Tower Power

Exotic approaches to naturalness and surprises from towers of states

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University of Arizona

Based on a long line of papers since my grad-school days, all sharing a common theme, with many excellent collaborators along the way...

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PASCOS 2025
Durham University, UK
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Mr. Tower

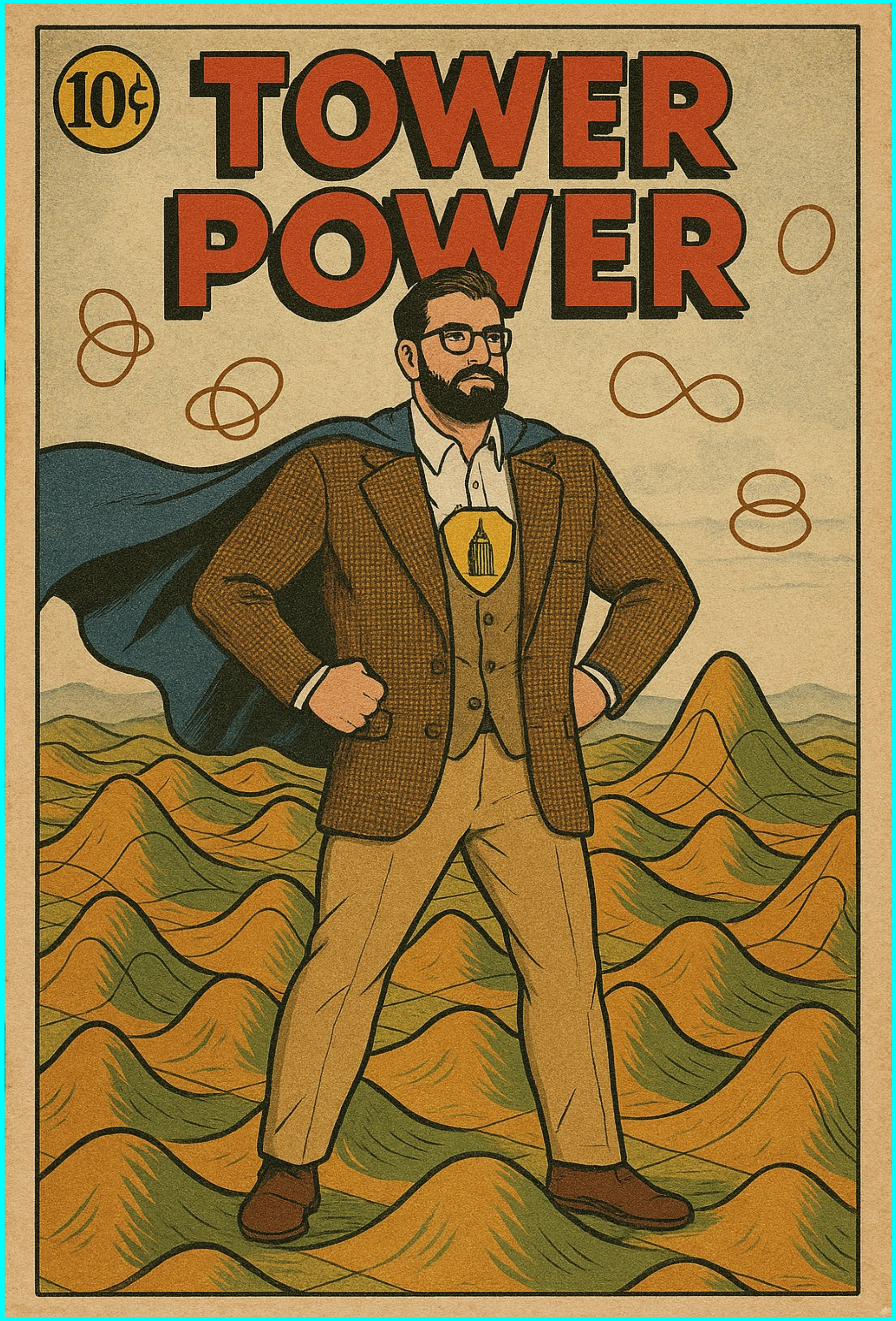
This talk has its very own superhero

(actually just “hero”, not “superhero”, since there is no SUSY)

accessories sold separately

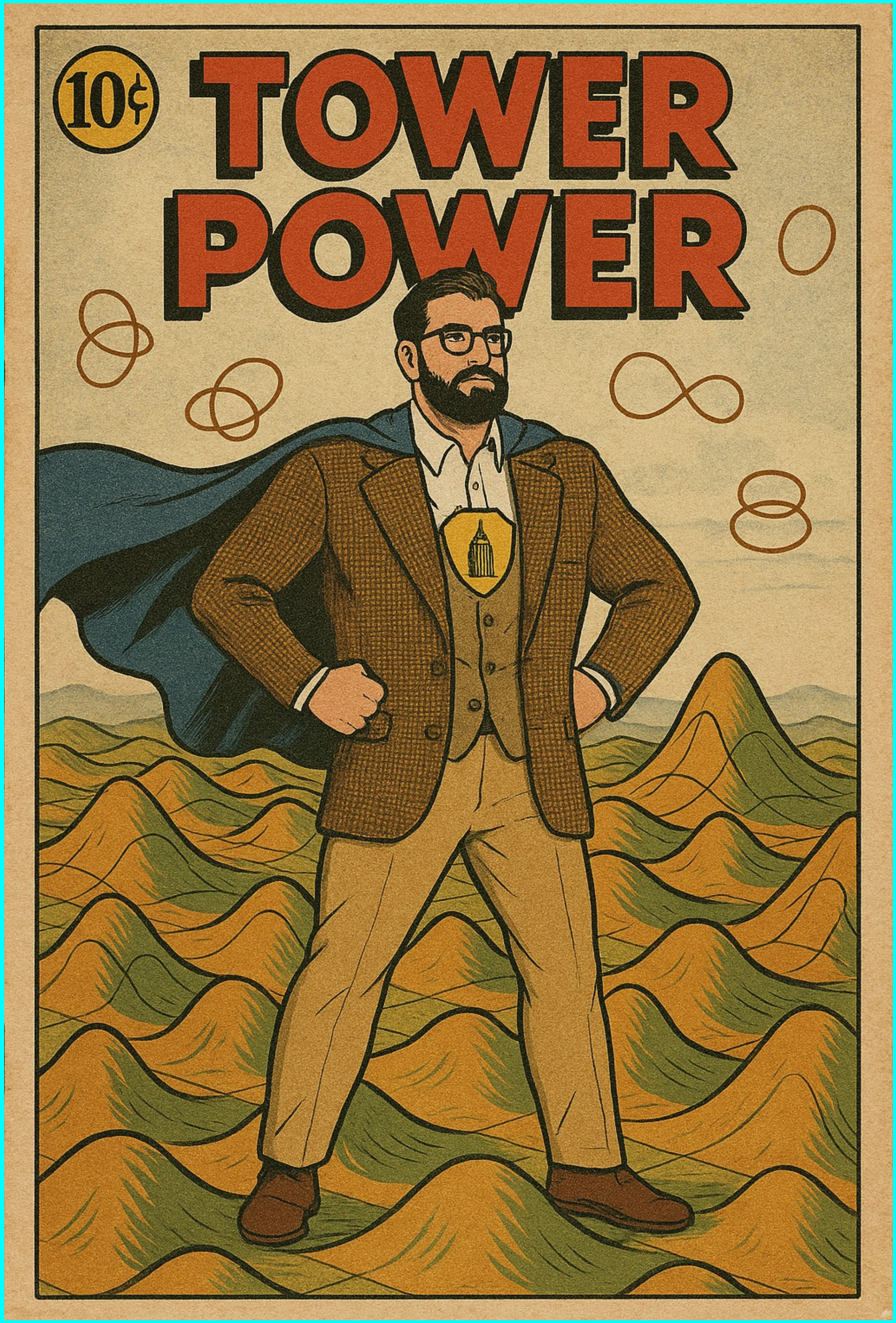
Mr. Tower

Professor Tower



Professor Tower

Full professorial accessories included: Tweed jacket, beard, glasses, shoes (no boots!), standing on a string landscape background.



Mr. Tower

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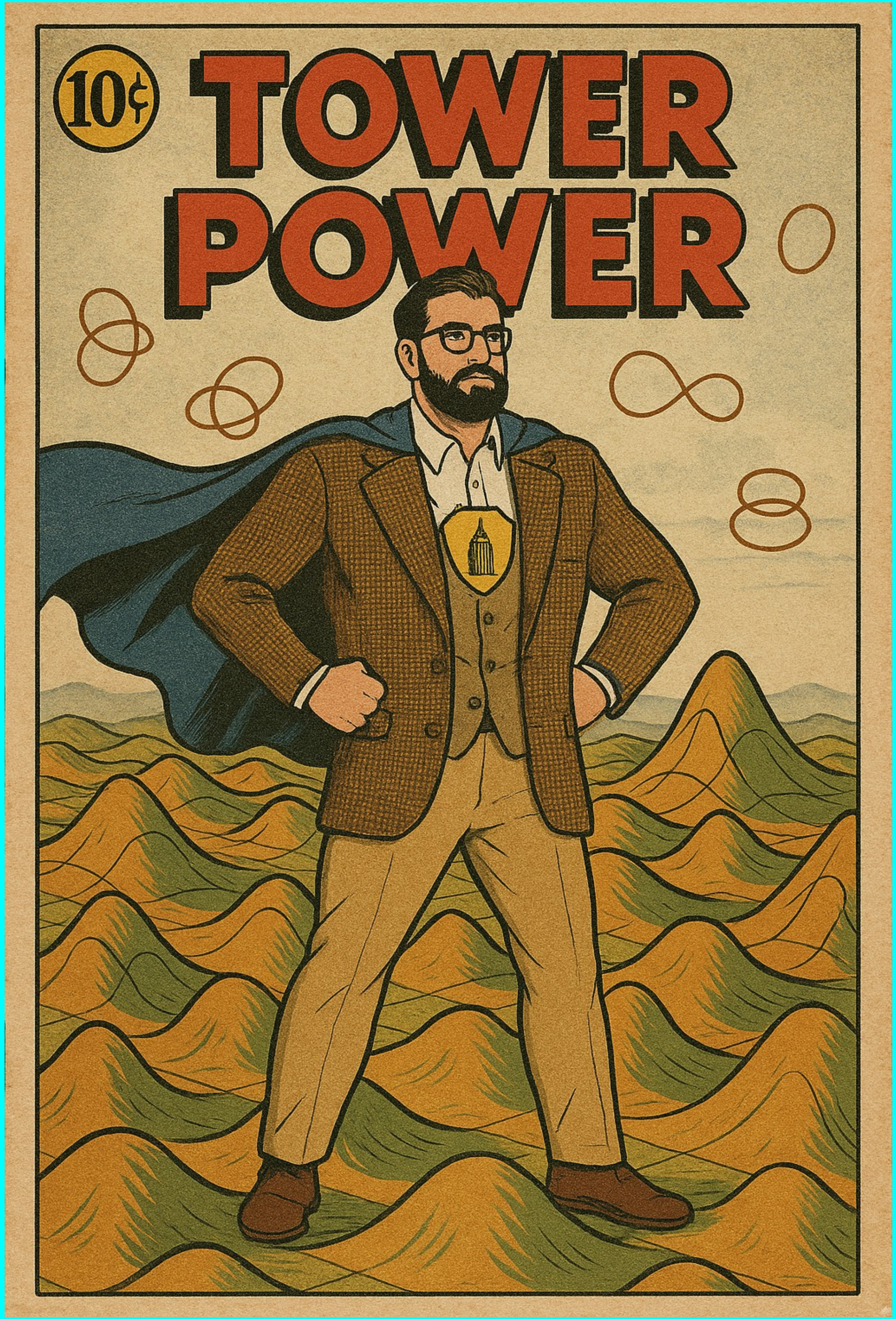
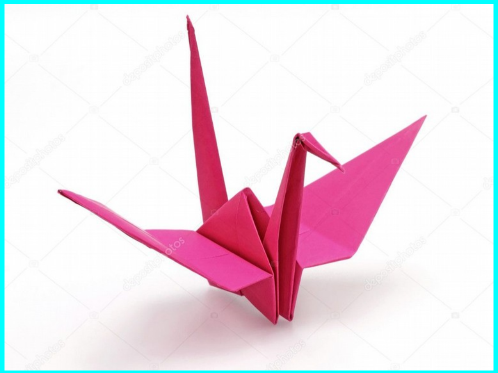


Mr. Tower

Professor Tower

This talk is dedicated to exploring a circle of ideas that are all connected to exotic approaches to naturalness...



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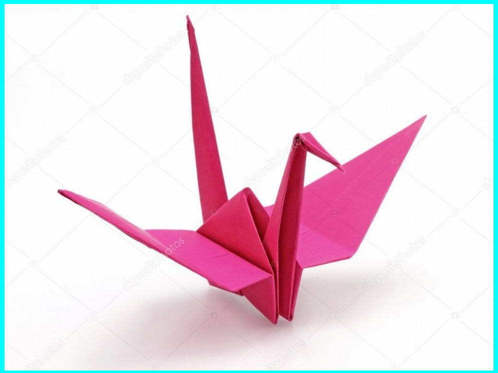
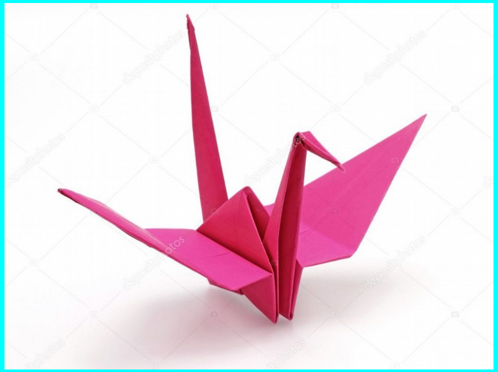


Naturalness concerns the existence and stability of widely separated energy scales with respect to quantum corrections
$\Rightarrow$ relations between UV and IR physics.

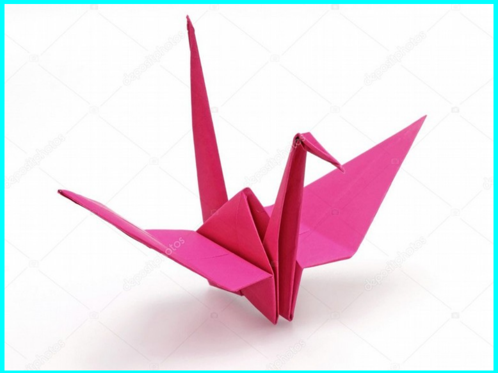
Therefore natural to explore ideas in which these interact and mix!

This talk is dedicated to exploring a circle of ideas that are all connected to exotic approaches to naturalness...



UV/IR mixing & associated symmetries

This talk is dedicated to exploring a circle of ideas that are all connected to exotic approaches to naturalness...

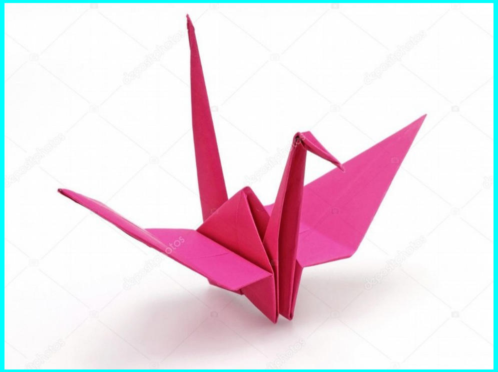


UV/IR mixing &
associated symmetries

Because these UV/IR symmetries involve physics at all scales, they will not be readily apparent to low-energy observers.

In such cases, the resulting finiteness would appear to be the result of "hidden" cancellations! Coefficients of dangerous terms would "magically" be zero,
akin to SUSY supertrace relations but *without* SUSY!

This talk is dedicated to exploring a circle of ideas that are all connected to exotic approaches to naturalness...



UV/IR mixing & associated symmetries

Finiteness through “hidden” cancellations

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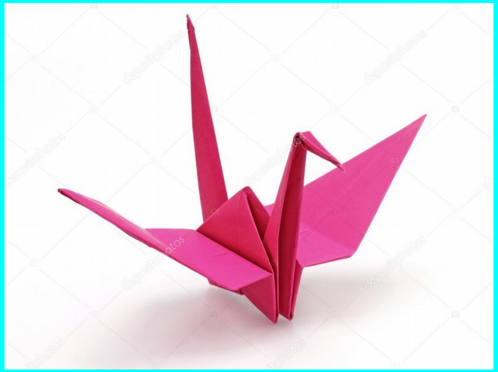


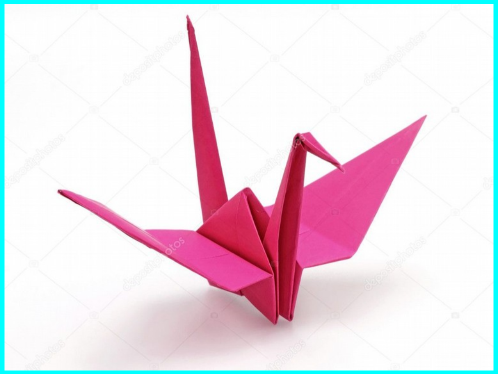
UV/IR mixing & associated symmetries

Finiteness through “hidden” cancellations

Because these cancellations are the result of (and protected by) UV/IR-mixed symmetries, they rely on conspiracies between physics *at all scales simultaneously*.

