

$B \rightarrow D^{(*)} \tau \nu$ and $b \rightarrow s$ penguin anomalies at Belle and Belle II

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for Belle and Belle II collaborations

PASCOS 2025, the 30th International Symposium on Particles, Strings and Cosmology
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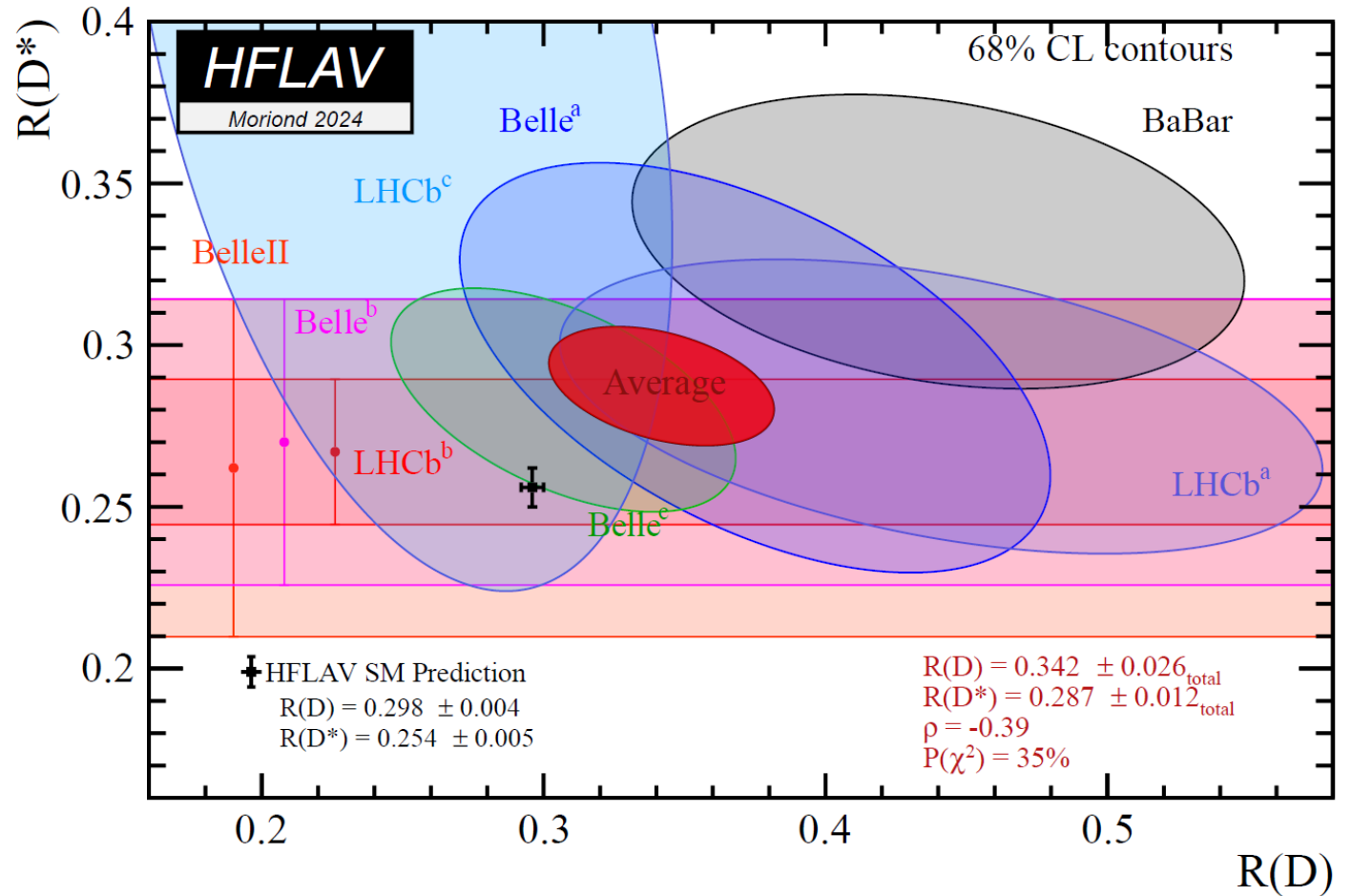


KEK



Lepton Flavor Universality Anomaly in $B \rightarrow D^{(*)}\tau\nu$ Decays

- $$R(D^{(*)}) = \frac{\mathcal{B}(B \rightarrow D^{(*)}\tau\bar{\nu})}{\mathcal{B}(B \rightarrow D^{(*)}\ell\bar{\nu})} \quad \ell = e, \mu$$
- Measurements so far show deviation from the SM expectation
 - $R(D) \sim 1.6\sigma$
 - $R(D^*) \sim 2.5\sigma$
 - **Combined $\sim 3.3\sigma$**



Evidence for $B \rightarrow K \nu \bar{\nu}$ Decays at Belle II

[[Phys. Rev. D **109**, 112006](#)]

- Signal **evidence of 3.5σ significance** by inclusive + hadronic tag

Inclusive tag : $[2.7 \pm 0.5(\text{stat}) \pm 0.5(\text{syst})] \times 10^{-5}$

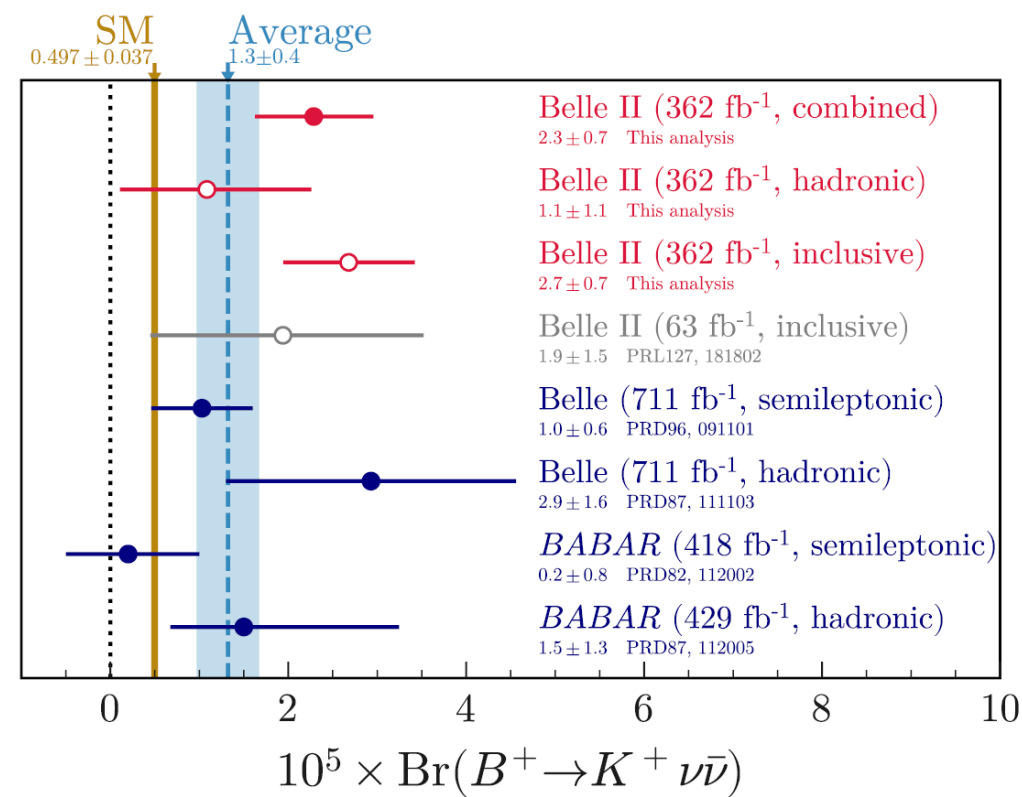
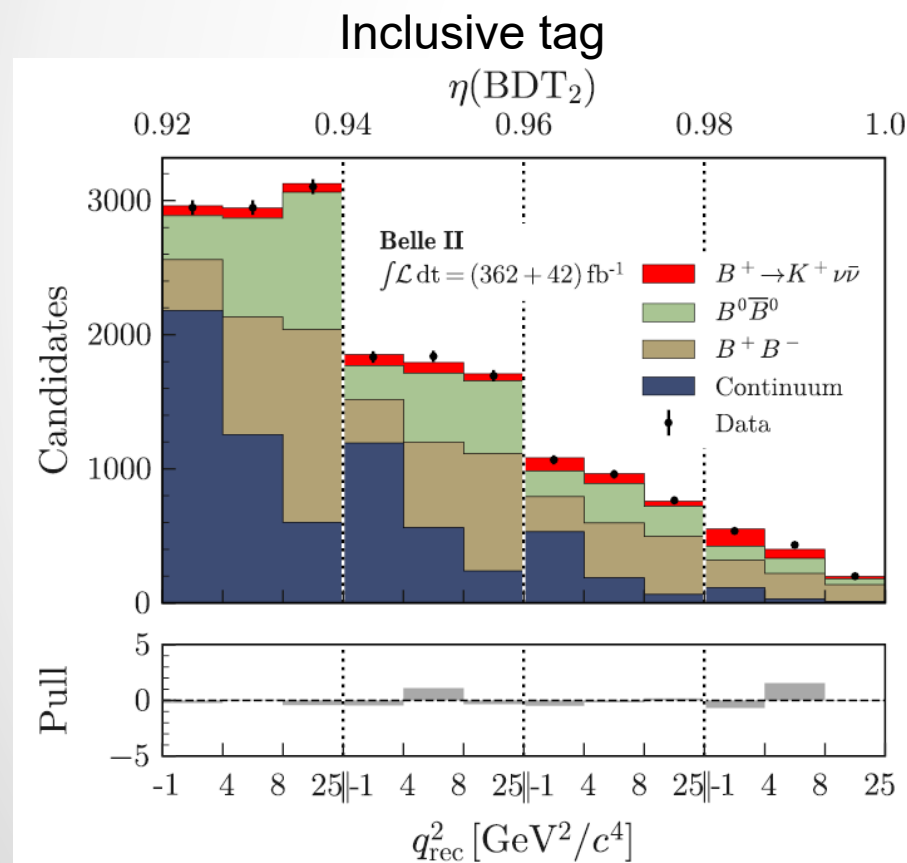
Hadronic Tag $[1.1^{+0.9}_{-0.8}(\text{stat})^{+0.8}_{-0.5}(\text{syst})] \times 10^{-5}$

Combined $(2.3 \pm 0.7) \times 10^{-5}$

SM expectation:

4.97×10^{-6}

$\updownarrow$ **2.7σ deviation**



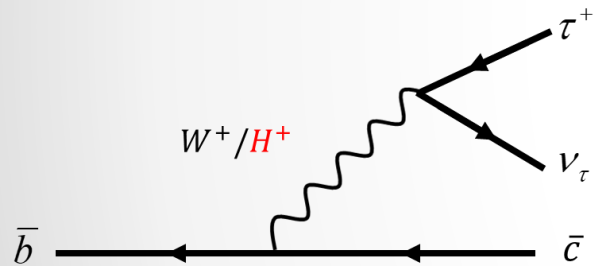
Possible New Physics

in $B \rightarrow D^{(*)}\tau\nu$ and $b \rightarrow s$ Penguin

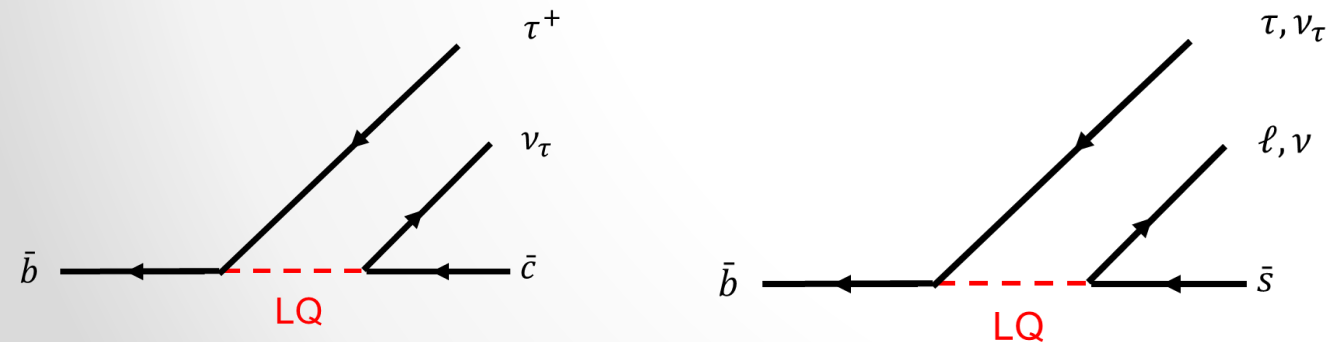
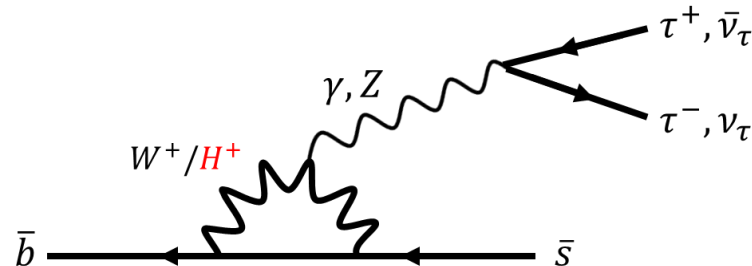
- Deviations from the SM may indicate **New Physics (NP) effects in B decays with tau flavor.**

- Charged Higgs
- Leptoquark
- ...

$$B^0 \rightarrow D^{(*)-}\tau^+\nu$$



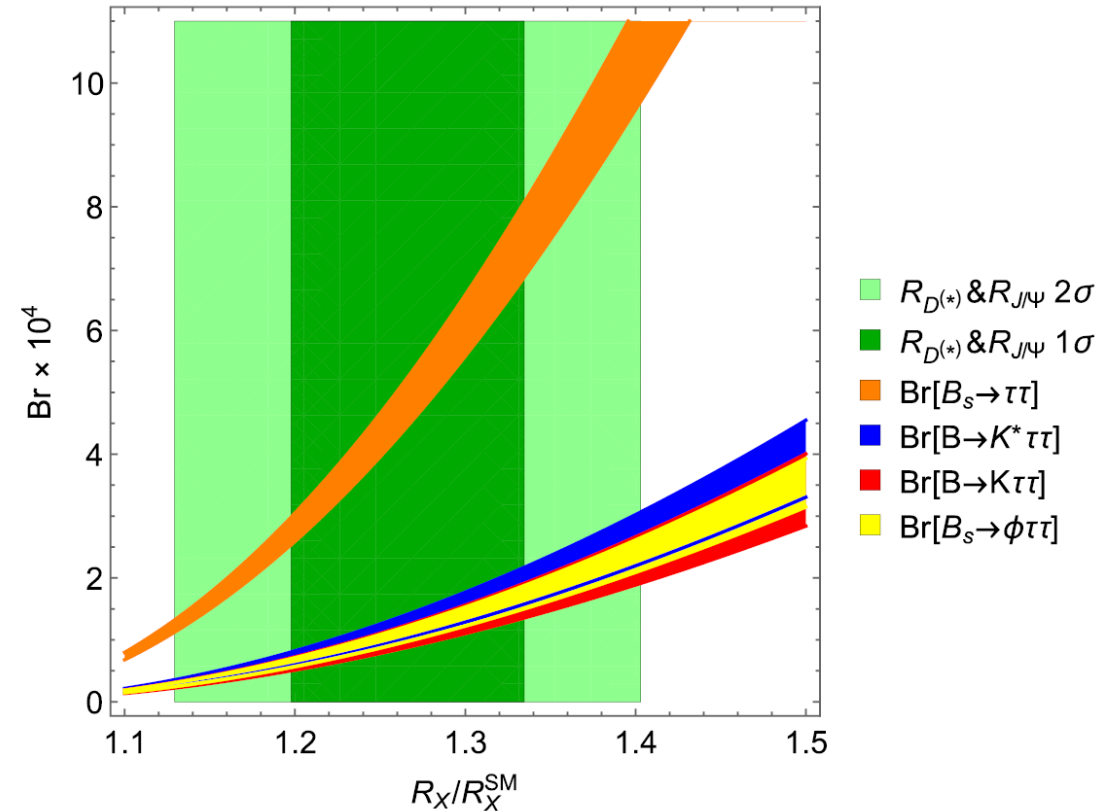
$$B^0 \rightarrow X_s \nu \bar{\nu}, K^{*0} \tau \tau, K^{(*)0} \tau \ell$$



PASCOS2025

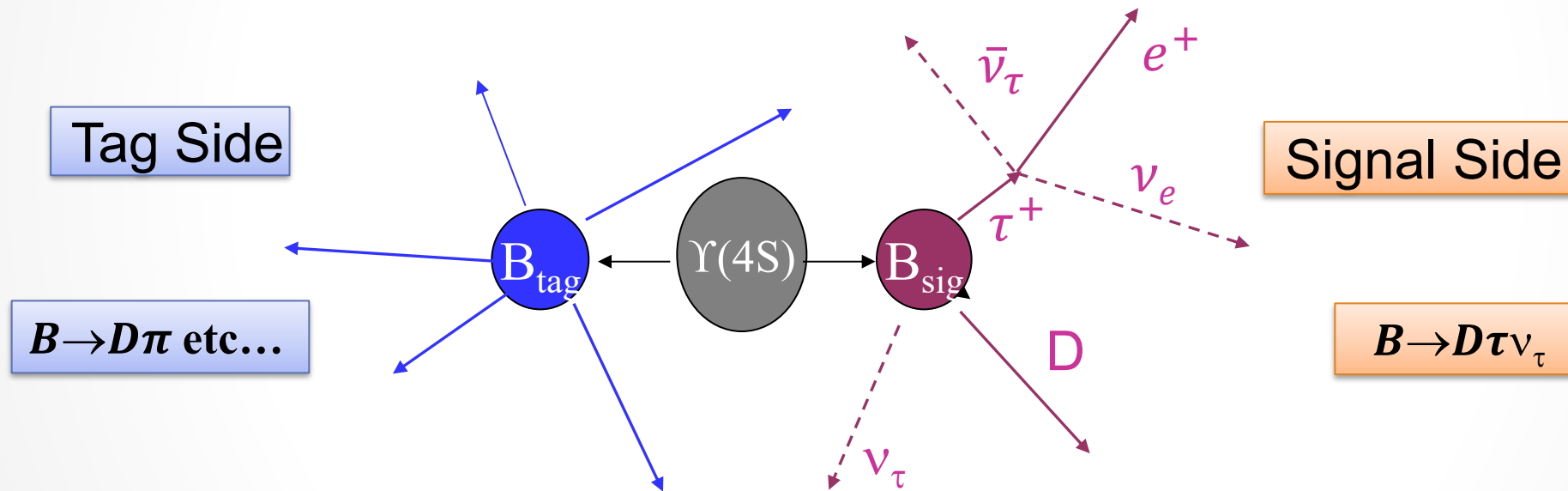
Possible NP enhancement of $\mathcal{B}(b \rightarrow s \tau^+ \tau^-)$ as a function of $R(X)$

[[B. Capdevila et al., PRL120, 181802\(2018\)](#)]



B Decay Analyses with Missing at B-factory

- **Difficulty due to the neutrino(s)** in the signal decay
→ Utilize the B factory specific feature : **Tagging one of the $B\bar{B}$ pair in the event**

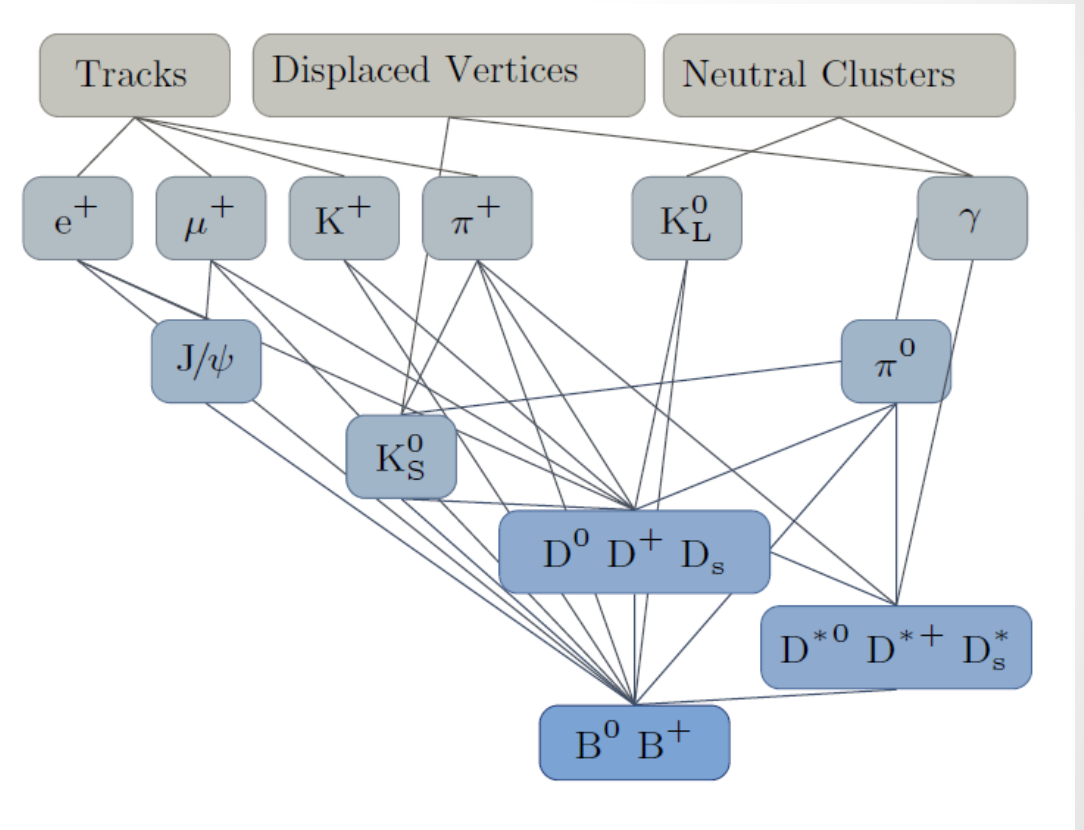


Reconstruct and remove B_{tag} from the event
→ Search for the signal decay in the remaining particles

