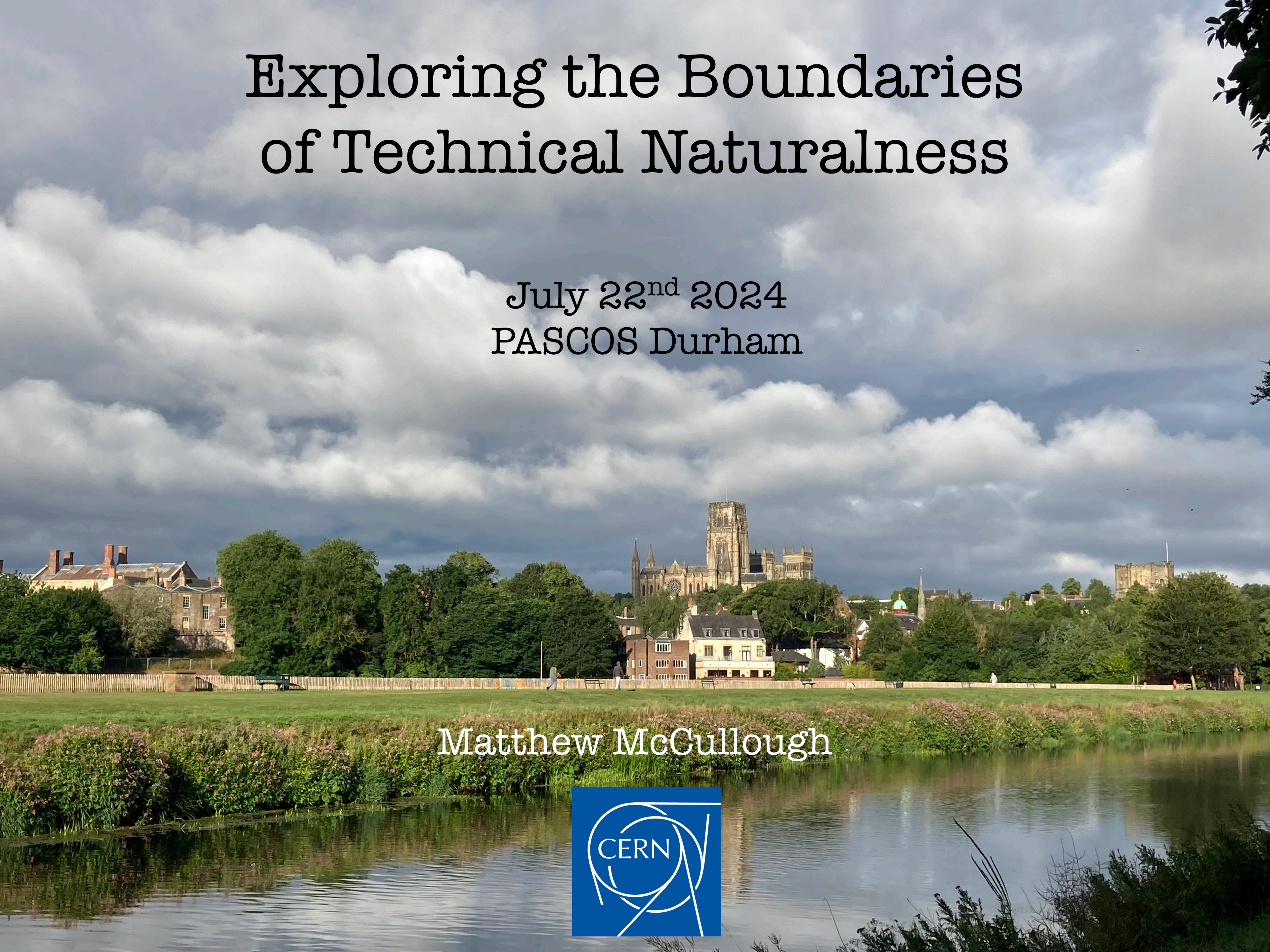
Exploring the Boundaries of Technical Naturalness

July 22nd 2024
PASCOS Durham

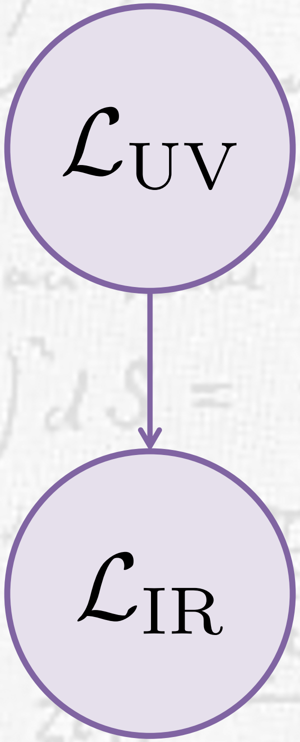


Matthew McCullough

CERN

Technical Naturalness

Whether or not we know a UV theory, we have tools to understand how its parameters may map into the IR EFT. Dimensional analysis, NDA, **Spurion Analysis**, symmetries...



Spurion Analysis

Basic idea: In UV or IR theory, treat parameters as if they were fields, or were vanishing. If this were the case, what global symmetries would there be?

$$\mathcal{L}(\text{Fields}, \text{Params})$$

Under these symmetries, the "charges" of the parameters dictate:

- How they enter into physical observables.
- The structure of quantum corrections.
- RG patterns.
- The structure of the IR EFT.

Spurion Analysis

Example 1: The Higgs mass.

The only symmetries that can protect a scalar mass in the absence of additional states are a shift symmetry (non-linearly realized global symm) or conformal symmetry. In

$$\mathcal{L}_{\mathrm{IR}}$$

if the Higgs mass parameter was the only one to break these symmetries, then it could be naturally small, just like the electron Yukawa.

Spurion Analysis

Example 1: The Higgs mass.

However, all of its interactions break shift and existence of the UV scale breaks any putative scale symmetry.

Thus there is nothing that can suppress corrections such as

$$\delta m_h^2 \sim \frac{\hbar y_t^2}{(4\pi)^2} M_{UV}^2$$

(Note this cannot be Planck scale, more like string scale...)

Where to look? My View

Global symmetries and their breaking play a central role in naturalness.

In particular, non-Abelian symmetry ubiquitous:

- Composite Higgs $\mathcal{G} \to \mathcal{H}$
- Flavour puzzle: $\mathrm{SU}(3)_{Q,U,D,L,E}$

Yet, often models have focused on U(1) toy models, or on **minimality** assumptions.

Where to look? My View

Here, by “minimality”, I mean something **very specific:** That global symmetries are explicitly broken by minimal irreducible representations.

This does not necessarily relate to “number of particles” for which we have no guidance.

Example: Scalar Quartic

Standard lore would have it that in a theory such as

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4$$

the cutoff should be at

$$\Lambda \lesssim \frac{(4\pi)^2}{\lambda}m^2$$

However, consider a U(1) pNGB with explicit breaking by operator of charge "n". EFT is:

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + \epsilon M^2 f^2 \cos\frac{n\phi}{f}$$

$$\approx \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}\epsilon n^2 M^2\phi^2 + \frac{1}{4!}\epsilon n^4 \frac{M^2}{f^2}\phi^4 + \ldots$$

Example: Scalar Quartic

EFT is:

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + \epsilon M^2 f^2 \cos\frac{n\phi}{f}$$
$$\approx \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}\epsilon n^2 M^2 \phi^2 + \frac{1}{4!}\epsilon n^4 \frac{M^2}{f^2}\phi^4 + \ldots$$

True cutoff is $\Lambda^2 \lesssim (4\pi)^2 f^2$

Naïve cutoff is $\Lambda^2 \lesssim (4\pi)^2 f^2/n^2$

Naïve estimate underestimates natural scale separation by factor n! Beware minimality!

Example: Weinberg Operator

Consider the Weinberg operator. Accidentally, lepton number perturbatively conserved in SM. Nothing to conserve it at dim-5:

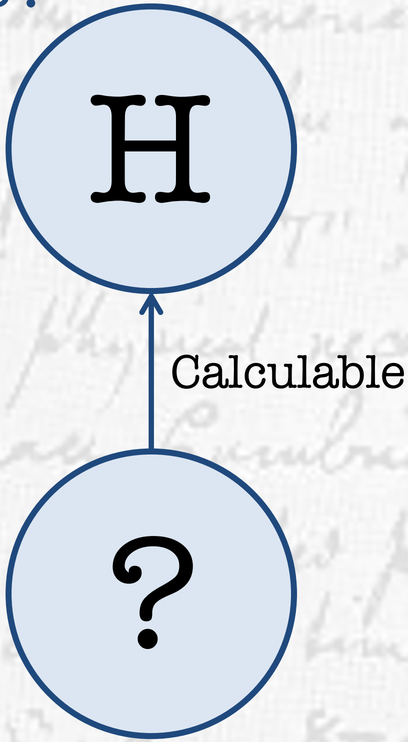
$$\mathcal{L}_W = \frac{(H \cdot L)(H \cdot L)}{\Lambda}$$

An observation much less considered is that this operator is in a non-minimal irrep of $SU(3)_L$. In fact, it is in the symmetric **6** irrep.

Majorana neutrino masses explicitly break $SU(3)_L$ non-minimally. (Credit: Conversations with Neal Weiner)

Back to the Higgs Boson

What about the Higgs?



The Standard Model, our best description of nature, breaks down at short distances: It is an effective field theory, to be replaced by something more fundamental at shorter distance scales.

Whence the Higgs Boson?

What about the Higgs?

Could the Higgs be a composite particle, like the Pion? "pNGB-like Higgs".

Question that's been asked many times… Kaplan, Georgi, Dimopoulos 1984 etc.

The Standard Model, our best descrip … e, breaks down at short distances: It is an effectiv… field theory, to be replaced by something more fundamental at shorter distance scales.

Generalising: “pNGB”

With general “pseudo-Nambu-Goldstone Bosons” there is a scale separation between their mass and the next microscopic scale:

$$\mathcal{G} \to \mathcal{H}$$

$$m^2_{\mathrm{pNGB}} \ll \Lambda_{UV}$$

$$\epsilon \ll 1$$

This is due to a spontaneously broken global symmetry, with potential from explicit breaking. Goldstone’s Theorem...

Pion-Like Higgs Circa 2025

For a Pion-like Higgs:

$$V(H) \approx \epsilon f^2 M^2 F(H/f)$$

Small parameter(s) whose magnitude depends entirely on magnitude of explicit symmetry breaking.