Such theories therefore have rich physics populating *all* scales, including infinite towers of states.

This talk is dedicated to exploring a circle of ideas that are all connected to exotic approaches to naturalness...

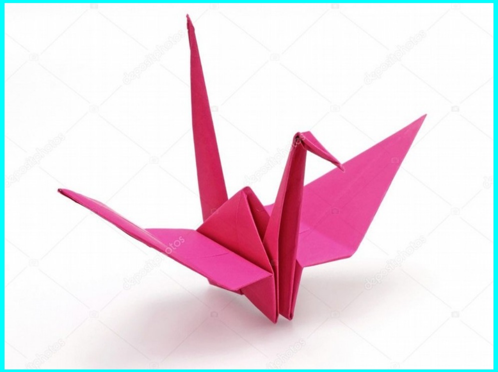


UV/IR mixing & associated symmetries

Finiteness through "hidden" cancellations

Infinite towers of states

This talk is dedicated to exploring a circle of ideas that are all connected to exotic approaches to naturalness...



UV/IR mixing & associated symmetries

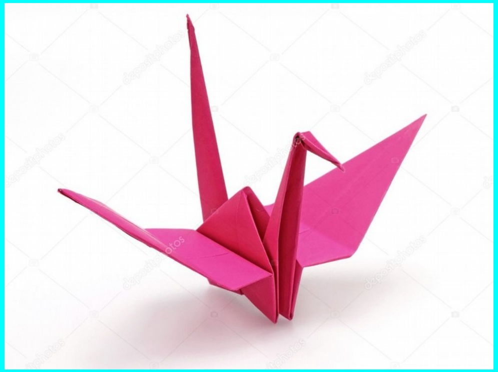
Finiteness through "hidden" cancellations

Infinite towers of states

Finally, if these theories have UV/IR mixing and achieve finiteness through conspiracies involving physics at all scales simultaneously, to what extent do they have low-energy EFT descriptions?

Is it possible to extract an EFT from such a theory?
What role would it play, and how could it be interpreted?

This talk is dedicated to exploring a circle of ideas that are all connected to exotic approaches to naturalness...



UV/IR mixing & associated symmetries

Finiteness through “hidden” cancellations

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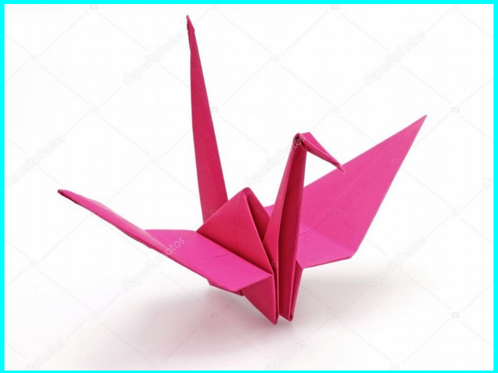
Extraction / role of EFTs

This talk is dedicated to exploring a circle of ideas that are all connected to exotic approaches to naturalness...



UV/IR mixing & associated symmetries

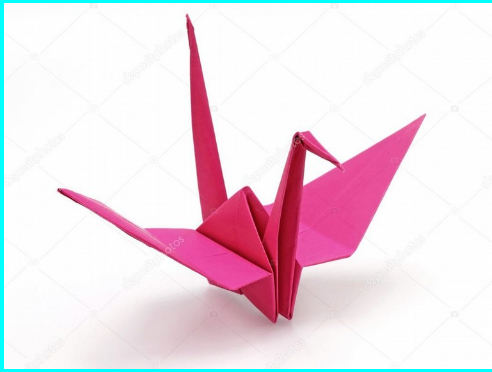
Finiteness through "hidden" cancellations

This is the circle of ideas we will be studying. But if we are talking about UV/IR-mixed theories, it is critical we study such ideas within frameworks that are UV-complete...

Infinite towers of states

Extraction / role of EFTs

This talk is dedicated to exploring a circle of ideas that are all connected to exotic approaches to naturalness...



UV/IR mixing & associated symmetries

Finiteness through “hidden” cancellations

UV completion:
String theory

Infinite towers of states

Extraction / role of EFTs

As we shall see, all of these features appear in string theory.

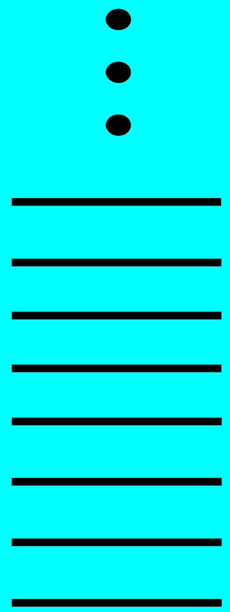
However, our goal will be discuss them in a generic way without focusing on any particular string theory in detail….

That said, behind the scenes we shall take our guidance from the physics (and the UV/IR-mixed theories, such as worldsheet modular invariance) that emerge within weakly-coupled heterotic strings. These strings readily furnish us with most of the physics that we expect to find in the low-energy world

- Rich particle content, including extended non-abelian gauge symmetries and chiral fermion representations, potentially even resembling the SM
- Unification with gravity
- UV completeness

So what lessons emerge from such strings?

At the level of explicit calculations, most of these features involving finiteness and UV/IR-symmetries are encapsulated within the *towers of states*.



Such towers of states appear in many/most theories of BSM physics

- KK modes for theories in extra dimensions
- Winding modes for closed string theories in extra dimensions
- Infinite towers of resonances in strongly coupled gauge theories
- Infinite towers of string resonances

Usually, when doing calculations, we simply ignore such heavy states, believing that they cannot really play a role for low-energy phenomenology.

?

- These states are usually at the Planck scale, or at the scales associated with the compactification geometry! ***How can they ever play an important role for low-energy phenomenology?***
- Can't they just be integrated out, leaving behind higher-dimensional operators suppressed by powers of these heavy scales?
- Wouldn't this justify the usual treatment?

Indeed, most studies of string phenomenology over the past 30 years have either ignored such states or integrated them out, treating them in an effective field-theoretic manner and rolling them into small threshold corrections…

However, there are reasons to take pause...

- We would not be integrating out one or two or three heavy states. We would be integrating out *infinite towers* of states!
- Even more severely, these towers of states have *degeneracies that grow exponentially* with their masses!
- *Can this still leave behind a power-law suppression of higher-dimensional operators? Usual EFT expectations about disregarding these infinite towers of states may not apply.*

Hagedorn

Indeed, it is natural to expect that these infinite towers of states would particularly affect quantities (such as the Higgs mass or the cosmological constant) which have positive mass dimension and are therefore sensitive to all mass scales in the theory. The same may also be true of hierarchy problems and apparent divergences in field-theory calculations!

Running theme throughout this talk:
Towers of states can be critical for low-energy physics, even if they are extremely massive.
As such, their role can transcend that of providing small "corrections" to low-energy physics. Instead, they can produce unexpected results which can completely change our perspective on low-energy physics and thereby potentially provide new solutions to long-standing problems.

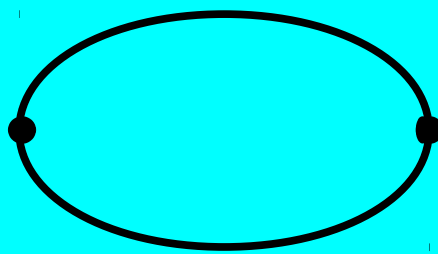
In this talk, I will provide snapshots of different examples in which towers of heavy states provide new insights, both in particle physics and cosmology. String theory will provide the background motivation for much of this talk, but I will keep things non-technical and avoid the mathematics as much as possible.

But first, what kinds of UV/IR-mixed symmetries are we talking about?

Without getting into details, let us consider a simplified toy example of such a UV/IR-mixed symmetry which will illustrate the main ideas.

Let us consider the zero-point one-loop amplitude (vacuum energy = CW effective potential = cosmological constant) Λ in field theory...

$$\Lambda \;=\; \tfrac{1}{2}\sum_n (-1)^{F_n}\, g_n \int \frac{d^D p}{(2\pi)^D}\, \log(p^2 + M_n^2)$$



Summation over spectrum

number of states
(# bosons - # fermions)
=0 for SUSY

Passing to a Schwinger proper-time formulation, we can rewrite this as

$$\Lambda \;=\; -\,\frac{1}{2}\left(\frac{\mu}{2\pi}\right)^D \int_0^\infty \frac{dt}{t}\, Z(t)$$

where

$$Z(t) \;=\; \frac{1}{t^{D/2}}\sum_n (-1)^{F_n}\, g_n\, e^{-\pi(M_n^2/\mu^2)t}$$

We thus have

$$\Lambda \;=\; -\frac{1}{2}\left(\frac{\mu}{2\pi}\right)^{D}\int_0^\infty \frac{dt}{t}Z(t)$$

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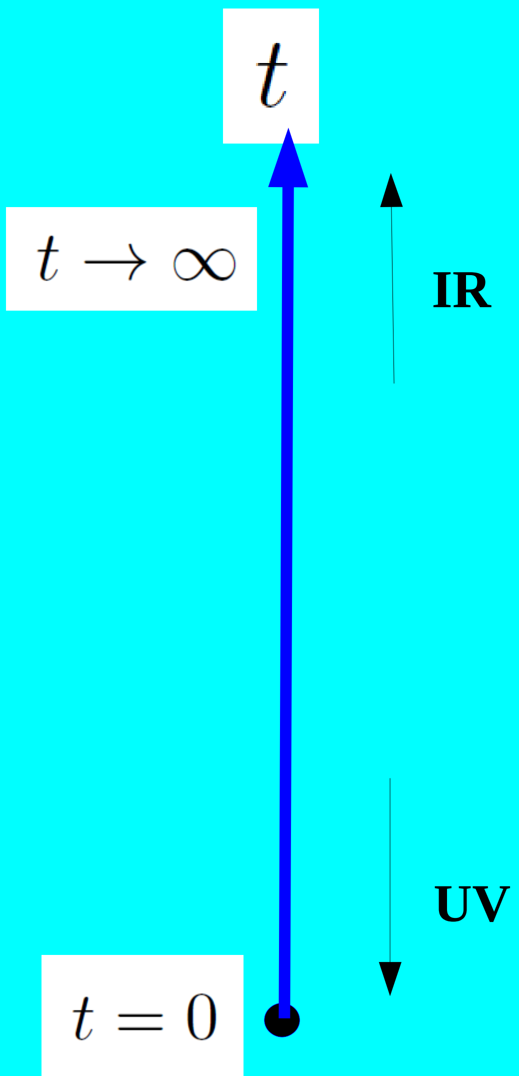

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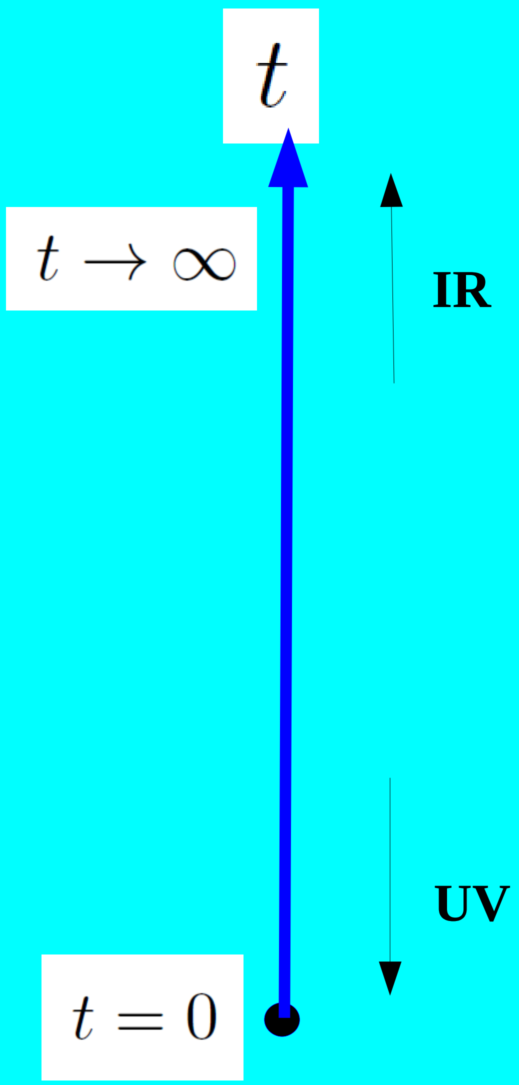


IR divergences arise here
(behavior of Z as $t \to$ infinity)

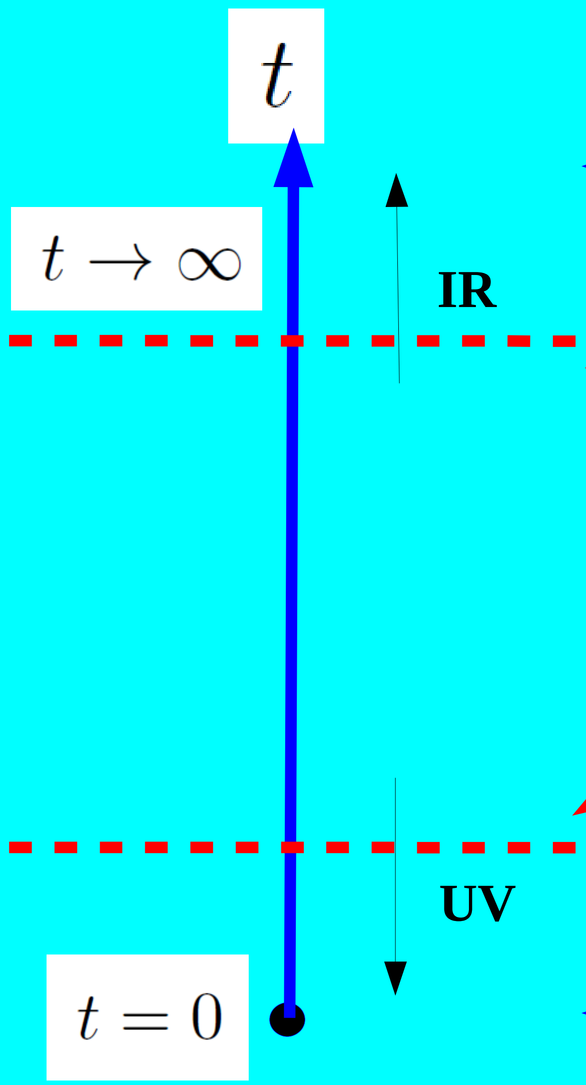
UV divergences arise here
(behavior of Z as $t \to 0$)

How to handle divergences?

We thus have

$$\Lambda \;=\; -\frac{1}{2}\left(\frac{\mu}{2\pi}\right)^{D}\int_0^\infty \frac{dt}{t}\,Z(t)$$

$$Z(t) \;=\; \frac{1}{t^{D/2}}\sum_n (-1)^{F_n} g_n\, e^{-\pi (M_n^2/\mu^2)t}$$



IR divergences arise here
(behavior of Z as $t \to$ infinity)

In ordinary QFT, we introduce cutoffs!

IR cutoff: $t \leq \Lambda_{\rm IR}^{-2}$

UV cutoff: $t \geq \Lambda_{\rm UV}^{-2}$

introduce either cutoff as needed → new mass scales

UV divergences arise here
(behavior of Z as $t \to 0$)

Thus far, we have stayed within traditional QFT.
But now let's ask a hypothetical question:

What if our theory had an exact symmetry under

$$t \longrightarrow \frac{1}{t} \quad ?$$

Such a symmetry is clearly not field-theoretic!
But let's pursue this anyway.

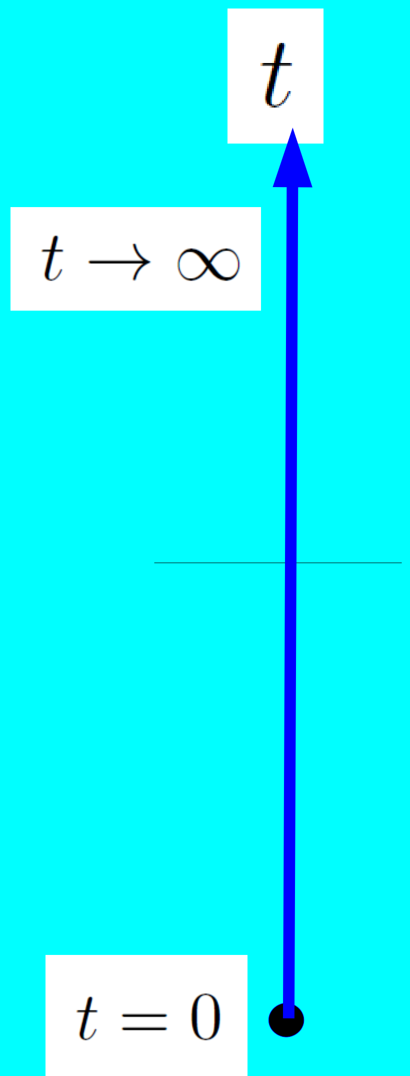
- What effects would this have?
- How could we interpret this?

