



How large can lepton mixing be?

José Santiago



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Universidad
de **Granada**

Based on work in progress with J. de Blas, C. Giuliano, G. Guedes, R. Sánchez





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No PMNS
mixing!



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How well do we know the electron?

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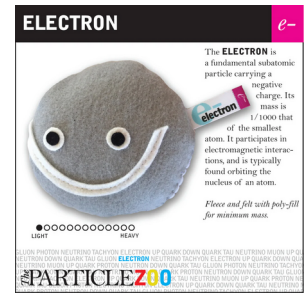
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How well do we know the electron?



- LEP and SLC measured the Z couplings to charged leptons to per-mille precision.
- Mixing with heavy VL leptons modify these couplings (very stringent limits on mixing).

Nuclear Physics B224 (1983) 107–136
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THE POSSIBILITY OF NEW FERMIONS WITH $\Delta I = 0$ MASS*

F. DEL AGUILA¹

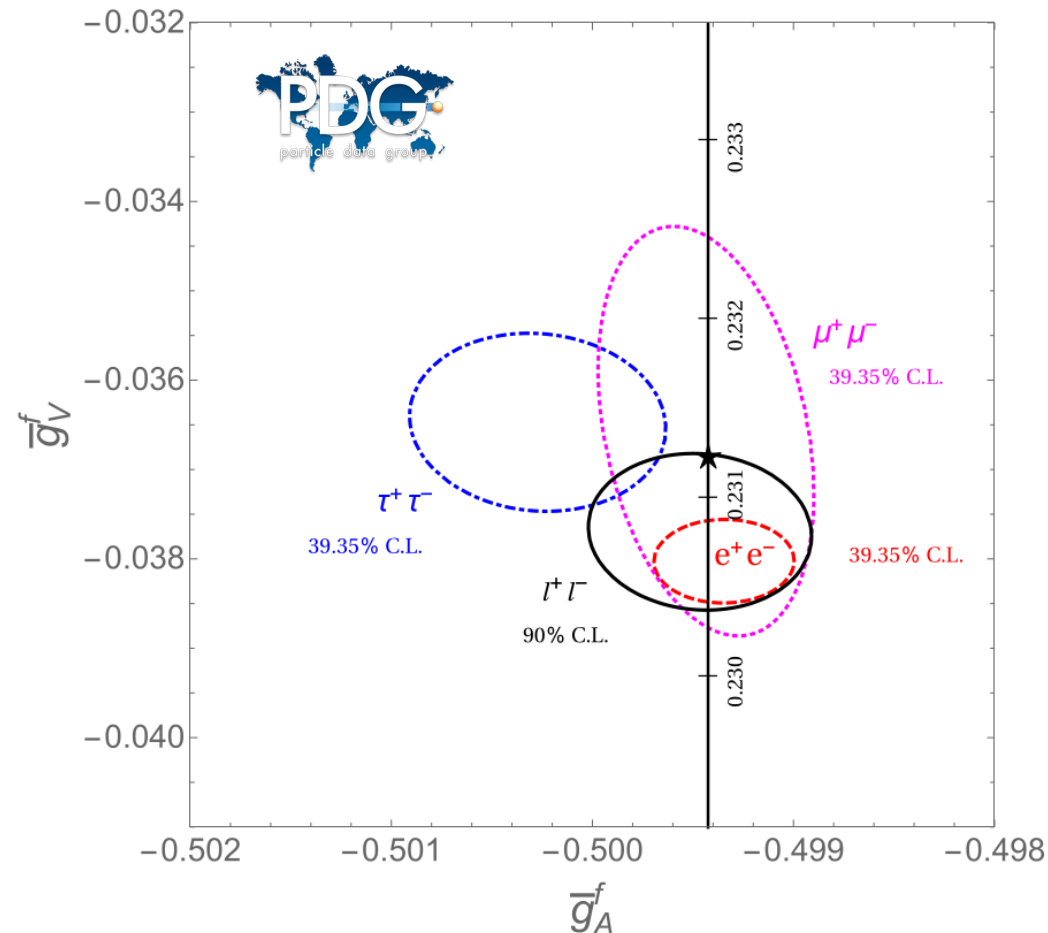
Physics Department, University of Florida, Gainesville, FL 32611, USA

M.J. BOWICK²

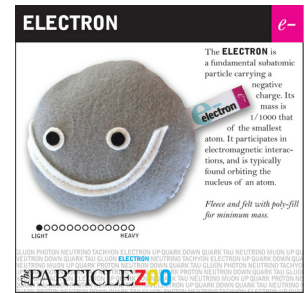
California Institute of Technology, Pasadena, CA 91125, USA and Physics Department, University of Florida, Gainesville, FL 32611, USA

Received 15 June 1982
(Revised 15 March 1983)

In the Glashow–Weinberg–Salam model the fermions have $\Delta I = \frac{1}{2}$ masses from the breaking of the weak SU(2) gauge symmetry. In many enlarged models, such as those from grand unified



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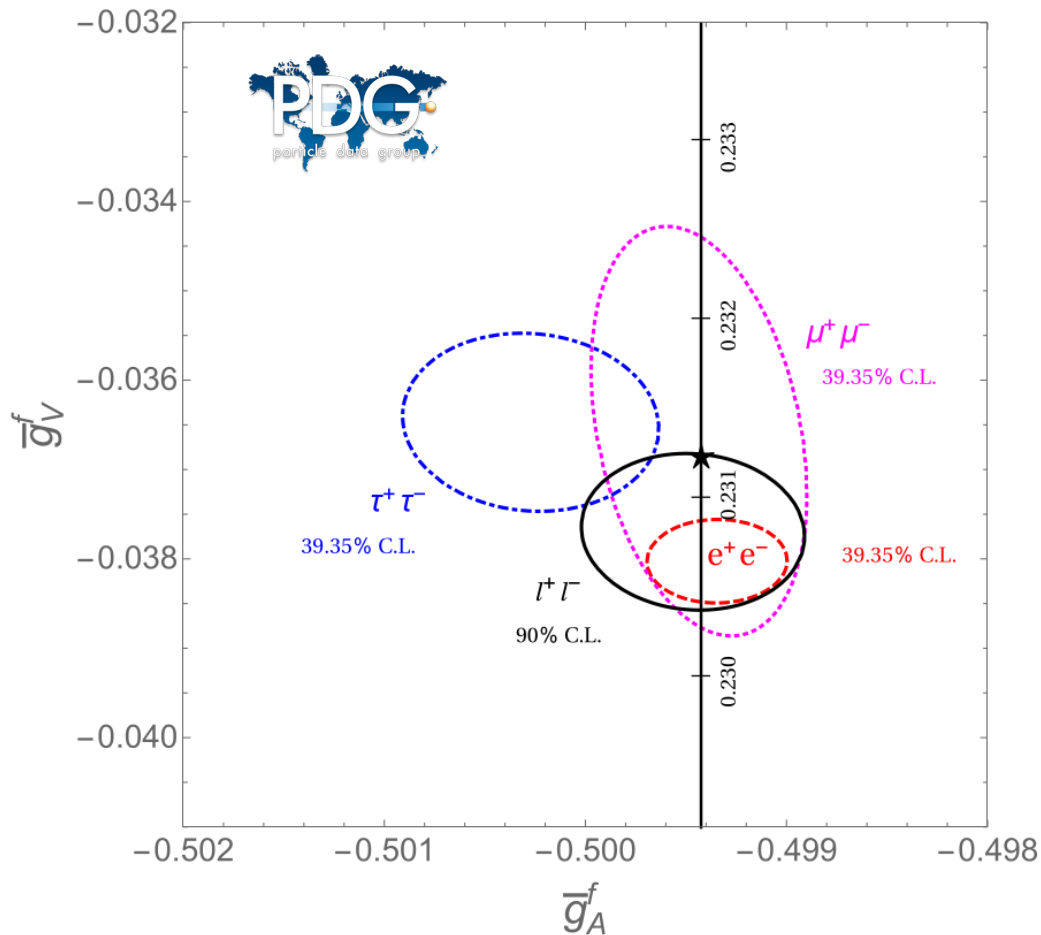
Electroweak limits on physics beyond the Standard Model

Jorge de Blas^{1,a}

¹ Department of Physics, University of Notre Dame, Notre Dame, IN 46556, USA

Lepton L	$(d_c, d_L)_Y$	95% C.L. EWPD limit on mixing $s_{L\ell}$		
		Only e	Only μ	Only τ
N	$(1, 1)_0$	0.041	0.030	0.087
E	$(1, 1)_{-1}$	0.021	0.030	0.033
$\begin{pmatrix} N \\ E^- \end{pmatrix}$	$(1, 2)_{-\frac{1}{2}}$	0.020	0.048	0.034
$\begin{pmatrix} E^- \\ E^{--} \end{pmatrix}$	$(1, 2)_{-\frac{3}{2}}$	0.028	0.028	0.046
$\begin{pmatrix} E^+ \\ N \\ E^- \end{pmatrix}$	$(1, 3)_0$	0.019	0.017	0.030

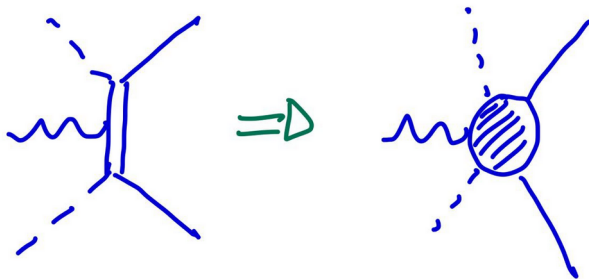
$$s^2 \lesssim 10^{-3}$$



How large is lepton mixing? J. Santiago (UGR)

How well do we know the electron?