B_{tag} : Full Event Interpretation (FEI)

[\[Computing and Software for Big Science 3, 6 \(2019\)\]](#)

- Reconstruct > 100 intermediate decays
→ $O(10,000)$ decay chains
- Multivariate classifier for signal probability
→ $O_{FEI} = 0 \sim 1$ for **BG** ~ **Sig**
 - Cut on O_{FEI} : Optimize eff. vs purity in each analysis
 - Additionally kinematical cuts are performed
 - B_{tag} invariant mass for hadronic decays
 - Missing $\sim$ neutrino for semileptonic decays
- **Hadronic** and **Semileptonic** Tagging
 - **Hadronic** ~ 0.5 % eff.
 - **Semileptonic** ~ 1 % eff. (eff. depends on O_{FEI} cut)



Recent Belle, Belle II Results

- $B^0 \rightarrow D^{(*)} \tau \nu$ $R(D^{(*)})$ with **Semileptonic Tag** at **Belle II**
[arXiv:2504.11220](https://arxiv.org/abs/2504.11220), accepted by PRD
- $B^0 \rightarrow K^{*0} \tau \tau$ with **Hadronic Tag** at **Belle II**
[arXiv:2504.10042](https://arxiv.org/abs/2504.10042), submitted to PRL
- $B^0 \rightarrow K_S^0 \tau \ell$ and $B^0 \rightarrow K^{*0} \tau \ell$ with **Hadronic Tag** at **Belle** + **Belle II**
[arXiv:2412.16470](https://arxiv.org/abs/2412.16470) ($K_S^0 \tau \ell$), accepted by PRL
[arXiv:2505.08418](https://arxiv.org/abs/2505.08418) ($K^{*0} \tau \ell$), accepted by JHEP
- $B^0 \rightarrow X_S \nu \bar{\nu}$ with **Hadronic Tag** at **Belle II**
- **Reinterpretation** of the $B^+ \rightarrow K^+ \nu \bar{\nu}$ **Belle II** Result
[arXiv:2507.12393](https://arxiv.org/abs/2507.12393), submitted to PRD

Analyzed data
Belle : 711fb⁻¹
Belle II : 365fb⁻¹

$B^0 \rightarrow D^{(*)}\tau\nu$ with Semileptonic Tag at Belle II

[\[arXiv:2504.11220\]](https://arxiv.org/abs/2504.11220) accepted by PRD

- First $R(D^{(*)})$ BelleII measurement using semileptonic B tagging
- 365fb^{-1} Belle II data
- Semileptonic FEI Tag, neutral B_{tag}
 - Charged B_{tag} is left for future study (require precise slow π^0 understanding)
- Leptonic tau decays: $\tau \rightarrow e\bar{\nu}_e\nu_\tau, \mu\bar{\nu}_\mu\nu_\tau$
- Fit signal $B \rightarrow D^{(*)}\tau\nu$ and normalization $B \rightarrow D^{(*)}l\nu$ simultaneously
→ obtain $R(D^*)$ and $R(D)$

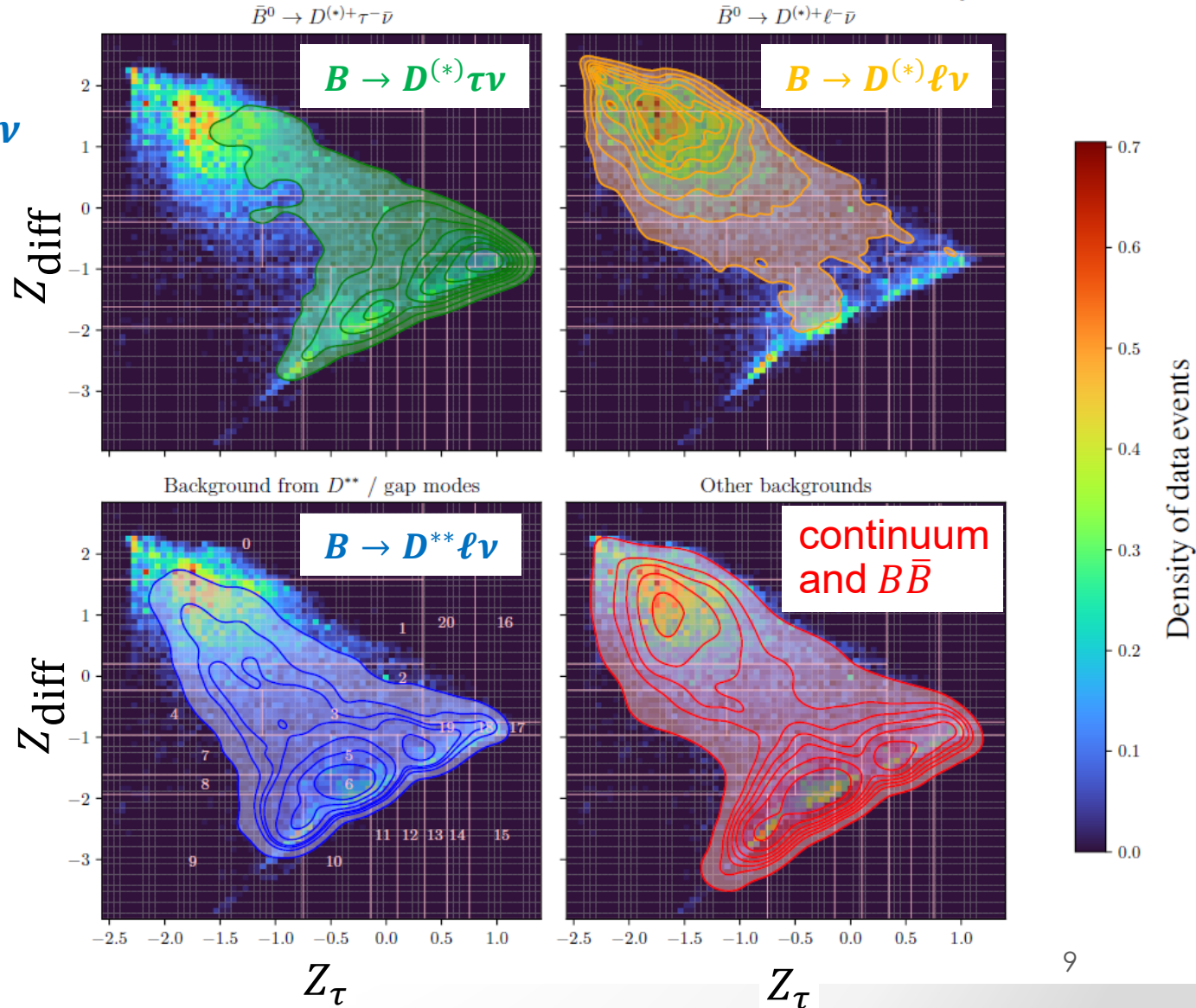
$B^0 \rightarrow D^{(*)}\tau\nu$ Signal Separation

BDT used to separate events

- Semitauonic signal events : $B \rightarrow D^{(*)}\tau\nu$
- Semileptonic events $B \rightarrow D^{(*)}\ell\nu$ and $B \rightarrow D^{**}\ell\nu$
- Background events: **continuum** and $B\bar{B}$
- Trained on 5 input variables
 - 2 specific decay angle correlations to semileptonic decays
 - Extra energy in the calorimeter
 - Signal side D and lepton momentum
- Output for each event: $Z_\tau, Z_\ell, Z_{\text{bkg}}$
- Use $Z_\tau, Z_{\text{diff}} \equiv Z_\ell - Z_{\text{bkg}}$ for the signal extraction