We thus have

$$\Lambda \;=\; -\frac{1}{2}\left(\frac{\mu}{2\pi}\right)^{D}\int_0^\infty \frac{dt}{t}\,Z(t)$$

$$Z(t) \;=\; \frac{1}{t^{D/2}}\sum_n (-1)^{F_n} g_n\, e^{-\pi(M_n^2/\mu^2)t}$$

- Integration domain (line) is invariant
- Measure is invariant
- Thus, partition function must also be invariant in such a theory!



Introducing a “flip” symmetry across t=1 line!

We thus have

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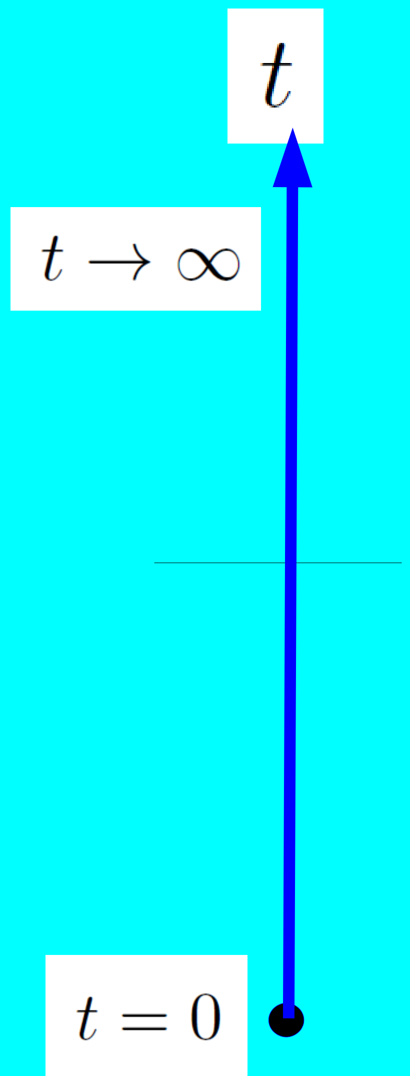


Introducing a “flip” symmetry across t=1 line!

- Physics from t >1 and t <1 integration regions becomes identical!
- These regions provide redundant descriptions of the same physics!
- Thus, UV divergence must also be the same as IR divergence, likewise attributable to same underlying physics!

Sound familiar?

Two well-known analogues…

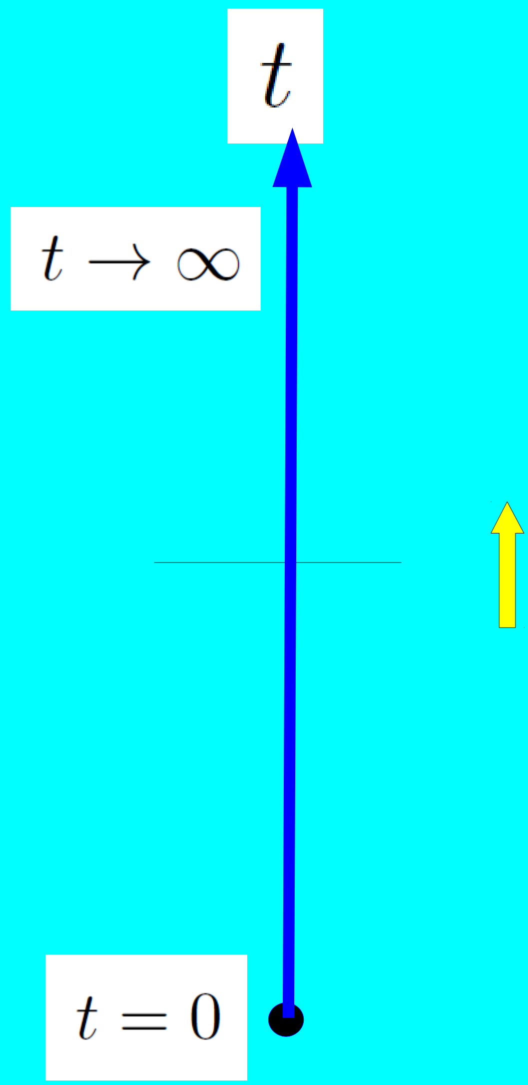
- Redundancy of description is like a *gauge* symmetry!
  Integrating over both $\mathcal{S}_1$ and $\mathcal{S}_2$ is like integrating over all of the gauge slices! Of course, there are only two gauge slices in this little example, and the overall factor of 2 is the "gauge volume". Still, the appropriate treatment is the same: **Effectively divide out by the gauge volume by choosing only *one* gauge slice!**

- Suppose we compactify a theory on a circle $y \sim y + 2\pi R$. Then we mod out by a Z_2 symmetry $y \rightarrow -y$ to construct an orbifold.
  What happens? The compactification volume of the orbifold is now only *half* that of the circle. We are effectively compactifying on the resulting line segment (= the orbifold). The point $y=0$ is self-dual and forms a new "edge".

We thus have

$$\Lambda = -\frac{1}{2}\left(\frac{\mu}{2\pi}\right)^{D}\int_0^{\infty}\frac{dt}{t}Z(t)$$

$$Z(t) = \frac{1}{t^{D/2}}\sum_n (-1)^{F_n} g_n\, e^{-\pi(M_n^2/\mu^2)t}$$

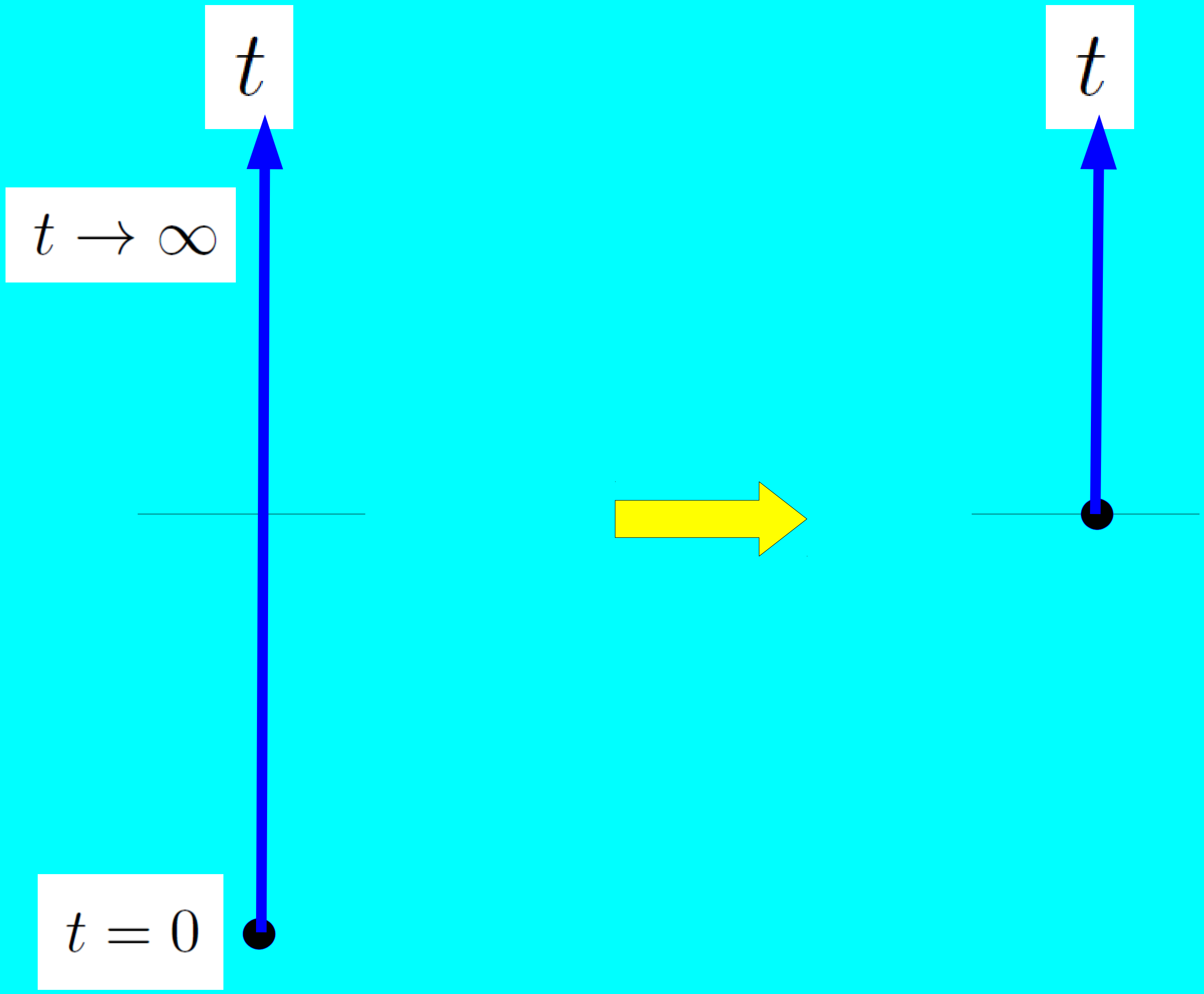


FOLD THE LINE ACROSS THIS POINT

We thus have

$$\Lambda \;=\; -\frac{1}{2}\left(\frac{\mu}{2\pi}\right)^{D}\int_0^\infty \frac{dt}{t}\,Z(t)$$

$$Z(t) \;=\; \frac{1}{t^{D/2}}\sum_n (-1)^{F_n} g_n\, e^{-\pi(M_n^2/\mu^2)t}$$



This becomes the new integration region!

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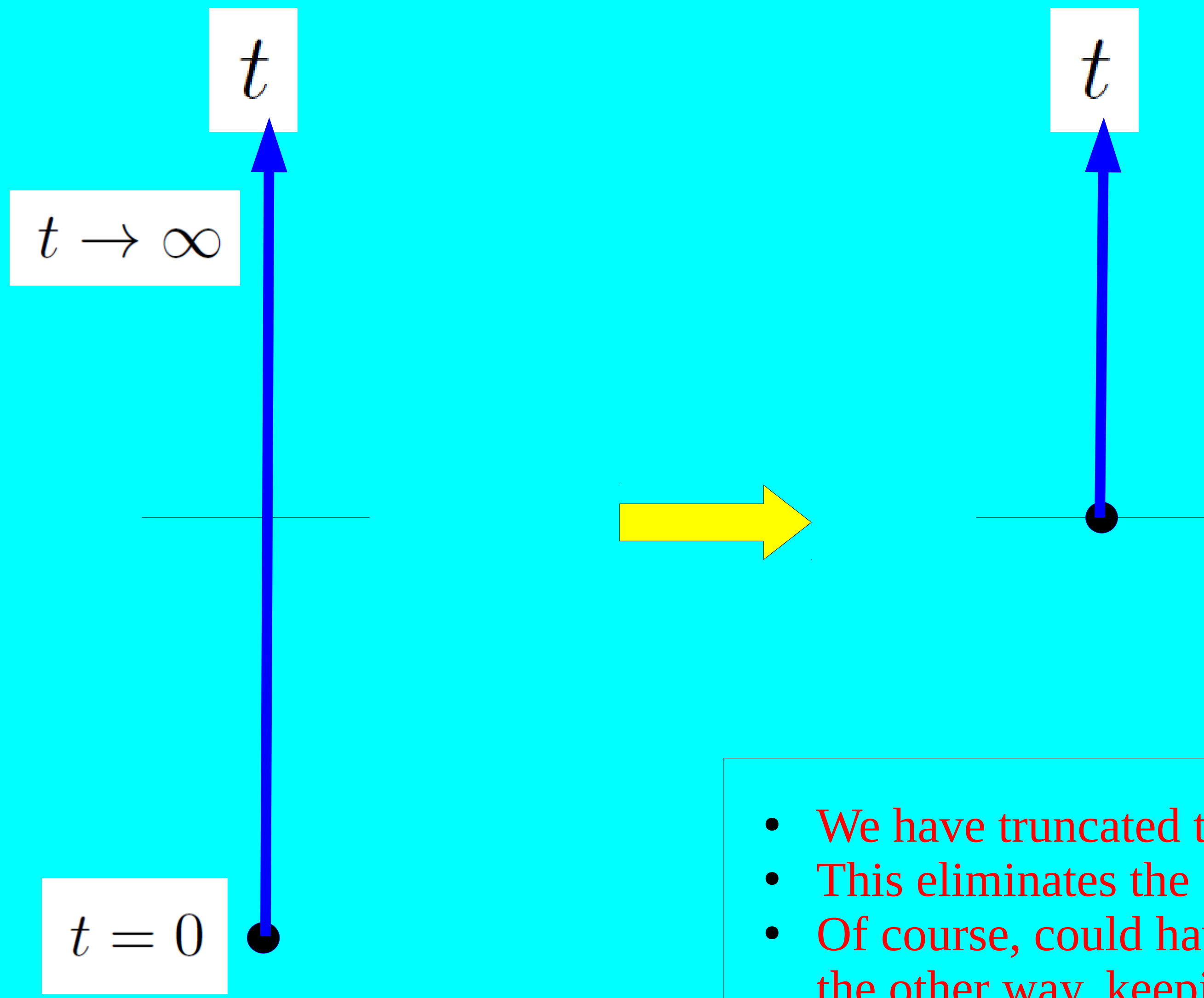

This becomes the new integration region!

- We have truncated the line to just one “slice”.
- This eliminates the spurious factor of 2.
- Of course, could have chosen to fold the line the other way, keeping t <1 instead.

We thus have

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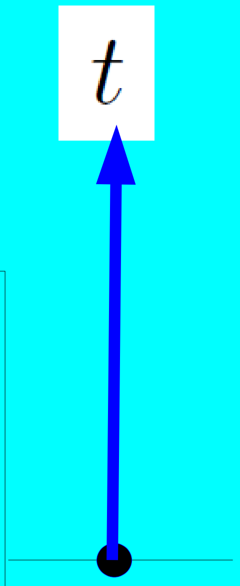

But what does this folding imply for UV versus IR?

- The bottom part is folded *onto* the top part.
- There is no longer a unique up or down direction on the remaining segment! No notion of increasingly UV or IR "directions" → all directionality is lost. "Non-orientable"
- The two divergences (UV and IR) have been folded on top of each other!
- Thus, ***there is only one divergence.*** You can call it UV or IR according to your choice/convention → meaningless distinction!

This becomes the new integration region!

Of course, this nightmare arises only if we have the $t \to 1/t$ symmetry.

Can this ever really happen in field theory?

Not likely...

$$\Lambda = -\frac{1}{2}\left(\frac{\mu}{2\pi}\right)^D \int_0^\infty \frac{dt}{t} Z(t)$$

where

$$Z(t) = \frac{1}{t^{D/2}} \sum_n (-1)^{F_n} g_n \, e^{-\pi(M_n^2/\mu^2)t}$$

- Line is invariant … **ALREADY TRUE**
- Measure is invariant … **ALREADY TRUE**
- Thus, Z would also need to be is invariant … **VERY HARD TO ARRANGE!**

Each *t*-factor in the exponential gets inverted!
Is there some mathematical identity?

$$\Lambda = -\frac{1}{2}\left(\frac{\mu}{2\pi}\right)^D \int_0^\infty \frac{dt}{t} Z(t)$$

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Each t-factor in the exponential gets inverted!
Is there some mathematical identity?

Yes!

$$\sum_{n=-\infty}^{\infty} e^{-\pi n^2/t} = \sqrt{t} \sum_{k=-\infty}^{\infty} e^{-\pi k^2 t}$$

Just one of a whole series of similar identities involving infinite sums of exponentials

"Poisson resummation"

<u>But this could only be useful if there were an *infinite* tower of states</u>! Rather sick from a field-theoretic perspective…. (Must also have very tight balancing of masses and degeneracies at each level in order for such identities to apply.)

But this is precisely what happens in string theory!

Indeed, in string theory we have the divergence structure

$$\underbrace{\mathrm{IR} = \mathrm{UV}}_{\text{string theory}} \quad \Longleftrightarrow \quad \underbrace{(\mathrm{IR} \oplus \mathrm{UV})}_{\text{field theory}} \;/\; \underbrace{\text{redundancy volume}}_{\infty}$$

- UV and IR divergences are identified, collapsed into one!
- This remaining divergence is softened, since we divide out by the volume!
- For actual string UV/IR symmetries, this volume is *infinite*.

As a result, many quantities which are divergent in field theory are actually finite in string theory!