- LEP and SLC measured the Z couplings to charged leptons to per-mille precision.
- Mixing with heavy VL leptons modify these couplings (very stringent limits on mixing).
- Only true if VLL contribute to the EFT at tree-level, dimension 6



PHYSICAL REVIEW D **78**, 013010 (2008)

Effects of new leptons in electroweak precision data

F. del Aguila,^{*} J. de Blas,⁺ and M. Pérez-Victoria[‡]

$$\begin{aligned}\mathcal{L}_6 = & (\alpha_{\phi l}^{(1)})_{ij}(\phi^\dagger iD_\mu \phi)(\bar{l}_L^i \gamma^\mu l_L^j) \\ & + (\alpha_{\phi l}^{(3)})_{ij}(\phi^\dagger i\sigma_a D_\mu \phi)(\bar{l}_L^i \sigma_a \gamma^\mu l_L^j) \\ & + (\alpha_{\phi e}^{(1)})_{ij}(\phi^\dagger iD_\mu \phi)(\bar{e}_R^i \gamma^\mu e_R^j) \\ & + (\alpha_{e\phi})_{ij}(\phi^\dagger \phi)\bar{l}_L^i \phi e_R^j + \text{H.c.}\end{aligned}$$

$$\delta g_L^\nu = \frac{1}{4}(-\alpha_{\phi l}^{(1)} + \alpha_{\phi l}^{(3)} + \text{H.c.}) \frac{v^2}{\Lambda^2},$$

$$\delta g_L^e = -\frac{1}{4}(\alpha_{\phi l}^{(1)} + \alpha_{\phi l}^{(3)} + \text{H.c.}) \frac{v^2}{\Lambda^2},$$

$$\delta g_R^e = -\frac{1}{4}(\alpha_{\phi e}^{(1)} + \text{H.c.}) \frac{v^2}{\Lambda^2},$$

$$\delta V_L^{e\nu} = (\alpha_{\phi l}^{(3)})^\dagger \frac{v^2}{\Lambda^2}.$$

How well do we know the electron?

- LEP and SLC measured the Z couplings to charged leptons to per-mille precision.



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Physics Letters B 641 (2006) 62–66

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- Mixing with heavy VL leptons modify these couplings (very stringent limits on mixing).
- Only true if VLL contribute to the EFT at tree-level, dimension 6.
- But mixing with several new VLL can induce cancellations (protected by symmetries)

A custodial symmetry for $Zb\bar{b}$

Kaustubh Agashe^a, Roberto Contino^{b,c,*}, Leandro Da Rold^d, Alex Pomarol^{d,e}

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^b Dipartimento di Fisica, Università di Roma "La Sapienza" and INFN, Sezione di Roma, I-00185 Roma, Italy

^c Department of Physics and Astronomy, Johns Hopkins University, Baltimore, MD 21218, USA

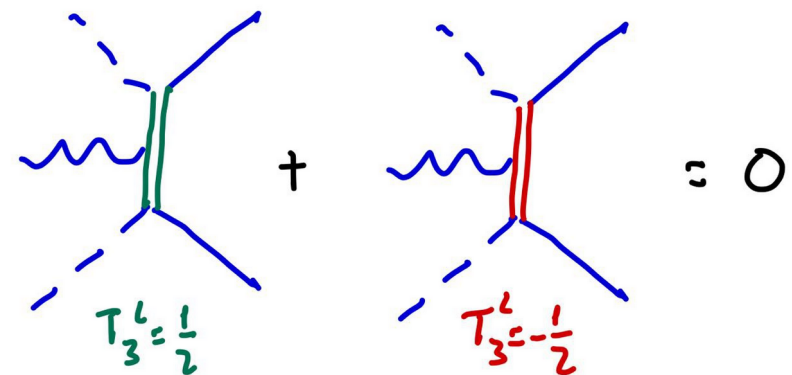
^d IFAE, Universitat Autònoma de Barcelona, E-08193 Bellaterra, Barcelona, Spain

^e Theory Division, CERN, CH-1211 Geneva 23, Switzerland

Received 15 June 2006; accepted 3 August 2006

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The model: degenerate bi-doublet

- Two new degenerate VLL with identical coupling to the SM leptons

$$\Delta_{1,LR} = \begin{pmatrix} N \\ E \end{pmatrix}_{-1/2}, \quad \Delta_{3,LR} = \begin{pmatrix} E' \\ Y \end{pmatrix}_{-3/2},$$

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \overline{\Delta}_1 [i\not{D} - M] \Delta_1 + \overline{\Delta}_3 [i\not{D} - M] \Delta_3 - \underline{[\lambda'_i (\overline{\Delta}_1 \phi + \overline{\Delta}_3 \tilde{\phi}) e_i + \text{h.c.}]},$$

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$$\mathcal{M}_0 = \begin{pmatrix} m_i \delta_{ij} & 0 & 0 \\ m'_i & M & 0 \\ m'_i & 0 & M \end{pmatrix}$$

$$U_L^\dagger \mathcal{M}_0 U_R = \mathcal{M}_D,$$

$$s^2 \sim 2 \left(\frac{m'_i}{M} \right)^2$$

$$m'_i = \lambda'_i v$$

$$U_L \approx \begin{pmatrix} U_L^{(3 \times 3)} & 0 & \sqrt{2} \frac{m_i m'_i}{M^2} \\ -\frac{m_i m'_i}{M^2} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{m_i m'_i}{M^2} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix},$$

$$U_R \approx \begin{pmatrix} U_R^{(3 \times 3)} (1 - \frac{m'_i m'_j}{M^2}) & 0 & \sqrt{2} \frac{m'_i}{M} \\ -\frac{m'_i}{M} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{m'_i}{M} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix},$$

The model: degenerate bi-doublet

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$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \bar{\Delta}_1 [i\not{D} - \textcircled{M}] \Delta_1 + \bar{\Delta}_3 [i\not{D} - \textcircled{M}] \Delta_3 - \underline{[\lambda'_i (\bar{\Delta}_1 \phi + \bar{\Delta}_3 \tilde{\phi}) e_i + \text{h.c.}]},$$

$$Z \rightarrow \bar{E}_{1n} e_n \sim \sqrt{2} \frac{m'_1}{M}$$

$$\text{---} \rightarrow e_L e_n \sim \lambda \left[1 - 2 \frac{m'_i m'_j}{M^2} \right]$$

$$W \rightarrow N_n, Y_n e_n \sim \sqrt{2} \frac{m'_1}{M}$$

$$H \rightarrow \bar{E}_{2L} e_n \sim \sqrt{2} \lambda'$$

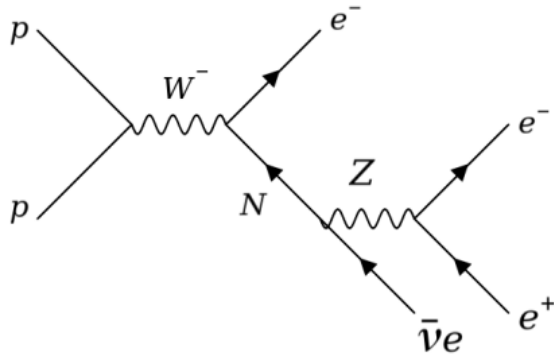
$$Z, W \rightarrow \bar{L} L' \sim O(\lambda)$$

The model: degenerate bi-doublet

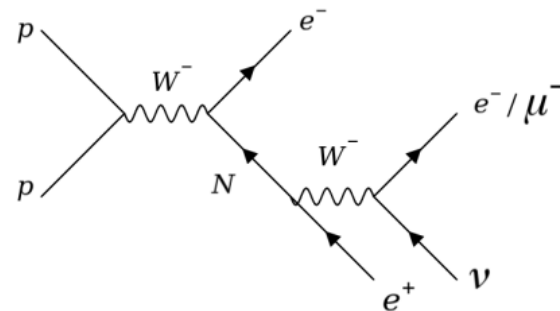
- Two new degenerate VLL with identical coupling to the SM leptons
- Tree-level, dimension 6 effects only in Yukawa couplings.
- What are the current (and future) constraints?
 - Direct searches: single and pair production.
 - Tree level, dimension 6 (precise measurement of lepton Yukawas).
 - Tree level, dimension 8.
 - One-loop dimension 6.
 - Theoretical constraints.
 - Future prospects.

Direct searches: single production

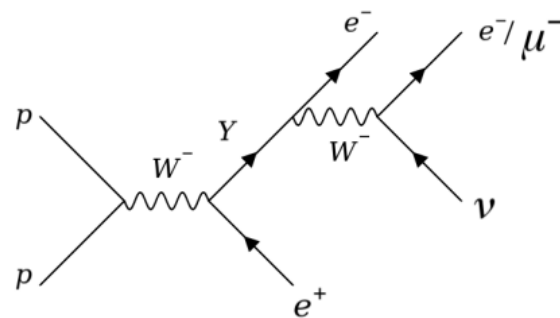
- Single production only considered for HNLs (large mixing).



