Non-Invertible Symmetries and Gapped Phases of Four Dimensional Quantum Fields

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UPPSALA UNIVERSITET

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SCGCS

Vetenskapsrådet

European Research Council
Established by the European Commission

Based on Joint works and discussions with

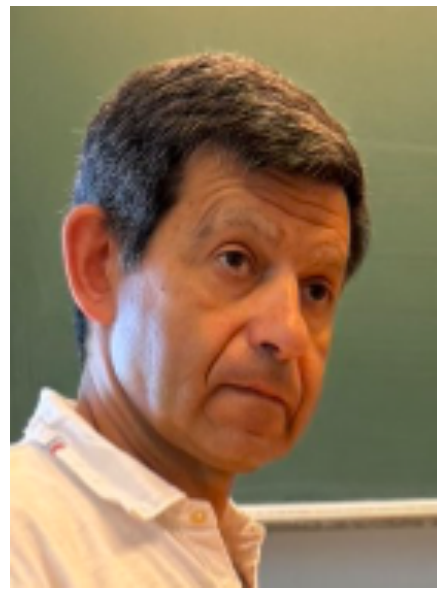
Elias Riedel Gårding

Azeem Hasan

Kantaro Ohmori

Federico Bonetti

Ruben Minasian



DZ, Riedel Gårding, Hasan 25

DZ, Ohmori *in preparation*

Bonetti, DZ, Minasian *in preparation*

Disclaimer

No references in this talk.

Apologies to the many experts in the audience!

For references I direct you to the papers (or feel free to ask me later)

Symmetries and RG flow

Key question in the theory of quantum fields: establish relations between microscopic and macroscopic physics

Microscopics — High Energies

(e.g. quarks: “easy” to compute)

Renormalization flow

Constraints?

Macroscopics — Low Energies

(e.g. hadrons: “easy” to measure)

Renormalization is (in principle) determined but also **very hard** to compute

$\longrightarrow$ **Symmetries:** one of the few known tools to constrain renormalization

TODAY’S TALK: implications of generalized (non-invertible) symmetries to constrain low energy **gapped** phases of 4d QFTs

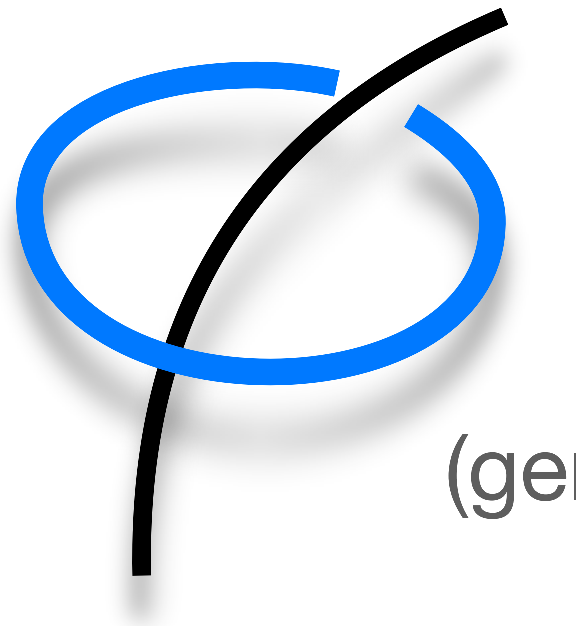
Generalized Symmetries

FACT: quantum fields have extended operators of various dimensions

e.g. Wilson lines, Vortex strings, etc..

Generalized symmetries introduced to capture corresponding quantum #s

p-form symmetry $\longleftrightarrow$ **charge** of operator of dimension p



linking action

(generalized Gauss's law)

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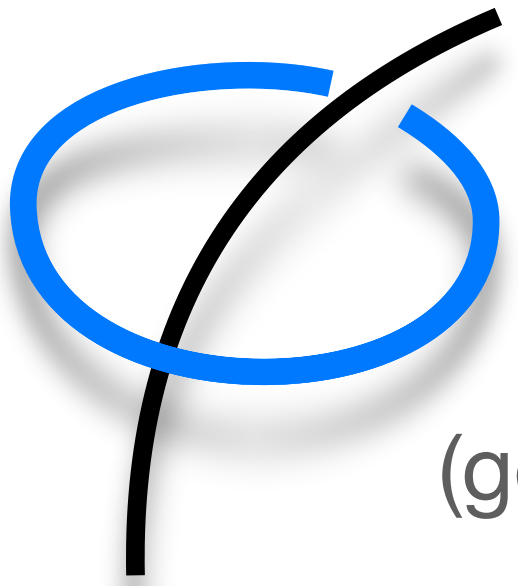

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(generalized Gauss's law)

Consider continuous case

$$\partial_\mu J^{[\mu\nu_1..\nu_p]} = 0 \quad \longrightarrow \quad exp\left(i\theta \int_{S^{d-p-1}} J^{[\mu\nu_1..\nu_p]} \epsilon_{\mu\nu_1\ldots\nu_p\nu_{p+2}\ldots\nu_d} dS^{\nu_{p+2}\ldots\nu_d} \right) \longrightarrow \quad U(1)^{(p)}$$

Eg. Maxwell field:

$$\partial_\mu F^{[\mu\nu]} = 0 \quad \& \quad \partial_\mu \tilde{F}^{[\mu\nu]} = 0 \quad \longrightarrow \quad U(1)_e^{(1)} \times U(1)_m^{(1)}$$

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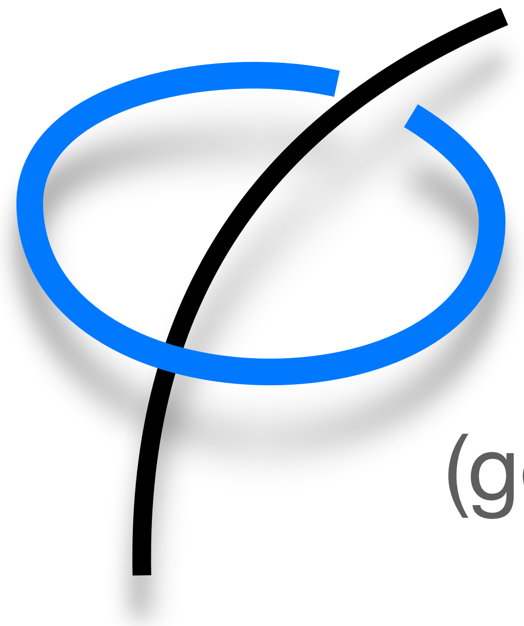

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Are **examples** of topological operators of **codimension d-p-1**

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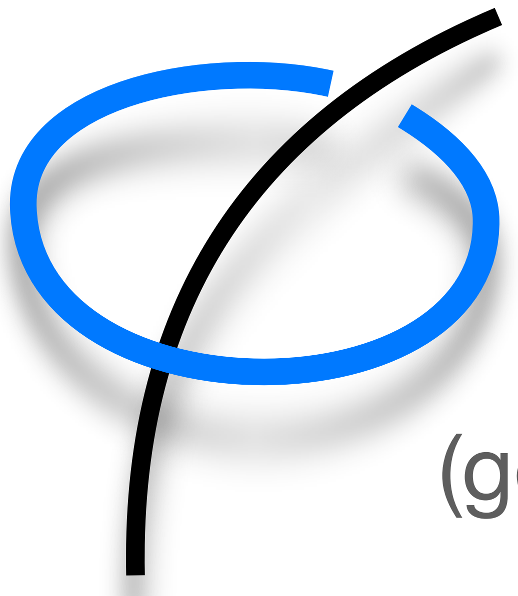

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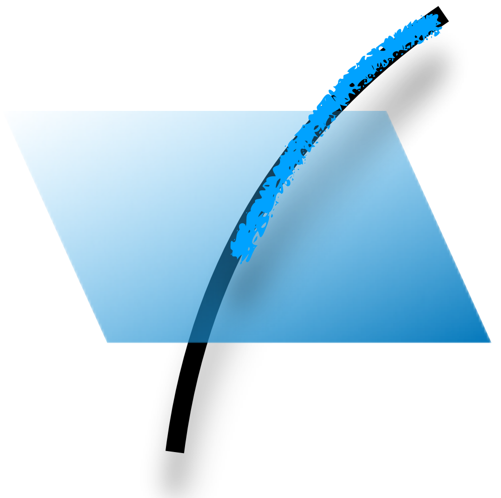
(generalized Gauss's law)

Are **examples** of topological operators of **codimension d-p-1**

- More quantum #s

e.g. crossing action…



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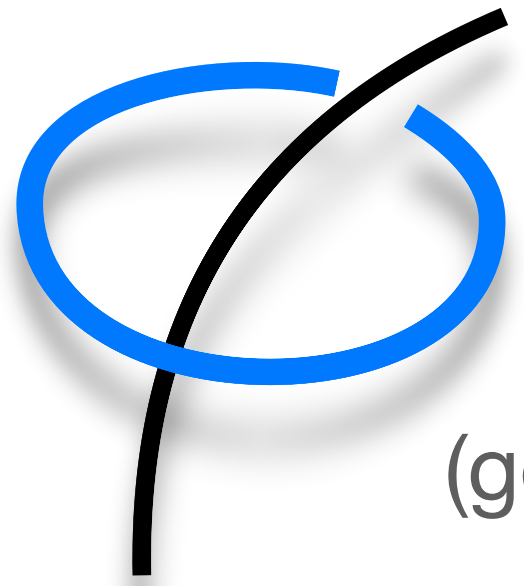