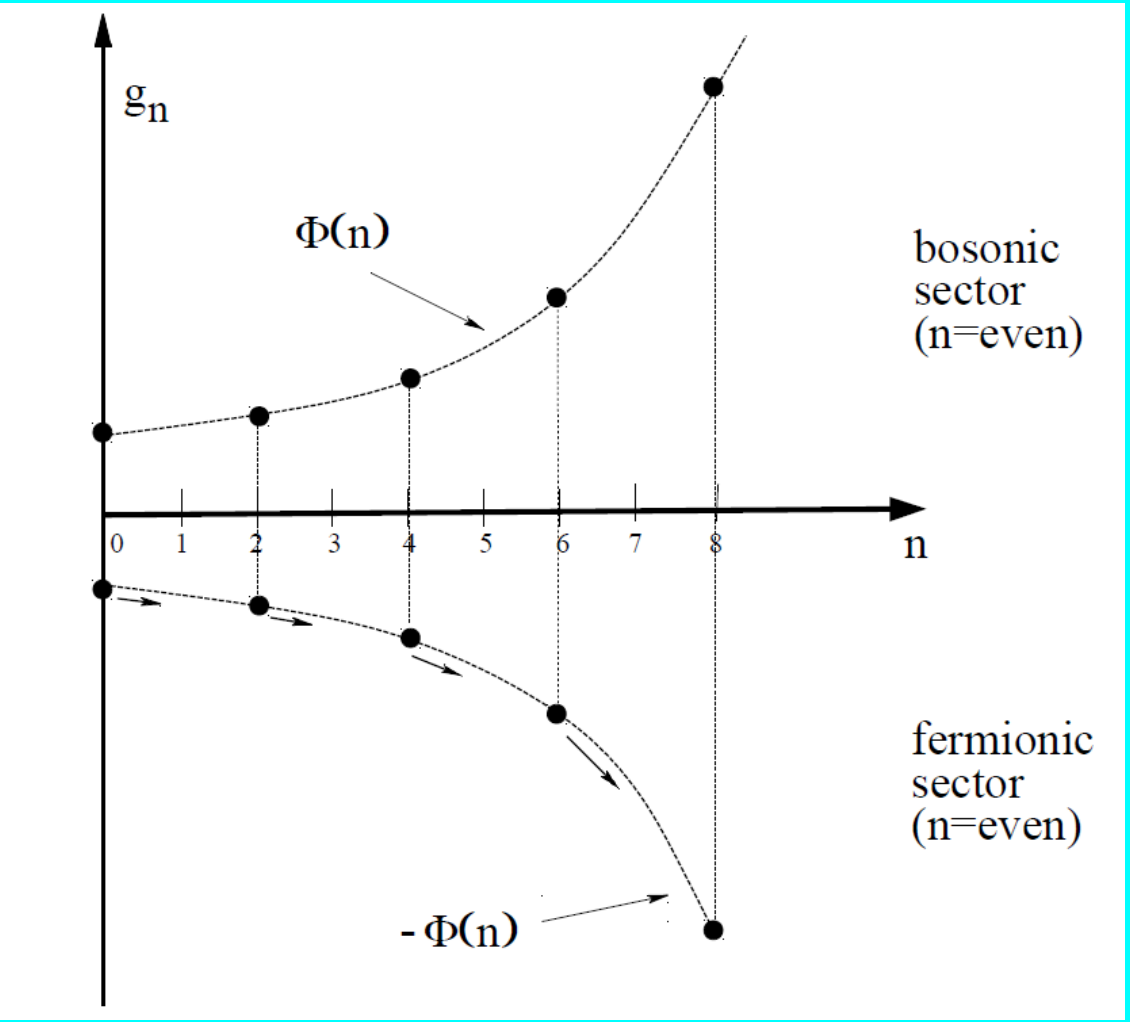
Indeed, they only ***appeared*** to be divergent because we were ignoring the UV/IR-mixed symmetries by throwing out the infinite towers of states and just looking at the low-lying states. Towers matter!

This is therefore an example in which the existence of an infinite tower of states --- rather than *exacerbating* a divergence --- actually eliminates it!

But how do these UV/IR symmetries actually constrain the distributions of bosons and fermions across the tower of states?

As expected, supersymmetric configurations at each mass level are allowed...

- Bosonic and fermionic functional forms $\Phi(n)$ are the same but have opposite signs.
- In this case, it is also true that the bosonic and fermionic sectors are "aligned", occurring at the same discrete values of n
- This implies that there are equal numbers of bosons and fermions at each mass level (the hallmark of SUSY).
- As a result, all SUSY cancellations occur in a pairwise manner, with the contributions from bosons at a given mass level cancelling against the contributions of fermions from the same mass level.



Unfortunately, this degenerate pairing of bosonic and fermionic states does not describe our world.

Yet the UV/IR mixing tightly controls what happens at all mass levels simultaneously across the infinite towers of states.

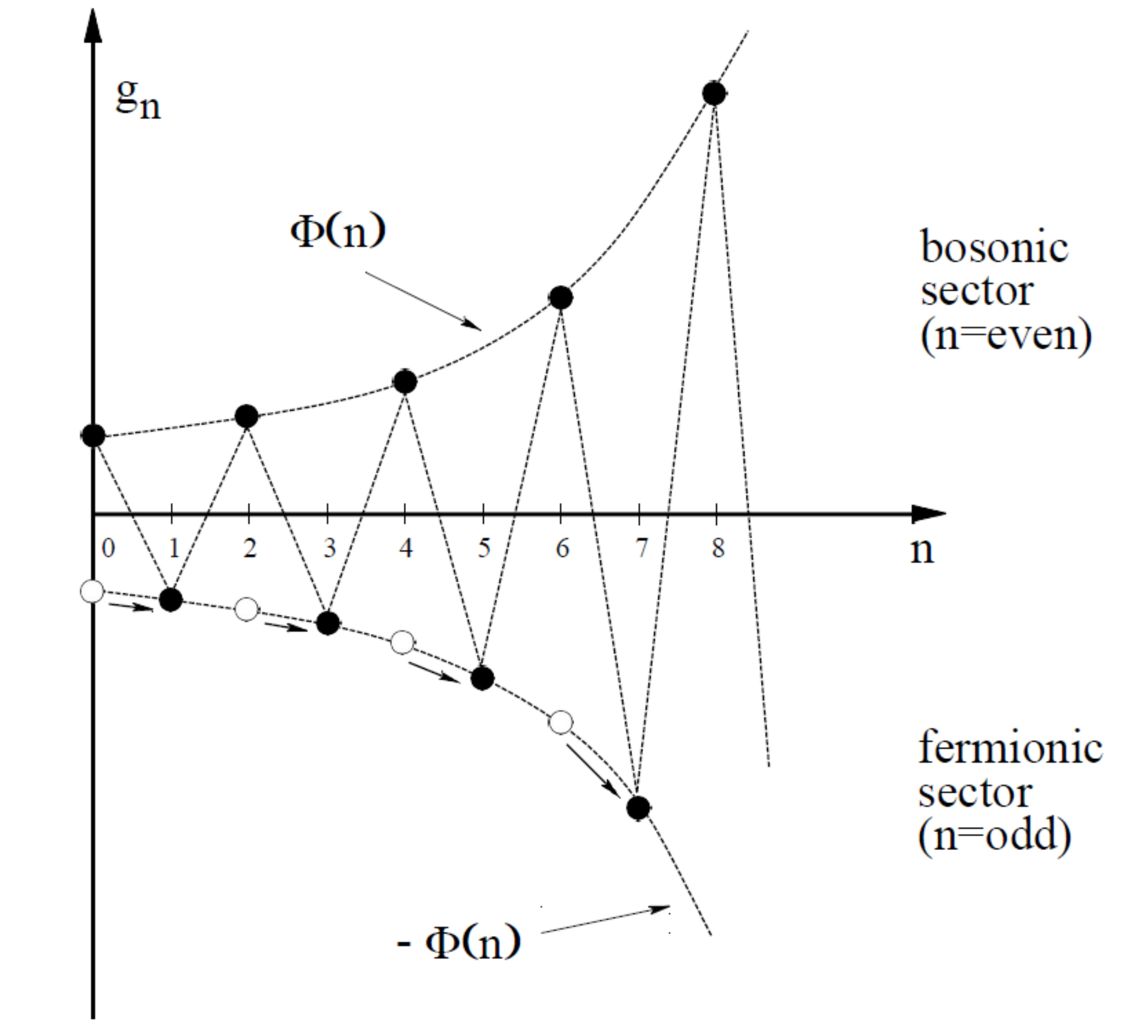
- To what extent can one disturb this SUSY picture while remaining consistent with the UV/IR-mixed symmetries?
- Indeed, what is the most general configuration of bosons and fermions across the entire tower of states that is allowed by the UV/IR-mixed symmetries?

Misaligned SUSY

- KRD, hep-th/9402006

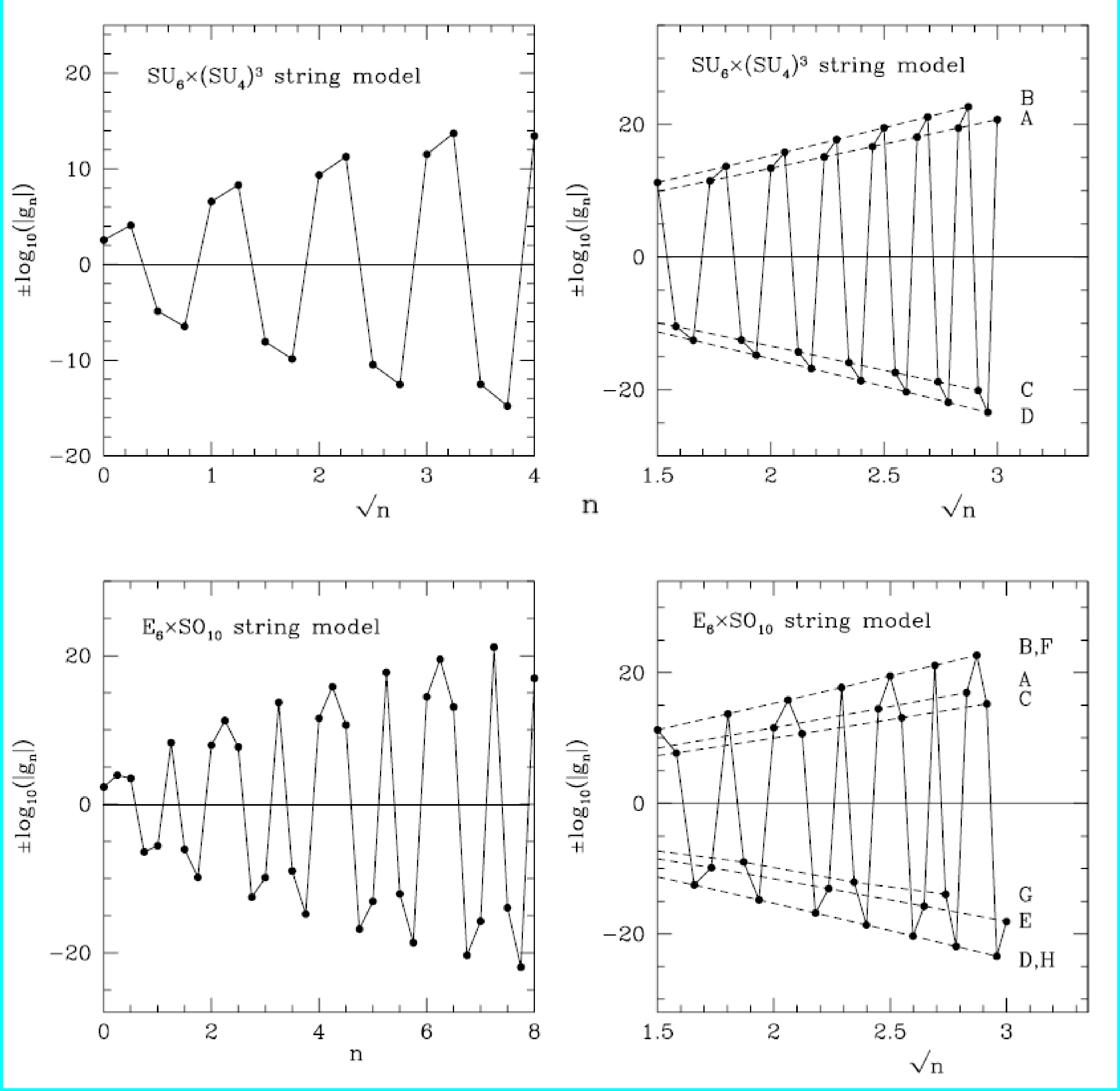
In any tachyon-free closed string theory, spacetime SUSY may be broken but a residual "**misaligned SUSY**" must always remain in the string spectrum!

- Bosonic and fermionic functional forms $\Phi(n)$ continue to cancel.
- **But now the bosonic and fermionic sectors become *misaligned*!**
- As sectors become misaligned, new states must populate each level to preserve $\Phi(n)$.
- *No pairwise cancellations* --- now all masses across the tower conspire together! This is the maximum degree to which SUSY may be broken.
- *This is how spectrum of a given string theory manages to configure itself at all mass levels so as to maintain finiteness --- even without SUSY.* **The towers matter!**



Actual string models...

- KRD, hep-th/9402006



Note: boson and fermion degeneracies never coincide, even asymptotically! No "asymptotic SUSY".

Supertraces over physical string states

- KRD, M. Moshe, R. Myers, hep-th/9503055

These configurations of bosonic and fermionic states enable an alternate, purely "on-shell" formulation of many string amplitudes directly and succinctly in terms of supertraces.

Define supertrace of any operator $\mathcal{X}$ as

$$\mathrm{Str}\,\mathcal{X} \equiv \lim_{y\to 0} \sum_n \underbrace{a_{nn}}_{\substack{\text{net numbers (\#B-\#F)}\\ \text{of physical string}\\ \text{states at level } n}} \, \underbrace{\mathcal{X}_{nn}}_{\substack{\text{eigenvalues}\\ \text{of operator } \mathcal{X}}} \, e^{-\alpha' \underbrace{M_n^2}_{\substack{\text{mass of}\\ \text{level } n}} y}$$

This regulator y guarantees finite results for infinite exponentially growing towers of string states, even as $y \to 0$.

- KRD, M. Moshe, R. Myers, hep-th/9503055

The spectrum of any 4D tachyon-free closed string theory then satisfies

- $$\mathrm{Str}\,\mathbf{1} \;=\; 0$$
  - If SUSY, satisfied trivially via boson/fermion pairing
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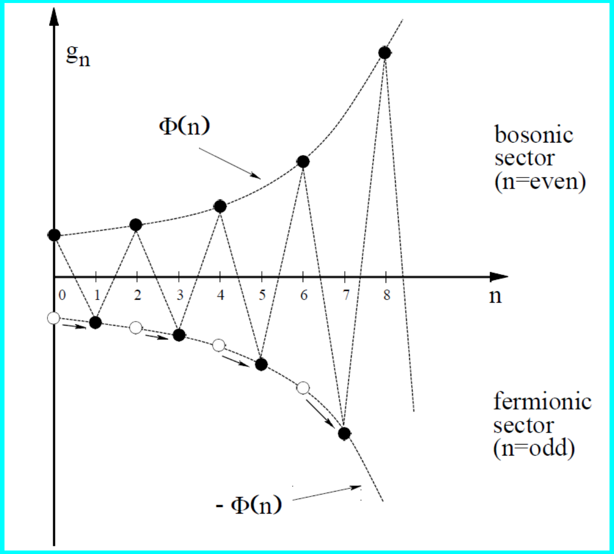

- KRD, M. Moshe, R. Myers, hep-th/9503055

The spectrum of any 4D tachyon-free closed string theory then satisfies

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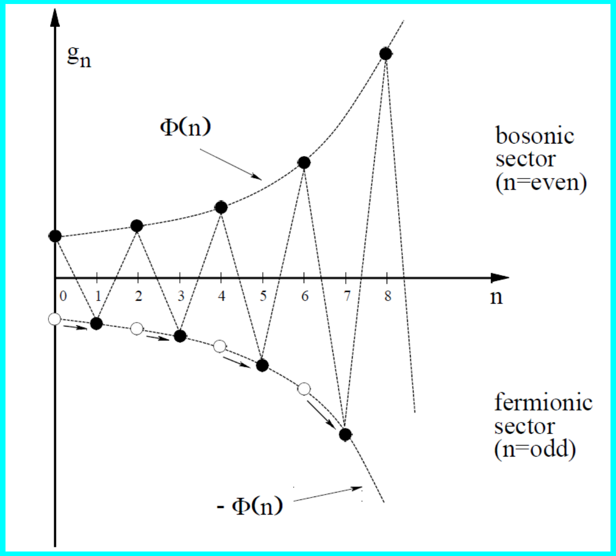
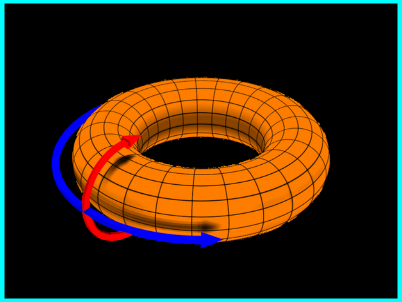


- $$\Lambda \;=\; \frac{1}{24}\,\mathcal{M}^2\,\mathrm{Str}\,M^2$$
  - **Cannot be understood in field theory,** where Str M^2 governs the quadratic divergence of Λ while Str 1 governs the quartic divergence.
  - But now **Str 1=0, and Str M^2 is the actual (finite) value of Λ itself!**
  - Sum involves only physical string states! Influence of unphysical states is felt through effects on the masses of the physical states.
  - Yields a purely "on-shell" formalism for Λ!

