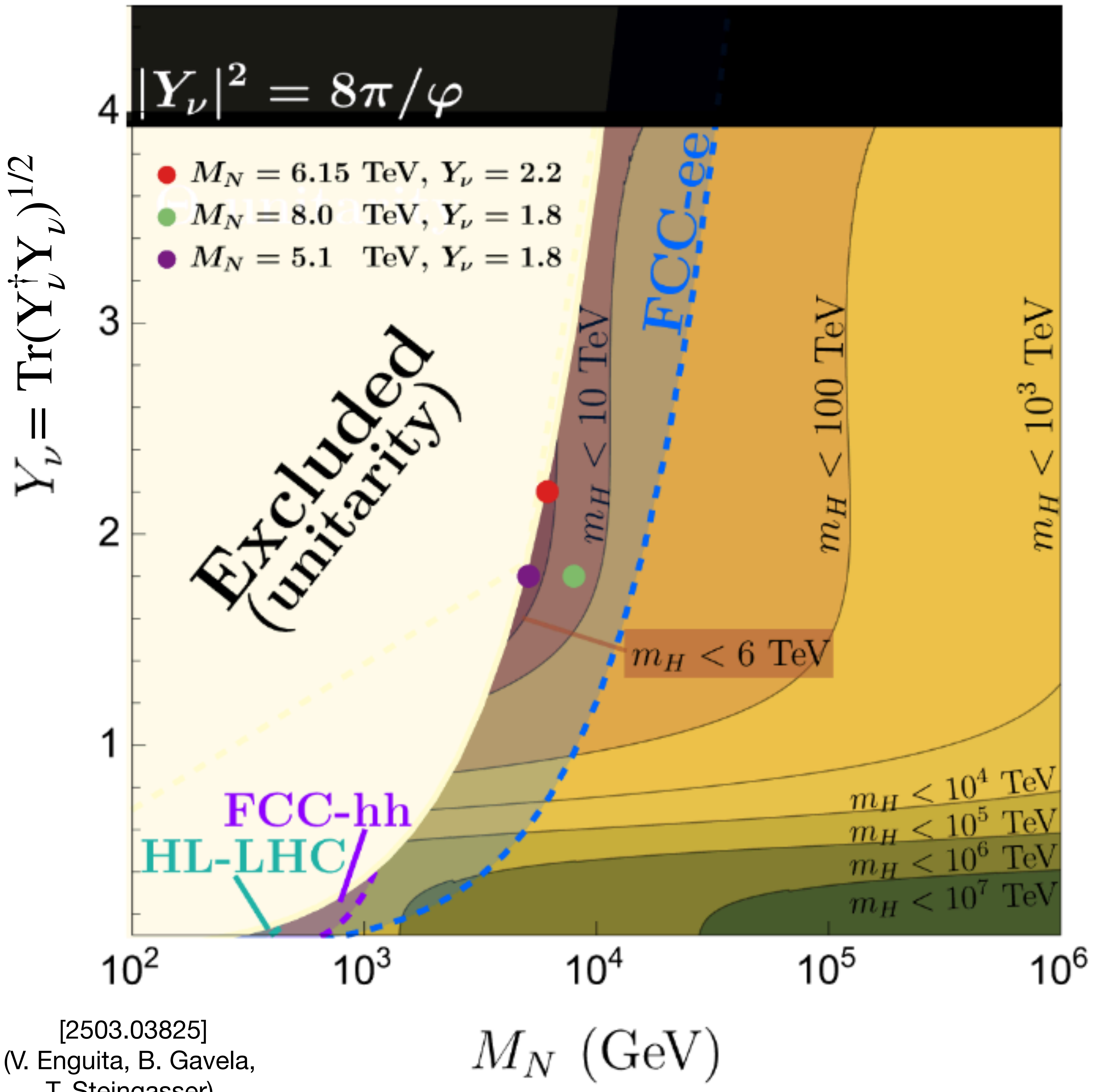
$$M_N < E < M_s$$

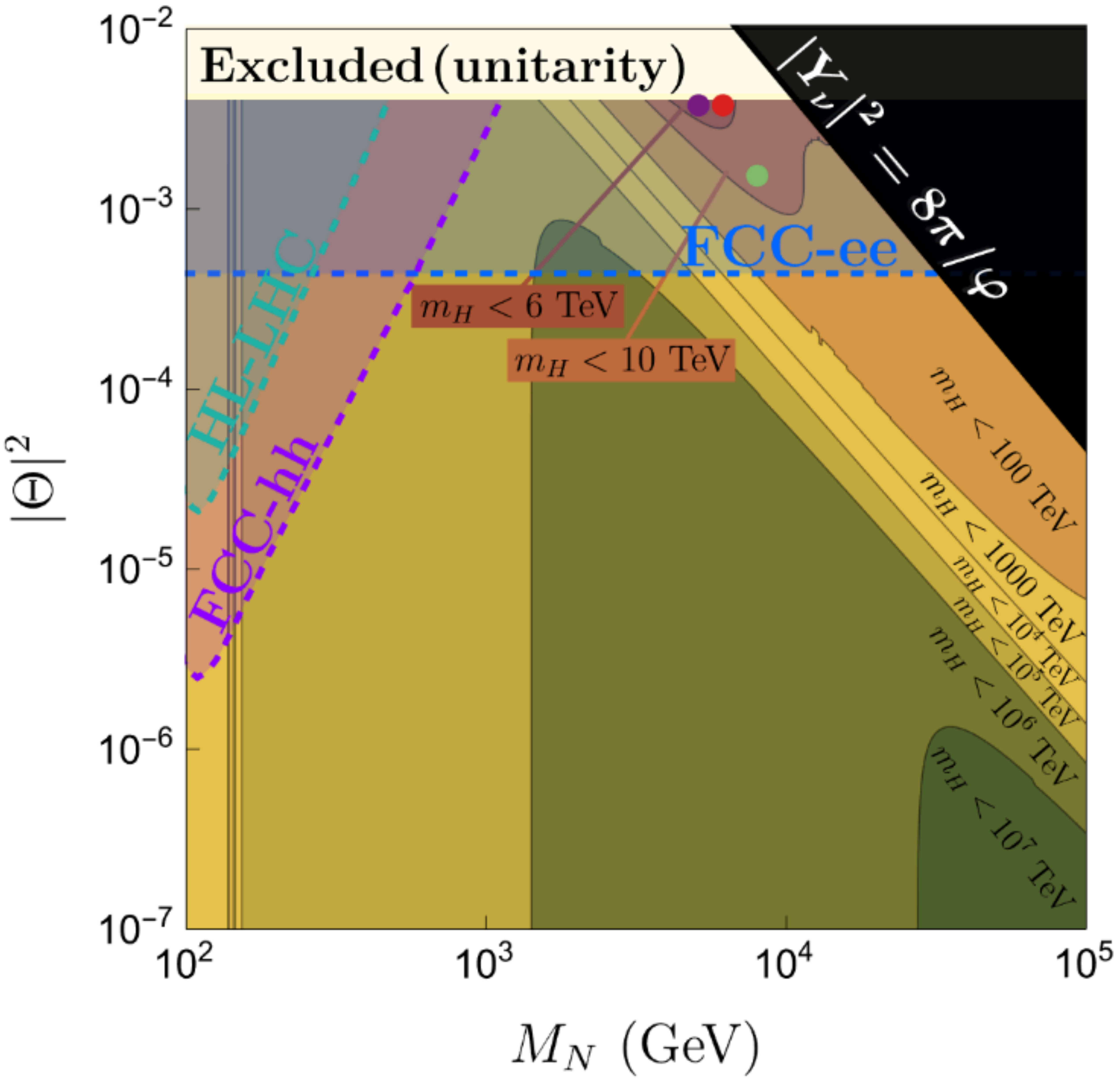
$\mathcal{O}_H = (H^\dagger H)^3$	$\mathcal{O}_{HW} = (H^\dagger H)W^a_{\mu\nu}W^{a\mu\nu}$
$\mathcal{O}_{H\Box} = (H^\dagger H)\Box(H^\dagger H)$	$\mathcal{O}_{HB} = (H^\dagger H)B_{\mu\nu}B^{\mu\nu}$
$\mathcal{O}_{HD} = (H^\dagger D^\mu H)^*(H^\dagger D_\mu H)$	$\mathcal{O}_{HWB} = (H^\dagger\sigma^a H)W^a_{\mu\nu}B^{\mu\nu}$
$\mathcal{O}_{uH} = (H^\dagger H)(\bar{q}u\widetilde{H})$	$\mathcal{O}_{He} = (H^\dagger i \overleftrightarrow{D} \mu H)(\bar{e}\gamma^\mu e)$
$\mathcal{O}_{dH} = (H^\dagger H)(\bar{q}dH)$	$\mathcal{O}^{(1)}_{Hq} = (H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{q}\gamma^\mu q)$
$\mathcal{O}_{eH} = (H^\dagger H)(\bar{\ell}eH)$	$\mathcal{O}^{(3)}_{Hq} = (H^\dagger i \overleftrightarrow{D}^a_\mu H)(\bar{q}\gamma^\mu\sigma^a q)$
$\mathcal{O}_{Hu} = (H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{u}\gamma^\mu u)$	$\mathcal{O}^{(1)}_{H\ell} = (H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{\ell}\gamma^\mu \ell)$
$\mathcal{O}_{Hd} = (H^\dagger i \overleftrightarrow{D} \mu H)(\bar{d}\gamma^\mu d)$	$\mathcal{O}^{(3)}_{H\ell} = (H^\dagger i \overleftrightarrow{D}^a_\mu H)(\bar{\ell}\gamma^\mu\sigma^a \ell)$