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- More quantum #s
- Higher structure

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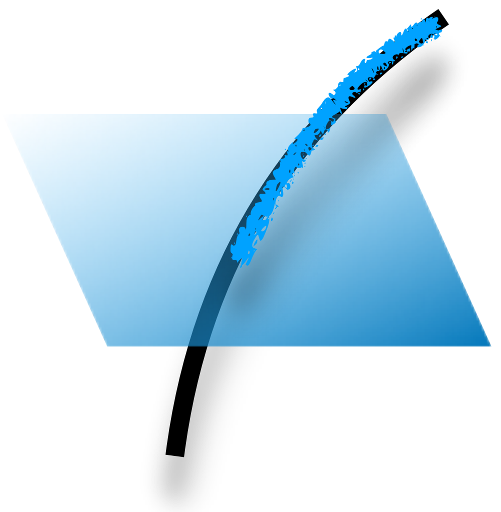
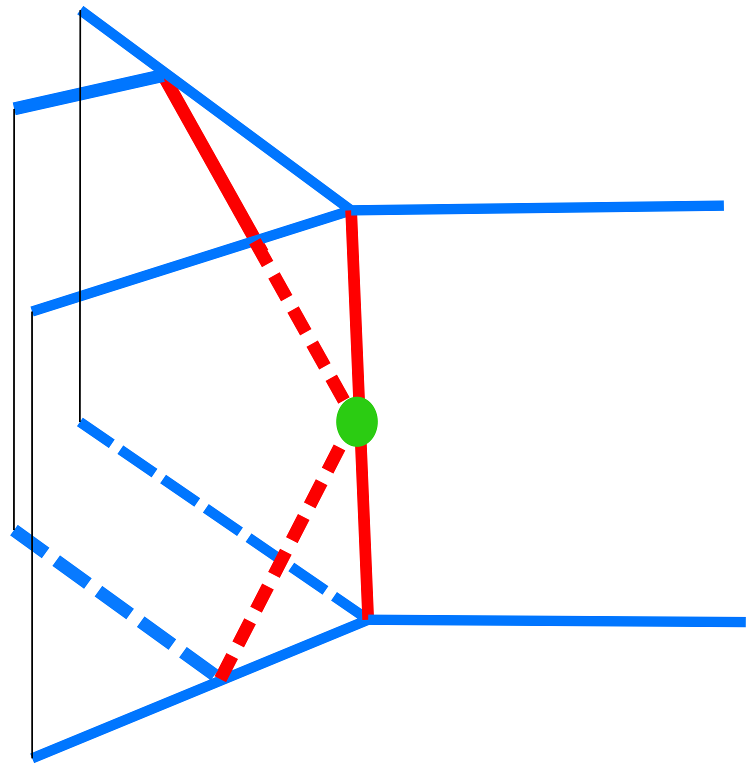


Extended topological operators can form intricated networks giving rise to further topological defects



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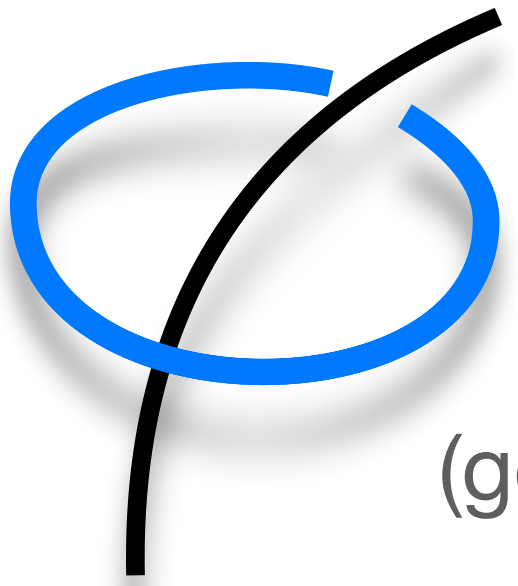

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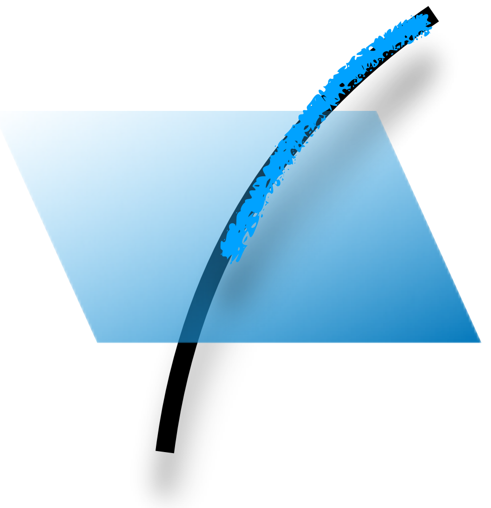

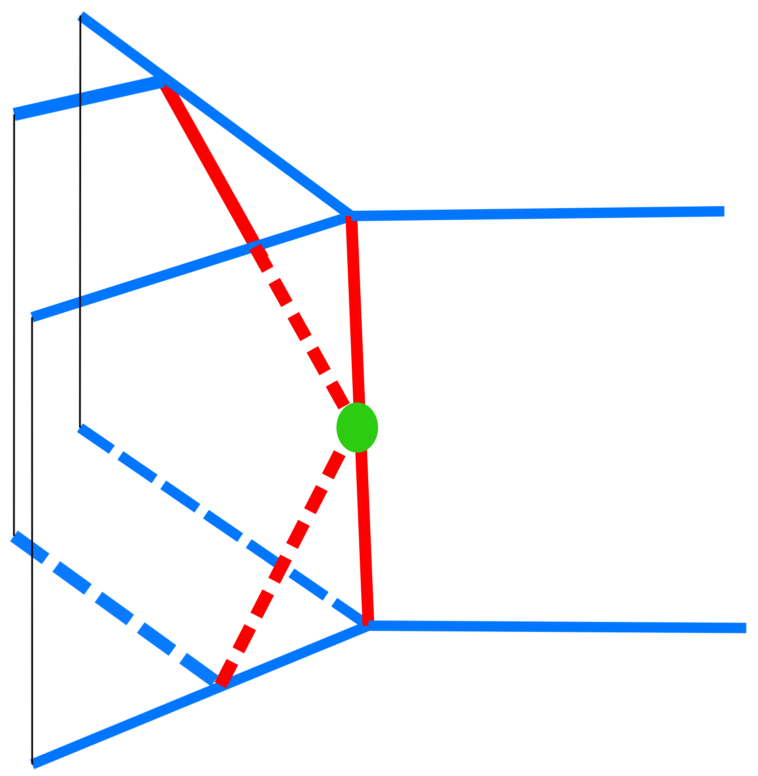

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Claim:

Topological operators/defects

define

Generalized symmetries

Topological Operators and Defects

TASK: characterize the subsector of the QFT of interest consisting of topological operators and defects and study their higher structure and action on the non-topological operators and defects

Two approaches:

- **Worldvolume approach**

  Compute explicitly the topological operators that one can couple to the theory

- **Bulk/boundary system**

  Topological Symmetry Theory or SymTFT

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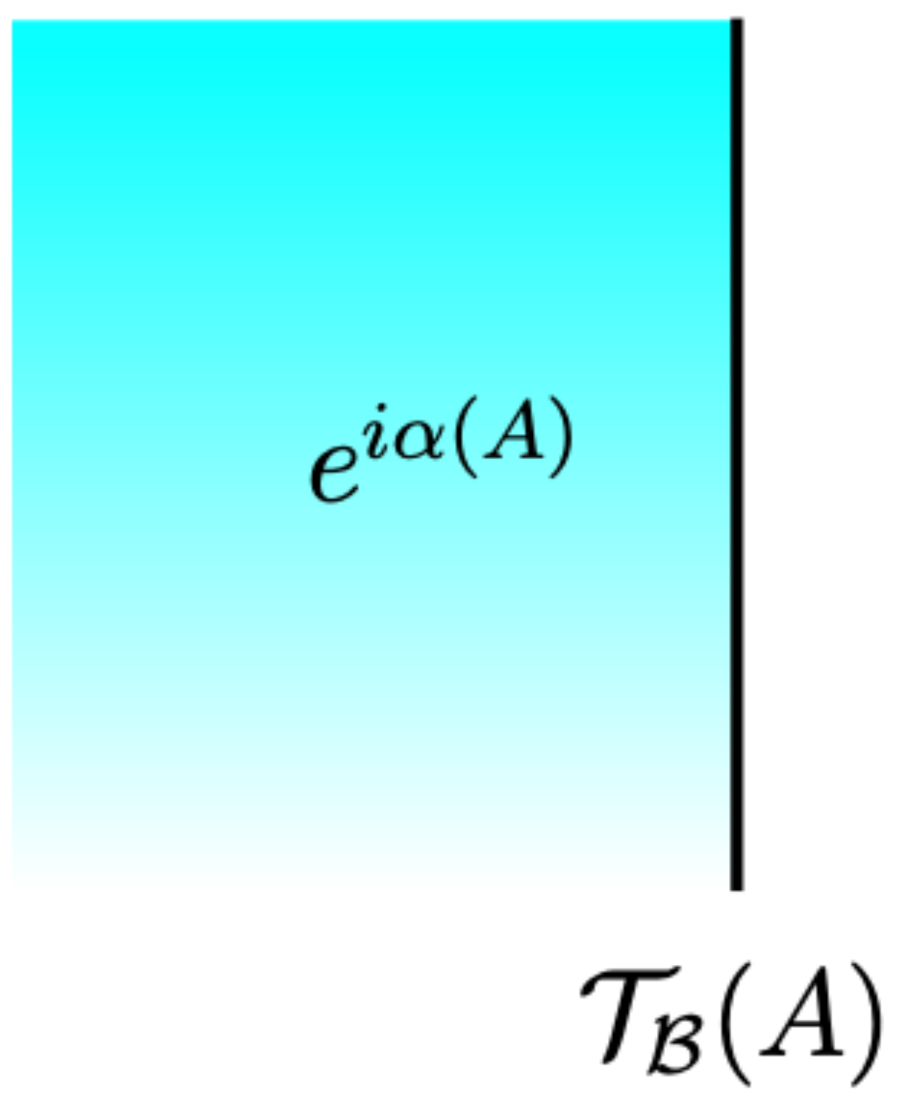
If symmetries are anomalous: anomaly inflow → bulk/boundary system