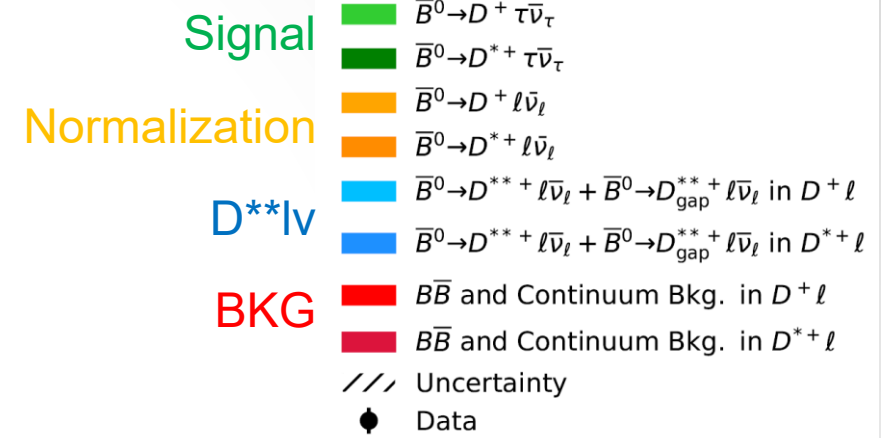
Belle II

$\int \mathcal{L} dt = 365 \text{ fb}^{-1}$



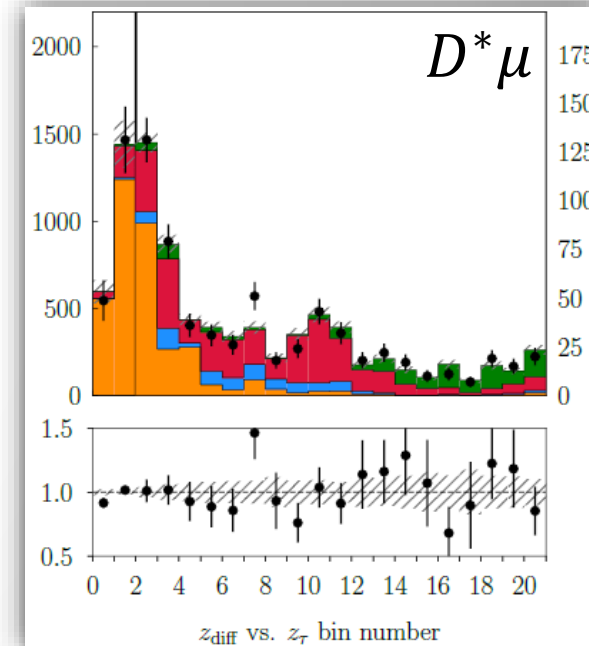
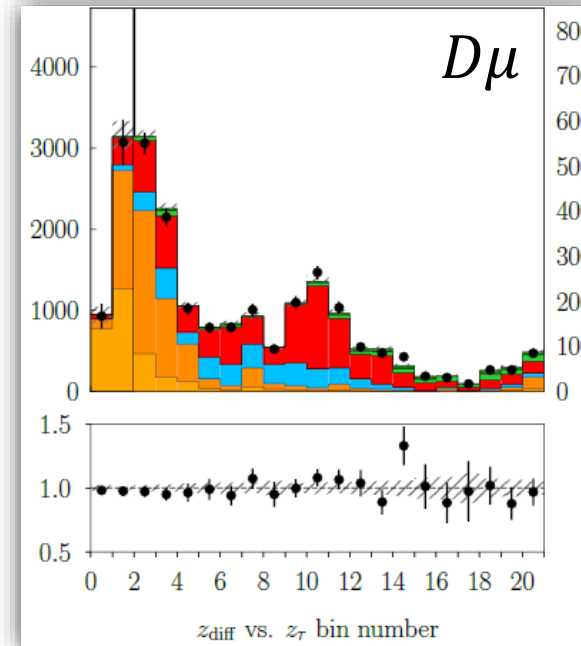
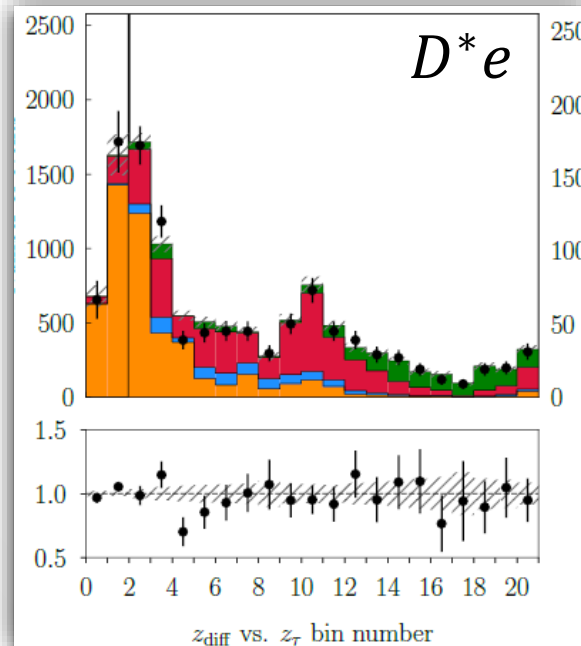
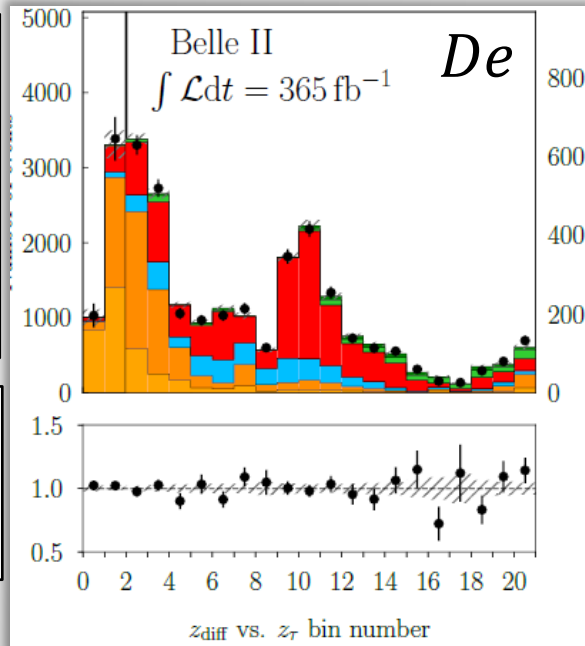
$B^0 \rightarrow D^{(*)}\tau\nu$ Signal Extraction Fit

- Fit is performed over 4 channels : $De, D^*e, D\mu, D^*\mu$
- 2D binned likelihood fits to Z_τ and Z_{diff} → projected to 1D
 - X=0,1 → large normalization events → left axis
 - Larger X → semitauonic signals → right axis
- 10 fit parameters : 2 signal, 2 normalization, 6 background



Number of events

Data/MC



$B^0 \rightarrow D^{(*)}\tau\nu$ $R(D^{(*)})$ Results

First Belle II $R(D^{(*)})$ Results with semileptonic tag

$$\mathcal{R}(D^+) = 0.418 \pm 0.074(\text{stat}) \pm 0.051(\text{syst})$$
$$\mathcal{R}(D^{*+}) = 0.306 \pm 0.034(\text{stat}) \pm 0.018(\text{syst})$$

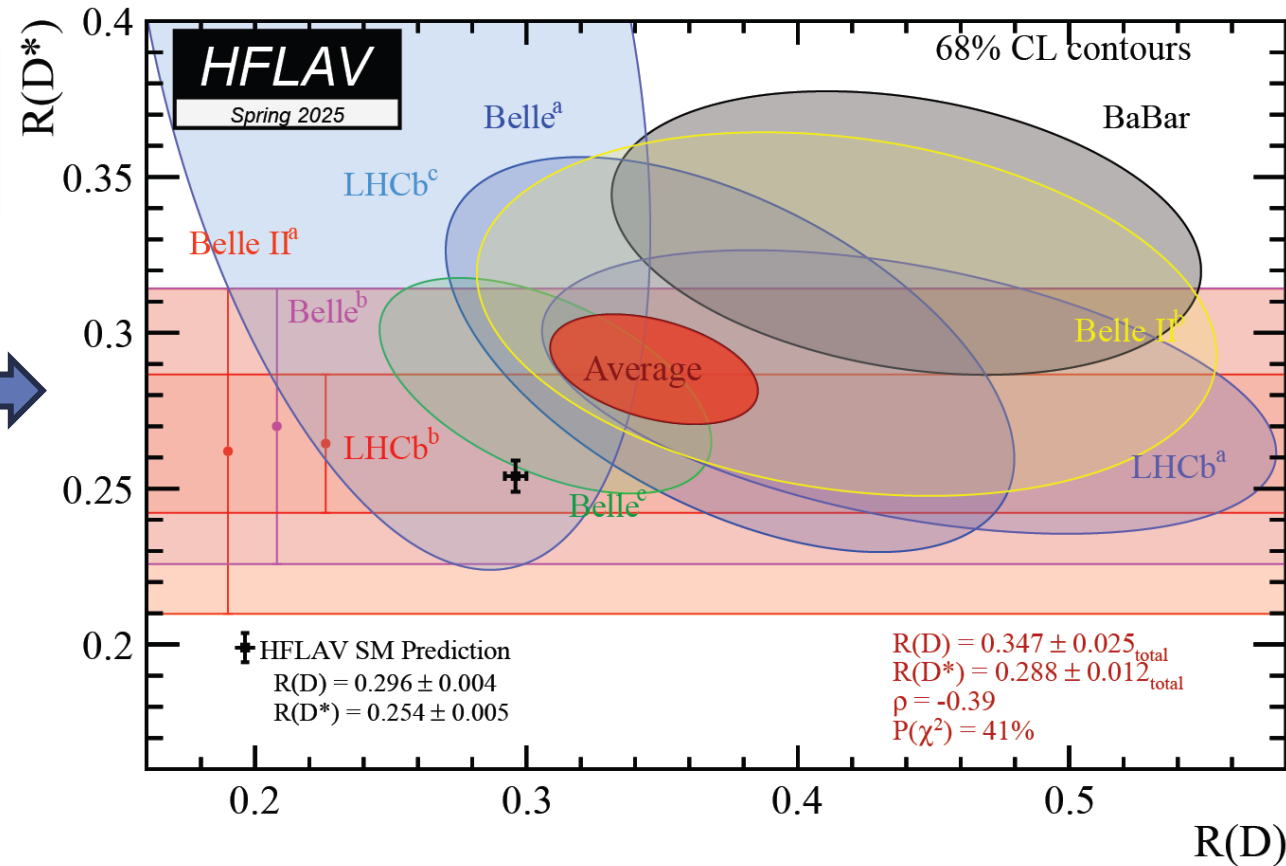
The tension between the $R(D^{(*)})$ measurements and the SM increases from 3.3σ to 3.8σ .



Also measured semi-electric to semi-muonic ratio

$$\mathcal{R}(D_{e/\mu}^+) = 1.07 \pm 0.05(\text{stat}) \pm 0.02(\text{syst})$$
$$\mathcal{R}(D_{e/\mu}^{*+}) = 1.08 \pm 0.04(\text{stat}) \pm 0.02(\text{syst})$$

consistent with 1 in 1.2 and 1.6 σ



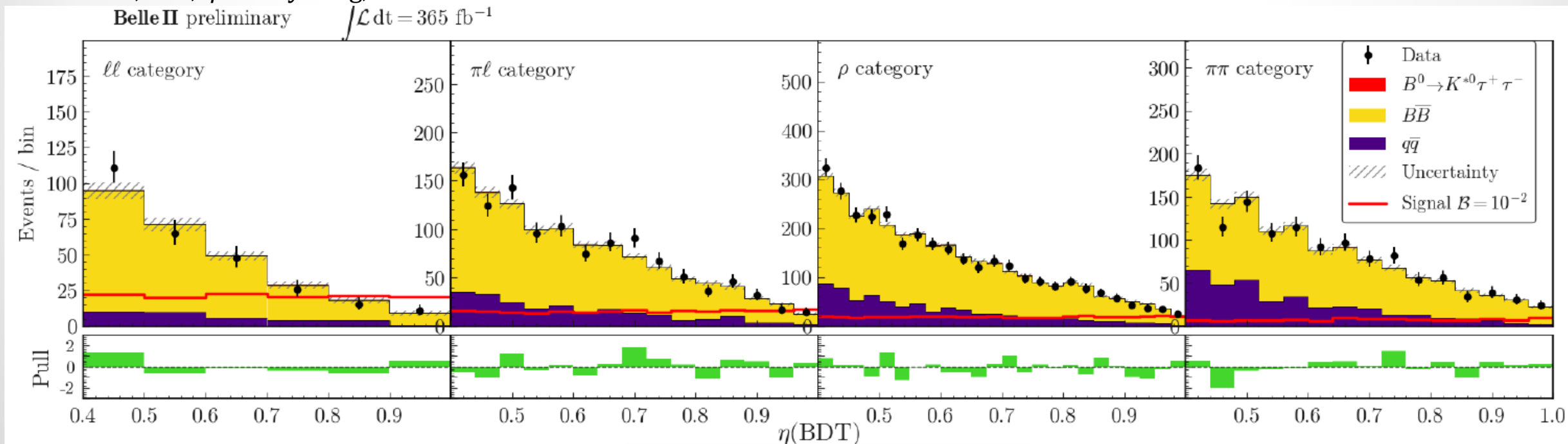
$B^0 \rightarrow K^{*0} \tau \tau$ with Hadronic Tag at Belle II