(a) eee signal through $N \rightarrow Z\nu_e$



(b) $eee/ee\mu$ signal through $N \rightarrow We$

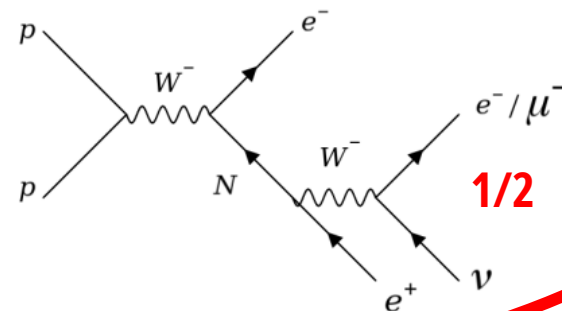
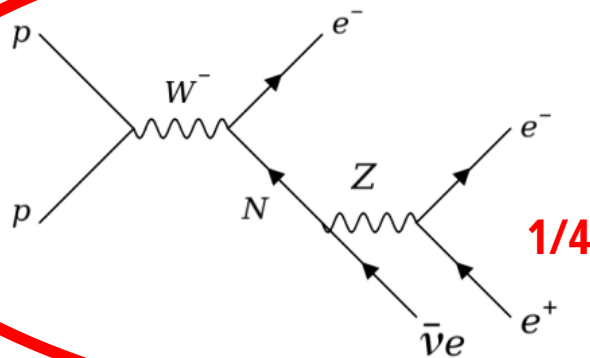


(c) $eee/ee\mu$ signal through $Y \rightarrow We$

Direct searches: single production

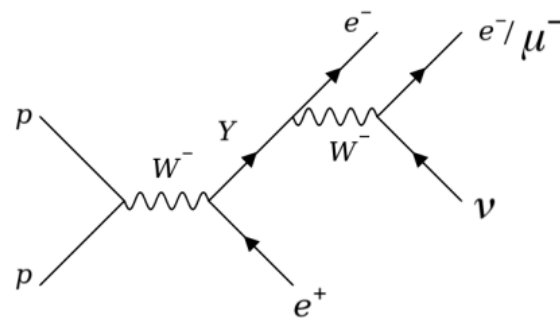
- Single production only considered for HNLs (large mixing).

HNLs



(a) eee signal through $N \rightarrow Z\nu$

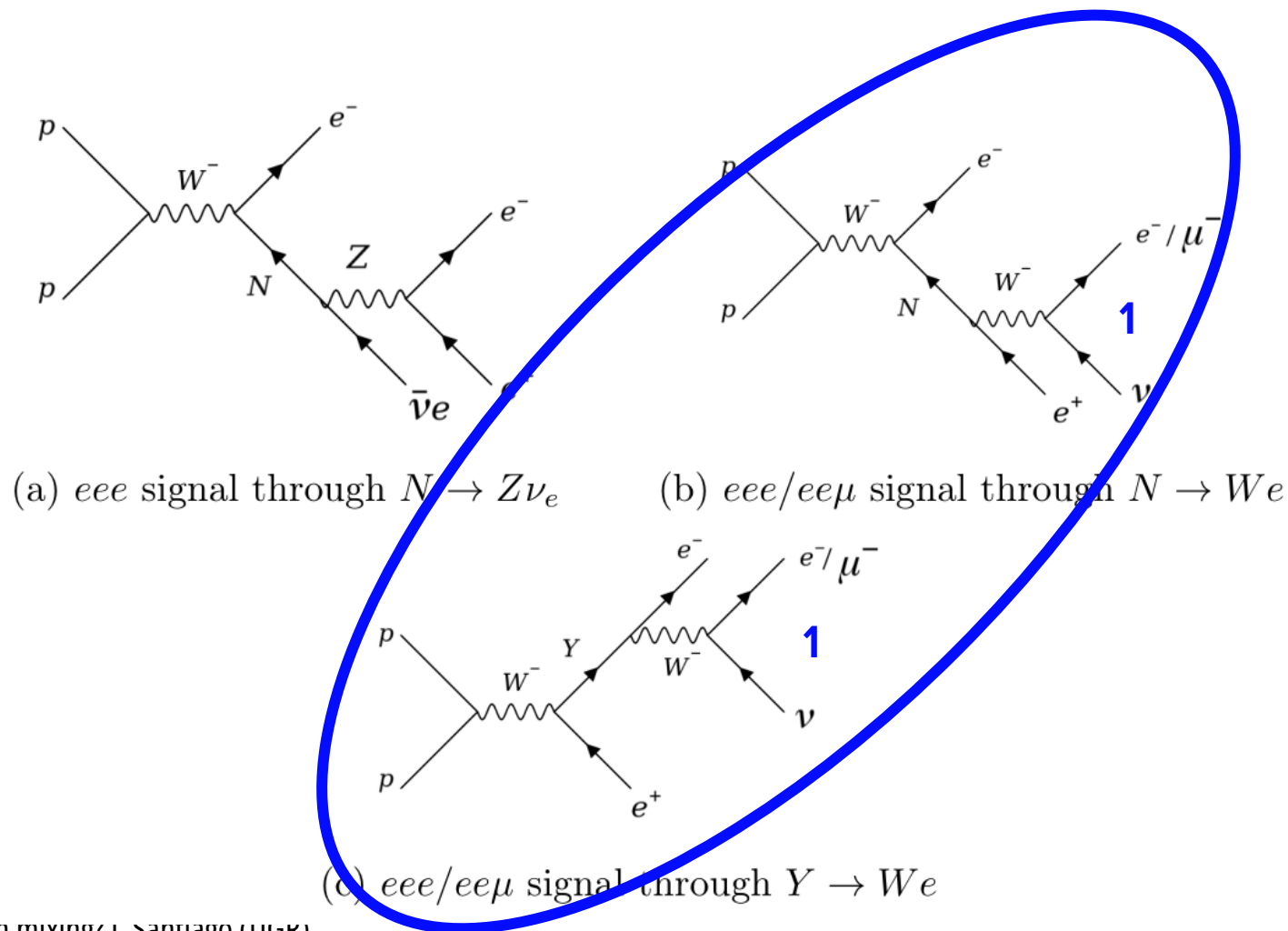
(b) $eee/ee\mu$ signal through $N \rightarrow We$



(c) $eee/ee\mu$ signal through $Y \rightarrow We$

Direct searches: single production

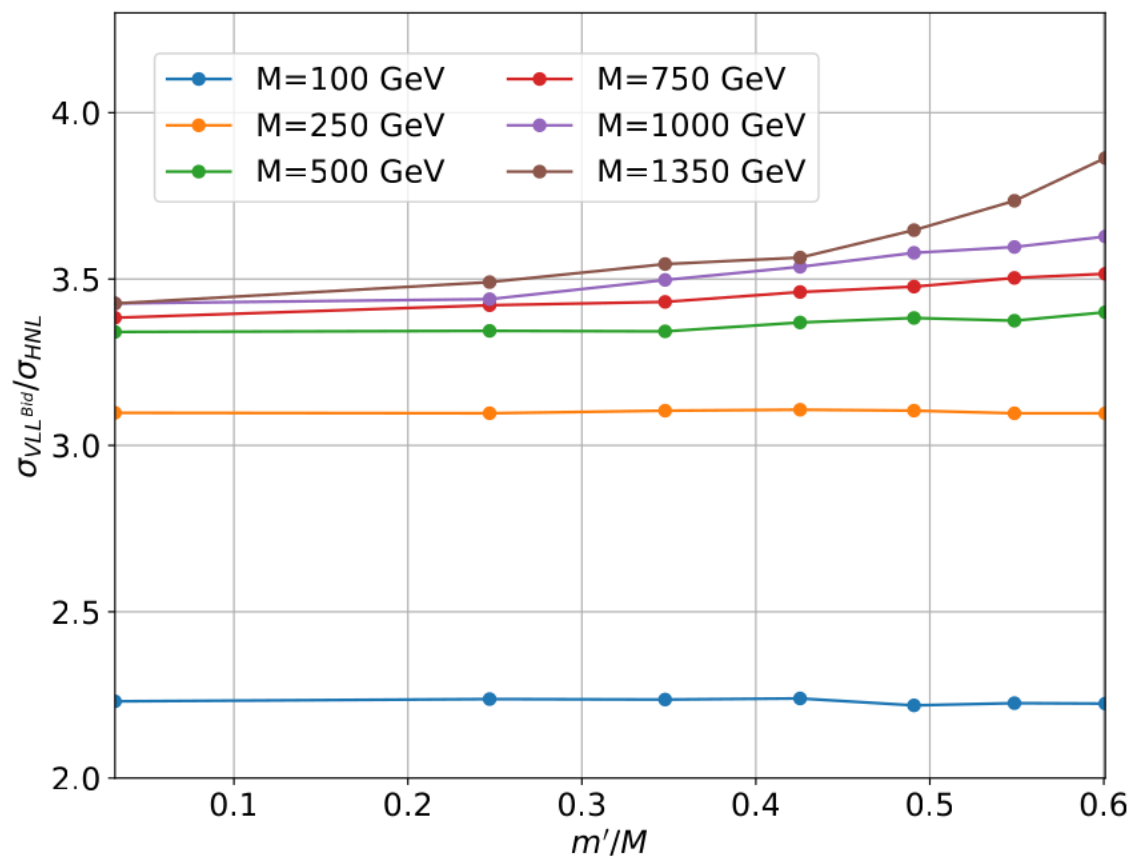
- Single production only considered for HNLs (large mixing).



