Multi-component Dark Matter in a simplified (E_6SSM)

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Why Go Beyond the Standard Model?

- Standard Model (SM) is successful but incomplete:
  - No explanation for neutrino masses
  - No dark matter candidate
  - Hierarchy and naturalness problems
  - Absence of gauge coupling unification
- Supersymmetry (SUSY) addresses many of these, but MSSM faces issues like the μ-problem and lack of rich collider signatures
- Local supersymmetry (SUSY) $\rightarrow$ Supergravity (SUGRA)
- SUGRA partially unifies gravity with SM interactions
- However, SUGRA is non-renormalizable $\rightarrow$ Treated as an effective low-energy theory

String Theory and Gauge Unification

- A UV-complete theory is needed beyond SUGRA
- Best candidate: 10D heterotic superstring theory based on $E_8 \times E_8'$
- In strong coupling: described by 11D SUGRA (M-theory)
- Compatible with unification scale M_X
- Compactification of extra dimensions breaks $E_8 \to E_6$ (or its subgroups)
- E_6 governs observable sector
- The second E_8' becomes the hidden sector, where SUSY breaking occurs.
- Generates soft SUSY-breaking terms in visible sector, characterized by the gravitino mass: $m_{3/2} \sim \mathcal{O}(\text{TeV})$
- Links string theory to realistic low-energy SUSY models

E_6 Grand Unified Theory

- E_6 is an exceptional complex Lie group that does not belong to the classical series: $SU(N)$, $SO(N)$, or $Sp(N)$.
- It has **rank** 6 and **dimension** 78.
- The fundamental representation is **27** (complex).
- The E_6 gauge group can be broken down to the SM gauge group as follows:

$$\begin{aligned} E_6 \quad &\longrightarrow \quad SO(10) \times U(1)_\psi \\ &\longrightarrow \quad SU(5) \times U(1)_\chi \times U(1)_\psi \\ &\longrightarrow \quad SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_\chi \times U(1)_\psi . \end{aligned}$$

- The low-energy gauge group is the SM extended by an additional $U(1)_N$ symmetry.
- This extra symmetry arises as:

$$U(1)_N = \cos\vartheta \, U(1)_\chi + \sin\vartheta \, U(1)_\psi$$

Decomposition and Low-Energy Content of $\mathbf{27}_i$

- The fundamental representation $\mathbf{27}_i$ of E_6, where $i = 1, 2, 3$, decomposes under $SU(5) \times U(1)_N$ as:

$$\mathbf{27}_i \to (10, \tfrac{1}{\sqrt{40}})_i + (\bar{5}, \tfrac{2}{\sqrt{40}})_i + (\bar{5}, \tfrac{-3}{\sqrt{40}})_i \\ + (5, \tfrac{-2}{\sqrt{40}})_i + (1, \tfrac{5}{\sqrt{40}})_i + (1, 0)_i$$

Field Assignments:

- $(10, \frac{1}{\sqrt{40}})_i$, $(\bar{5}, \frac{2}{\sqrt{40}})_i$: Standard Model matter
- $(\bar{5}, \frac{-3}{\sqrt{40}})_i$, $(5, \frac{-2}{\sqrt{40}})_i$: Higgs doublets H_{di}, H_{ui} and exotic quarks $\bar{D}_i$, D_i
- $(1, \frac{5}{\sqrt{40}})_i$: SM singlets S_i
- $(1, 0)_i$: Right-handed neutrinos

SUSY Breaking and the Inert Sector

- At low energies, $U(1)_N$ is spontaneously broken by the VEV of the singlet S_3,

$$\langle S_3 \rangle = \frac{s}{\sqrt{2}},$$

  generating a Z' boson with mass $\sim 1\,\text{TeV}$.

- Anomaly cancellation is automatic with three complete **27**-plets surviving down to low energies.
- Only the third generation Higgs doublets and singlet acquire VEVs:

$$\langle H_{d3}^0 \rangle = \frac{v_d}{\sqrt{2}}, \quad \langle H_{u3}^0 \rangle = \frac{v_u}{\sqrt{2}}, \quad \langle S_3 \rangle = \frac{s}{\sqrt{2}}$$

- The first two generations $H_{d\alpha}, H_{u\alpha}, S_\alpha$ $(\alpha = 1, 2)$ remain inert, suppressing FCNCs due to their weak Yukawa couplings.