Consider a QFT: $\mathcal{T}_{\mathcal{B}}$

Couple it to background gauge fields: $\mathcal{T}_{\mathcal{B}}(A)$

Anomaly: failure to respect background gauge transformation

Cured with invertible field theory in one higher dimension

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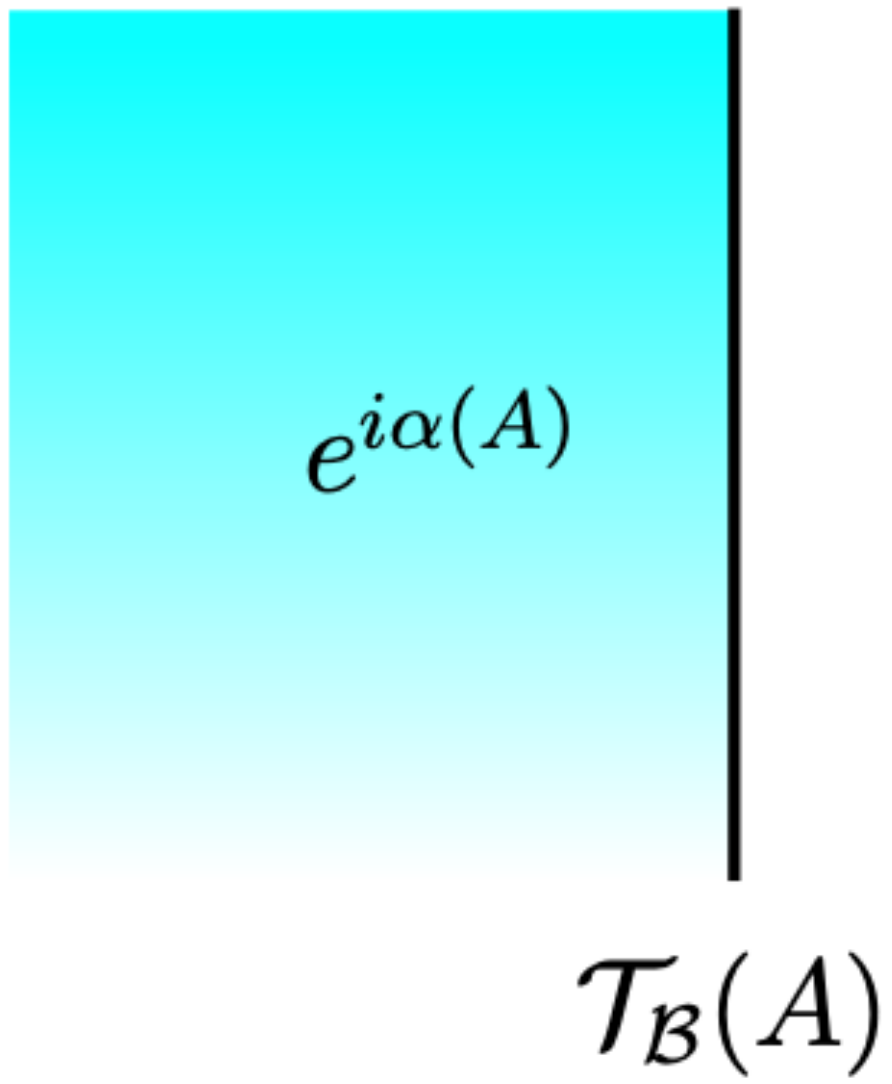

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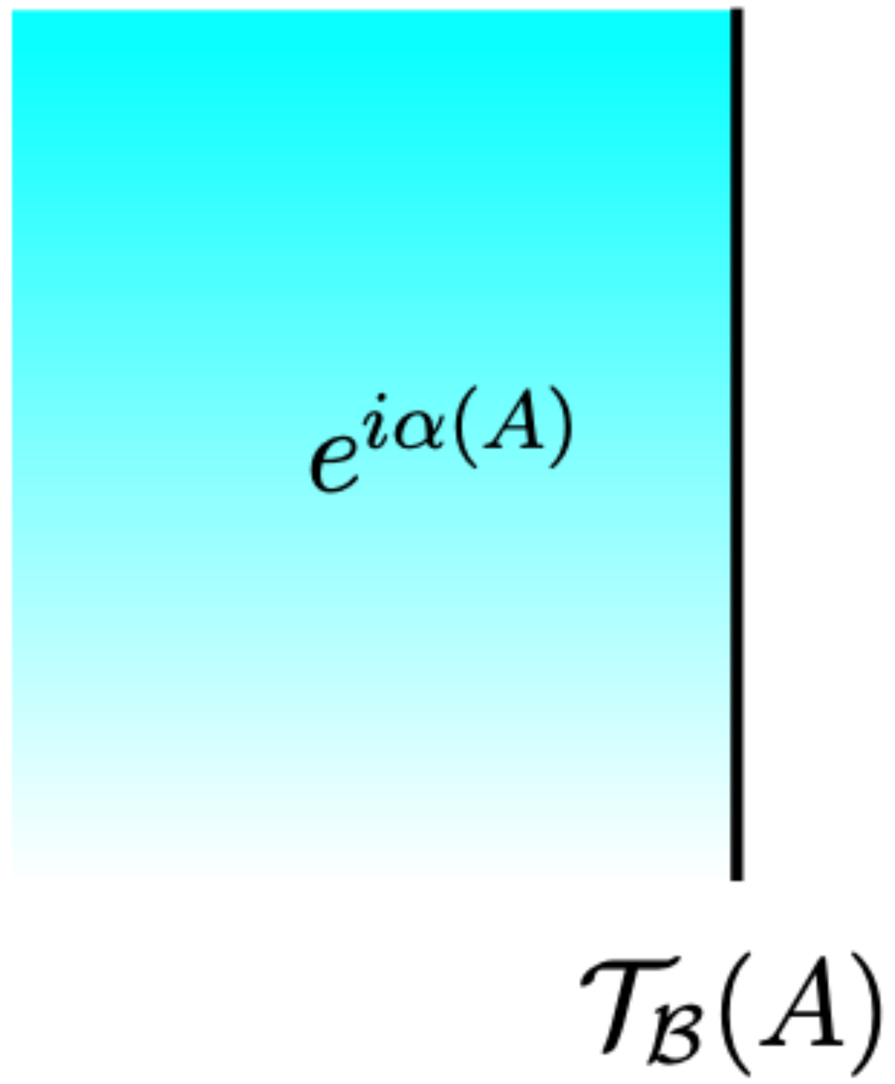

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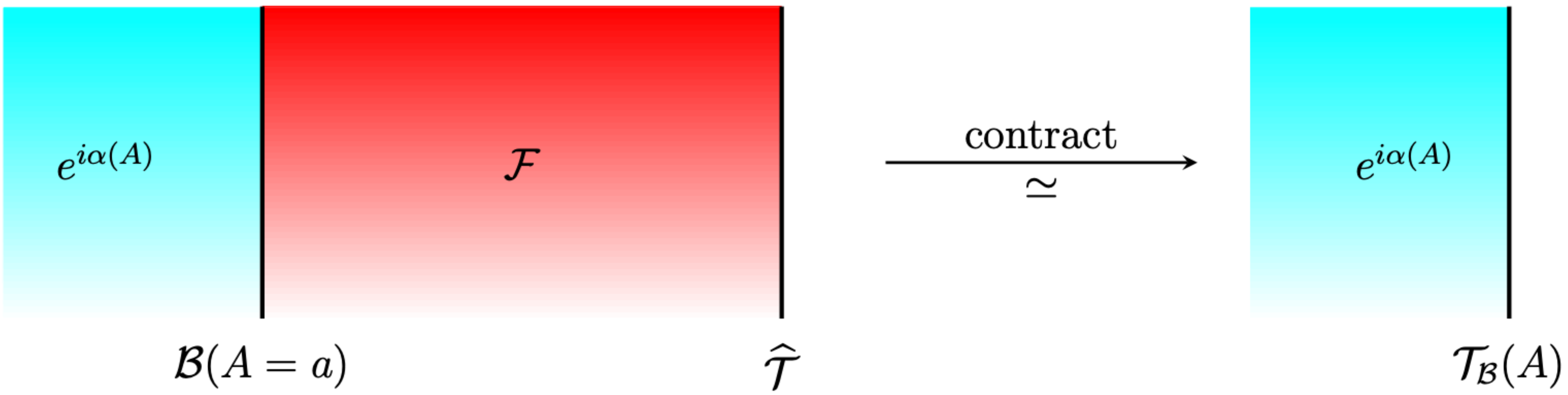

