Symmetry constraints on defect RG flows

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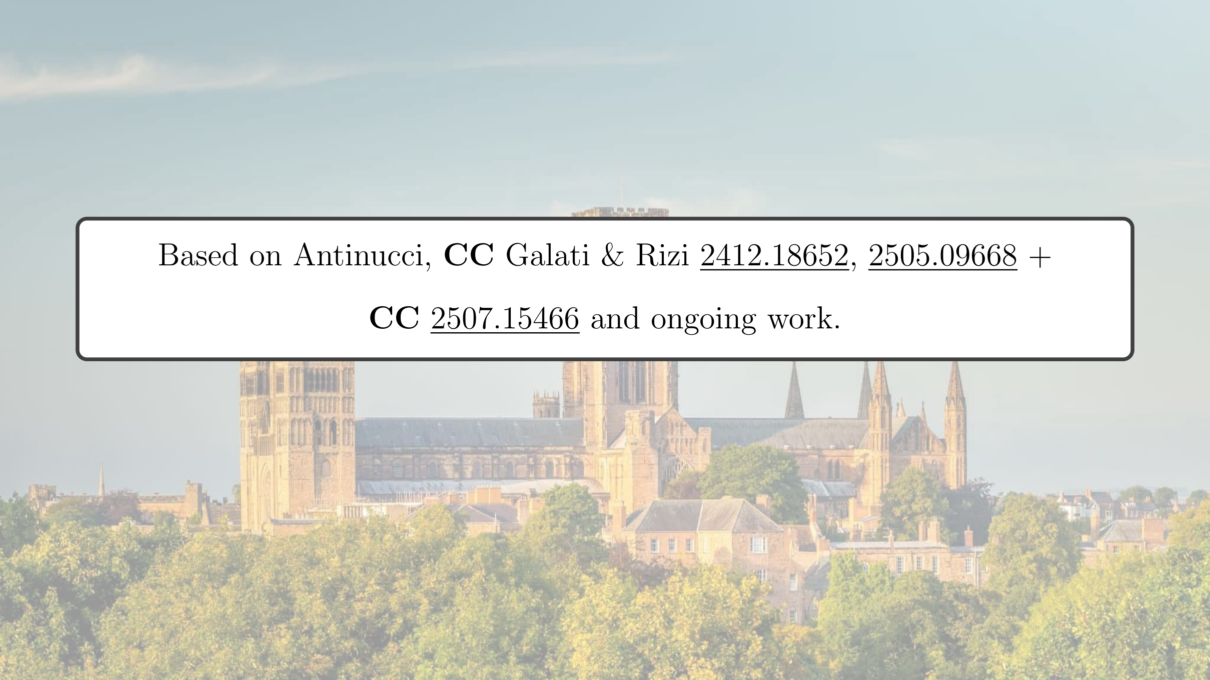
PASCOS 2025
Durham

Science and Technology Facilities Council



Based on Antinucci, **CC** Galati & Rizi 2412.18652, 2505.09668 +

CC 2507.15466 and ongoing work.

Why Defects?

Defects and their RG flows are ubiquitous in Physics:

HEP-TH
- Wilson lines and 't Hooft operators in gauge theories[Polchinski,Sully '11] [Aharony,Cuomo,Komargodski,Mezei,Raviv-Moshe'23] .
- Pinning field defects in $O(N)$ CFT[Cuomo,Komargodski,Mezei '21 + Raviv-Moshe '22] [Raviv-Moshe, Zhong ´23] [Giombi,Liu '23] .
- Domain walls in SSB scenarios.
- Monodromy defects for free theories[Bianchi,Chalabi,Prochazka,Robinson,Sisti '21] [Giombi,Helfenberger,Ji,Khanchandani '21] [Herzog,Shresta '22] .

COND-MAT
- Lattice impurities (Kondo problem)[Anderson'70,Wilson'75,Affleck,Ludwig'90...]
- Disclocations and Disclination [Barkeshli,Fechisin,Komargodski,Zhong '25] .
- Pinning defects in ferromagnets [Assaad,Herbut '13] [Parisen,Assaad,Wessell '16] .

GEN-SYM
- Topological defects describe Generalized Symmetries [Gaiotto,Kapustin,Seiberg,Willet'14] ...

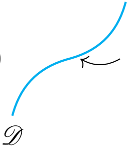
The List goes on...

Window into strongly coupled dynamics (e.g. confinement).

Bulk-defect systems are inherently strongly coupled → few analytic results.