+ …

[2503.03825]
(V. Enguita, B. Gavela,
T. Steingasser)

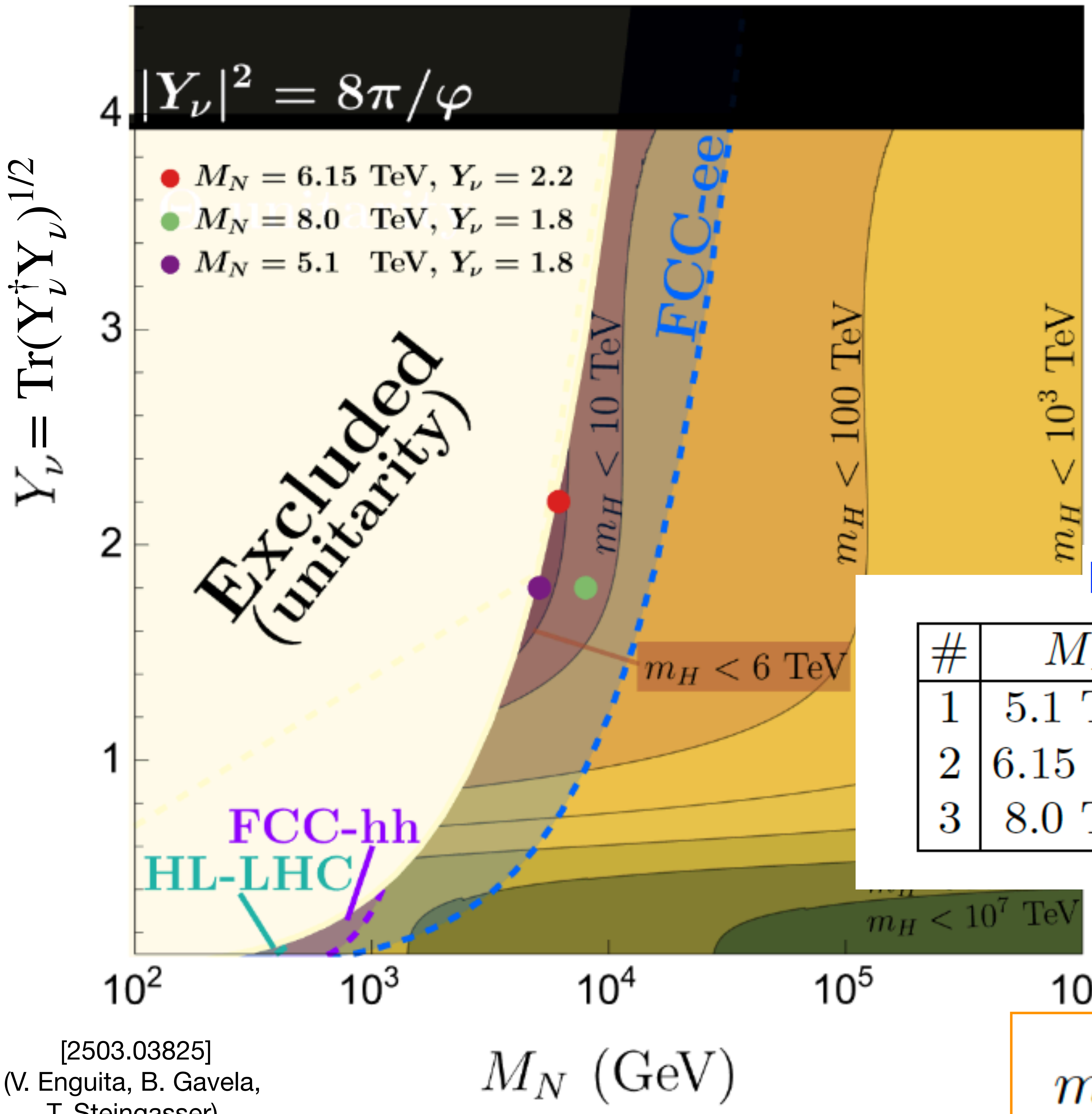
Metastability bound @FCC-ee Heavy sterile neutrinos

