Effective description of Quantum Black Holes

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Executive summary

Effective (physical) metric descriptions of black hole metrics

General (quantum) corrected Hawking Temperature

Universal constraints for models of (quantum) gravity

Relevant for parametrising pheno studies

Consistent bottom-up approach for (quantum) gravity

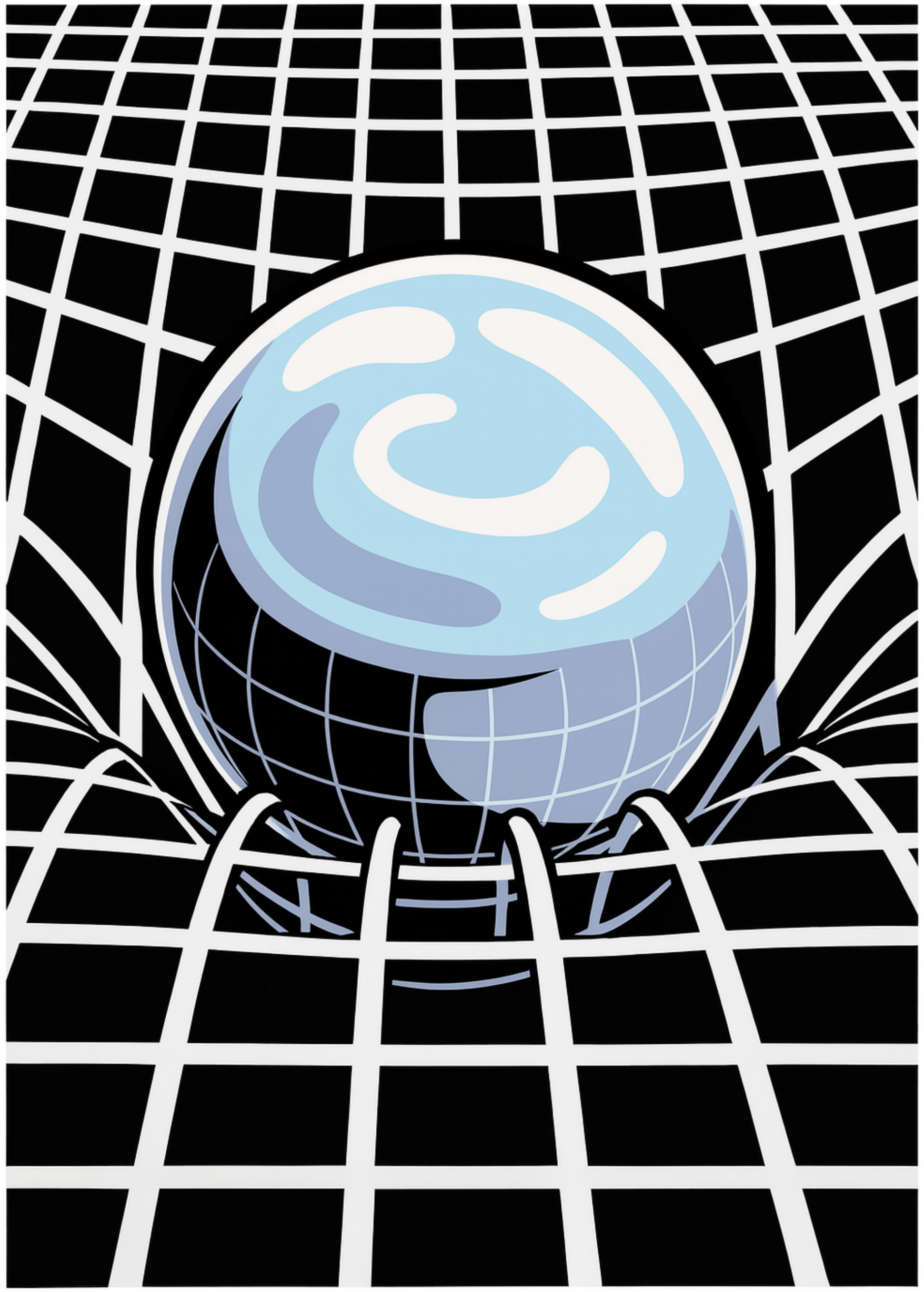
Einstein's General Relativity

1915

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + g_{\mu\nu} \Lambda = 8\pi T_{\mu\nu}$$

Λ = cosmological constant

$T_{\mu\nu}$ = matter contribution



Space - time warping

Gravity as a geometric force

Profound but not fundamental

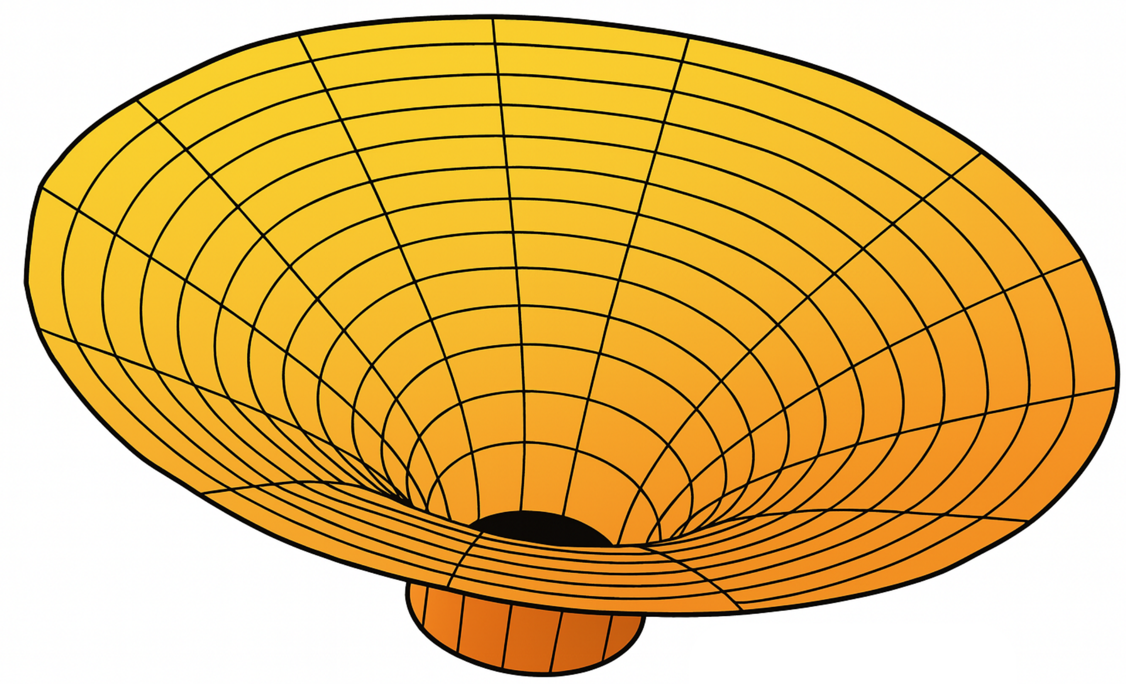
Schwarzschild's Black Hole

1915

$$ds^2 = -f_0(r)dt^2 + \frac{dr^2}{f_0(r)} + r^2 d\theta^2 + r^2 \sin^2\theta\, d\phi^2$$

$$f_0(r) = 1 - \frac{2G_N M}{r}$$

G_N	Newton's constant
M	Mass of gravitating object
$r_S = 2G_N M$	Schwarzschild radius