Compositeness scale

UV Mass Scale

A periodic function whose form depends entirely on the nature of explicit symmetry breaking.

Higgs mass depends on ε and the curvature of F.

Higgs vev is independent of ε, only cares about location of global minimum of F.

Pion-Like Higgs Circa 2025

For a Pion-like Higgs:

Also… due to operators like

$$\frac{1}{f^2}\left(\partial|H|^2\right)^2$$

Higgs couplings are modified, relative to SM, by O(1) factors of v^2/f^2.

nature of explicit symmetry breaking.

depends on ε and the curvature of F.

Higgs vev is independent of ε, only cares about location of global minimum of F.

Pion-Like Higgs Circa 2024

Assumption until now(ish): Sources of explicit symmetry breaking are top and gauge, so

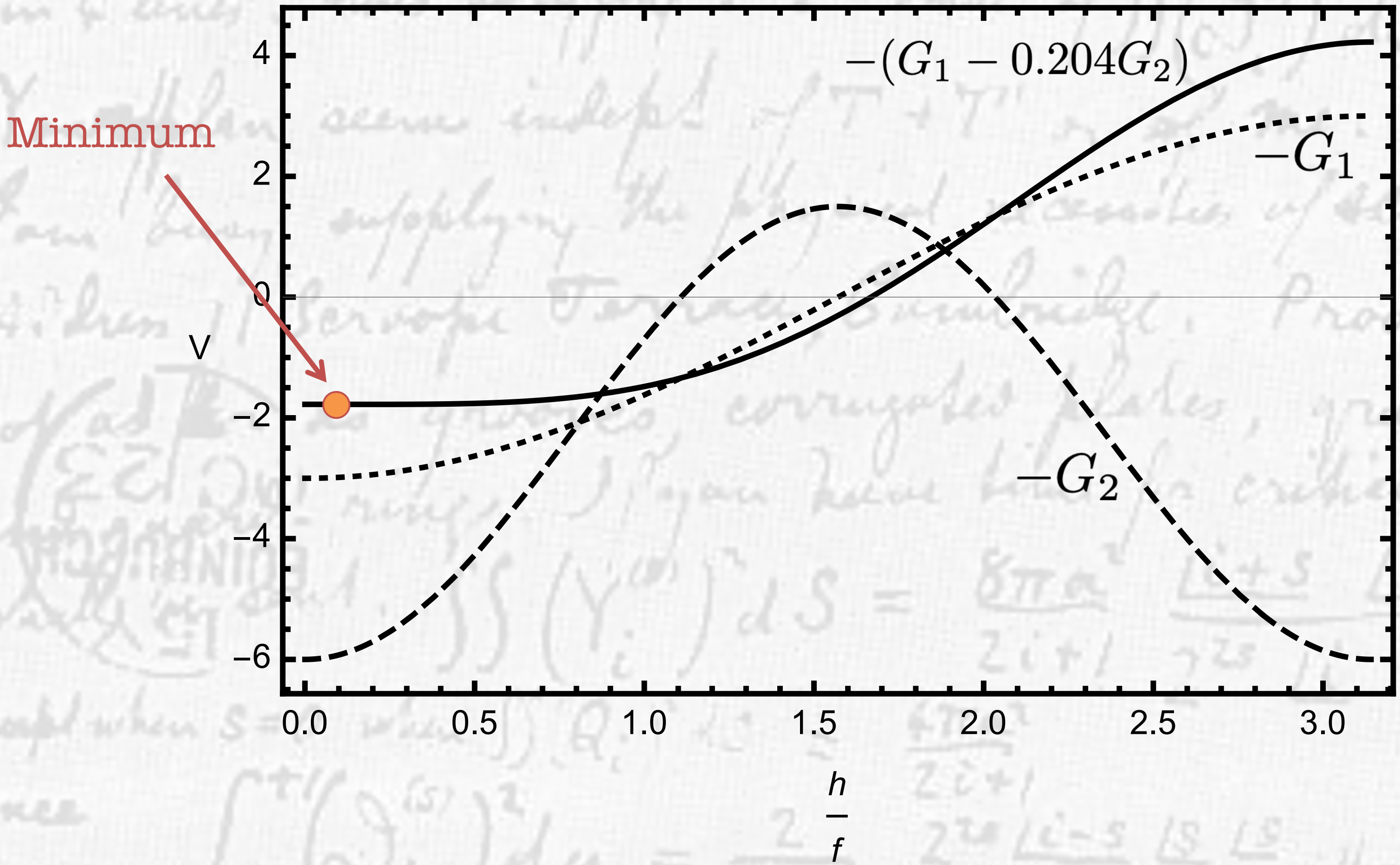
$$F(H/f) \sim \cos(H/f)\ ,\ \cos^2(H/f)\ ,\ \cos^4(H/f)\ ,$$

and

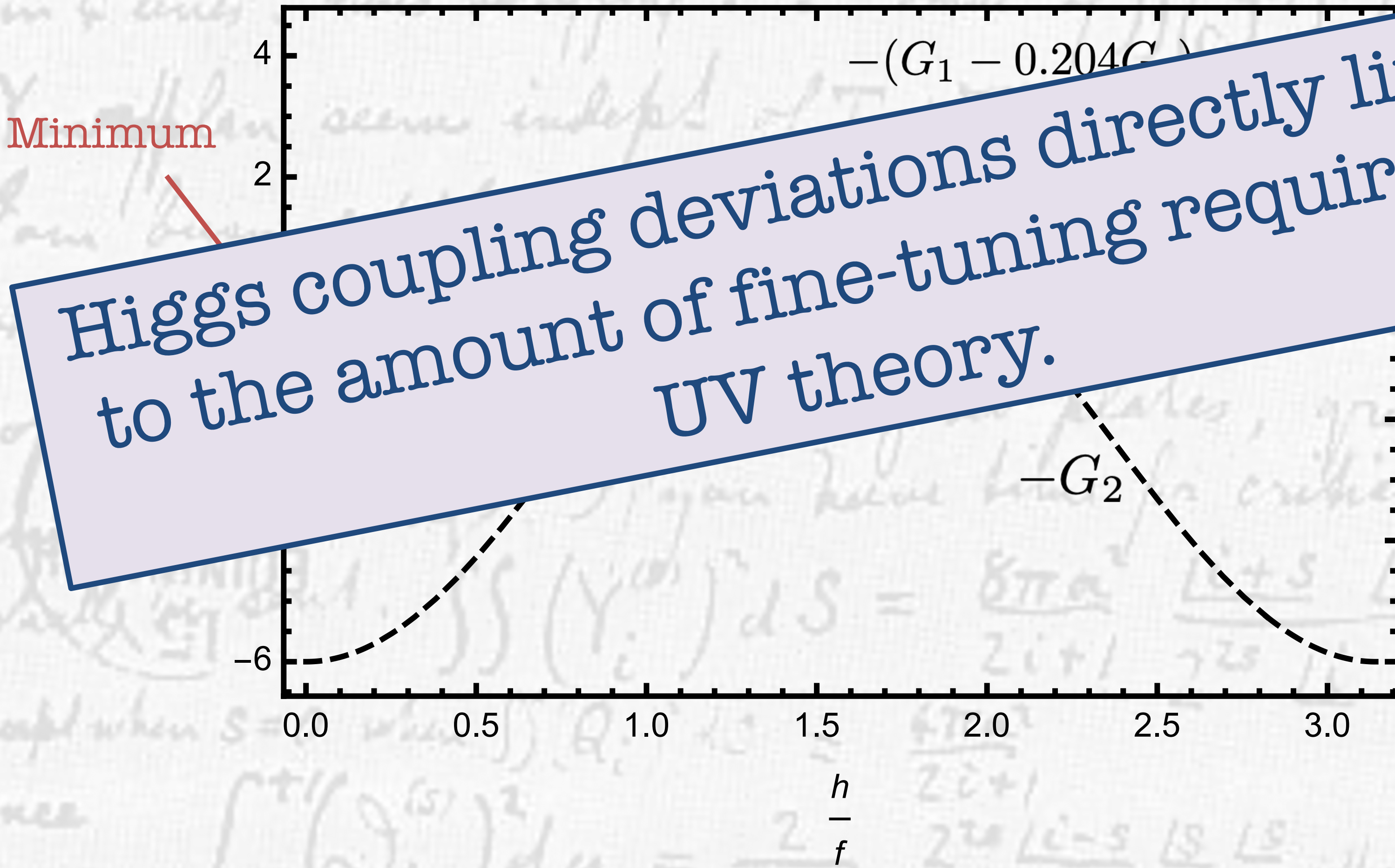
$$\epsilon \sim \frac{3\lambda_t^2}{8\pi^2}\ ,\ \frac{g^2}{16\pi^2}$$

Leading to a "universal" source of fine-tuning: Only way to get $v \ll f$ is to have different contributions (loops or whatever) and fine-tune coefficients so that minimum is not at 0 or πf.

Pion-Like Higgs Circa 2024



Pion-Like Higgs Circa 2024



Higgs coupling deviations directly linked to the amount of fine-tuning required in UV theory.