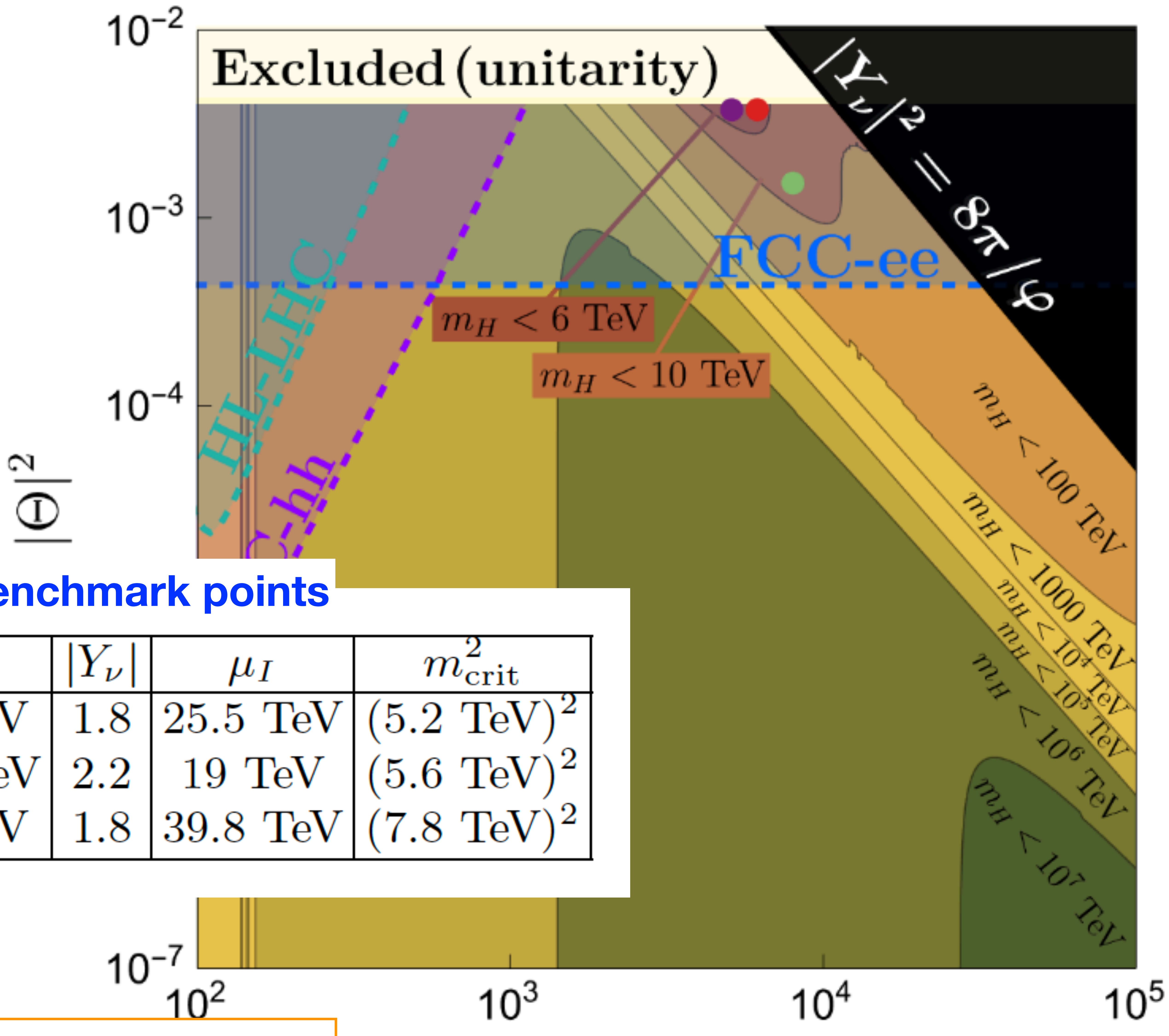




[2503.03825]
(V. Enguita, B. Gavela, T. Steingasser)

Metastability bound @FCC-ee Heavy sterile neutrinos





Benchmark points

#	M_N	$\lvert Y_\nu \rvert$	μ_I	m^2_{crit}
1	5.1 TeV	1.8	25.5 TeV	$(5.2\ \mathrm{TeV})^2$
2	6.15 TeV	2.2	19 TeV	$(5.6\ \mathrm{TeV})^2$
3	8.0 TeV	1.8	39.8 TeV	$(7.8\ \mathrm{TeV})^2$

$$m_N < \mu_I < M_s$$

[2503.03825]
(V. Enguita, B. Gavela, T. Steingasser)

$$\beta_\lambda \sim \lambda^2 + \lambda(y_t^2 - \text{gauge terms}) - y_t^4 + \text{gauge terms}$$

$$\beta_{y_t} \sim -y_t(\text{gauge terms} - y_t^2)$$

$$\lambda \to \lambda(\mu, \eta) \equiv \lambda(\mu) + \delta\lambda(\eta, \mu)$$

$$\begin{aligned}(4\pi)^2\delta\lambda(\mu,\eta) = &- \frac{15g^4}{32} - \frac{5g^2(g')^2}{16} - \frac{5(g')^4}{32} + \\ &+ \frac{9y_t^4}{2} + \frac{3}{8}g^4 \log\left(\frac{g^2}{4}\frac{\eta^2}{\mu^2}\right) + \\ &+ \frac{3}{16}\left(g^2 + (g')^2\right)^2 \log\left(\frac{g^2 + (g')^2}{4}\frac{\eta^2}{\mu^2}\right) + \\ &- 3y_t^4 \log\left(\frac{y_t^2}{2}\frac{\eta^2}{\mu^2}\right) + \frac{3|Y_\nu|^4}{2} - \\ &- |Y_\nu|^4 \log\left(\frac{|Y_\nu|^2}{2}\frac{\eta^2}{\mu^2}\right) . \qquad (4\end{aligned}$$

Heavy scalar sector

$$\beta^{(1)}_{\lambda_H} = +\frac{27}{200}g_1^4 + \frac{9}{20}g_1^2 g_2^2 + \frac{9}{8}g_2^4 + \lambda_{H\sigma}^2 - \frac{9}{5}g_1^2\lambda_H - 9g_2^2\lambda_H + 24\lambda_H^2 + 12\lambda_H y_t^2 + 4\lambda_H \mathrm{Tr}\left(Y_\nu Y_\nu^\dagger\right) - 6y_t^4 - 2\mathrm{Tr}\left(Y_\nu Y_\nu^\dagger Y_\nu Y_\nu^\dagger\right), \tag{C1}$$

Metastability bounds - Scalar sector of Majoron model

$$V = -m_H^2|H|^2 + \lambda_H|H|^4 - m_S^2|S|^2 + \lambda_S|S|^4 + \kappa|H|^2|S|^2$$

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$$\mathcal{L}_{\text{eff}} \supset \frac{C_H}{\Lambda^2}\mathcal{O}_H + \frac{C_{H\Box}}{\Lambda^2}\mathcal{O}_{H\Box} + \frac{C_{HD}}{\Lambda^2}\mathcal{O}_{HD}$$

$$\mathcal{O}_H \equiv (H^\dagger H)^3\,, \quad \mathcal{O}_{H\Box} \equiv (H^\dagger H)\Box(H^\dagger H)\,, \quad \mathcal{O}_{HD} \equiv (H^\dagger D^\mu H)^*(H^\dagger D_\mu H)$$

$$C_H^{(0)} = 0\,, \quad C_{H\Box}^{(0)} = -\frac{\kappa^2}{4\,\lambda_S}\,, \quad C_{HD}^{(0)} = 0\,,$$

$$g_{hhh} = 1 + \frac{3}{\sqrt{2}}\frac{1}{G_F M_s^2}C_{H\Box}^{(0)}$$

$$g_{hhhh} = 1 + \frac{50}{3\sqrt{2}}\frac{1}{G_F M_s^2}C_{H\Box}^{(0)}$$

$$C_H^{(0)} = 0\,, \qquad C_{H\Box}^{(0)} = -\frac{\kappa^2}{4\,\lambda_S}\,, \qquad C_{HD}^{(0)} = 0\,, \tag{31}$$

where the superscript (0) signals tree-level quantities. In addition, the term quadratic term |H| and the Higgs self-coupling in Eq. (4) also receive tree-level corrections:

$$\mu_H^2 \to \mu_H^2 - \frac{\kappa}{2\lambda_S}\,, \qquad \lambda_H \to \lambda \equiv \lambda_H - \frac{\kappa^2}{4\lambda_S}\,. \tag{32}$$

[6] For the SM couplings, we will use as initial conditions their values at the top mass scale as given in Ref. [71] and integrate their beta functions at three-loop accuracy, including the most important four-loop term for the strong gauge coupling g_s. At $\mu = M_N$, we additionally take into account the threshold corrections given in Ref. [72].