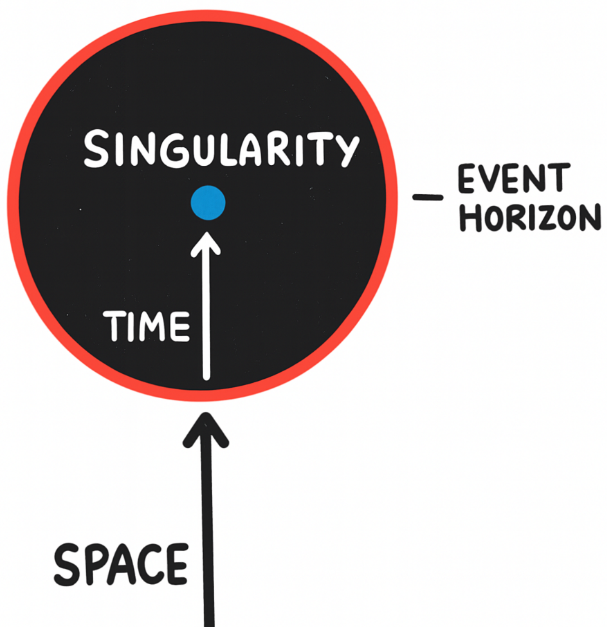


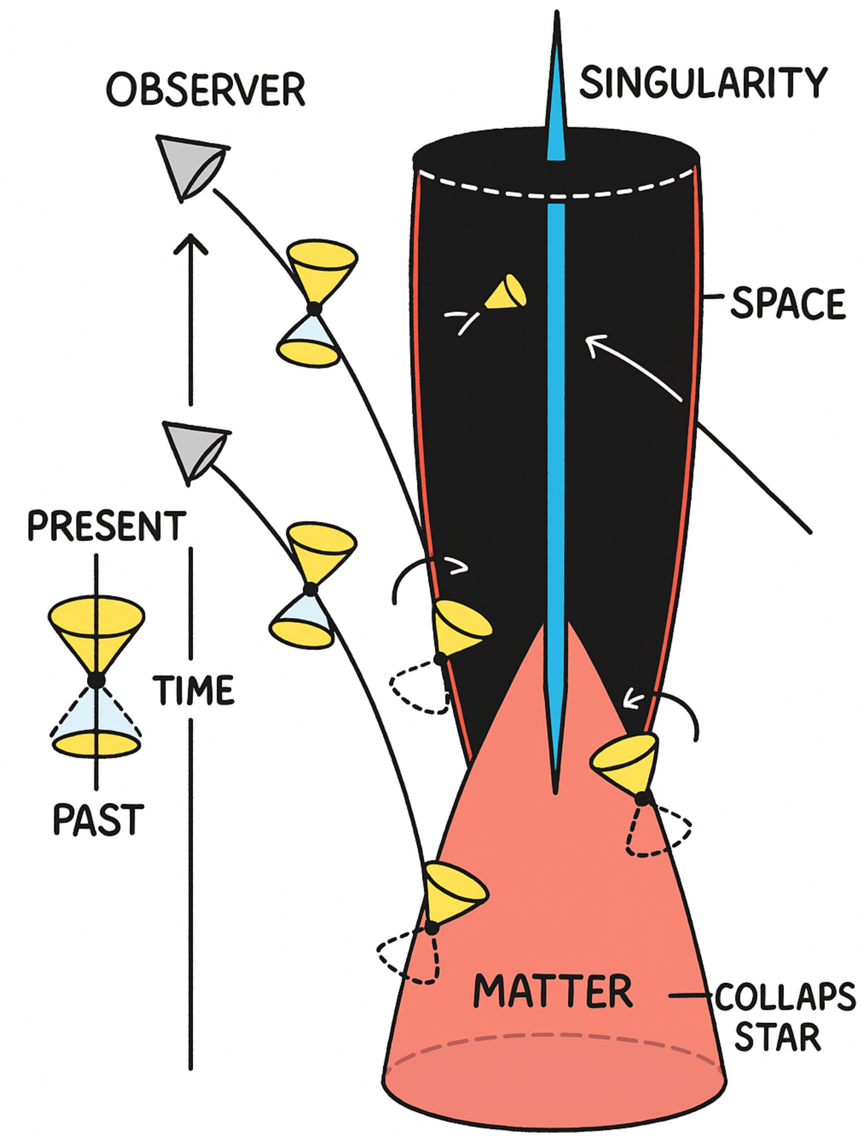
First exact solution of GR equations

Two papers on GR and another on quantum mechanics written at the front!

Penrose's BH formation

1965





GR in the BH forces matter towards the singularity

Singularity in BH, event in time not a point in space

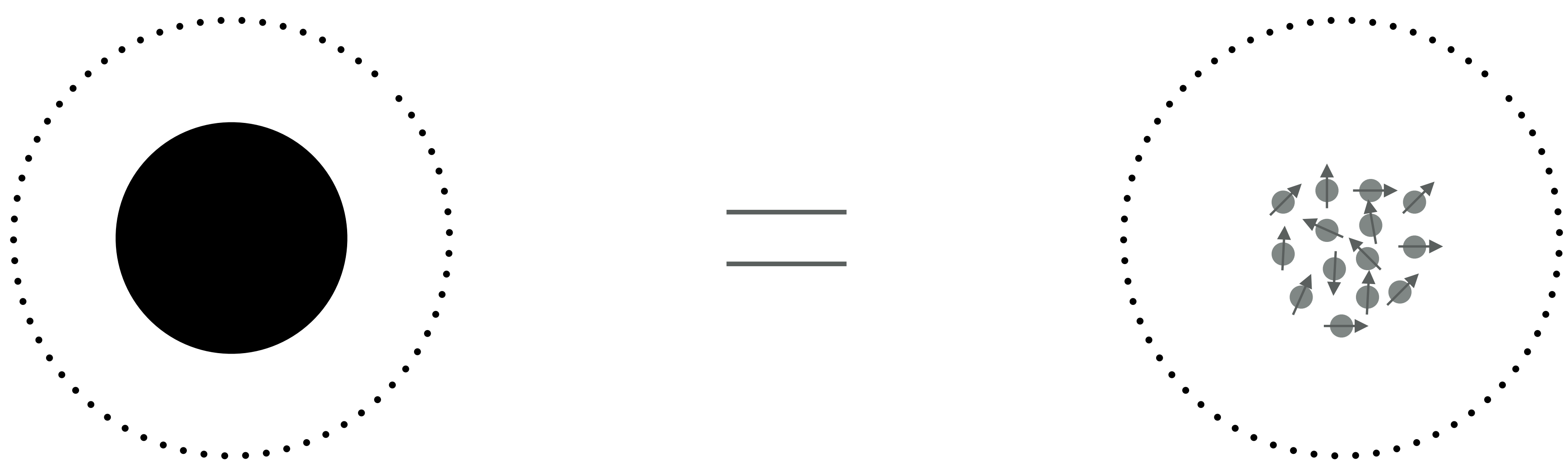
That is why in BH space and time switch roles

At the singularity matter density is infinite and time ends!

BHs as simple quantum systems?

Seen from the *outside*, a BH can be viewed as a quantum system with $A/(4G_N)$ degrees of freedom, which evolves unitary under time evolution

$G_N = \ell_P^2$ is the Newton's constant



Central Dogma

Almheiri, Hartman, Maldacena, Shaghoulian, Tajdini, Rev.Mod.Phys.93 (2021) 3, 035002