Our model

Direct searches: single production

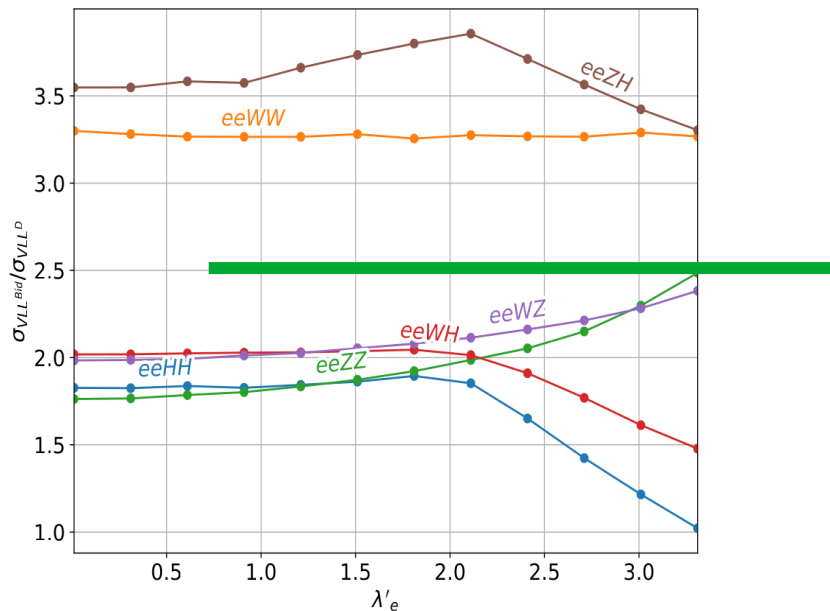
- Single production only considered for HNLs (large mixing).
- We expect a factor of 4 (2 channels twice BR) except for the Z contribution.



We use these factors to rescale the current reach from single production in our model.

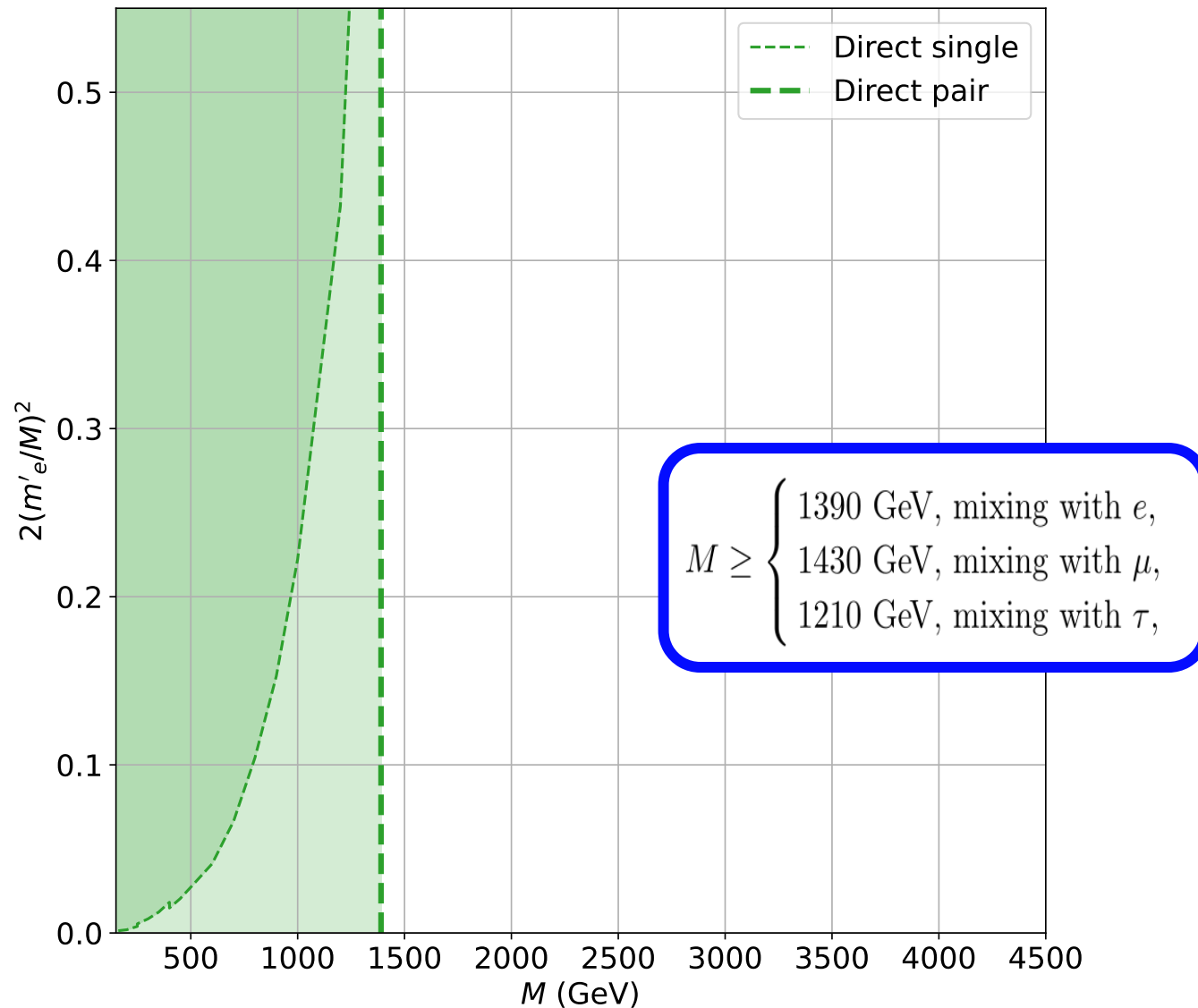
Direct searches: pair production

- Pair production is independent of the mixing.
- We can recast VLL doublet searches in pair production (we have more states with different couplings and BRs)



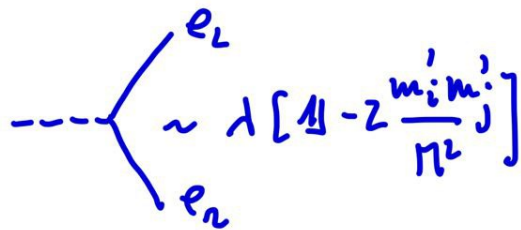
$$M \geq \begin{cases} 1390 \text{ GeV, mixing with } e, \\ 1430 \text{ GeV, mixing with } \mu, \\ 1210 \text{ GeV, mixing with } \tau, \end{cases}$$

How well do you know the electron?



Tree level dimension 6

- At tree level dimension 6 we only modify Yukawa couplings



$$\kappa_l \equiv \frac{Y_l}{y_l^{\text{SM}}} = 1 - 2 \left(\frac{m'_l}{M} \right)^2,$$

$$\kappa_\tau = 0.93 \pm 0.07, \quad 68\% \text{CL.} \quad [\text{ATLAS 2207.00092}]$$

$$2 \left(\frac{m'_\tau}{M} \right)^2 \lesssim 0.14. \quad @ 95\% \text{ CL}$$

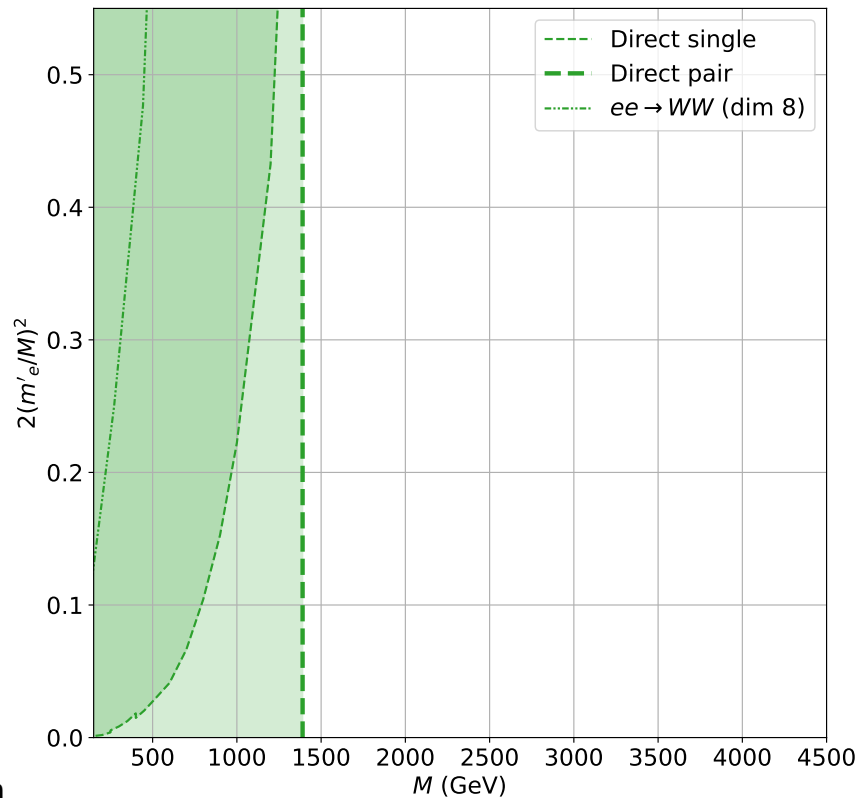
No relevant bounds for electron or muon

- Higgs mediates flavour violating processes (suppressed by light Yukawa couplings). Only mu-e conversion place a constraint.

$$\frac{m'_\mu m'_e}{M^2} \leq 0.065.$$

Tree level dimension 8

- At tree level dimension 8 we generate many operators but no contributions to 3-point vertices.
- Custodial protection partially survives at dimension 8.
- Leading constraints from $e^+e^- \rightarrow W^+ W^-$ (still not competitive).