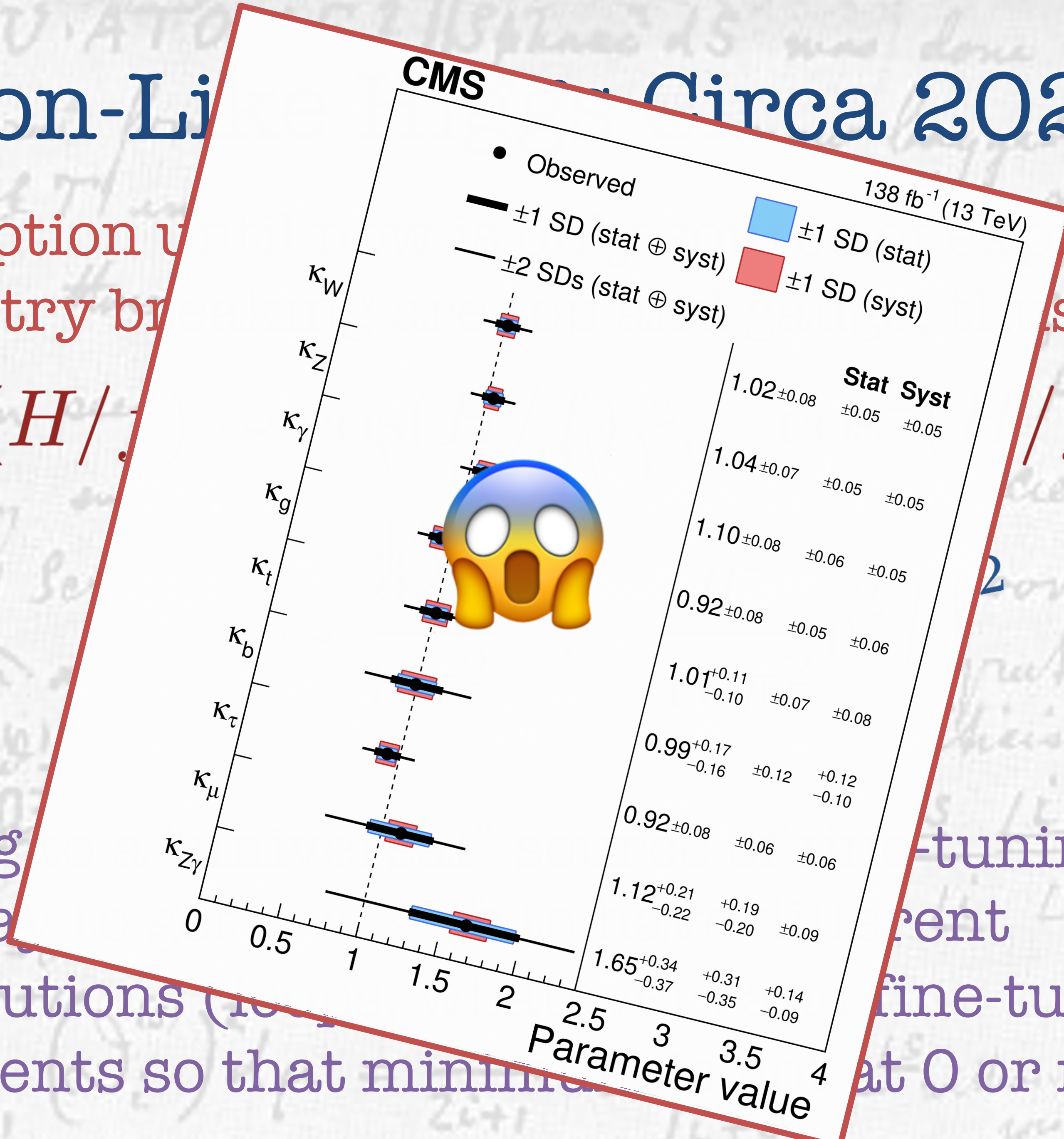
Pion-Li… Circa 2024

Assumption u… …icit symmetry br… …s

$F(H/$ … $/f)$

and … 2

Leading … -tuning. Only wa… …rent contributions (lo… …fine-tune coefficients so that minim… …at 0 or f.



Pion-Like Higgs Circa 2024

Questions concerning paradigm…

What if we take technical naturalness, not our own **aesthetics** or **minimality**, as our guide?

Why shouldn't there be more sources of explicit symmetry breaking for pNGB Higgs? Quark masses and QED coupling both contribute to pion potential, with completely different UV origins.

Based on various published and unpublished work with
Durieux and Salvioni
Salvioni and Menkara
Durieux, Kang, Quevillon.

Question

If we live in the IR and have some set of light pNGBs

$$\Pi$$

with a scalar potential

$$V(\Pi)$$

how do we essentially "organise" or structurally understand the pNGB potential without recourse to the UV symmetries and irreps?

Answer

pNGBs parameterise coordinates on G/H. To understand functions on a manifold, work with harmonic functions.

Lots of pNGB examples live on the sphere:

$$\frac{\mathcal{G}}{\mathcal{H}} \simeq \mathcal{S}^N$$

For SO(N+1)→SO(N), also SU(N')→SU(N'-1), Sp(N'')→Sp(N''-1). Thanks to Joe Davighi who emphasized breadth to us.

Note that it also applies to some special groups.

Decomposing pNGB Potentials

For any spontaneous (internal) symmetry breaking pattern with coset

$$\frac{\mathcal{G}}{\mathcal{H}} \simeq \mathcal{S}^N$$

for which there is also explicit symmetry breaking that preserves an SO(N) subgroup in H, then can decompose <u>any</u> pNGB potential as a sum of harmonics

$$V(\Pi) = \epsilon M^2 f^2 \sum_n a_n G_n^{\frac{N-1}{2}} (\cos \Pi / f)$$

These are the natural functions for this manifold (and a complete basis).

Back to Multipoles

For SO(N+1) →SO(N) can show each polynomial uniquely corresponds to n-index symmetric irrep spurion, i.e. multipole!

$$\epsilon \frac{M^2}{f^{n-2}} S^{a_1 a_2 \ldots a_n} \Sigma_{a_1} \Sigma_{a_1} \ldots \Sigma_{a_n} \to G_n^{\frac{N-1}{2}} (\cos \Pi / f)$$

They are the same thing.

For N-sphere cosets from other breaking patterns we know that Gegenbauers are still the appropriate functions in IR.

Technical Naturalness

Conclusion: A pNGB potential of the form

$$V = \epsilon M^2 f^2 \left(G_n \left(\cos \Pi / f\right) + \epsilon \sum_{p=0}^{2n} a_p G_p \left(\cos \Pi / f\right) + \mathcal{O}(\epsilon^2) + \ldots \right)$$

is technically natural.

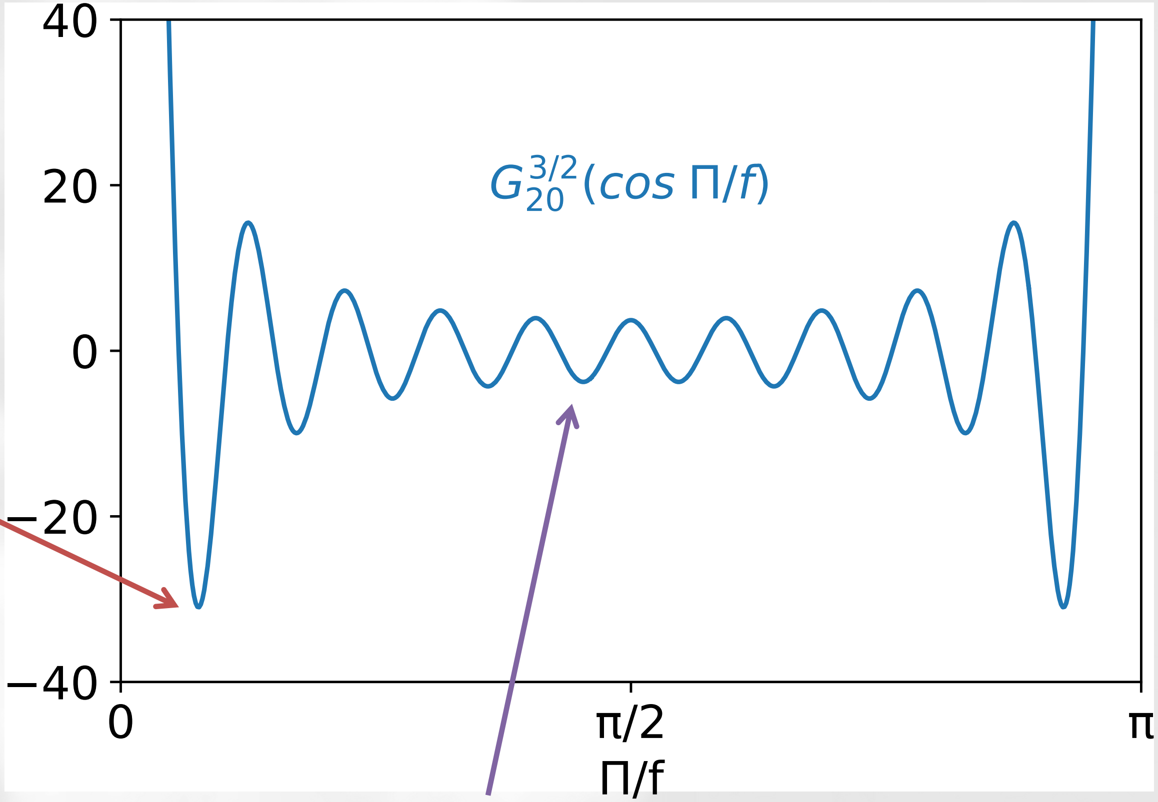
Can see this in many ways. For instance, for SO(N+1) case start with spurion S and write down every allowed operator. Construction radiatively stable at **all loop orders, in IR and in UV**.

Getting to know Gegenbauer

The Gegenbauer potential looks like:

Global minimum at naturally small field values:

$$\frac{\langle \Pi \rangle}{f} \approx \frac{j_{\lambda+1/2,1}}{n+\lambda} \approx \frac{5.1}{n}$$



Approximately periodic:

$$G_n^\lambda\left(\cos\frac{\Pi}{f}\right) \xrightarrow{n\gg 1} \frac{J_{\lambda-1/2}\left((n+\lambda)\frac{\Pi}{f}\right)}{\Pi^{\lambda-1/2}} \xrightarrow{\frac{\Pi}{f}\gg\frac{1}{n}} \frac{\cos\left((n+\lambda)\frac{\Pi}{f}-\lambda\frac{\pi}{2}\right)}{\Pi^\lambda}$$

Application

Consider some standard pNGB Higgs construction and, inspired by pions, allow for an additional source of explicit symmetry breaking, in n-index irrep of global symmetry.

$$\mathcal{L} = \mathcal{L}_{\mathrm{Old}} + \epsilon \mathcal{L}_{S_n \neq 0}$$

What happens?

Gegenbauer's Twin

Generalising Gegenbauer story to pNGB Twin Higgs for SO(8) $\rightarrow$ SO(7) and going to Unitary gauge the top-sector contributions to the Higgs potential are

$$V_t \approx \frac{3y_t^4 f^4}{64\pi^2}\left[\sin^4\frac{h}{f}\,\log\frac{a}{\sin^2 h/f}+\cos^4\frac{h}{f}\,\log\frac{a}{\cos^2 h/f}\right]$$

Whereas the symmetric n-index irrep gives

$$V_G^{(n)} = \epsilon\, m_\rho^2 f^2 G_n^{3/2}\left(\cos 2h/f\right)$$

Note: This is radiatively stable at all scales.