Recall

$$\Lambda \;\equiv\; -\frac{\mathcal{M}^4}{2}\int_{\mathcal{F}}\frac{d^2\tau}{\tau_2^2}\,\tau_2^{-1}\sum_{m,n}\overline{q}^m q^n$$



A one-loop amplitude just from a weighted counting of states!

- KRD, M. Moshe, R. Myers, hep-th/9503055

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These results were known since 1995.

But more recently we found that they generalize significantly!

- KRD, M. Moshe, R. Myers, hep-th/9503055
- S. Abel, KRD, L. Nutricati, 2303.08534

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- $$\mathrm{Str}\,\mathcal{X} \;=\; 0$$
  - Holds for *any* operator insertion $\mathcal{X}$ leading to a finite modular-invariant amplitude $<\mathcal{X}>$
  $$\langle \mathcal{X} \rangle \;\equiv\; \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2}\,\tau_2^{-1} \sum_{m,n} a_{mn}\,\mathcal{X}_{mn}\,\overline{q}^m q^n$$
  - Can even be taken as an on-shell *definition* of when an insertion $\mathcal{X}$ is modular invariant

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- $$\langle \mathcal{X} \rangle \;=\; \frac{\pi}{3}\left[\mathrm{Str}\,\frac{d\mathcal{X}}{d\tau_2} \;-\; \frac{1}{4\pi\mathcal{M}^2}\,\mathrm{Str}\,(\mathcal{X} M^2)\right]$$
  - All corresponding amplitudes take this form
  - Λ is special case with $\mathcal{X}=\mathbf{1}$:
  $$\Lambda \;\equiv\; -\,\frac{\mathcal{M}^4}{2}\,\langle \mathbf{1} \rangle$$
  - No need for one-loop integrals, depends only on physical states!

But how are such supertrace relations even possible?

Once again, it’s the magic of the towers --- and the structure of the misaligned SUSY –- which allows such results to emerge.

- **If we were to break SUSY in field theory**, bosonic and fermionic states would still paired, but their degeneracy would be lifted:

$$f^{(B)}(y) = \sum_n g(n)\, e^{-ny}$$
$$f^{(F)}(y) = \sum_n g(n)\, e^{-(n+\delta n)y}$$

$$f_{\text{field}}(y) = \sum_n g(n) \left[e^{-ny} - e^{-(n+\delta n)y}\right]$$

- **In string theory,** by contrast, the UV/IR-mixing forces new states to come down from infinity [thereby shifting the corresponding *g(n)* function] and the pairing is destroyed:

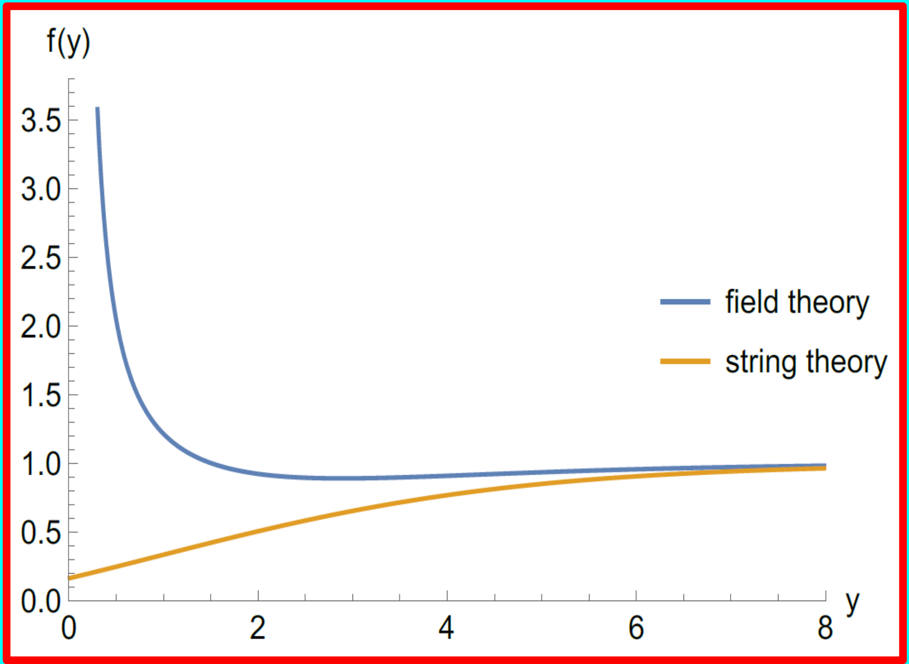
$$g(n) \longrightarrow g(n+\delta n)$$

$$f_{\text{string}}(y) = \sum_n \left[g(n)e^{-ny} - g(n+\delta n)e^{-(n+\delta n)y}\right]$$

For δn = 1/2:

$$f_{\text{string}}(y) = \sum_{n\in\mathbb{Z}/2} (-1)^{2n} g(n)\, e^{-ny}$$

Misaligned SUSY!



The shifting of the *g(n)* function is the critical extra feature of misaligned SUSY. **Infinite towers then produce a finite y → 0 limit, enabling supertraces to represent physical quantities directly!**

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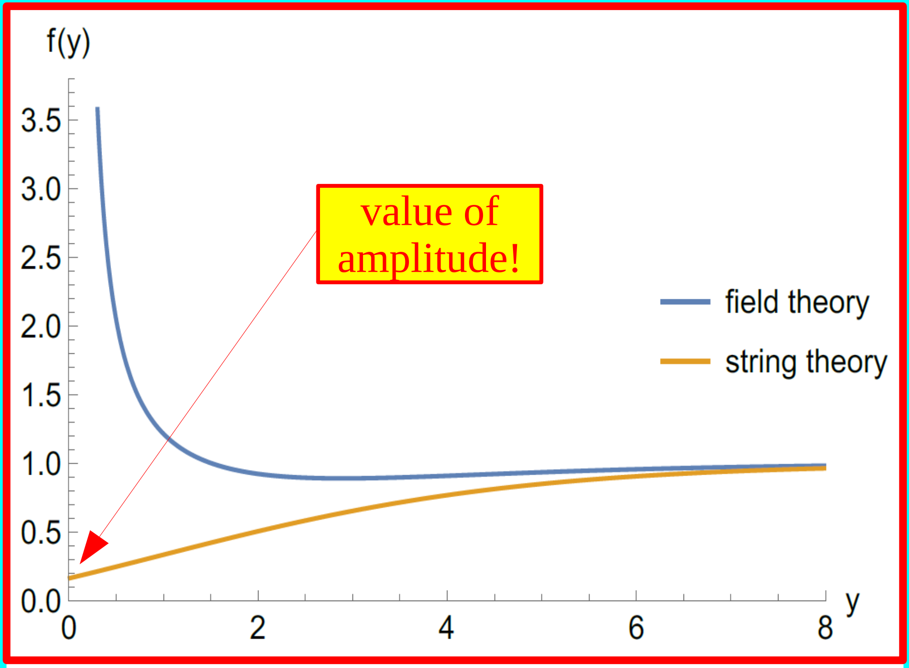
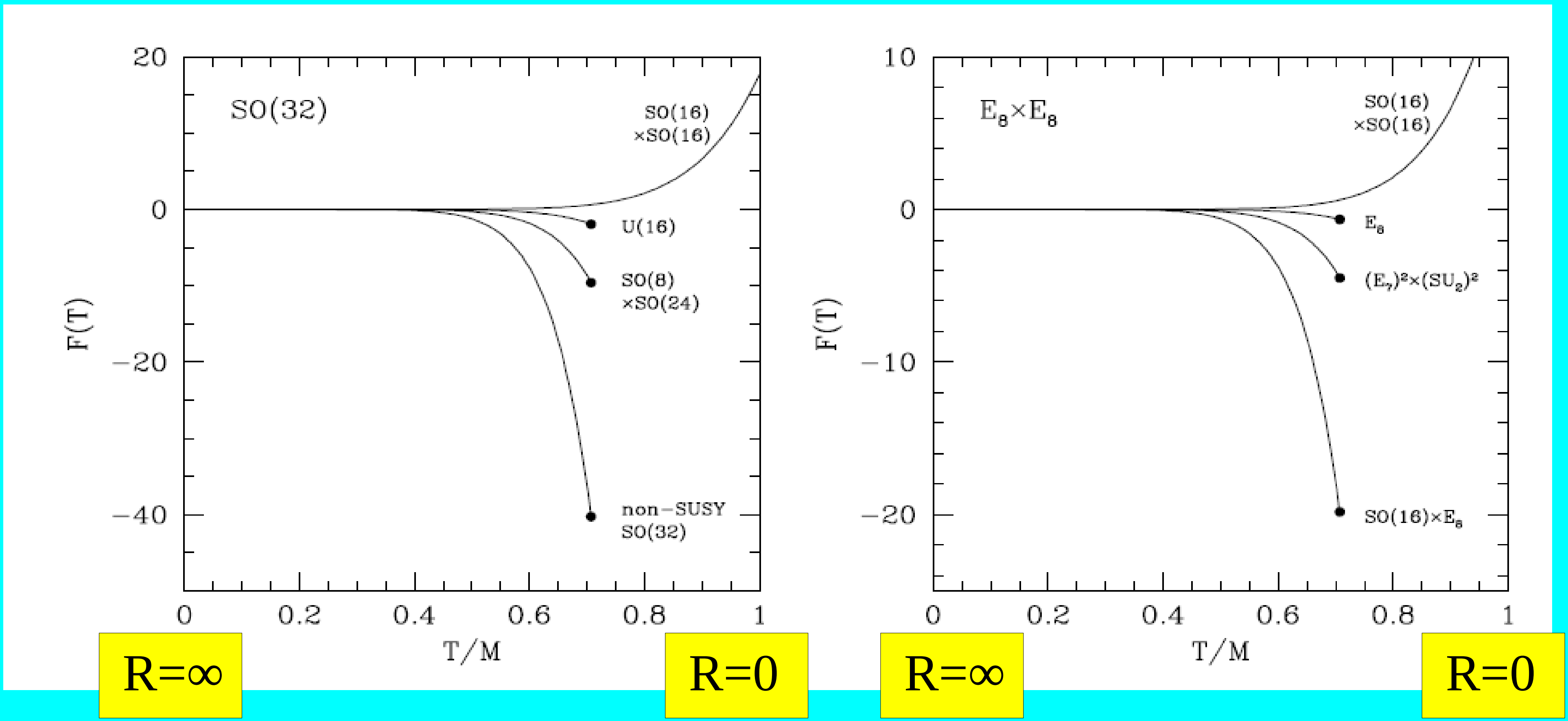


The shifting of the *g(n)* function is the critical extra feature of misaligned SUSY. **Infinite towers then produce a finite y → 0 limit, enabling supertraces to represent physical quantities directly!**

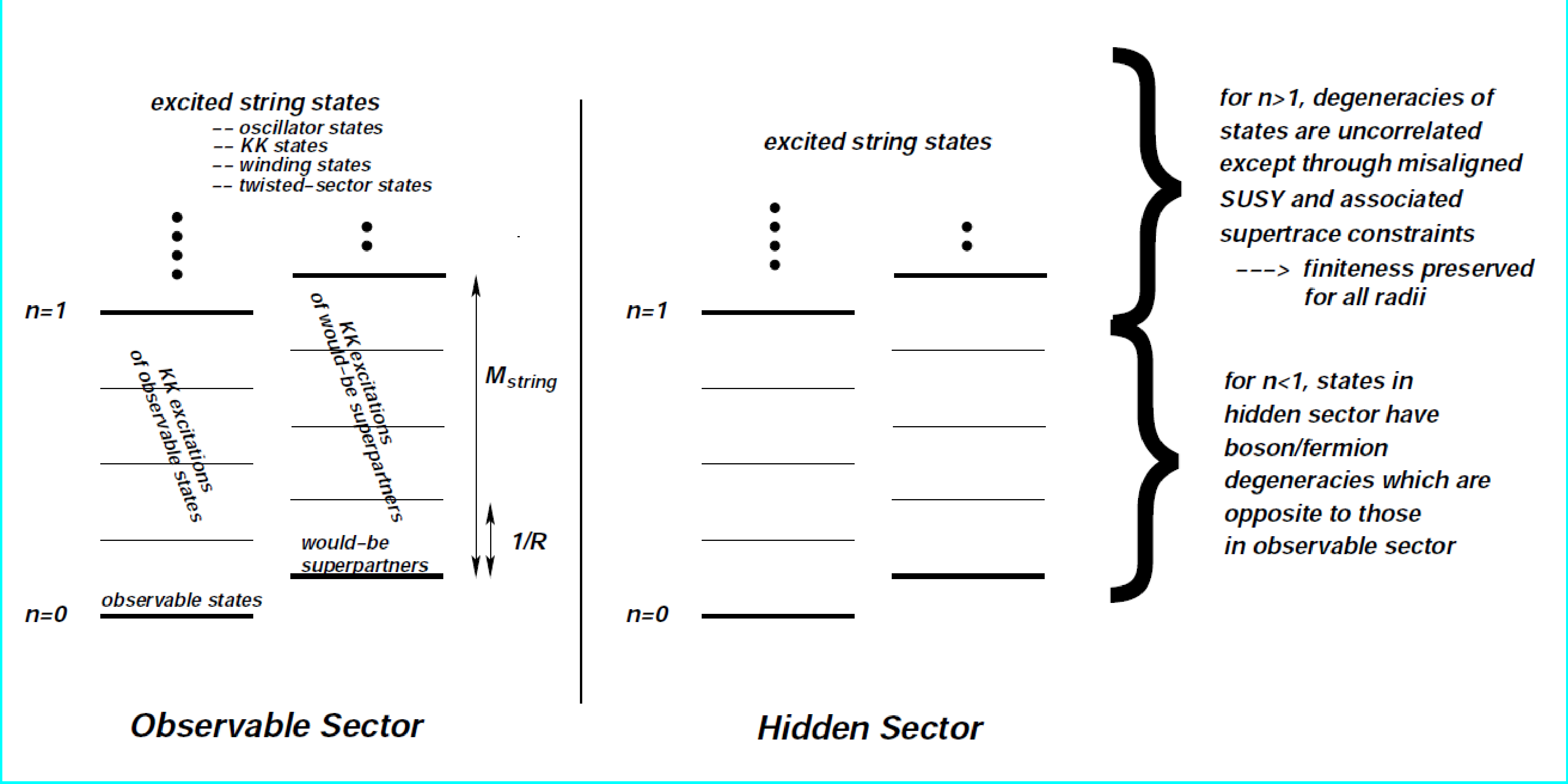
One way to break SUSY is to compactify a SUSY theory in conjunction with a SUSY-breaking twist. For example, for D=10 we have...



Choice of twist determines the opposite (R=0) endpoint and thereby determines how SUSY is broken (*i.e.*, which states are preserved, which are projected out).

The spectra of such models have a generic structure in the large-radius limit...

- S. Abel, KRD, E. Mavroudi, 1502.03087



Between each string level are large towers of KK states!

Once again, these towers play an amazing role!

All of these string theories share a universal behavior as they approach the decompactification $R \to \infty$ (or $M_{\mathrm{KK}} \to 0$) SUSY limit:

$$\Lambda(R) = \left[N_b^{(0)} - N_f^{(0)}\right] \left(\frac{M_{\mathrm{KK}}}{M_s}\right)^D + \left(\frac{M_{\mathrm{KK}}}{M_s}\right)^{D/2} \sum_{n=1}^{\infty} n^{D/4} \left[N_b^{(n)} - N_f^{(n)}\right] e^{-4\pi\sqrt{n} M_s / M_{\mathrm{KK}}}$$

expected FT result from massless states

result from summing over large KK tower within string levels (n,n+1)

The summation over the tower has inverted the expected form of the exponential! Now depends on M_s / M_{KK}!
This exponential is *extremely* suppressed!

- S. Abel, KRD, E. Mavroudi, 1502.03087, 1712.06894

Why is this important?

(and we can!)

If we can build string models in this class with $N_b^{(0)} = N_f^{(0)}$, then such models would have

$$\boxed{M_{\cancel{\text{SUSY}}} \sim M_{\text{KK}} \quad \text{but} \quad \Lambda \sim e^{-4\pi M_s/M_{\text{KK}}}}$$

⇒ **We can thus have *O* (TeV) SUSY-breaking while simultaneously suppressing Λ (or equivalently suppressing the dilaton tadpole) *exponentially*!**

This provides a new approach to non-SUSY string model-building. Vacua are not perfectly stable but *metastable*! They slowly roll, but are essentially stable on cosmological timescales, with $\mathbf{\Lambda} \sim 0$!