How defects are defined

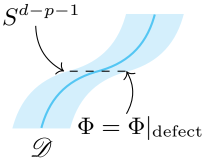
[Electric]: $S_{\text{bulk}} = \int d^d\mathbf{x}\, \mathscr{L}_{\text{bulk}}(\Phi)$



$\int d^p x\, (\, \mathscr{L}(\varphi) + F(\Phi, \varphi))$

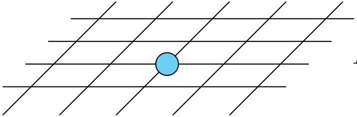
Ex. Wilson lines $\mathscr{D} = P \exp\left(i \int A\right)$, $O(N)$ defect $\mathscr{D} = \exp\left(n_i \int \phi^i\right)$ ...

[Magnetic]:



Ex. 't Hooft (disorder) operators $\frac{1}{2\pi} \int_{S^2} F = 1$.

[Impurities]:



$H = H_0 + H_{\text{impurity}}$

Ex. Kondo problem.

Defect RG flows

We will focus on the IR fate $\mathscr{D}_{IR}$ of a UV defect/impurity.

The following are common scenarios:

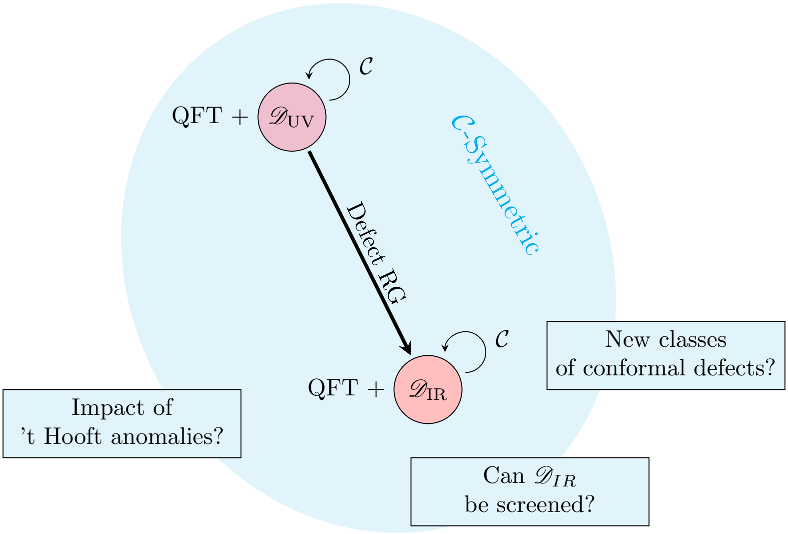
Screening Bulk and defect decouple completely $\mathscr{D}_{IR} = \mathbb{1}_p$.

Conformal $\mathscr{D}_{IR}$ preserves $SO(2,p) \times SO(d-p)$ conformal group with a single vacuum. [Billó,Gonçalves,Lauria,Meineri '16]

Topological $\mathscr{D}_{IR}$ is a nontrivial topological defect in the theory.

> **This Talk:** If the bulk has a symmetry $\mathcal{C}$, does it constrain $\mathscr{D}_{IR}$?

⚠ Common setup: bulk CFT fixed → **Defect RG flow**. Our results hold regardless of this assumption, provided we assume that $\mathcal{C}$ acts **faithfully** along the RG.



Related comments in [CC '24] [Choi,Rayhaun,Zheng '24]

Symmetry & Defects I: Symmetric defects

Consider a bulk system with symmetry G. For concreteness $G = U(1)$.

In the presence of a defect $\mathscr{D}$ the Ward identities for the G current are modified: [Padayasi,Krishnan,Metlitski,Gruzberg,Meineri '21] [Drukker,Kong,Sakkas '22] [Herzog,Schaub '23] [CC,DiPietro,Ji,Komatsu '23] [Cuomo,Zhang '23] :

$$\partial_\mu J^\mu(\mathbf{x}) = t(x)\,\delta(\Sigma_{\mathscr{D}})\,,$$

a nontrivial **tilt operator** $t(x)$ signals symmetry breaking by the defect.

In order for $\mathscr{D}$ to preserve the symmetry we will need the tilt to trivialize:

$$t(x) = \partial_a\, j^a(x)\,, \quad \text{for some defect current } j^a\,.$$

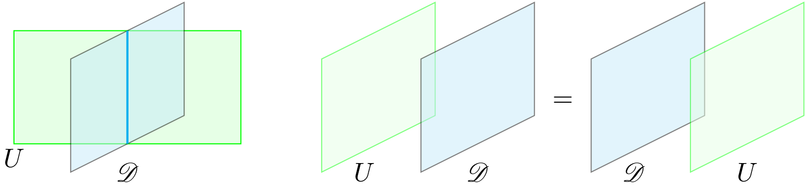
In this case we'll say that $\mathscr{D}$ is **symmetric** wrt G.