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Whereas the symmetric n-index irrep gives

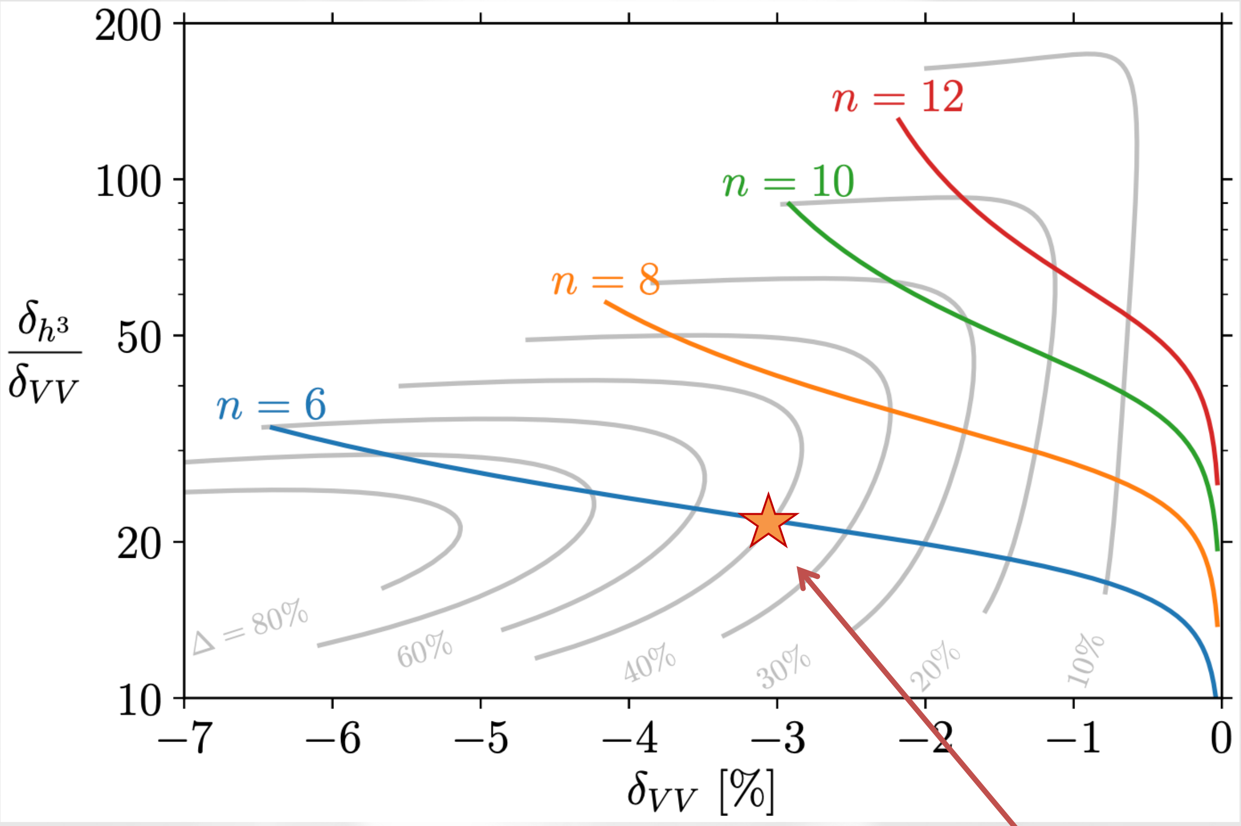
Two model parameters.

$$V_G^{(n)} = \epsilon\, m_\rho^2 f^2 G_n^{3/2}\left(\cos 2h/f\right)$$

Note: This is radiatively stable at all scales.

Gegenbauer’s Twin

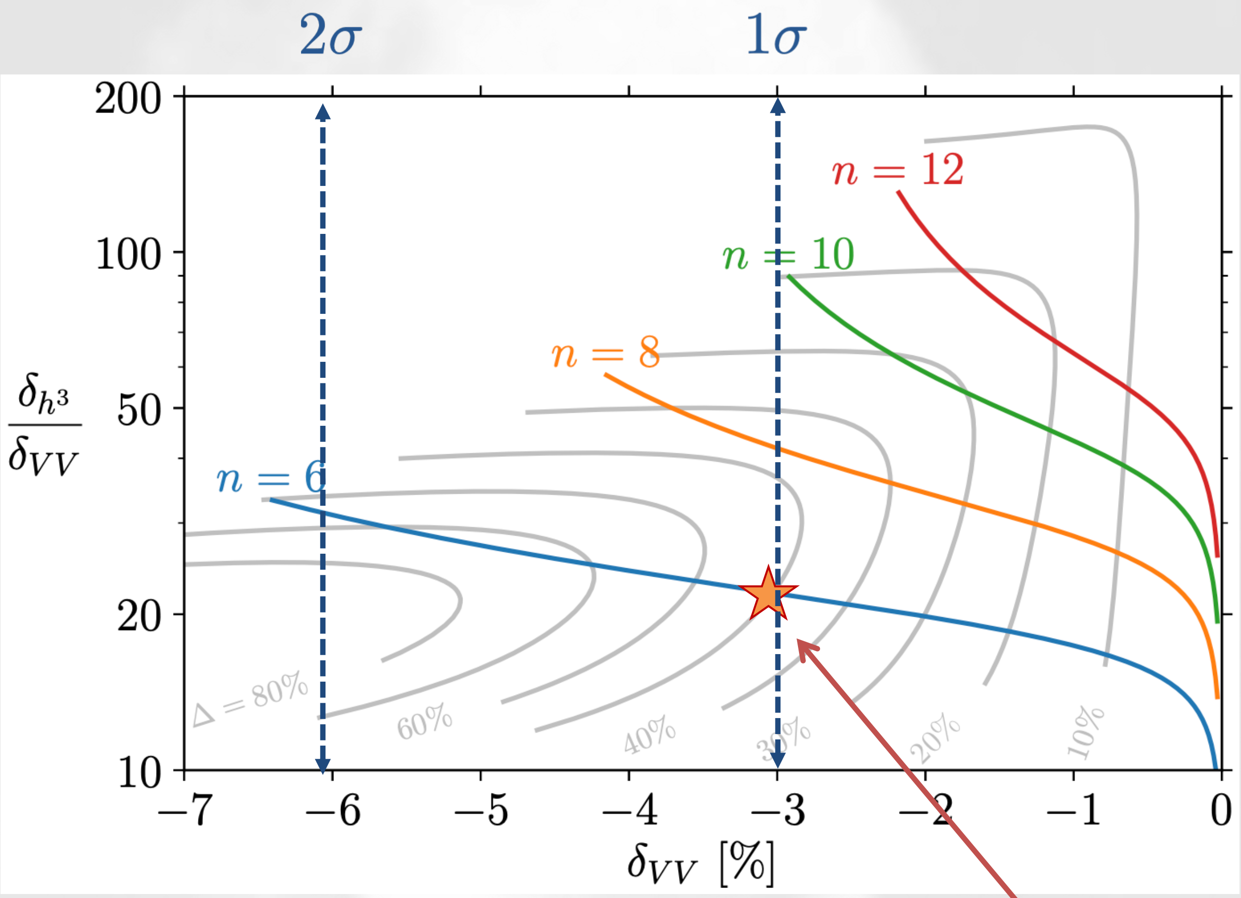
Predictions, in absolute terms:



Example point. Low tuning, 3% single-coupling correction, 70% self-coupling correction.

Gegenbauer's Twin

Present Limits



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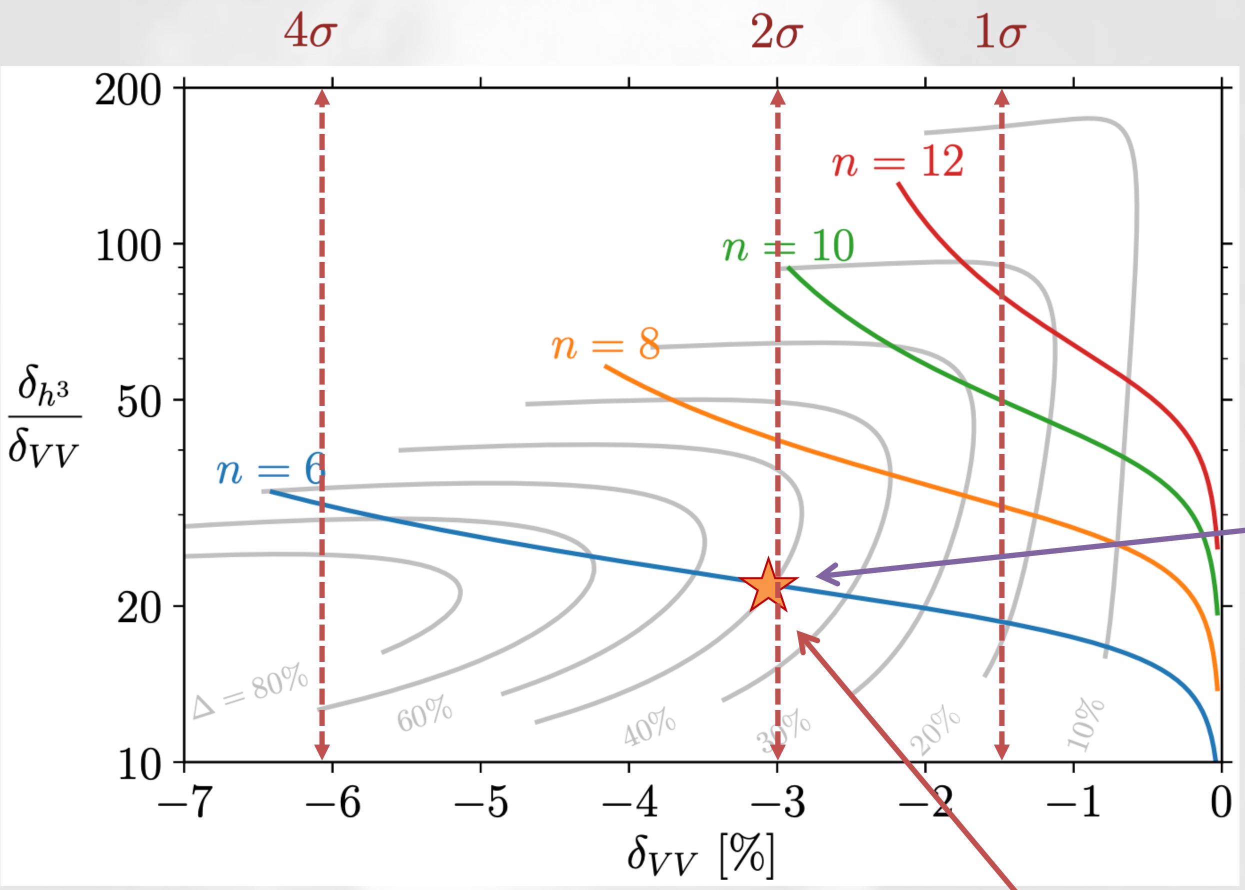
Gegenbauer's Twin