[[arXiv:2504.10042](#)], submitted to PRL

- $b \rightarrow s$ penguin and box diagrams
- SM Prediction $(0.98 \pm 0.10) \times 10^{-7}$
[[J. L. Hewett, Phys. Rev. D 53, 4964 \(1996\)](#), [B. Capdevila et al., Phys. Rev. Lett. 120, 181802 \(2018\)](#)]
- 365 fb^{-1} Belle II data
- Hadronic FEI Tag
- One prong tau decays: $\tau \rightarrow e \bar{\nu}_e \nu_\tau$, $\mu \bar{\nu}_\mu \nu_\tau$, $\pi \nu_\tau$, $\rho \nu_\tau (\rho \rightarrow \pi \pi^0)$
- $K^{*0} \rightarrow K^+ \pi^-$
- Previous result: Belle 711 fb^{-1} $\mathcal{B}(B^0 \rightarrow K^{*0} \tau \tau) < 3.1 \times 10^{-3}$ (90% C.L.)
[[Phys. Rev. D 108, L011102 \(2023\)](#)]

$B^0 \rightarrow K^{*0} \tau \tau$ Result

- Fit BDT Output to extract signals
 - Event shape variables
 - q^2 , kinematics of K^* and τ candidates
 - Missing energy and momentum, extra energy in calorimeter
- Calibration by control samples such as off-resonance data, $B \rightarrow K^* J/\psi$ events
- Separate events in 4 categories depending $\tau\tau$ daughters
 - $\ell\ell$, $\pi\ell$, ρ + anything, $\pi\pi$



No significant signal $\rightarrow$ set upper limit (90% C.L.)

$$\mathcal{B}(B^0 \rightarrow K^{*0} \tau \tau) < 1.8 \times 10^{-3}$$

$B^0 \rightarrow K_S^0 \tau \ell$ and $B^0 \rightarrow K^{*0} \tau \ell$ with Hadronic Tag at Belle + Belle II

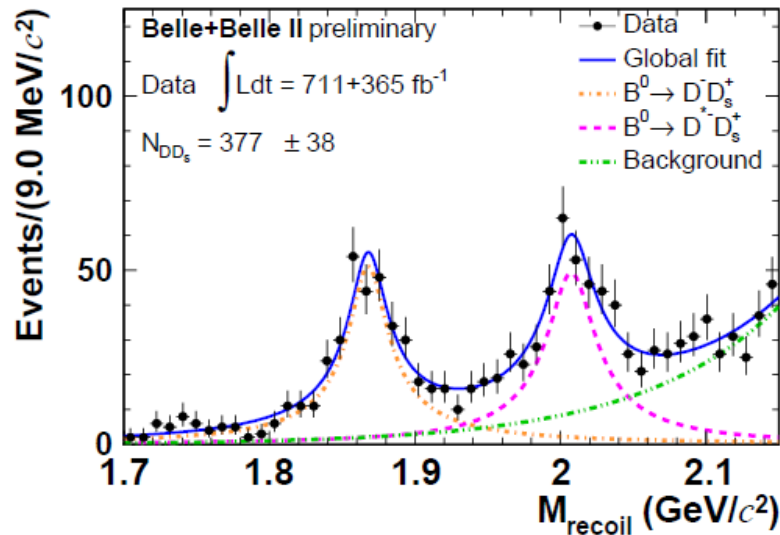
[\[arXiv:2412.16470\]](#) ($K^0 \tau \ell$), accepted by PRL
[\[arXiv:2505.08418\]](#) ($K^{*0} \tau \ell$), accepted by JHEP

- $b \rightarrow s$ lepton flavor violation
- Combine Belle and BelleII Data
 - 711fb⁻¹ Belle
 - 365fb⁻¹ Belle II
- Hadronic FEI Tag
- Tau candidate: require one charged track
 - For $K_S^0 \tau \ell$, $e, \mu, \pi, \rho(\rightarrow \pi\pi^0)$ for one prong τ decays
 - For $K^{*0} \tau \ell$, no explicit particle ID required
- $K_S^0 \rightarrow \pi^+ \pi^-$, $K^{*0} \rightarrow K^+ \pi^-$
- Previously published results:
 - BaBar $B^+ \rightarrow K^+ \tau \ell$ upper limits at $[1.5, 4.5] \times 10^{-5}$
[[PRD 86, 012004 \(2012\)](#)]
 - Belle $B^+ \rightarrow K^+ \tau \ell$
most stringent upper limit for $B^+ \rightarrow K^+ \tau^+ \mu^-$ at 6×10^{-6}
[[PRL 130, 261802 \(2023\)](#)]
 - LHCb $B^+ \rightarrow K^{*0} \tau \mu$ upper limits at $[0.8, 1.0] \times 10^{-5}$
[[JHEP 06 2023 143](#)]
(LHCb reported (preliminary) world best upper limit of $B^0 \rightarrow K^{*0} \tau e$ at Moriond 2025 [[arXiv:2506.15347](#)])

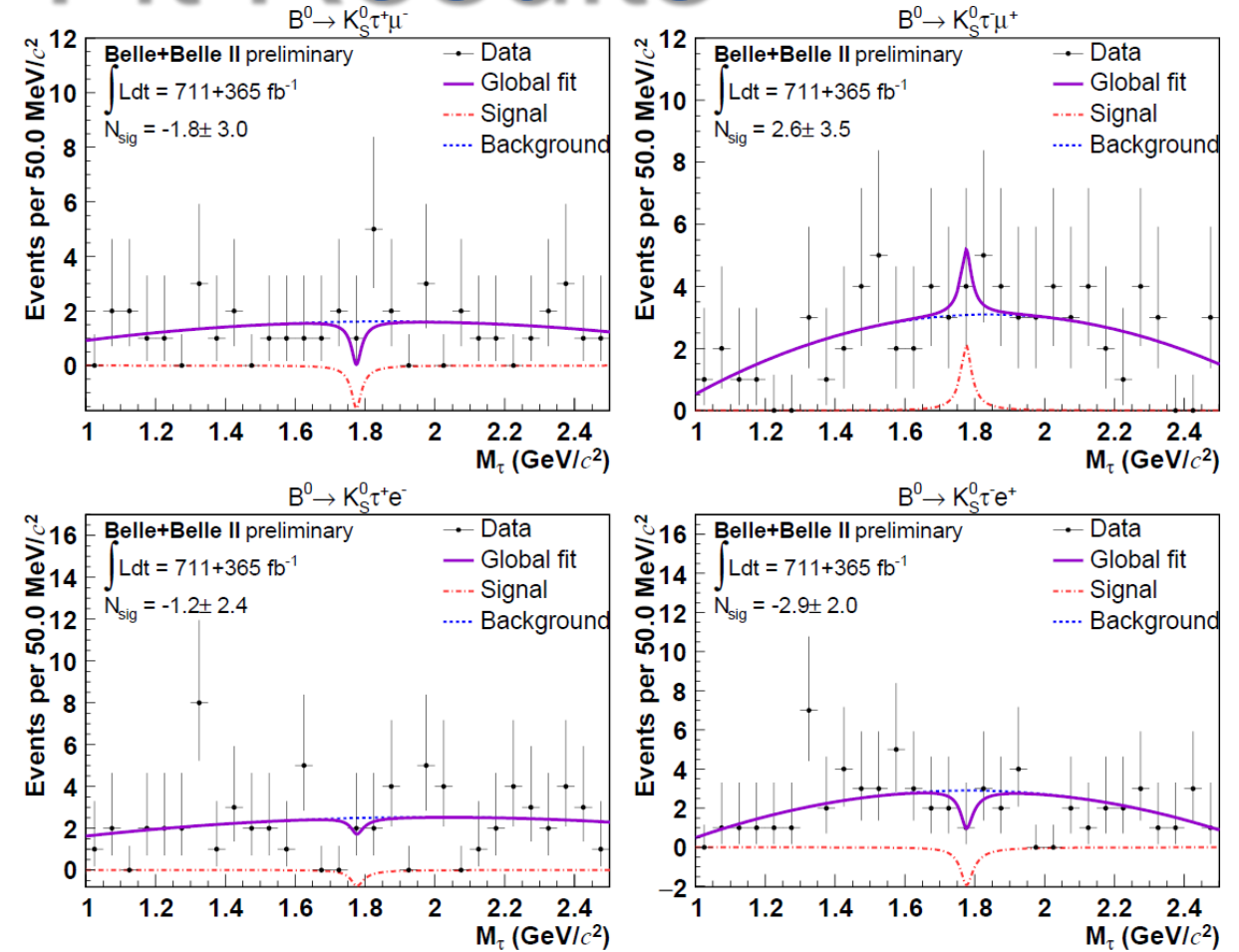
$B^0 \rightarrow K_S^0 \tau \ell$ Fit Results

- Signal is extracted by fit to the recoiling τ mass
- $$M_\tau^2 = (E_{e^+e^-} - p_\ell - p_{K_S^0} - p_{B_{tag}})^2$$
- Calibration by recoiling D mass in $B \rightarrow DD_S$ control sample

$B \rightarrow DD_S$ control sample



No significant signal $\rightarrow$ set upper limits (90% C.L.)

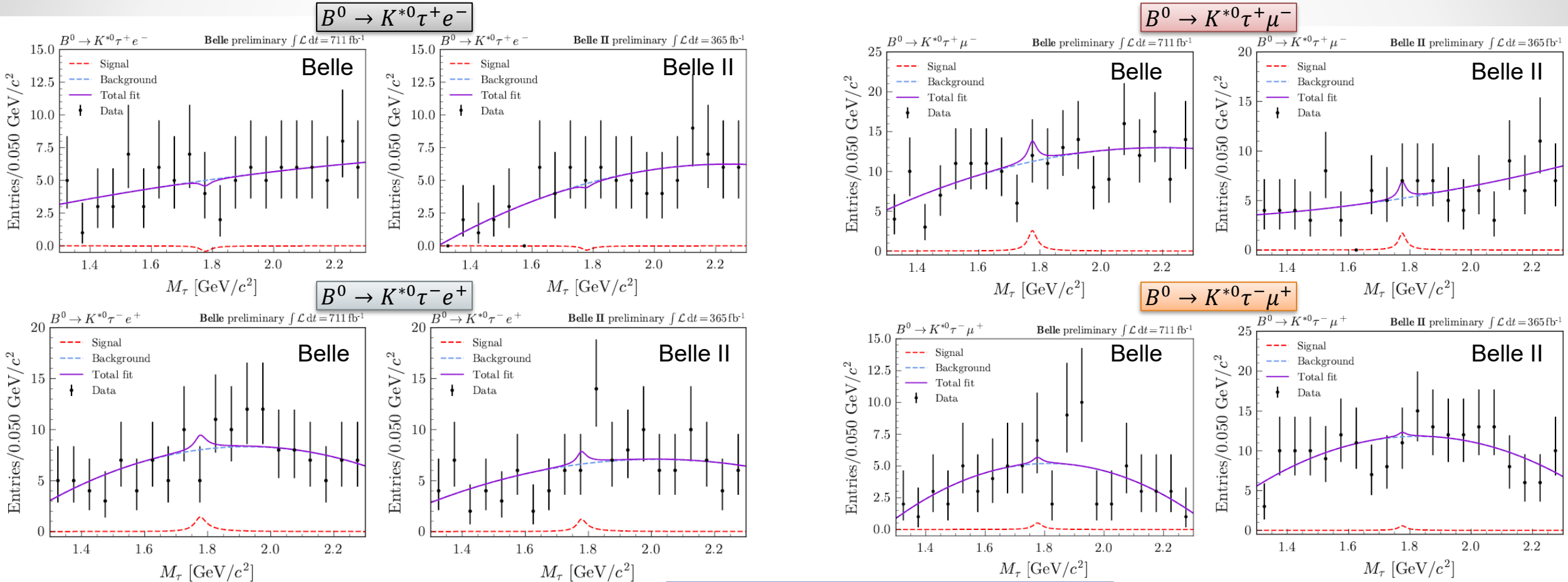


$$\begin{aligned} \mathcal{B}(B^0 \rightarrow K_S^0 \tau^+ \mu^-) &< 1.1 \times 10^{-5} \\ \mathcal{B}(B^0 \rightarrow K_S^0 \tau^- \mu^+) &< 3.6 \times 10^{-5} \\ \mathcal{B}(B^0 \rightarrow K_S^0 \tau^+ e^-) &< 1.5 \times 10^{-5} \\ \mathcal{B}(B^0 \rightarrow K_S^0 \tau^- e^+) &< 0.8 \times 10^{-5} \end{aligned}$$