	I	II	II'	II"	III	III'	IV	IV'	V
MAXIMA	$\boldsymbol{P_0}$	$\boldsymbol{P_0}$	–	–	$\boldsymbol{P_0}$	–	$\boldsymbol{P_0}$	–	–
MINIMA	$\boldsymbol{P_1}, \boldsymbol{P_2}$	$\boldsymbol{P_3}$	$\boldsymbol{P_3}$	$\boldsymbol{P_3}$	$\boldsymbol{P_2}$	$\boldsymbol{P_2}$	$\boldsymbol{P_1}$	$\boldsymbol{P_1}$	$\boldsymbol{P_0}$
SADDLES	$\boldsymbol{P_3}$	$\boldsymbol{P_1}, \boldsymbol{P_2}$	$\boldsymbol{P_0}, \boldsymbol{P_2}$	$\boldsymbol{P_0}, \boldsymbol{P_1}$	$\boldsymbol{P_1}$	$\boldsymbol{P_0}$	$\boldsymbol{P_2}$	$\boldsymbol{P_0}$	–

$$\overline{\boldsymbol{P}}_0 = (0, 0) \ ,$$

$$\overline{\boldsymbol{P}}_1 = \left(\sqrt{\text{sign}[\mu_H^2]},\, 0\right) \ ,$$

$$\overline{\boldsymbol{P}}_2 = \left(0,\, \sqrt{\text{sign}[\mu_S^2]}\right) \ ,$$

$$\overline{\boldsymbol{P}}_3 = \left(\sqrt{\text{sign}[\mu_H^2]\, \frac{\Omega_H\,(\Omega_S - 1)}{\Omega_H\,\Omega_S - 1}}, \sqrt{\text{sign}[\mu_S^2]\, \frac{\Omega_S\,(\Omega_H - 1)}{\Omega_H\,\Omega_S - 1}}\right)$$

where

$$\Omega_H \equiv 2\,\frac{\mu_S^2}{\mu_H^2}\cdot\frac{\lambda_H}{\kappa}\,, \quad \text{and} \quad \Omega_S \equiv 2\,\frac{\mu_H^2}{\mu_S^2}\cdot\frac{\lambda_S}{\kappa}\,.$$

$$\lambda_H, \lambda_S > 0\,, \ \text{for } \kappa \geq 0\,,$$

$$\lambda_H, \lambda_S > 0 \ \text{ and } \ \lambda_S\,\lambda_H > \frac{\kappa^2}{4}\,, \ \text{for } \kappa < 0\,.$$

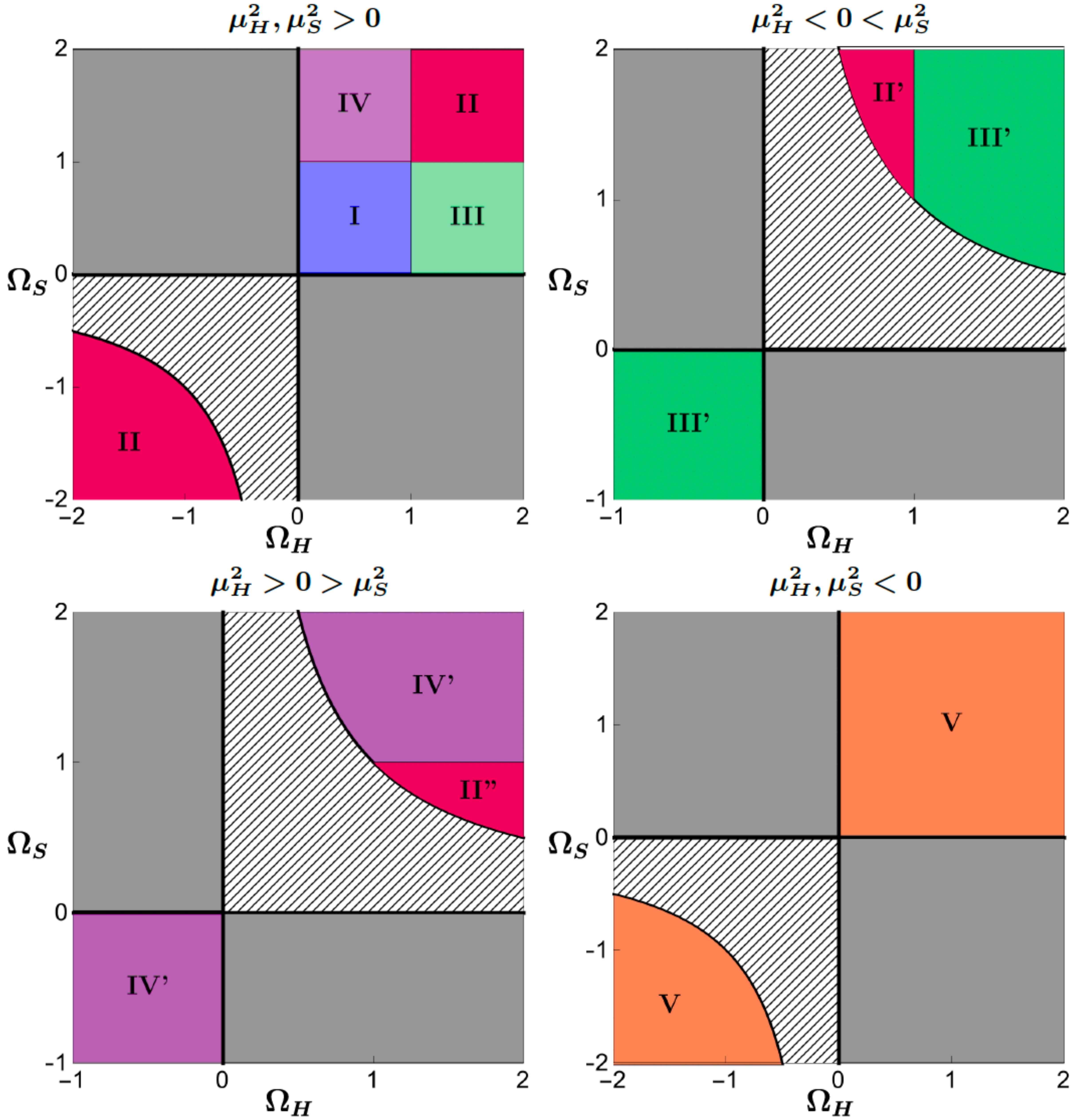


FIG. 1: Phase space (Ω_H, Ω_S). The Phases **I-V**, **II'-IV'** and **II"** correspond to all possible configurations of the stationary points of the potential in Eq. (4) under the assumption of stability. Grey regions are not accessible under that assumption. Hatched regions are unstable, see text.