HL-LHC Expectations

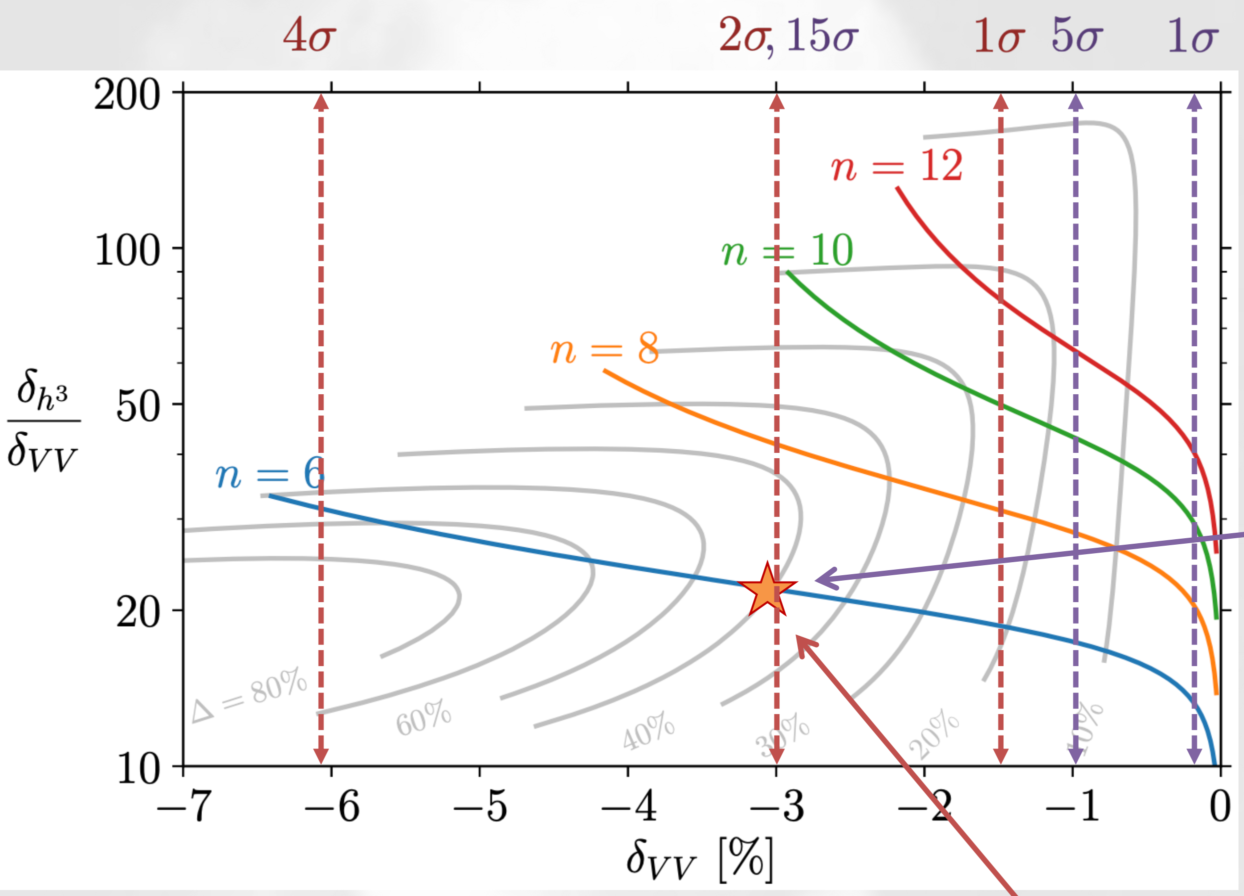


Only 2σ at HL-LHC.

Example point. Low tuning, 3% single-coupling correction, 70% self-coupling correction.

Gegenbauer's Twin

HL-LHC Expectations & FCC-ee



Only 2σ at HL-LHC but 15σ at FCC-ee!

Example point. Low tuning, 3% single-coupling correction, 70% self-coupling correction.

Implication

If we take technical naturalness alone as a guide then it seems, to me, there is no naturalness crisis (yet). Aesthetics crisis? Perhaps...

Some technically natural pNGB Higgs scenarios are consistent with LHC bounds and may even be very difficult to probe at HL-LHC. Future colliders would do better.

Higgs precision era will be necessary to answer natural.

What about Flavour?

Fermion Yukawas explicitly break flavour symm

$$\mathrm{SU}(3)_Q \times \mathrm{SU}(3)_U \to \mathrm{U}(1)^3$$

Yukawa coupling is a spurion in the irrep

$$(\mathbf{3}, \overline{\mathbf{3}})$$

UV scenarios for generating this include:

- MFV
- Froggatt-Nielsen
- Universal
- Aligned...

All of which correspond to minimal irreps for underlying UV spurion.

What about non-minimal Flavour?

Work to appear soon with Banks, Crawford, Sutherland... Suppose the UV breaking is through a non-minimal spurion:

$$\mathcal{Y} \sim \mathbf{6} \times \overline{\mathbf{6}}$$

To build the $(\mathbf{3}, \overline{\mathbf{3}})$ Yukawa coupling need at least three insertions

$$y \sim \mathcal{Y}^2 \overline{\mathcal{Y}} + \ldots \ ,$$

Thus, a small hierarchy in breaking...

$$\mathrm{SU}(3) \to \mathrm{SU}(2) \to \mathrm{U}(1)$$

ends up cubed in the Yukawa coupling!

What about non-minimal Flavour?

For families of non-hierarchical and non-tuned values of spurion, accidentally get rank-1 at $\mathcal{O}(\mathcal{Y}^3)$ rank-2 at $\mathcal{O}(\mathcal{Y}^4)$ and rank-3 at $\mathcal{O}(\mathcal{Y}^5)$. "Magic".

To build the $(\mathbf{3}, \overline{\mathbf{3}})$ Yukawa coupling need

$$y \sim \mathcal{Y}^2\overline{\mathcal{Y}} + \epsilon\left(\mathcal{Y}_1^4 + \mathcal{Y}_2^4 + \mathcal{Y}\overline{\mathcal{Y}}^3\right) + \epsilon^2\left(\overline{\mathcal{Y}}^5 + \mathcal{Y}^3\overline{\mathcal{Y}}^2 + \ldots\right) + \ldots \quad ,$$

Consecutive orders naturally suppressed as correspond to higher orders in EFT expansion.

Not FN: New mechanism to explain flavour!

Experimental Consequences

SMEFT four-fermion operators are in **1**, **8**, **27** irreps of the SU(3)'s. The accidental symmetry magic doesn't occur.

Typically* generate $\mathcal{O}(1)$ flavour violation in observables, not proportional to Yukawas, unlike other flavour scenarios.

Punchline: Looking beyond a "minimal" assumption for the irreps that break flavour, new territory opens up. But, many questions remain, not least the origin of non-minimality.

Conclusions

The path to the UV-completion of the SM could be long. Naturalness is our headlamp, but we should remember look in all directions.

Many thanks to our kind hosts, **IPPP**.

Whence the Higgs Boson?

Standard Model is the “IR” of some “UV-completion”.

Unless reductionism ends now, SM at the weak scale (IR) calculable from within UV theory.

Hence Higgs potential is predicted from UV:

$$V(H)$$

A “natural” UV-completion generates two “IR” parameters without tuning “UV” parameters:

$$\langle H \rangle \qquad\qquad m_h$$

Pion Reminder

With the composite pions there is a scale separation between their mass and the next microscopic scale (QCD)...

$$\boxed{SU(2)_L \times SU(2)_R \to SU(2)_V}$$

$$m_\pi^2 \ll m_p^2$$

$$m_q \ll \Lambda_{QCD}$$

This is due to a spontaneously broken global symmetry, with potential from explicit breaking. Goldstone's Theorem...

Internal Symmetries

One could consider a standard CCWZ-like procedure, in terms of G and H. Would have

$$V_{\mathrm{pNGB}} = \ldots + \epsilon \frac{M^2}{f^{n-2}} S^{a_1 a_2 \ldots a_n} \Sigma_{a_1} \Sigma_{a_1} \ldots \Sigma_{a_n}$$

Where the S is explicit breaking in n-index symmetric irrep (literally the multipoles), e.g. for SO(N). ε is a small parameter.

But this is cumbersome. After all, the pNGBs don't really care about G and H, but G/H...

Identifying Harmonics

Harmonics are eigenfunctions of Laplace-Beltrami operator

$$\Delta_{\mathcal{S}^N} V_\epsilon(\Pi_j)$$

Now consider a scenario where the potential is a function of

$$\Pi = \left(\sum_{j=1}^{N} \Pi_j^2 \right)^{1/2}$$

such that the IR respects an SO(N) symmetry acting on the pNGBs. Note that this does not necessarily imply H=SO(N)...

Identifying Harmonics

In this case we have

$$\Delta_{\mathcal{S}^N} V_\epsilon(\Pi) = \frac{\partial^2 V_\epsilon(\Pi)}{\partial \Pi^2} + \frac{N-1}{f} \cot \frac{\Pi}{f} \frac{\partial V_\epsilon(\Pi)}{\partial \Pi}$$

Geometry

and eigenfunctions are

$$\Delta_{\mathcal{S}^N} G_n^{(N-1)/2}(\cos \Pi/f) = -\frac{n(n+N-1)}{f^2} G_n^{\frac{N-1}{2}}(\cos \Pi/f)$$

Gegenbauer polynomials.

Physical Interpretation

Gegenbauer polynomials are n-dimensional generalisation of Legendre polynomials... which are 3D angular momentum eigenfunctions.

Calling some axes the "z" direction we have

$$G_n^{\frac{N-1}{2}}(\cos\Pi/f) \sim |n,\mathbf{0}\rangle$$

where the "angular momentum" is in the internal manifold. In this way we can think of the spurion

$$S^{a_1 a_2 \ldots a_n}$$

as a flux carrying angular momentum "n".

Organising a pNGB Potential

Start with simple scenario. Suppose

SO(N+1) →SO(N)

spontaneous breaking with a small spurion
sourcing explicit SO(N+1) →SO(N) breaking,

$$\epsilon S^{a_1 a_2 \ldots a_n}$$

with "n" internal ang. mom. with magnitude "ε".