- S. Abel and KRD, 2106.04622

Another surprise

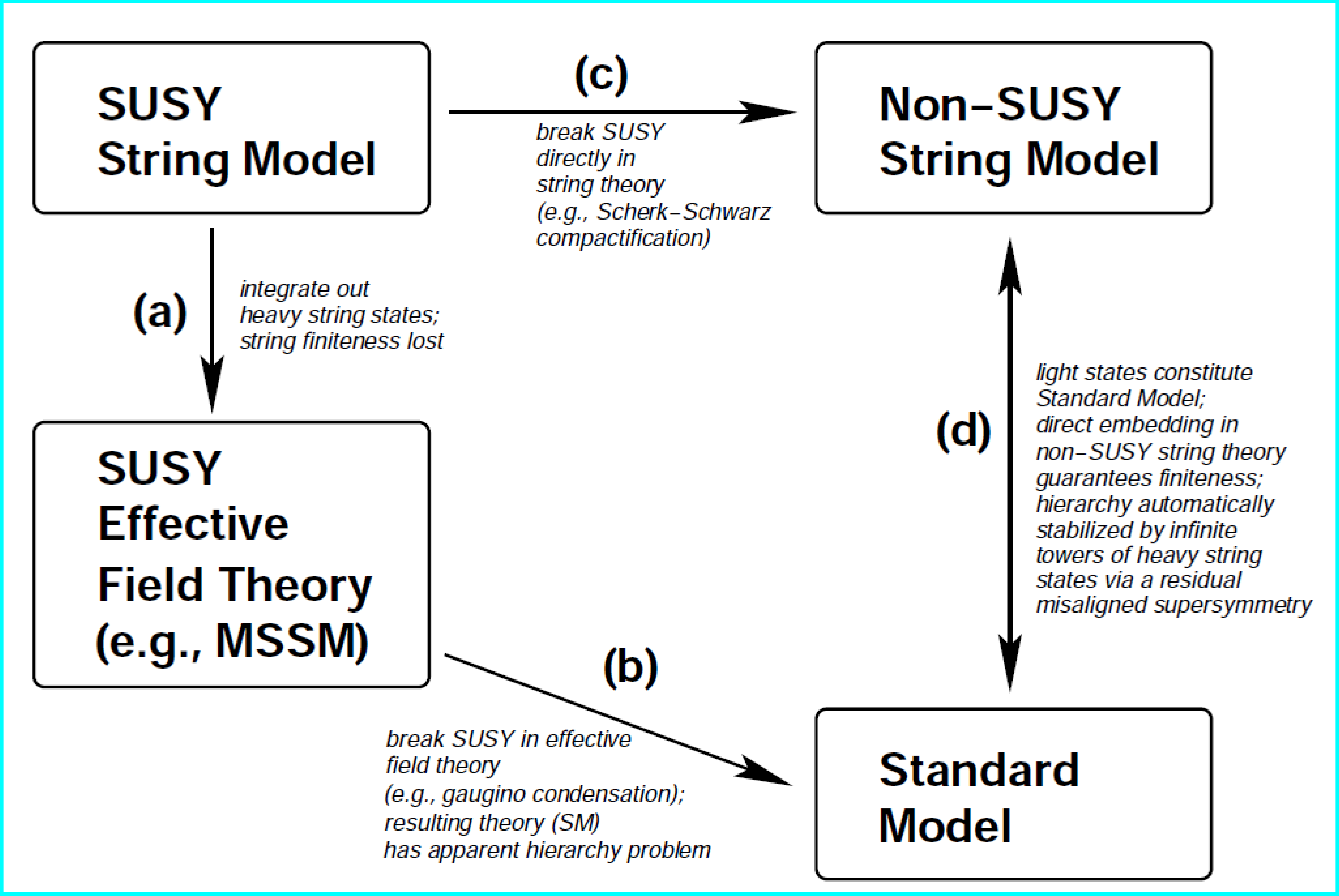
If one calculates the Higgs mass within these string models by taking into account the full towers of states, one finds

$$\boxed{m_\phi^2 \;=\; \frac{\xi}{4\pi^2}\frac{\Lambda}{\mathcal{M}^2} - \frac{\mathcal{M}^2}{2}\left\langle \tau_2\,\mathbb{X}_1 + \tau_2^2\,\mathbb{X}_2 \right\rangle}$$

This relation applies for all scalars ϕ in the theory (specific choice governed by ξ and the X's).

This is a one-loop relation, but it joins together precisely the two quantities (m_ϕ and Λ) whose values lie at the heart of the two most pressing hierarchy problems in modern physics!

Such non-SUSY models lead to a new approach to string phenomenology!



- KRD, hep-ph/ 0104274

...where all physical quantities must be calculated taking into account the entire towers of states and their accompanying symmetries! Divergence structure is greatly altered and hierarchy problems may no longer arise.

- S. Abel, KRD, L. Nutricati, 2407.11160

An even bigger surprise

To understand this, we begin by noting that as the spacetime dimensionality of an ordinary QFT increases, the QFT tends to become more finite in the IR but more divergent in the UV.

By contrast, for a UV/IR-mixed theory such as string theory, there is only one divergence. This divergence tends to become *less* severe as the spacetime dimensionality increases.

This is because higher-dimensional string theories have more "internal" constraints/cancellations within their spectra than 4D theories have.

Given this, we are then able to prove a number of surprising results.

Any 4D closed string theory which can be realized as a geometric compactification of a higher-*D* theory will inherit the precise stricter internal cancellations of the higher-*D* theory **<u>despite the compactification</u>!**

Moreover, this remains true even if the 4D theory is nowhere near the decompactification limit! These are surprising hidden cancellations that exist simply because the theory has a decompactification limit in ***<u>some</u>*** region of parameter space!

So what are the consequences of this?

A boatload of new supertrace identities that the spectra of such theories must satisfy when their entire towers of states are included in the sums!

For example,

$\delta = 0:$ **4D**

$$\Delta_G = \frac{\xi}{6}\left[\mathrm{Str}\,\overline{Q}_H^2 - \frac{1}{12}\mathrm{Str}_E\,\mathbf{1}\right]$$

$\delta = 2:$ **6D**

$$\begin{cases} \mathrm{Str}'\,\overline{Q}_H^2 - \frac{1}{12}\mathrm{Str}'_E\,\mathbf{1} = 0 \\ \Delta_G \approx \frac{\pi}{3}\widetilde{V}_T\left[-2\,\mathrm{Str}'\,(Q_G^2\overline{Q}_H^2) + \frac{1}{6}\mathrm{Str}'_E\,Q_G^2 - \frac{\xi}{2\pi}\mathrm{Str}'\left(\overline{Q}_H^2\widetilde{M}^2\right) + \frac{\xi}{24\pi}\mathrm{Str}'_E\,\widetilde{M}^2\right] \end{cases}$$

$\delta = 4:$ **8D**

$$\begin{cases} \mathrm{Str}'\,\overline{Q}_H^2 - \frac{1}{12}\mathrm{Str}'_E\,\mathbf{1} = 0 \\ -2\,\mathrm{Str}'\,(Q_G^2\overline{Q}_H^2) + \frac{1}{6}\mathrm{Str}'_E\,Q_G^2 - \frac{\xi}{2\pi}\mathrm{Str}'\left(\overline{Q}_H^2\widetilde{M}^2\right) + \frac{\xi}{24\pi}\mathrm{Str}'_E\,\widetilde{M}^2 = 0 \\ \Delta_G \approx \frac{\pi}{3}\widetilde{V}_T\left[2\,\mathrm{Str}'\left(\overline{Q}_H^2 Q_G^2\widetilde{M}^2\right) - \frac{1}{6}\mathrm{Str}'_E\left(Q_G^2\widetilde{M}^2\right) + \frac{\xi}{4\pi}\mathrm{Str}'\left(\overline{Q}_H^2\widetilde{M}^4\right) - \frac{\xi}{48\pi}\mathrm{Str}'_E\,\widetilde{M}^4\right] \end{cases}$$

- S. Abel, KRD, L. Nutricati, 2407.11160

These new constraints are apparent only when the entire towers of states are included in the sums!
However, these constraints have a huge effect...

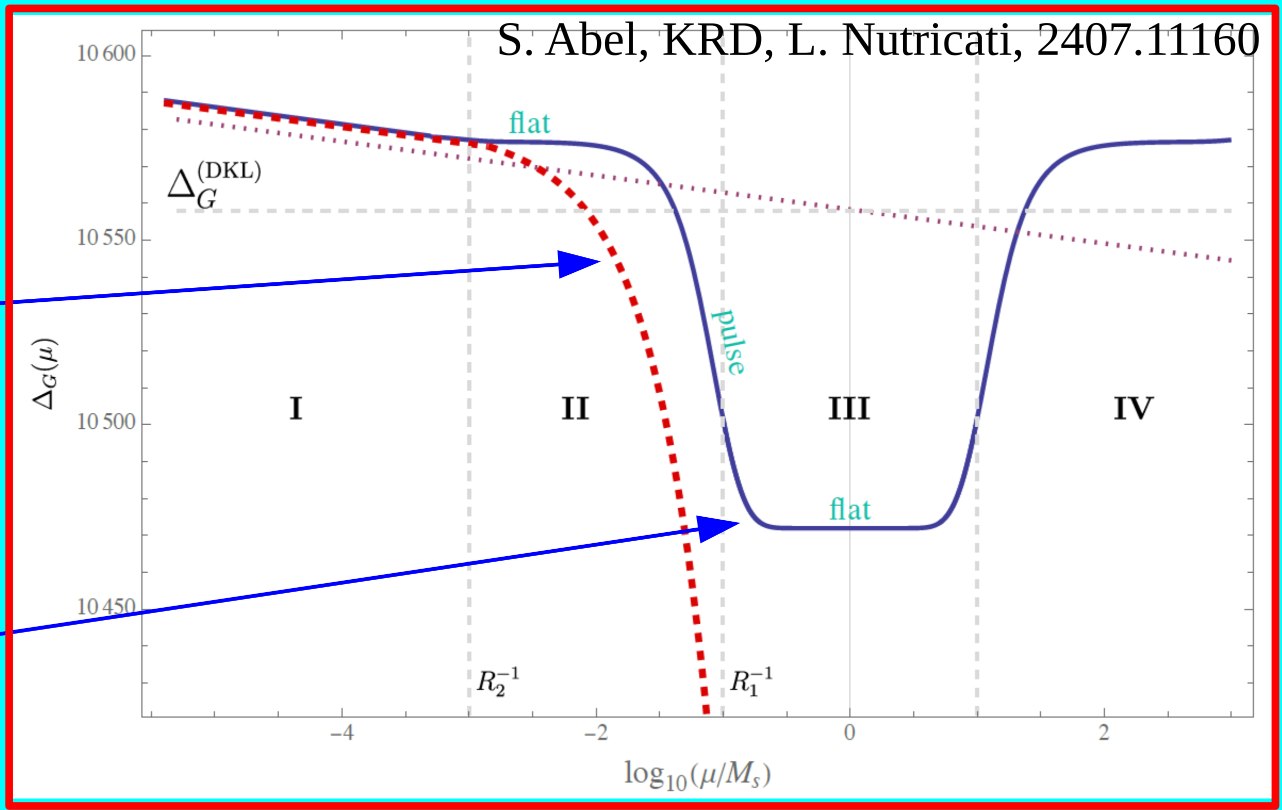
A New Non-Renormalization Theorem from UV/IR Mixing

These new constraints kill the running of quantities such as gauge couplings, and the theory necessarily enters a fixed-point regime! More specifically, if the theory has δ extra dimensions opening up at $M_{KK}=1/R$, then

- For δ>2, all running is killed both above and even below 1/*R*!
- For δ=2, all running is killed above 1/*R*.
  Below 1/*R*, at most logarithmic running survives.

Expected power-law running

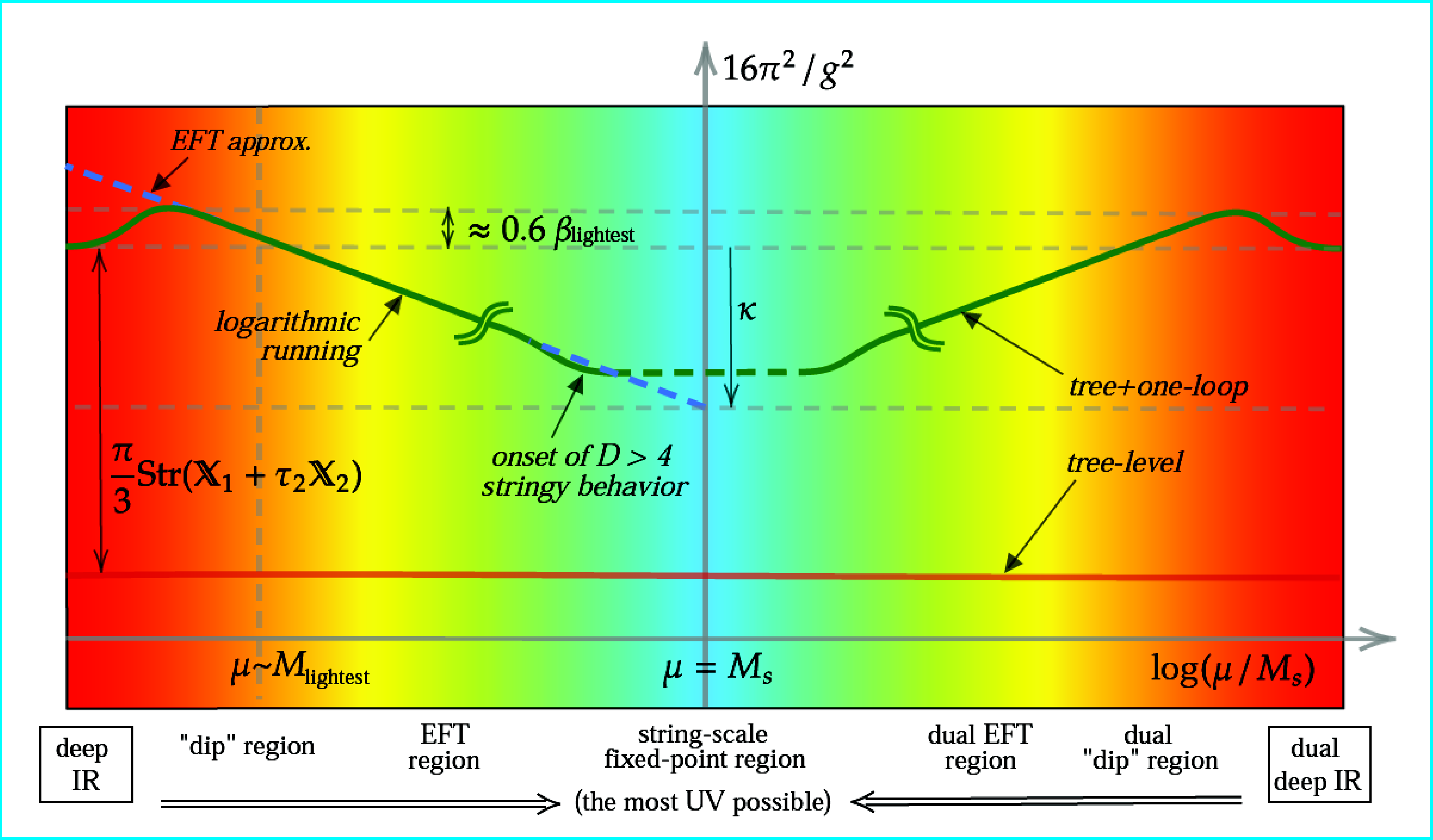
Actual "running"



Toroidal compactification with radii (R_1,R_2). See paper for details.

With all the bells and whistles...