One-loop dimension 6: flavour violation

- At one loop, mass dimension 6 order many contributions are generated.
- Lepton flavour is violated when mixing with more than one generation (only $\Delta L = 1$)

$$BR(\mu \rightarrow 3e) \leq 10^{-12} \text{ [22]}$$

$$BR(\mu \rightarrow e\gamma) \leq 3.1 \cdot 10^{-13} \text{ [24]}$$

$$BR(\mu\text{Au} \rightarrow e\text{Au}) \leq 7 \cdot 10^{-13} \text{ [26]}$$

$$BR(\tau \rightarrow 3\mu) \leq 1.9 \cdot 10^{-8} \text{ [28]}$$

$$BR(\tau \rightarrow 3e) \leq 2.7 \cdot 10^{-8} \text{ [23]}$$

$$BR(\tau \rightarrow e\gamma) \leq 3.33 \cdot 10^{-8} \text{ [25]}$$

$$BR(\tau \rightarrow \mu\gamma) \leq 4.2 \cdot 10^{-8} \text{ [27]}$$

$$BR(\tau \rightarrow \mu ee) \leq 1.8 \cdot 10^{-8} \text{ [23]}$$

One-loop dimension 6: flavour violation

- At one loop, mass dimension 6 order many contributions are generated.
- Lepton flavour is violated when mixing with more than one generation (only $\Delta L = 1$)

$$\frac{m'_\mu m'_e}{M^2} \leq \begin{cases} 1.58 \cdot 10^{-5}, & [\mu \rightarrow e\gamma], \\ 2.70 \cdot 10^{-5}, & [\mu \rightarrow e \text{ conversion in Au}], \\ 2.89 \cdot 10^{-4}, & [\mu \rightarrow 3e], \end{cases}$$

$$\frac{m'_\tau m'_e}{M^2} \leq \begin{cases} 0.012, & [\tau \rightarrow e\gamma], \\ 0.091, & [\tau \rightarrow 3e], \end{cases}$$

$$\frac{m'_\tau m'_\mu}{M^2} \leq \begin{cases} 0.014, & [\tau \rightarrow \mu\gamma], \\ 0.113, & [\tau \rightarrow \mu ee], \\ 0.116, & [\tau \rightarrow 3\mu]. \end{cases}$$

One-loop dimension 6: flavour preserving

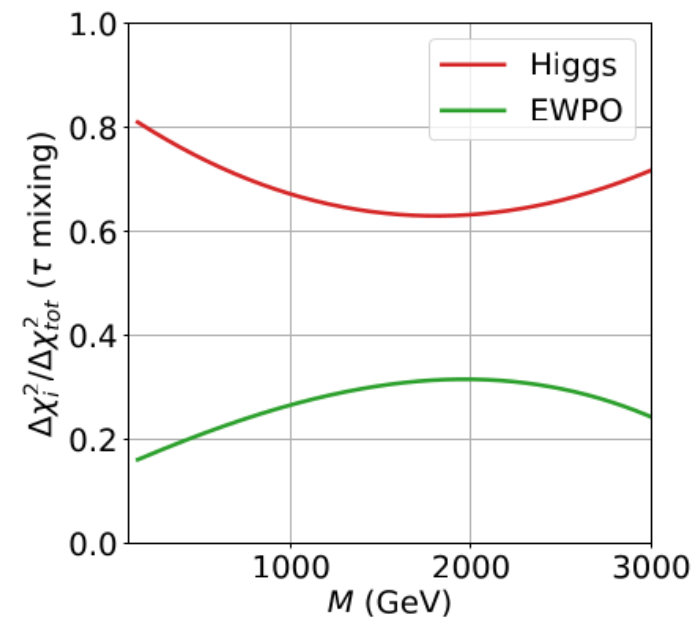
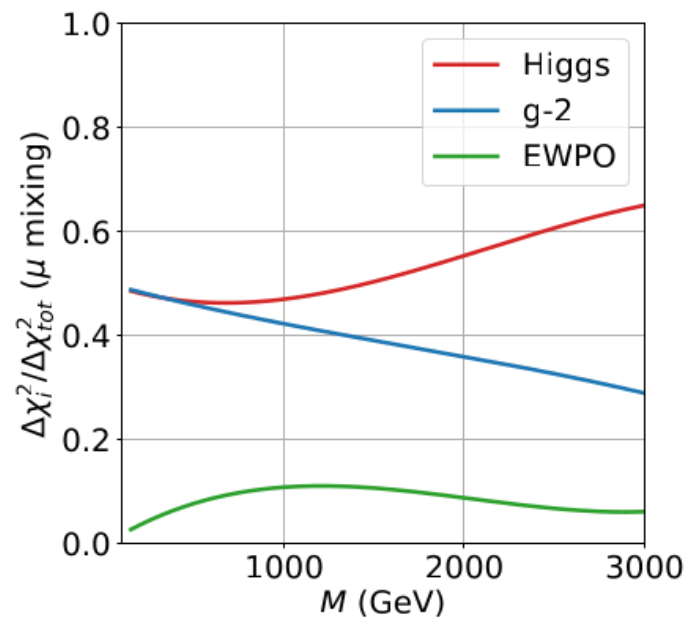
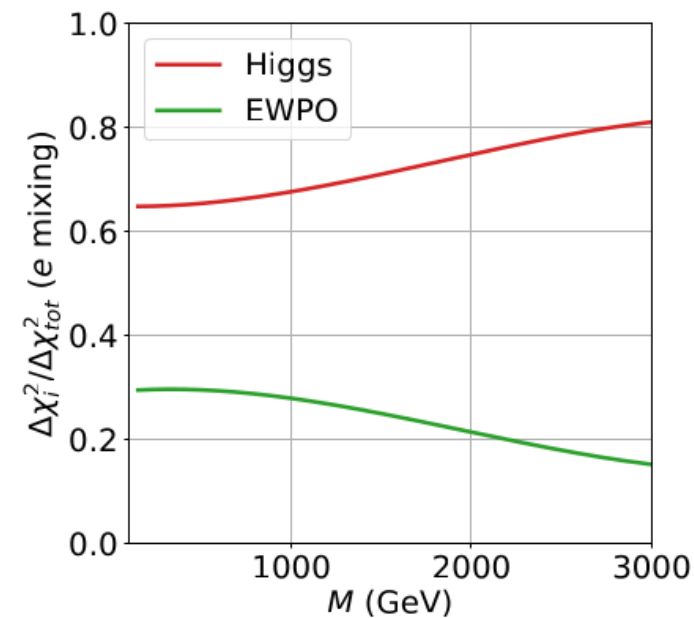
- At one loop, mass dimension 6 order many contributions are generated.
- EWPD and Higgs physics are the most constraining (in general).

$$\begin{aligned}
 16\pi^2 \alpha_{HB} &= \frac{2g_1^2}{3} \sum_k \frac{|\lambda'_k|^2}{M^2} + \dots, \\
 16\pi^2 \alpha_{H\Box} &= \left[\frac{g_1^2}{9} + \frac{g_2^2}{3} \right] \sum_k \frac{|\lambda'_k|^2}{M^2} - \frac{1}{3} \sum_{kl} \frac{|\lambda'_k|^2 |\lambda'_l|^2}{M^2} + \dots, \\
 16\pi^2 \alpha_{HD} &= \frac{4g_1^2}{9} \sum_k \frac{|\lambda'_k|^2}{M^2} + \dots, \\
 16\pi^2 (\alpha_{He})_{ij} &= -\frac{2g_1^2}{9} \sum_k \frac{|\lambda'_k|^2}{M^2} \delta_{ij} + g_1^2 \left[\frac{19}{9} + 3 \log \left(\frac{\mu^2}{M^2} \right) \right] \frac{\lambda'^*_i \lambda'_j}{M^2} + \dots, \\
 16\pi^2 (\alpha_{H\psi}^{(1)})_{ij} &= \frac{2g_1^2}{9} Y_\psi \sum_k \frac{|\lambda'_k|^2}{M^2} \delta_{ij} + \dots, \\
 16\pi^2 (\alpha_{Hl}^3)_{ij} &= 16\pi^2 (\alpha_{Hq}^3)_{ij} = \frac{g_2^2}{9} \sum_k \frac{|\lambda'_k|^2}{M^2} \delta_{ij} + \dots, \\
 16\pi^2 (\alpha_{eW})_{ij} &= -y_i \frac{g_2}{8} \frac{\lambda'^*_i \lambda'_j}{M^2}, \\
 16\pi^2 (\alpha_{eB})_{ij} &= -y_i \frac{13g_1}{24} \frac{\lambda'^*_i \lambda'_j}{M^2}.
 \end{aligned}$$

} **Higgs**
} **EWPD**
} **Dipoles (g-2)**

One-loop dimension 6: flavour preserving

- At one loop, mass dimension 6 order many contributions are generated.
- EWPD and Higgs physics are the most constraining (in general).