Organising a pNGB Potential

$O(\varepsilon)$ $$V_\epsilon = \epsilon M^2 f^2 G_n^{\frac{N-1}{2}}(\cos\Pi/f)$$

$O(\varepsilon^2)$ $$V_{\epsilon^2} = \epsilon^2 M^2 f^2 \sum_{p=0}^{2n} a_p G_p^{\frac{N-1}{2}}(\cos\Pi/f)$$

$O(\varepsilon^3)$ $$V_{\epsilon^3} = \epsilon^3 M^2 f^2 \sum_{p=0}^{3n} c_p G_p^{\frac{N-1}{2}}(\cos\Pi/f)$$

$O(\varepsilon^4)$ $$V_{\epsilon^4} = \epsilon^4 M^2 f^2 \sum_{p=0}^{4n} d_p G_p^{\frac{N-1}{2}}(\cos\Pi/f)$$

Organising a pNGB Potential

$O(\varepsilon)$ $$V_\epsilon = \epsilon M^2 f^2 G_n^{\frac{N-1}{2}}\left(\cos\Pi/f\right)$$

$O(\varepsilon^2)$ $$\dots M^2 f^2 \sum^{2n} a_p G_p^{\frac{N-1}{2}}\left(\cos\Pi/f\right)$$

Why? Undergrad memories...

$O(\varepsilon^3)$ $$V_{\epsilon^3} = \epsilon^3 M^2 f^2 \sum_{p=0}^{3n} c_p G_p^{\frac{N-1}{2}}\left(\cos\Pi/f\right)$$

$O(\varepsilon^4)$ $$V_{\epsilon^4} = \epsilon^4 M^2 f^2 \sum_{p=0}^{4n} d_p G_p^{\frac{N-1}{2}}\left(\cos\Pi/f\right)$$

Tensor Products

Internal "angular momentum" adds in the same way as in 3D. Hence

$$|n,\mathbf{0}\rangle \otimes |n,\mathbf{0}\rangle = \sum_{p=0}^{2n} a_p |p,\mathbf{0}\rangle$$

or, more generally

$$\left(|n,\mathbf{0}\rangle\right)^P = \sum_{p=0}^{Pn} a_p |p,\mathbf{0}\rangle$$

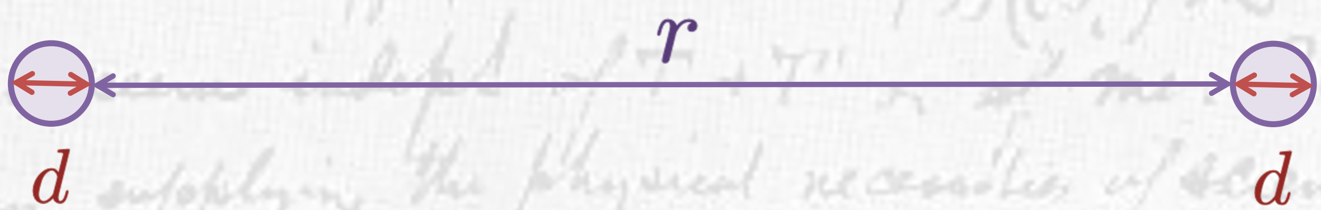
So know which IR operators arise and, for SO(N+1) case, we know which Wilson coefficients in UV are allowed by symmetries.

But first, I want to talk about Methane…


BURP!
GETTY IMAGES

Effective Field Theory

Consider the electrostatic potential between two identical objects far separated:



We may capture all effects of substructure(=microphysics) in an EFT expansion of electrostatic potential:

$$V(r) = \frac{a_0}{r} + \frac{a_1 d}{r^2} + \frac{a_2 d^2}{r^3} + \frac{a_3 d^3}{r^4} + \frac{a_4 d^4}{r^5} + \ldots$$

Effective Field Theory

Each term in

$$V(r) = \frac{a_0}{r} + \frac{a_1 d}{r^2} + \frac{a_2 d^2}{r^3} + \frac{a_3 d^3}{r^4} + \frac{a_4 d^4}{r^5} + \dots$$

comes from a multipole-multipole electrostatic interaction, wherein:

$$V_{l_1 l_2}(r) = \frac{a_{l_1+l_2} d^{l_1+l_2-1}}{r^{1+l_1+l_2}}$$

We may think of each multipole as a small "spurion" parameter in an irrep which explicitly breaks spatial SO(3) symmetry. (Think, for example, of charge configurations).

Effective Field Theory

Theoretical chemist's picture...

	dipole (p) $l = 1$	quadrupole (d) $l = 2$	octopole (f) $l = 3$
C_1	$3A$	$5A$	$7A$
C_s	$2A' + A''$	$3A' + 2A''$	$4A' + 3A''$
C_i	$3A_u$	$5A_g$	$7A_u$
C_2	$A + 2B$	$3A + 2B$	$3A + 4B$
C_3	$A + E$	$A + 2E$	$3A + 2E$
C_4	$A + E$	$A + 2B + E$	$A + 2B + 2E$
C_5	$A + E_1$	$A + E_1 + E_2$	$A + E_1 + 2E_2$
C_6	$A + E_1$	$A + E_1 + E_2$	$A + 2B + E_1 + E_2$
C_7	$A + E_1$	$A + E_1 + E_2$	$A + E_1 + E_2 + E_3$
C_8	$A + E_1$	$A + E_1 + E_2$	$A + E_1 + E_2 + E_3$
D_2	$B_1 + B_2 + B_3$	$2A + B_1 + B_2 + B_3$	$A + 2B_1 + 2B_2 + 2B_3$
D_3	$A_2 + E$	$A_1 + 2E$	$A_1 + 2A_2 + 2E$
D_4	$A_2 + E$	$A_1 + B_1 + B_2 + E$	$A_2 + B_1 + B_2 + 2E$
D_5	$A_2 + E_1$	$A_1 + E_1 + E_2$	$A_2 + E_1 + 2E_2$
D_6	$A_2 + E_1$	$A_1 + E_1 + E_2$	$A_2 + B_1 + B_2 + E_1 + E_2$
C_{2v}	$A_1 + B_1 + B_2$	$2A_1 + A_2 + B_1 + B_2$	$2A_1 + A_2 + 2B_1 + 2B_2$
C_{3v}	$A_1 + E$	$A_1 + 2E$	$2A_1 + A_2 + 2E$
C_{4v}	$A_1 + E$	$A_1 + B_1 + B_2 + E$	$A_1 + B_1 + B_2 + 2E$
C_{5v}	$A_1 + E_1$	$A_1 + E_1 + E_2$	$A_1 + E_1 + 2E_2$
C_{6v}	$A_1 + E_1$	$A_1 + E_1 + E_2$	$A_1 + B_1 + B_2 + E_1 + E_2$
C_{2h}	$A_u + 2B_u$	$3A_g + 2B_g$	$3A_u + 4B_u$
C_{3h}	$A'' + E'$	$A' + E' + E''$	$2A' + A'' + E' + E''$
C_{4h}	$A_u + E_u$	$A_g + 2B_g + E_g$	$A_u + 2B_u + 2E_u$
C_{5h}	$A'' + E_1'$	$A' + E_2' + E_1''$	$A'' + E_1' + E_2' + E_2''$
C_{6h}	$A_u + E_{1u}$	$A_g + E_{1g} + E_{2g}$	$A_u + 2B_u + E_{1u} + E_{2u}$
D_{2h}	$B_{1u} + B_{2u} + B_{3u}$	$2A_g + B_{1g} + B_{2g} + B_{3g}$	$A_u + 2B_{1u} + 2B_{2u} + 2B_{3u}$
D_{3h}	$A_2'' + E'$	$A_1' + E' + E''$	$2A_1' + A_2' + A_1'' + A_2'' + E' + E''$
D_{4h}	$A_{2u} + E_u$	$A_{1g} + B_{1g} + B_{2g} + E_g$	$A_{2u} + B_{1u} + B_{2u} + 2E_u$
D_{5h}	$A_2'' + E_1'$	$A_1' + E_2' + E_1''$	$A_2'' + E_1' + E_2' + E_2''$
D_{6h}	$A_{2u} + E_{1u}$	$A_{1g} + E_{1g} + E_{2g}$	$A_{2u} + B_{1u} + B_{2u} + E_{1u} + E_{2u}$
D_{8h}	$A_{2u} + E_{1u}$	$A_{1g} + E_{1g} + E_{2g}$	$A_{2u} + E_{1u} + E_{2u} + E_{3u}$
D_{2d}	$B_2 + E$	$A_1 + B_1 + B_2 + E$	$A_1 + A_2 + B_2 + 2E$
D_{3d}	$A_{2u} + E_u$	$A_{1g} + 2E_g$	$A_{1u} + 2A_{2u} + 2E_u$
D_{4d}	$B_2 + E_1$	$A_1 + E_2 + E_3$	$B_2 + E_1 + E_2 + E_3$
D_{5d}	$A_{2u} + E_{1u}$	$A_{1g} + E_{1g} + E_{2g}$	$A_{2u} + E_{1u} + 2E_{2u}$
D_{6d}	$B_2 + E_1$	$A_1 + E_2 + E_5$	$B_2 + E_1 + E_3 + E_4$
S_4	$B + E$	$A + 2B + E$	$2A + B + 2E$
S_6	$A_u + E_u$	$A_g + 2E_g$	$3A_u + 2E_u$
S_8	$B + E_1$	$A + E_2 + E_3$	$B + E_1 + E_2 + E_3$
T	T	$E + T$	$A + 2T$
T_h	T_u	$E_g + T_g$	$A_u + 2T_u$
T_d	T_2	$E + T_2$	$A_1 + T_1 + T_2$
O	T_1	$E + T_2$	$A_2 + T_1 + T_2$
O_h	T_{1u}	$E_g + T_{2g}$	$A_{2u} + T_{1u} + T_{2u}$
$C_{\infty v}$	$\Sigma^+ + \Pi$	$\Sigma^+ + \Pi + \Delta$	$\Sigma^+ + \Pi + \Delta + \Phi$
$D_{\infty h}$	$\Sigma_u^+ + \Pi_u$	$\Sigma_g^+ + \Pi_g + \Delta_g$	$\Sigma_u^+ + \Pi_u + \Delta_u + \Phi_u$
I	T_1	H	$T_2 + G$
I_h	T_{1u}	H_g	$T_{2u} + G_u$