This equation shows that the requirements of a small Higgs mass and a negative quartic coupling in the intermediate energy region imply

$$\mu_H^2 \gtrsim \frac{\kappa}{2\lambda_S}\mu_S^2 \qquad \text{and} \qquad \kappa^2 > 4\lambda_H\lambda_S. \tag{37}$$

In terms of the parameters Ω_H and Ω_S, this corresponds to

$$\Omega_H \lesssim 4\frac{\lambda_S\lambda_H}{\kappa^2} < 1\,, \quad \Omega_S \gtrsim 1 \quad \text{and} \quad \kappa > 0\,, \tag{38}$$

see Eqs. (13)-(15). The conditions in Eq. (38) eliminate two of the three vacuum configurations identified as *a priori* suitable in Sec. II: the first and third condition are not simultaneously satisfied by phase **II**, while the last one is not satisfied by phase **II**', see Fig. 2. This leaves **IV** as the optimal tree-level configuration.

Metastability bounds - Scalar sector of Majoron model

$$V = -m_H^2|H|^2 + \lambda_H|H|^4 - m_S^2|S|^2 + \lambda_S|S|^4 + \kappa|H|^2|S|^2$$

Compute the Euclidean action of the two-field instanton (*H_I*, *S_I*)

$$\frac{\mathrm{d}^2}{\mathrm{d}\rho^2}(H_I, S_I) + \frac{3}{\rho}\frac{\mathrm{d}}{\mathrm{d}\rho}(H_I, S_I) = \nabla_{H,S}V(H_I, S_I)$$

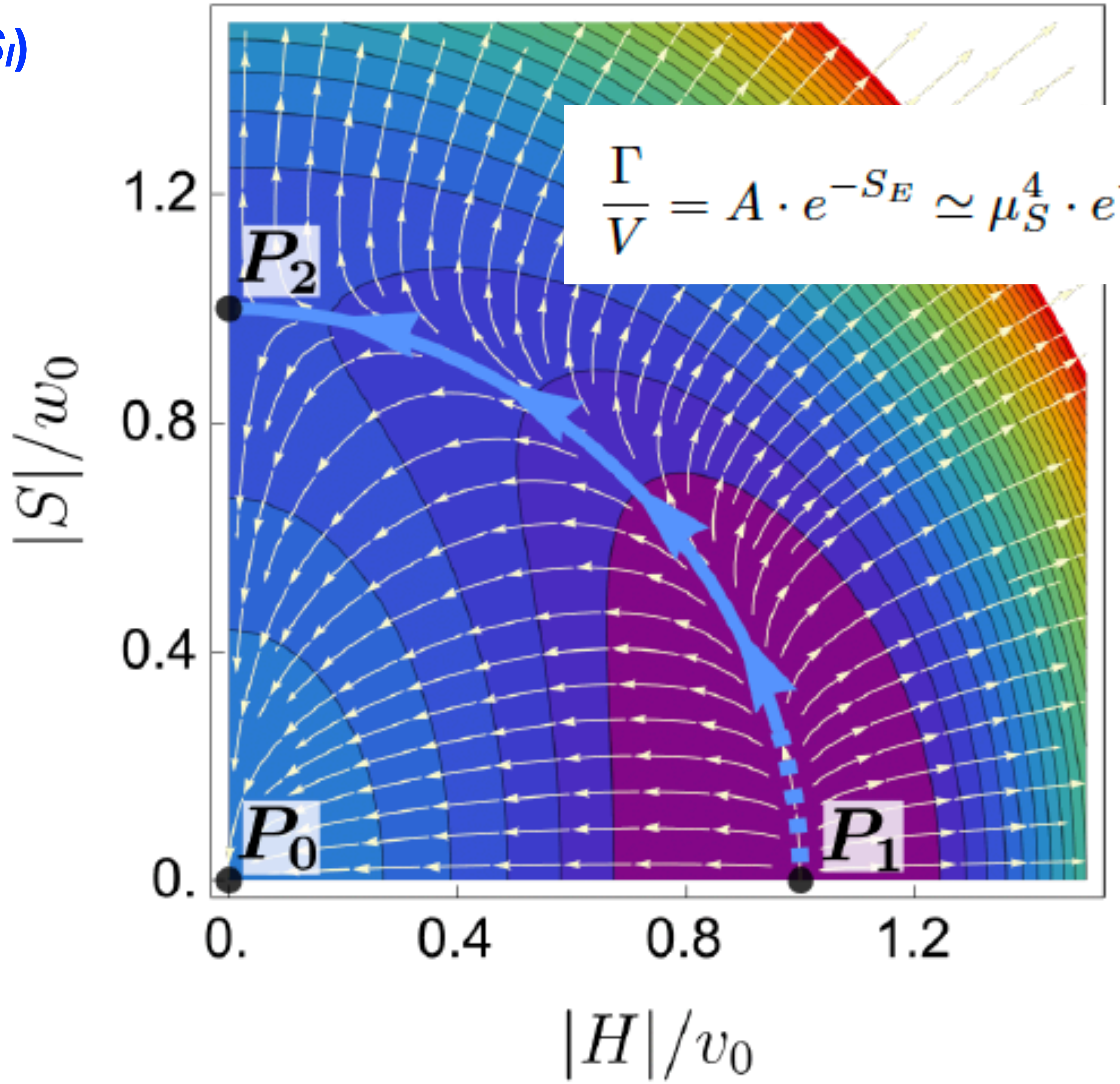
O(4)-symmetric ansatz $\rho^2 = t^2 + \mathbf{x}^2$

Approximate $(H_I(\rho), S_I(\rho) = H_\gamma(\eta_I(\rho)), S_\gamma(\eta_I(\rho)))$

where $\eta_I(\rho)$ is the instanton in the effect. potential $\boldsymbol{V}_\gamma$

$$\frac{\mathrm{d}^2}{\mathrm{d}\rho^2}\eta_I + \frac{3}{\rho}\frac{\mathrm{d}}{\mathrm{d}\rho}\eta_I = \frac{\mathrm{d}}{\mathrm{d}\eta}V_\gamma(\eta_I)$$

$$S_E = \int \mathrm{d}^4x\, \frac{1}{2}\dot{\eta}_I^2 + \frac{1}{2}|\nabla\eta_I|^2 + V_\gamma(\eta_I)$$



$$\frac{\Gamma}{V} = A \cdot e^{-S_E} \simeq \mu_S^4 \cdot e^{-S_E}$$

Hence, it is necessary to compute the Euclidean action of the two-field instanton (H_I, S_I) connecting the metastable vacuum near $\boldsymbol{P_2}$ with the basin surrounding the true vacuum $\boldsymbol{P_1}$. Using the standard Ansatz of an $O(4)$-symmetric solution, i.e., $H_I(\rho)$ and $S_I(\rho)$ with $\rho^2 = t^2 + \mathbf{x}^2$, the Euclidean equations of motion can be interpreted as the motion of a point particle in the inverted two-dimensional potential while subject to a time-dependent friction,

$$\frac{\mathrm{d}^2}{\mathrm{d}\rho^2}(H_I, S_I) + \frac{3}{\rho}\frac{\mathrm{d}}{\mathrm{d}\rho}(H_I, S_I) = \nabla_{H,S} V(H_I, S_I). \tag{58}$$