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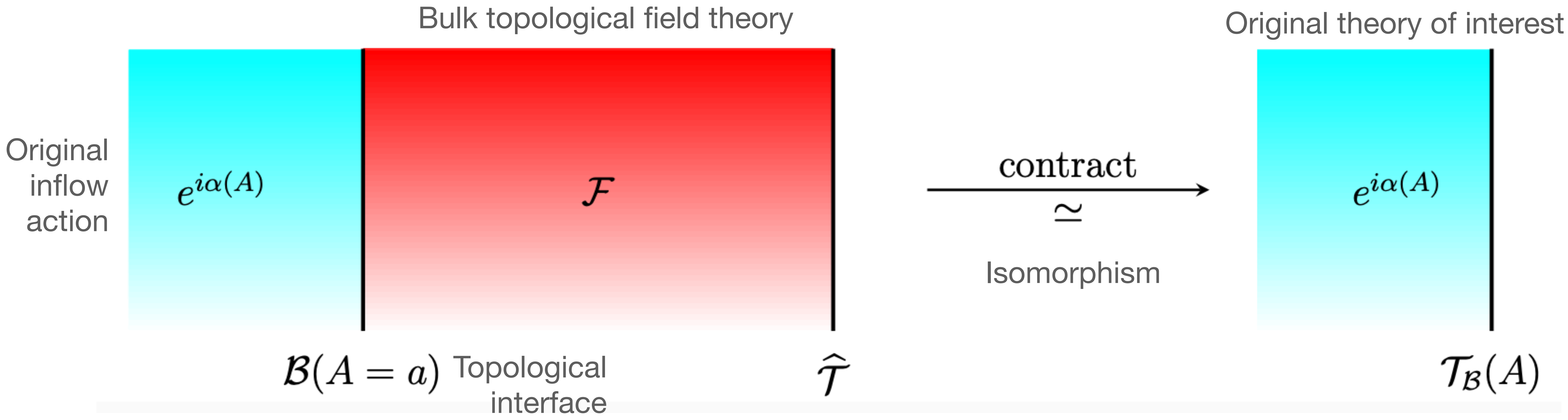

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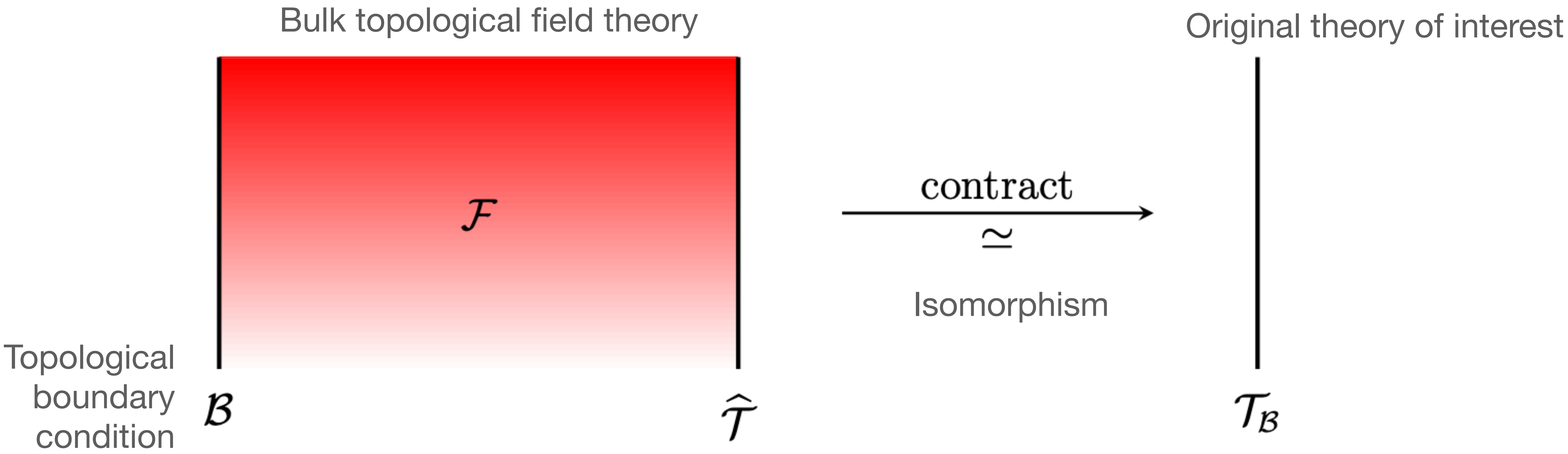

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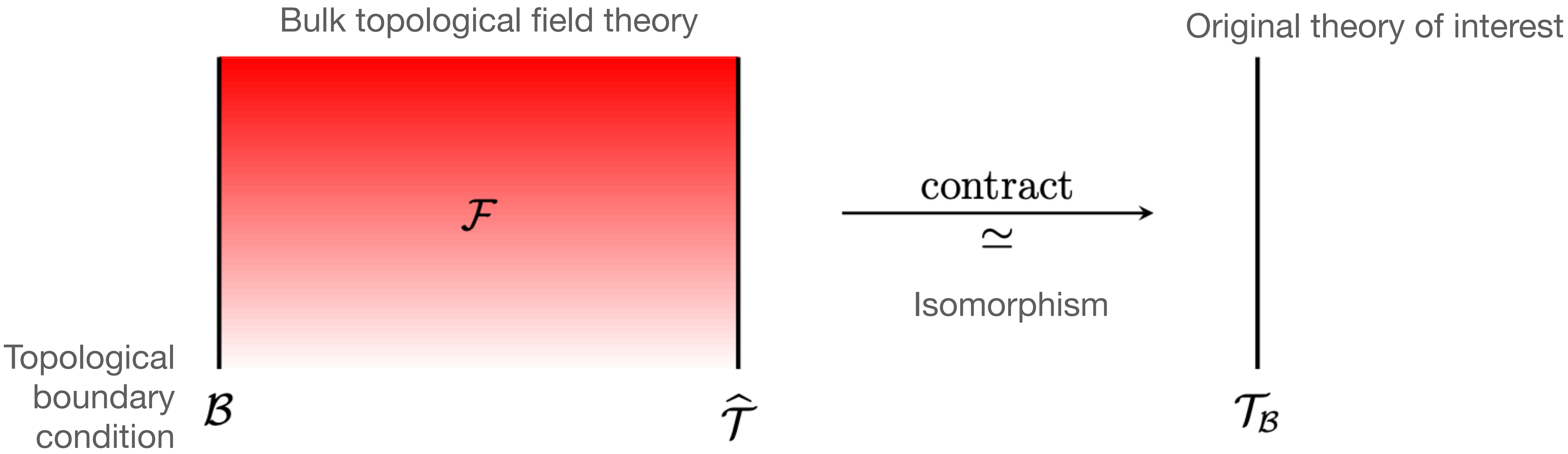

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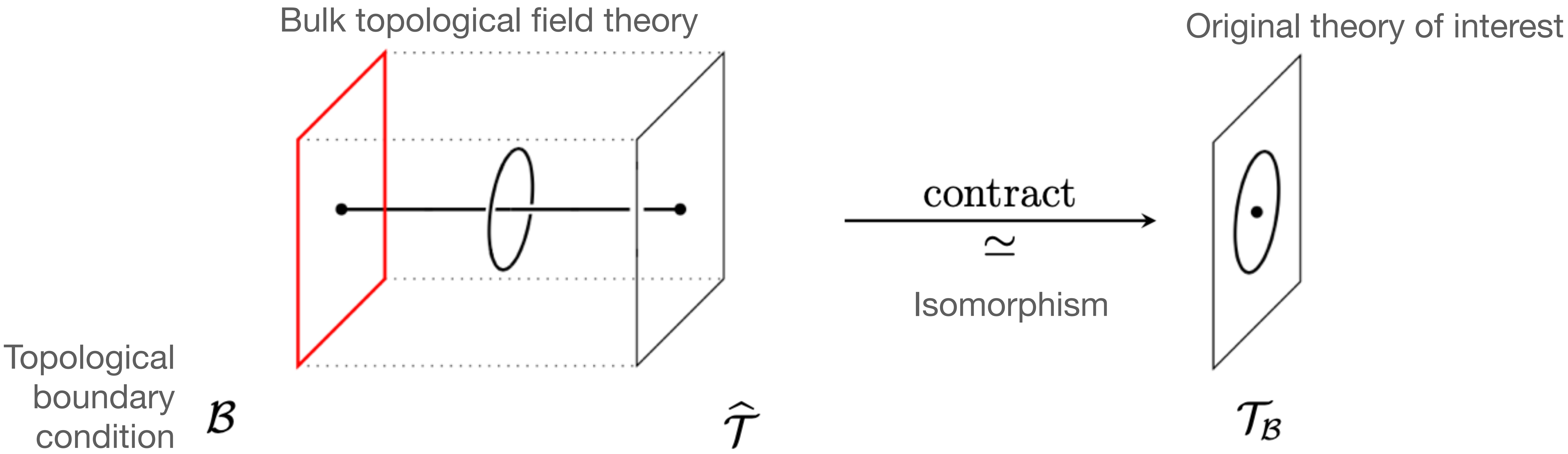