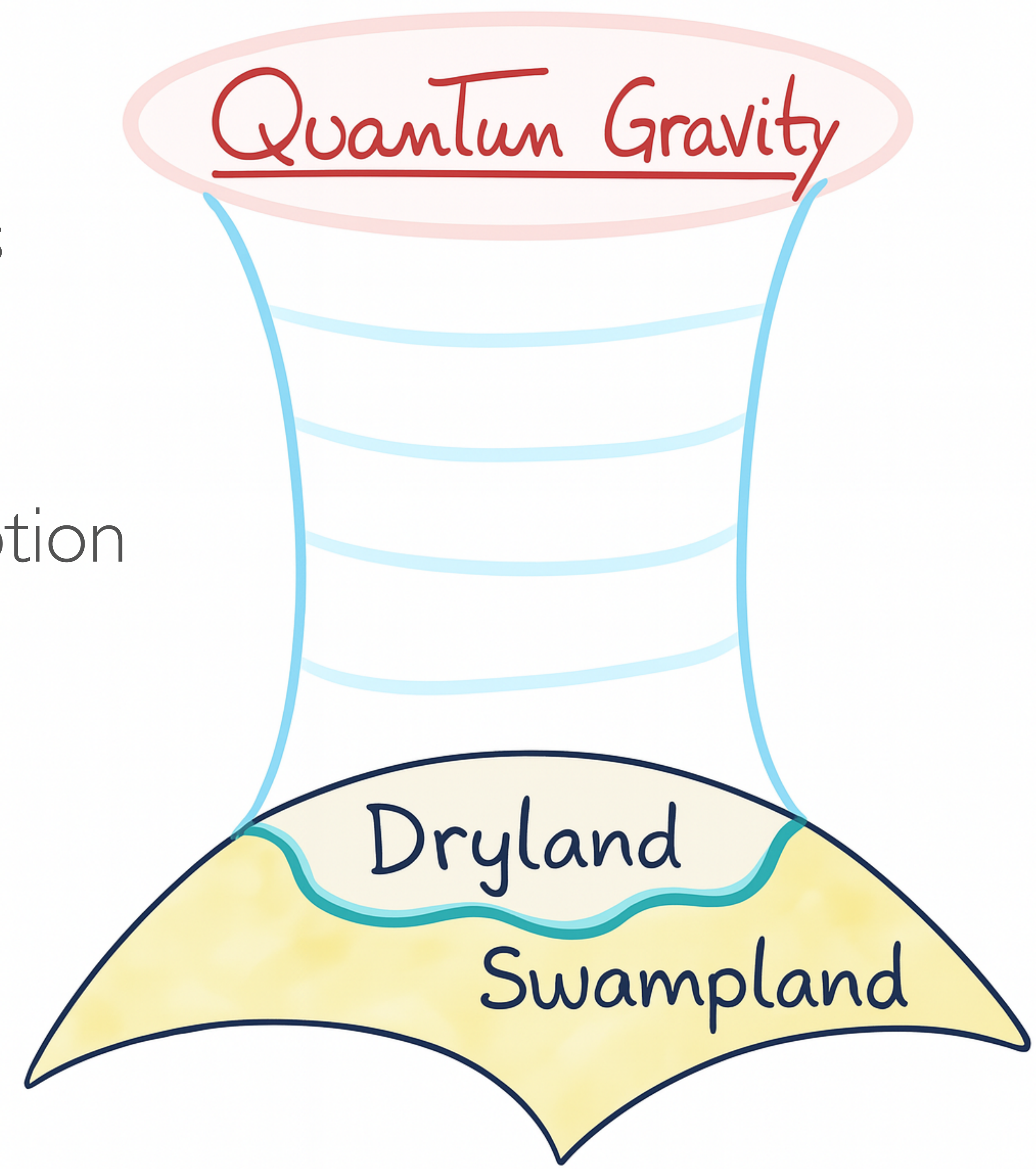
How to move forward?

Quantum gravity at odds with QFT

A more profound description exists

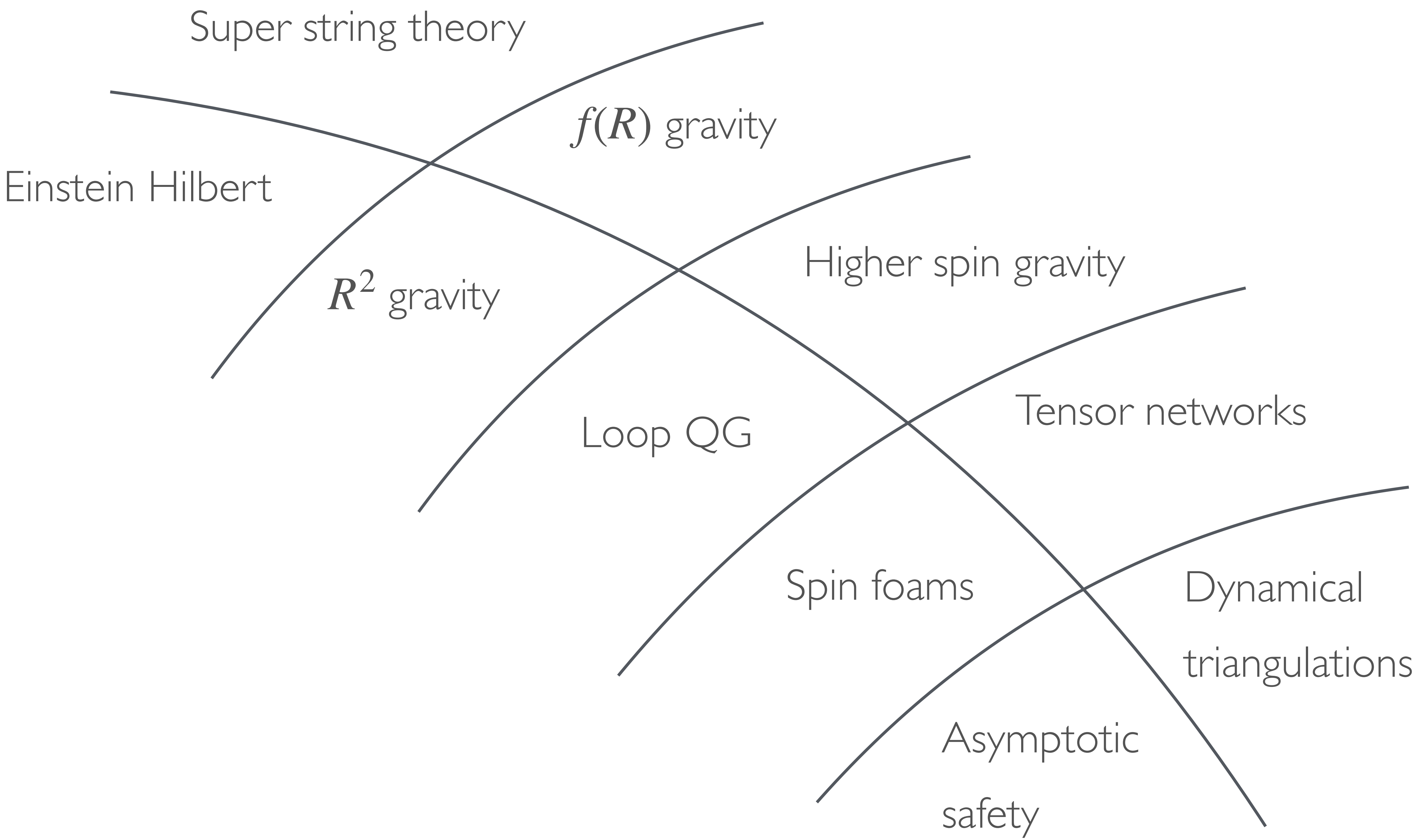
Might not have a Lagrangian description

Symmetries shape QG corrections



Top-down

Which QG?


Super string theory
f(R) gravity
Einstein Hilbert
R² gravity
Higher spin gravity
Loop QG
Tensor networks
Spin foams
Dynamical triangulations
Asymptotic safety

Agnostic approach

… but effective

Black holes as quantum lampposts

Exploit symmetries

Asymptotic behaviours

Derive general results

Effective metric descriptions of Quantum Black Holes

with Del Piano and Hohenegger 2203.13515, 2307.13489, 2403.12679

BTZ 3-D BH, with Paciarini, Myszkowski and Hohengger 2412.15960

Charged BH, with Damia Paciarini, Del Piano, and Hohenegger 2504.20810

Regular Spacetimes with Damia Paciarini, Myszkowski, and Vellucci 2506.12620

Schwarzschild's properties

Static/time independent

Spherically symmetric

Symmetries

Asymptotic behaviours

Flat at large distance

Event horizon

Singular at the origin

Solution of Einstein's GR equations in vacuum

Quantum BH properties

Static/time independent

Spherically symmetric

Symmetries

Asymptotic behaviours

Flat at large distance

Event horizon

Singular at the origin ?

Solution of quantum Einstein's GR equations in vacuum ?