Discrete Symmetries in the E_6SSM

In addition to gauge symmetries, the model includes four discrete $\mathbb{Z}_2$ symmetries that control flavor, proton stability, and supersymmetric interactions:

Field	Z_2^H	Z_2^L	Z_2^B	$Z_2^M \equiv R$
S_α	$-$	$+$	$+$	$+$
$H_{d\alpha}, H_{u\alpha}$	$-$	$+$	$+$	$+$
S_3	$+$	$+$	$+$	$+$
H_{d3}, H_{u3}	$+$	$+$	$+$	$+$
Q_i, u_i^c, d_i^c	$-$	$-$	$-$	$-$
L_i, e_i^c	$-$	$-$	$-$	$-$
$\bar{D}_i, D_i$	$-$	$+$	$-$	$+$

- Z_2^H distinguishes the third generation from inert generations, suppressing flavor-changing interactions (e.g., forbids $\lambda_{\alpha 33}, \lambda_{3\alpha 3}, \lambda_{\alpha\beta\gamma}$ with $\alpha, \beta, \gamma = 1, 2$).
- Z_2^L **or** Z_2^B ensures proton stability by forbidding baryon- and lepton-number violating terms.
- $Z_2^M \equiv R$**-parity** is automatically conserved due to $U(1)_N$ and stabilizes the Lightest Supersymmetric Particle (LSP).

Solving the μ-Problem

- In this case, the low-energy effective superpotential takes the form

$$W = Y_u Q U^c H_u + Y_d Q D^c H_d + Y_e L E^c H_d + Y_\nu L \nu^c H_u + \lambda S H_d H_u,$$

where the last term $\lambda S H_d H_u$ represents the combination $\lambda_{ijk} S_i H_{d_j} H_{u_k}$.

- As a result, the effective μ-parameter is dynamically generated via

$$\mu = \lambda_{333} \frac{s}{\sqrt{2}}$$

leading to the term $\mu H_{d3} H_{u3}$ in the superpotential.

- This mechanism naturally resolves the so-called μ-problem of the MSSM: ($\mu H_u H_d$) is required to be $\mathcal{O}(M_Z)$.

- Its origin is unexplained: fine-tuning problem

Active Neutralino States in the E_6SSM

- The neutralinos $\tilde{\chi}_i^0$ $(i = 1, \ldots, 6)$ are mass eigenstates formed from:
  - Gauginos: $\tilde{B}$ (bino), $\tilde{W}^0$ (wino), $\tilde{B}'$ (B'ino)
  - Higgsinos: $\tilde{H}_d^0$, $\tilde{H}_u^0$
  - Singlino: $\tilde{S}$
- In the basis $(\lambda_{\tilde{B}}, \tilde{W}^0, \tilde{H}_d^0, \tilde{H}_u^0, \tilde{S}, \lambda_{B'})$, the neutralino mass matrix is 6×6:

$$m_{\tilde{\chi}^0} = \begin{pmatrix} M_1 & 0 & -M_Z s_W c_\beta & M_Z s_W s_\beta & 0 & 0 \\ 0 & M_2 & M_Z c_W c_\beta & M_Z c_W s_\beta & 0 & 0 \\ -M_Z s_W c_\beta & M_Z c_W c_\beta & 0 & -\frac{1}{\sqrt{2}} v_s \lambda & -\frac{1}{\sqrt{2}} \lambda v s_\beta & m_{\lambda_{B'} \tilde{H}_d^0} \\ M_Z s_W s_\beta & M_Z c_W c_\beta & -\frac{1}{\sqrt{2}} v_s \lambda & 0 & -\frac{1}{\sqrt{2}} \lambda v c_\beta & m_{\lambda_{B'} \tilde{H}_u^0} \\ 0 & 0 & -\frac{1}{\sqrt{2}} \lambda v s_\beta & -\frac{1}{\sqrt{2}} \lambda v c_\beta & 0 & \frac{1}{2}\sqrt{\frac{5}{2}} g_N v_s \\ 0 & 0 & m_{\tilde{H}_d^0 \lambda_{B'}} & m_{\tilde{H}_u^0 \lambda_{B'}} & \frac{1}{2}\sqrt{\frac{5}{2}} g_N v_s & M_1' \end{pmatrix}$$

- The matrix is diagonalized via a unitary matrix N: $N^* m_{\tilde{\chi}^0} N^\dagger = \mathrm{diag}(m_{\tilde{\chi}_i^0})$
- The lightest state (LSP) is:

$$\tilde{\chi}_1^0 = \sum_{i=1}^{6} N_{1i} \psi_i$$

where ψ_i denotes the gauge/Higgs eigenstates.