Recovers topological defects and their action — here we see e.g. the linking

Topological Interfaces from Bulk/Boundary

When SymTFT itself has a 0-form symmetry whose defects are **endable and act non-trivially on the topological boundary conditions**

$\longrightarrow$ streamlines construction of topological interfaces

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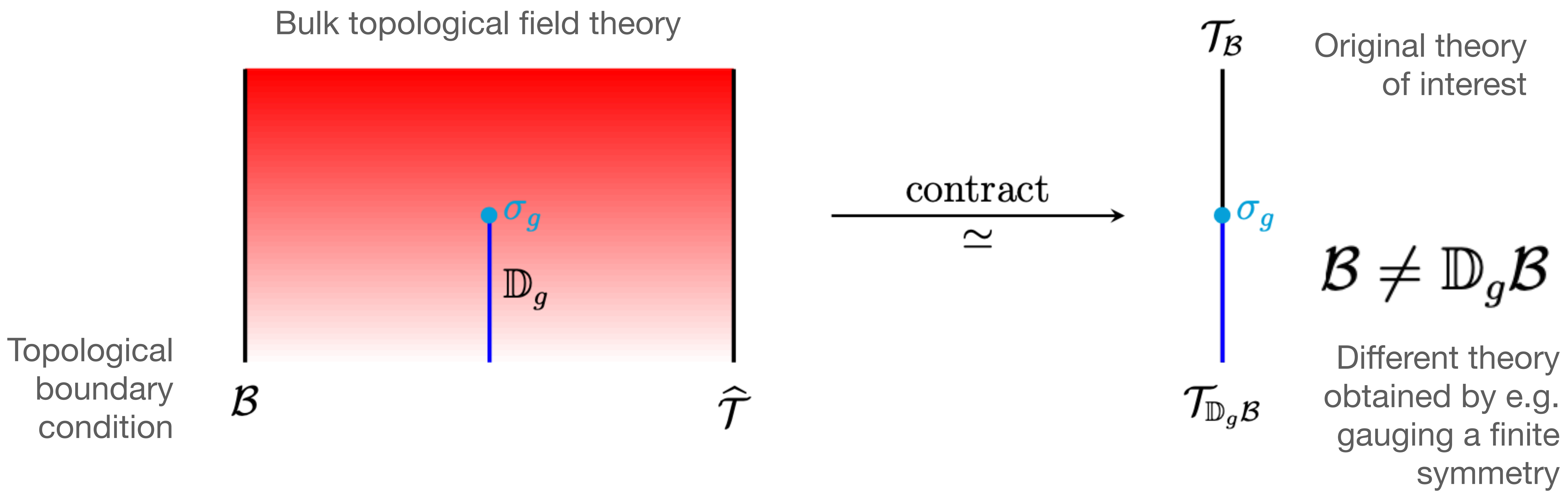

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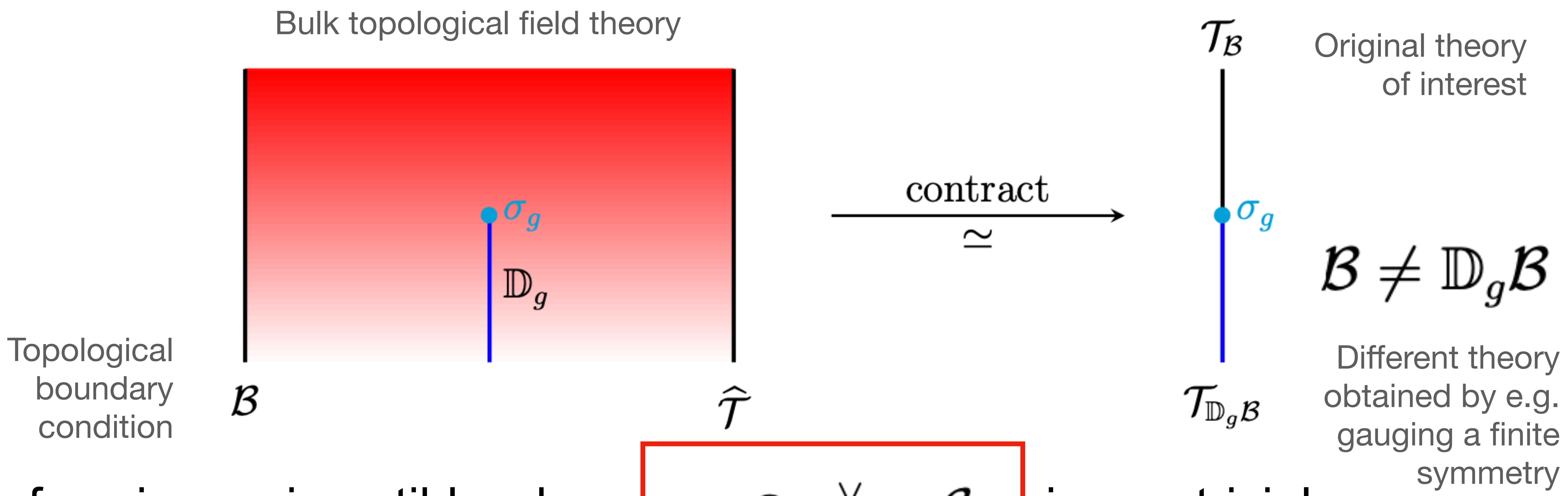

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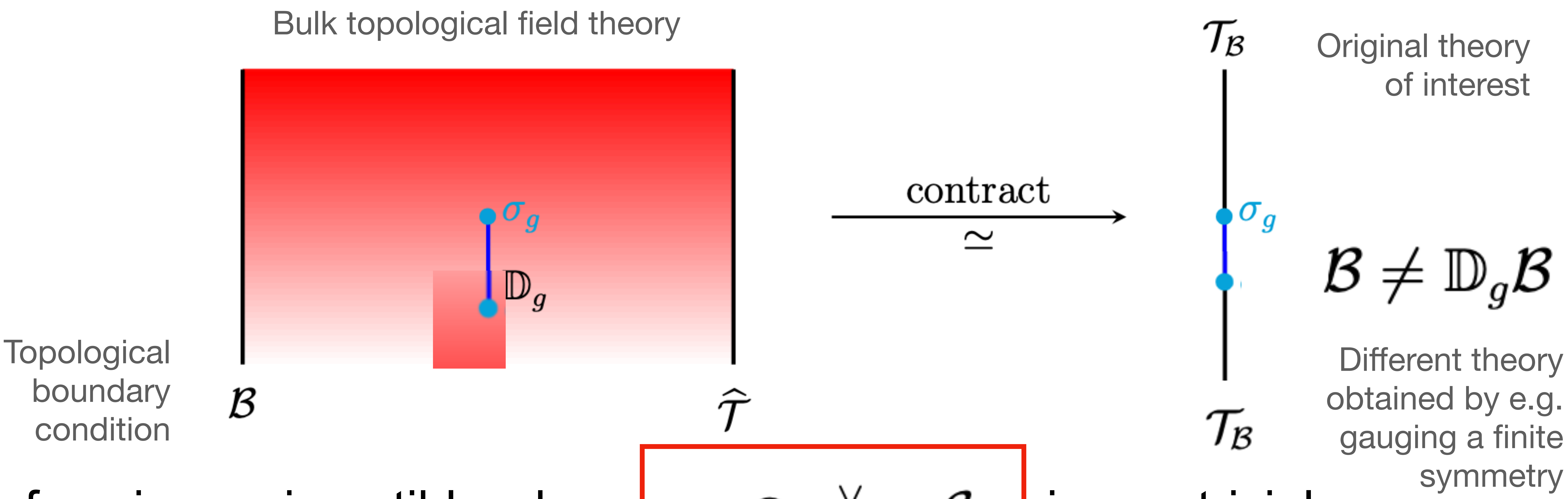

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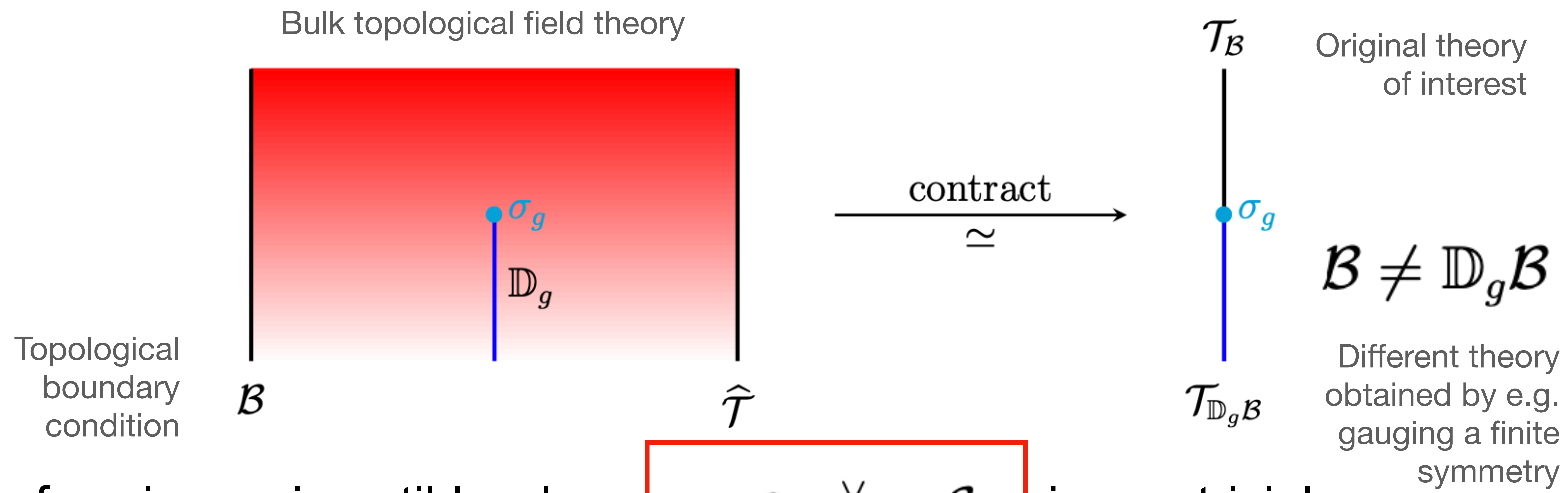