- S. Abel, KRD, L. Nutricati, 2303.08534



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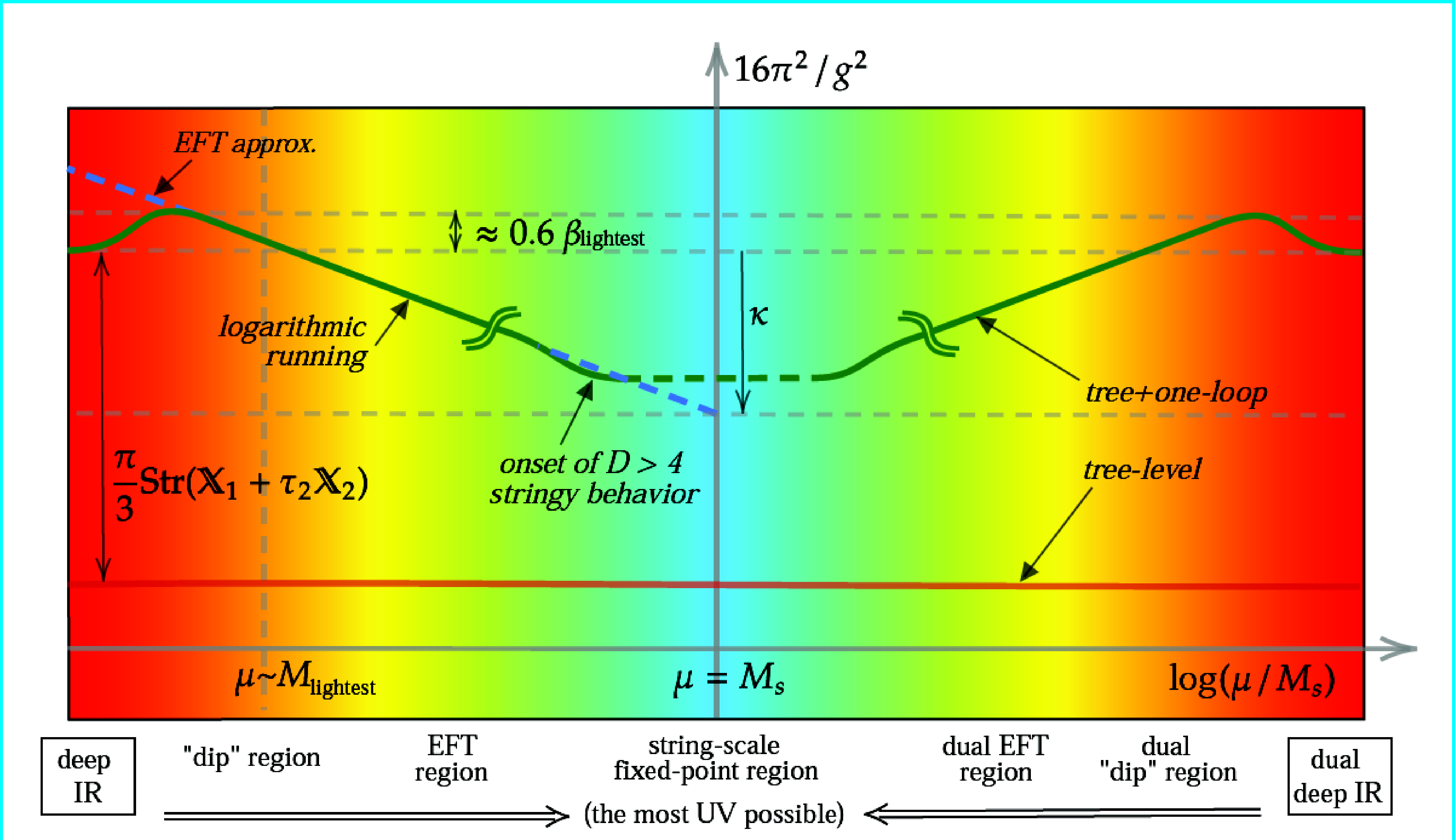


$$\mu \rightarrow M_s^2 / \mu$$

Scale duality!

Another feature arising from the full towers of states!

Two comments ---

1. It may seem strange that winding modes can affect the running even *below* M_s, where no winding modes have yet appeared.

However, we are used to the idea that KK modes can affect the running *above* M_s.

So under scale duality inversion, winding modes should likewise affect physics *below* M_s!

2. *Whither EFTs*? To what extent do EFTs provide relevant low-energy descriptions of such theories?

- One must *spontaneously break the UV/IR-mixed symmetries* in order to build an appropriate EFT.
- Certainly for $\mu << M_s$, the features associated with scale duality are "far away", not directly relevant.
- Thus, within certain range of scales, the theory then behaves as one would expect for an EFT ***except***
    - Divergences are softened, running can be different (*e.g.*, log running for Higgs)
    - Even at low energies the theory is still sensitive to the *infinite* towers of states. Running governed by supertraces over *all* states.
    - EFT-like behavior also cuts off as one approaches the deep IR --- required since theory must remain sensitive to infinite towers and match the "dual" deep IR in which all states contribute.

Caution advised, must understand the context and purpose.
(Consult a professional near you.)

- KRD, E. Dudas, T. Gherghetta, B. Thomas, 2508.xxxxx.

Decoherence in the Dark

If the states in the tower are all dark, then it may happen that only one linear combination ϕ' of such ϕ-states actually couples to the visible sector.

$$\phi' \equiv \sum_{\ell=0}^{N} c_\ell \phi_\ell$$

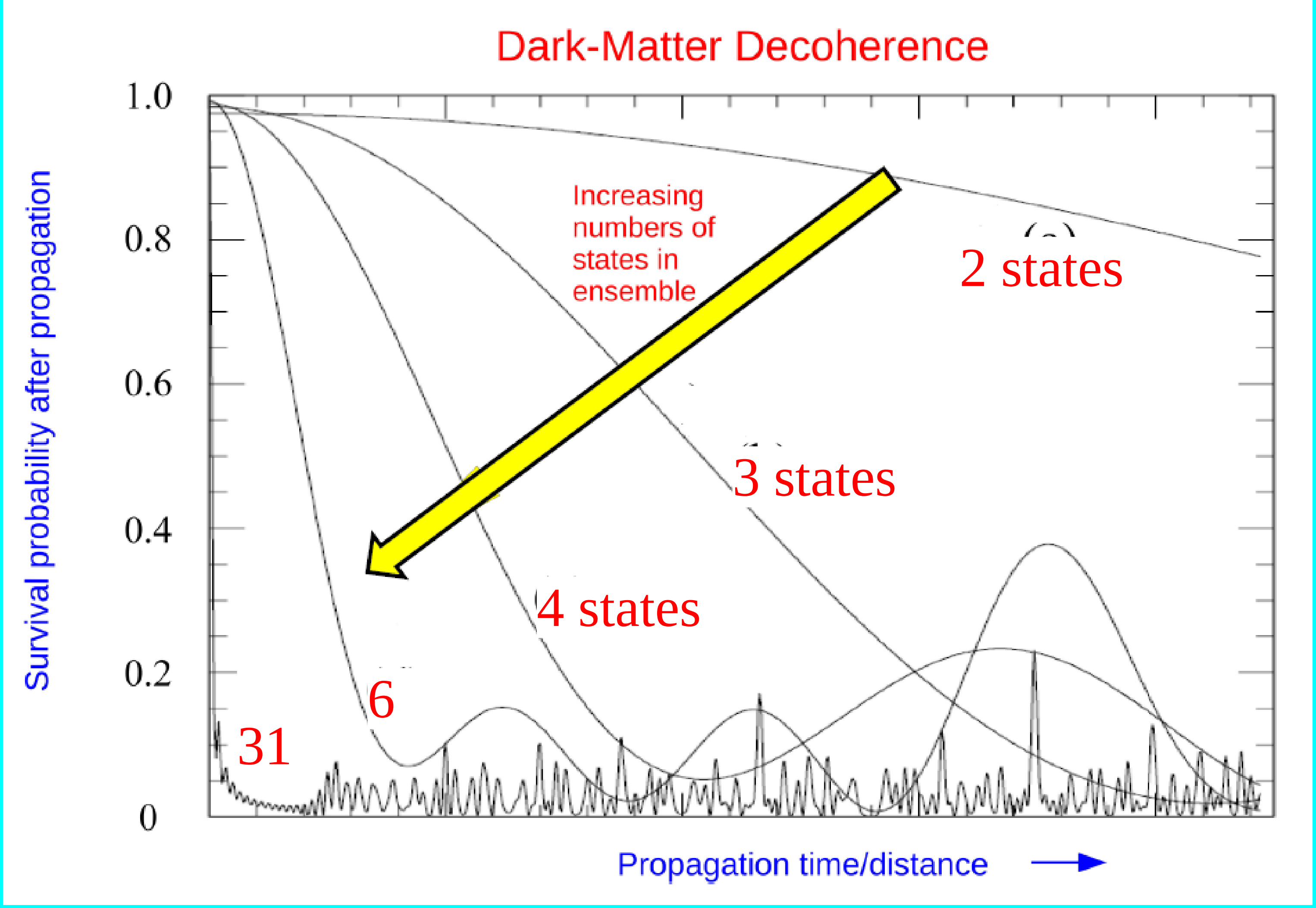
e.g., this situation arises directly if the SM lives on a brane embedded within a higher-dimensional bulk. Such bulk ϕ-fields would be SM-neutral ("dark") and could be members of the supergravity multiplet, string-theory moduli, axions, RH neutrinos, *etc.*

In such cases, any DM production process involving the visible sector (*e.g.*, in the lab, or in a distant star) will only produce DM in the ϕ' linear combination. Likewise, any subsequent detection process involving interactions with the visible sector will also only detect the ϕ' linear combination.

The probability to detect ϕ’ after time t is given by

$$P_{\phi' \to \phi'}(t)$$

But the different ϕ components within the tower will have different masses and thus different phases under time evolution. The ϕ’ state will quickly *decohere* after being produced and escape further detection.



Thus the existence of the tower provides a new mechanism which can help the dark sector stay dark!

PASCOS

Thus far we have concentrated on how **S** differs from the expectations from **PA**.

But what about the implications of the towers of states for **COS ?**

Two critical questions

- How are these towers produced / populated in the early universe? More specifically, how much "abundance" Ω_ℓ do they carry?

abundance = fraction of total energy density ρ_{crit}

Must not overclose the universe!
Important (but largely ignored) constraint for string phenomenology / landscape!

- What happens when these towers of states decay?

Two features tend to govern these decays
- Heavy states at top of tower tend to have largest decay widths and decay first, then lighter ones. Decays thus proceed "down the tower".
- For any state, the dominant decay mode is to the lightest states available. Such decay products are therefore produced with huge amounts of kinetic energy (relativistic), and are effectively radiation.

So what is the effect of such infinite towers of states on early-universe cosmology?

These decays establish a sequential process working its way down the tower which continually converts matter into radiation.

This may seem rather trivial, but there is actually a competing effect which pushes the other way: *cosmological expansion*!

- radiation scales as a^{-4} (a = FRW scale factor)
- matter scales as a^{-3}

Thus, *even if nothing else happens*, cosmological expansion causes the *relative* fractional energy densities ("abundances") of matter and radiation to change

- abundance of radiation Ω_γ *drops*
- abundance of matter Ω_M *rises*

(Total remains fixed at 1 for a matter/radiation universe.)

Indeed, this is how a radiation-dominated universe becomes matter-dominated universe *simply as the result of cosmic expansion*.

We thus see that

- *decays along tower*: convert $\Omega_M \rightarrow \Omega_\gamma$
- *cosmic expansion*: converts $\Omega_\gamma \rightarrow \Omega_M$

Can these two effects cancel?

This would be a way of keeping the matter and radiation abundances *fixed* --- at least through the time interval (which may stretch across many *e*-folds) during which the decays are proceeding sequentially down the tower.

Seems like too much to ask for!

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Seems like too much to ask for!

But….

… they CAN balance
… they DO balance
… *even if* they don't start out by balancing,
the balanced solution is an ***attractor***
and the system will quickly come into balance all by itself!

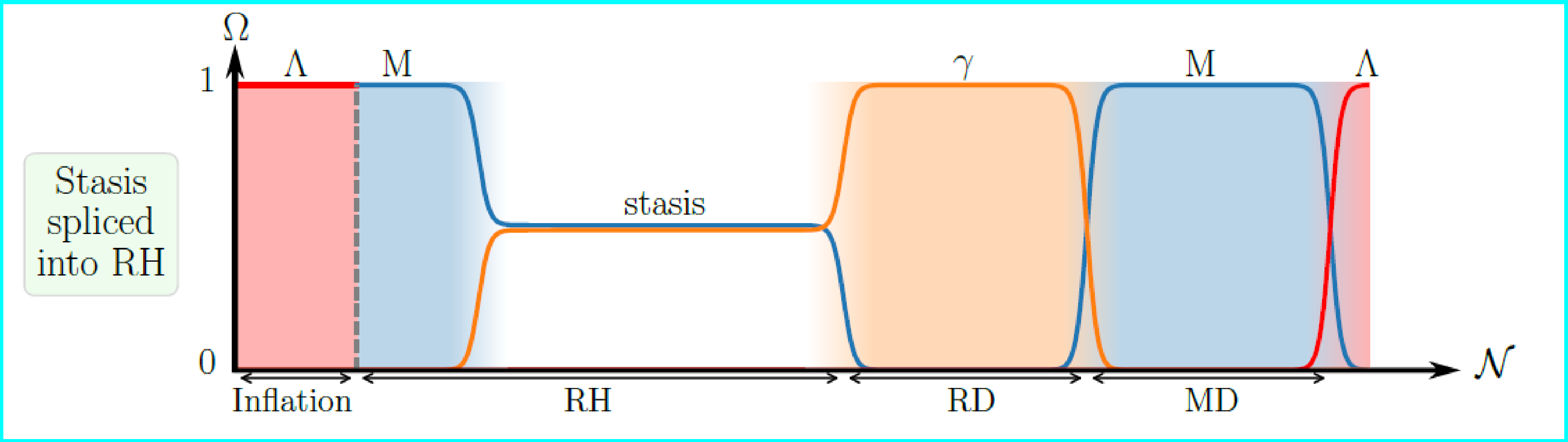
Especially remarkable because particle decay and cosmological expansion are very different things ---
one is particle physics, the other cosmology!

- KRD, L. Heurtier, F. Huang, D. Kim, T. Tait, B. Thomas, 2011.04753

What emerges, then, is an epoch of **stasis**:

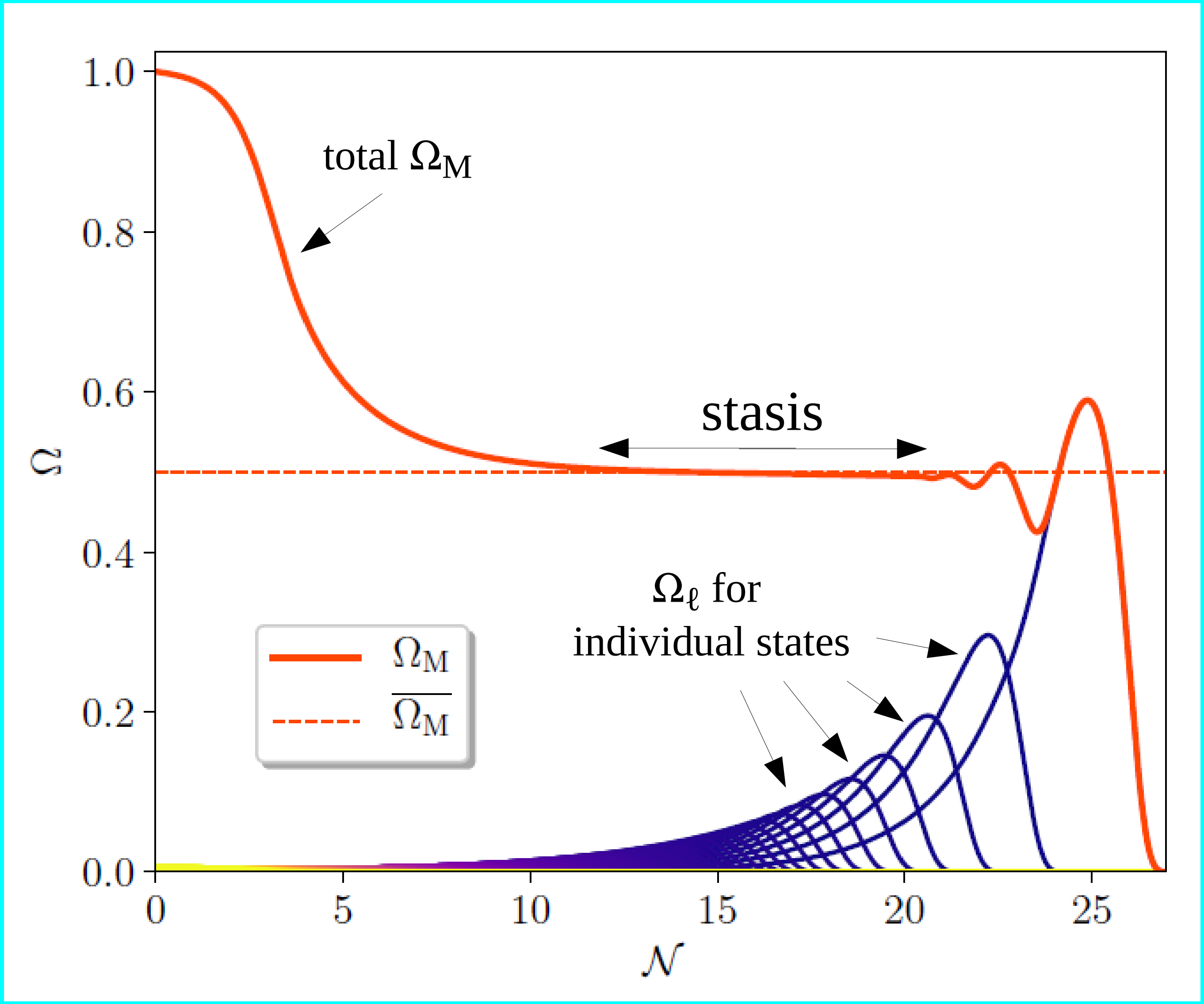
A cosmological epoch during which the abundances of different energy components (matter, radiation, vacuum energy, etc.) remain constant despite cosmological expansion.