First in $B^0 \rightarrow K_S \tau \ell$
Best limit for $b \rightarrow s \tau e$

$B^0 \rightarrow K^{*0} \tau \ell$ Fit Results

- Simultaneous fit of Belle and Belle II data



No significant signal $\rightarrow$ set upper limits (90% C.L.)

$$\begin{aligned} \mathcal{B}(B^0 \rightarrow K^{*0} \tau^+ e^-) &< 2.9 \times 10^{-5} \\ \mathcal{B}(B^0 \rightarrow K^{*0} \tau^- e^+) &< 6.4 \times 10^{-5} \\ \mathcal{B}(B^0 \rightarrow K^{*0} \tau^+ \mu^-) &< 4.2 \times 10^{-5} \\ \mathcal{B}(B^0 \rightarrow K^{*0} \tau^- \mu^+) &< 5.6 \times 10^{-5} \end{aligned}$$

**First at B-factory
in $B^0 \rightarrow K^{*0} \tau \ell$**

Search for $B^0 \rightarrow X_s \nu \bar{\nu}$ with Hadronic Tag at Belle II

- Inclusive $b \rightarrow s$ decays $\rightarrow$ Sensitive to the different NP parameters
[[T. Felkl et al., JHEP 12, 118 \(2021\)](#)]
- SM Prediction $(2.9 \pm 0.3) \times 10^{-5}$ [[A. J. Buras et al., JHEP 02, 184 \(2015\)](#)]
- Previous result: ALEPH, $\mathcal{B}(b \rightarrow s \nu \bar{\nu}) < 6.4 \times 10^{-4}$ (90% *C.L.*)
[[Eur. Phys. J. C 19, 213 \(2001\)](#)]
- 365fb⁻¹ Belle II data
- Hadronic FEI Tag

$B^0 \rightarrow X_S \nu \bar{\nu}$ Results

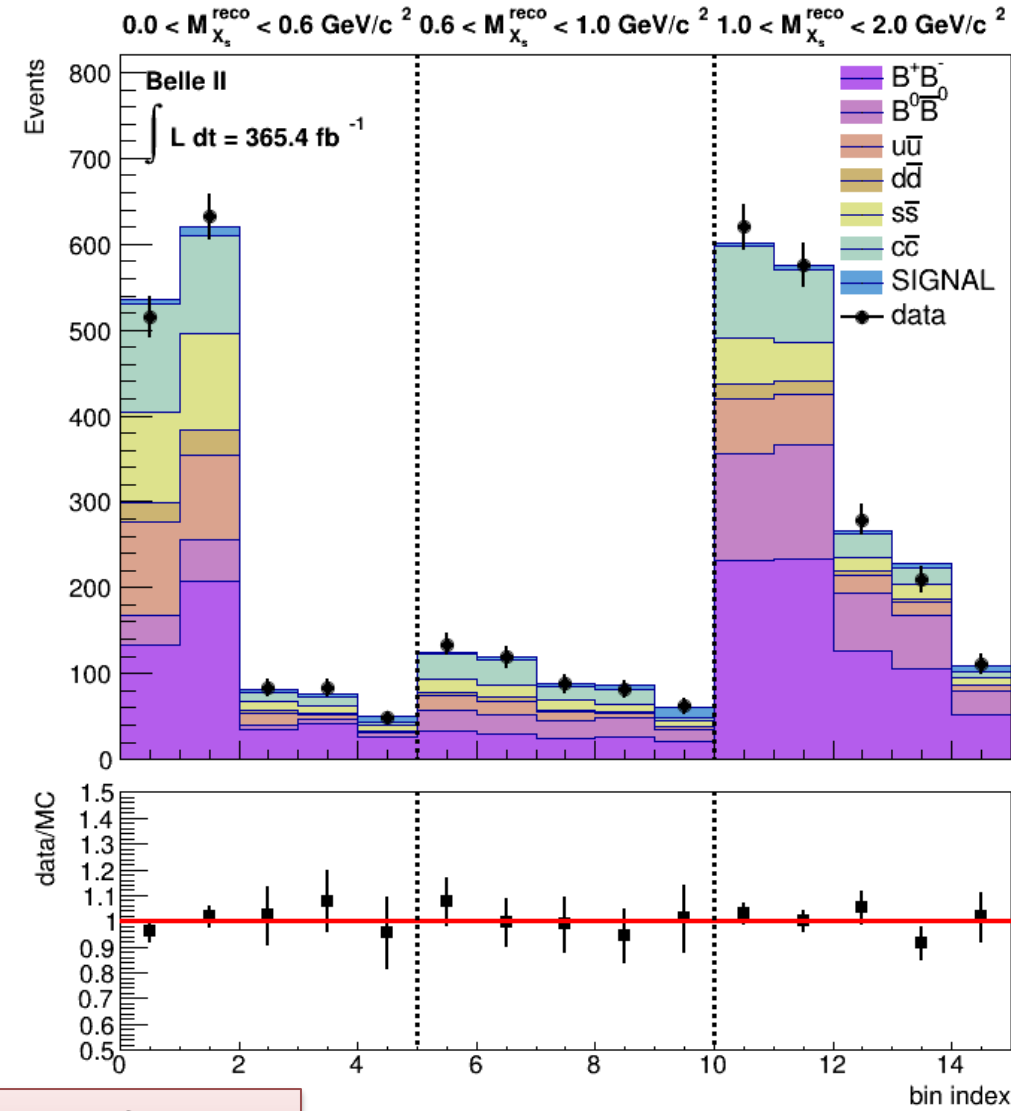
- X_S reconstructed in 30 decay modes:
K, $K\pi$, $K2\pi$, $K3\pi$, $K4\pi$, $3K$, $3K\pi$
covers 93% of the inclusive modes
- Requires no remaining particles
- Background suppression by BDT
- Fit the BDT output in 3 M_{X_S} regions

no significant signal $\rightarrow$ set upper limits (90% C.L.)

$$\mathcal{B}(B^0 \rightarrow X_S \nu \bar{\nu}) < \begin{cases} 2.5 \times 10^{-5} & (0.0 < M_{X_S} < 0.6 \text{ GeV}/c^2) \\ 1.0 \times 10^{-4} & (0.6 < M_{X_S} < 1.0 \text{ GeV}/c^2) \\ 3.5 \times 10^{-4} & (1.0 \text{ GeV}/c^2 < M_{X_S}) \end{cases}$$

For entire M_{X_S} region

$$\mathcal{B}(B^0 \rightarrow X_S \nu \bar{\nu}) < 3.6 \times 10^{-4}$$



World best limit and first at B-factory

Compatible with the hadronic tagged contribution to Belle II $B^+ \rightarrow K^+ \nu \bar{\nu}$

Reinterpretation of $B^+ \rightarrow K^+ \nu \bar{\nu}$

[[arXiv:2507.12393](https://arxiv.org/abs/2507.12393)], submitted to PRD

Belle II Inclusive + Hadronic Tag Analysis Result

$$\mathcal{B}(B^+ \rightarrow K^+ \nu \bar{\nu}) = [2.3 \pm 0.5(\text{stat})_{-0.4}^{+0.5}(\text{syst})] \times 10^{-5}$$

3.5 σ above the bkg-only hypothesis

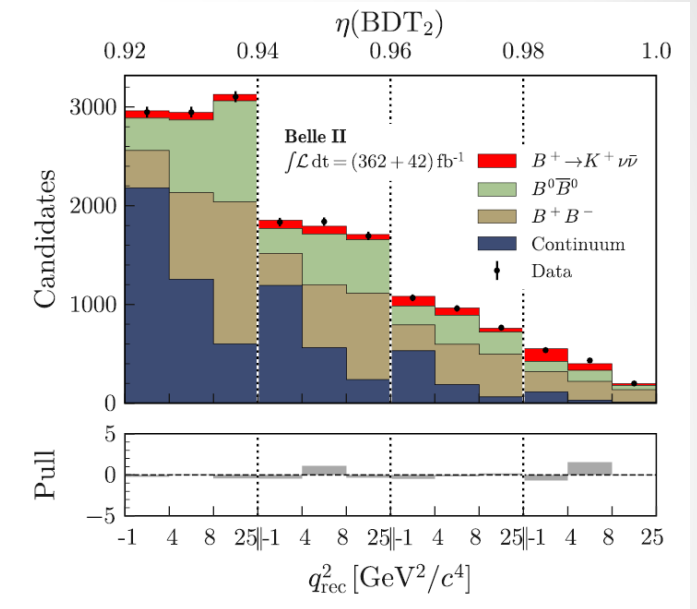
2.7 σ above the SM prediction

SM shape is assumed $\rightarrow$ How to interpret it in the new physics models?