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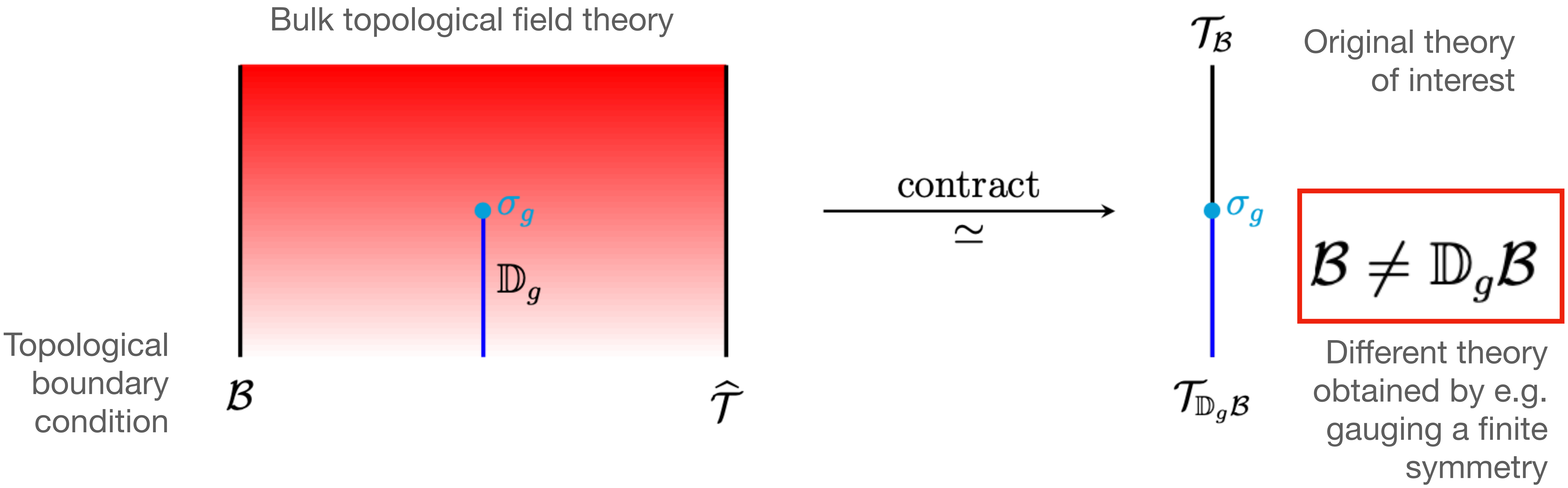

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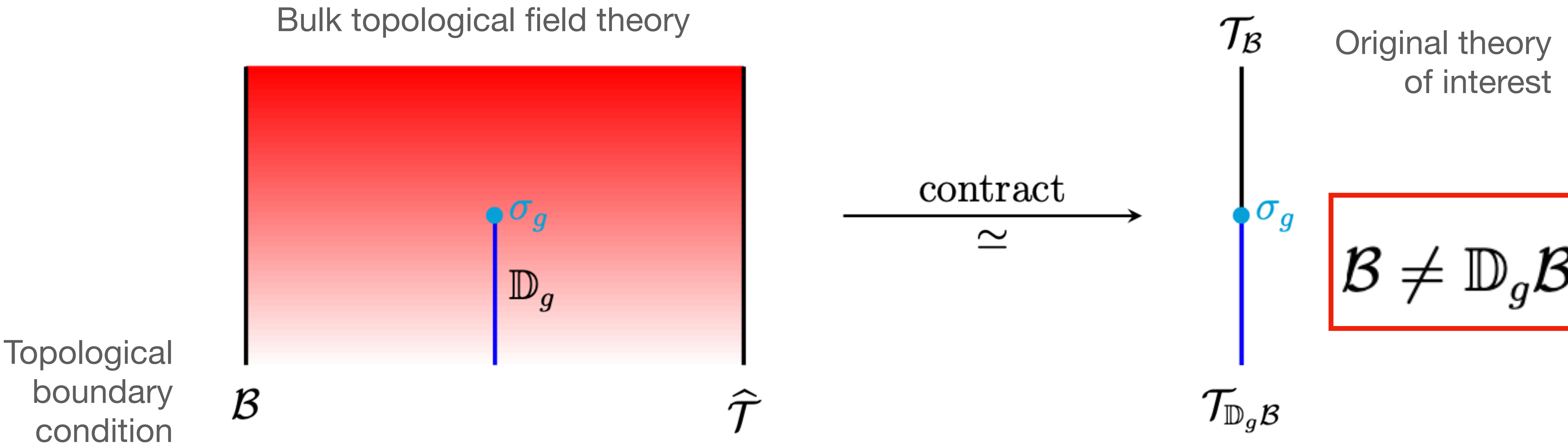

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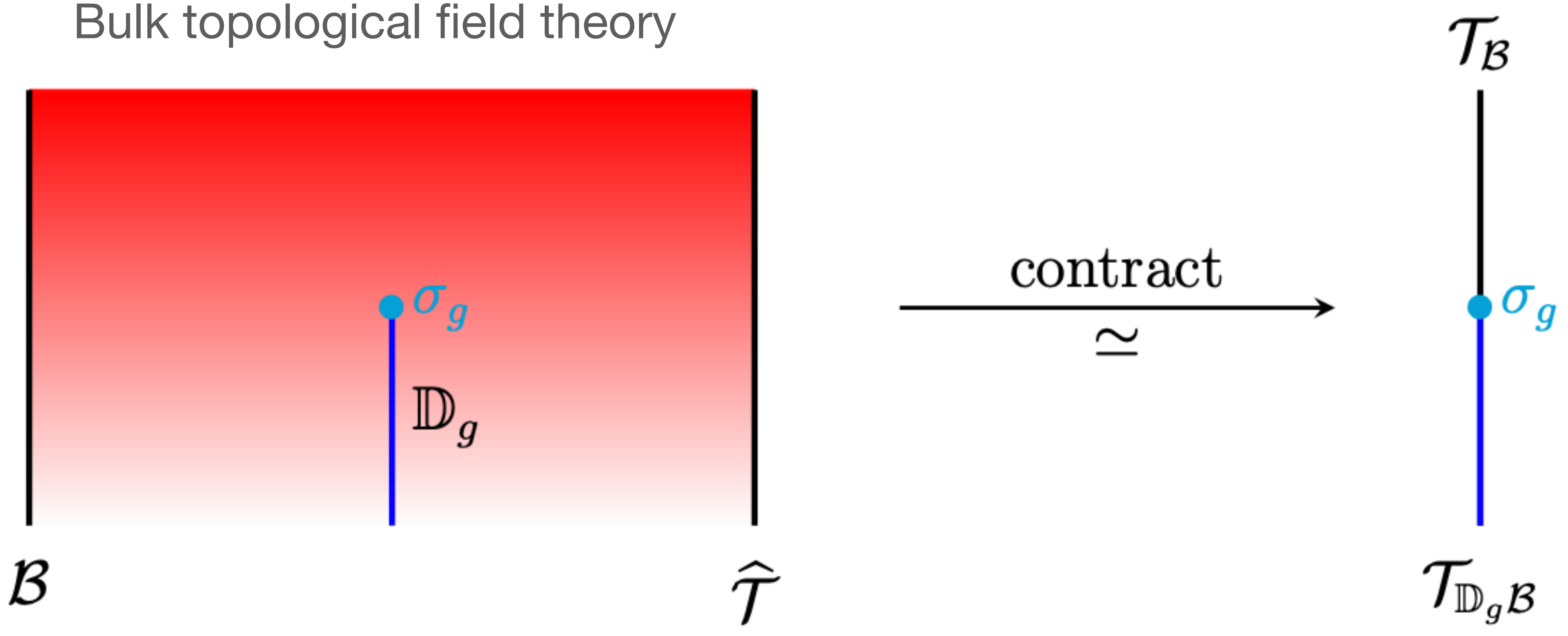
If moreover, $\boxed{\mathbb{D}_g \widehat{\mathcal{T}} \simeq \widehat{\mathcal{T}}}$ $\longrightarrow$ duality isomorphism $\mathsf{S}_{g^{-1}} : \mathcal{T}_{\mathbb{D}_g \mathcal{B}} \simeq \mathcal{T}_{\mathcal{B}}$