Point Groups = Isometry Groups

	hexadecapole (g) $l = 4$	32-pole (h) $l = 5$	64-pole (i) $l = 6$
C_1	$9A$	$11A$	$13A$
C_s	$5A' + 4A''$	$6A' + 5A''$	$7A' + 6A''$
C_i	$9A_g$	$11A_u$	$13A_g$
C_2	$5A + 4B$	$5A + 6B$	$7A + 6B$
C_3	$3A + 3E$	$3A + 4E$	$5A + 4E$
C_4	$3A + 2B + 2E$	$3A + 2B + 3E$	$3A + 4B + 3E$
C_5	$A + 2E_1 + 2E_2$	$3A + 2E_1 + 2E_2$	$3A + 3E_1 + 2E_2$
C_6	$A + 2B + E_1 + 2E_2$	$A + 2B + 2E_1 + 2E_2$	$3A + 2B + 2E_1 + 2E_2$
C_7	$A + E_1 + E_2 + 2E_3$	$A + E_1 + 2E_2 + 2E_3$	$A + 2E_1 + 2E_2 + 2E_3$
C_8	$A + 2B + E_1 + E_2 + E_3$	$A + 2B + E_1 + E_2 + 2E_3$	$A + 2B + E_1 + 2E_2 + 2E_3$
D_2	$3A + 2B_1 + 2B_2 + 2B_3$	$2A + 3B_1 + 3B_2 + 3B_3$	$4A + 3B_1 + 3B_2 + 3B_3$
D_3	$2A_1 + A_2 + 3E$	$A_1 + 2A_2 + 4E$	$3A_1 + 2A_2 + 4E$
D_4	$2A_1 + A_2 + B_1 + B_2 + 2E$	$A_1 + 2A_2 + B_1 + B_2 + 3E$	$2A_1 + A_2 + 2B_1 + 2B_2 + 3E$
D_5	$A_1 + 2E_1 + 2E_2$	$2A_1 + A_2 + 2E_1 + 2E_2$	$A_1 + 2A_2 + 3E_1 + 2E_2$
D_6	$A_1 + B_1 + B_2 + E_1 + 2E_2$	$A_2 + B_1 + B_2 + 2E_1 + 2E_2$	$2A_1 + A_2 + B_1 + B_2 + 2E_1 + 2E_2$
C_{2v}	$3A_1 + 2A_2 + 2B_1 + 2B_2$	$3A_1 + 2A_2 + 3B_1 + 3B_2$	$4A_1 + 3A_2 + 3B_1 + 3B_2$
C_{3v}	$2A_1 + A_2 + 3E$	$2A_1 + A_2 + 4E$	$3A_1 + 2A_2 + 4E$
C_{4v}	$2A_1 + A_2 + B_1 + B_2 + 2E$	$2A_1 + A_2 + B_1 + B_2 + 3E$	$2A_1 + A_2 + 2B_1 + 2B_2 + 3E$
C_{5v}	$A_1 + 2E_1 + 2E_2$	$2A_1 + A_2 + 2E_1 + 2E_2$	$2A_1 + A_2 + 3E_1 + 2E_2$
C_{6v}	$A_1 + B_1 + B_2 + E_1 + 2E_2$	$A_1 + B_1 + B_2 + 2E_1 + 2E_2$	$2A_1 + A_2 + B_1 + B_2 + 2E_1 + 2E_2$
C_{2h}	$5A_g + 4B_g$	$5A_u + 6B_u$	$7A_g + 6B_g$
C_{3h}	$A' + 2A'' + 2E' + E''$	$2A' + A'' + 2E' + 2E''$	$3A' + 2A'' + 2E' + 2E''$
C_{4h}	$3A_g + 2B_g + 2E_g$	$3A_u + 2B_u + 3E_u$	$3A_g + 4B_g + 3E_g$
C_{5h}	$A' + E_1' + E_2' + E_1'' + E_2''$	$2A' + A'' + E_1' + E_2' + E_1'' + E_2''$	$A' + 2A'' + 2E_1' + E_2' + E_1'' + E_2''$
C_{6h}	$A_g + 2B_g + E_{1g} + 2E_{2g}$	$A_u + 2B_u + 2E_{1u} + 2E_{2u}$	$3A_g + 2B_g + 2E_{1g} + 2E_{2g}$
D_{2h}	$3A_g + 2B_{1g} + 2B_{2g} + 2B_{3g}$	$2A_u + 3B_{1u} + 3B_{2u} + 3B_{3u}$	$4A_g + 3B_{1g} + 3B_{2g} + 3B_{3g}$
D_{3h}	$A_1' + A_1'' + A_2'' + 2E' + E''$	$A_1' + A_2' + A_2'' + 2E' + 2E''$	$2A_1' + A_2' + A_1'' + A_2'' + 2E' + 2E''$
D_{4h}	$2A_{1g} + A_{2g} + B_{1g} + B_{2g} + 2E_g$	$A_{1u} + 2A_{2u} + B_{1u} + B_{2u} + 3E_u$	$2A_{1g} + A_{2g} + 2B_{1g} + 2B_{2g} + 3E_g$
D_{5h}	$A_1' + E_1' + E_2' + E_1'' + E_2''$	$2A_1' + A_2'' + E_1' + E_2' + E_1'' + E_2''$	$A_1' + 2A_2'' + 2E_1' + E_2' + E_1'' + E_2''$
D_{6h}	$A_{1g} + B_{1g} + B_{2g} + E_{1g} + 2E_{2g}$	$A_{2u} + B_{1u} + B_{2u} + 2E_{1u} + 2E_{2u}$	$2A_{1g} + A_{2g} + B_{1g} + B_{2g} + 2E_{1g} + 2E_{2g}$
D_{8h}	$A_{1g} + B_{1g} + B_{2g} + E_{1g} + E_{2g} + E_{3g}$	$A_{2u} + B_{1u} + B_{2u} + E_{1u} + E_{2u} + 2E_{3u}$	$A_{1g} + B_{1g} + B_{2g} + E_{1g} + 2E_{2g} + 2E_{3g}$
D_{2d}	$2A_1 + A_2 + B_1 + B_2 + 2E$	$A_1 + A_2 + B_1 + 2B_2 + 3E$	$2A_1 + A_2 + 2B_1 + 2B_2 + 3E$
D_{3d}	$2A_{1g} + A_{2g} + 3E_g$	$A_{1u} + 2A_{2u} + 4E_u$	$3A_{1g} + 2A_{2g} + 4E_g$
D_{4d}	$A_1 + B_1 + B_2 + E_1 + E_2 + E_3$	$A_1 + A_2 + B_2 + E_1 + E_2 + 2E_3$	$A_1 + B_1 + B_2 + 2E_1 + 2E_2 + E_3$
D_{5d}	$A_{1g} + 2E_{1g} + 2E_{2g}$	$A_{1u} + 2A_{2u} + 2E_{1u} + 2E_{2u}$	$2A_{1g} + A_{2g} + 3E_{1g} + 2E_{2g}$
D_{6d}	$A_1 + E_2 + E_3 + E_4 + E_5$	$B_2 + E_1 + E_2 + E_3 + E_4 + E_5$	$A_1 + B_1 + B_2 + E_1 + E_2 + E_3 + E_4 + E_5$
S_4	$3A + 2B + 2E$	$2A + 3B + 3E$	$3A + 4B + 3E$
S_6	$3A_g + 3E_g$	$3A_u + 4E_u$	$5A_g + 4E_g$
S_8	$A + 2B + E_1 + E_2 + E_3$	$2A + B + E_1 + E_2 + 2E_3$	$A + 2B + 2E_1 + 2E_2 + E_3$
T	$A + E + 2T$	$E + 3T$	$2A + E + 3T$
T_h	$A_g + E_g + 2T_g$	$E_u + 3T_u$	$2A_g + E_g + 3T_g$
T_d	$A_1 + E + T_1 + T_2$	$E + T_1 + 2T_2$	$A_1 + A_2 + E + T_1 + 2T_2$
O	$A_1 + E + T_1 + T_2$	$E + 2T_1 + T_2$	$A_1 + A_2 + E + T_1 + 2T_2$
O_h	$A_{1g} + E_g + T_{1g} + T_{2g}$	$E_u + 2T_{1u} + T_{2u}$	$A_{1g} + A_{2g} + E_g + T_{1g} + 2T_{2g}$
$C_{\infty v}$	$\Sigma^+ + \Pi + \Delta + \Phi + \Gamma$	$\Sigma^+ + \Pi + \Delta + \Phi + \Gamma + H$	$\Sigma^+ + \Pi + \Delta + \Phi + \Gamma + H + I$
$D_{\infty h}$	$\Sigma_g^+ + \Pi_g + \Delta_g + \Phi_g + \Gamma_g$	$\Sigma_u^+ + \Pi_u + \Delta_u + \Phi_u + \Gamma_u + H_u$	$\Sigma_g^+ + \Pi_g + \Delta_g + \Phi_g + \Gamma_g + H_g + I_g$
I	$G + H$	$T_1 + T_2 + H$	$A + T_1 + G + H$
I_h	$G_g + H_g$	$T_{1u} + T_{2u} + H_u$	$A_{1g} + T_{1g} + G_g + H_g$

Gelessus, Thiel, Weber, 1995

Effective Field Theory

Theoretical chemist's picture...