Effective Metric of Quantum Black Holes

$$ds^2 = -h(z)dt^2 + \frac{dz^2}{f(z)} + z^2 d\theta^2 + z^2 \sin^2\theta\, d\phi^2$$

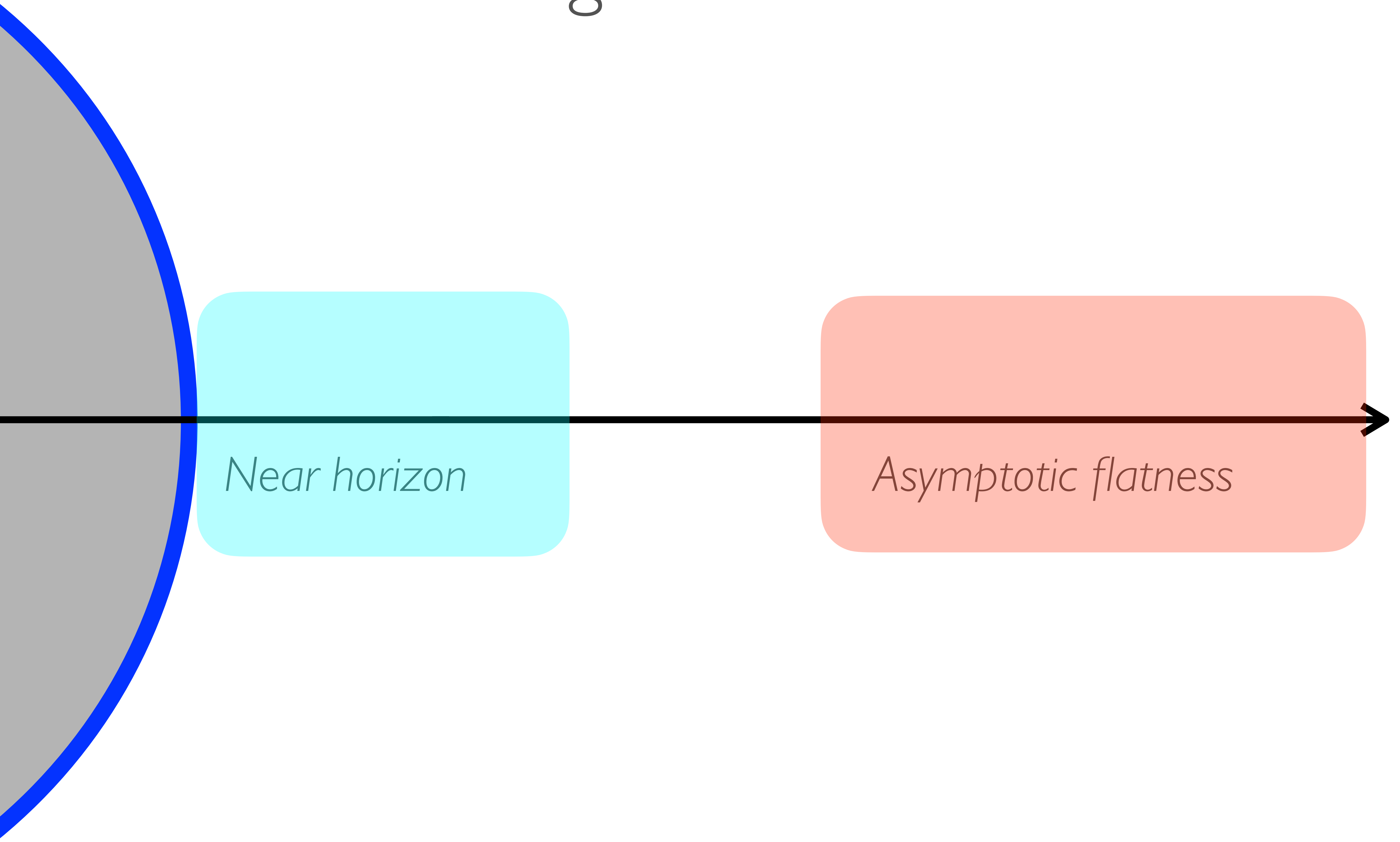
$$\chi = \frac{M}{M_{pl}} \qquad z = \frac{r}{\ell_{pl}}$$

$$f(z) = 1 - \frac{2\chi}{z}\,\Phi\left(d\right) \qquad h(z) = 1 - \frac{2\chi}{z}\,\Psi\left(d\right)$$

Proper distance measured in units of the quantum length ℓ_P

$$d(z) := \frac{1}{\ell_{\mathrm{P}}}\int_0^{z\ell_{\mathrm{P}}} \sqrt{|ds^2|} = \int_0^z \frac{dz'}{\sqrt{|f(z')|}}$$