Bulk topological field theory

Original theory of interest



Topological boundary condition

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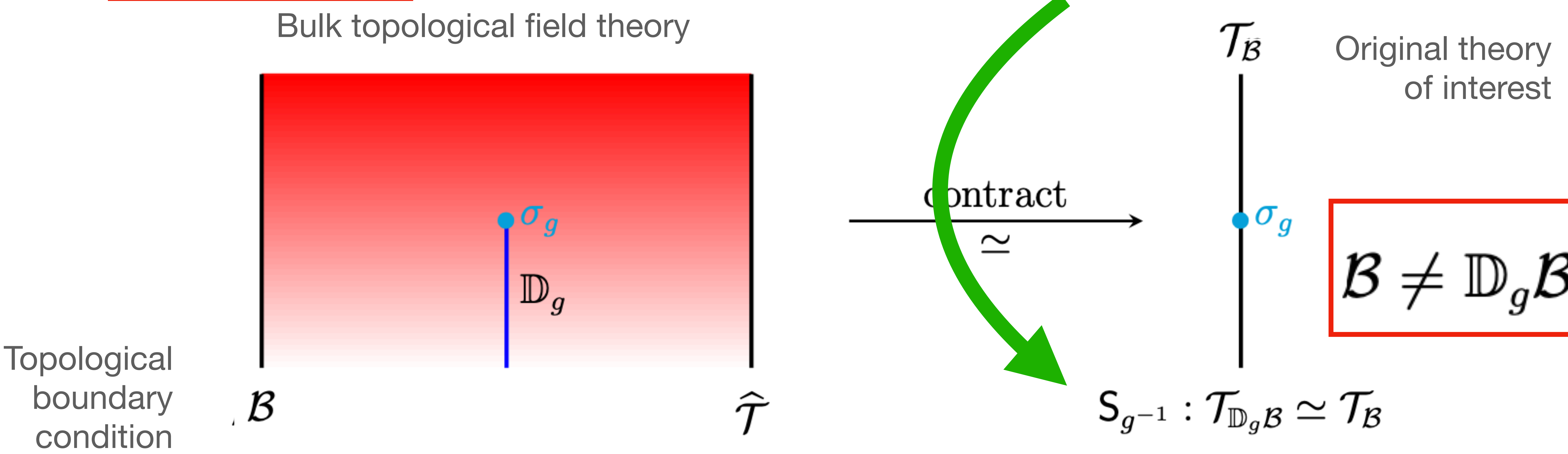

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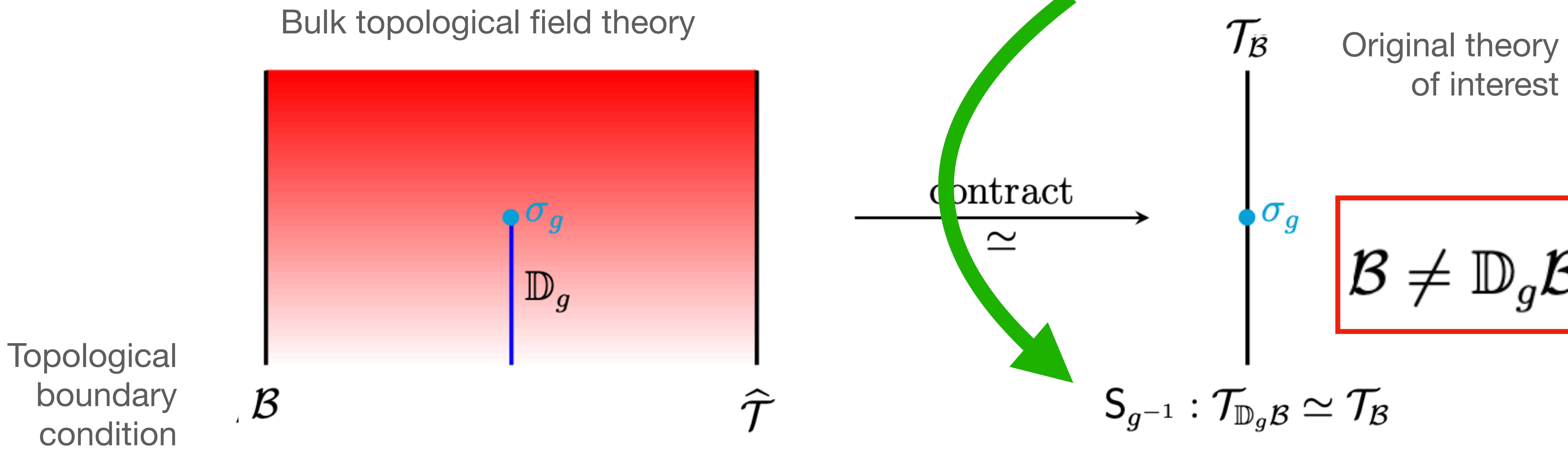

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Need SymTFT with 0-form symmetry such that these 3 conditions are met

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$\longrightarrow$ Needs a **new SymTFT** by **gauging 0-form symmetry**

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The 4d Case

This construction applies to **4d QFTs with 1-form symmetries**

e.g. Maxwell theory, various supersymmetric gauge theories with EM duality, more general 4d N=2 QFTs,…

It can be proven that the SymTFT in this case is always equivalent to a topological BF theory with gauge group

$$\mathbb{A} := \prod_{p\in P} \mathbb{A}_p \qquad \mathbb{A}_p := \prod_{i=1}^{\ell} \mathbb{Z}_{p^{k_i}}^{r_i}$$

With topological surface defects with braiding

$$\Phi(M_2)\Phi(M_2') = M_2(M_2')\Phi(M_2')\Phi(M_2) \qquad M_2 \in H_2(M_5, \mathbb{A}\times\mathbb{A}^\vee)$$

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Topological boundary conditions → Lagrangian subgroups L of $\mathbb{A} \times \mathbb{A}^\vee$

Theory with 1-form symmetry $\rightarrow (\mathbb{A} \times \mathbb{A}^\vee)/L$

The 4d Case - continued

In this case we have a bulk 0-form symmetry arising from

$$\mathrm{Sp}(\mathbb{A} \times \mathbb{A}^\vee)$$

And one can check **conditions 1,2,3 outlined above are met**.

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Original twist defects + Wilson lines of the gauged 0-form symmetry

$\rightarrow$ **Gauged SymTFT has more boundary conditions!**

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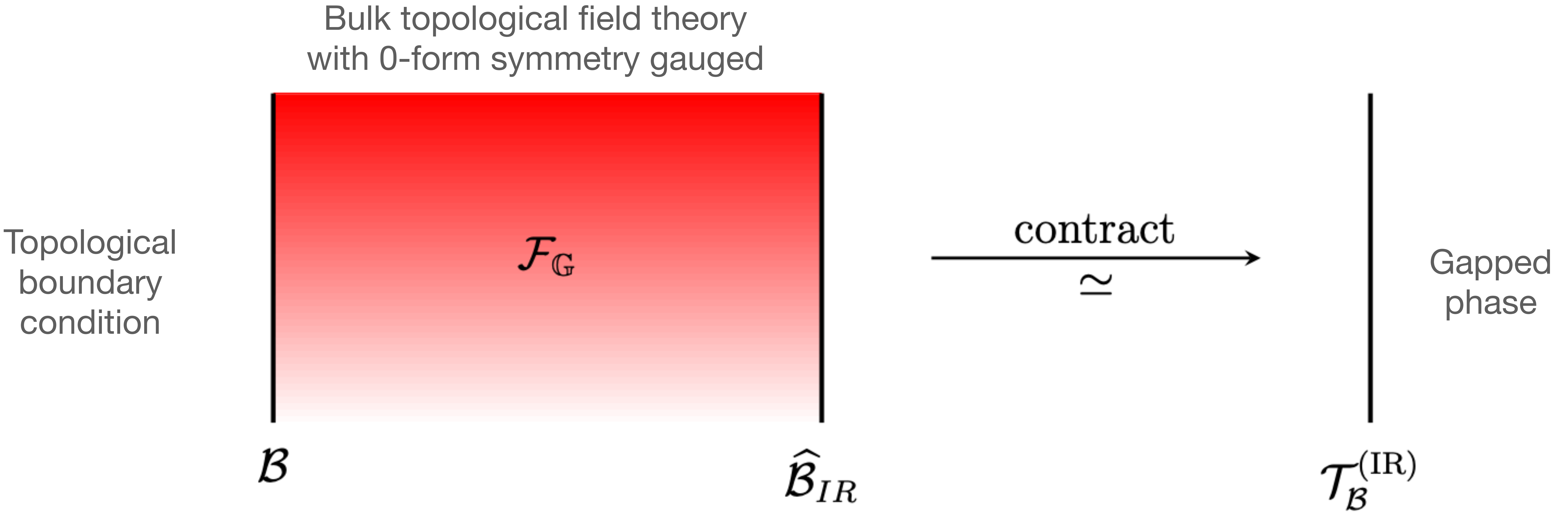
Crossing action on surface defects is needed to classify these

$$\sigma(g \otimes M_3)\Phi(q \otimes M_2) = \Phi((g^{-1}q) \otimes M_2)\sigma(g \otimes M_3)$$

Gapped phases with Non-Invertible symmetry in 4d

Now consider the SymTFT description of a gapped phase with non-invertible symmetry of the types we discussed above:



Other topological boundary condition such that

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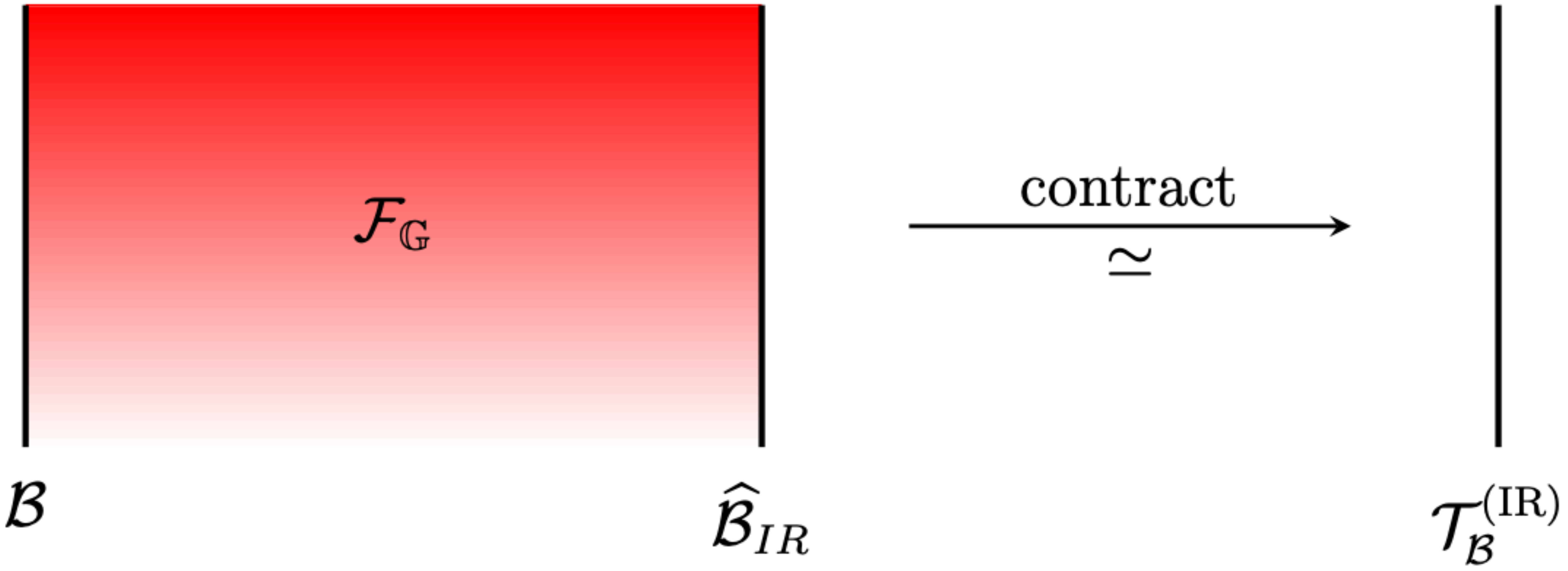

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Simplest case: $\mathbb{A} = (\mathbb{Z}_N)^r$, $\mathbb{G} = \langle g \rangle$ cyclic:

- Chinese Remainder Theorem: Enough to consider $N = p^k$ with p prime.
- Relate Lagrangians over $\mathbb{Z}_{p^k}$ to invariant subspaces of $\mathbb{Q}_p^{2r}$ ($\mathbb{Q}_p$ the p-adic rationals)
- Linear algebra over fields $\mathbb{F}_p$ or $\mathbb{Q}_p$, governed by the characteristic polynomial $\det(x - g)$.
- For finite order g: $\det(x - g)$ is a product of cyclotomic polynomials $\Phi_n(x)$.
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Example: if *m*,*k* odd and

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Invariant Lagrangian subgroups exist if -1 is not a power of p mod n

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n \ p mod n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
3	✓	✗	·																
4	✓	·	✗																
5	✓	✗	✗	✗	·														
6	✓	⊗	·		✗														
7	✓	✓	✗	✓	✗	✗	·												
8	✓	·	✓		✓		✗												
9	✓	✗	·	✓	✗		✓	✗											
10	✓	⊗	✗		·		✗		✗										
11	✓	✗	✓	✓	✓	✗	✗	✗	✓	✗	·								
12	✓	·	·		✓		✓				✗								
13	✓	✗	✓	✗	✗	✗	✗	✗	✓	✗	✗	✗	·						
14	✓	⊙	✗		✗		·		✓		✓		✗						
15	✓	✓	·	✓	·		✓	✓			✓		✓	✗					
16	✓	·	✓		✓		✓		✓		✓		✓		✗				
17	✓	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	·		
18	✓	⊗	·		✗		✓				✗		✓				✗		
19	✓	✗	✗	✓	✓	✓	✓	✗	✓	✗	✓	✗	✗	✗	✗	✓	✓	✗	·
20	✓	·	✓		·		✓		✓		✓		✓				✓		✗

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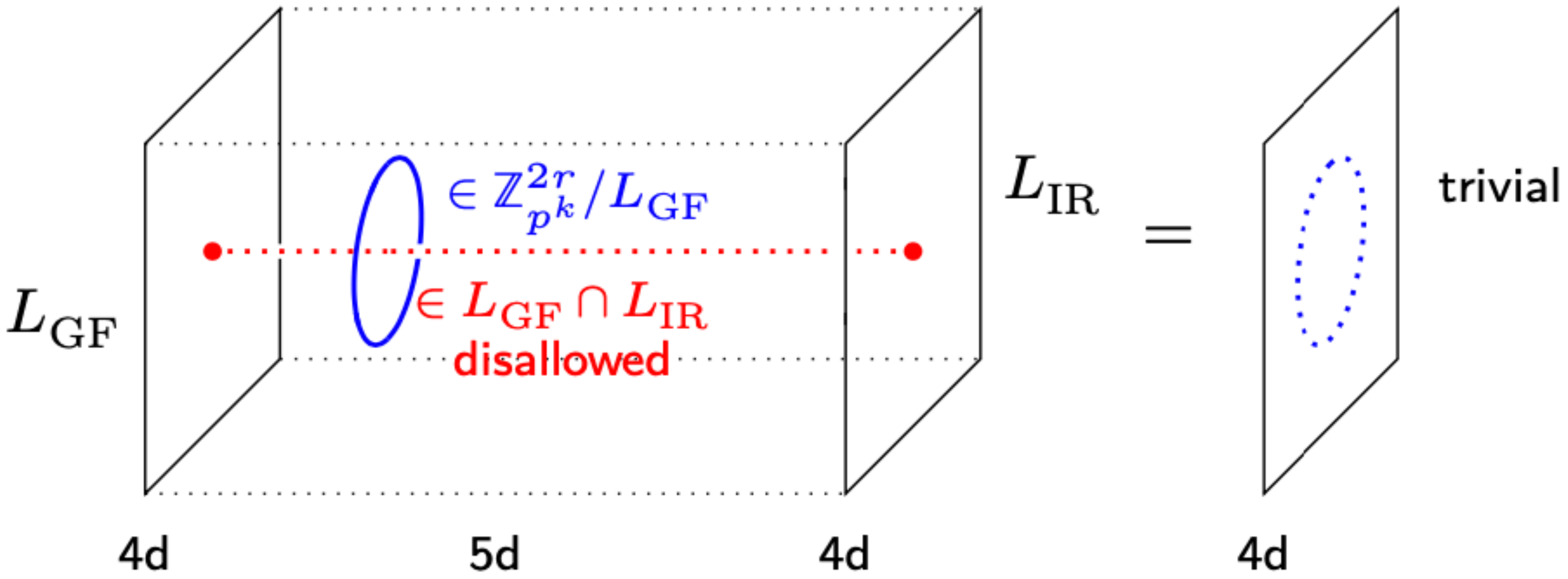
Back to the SymTFT!

Easy to calculate!

Gapped phases with Non-Invertible symmetry in 4d

Can the gapped phase be trivial? Consider the intersection:



If there are no charged line operators the phase is trivial otherwise we can have a topological order

SSB of Non-Invertible symmetry in 4d

How about spontanous symmetry breaking of the non-invertible duality symmetries?

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In the gapped case: encoded in the further boundary conditions of the SymTFT with the 0-form symmetry gauged!

Arise from orbits of gapped boundary conditions of the original SymTFT

$$\mathbb{D}_g(\mathcal{B}_1 \oplus \cdots \oplus \mathcal{B}_\ell) \simeq \mathcal{B}_1 \oplus \cdots \oplus \mathcal{B}_\ell$$

Structure of the different vacua is encoded in the overlaps $\mathcal{B} \otimes_{\mathcal{F}} \mathcal{B}_K$

This reproduces nicely the structure of vacua of various supersymmetric gauge theories (e.g. N=1*)

Possible Generalizations

We have just started analyzing the structure of phases with non-invertible duality symmetries

The case of non-finite non-invertible symmetries is richer

→ Matching higher structures inform the stability of solitons for non-linear sigma models

→ Classification of corresponding gapped phases is missing

Possible application of SymTFT for continuous symmetries

→ Interesting in particular the application to SSB which for continuous symmetries is radically different

Conclusions

We have discussed an example of applications of topological field theory techniques to inform questions about dynamics of gauge theory

Symmetries with a non-trivial higher structure are extremely constraining: so constraining the we can classify the possible gapped phases and determine coarsely the structure of the corresponding vacua: trivial (SPTs) or non-trivial (Topological Orders)

Many open questions remain: our classification is a classification of possibilities — how to inform the dynamics of SSB of non-invertible symmetries?

In the classification: new boundary conditions corresponding to models with duality symmetries that are gauged. Are these realized?

e.g. Self-dual Maxwell theory? Argyres-Douglas fixed points?

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Thank you for your attention!