One-loop dimension 6: flavour preserving

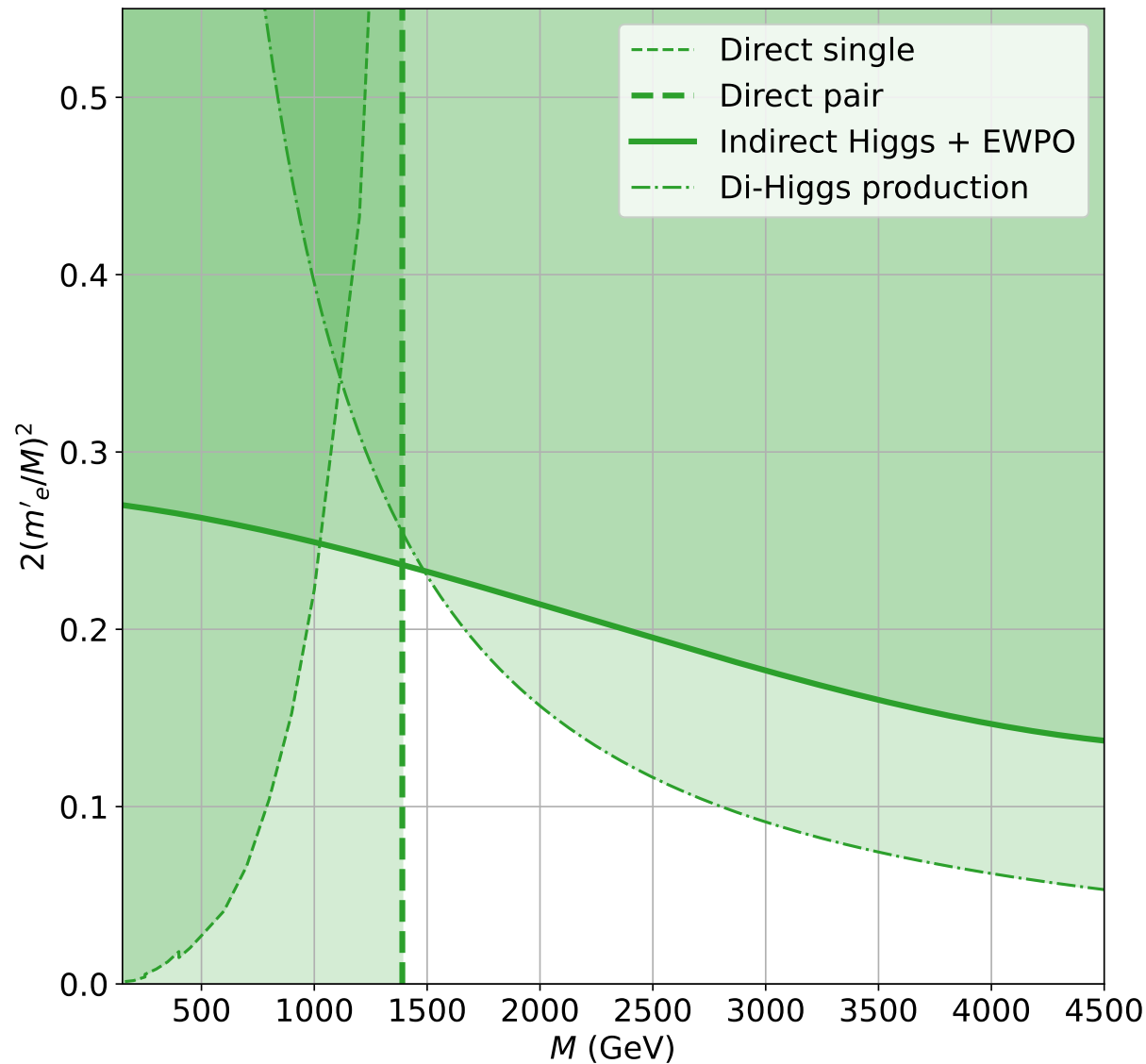
- At one loop, mass dimension 6 order many contributions are generated.
- EWPD and Higgs physics are the most constraining (in general).
- Double Higgs production starting to become relevant.

$$\kappa_\lambda = 1 - \frac{8v^4\alpha_H}{m_H^2} + \dots = 1 - \frac{1}{6\pi^2} \left(2\frac{m'^2}{M^2}\right)^3 \frac{M^4}{m_H^2 v^2}$$

$$-1.2 \leq \kappa_\lambda \leq 7.2. \quad [\text{ATLAS 2406.09971}]$$

$$2\frac{m'^2}{M^2} \leq \left((1 + |\kappa_{\min}^{\text{exp}}|)6\pi^2 \frac{v^2 m_H^2}{M^4}\right)^{\frac{1}{3}} \approx \left(\frac{0.5 \text{ TeV}}{M}\right)^{\frac{4}{3}}$$

One-loop dimension 6: flavour preserving



Theoretical constraints

- Mild constraints on the mixing imply large values of couplings being probed (strong coupling?).
- Perturbative unitarity: $\lambda' \lesssim 4$ [\[Allwicher, Arnan, Barducci, Nardecchia 2108.00013\]](#)
- Stability of the potential: $\lambda' \lesssim 2$

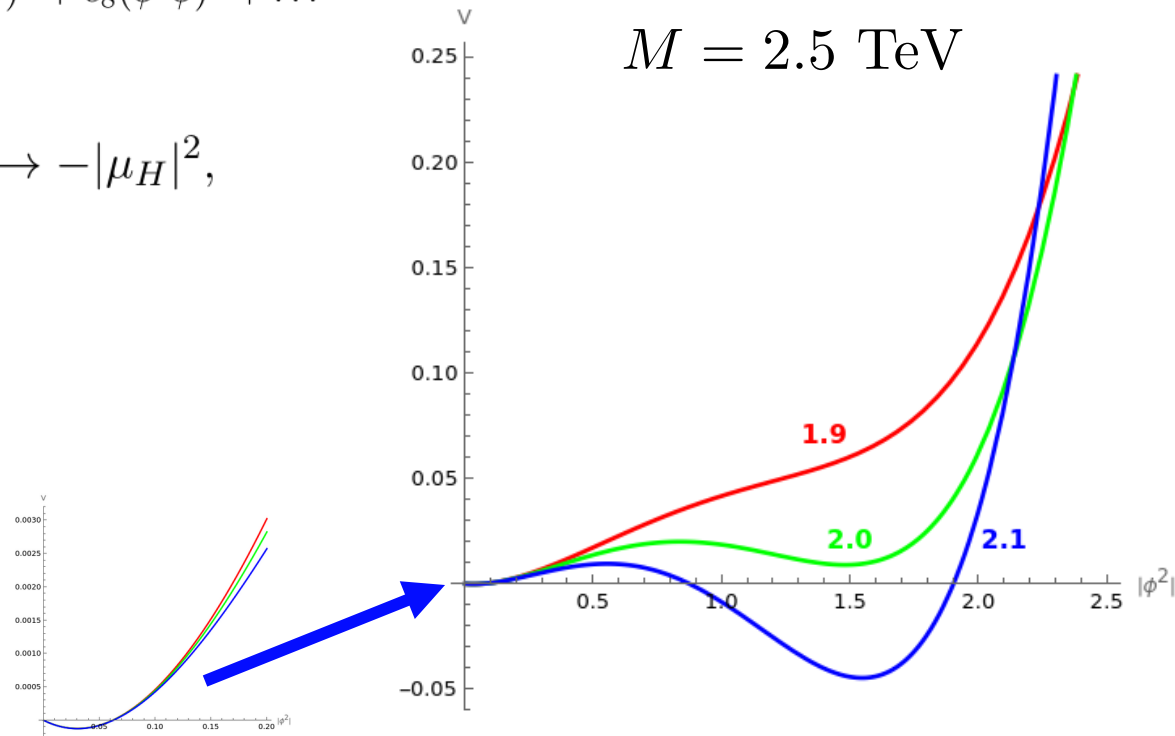
$$V(\phi) = c_2 (\phi^\dagger \phi) + c_4 (\phi^\dagger \phi)^2 + c_6 (\phi^\dagger \phi)^3 + c_8 (\phi^\dagger \phi)^4 + \dots$$

$$c_2 \sim -|\mu_H|^2 + \frac{\lambda'^2 M^2}{4\pi^2} \rightarrow -|\mu_H|^2,$$

$$c_4 \sim \lambda - \frac{5}{24\pi^2} \frac{\lambda'^4 |\mu_H^2|}{M^2},$$

$$c_6 \sim -\frac{\lambda'^6}{6\pi^2 M^2},$$

$$c_8 \sim \frac{\lambda'^8}{12\pi^2 M^4}.$$



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[\[Durieux, McCullough, Salvioni, 2209.00666\]](#)

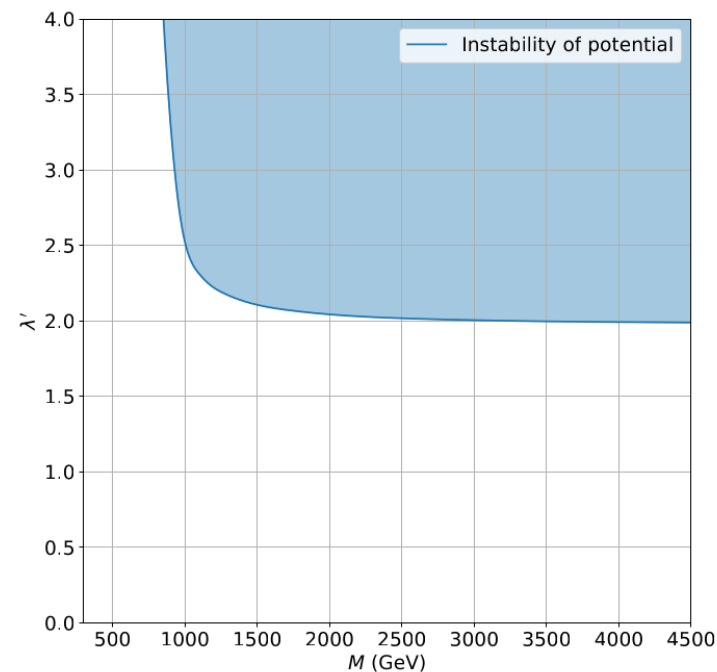
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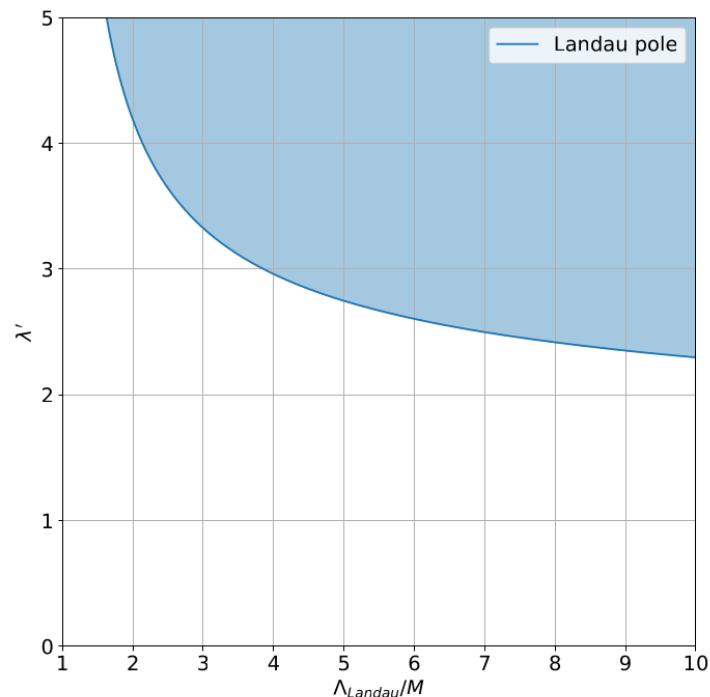
$$c_6 \sim -\frac{\lambda'^6}{6\pi^2 M^2},$$