Natural regions



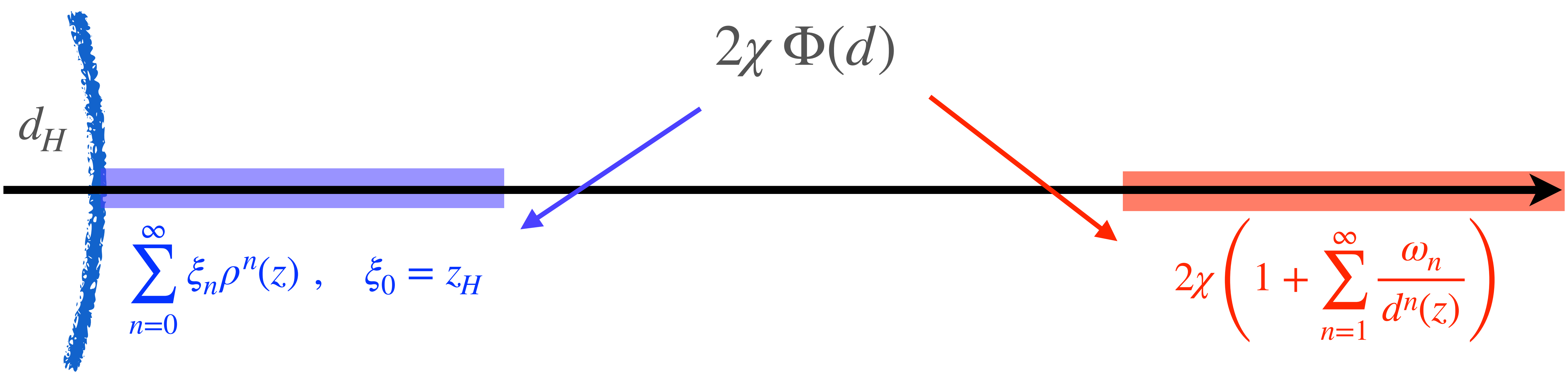
Rationale

(Quantum) metric written via proper distance

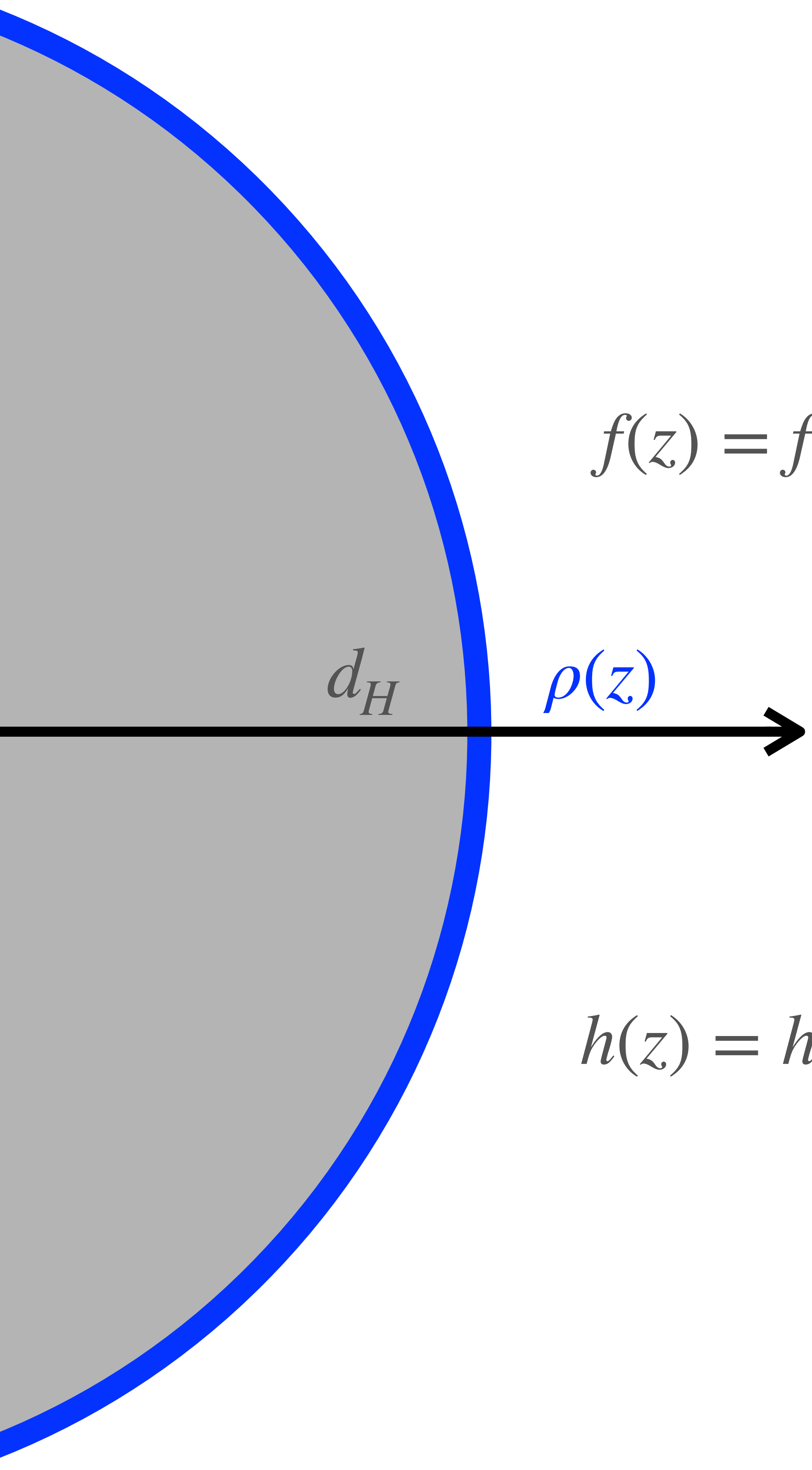
Difficult task

Metric deformations in terms of the physical distance

Solve in two limits



Near horizon expansion

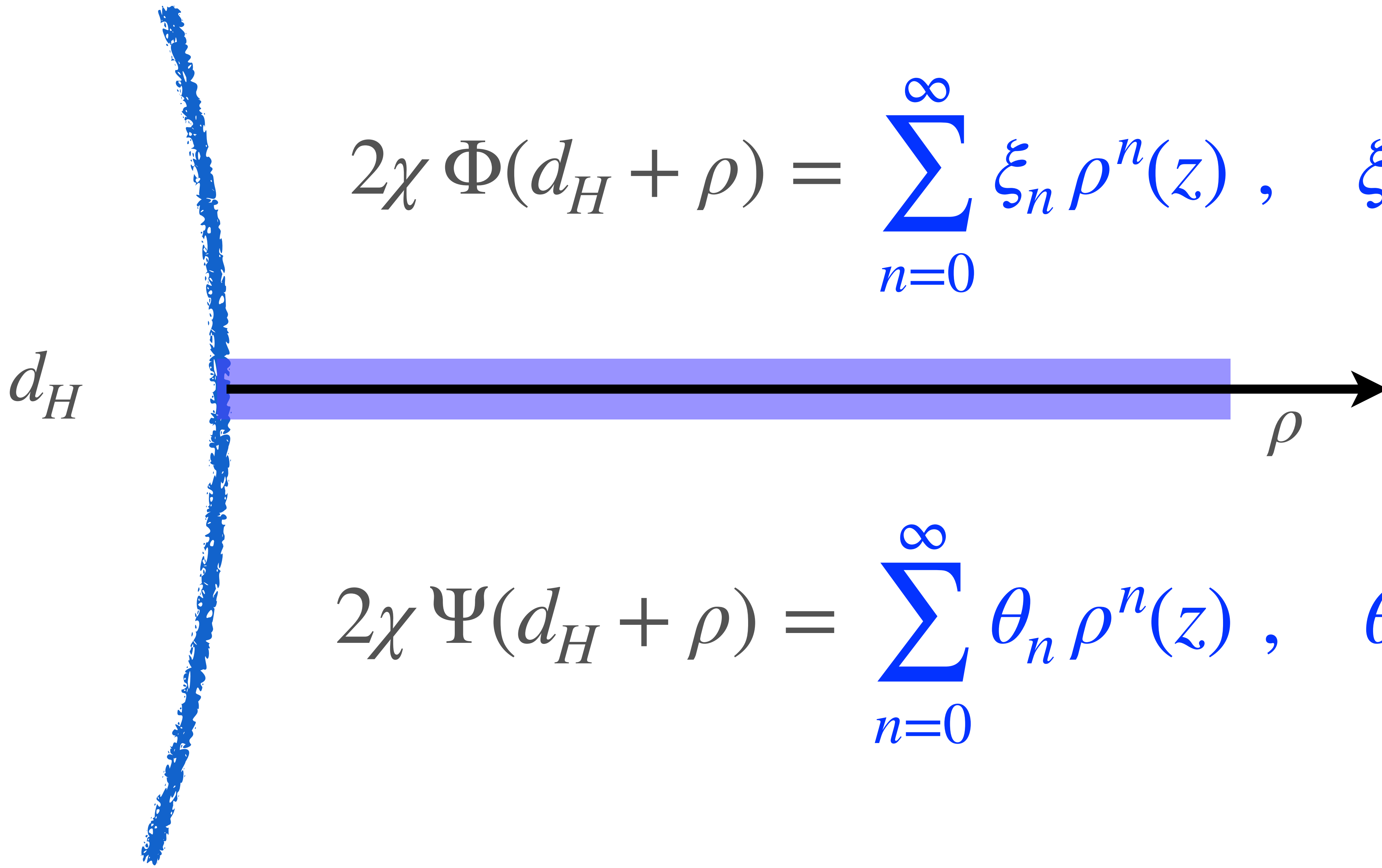


$$f(z) = f_H^{(1)}(z - z_h) + \frac{f_H^{(2)}}{2}(z - z_h)^2 + \mathcal{O}((z - z_h)^3)$$

$$h(z) = h_H^{(1)}(z - z_h) + \frac{h_H^{(2)}}{2}(z - z_h)^2 + \mathcal{O}((z - z_h)^3)$$

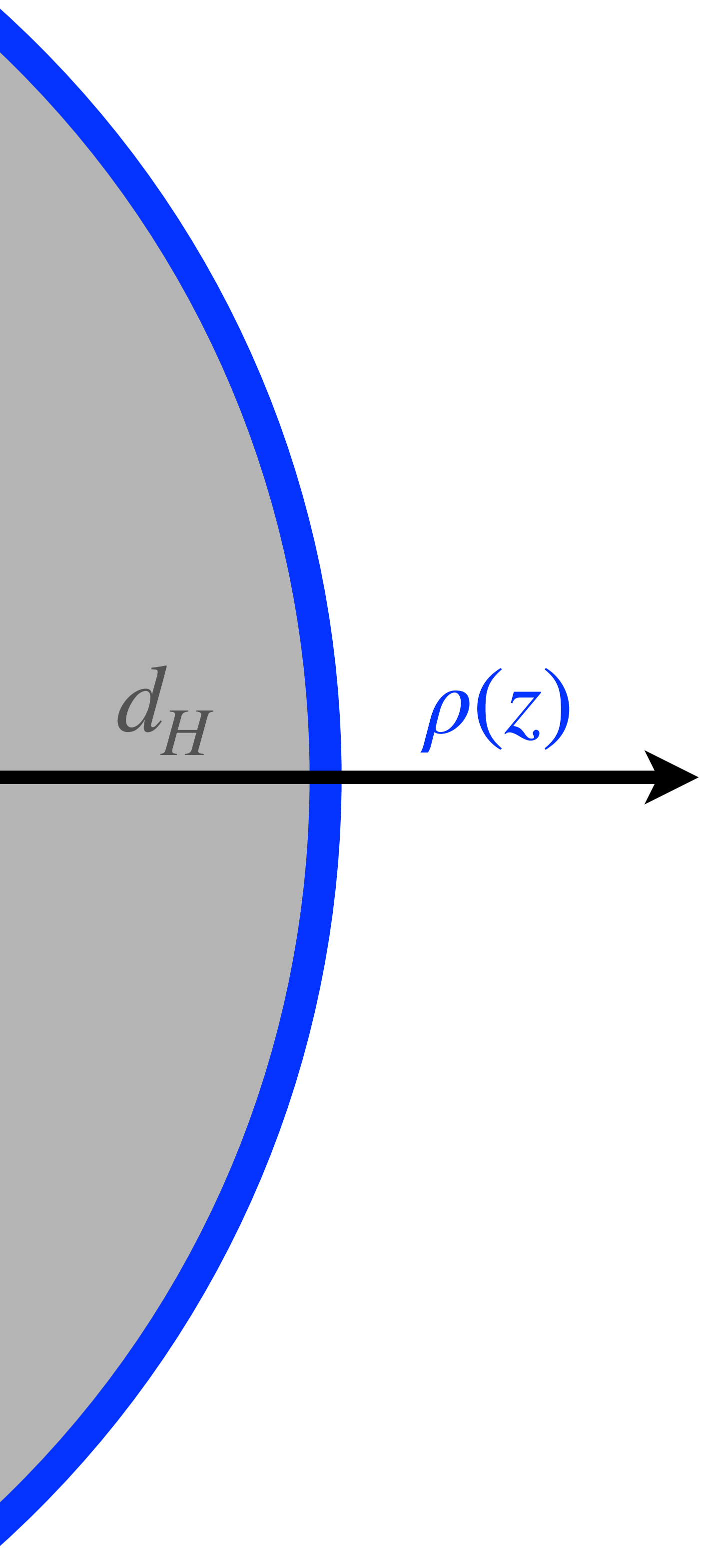
Imposing finiteness of physical observables