	dipole (p) $l = 1$	quadrupole (d) $l = 2$	octopole (f) $l = 3$
C_1	$3A$	$5A$	$7A$
C_s	$2A' + A''$	$3A' + 2A''$	$4A' + 3A''$
C_i	$3A_u$	[illegible]	$7A_u$
C_2	$A +$ [illegible]	[illegible]	$3A + 4B$
C_3	$A +$ [illegible]	[illegible]	[illegible] $2E$
C_4	$A +$ [illegible]	[illegible]	[illegible]
C_5	A [illegible]	[illegible]	[illegible]
C_6	A [illegible]	[illegible]	[illegible]
C_7	A [illegible]	[illegible]	[illegible]
C_8	[illegible]	[illegible]	[illegible]
D_2	[illegible]	[illegible]	[illegible]
D_3	[illegible]	[illegible]	[illegible]
D_4	[illegible]	[illegible]	[illegible]
D_5	[illegible]	[illegible]	[illegible]
D_6	[illegible]	[illegible]	[illegible]
C_{2v}	[illegible]	[illegible]	[illegible]
C_{3v}	[illegible]	[illegible]	[illegible]
C_{4v}	[illegible]	[illegible]	[illegible]
C_{5v}	[illegible]	[illegible]	[illegible]
C_{6v}	$A_1 + E_1$	[illegible]	[illegible]
C_{2h}	$A_u + 2B_u$	$3A_g +$ [illegible]	[illegible]
C_{3h}	$A'' + E'$	$A' + E' + E''$	[illegible]
C_{4h}	$A_u + E_u$	$A_g + 2B_g + E_g$	$A_u + 2B_u$ [illegible]
C_{5h}	$A'' + E_1'$	$A' + E_2' + E_1''$	$A'' + E_1' + E_2' + E_2''$
C_{6h}	$A_u + E_{1u}$	$A_g + E_{1g} + E_{2g}$	$A_u + 2B_u + E_{1u} + E_{2u}$
D_{2h}	$B_{1u} + B_{2u} + B_{3u}$	$2A_g + B_{1g} + B_{2g} + B_{3g}$	$A_u + 2B_{1u} + 2B_{2u} + 2B_{3u}$
D_{3h}	$A_2'' + E'$	$A_1' + E' + E''$	$2A_1' + A_2' + A_1'' + A_2'' + E' + E''$
D_{4h}	$A_{2u} + E_u$	$A_{1g} + B_{1g} + B_{2g} + E_g$	$A_{2u} + B_{1u} + B_{2u} + 2E_u$
D_{5h}	$A_2'' + E_1'$	$A_1' + E_2' + E_1''$	$A_2'' + E_1' + E_2' + E_2''$
D_{6h}	$A_{2u} + E_{1u}$	$A_{1g} + E_{1g} + E_{2g}$	$A_{2u} + B_{1u} + B_{2u} + E_{1u} + E_{2u}$
D_{8h}	$A_{2u} + E_{1u}$	$A_{1g} + E_{1g} + E_{2g}$	$A_{2u} + E_{1u} + E_{2u} + E_{3u}$
D_{2d}	$B_2 + E$	$A_1 + B_1 + B_2 + E$	$A_1 + A_2 + B_2 + 2E$
D_{3d}	$A_{2u} + E_u$	$A_{1g} + 2E_g$	$A_{1u} + 2A_{2u} + 2E_u$
D_{4d}	$B_2 + E_1$	$A_1 + E_2 + E_3$	$B_2 + E_1 + E_2 + E_3$
D_{5d}	$A_{2u} + E_{1u}$	$A_{1g} + E_{1g} + E_{2g}$	$A_{2u} + E_{1u} + 2E_{2u}$
D_{6d}	$B_2 + E_1$	$A_1 + E_2 + E_5$	$B_2 + E_1 + E_3 + E_4$
S_4	$B + E$	$A + 2B + E$	$2A + B + 2E$
S_6	$A_u + E_u$	$A_g + 2E_g$	$3A_u + 2E_u$
S_8	$B + E_1$	$A + E_2 + E_3$	$B + E_1 + E_2 + E_3$
T	T	$E + T$	$A + 2T$
T_h	T_u	$E_g + T_g$	$A_u + 2T_u$
T_d	T_2	$E + T_2$	$A_1 + T_1 + T_2$
O	T_1	$E + T_2$	$A_2 + T_1 + T_2$
O_h	T_{1u}	$E_g + T_{2g}$	$A_{2u} + T_{1u} + T_{2u}$
$C_{\infty v}$	$\Sigma^+ + \Pi$	$\Sigma^+ + \Pi + \Delta$	$\Sigma^+ + \Pi + \Delta + \Phi$
$D_{\infty h}$	$\Sigma_u^+ + \Pi_u$	$\Sigma_g^+ + \Pi_g + \Delta_g$	$\Sigma_u^+ + \Pi_u + \Delta_u + \Phi_u$
I	T_1	H	$T_2 + G$
I_h	T_{1u}	H_g	$T_{2u} + G_u$

By inspecting character tables can see what is the first multipole transforming as identity for a given isometry group.

Isometry Groups

	hexadecapole (g) $l = 4$	32-pole (h) $l = 5$	64-pole (i) $l = 6$
C_1	$9A$	$11A$	$13A$
C_s	$5A' + 4A''$	$6A' + 5A''$	$7A' + 6A''$
C_i	$9A_g$	$11A_u$	$13A_g$
C_2	$5A + 4B$	$5A + 6B$	$7A + 6B$
C_3	$3A + 3E$	$3A + 4E$	$5A + 4E$
C_4	$3A + 2B + 2E$	$3A + 2B + 3E$	$3A + 4B + 3E$
C_5	$A + 2E_1 + 2E_2$	$3A + 2E_1 + 2E_2$	$3A + 3E_1 + 2E_2$
C_6	$A + 2B + E_1 + 2E_2$	$A + 2B + 2E_1 + 2E_2$	$3A + 2B + 2E_1 + 2E_2$
C_7	$A + E_1 + E_2 + 2E_3$	$A + E_1 + 2E_2 + 2E_3$	$A + 2E_1 + 2E_2 + 2E_3$
C_8	$A + 2B + E_1 + E_2 + E_3$	$A + 2B + E_1 + E_2 + 2E_3$	$A + 2B + E_1 + 2E_2 + 2E_3$
D_2	$3A + 2B_1 + 2B_2 + 2B_3$	$2A + 3B_1 + 3B_2 + 3B_3$	$4A + 3B_1 + 3B_2 + 3B_3$
[illegible]	$2A_1 + A_2 + 3E$	$A_1 + 2A_2 + 4E$	$3A_1 + 2A_2 + 4E$
[illegible]	[illegible] $A_2 + B_1 + B_2 + 2E$	$A_1 + 2A_2 + B_1 + B_2 + 3E$	$2A_1 + A_2 + 2B_1 + 2B_2 + 3E$
[illegible]	[illegible]	$2A_1 + A_2 + 2E_1 + 2E_2$	$A_1 + 2A_2 + 3E_1 + 2E_2$
[illegible]	[illegible]	$A_2 + B_1 + B_2 + 2E_1 + 2E_2$	$2A_1 + A_2 + B_1 + B_2 + 2E_1 + 2E_2$
[illegible]	[illegible]	[illegible] $3B_1 + 3B_2$	$4A_1 + 3A_2 + 3B_1 + 3B_2$
[illegible]	[illegible]	[illegible]	$3A_1 + 2A_2 + 4E$
[illegible]	[illegible]	[illegible]	$2A_1 + A_2 + 2B_1 + 2B_2 + 3E$
[illegible]	[illegible]	[illegible]	[illegible] $2E_2$
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible] E''
[illegible]	[illegible]	[illegible]	[illegible] E_2''
D_{6h}	[illegible]	[illegible]	[illegible] $+ 2E_{2g}$
D_{8h}	$A_{1g} + B_{1g} + B_{2g}$ [illegible]	[illegible]	[illegible] $+ 2E_{3g}$
D_{2d}	$2A_1 + A_2 + B_1 + B_2 + 2E$	[illegible]	[illegible]
D_{3d}	$2A_{1g} + A_{2g} + 3E_g$	A_{1u} [illegible]	[illegible]
D_{4d}	$A_1 + B_1 + B_2 + E_1 + E_2 + E_3$	$A_1 + A_2 + B_2 + E_1 +$ [illegible]	[illegible] E_3
D_{5d}	$A_{1g} + 2E_{1g} + 2E_{2g}$	$A_{1u} + 2A_{2u} + 2E_{1u} + 2E_{2u}$	[illegible]
D_{6d}	$A_1 + E_2 + E_3 + E_4 + E_5$	$B_2 + E_1 + E_2 + E_3 + E_4 + E_5$	$A_1 + B_1 +$ [illegible] $E_3 + E_4 + E_5$
S_4	$3A + 2B + 2E$	$2A + 3B + 3E$	$3A + 4B + 3E$
S_6	$3A_g + 3E_g$	$3A_u + 4E_u$	$5A_g + 4E_g$
S_8	$A + 2B + E_1 + E_2 + E_3$	$2A + B + E_1 + E_2 + 2E_3$	$A + 2B + 2E_1 + 2E_2 + E_3$
T	$A + E + 2T$	$E + 3T$	$2A + E + 3T$
T_h	$A_g + E_g + 2T_g$	$E_u + 3T_u$	$2A_g + E_g + 3T_g$
T_d	$A_1 + E + T_1 + T_2$	$E + T_1 + 2T_2$	$A_1 + A_2 + E + T_1 + 2T_2$
O	$A_1 + E + T_1 + T_2$	$E + 2T_1 + T_2$	$A_1 + A_2 + E + T_1 + 2T_2$
O_h	$A_{1g} + E_g + T_{1g} + T_{2g}$	$E_u + 2T_{1u} + T_{2u}$	$A_{1g} + A_{2g} + E_g + T_{1g} + 2T_{2g}$
$C_{\infty v}$	$\Sigma^+ + \Pi + \Delta + \Phi + \Gamma$	$\Sigma^+ + \Pi + \Delta + \Phi + \Gamma + H$	$\Sigma^+ + \Pi + \Delta + \Phi + \Gamma + H + I$
$D_{\infty h}$	$\Sigma_g^+ + \Pi_g + \Delta_g + \Phi_g + \Gamma_g$	$\Sigma_u^+ + \Pi_u + \Delta_u + \Phi_u + \Gamma_u + H_u$	$\Sigma_g^+ + \Pi_g + \Delta_g + \Phi_g + \Gamma_g + H_g + I_g$
I	$G + H$	$T_1 + T_2 + H$	$A + T_1 + G + H$
I_h	$G_g + H_g$	$T_{1u} + T_{2u} + H_u$	$A_{1g} + T_{1g} + G_g + H_g$

Gelessus, Thiel, Weber, 1995

CH_4

For methane the point group is Cubic:

$$V_T(r) \propto \frac{d^6}{r^7}$$

This is a steep potential generated by a spurion in the 3-index irrep of SO(3) which explicitly breaks SO(3) to the Cubic group.

Methane isn't that special though. Ask cows.

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Punchline: Explicit global symmetry breaking by spurions in large irreps happens in nature already.

Methane isn't that special though. Ask cows.

Spurion Analysis

Example 1: The electron mass.

The electron Yukawa is the only parameter in

$$\mathcal{L}_{\mathrm{IR}}$$

which breaks a U(1) electron chiral symmetry. Thus, within the IR it can only be renormalized proportional to itself.

Spurion Analysis

Example 1: The electron mass.

Furthermore, if there is only one parameter in

$$\mathcal{L}_{\mathrm{UV}}$$

which breaks the chiral symmetry (and gives rise to the Yukawa), then even in the UV the renormalization of the operator responsible for the IR electron Yukawa is also renormalized proportional to itself.

Can start small, stay small, at all scales.

('t Hooft Natural)