$$c_8 \sim \frac{\lambda'^8}{12\pi^2 M^4}.$$

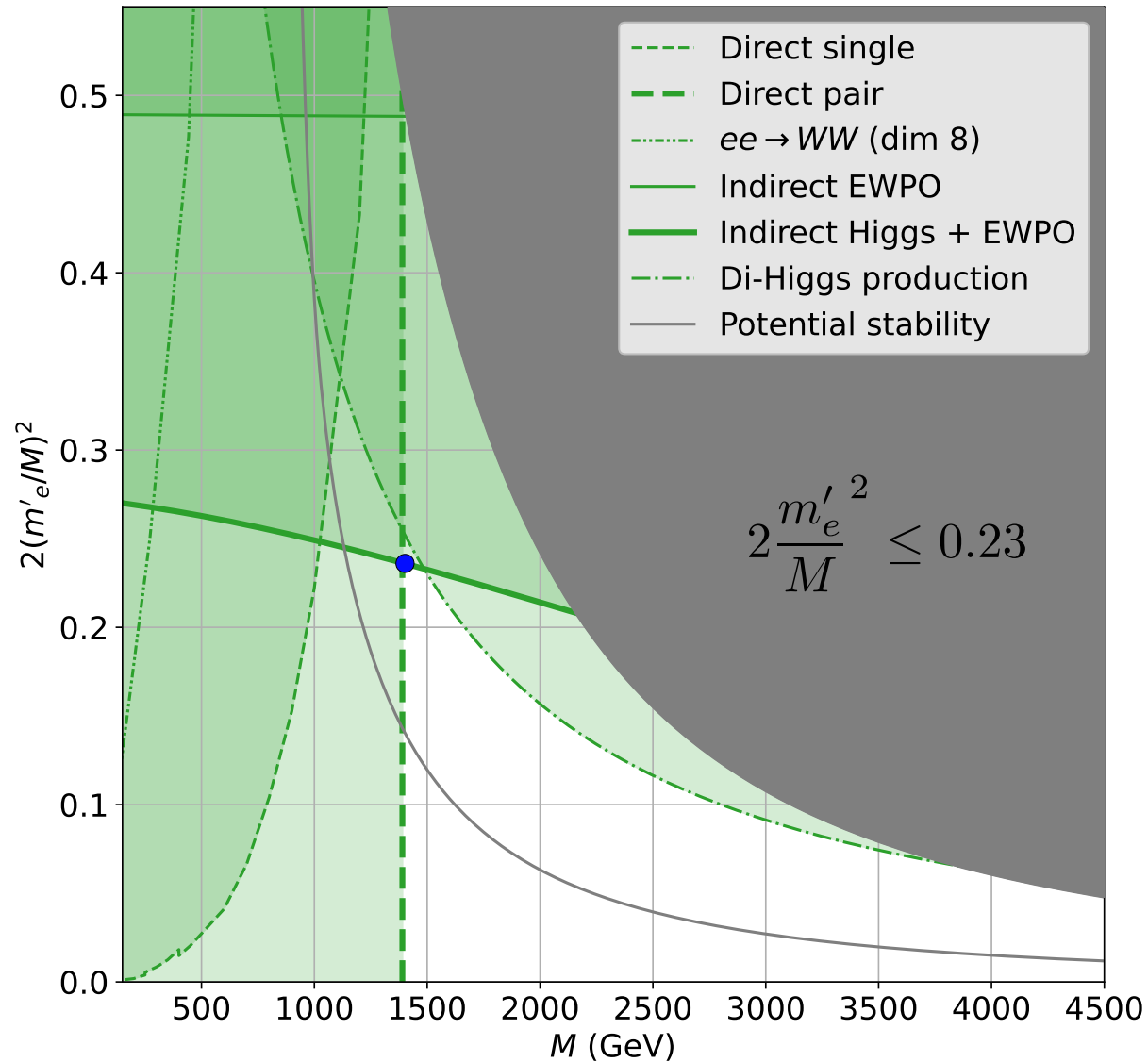


Theoretical constraints

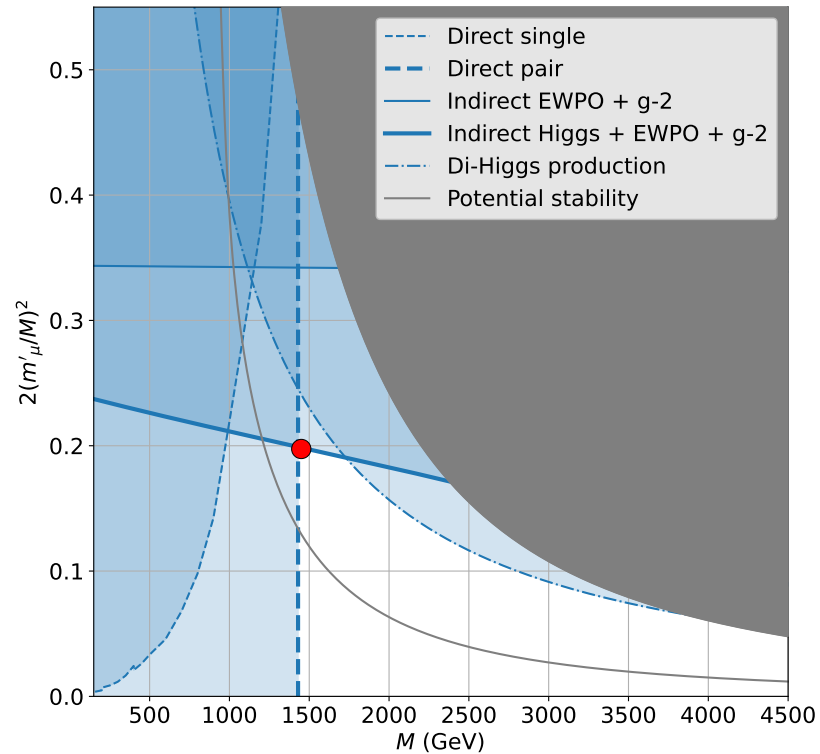
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- Stability of the potential: $\lambda' \lesssim 2$ (can be lifted with scalar quadruplet)
- Landau Pole: $\lambda' \lesssim 2 - 4$ [\[Durieux, McCullough, Salvioni, 2209.00666\]](#)



Current constraints

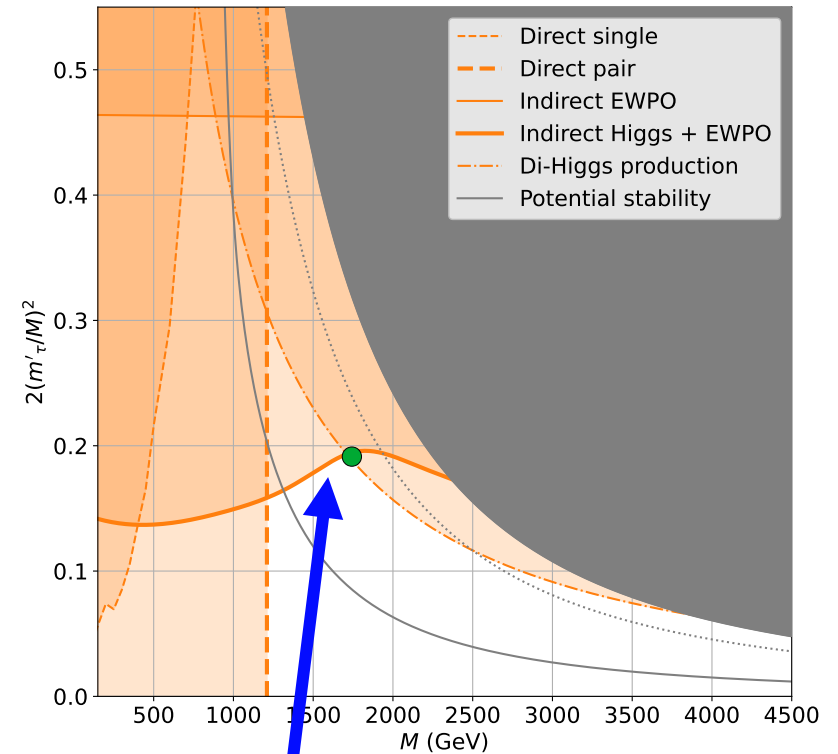


Current constraints



$$2 \frac{m'^2_\mu}{M} \leq 0.20$$

$$(\alpha_{eH})_\tau = y_\tau \frac{\lambda'^2_\tau}{M^2} \left[1 - \frac{79}{96\pi^2} \lambda'^2_\tau \right] + \dots$$