Reinterpretation with model-agnostic likelihoods

based on [[L. Gärtner et al., Eur. Phys. J. C 84, 693 \(2024\)](#)]

- Number density for SM: $n_0(x) = L \int \varepsilon(x|q^2) \sigma_0(q^2) dq^2 \rightarrow \sum_{q^2 \text{ bins}} n_{0,q^2}(x)$,
 - Number density for alternative model: $n_1(x) = \sum_{q^2 \text{ bins}} n_{0,q^2}(x) w(q^2)$, $w(q^2) = \sigma_1(q^2)/\sigma_0(q^2)$
- $$\left\{ \begin{array}{l} x = \text{fitting variable} \\ \varepsilon = \text{efficiency} \\ \sigma_0 = \text{cross section density} \leftarrow \text{Model dependent} \\ L = \text{luminosity} \end{array} \right.$$



Application to $B^+ \rightarrow K^+ \nu \bar{\nu}$

Number density for the alternative model:

$$n_1(x) = \sum_{q^2 \text{ bins}} n_{0,q^2}(x) w(q^2)$$

$$w(q^2) = \sigma_1(q^2) / \sigma_0(q^2)$$

- $n_{0,q^2}(x)$: calculated for the SM

a map for $q^2 \rightarrow$ histogram bins of reconstructed q^2 and $\eta(\text{BDT}_2)$

- Weight $w(q^2)$ calculated for the NP model

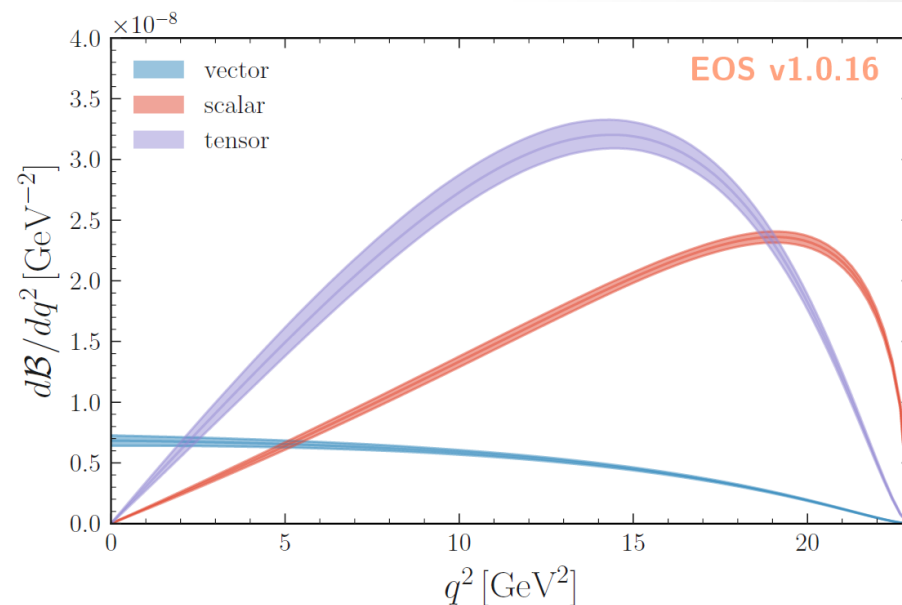
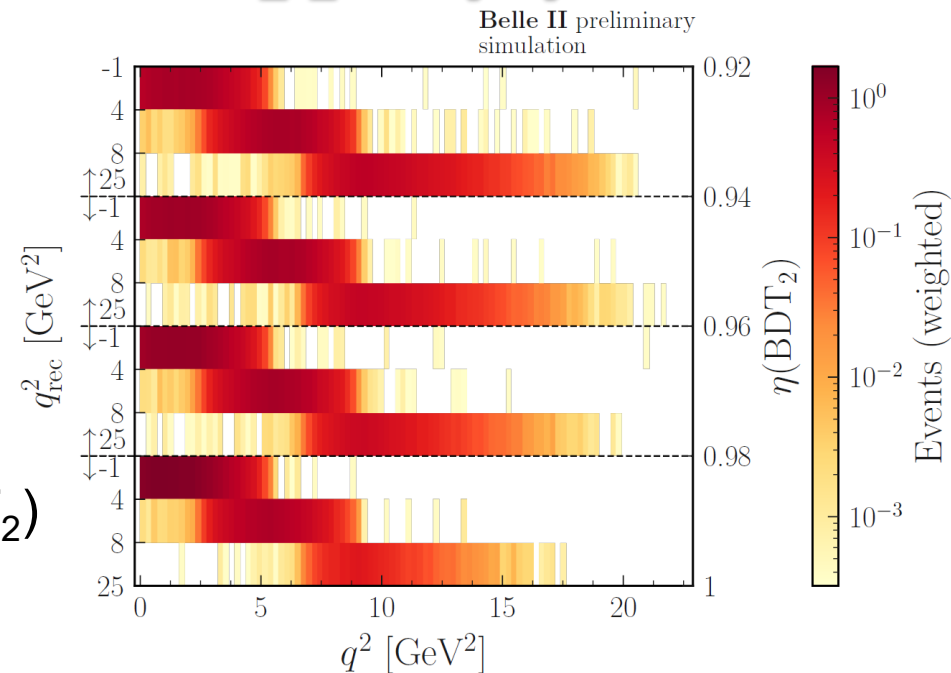
Differential cross section calculated with Weak Effective Theory (WET)

including **vector**, **scalar** and **tensor** contributions

(SM: C_{VL} only)

$$\frac{dB}{dq^2} = \alpha(q^2) |C_{VL} + C_{VR}|^2 + \beta(q^2) |C_{SL} + C_{SR}|^2$$

$$+ \gamma(q^2) |C_{TL}|^2$$



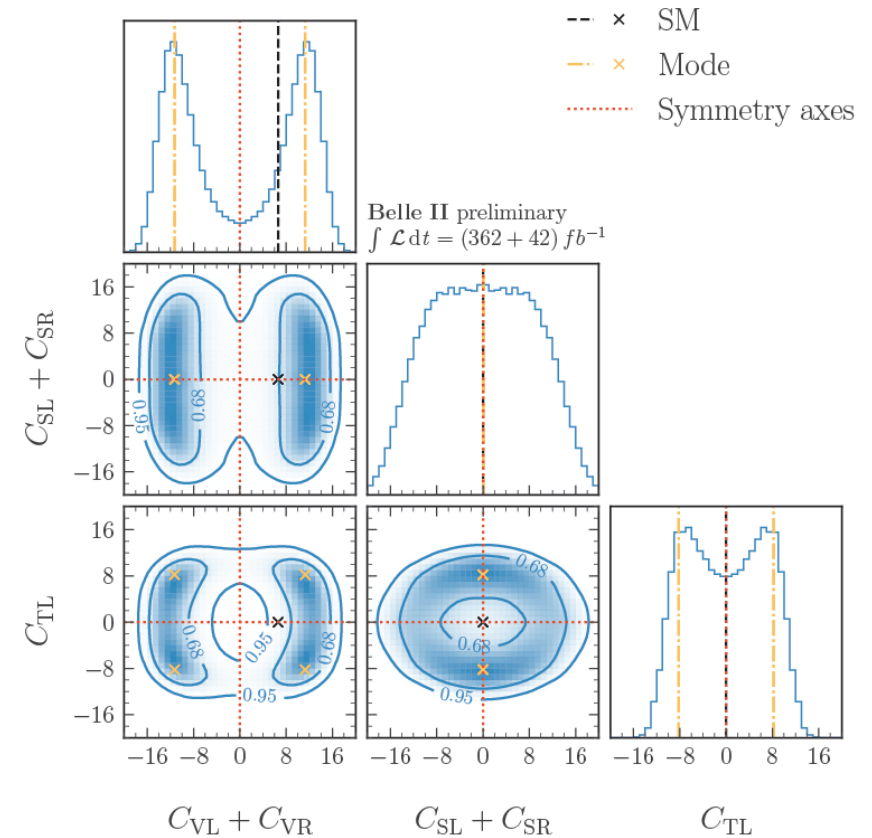
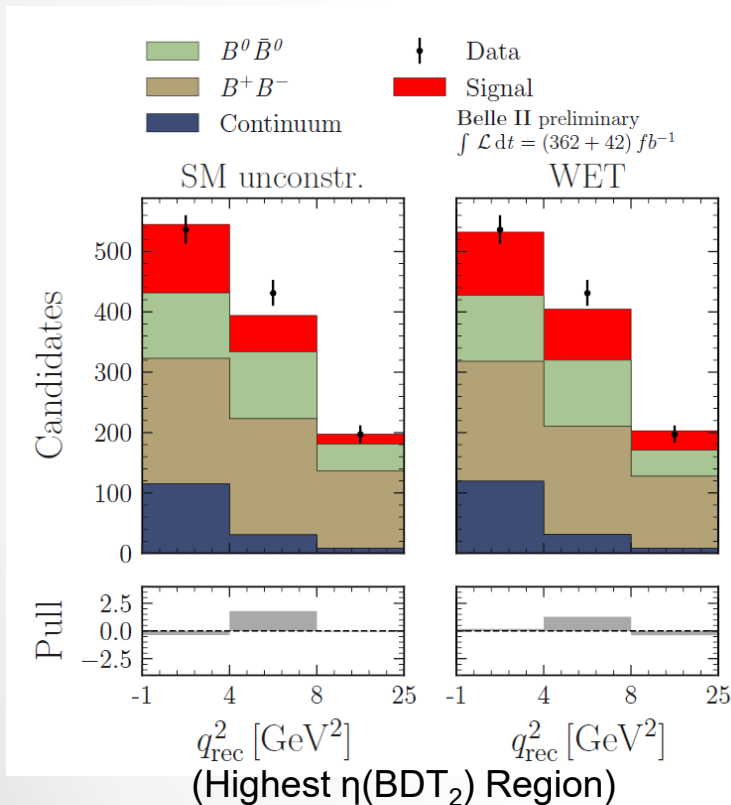
NP Interpretation of $B^+ \rightarrow K^+ \nu \bar{\nu}$

- Fit with the reweighted number density

Three parameters of interest (taken as real) :

$$C_{VL} + C_{VR}, C_{SL} + C_{SR}, C_{TL}$$

- 3.3σ significance v.s. bkg only



Vector + Tensor solution is preferred

Parameters	Mode	68% HDI	95% HDI
$ C_{VL} + C_{VR} $	11.3	[7.82, 14.6]	[1.86, 16.2]
$ C_{SL} + C_{SR} $	0.00	[0.00, 9.58]	[0.00, 15.4]
$ C_{TL} $	8.21	[2.29, 9.62]	[0.00, 11.2]

Summary

- Anomaly in $R(D^*)$ and $R(D)$ and large $B \rightarrow K\nu\bar{\nu}$ signal
→ Correlated NP may be in $B \rightarrow D^{(*)}\tau\nu$ and $b \rightarrow s$ penguin decays

New results from Belle+BelleII with analyses with the improved sensitivity are reported.

- Tension in $R(D^{(*)})$ increased to 3.8σ by adding the BelleII semileptonic tag results
- Searches for $b \rightarrow s$ penguin decays with τ and ν
 - Best upper limit of $B^0 \rightarrow K^*\tau\tau$
 - First search of $B^0 \rightarrow K_S^0\tau\ell$
 - First search at B-factories of $B^0 \rightarrow K^{*0}\tau\ell$
 - Best upper limit of $B^0 \rightarrow X_S\nu\bar{\nu}$
- Reinterpretation of $B^+ \rightarrow K^+\nu\bar{\nu}$ with New Physics by WET calculation is performed
 - Vector + Tensor solution is preferred.
 - The tools for further reinterpretation with other any NP possibilities will be provided.

More results will come with increasing Belle II data

$B^0 \rightarrow D^{(*)} \tau \nu \ R(D^{(*)})$ Syst. Errors

- Multiplicatives are small
→ cancel by taking Ratio
- MC sample size is the largest source
- $B \rightarrow D^{**} \ell \nu$ understanding (written as “Gap B ”) and the semileptonic/tauonic form factors are the next largest

TABLE I. Systematic uncertainties on $\mathcal{R}(D^+)$ and $\mathcal{R}(D^{*+})$ ranked by the magnitude of the uncertainty on $\mathcal{R}(D^+)$. The percentage values in brackets indicate the relative uncertainty.