A symmetric defects allows for an improvement of the symmetry generator $U_\alpha(Y) = \exp\left(i\alpha \int_Y \star J\right)$:

$$U_\alpha(Y) \longrightarrow U_\alpha(Y)\ \exp\left(-i\alpha \int_{Y\cap\Sigma_p} \star j\right)$$

Such that $U_\alpha(Y)$ remains topological in the presence of $\mathscr{D}$. In other words:

$$U_\alpha\ \mathscr{D} = \mathscr{D}\ U_\alpha$$



We can then carry out many of the familiar hep-th procedures, such as turning on gauge fields for the defect symmetry G.

A defect $\mathscr{D}$ being **symmetric** does not itself give strong constraints on defect RG.

This follows from the fact that the identity $\mathbb{1}_p$ itself is a symmetric defect:

$$\mathbb{1}_p \, U = U \, \mathbb{1}_p \, .$$

To derive constraints we will need to specify more details about the symmetry representation on $\mathscr{D}$. The set of representations can be derived from the correct generalization of the Strip Algebra [Kong,Kitaev '11] [C. Cordova,Garcia-Sepulveda,Holfester,Ohmori'24] [CC,L. Cordova,Komatsu '24] [Gagliano,Grigoletto,Ohmori'25] . Connects to the idea of generalized charges/ higher tube algebras [Bhardwaj,Schafer-Nameki '23] [Bartsch,Bullimore,Grigoletto'23] .

These representations have very interesting features, such as non-trivial **defect anomalies**: 't Hooft anomalies localized on the defect's worldvolume.

We will now focus on a specific (but interesting) example.

Symmetry & Defects II: Symmetry-Reflecting defects

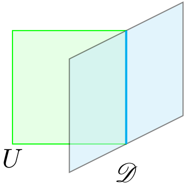
A natural generalization of this concept is what we call **symmetry reflecting defects**:[1]

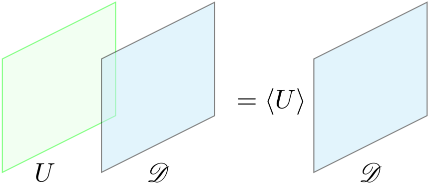
$$U\,\mathscr{D} = \mathscr{D}\,U = \langle U\rangle\,\mathscr{D}\,.$$

The symmetry defects are **absorbed** by $\mathscr{D}$. For concreteness we focus on $p = d - 1$. In terms of the current J it means that, on the defect's worldvolume:

$$J_\perp(x) = \partial_a\,\eta^a(x)\,.$$

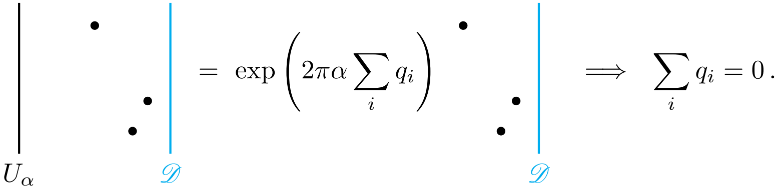
Alternatively, the topological operator U can terminate **topologically** on $\mathscr{D}$.



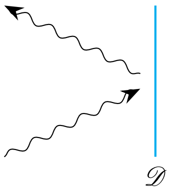


[1]A similar concept for boundary conditions appeared in [Choi,Rayhaun,Sanghavi,Shao '23]

A symmetry reflecting interface preserves the G symmetry **independently** on the two sides. The total symmetry in this case is **at least** $G_L \times G_R$.



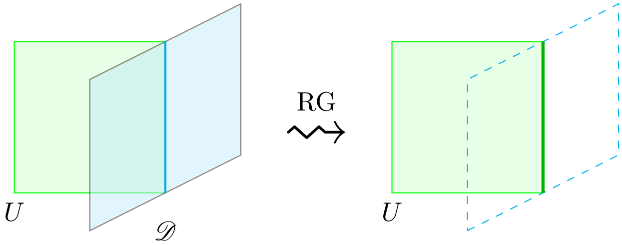
All of the symmetry charge scattering on $\mathscr{D}$ is thus reflected back. $\mathscr{D}$ acts as an **hard wall** for charged objects.



Similar ideas can be formulated using the Defect OPE of charged bulk fields.

Consequences

A symmetry reflecting defect cannot be screened in the IR.