We can now simplify this system of equations by recalling that the true and false vacuum are connected through a steep valley γ, which translates to a narrow ridge in the imaginary-time picture. It is now easy to see that the only path along which the "particle" can roll towards the true vacuum needs to be close to γ, as it would otherwise develop some runaway behavior away from the false vacuum, see Fig. 8. This suggests that, to leading order, we can approximate the shape of the instanton by $(H_I(\rho), S_I(\rho) = H_\gamma(\eta_I(\rho)), S_\gamma(\eta_I(\rho)))$, where $\eta_I(\rho)$ is the instanton in the effective potential V_γ [8],

$$\frac{\mathrm{d}^2}{\mathrm{d}\rho^2}\eta_I + \frac{3}{\rho}\frac{\mathrm{d}}{\mathrm{d}\rho}\eta_I = \frac{\mathrm{d}}{\mathrm{d}\eta}V_\gamma(\eta_I). \tag{59}$$

The Euclidean action along this contour is then defined through

$$S_E = \int \mathrm{d}^4x \, \frac{1}{2}\dot{\eta}_I^2 + \frac{1}{2}|\nabla\eta_I|^2 + V_\gamma(\eta_I)\,. \tag{60}$$

$$\frac{\Gamma}{V} = A \cdot e^{-S_E} \simeq \mu_S^4 \cdot e^{-S_E}$$

SM: $$S_E = \frac{8\pi^2}{3|\lambda(\mu_S)|}$$

For a given vacuum decay rate per unit volume, the lifetime of the vacuum is defined as the time after which the probability that a vacuum bubble has nucleated within the past lightcone of any observer is ~1

$$1 \sim \int_{\mathcal{P}} \mathrm{d}^4x \, \frac{\Gamma}{V}$$

$$V_{\mathcal{P}} = \frac{0.15}{H_0^4} = 2.2 \cdot 10^{163} (\mathrm{GeV})^{-4}.$$

Lower bound on the euclidean action: $$S_E > 367.104 + 4 \ln \left(\frac{\mu_S}{\mathrm{GeV}} \right)$$

Metastability and the ALP-SMEFT connection

$$\mathcal{L}^{D\leq 6}_{\rm SM+ALP} = c_{GG}\,\frac{a}{f}\,\frac{\alpha_s}{4\pi}\,G^A_{\mu\nu}\,\tilde{G}^{\mu\nu\,A} + c_{WW}\,\frac{\alpha_L}{4\pi}\,\frac{a}{f}\,W^I_{\mu\nu}\,\tilde{W}^{\mu\nu\,I} + c_{BB}\,\frac{\alpha_Y}{4\pi}\,\frac{a}{f}\,B_{\mu\nu}\,\tilde{B}^{\mu\nu}$$
$$+ \frac{\partial^\mu a}{f}\sum_F \bar{\psi}_F\,\boldsymbol{c}_F\,\gamma_\mu\,\psi_F + \frac{C_{HH}}{f^2}\,(\partial^\mu a)(\partial_\mu a)\,H^\dagger H\,. \qquad ($$

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Heavy fermion sector

An example of U(1)$_L$- protected low-scale seesaw

$$\boldsymbol{Y}_R = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & Y_R \\ 0 & Y_R & 0 \end{pmatrix} , \quad \boldsymbol{Y}_\nu = \begin{pmatrix} 0 & Y_{12} & 0 \\ 0 & Y_{22} & 0 \\ 0 & Y_{32} & 0 \end{pmatrix}$$

I.e. only the second RHN couples to the SM

$$\mathrm{Tr}(\boldsymbol{Y}_\nu^\dagger \boldsymbol{Y}_\nu \boldsymbol{Y}_\nu^\dagger \boldsymbol{Y}_\nu) = \mathrm{Tr}(\boldsymbol{Y}_\nu^\dagger \boldsymbol{Y}_\nu)^2 = |Y_\nu|^4$$

$$|Y_\nu|^2 \equiv |Y_{12}|^2 + |Y_{22}|^2 + |Y_{32}|^2$$

$$\Theta_\nu^a = \frac{\boldsymbol{Y}_\nu^{*a2}}{\sqrt{2}} \frac{v}{M_N} \quad \text{and} \quad |\Theta_\nu|^2 \equiv \sum_a |\Theta^a|^2$$

Ref. [40]. In the latter, the approximate symmetry enforces a vanishingly small value for the combination of matrices in Eq. (20), while the individual entries in the $\boldsymbol{Y}_\nu$ matrix can be $\mathcal{O}(1)$. The main results of our paper will hold irrespective of the specific choice of low-scale Majoron model. In practice, for the numerical analyses we will use a version of the SPSS where the approximate $U(1)_L$ symmetry is displayed by the choice:

$$\boldsymbol{Y}_R = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & Y_R \\ 0 & Y_R & 0 \end{pmatrix}, \qquad \boldsymbol{Y}_\nu = \begin{pmatrix} 0 & Y_{12} & 0 \\ 0 & Y_{22} & 0 \\ 0 & Y_{32} & 0 \end{pmatrix}, \tag{21}$$

in which only $\nu_{R_{2,3}}$ couple to the SM and receive heavy degenerate masses,

$$M_N \equiv \frac{Y_R\, w}{2\sqrt{2}}, \tag{22}$$

while ν_{R_1} remains secluded as well as massless. This choice leads to very simple analytical expressions, because the RG analysis is only sensitive to the combinations [10]

$$\mathrm{Tr}(\boldsymbol{Y}_\nu^\dagger \boldsymbol{Y}_\nu \boldsymbol{Y}_\nu^\dagger \boldsymbol{Y}_\nu) = \mathrm{Tr}(\boldsymbol{Y}_\nu^\dagger \boldsymbol{Y}_\nu)^2 = |Y_\nu|^4, \tag{23}$$

where

$$|Y_\nu|^2 \equiv |Y_{12}|^2 + |Y_{22}|^2 + |Y_{32}|^2. \tag{24}$$

In turn, the $\nu_{R_i} - \nu_{L_i}$'s mixing is then characterized by the following mixing angles:

$$\Theta_\nu^a = \frac{\boldsymbol{Y}_\nu^{*a2}}{\sqrt{2}} \frac{v}{M_N} \qquad \text{and} \qquad |\Theta_\nu|^2 \equiv \sum_a |\Theta^a|^2, \tag{25}$$

νSMEFT

$$M_N \; < \; E \; < \; M_s$$

d=5:

$$\mathcal{O}_{NH} = \bar{N}_i N_j^c H^\dagger H + h.c.\,,$$

$$\text{with} \qquad \frac{\boldsymbol{C}_{NH}^{(0)}}{\Lambda} = -2\sqrt{2}\,\kappa\,\frac{\boldsymbol{M}_N}{M_s^2}$$

d=6:

$$\mathcal{O}_{NN} = (\bar{N} N^c)(\bar{N} N^c)\,,$$

$$\text{with} \qquad \frac{\boldsymbol{C}_{NN}^{(0)}}{\Lambda^2} = 24\,\lambda_S\,\frac{\boldsymbol{M}_N^2}{M_s^4}$$