$$2\chi\,\Phi(d_H+\rho) = \sum_{n=0}^{\infty} \xi_n\,\rho^n(z)\ , \qquad \xi_0 = z_H$$



$$2\chi\,\Psi(d_H+\rho) = \sum_{n=0}^{\infty} \theta_n\,\rho^n(z)\ , \qquad \theta_0 = z_H$$

Near horizon expansion



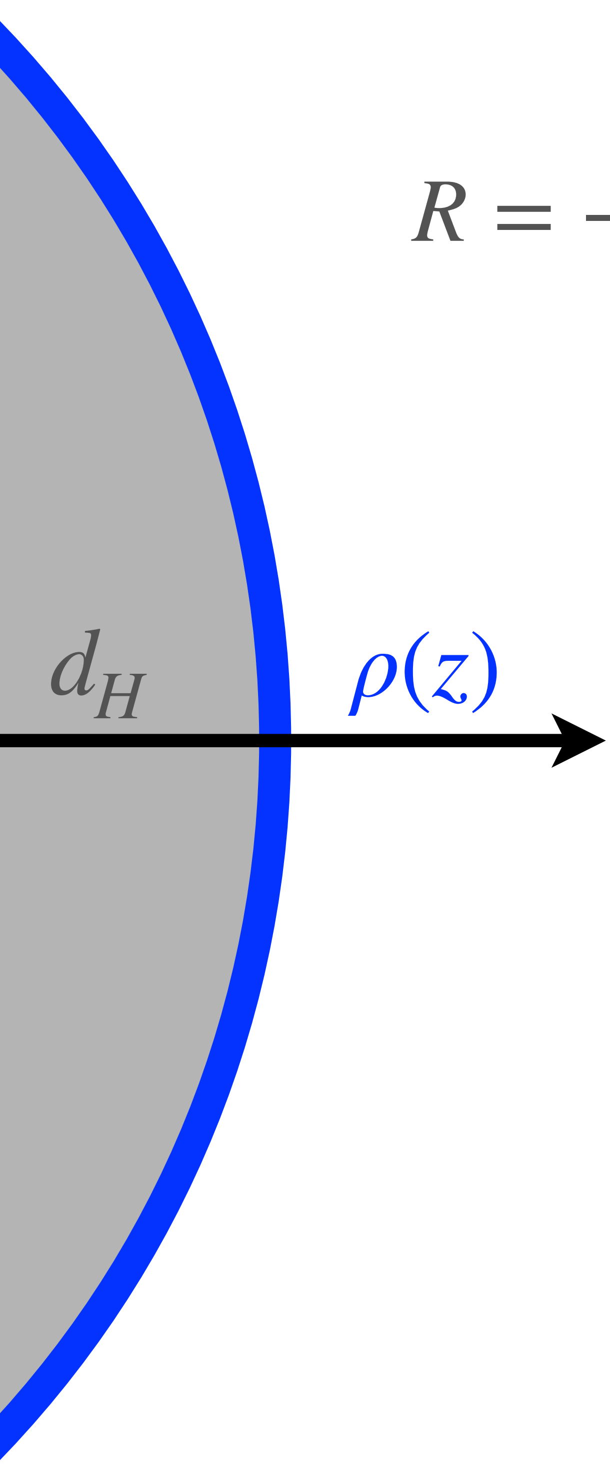
$$\rho(z) \approx \frac{2\sqrt{z - z_h}}{\sqrt{f_H^{(1)}}} - \frac{f_H^{(2)}(z - z_h)^{3/2}}{6(f_H^{(1)})^{3/2}}$$

Invert

$$f(\rho) = \frac{(f_H^{(1)})^2}{4}\rho^2 + \frac{(f_H^{(1)})^2 f_H^{(2)}}{24}\rho^4 + \mathcal{O}(\rho^5)$$

Finiteness conditions

$$R = -\frac{\left(z f^{(1)} + 4f\right) h^{(1)}}{2z\,h} + \frac{f\,h^{(2)}}{h} - \frac{2\left(z f^{(1)} + f - 1\right)}{z^2} + \frac{f(h^{(1)})^2}{2h^2}$$



$$f^{(1)}(\rho) \propto \frac{\xi_1}{\rho} + \text{non singular terms}$$

Divergent at the horizon!

Therefore

$$\xi_1 = 0 \qquad\qquad \theta_1 = 0$$

from $f^{(1)}$ *from* $h^{(1)}$

Physical conditions from event horizon

$$\xi_1 = \theta_1 = 0 \ , \quad \xi_3 = -\frac{1}{2}(1 + 3\overline{\varpi})\theta_3 \ , \quad \xi_2 \leq \frac{1}{16 z_H} \ , \quad \theta_2 < \frac{1 + \overline{\varpi}}{8 z_H}$$

$$\bar{\varpi} = \sqrt{1 - 16 z_H \xi_2}$$

Conditions for finite Ricci & bounds for positive metric functions

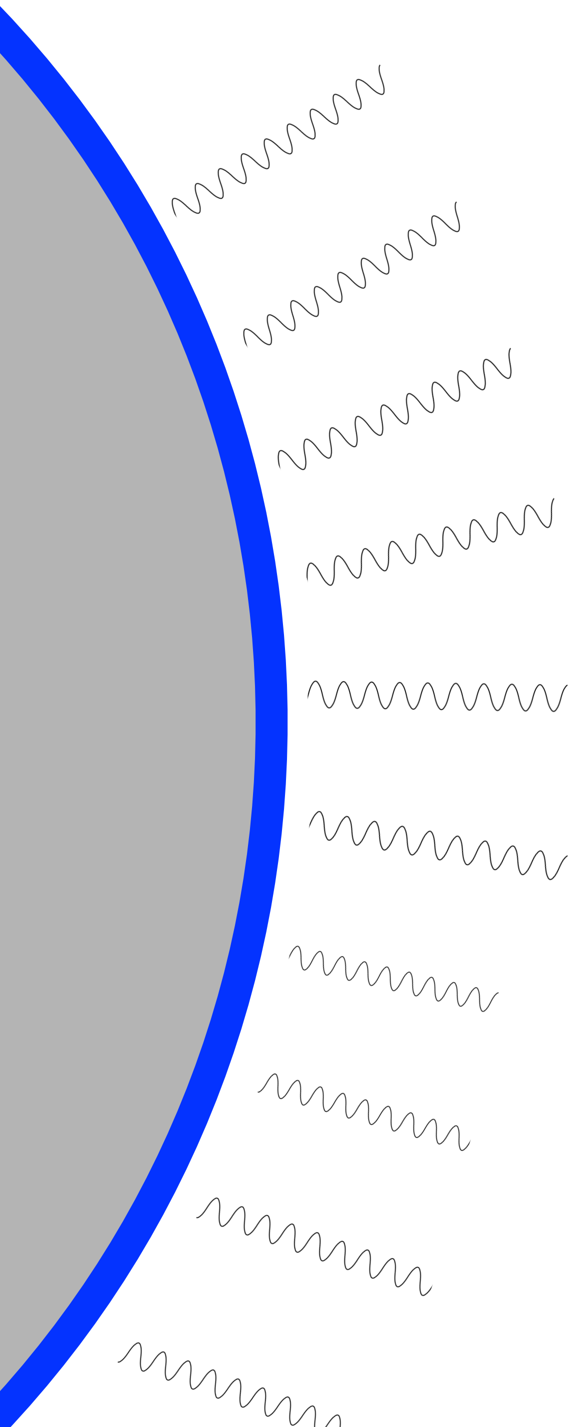
Universal constraints for physical (quantum) metrics