Systematic Uncertainty	$\Delta\mathcal{R}(D^+)$	$\Delta\mathcal{R}(D^{*+})$
Additive		
MC sample size	0.033 (8.0%)	0.014 (4.7%)
Gap $\mathcal{B}$	0.027 (6.4%)	0.001 (0.1%)
LID efficiency (μ)	0.022 (5.1%)	0.001 (0.1%)
Fake rates (e)	0.012 (2.9%)	0.003 (0.9%)
$\pi^\pm$ from $D^* \rightarrow D\pi$	0.003 (0.7%)	0.001 (0.1%)
Continuum fraction	0.002 (0.6%)	0.001 (0.2%)
$\bar{B} \rightarrow D^{(*)} \ell \bar{\nu}_\ell / \tau \bar{\nu}_\tau$ FFs	0.002 (0.5%)	0.002 (0.7%)
Gap FFs	0.002 (0.5%)	0.001 (0.2%)
$\mathcal{B}(\bar{B} \rightarrow D^{**} \ell \bar{\nu}_\ell)$	0.002 (0.5%)	0.001 (0.1%)
$\bar{B} \rightarrow D^{**} \ell \bar{\nu}_\ell$ FFs	0.001 (0.3%)	0.001 (0.2%)
BDT modeling	0.001 (0.3%)	0.001 (0.2%)
LID efficiency (e)	0.001 (0.1%)	0.001 (0.2%)
Fake rates (μ)	0.001 (0.1%)	0.001 (0.1%)
Total Additive Uncertainty	0.050 (12%)	0.015 (4.8%)
Multiplicative		
$\bar{B} \rightarrow D^{(*)} \ell \bar{\nu}_\ell / \tau \bar{\nu}_\tau$ FFs	0.009 (2.1%)	0.011 (3.5%)
MC sample size	0.007 (1.7%)	0.004 (1.2%)
LID efficiency (e)	0.001 (0.2%)	0.001 (0.2%)
$\mathcal{B}(\tau^- \rightarrow \ell^- \bar{\nu}_\ell \nu_\tau)$	0.001 (0.2%)	0.001 (0.2%)
LID efficiency (μ)	0.001 (0.1%)	0.001 (0.1%)
Tracking efficiency	0.001 (0.1%)	0.001 (0.1%)
$\pi^\pm$ from $D^* \rightarrow D\pi$	– (–)	0.001 (0.2%)
Total Multiplicative Uncertainty	0.012 (2.8%)	0.011 (3.7%)
Total Syst. Uncertainty	0.051 (12%)	0.018 (6.2%)
Total Stat. Uncertainty	0.074 (18%)	0.034 (11%)
Total Uncertainty	0.090 (22%)	0.039 (13%)

$B^0 \rightarrow K^{*0} \tau \tau$ Efficiency and Syst. Errors

Signal efficiencies and expected background yields

Signal category	$\varepsilon \times 10^5$	$B\bar{B}$	$q\bar{q}$
$\ell\ell$	4.0	275	39
$\pi\ell$	7.6	1058	230
ρ	15.5	3279	845
$\pi\pi$	4.0	1077	424

Systematic errors

Source	Impact on $\mathcal{B} \times 10^{-3}$
$B \rightarrow D^{**} \ell / \tau \nu$ branching fractions	0.29
Simulated sample size	0.27
$q\bar{q}$ normalization	0.18
ROE cluster multiplicity	0.17
π and K ID	0.14
B decay branching fraction	0.11
Combinatorial $B\bar{B}$ normalization	0.09
Signal and peaking $B^0 \bar{B}^0$ normalization	0.07
Lepton ID	0.04
π^0 efficiency	0.03
f_{00}	0.01
$N_{\Upsilon(4S)}$	0.01
$D \rightarrow K_L^0$ decays	0.01
Signal form factors	0.01
Luminosity	< 0.01
Total systematics	0.52
Statistics	0.86

$B^0 \rightarrow K_S^0 \tau \ell$ Efficiency and Syst. Errors

TABLE I. Efficiencies (ϵ), signal yields (N_{sig}) of the data fit, central value of the branching fractions and the observed $\mathcal{B}^{\text{UL}}$ at 90% CL. The first uncertainty of the central value is statistical and the second is systematic.

Channels	$\epsilon(10^{-4})$	N_{sig}	$\mathcal{B}(10^{-5})$	
			Central value	UL
$B^0 \rightarrow K_S^0 \tau^+ \mu^-$	1.7	-1.8 ± 3.0	$-1.0 \pm 1.6 \pm 0.2$	1.1
$B^0 \rightarrow K_S^0 \tau^- \mu^+$	2.1	2.6 ± 3.5	$1.1 \pm 1.6 \pm 0.3$	3.6
$B^0 \rightarrow K_S^0 \tau^+ e^-$	2.0	-1.2 ± 2.4	$-0.5 \pm 1.1 \pm 0.1$	1.5
$B^0 \rightarrow K_S^0 \tau^- e^+$	2.1	-2.9 ± 2.0	$-1.2 \pm 0.9 \pm 0.3$	0.8

Systematic errors

- BDT selections 16-18 %
 - Signal PDF 15%
 - B_{tag} efficiency 4%
 - Fitting procedure 0.8-1.6%
 - K_S reconstruction 1.1%
 - PID 0.3-1.0%
 - π^0 reconstruction 1.3%
 - Requirement of no additional π^0 1.0 %
 - $N_{B\bar{B}}$ 1.1%
 - $f_{+/-00}$ 1.5%
 - $\mathcal{B}$ of K_S, τ, ρ, π^0 0.7%
- $B \rightarrow D_S D^-$ sample

$B^0 \rightarrow K^{*0} \tau \ell$ Efficiency and Syst. Errors

B flavor and ℓ charge relations

$$OS\ell : B^0 \rightarrow K^{*0} \tau^+ \ell^-$$

$$SS\ell : B^0 \rightarrow K^{*0} \tau^- \ell^+$$

Signal efficiencies

	OSe	SSe	$OS\mu$	$SS\mu$
Belle	0.046	0.038	0.052	0.024
Belle II	0.075	0.056	0.060	0.051

Systematic errors

Source	Belle				Belle II			
	OSe	SSe	$OS\mu$	$SS\mu$	OSe	SSe	$OS\mu$	$SS\mu$
FEI efficiency [%]	4.9	4.9	4.9	4.9	6.2	6.1	6.1	6.2
Lepton ID efficiency [%]	2.0	2.4	2.2	2.2	0.7	1.1	0.7	0.6
Hadron ID efficiency [%]	1.9	2.0	1.9	2.0	3.7	3.7	3.6	3.7
BDT efficiency [%]	27	21	18	23	29	31	34	31
Tracking efficiency [%]	1.4				1.1			
Total efficiency [%]	27.6	21.8	18.9	23.7	29.8	31.8	34.7	31.7
Signal PDF μ [%]	0.1				0.2			
Signal PDF λ [%]	21				59			
$N_{\Upsilon(4S)}$ [%]	1.4				1.6			
f^{00} [%]	0.8							
Background PDF ($\times 10^{-5}$)	0.11	0.28	0.09	0.02	0.11	0.28	0.09	0.02
Total impact on UL ($\times 10^{-5}$)	0.3	0.9	0.4	0.5	0.3	0.9	0.4	0.5

$B^0 \rightarrow X_s \nu \bar{\nu}$ Efficiency and Syst. Errors

Explicit 30 Decay modes

	$B^0 \bar{B}^0$			$B^\pm$		
K	K_S^0			$K^\pm$		
$K\pi$	$K^\pm \pi^\mp$	$K_S^0 \pi^0$		$K^\pm \pi^0$	$K_S^0 \pi^\pm$	
$K2\pi$	$K^\pm \pi^\mp \pi^0$	$K_S^0 \pi^\pm \pi^\mp$	$K_S^0 \pi^0 \pi^0$	$K^\pm \pi^\mp \pi^\pm$	$K_S^0 \pi^\pm \pi^0$	$K^\pm \pi^0 \pi^0$
$K3\pi$	$K^\pm \pi^\mp \pi^\pm \pi^\mp$	$K_S^0 \pi^\pm \pi^\mp \pi^0$	$K^\pm \pi^\mp \pi^0 \pi^0$	$K^\pm \pi^\mp \pi^\pm \pi^0$	$K_S^0 \pi^\pm \pi^\mp \pi^\pm$	$K_S^0 \pi^\pm \pi^0 \pi^0$
$K4\pi$	$K^\pm \pi^\mp \pi^\pm \pi^\mp \pi^0$	$K_S^0 \pi^\pm \pi^\mp \pi^\pm \pi^\mp$	$K_S^0 \pi^\pm \pi^\mp \pi^0 \pi^0$	$K^\pm \pi^\mp \pi^\pm \pi^\mp \pi^\pm$	$K_S^0 \pi^\pm \pi^\mp \pi^\pm \pi^0$	$K^\pm \pi^\mp \pi^\pm \pi^0 \pi^0$
$3K$	$K^\pm K^\mp K_S^0$			$K^\pm K^\mp K^\pm$		
$3K\pi$	$K^\pm K^\mp K^\pm \pi^\mp$	$K^\pm K^\mp K_S^0 \pi^0$		$K^\pm K^\mp K^\pm \pi^0$	$K_S^0 K^\pm K^\mp \pi^\pm$	

Branching fractions and efficiencies

		$\mathcal{B} [10^{-5}]$				
$M_{X_s} [\text{GeV}/c^2]$	ϵ	N_{sig}	Central value	UL_{obs}	UL_{exp}	
[0, 0.6]	0.25%	10^{+18+18}_{-17-16}	$0.5^{+0.9+0.9}_{-0.8-0.8}$	2.5	2.4	
[0.6, 1.0]	0.11%	36^{+27+31}_{-25-26}	$3.8^{+2.8+3.2}_{-2.6-2.7}$	10.0	7.2	
[1.0, $M_{X_s}^{\text{max}}$)	0.06%	33^{+44+64}_{-42-53}	$7.2^{+9.6+13.9}_{-9.2-11.6}$	35.3	28.3	
Full range	0.11%	80^{+61+93}_{-59-79}	$11.5^{+8.9+13.5}_{-8.5-11.4}$	35.6	27.9	

Systematic errors

Source	Uncertainty [10^{-5}]
MC statistics	+7.0 -5.9
Background normalization	+6.2 -6.1
Branching ratio of major B meson decay	+2.9 -2.1
Fragmentation	+2.7 -1.8
Photon multiplicity correction	+2.5 -1.8
$\mathcal{O}$ selection efficiency	+3.3 -0.9
Non-resonant $X_s \nu \bar{\nu}$ generation point	+3.3 -0.7
Other subdominant contributions	+3.7 -2.7
Total systematic uncertainty	+13.5 -11.4