Metastability bound —> BSM

[2105.08617]
(G. Giudice, M. McCullough, T. You)

"Self-organized localisation"

[2108.09315]
(J. Khoury, TS)

Metastability +SSB*

$$m_h^2 \lesssim |\beta_\lambda(\mu_I)|\,\mu_I^2 \ll \Lambda_{\mathrm{UV}}^2$$

Landscape +crunching

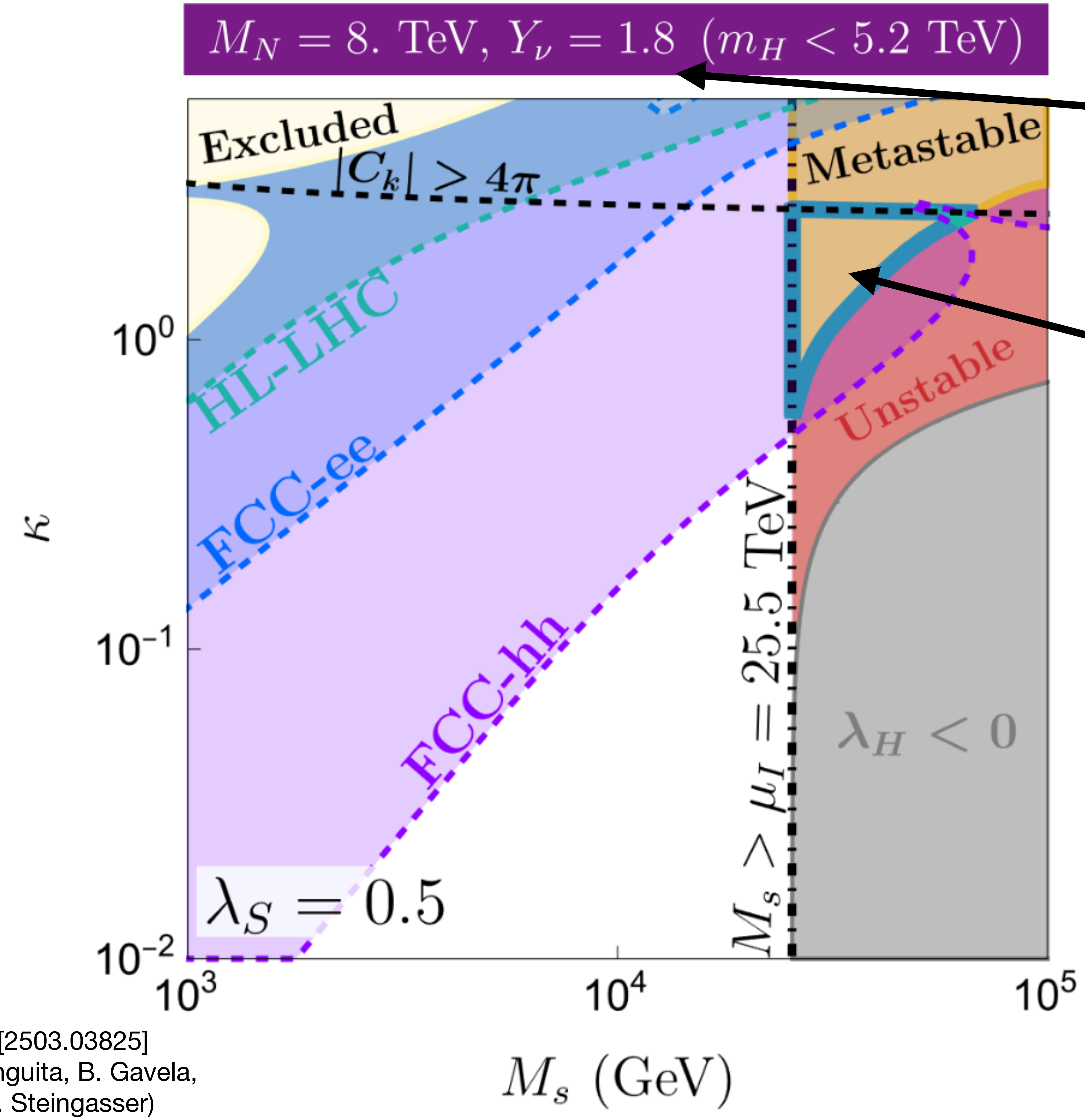
Landscape

[2502.07876]
(S. Benevedes, A. Ismail, TS)

[1907.07693]
(J. Khoury, O. Parrikar)

Metastability bound @FCC - hh Majoron scalar sector

$m_N < \mu_I < M_s$



[2503.03825]
(V. Enguita, B. Gavela, T. Steingasser)

strong destabilization

strong stabilization

large κ or "small" M_S

strong signal:

$$\mathscr{L}_{H\Box} = -\frac{\kappa^2}{4\lambda_S M_S^2} |H|^2 \Box |H|^2$$

Metastability bounds - BSM features for fermion path

General: Smaller μ_I → Shorter lifetime

$\mu_I \sim \mathcal{O}(\text{TeV})$ → lifetime < age of the universe

Ingredients for a strong Higgs bound and viable universe:

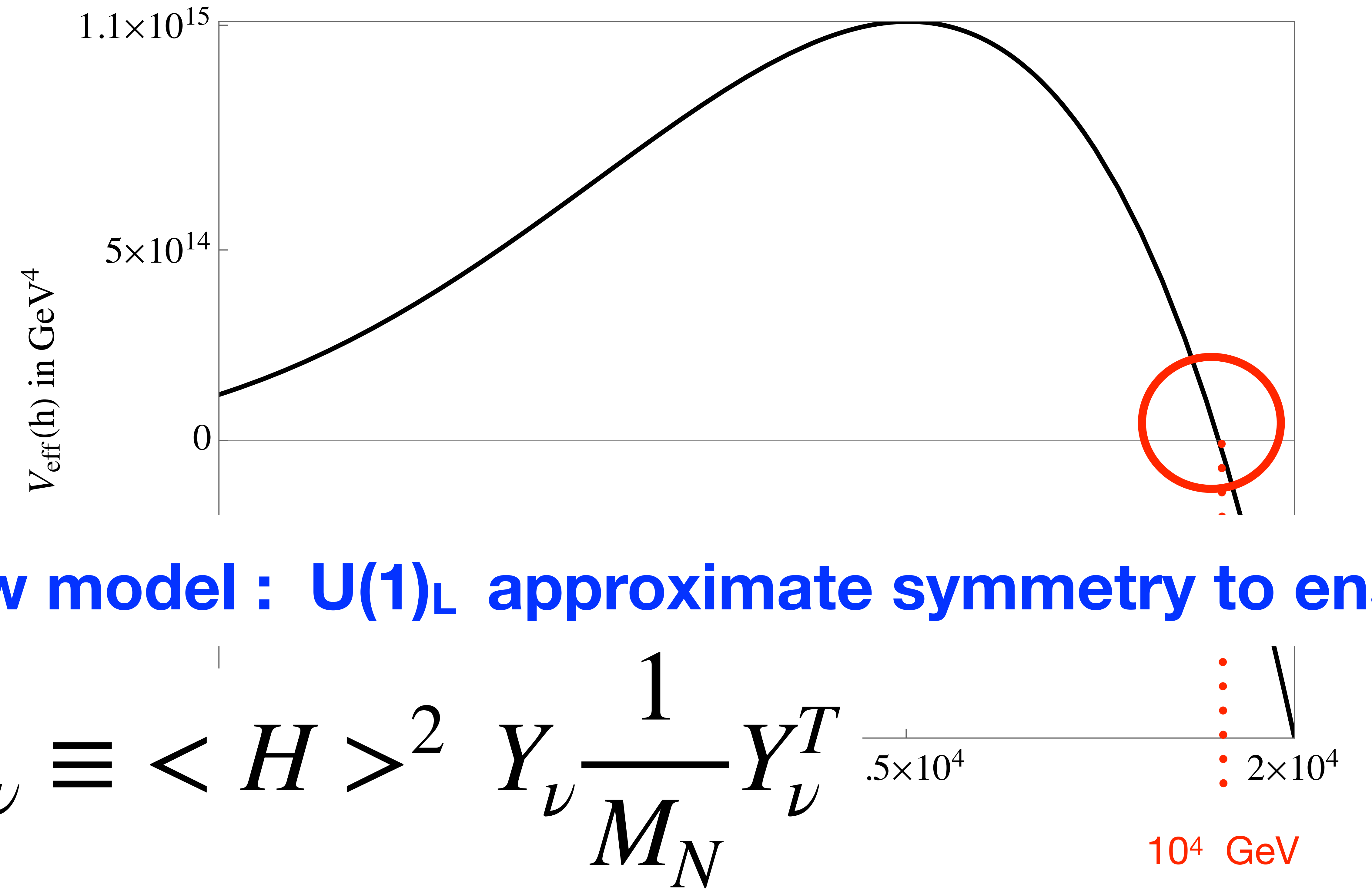
* Heavy fermions to lower μ_I to $\mu_I \sim \mathcal{O}(\text{TeV})$
* Some scalar input to stabilise the vacuum enough

Can you lower the instability scale? —> BSM

Higgs Potential: $V_{\rm eff}(H) = -\frac{1}{2} m_{\rm eff}^2 |H|^2 + \frac{1}{4}\lambda_{\rm eff} |H|^4$ +Type-I seesaw

$$\mathcal{L}_{N_R} = \overline{N_R}\partial\!\!\!/ N_R - \bar{\ell}_L \mathbf{Y}_\nu \tilde{H} N_R - \frac{1}{2}\overline{N_R^c}\mathbf{Y}_R M_N N_R + \text{h.c.}$$

Very easy: e.g.
heavy neutrinos



$\mathcal{O}$ (TeV) **scale requires a low-scale seesaw model : U(1)$_L$ approximate symmetry to ensure**

light neutrino masses even with Y_ν~1: $M_\nu \equiv \langle H \rangle^2 \; Y_\nu \frac{1}{M_N} Y_\nu^T$

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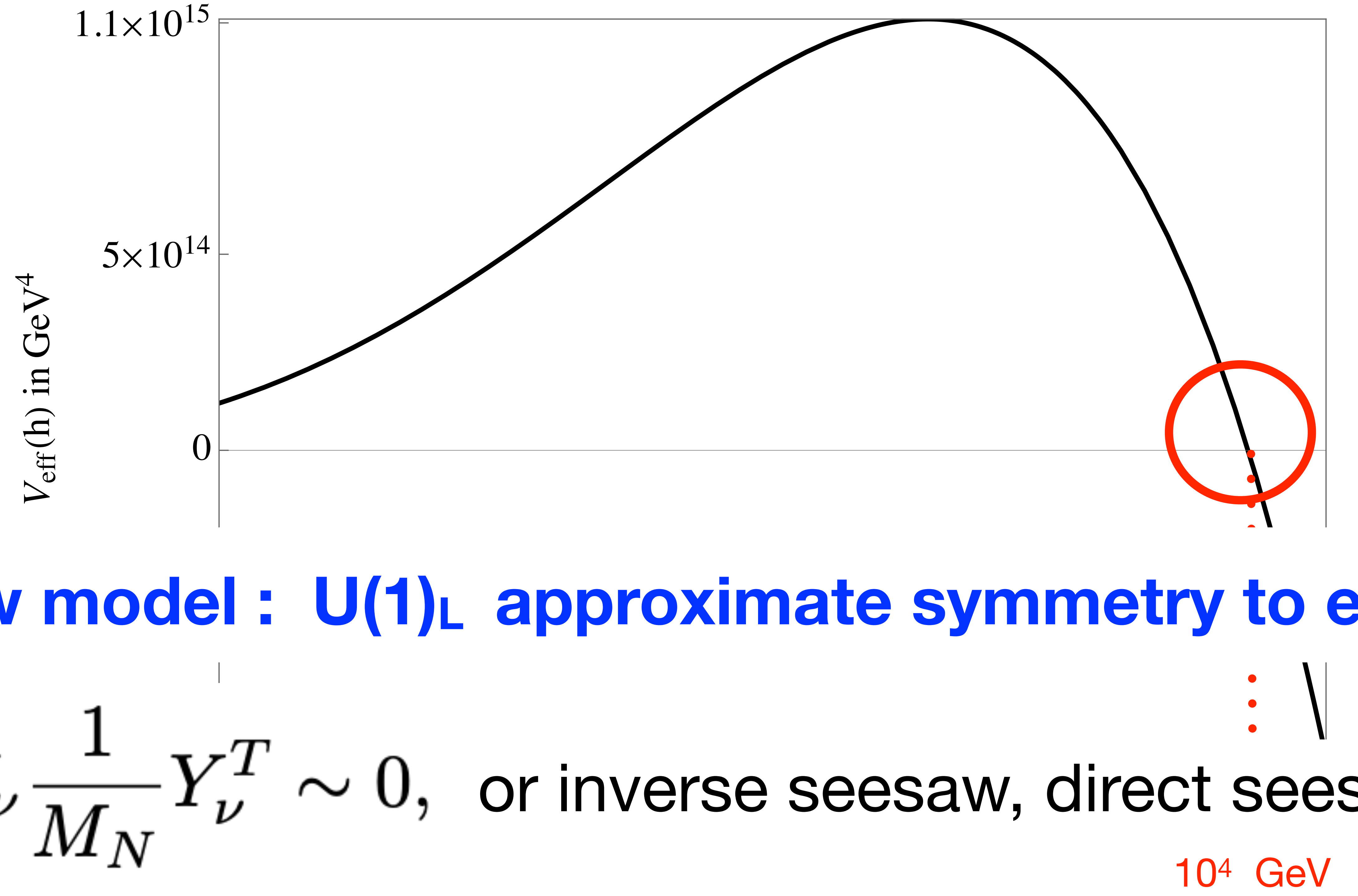

$O\ (TeV)$ **scale requires a low-scale seesaw model : U(1)$_L$ approximate symmetry to ensure**

light neutrino masses even with Y_ν~1: $Y_\nu \frac{1}{M_N} Y_\nu^T \sim 0,$ or inverse seesaw, direct seesaw etc..