The IR fixed point can either be:

- A nontrivial symmetry reflecting conformal defect.
- A nontrivial (and non-invertible) topological defect.
- A theory of Defect Goldstone modes.

Construction I: Coupling via an 't Hooft anomaly

Let $G = U(1)$. We define a $p = 2$ $\mathscr{D}_{UV}$ by electrically coupling $U(1)$ symmetric matter, whose $U(1)$ symmetry has an 't Hooft anomaly:

$$\partial_a \eta^a = \frac{1}{2\pi} F \,.$$

Setting $F = 2\pi J_\perp$ defines a UV symmetry reflecting defect.

Example: compact scalar $\theta \sim \theta + 2\pi$ on the defect, axion coupling:

$$S_{\text{int.}} = i \int d^2x \, \theta J_\perp \,.$$

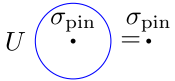
This allows to construct interesting un-screenable defects in (2+1)d and (3+1)d gauge theories (with matter) with a $U(1)_T$ symmetry:

$$\star J = \frac{1}{2\pi} da \,.$$

Construction II: Deforming Topological defects

A wide class of conformal defects are obtained by the “pinning field” construction:

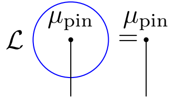
$$\mathscr{D} = \mathbb{1}_p + \lambda \int d^p x \; \sigma_{\text{pin}} \,, \qquad \Delta(\sigma_{\text{pin}}) < p \,.$$

These defects are symmetric if  .

A symmetry reflecting defect can be constructed in a similar manner by deforming a topological defect $\mathscr{N}$ (related ideas [Kormos,Runkel,Watts ’09] [Makabe, Watts ’17]):

$$\mathcal{L}\,\mathscr{N} = \mathscr{N}\,\mathcal{L} = d_{\mathcal{L}}\,\mathscr{N}$$

$$\mathscr{D} = \mathscr{N} + \lambda \int d^p x \, \mu_{\text{pin}} \,,$$

 . $\quad \forall\ \mathcal{L} \in \mathcal{C} \,.$

Interestingly, μ_{pin} can be a **nonlocal (twisted)** operator living at the end of an $\mathcal{L}$ line.

Example:

$$\text{(1+1)d Ising CFT}\,,\;\; G=\mathbb{Z}_2: \qquad \mathscr{D} = \Big|_{\,{\color{blue}1+\eta}} + \lambda \int dx\, \mu_{\frac{1}{16},\frac{1}{16}}\,.$$

The flow can be "bootstrapped" exactly:

$$\underset{\text{KW duality}}{\mathcal{N}} \times \mathscr{D} = \left(\Big\vdots_{\,\mathbb{1}_p} + \lambda \int dx\; \sigma_{\frac{1}{16},\frac{1}{16}} \;\oplus\; \Big\vdots_{\,\mathbb{1}_p} - \lambda \int dx\; \sigma_{\frac{1}{16},\frac{1}{16}} \right) \times \mathcal{N}\,.$$

The term in () brackets flows to $|+\rangle\langle+| \oplus |-\rangle\langle-|$ where $\{|+\rangle, |-\rangle, |f\rangle\}$ are the Cardy states for Ising. Using $\mathcal{N}|\pm\rangle = |f\rangle$, $\mathcal{N}|f\rangle = |+\rangle + |-\rangle$, we conclude that:

$$\mathscr{D}_{IR} = |f\rangle\langle f|\,.$$

Symmetry and Defects III: Symmetry Breaking and Modulation

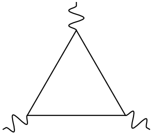
We now consider symmetry-breaking defects, in which case $t(x)$ is nontrivial.

Breaking a continuous symmetry G defines a family of defect $\mathscr{D}_\sigma$ by the deformation:

$$i \int_{\mathscr{D}} \operatorname{Tr} \sigma\, t(x)\,, \quad g \in G = e^{i\sigma}\,.$$

For boundary conditions $\mathscr{B}$, if G suffers from an 't Hooft anomaly (e.g. the $SU(N_f)$ symmetry of N_f Weyl fermions),

$$Z[A + d_A \lambda] = e^{i \int \omega(A,\lambda)} Z[A]\,,$$



then $\mathscr{B} = \mathscr{B}_\sigma$ must break the symmetry.

We call this breaking **anomaly-enforced** [CC '25] , see also [Choi et al '25]

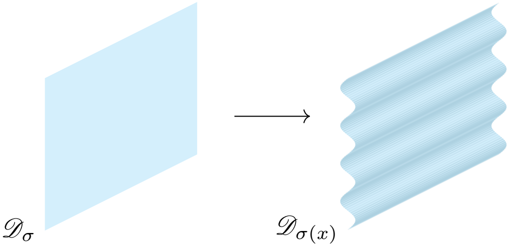
A natural question is whether an anomaly-enforced breaking fundamentally differs from a vanilla one.