For example,



The universe continues to expand, but the abundances stay fixed!
Time passes as measured in *e*-folds, but not as measured by abundances!

For example, for certain parameter choices one can obtain an extended epoch with matter-radiation equality...



Parameter choices

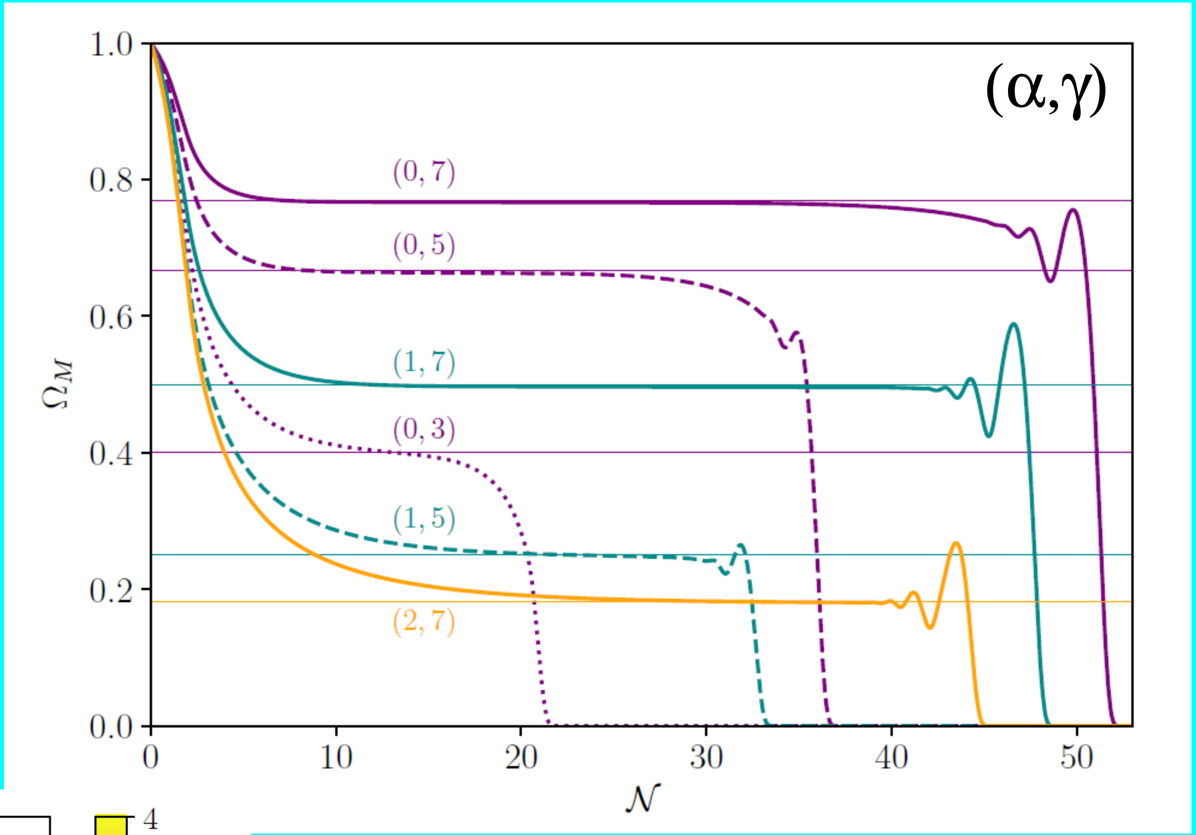
$(\alpha,\gamma,\delta)=(1,7,1)$

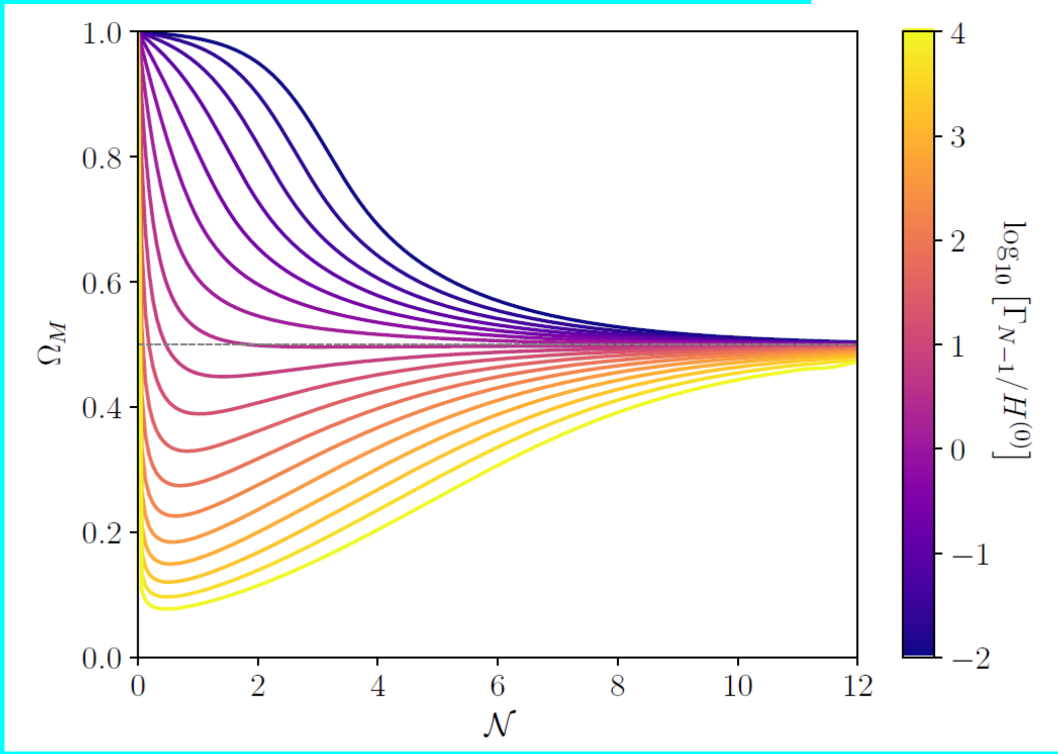
$N=300$
$m_0/\Delta m=1$
$\Gamma_{N-1}/H^{(0)} = 0.1$

Exact numerical solution using Boltzmann code, no approximations.

- KRD, L. Heurtier, F. Huang, D. Kim, T. Tait, B. Thomas, 2011.04753

Indeed, we obtain cosmological stasis *regardless* of the initial values of system. It's a global attractor!

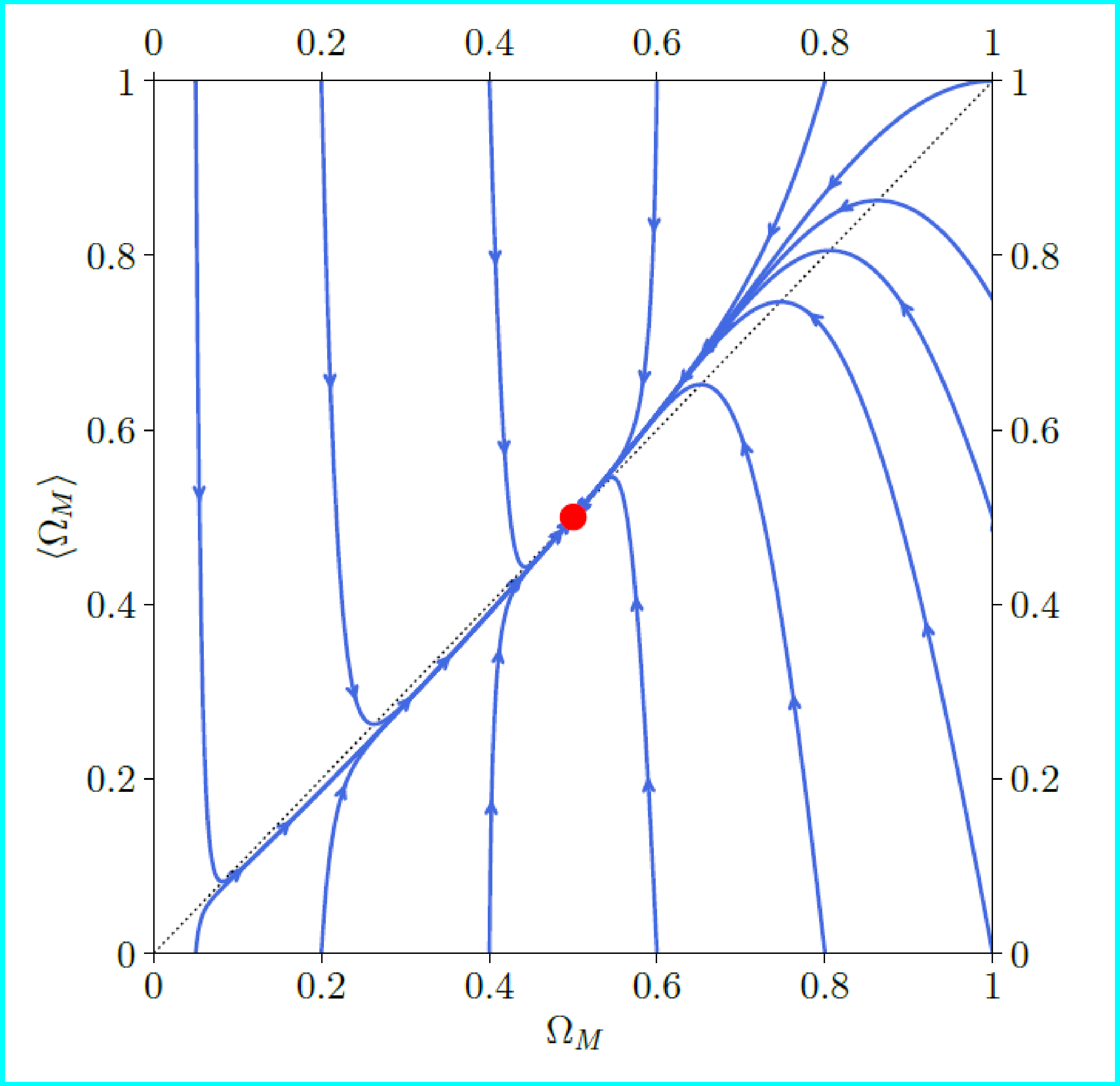




Can also vary $\Gamma_{N\text{-}1}/H^{(0)}$ = rate of decays relative to cosmological expansion. Affects initial behavior but stasis always emerges with same abundance!

Indeed, matter/radiation stasis is a *global* attractor within such BSM cosmologies…

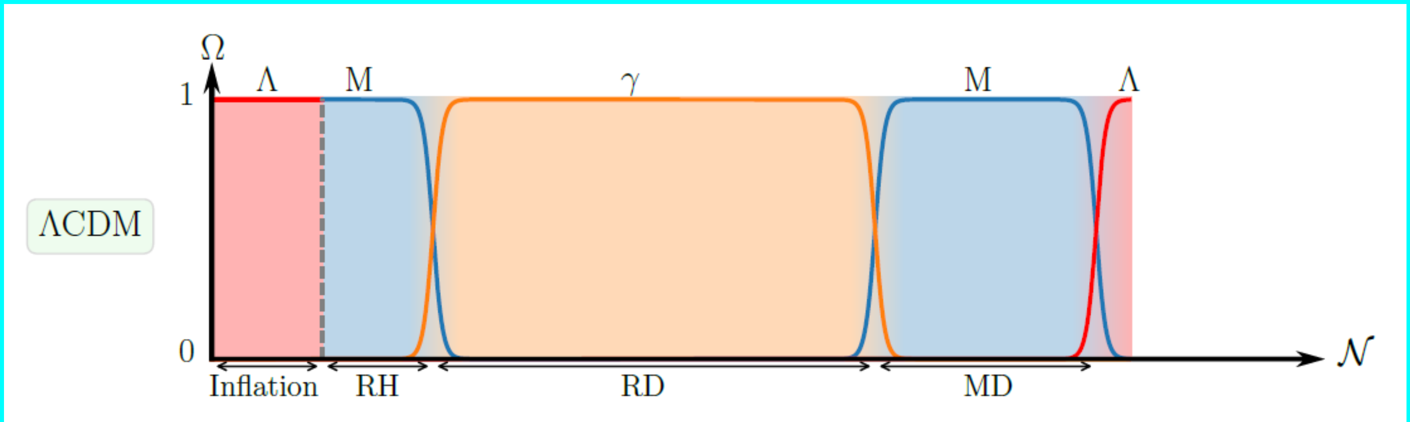
time-averaged *history* of abundance

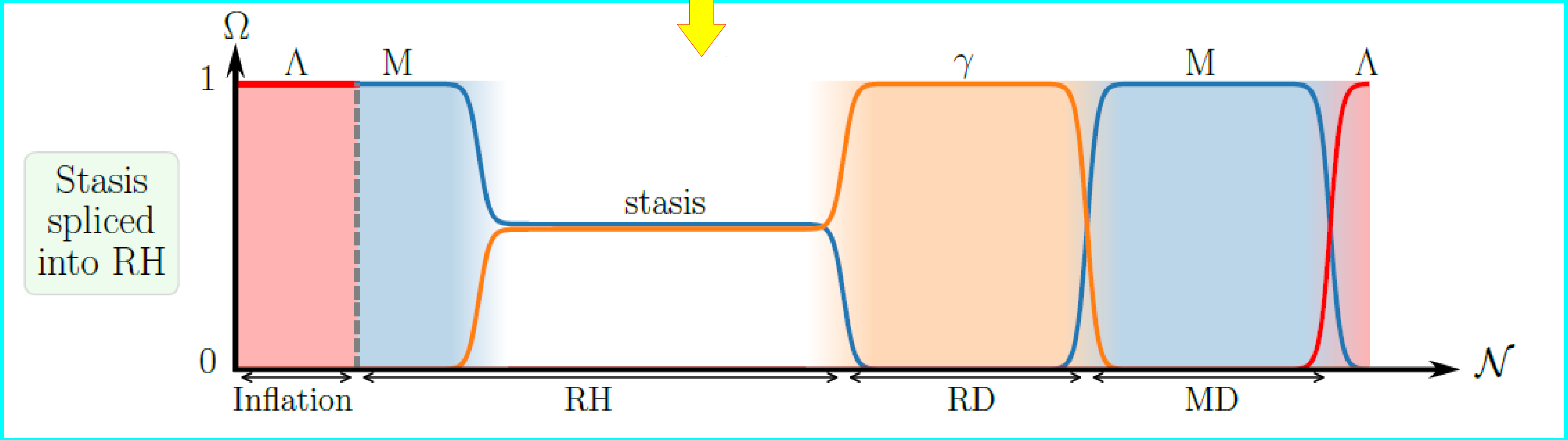


instantaneous abundance

- KRD, L. Heurtier, F. Huang, D. Kim, T. Tait, B. Thomas, 2011.04753

Stasis is nothing less than a new kind of cosmological epoch --- one which is fairly general in most BSM cosmologies involving towers of states.





… changes the entire cosmological timeline!

Lots of subsequent work on stasis over the past 5 years...

- **Pairwise stases** between any two energy components (vacuum energy, matter, radiation, kination, PBHs, *etc*.)
- **Triple stasis** involving vacuum energy, matter, radiation simultaneously
- **Stasis inflation**: could the inflationary epoch be a stasis epoch?
- **Thermal** stasis
- **Observational signatures** of stasis: gravity waves, density perturbations

- KRD, Lucien Heurtier, Fei Huang, Doojin Kim, Tim M.P. Tait, and Brooks Thomas
  - arXiv:2111.04753
  - arXiv:2212.01369
- KRD, Lucien Heurtier, Fei Huang, Tim M.P. Tait, and Brooks Thomas
  - arXiv:2309.10345
  - arXiv:2406.06830
  - arXiv:2503.19959 also w/ D. Hoover, A. Paulsen
- Jonah Barber, KRD, Brooks Thomas
  - arXiv:2408.16255
  - arXiv:2412.09123

Stasis represents nothing less than a new kind of cosmological epoch --- one which is fairly general in most BSM cosmologies involving towers of states. Moreover, the attractor behavior survives across a wide swath of BSM models, and for a wide range of parameters within each model. As a result, stasis epochs are a rather generic feature of such BSM cosmologies, and if you don't take them into account in your favorite BSM model then you should be prepared to explain why they fail to arise.

Conclusion: Two final comments

- Many fundamental questions in QFT and particle physics in general assume traditional linear relationships between IR and UV.   This is especially true for hierarchy problems, which have been a primary motivation for new physics over the past 50 years. But string suggests other ways of thinking about such problems: UV/IR mixing, softened divergences (even finiteness), scale duality, *etc*.   **Thus hierarchy problems may not be fundamental or survive in the manner we normally assume.**

- The existence of a stasis epoch within BSM cosmologies is likely to give rise to a host of new theoretical possibilities across the entire cosmological timeline, ranging from potential implications for primordial density perturbations, dark-matter production, and structure formation all the way to new inflation scenarios, modified reheating, and even the age of the universe. BSM cosmologies may therefore be much richer than previously imagined, and serious rethinking is required.