Del Piano, Hohenegger, Sannino 2307.13489

Temperature

Hawking's Radiation

1974



$$T = \frac{1}{8\pi M}$$

Black holes evaporate

$$t_{evaporate} \approx 2.140 \times 10^{67} \text{ years} \left(\frac{M}{M_\odot}\right)^3$$

$$t_{Universe-age} \approx 1.40 \times 10^{10} \text{ years}$$

The smaller the BH the higher the temperature

No chance to observe HR for any BH with $M \geq M_\odot$

Quantum Hawking temperature

Defined via surface gravity

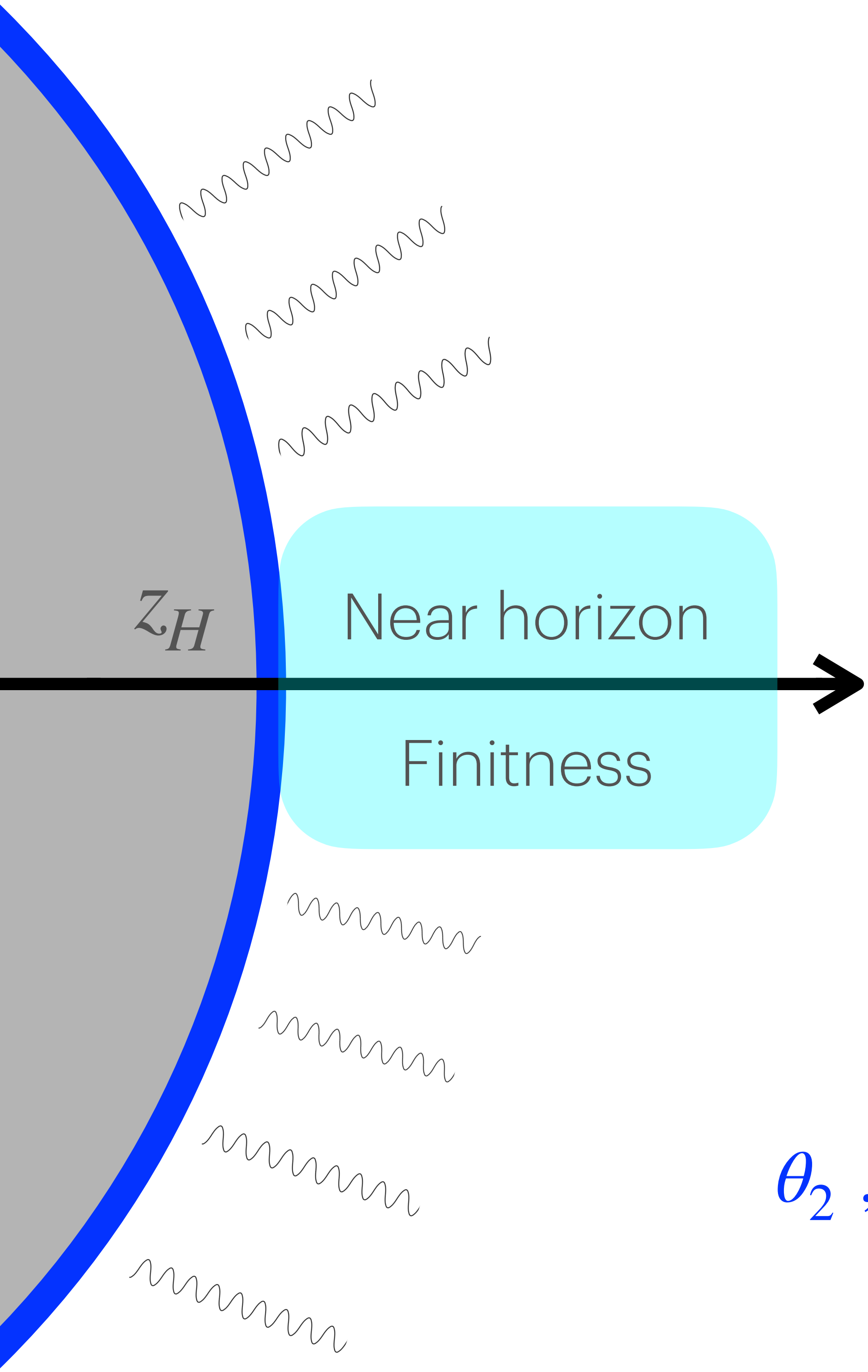
$$T_{\mathrm{H}} = \frac{\kappa}{2\pi} = \frac{1}{4\pi\ell_p}\sqrt{f_H^{(1)} h_H^{(1)}}$$

$$\ell_P^2 \kappa^2 = -\frac{1}{2} \nabla_\mu (K^t)_\nu \nabla^\mu (K^t)^\nu \Big|_{z=z_H} = \frac{1}{4} \frac{f \cdot (h^{(1)})^2}{h} \Big|_{z=z_H}$$

$$(K^t)^\mu = \delta^{\mu 0}$$

Time-like Killing vector

Quantum Hawking Temperature



$$T_{\text{Hawking}} = \frac{1}{8\,\pi\,M} = \frac{1}{4\,\pi\,z_S}$$

$$\Downarrow$$

$$T_{\text{QBH}} = \frac{\sqrt{1 + \overline{\varpi} - 8 z_H \theta_2}}{4\pi\sqrt{2} z_H}$$

$$\bar{\varpi} = \sqrt{1 - 16 z_H \xi_2}$$

z_H Quantum horizon

θ_2 , ξ_2 Parameters specifying the underlying QG theory

Del Piano, Hohenegger, Sannino 2023,

Living on the edge: Quantum Black Hole Physics from the Event Horizon

Entropy

Fine - grained entropy

Via von Neuman

$$S_{vN} = -Tr[\rho \log \rho]$$

ρ is the density matrix

Invariant under unitary time evolution $\rho \to U\rho U^{-1}$

Quantifies our ignorance about the quantum state of the system

Coarse - grained entropy

Measure a subset of observables A_i

Density matrices $\tilde{\rho}$ yielding the same result as ρ

$$Tr[\tilde{\rho} A_i] = Tr[\rho A_i]$$

Maximise von Neuman S for all $\tilde{\rho}$

Example: Thermodynamic S with A_i ~ energy and volume

Obeys 2nd law of thermodynamics, increases under time evolution

We shall use

The first law of thermodynamics

$$\mathrm{d}M = M_\mathrm{P}\,\mathrm{d}\chi = T_\mathrm{H}\,\mathrm{d}S$$

$$S = M_\mathrm{P}\int \frac{\mathrm{d}\chi}{T_\mathrm{H}(\chi)}$$

Need also dependence on mass

$$d_H(\chi) \qquad z_H(\chi) \qquad \xi_2(\chi) \qquad \theta_2(\chi)$$

To be expected, information on the quantum states required

Classical Hawking thermodynamic limit

$$\xi_2 = \theta_2 = 0 \qquad d_H = d_0 = \pi\chi \qquad z_H = 2\chi$$

Direct dependence on d_H disappears

$$T_{\mathrm{H}}^{\mathrm{Classical}} = \frac{1}{8\ell_P \pi \chi} = \frac{1}{8\,\pi\, M}$$

$$\Downarrow$$

$$S^{\mathrm{Classical}} = M_{\mathrm{P}} \int \frac{\mathrm{d}\chi}{T_{\mathrm{H}}(\chi)} = 4\pi\chi^2$$