To answer this we would like to couple the bulk + defect system to a gauge field A.

Naively this is not possible, as $A \to A + d_A\lambda$ gives rise to a boundary term

$$i \int_{\mathscr{B}} \mathrm{Tr}\, \lambda(x) t(x)\,.$$

This can be circumvented provided we consider coupling the defect to a **modulated** coupling $\sigma(x)$.



The bulk + boundary system can be made gauge invariant by a non-linear transformation for σ:

$$A \to A + d\lambda\,, \quad \sigma \to \sigma - \lambda\,.$$

The defect free-energy now depends on A, σ and the invariant combination $\omega_A = g^{-1}(d + A)g \quad .$

For anomalous symmetries in the presence of a boundary, the Wess-Zumino consistency condition is violated by a boundary term:

$$\delta_{\lambda_1}\omega(A,\lambda_2) - \delta_{\lambda_2}\omega(A,\lambda_1) - \omega(A,[\lambda_1,\lambda_2]) = d\beta(\lambda_1,\lambda_2,A)\,.$$

The presence of a boundary term forces the symmetry breaking [Jensen, Yarom '19] .

However, for modulated defects, β can be cancelled by the modulated free energy $F_{\mathscr{B}}$:

$$\beta(\lambda_1,\lambda_2,A) = \delta_{\lambda_1}F_{\mathscr{B}}(\lambda_2,A) - \delta_{\lambda_2}F_{\mathscr{B}}(\lambda_1,A)\,.$$

This fixes universal, anomaly-induced terms in $F_{\mathscr{B}}$.

Boundary Transport and SPT pumping

Consider $G = U(1)$ the anomalies:

$$\omega_{(1+1)} = \frac{\chi_{(1+1)}}{2\pi} \int d\lambda\, A\,, \qquad \omega_{(3+1)} = \frac{\chi_{(3+1)}}{24\pi^2} \int d\lambda A\, dA\,,$$

Fix:

$$F_{\mathscr{B},(1+1)} = \frac{\chi_{(1+1)}}{2\pi} \int \sigma(A + d\sigma) + \ldots\,, \quad F_{\mathscr{B},(3+1)} = \frac{\chi_{(3+1)}}{24\pi^2} \int \sigma(A + d\sigma) dA + \ldots$$

Give rise to the following (Hall) boundary currents:

$$Q_{\mathscr{B})} = \chi_{(1+1)} \frac{\sigma}{2\pi}\,, \qquad \mathcal{J}^i_{\mathscr{B}} = \chi_{(3+1)} \frac{\sigma}{8\pi^2} \epsilon^{ijk}\, F_{jk}\,.$$

As we wind around the circle $\sigma \to \sigma + 2\pi$ charge is deposited on the boundary.

This is a Thouless-pump phenomenon and correspond to the stacking of $U(1)$ SPTs

$$i\chi_{(1+1)} \int_{\mathscr{B}} A\,, \qquad\qquad i\frac{\chi}{4\pi}\int_{\mathscr{B}} AdA\,,$$

Which describe Integer Quantum Hall states in (0+1) and (2+1) dimensions.

This shows a deep interplay between bulk 't Hooft anomalies and the topology of families of defects related by anomaly-enforced symmetry breaking.

Thank you!

- How do symmetry-refined versions of defect entropy interplay with the possible representations of symmetry on $\mathscr{D}$? ([Karch,Kusuki,Ooguri,Sun,Wang '23] for recent studies of defect entropy and [Choi, Rayhaun, Zheng '24] [Heymann,Quella '24] [Kusuki,Murciano,Ooguri,Pal '24] [Bastida,Das,Sierra,Molina-Vilaplana '24] symmetry resolved entropy)

- Does (generalized) symmetry allow to constrain/bootstrap defect fusion rules? [Bachas,Brunner '07] [Konechny '15] [Soderberg '21] [Diatlyk,Khanchandani,Popov,Wang '24] [Kravchuck,Radcliffe,Sinha '24] .

- "Anomalies in the space of couplings" [Cordova,Freed,Lam,Seiberg '19] for defect RG flows? Relation with [Debray,Devalapurkar,Krulewski,Liu,Pacheco-Tallaj,Thorngren '23] ?

- Application to lattice impurities (generalized Kondo)? How is the representation of symmetry on the defect encoded in the lattice formulation?