$$2 \frac{m'^2_\tau}{M} \leq 0.19$$

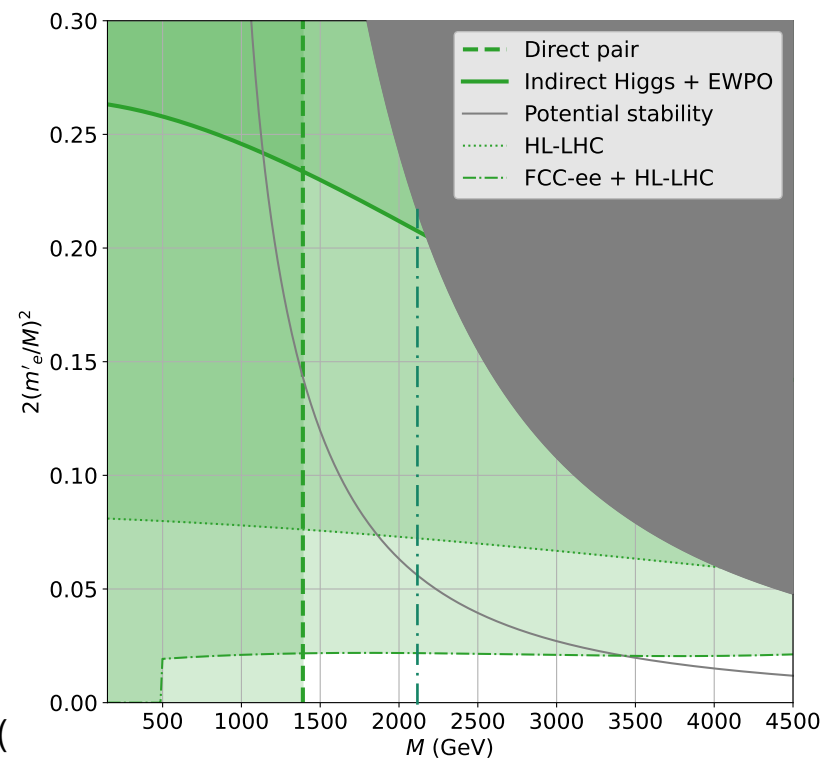
Interplay of tree-level and one-loop contribution to tau Yukawa

Future constraints

Preliminary

- Future experiments will further constrain lepton mixing:
 - HL-LHC: increased reach in direct searches and more precise Higgs measurements
 - FCC-ee: increased precision on EWPD
 - FCC-hh: increased reach in direct searches and high precision Higgs physics.

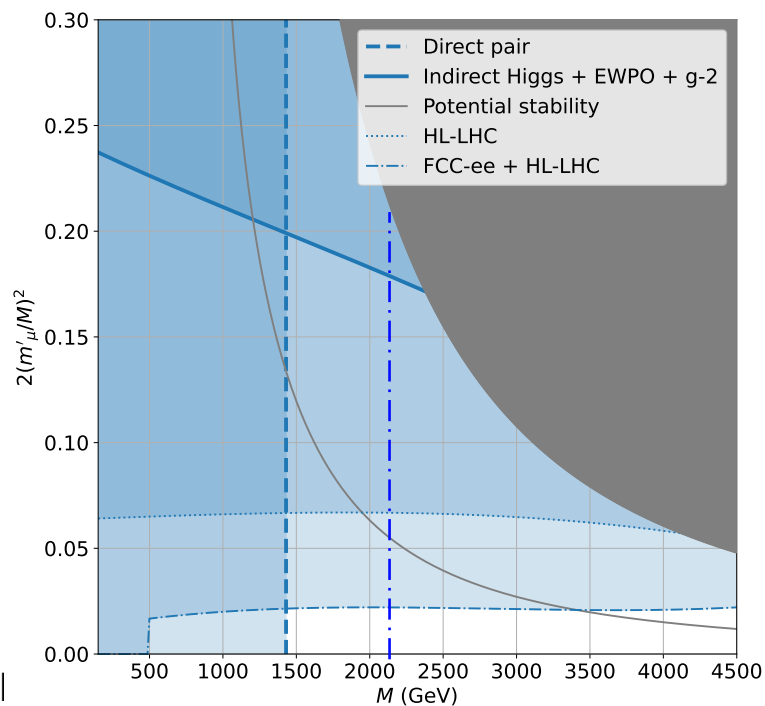
Collider reach ^{β} [Salam, Weiler]



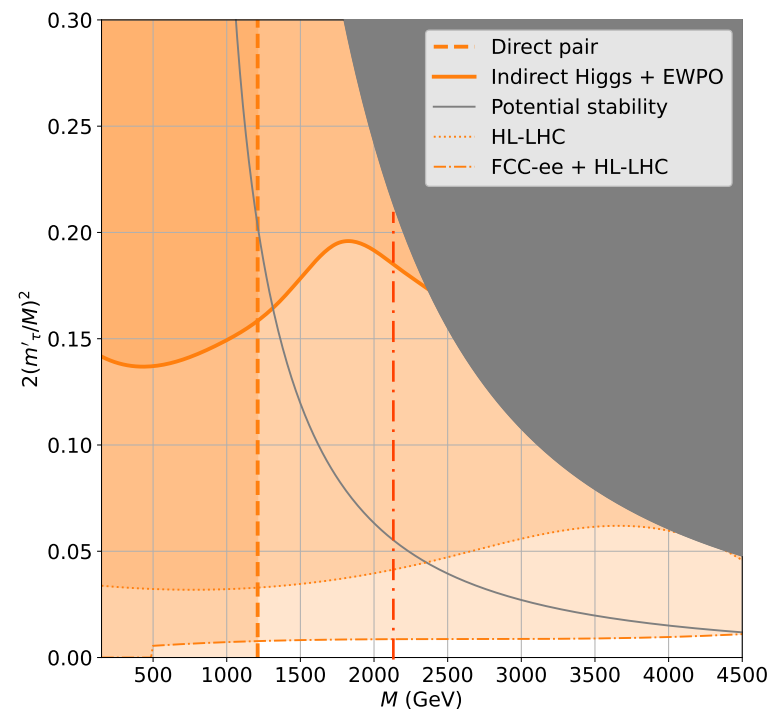
Future constraints

Preliminary

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How large is μ



- Fairly well

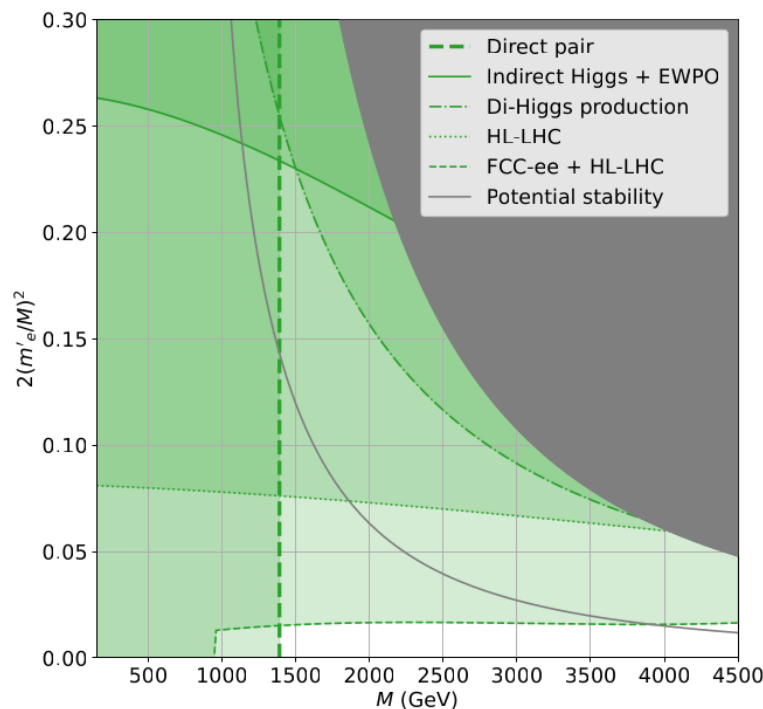
$$s^2 \lesssim 10^{-3}$$

Lepton		95% C.L. EWPD limit on mixing $s_{L\ell}$		
L	$(d_c, d_L)_Y$	Only e	Only μ	Only τ
N	$(1, 1)_0$	0.041	0.030	0.087
E	$(1, 1)_{-1}$	0.021	0.030	0.033
$\begin{pmatrix} N \\ E^- \end{pmatrix}$	$(1, 2)_{-\frac{1}{2}}$	0.020	0.048	0.034
$\begin{pmatrix} E^- \\ E^{--} \end{pmatrix}$	$(1, 2)_{-\frac{3}{2}}$	0.028	0.028	0.046
$\begin{pmatrix} E^+ \\ N \\ E^- \end{pmatrix}$	$(1, 3)_0$	0.019	0.017	0.030

How well do we know the electron?

- Not as well as we thought!
- Tailored searches might carve out part of parameter space.
- Future colliders will shrink the window significantly.

$$s^2 \lesssim 0.23$$



How la