So far…

Event horizon finiteness conditions

Ricci and Kretschmann scalars finite at the (outer) horizon

General expression for (quantum) Hawking temperature

Power of constraints

The Unsafest Safe metric in Quantumland

Del Piano, Hohenegger, Sannino 2307.13489

Bonanno Reuter hep-th/0002196

$$G_{\text{Newton}} \longrightarrow G(k) = \frac{G(k=0)}{1 + \omega\, G(k=0)\, k^2}$$

$$k(z) = \frac{\xi}{d(z)}$$

$$h(z) = f(z) =:= 1 - \frac{2\chi}{z} \frac{1}{1 + \widetilde{\omega}/d(z)^2}$$

Φ

$$\widetilde{\omega} = \frac{118}{15\pi} ??$$

$$d(z) = \int_0^z \frac{1}{\sqrt{|f(z)|}}$$

Violates the finiteness conditions

$$\xi_1 = \theta_1 \neq 0$$

This is "not" a black hole

Minimal (quantum) metric

A (novel) solution of the fitness conditions is

$$\log[\Phi\left(d_H+\rho\right)] = \log[\Psi\left(d_H+\rho\right)] = -\frac{3\phi_2}{2d_H^2} + \frac{\rho^2(d_H+3\rho)\phi_2}{2d_H^2(d_H+\rho)^3} \qquad \phi_2 \in \Re$$

Further assuming

$$z_H = 2\chi \exp\left(-\frac{3\phi_2}{2d_H^2}\right)$$

$$\phi_2 = \mathcal{O}(\chi^0) \qquad d_H = \pi\chi + \mathcal{O}(\chi^0)$$

$$T_{\rm H} = \frac{1}{8\pi\chi}\left[1 + \frac{(3\pi^2-16)\phi_2}{2\pi^4\chi^2} + \mathcal{O}(\chi^{-3})\right]$$

$$S_{\rm H} = 4\pi\chi^2\left[1 - \frac{\left(3\pi^2-16\right)\phi_2}{3\pi^2\chi^2}\log(\chi^2) + \mathcal{O}\left(\frac{1}{\chi^4}\right)\right] + \text{const}$$

Del Piano, Hohenegger, Sannino 2307.13489

Stepping away from the abyss

Del Piano, Hohenegger, Sannino

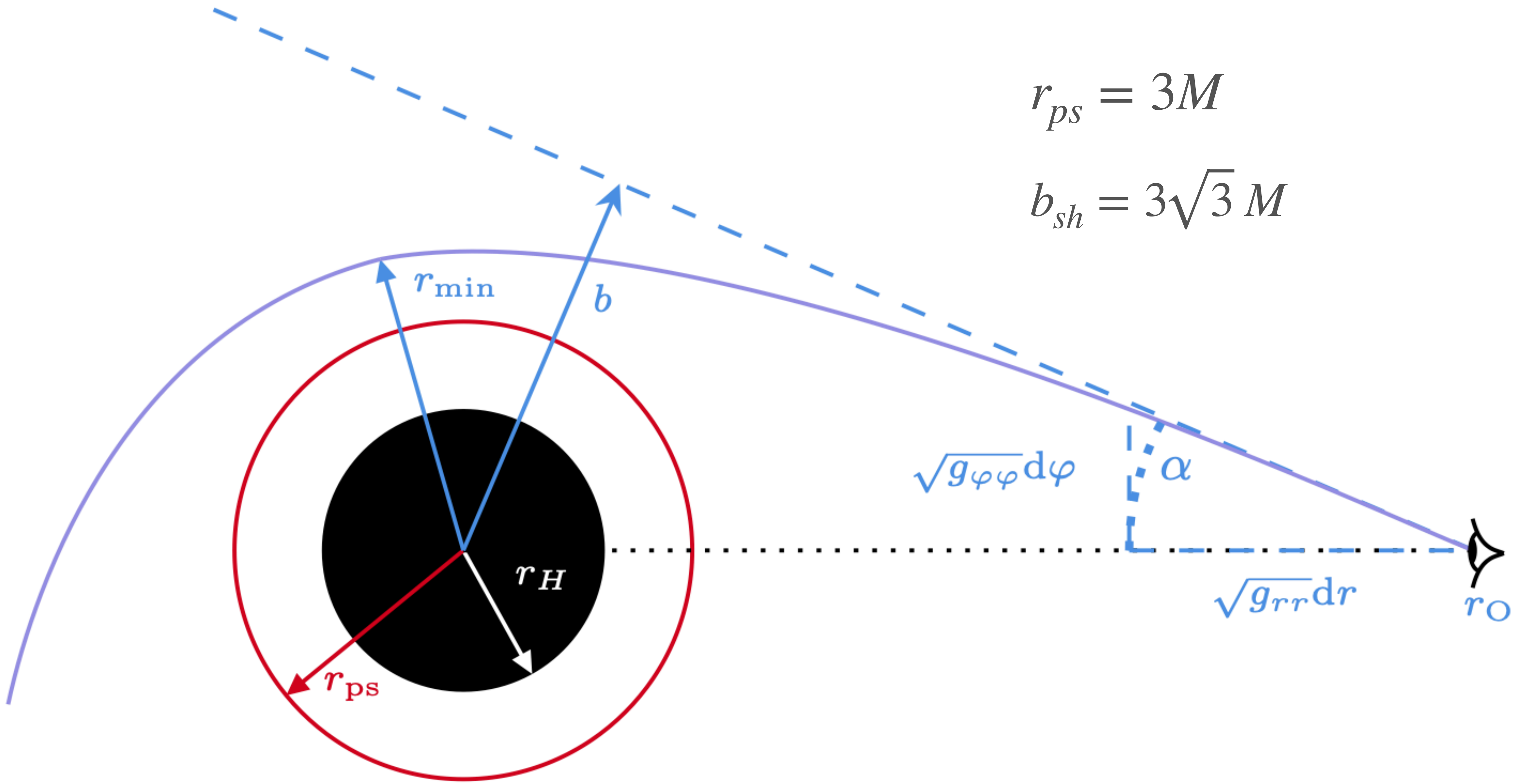
2412.13673

Photon sphere and shadow

Schwarzschild

$$r_{ps} = 3M$$

$$b_{sh} = 3\sqrt{3}\,M$$



Perlick, Phys. Rept., 947:1–39, 2022

Closed form result via Padé

Del Piano, Hohenegger, Sannino 2412.13673

$$b_{sh} = 3\sqrt{3}\, M$$

$$\downarrow$$

$$\lim_{N\to\infty} \frac{b_{sh}(N)}{M} = \frac{b_{sh}}{M} \sim \frac{3\sqrt{3}}{2M}\Psi(2M\, s) \qquad s \sim 1.5245 \sim \frac{\rho_{ps}^{class}}{2M}$$

$$\left| \frac{\Psi(2M_{87*}\, s)}{2M_{87*}} - 1 \right| \leq 0.17$$

Reconstruct quantum deviations

Kocherlakota, Phys. Rev. D 103(20):104047

Charged BHs

At the classical level can be extremal

Proper time of free-falling observer better for EMD description

Applications to the Weak Gravity conjecture

Damia Paciarini, Del Piano, Hohenegger, Sannino

2504.20810

BTZ - 3D rotating BH

Bañados–Teitelboim–Zanelli (BTZ): negative curvature & no singularity

Universal constraints near the horizon via EMD

General form of the Hawking temperature

Effective description also near the origin

Potential applications: AdS/CFT

Damia Paciarini, Myszkowski, Hohenegger, Sannino

2412.15960

What next

Comparing with other effective approaches
[Johannsen-Psaltis, Rezzolla-Zhidenko, PPN, etc…]

General constraints for non-singular EMD metrics, 2506.12620

Corrections to BH (mergers) & gravity waves (QNM,…)

Extended gravitational objects

Wormholes and cosmology

Doubly Quantum Entanglement

Applications

Universal constraints on quantum gravity landscape via:

Monitoring star orbits around BHs

Quantum corrections to gravity waves in BH mergers

Quantum evaporation for primordial BHs


Thanks IPPP @ Durham for PASCOS

Thank you

Space, time and matter are emerging concepts