Inert Neutralino States

- The inert sector includes additional neutralinos from the first two generations of Higgsino doublets:

$$(\tilde{h}_{d1}^{0,I}, \tilde{h}_{d2}^{0,I}, \tilde{h}_{u1}^{0,I}, \tilde{h}_{u2}^{0,I})$$

- The 4×4 mass matrix is:

$$m_{\tilde{\chi}^{0,I}} = -\frac{v_s}{\sqrt{2}} \begin{pmatrix} 0 & 0 & \lambda_{311} & \lambda_{312} \\ 0 & 0 & \lambda_{321} & \lambda_{322} \\ \lambda_{311} & \lambda_{312} & 0 & 0 \\ \lambda_{321} & \lambda_{322} & 0 & 0 \end{pmatrix}$$

- Inert singlinos are neglected in this simplified model:
  - No Yukawa couplings
  - Remain massless and decoupled

Active Higgs Sector in the E_6SSM

- The scalar sector includes active doublets H_u, H_d and a singlet S.
- Mixing leads to 3×3 mass matrices for both:
  - CP-even scalars (including the SM-like Higgs with $m \simeq 125$ GeV)
  - CP-odd pseudoscalars
- Extra Higgs states are typically heavier than the SM-like Higgs.

Inert Scalar Sector

- Inert neutral scalars form mass eigenstates in the basis:

$$\left(h_1^{0I}, h_2^{0I*}\right), \quad \left(h_1^{0I*}, h_2^{0I}\right)$$

- Mass matrix:

$$m_h^{0I} = \begin{pmatrix} m_{11}^2 & m_{12}^2 \\ m_{12}^{2T} & m_{22}^2 \end{pmatrix}$$

- Z_2^H symmetry ensures CP conservation:
  - CP-even and CP-odd inert scalars are degenerate in mass
  - Physical states are complex scalars: h_i

- Inert singlet scalars are decoupled with mass matrix:

$$m_{s^I}^2 = \left[-\frac{1}{16} g_N^2 \left(2v_2^2 + 3v_1^2 - 5v_s^2\right) + m_s^2\right] I_{2\times 2}$$

- Mass is controlled by $g_N^2 v_s^2$ (related to Z' scale) and m_s^2:
  - Inert singlet scalars are always heavy if $m_s^2 > 0$

Two-Component Dark Matter Scenario

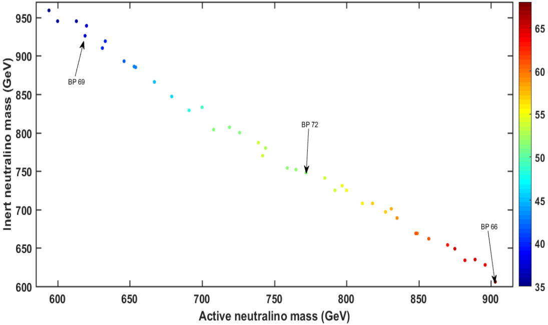
- **Setup:** Two distinct DM candidates: one stabilized by R-parity (e.g., higgsino or wino LSP), and the other by a Z_2 symmetry (e.g., inert higgsino or scalar).
- **Motivation:** Both components are typically underabundant individually (e.g., higgsino/wino), making them ideal for multi-component DM.
- **Thermal History:** Heavier DM freezes out first. If coannihilation between components is allowed, it can drastically alter relic abundances.
- **Constraints:** Inert (pseudo)scalars interact via Z-boson and are excluded by Direct Detection bounds unless subdominant.
- **Tools Used:**
  - SARAH 4.14.1 model implementation
  - SPheno 4.0.3 spectrum generation
  - MicrOmegas 5.0.8 relic abundance & DM observables

Two Higgsinos as Dark Matter Candidates

- Sub-TeV MSSM higgsinos lead to underabundant relic density in the single-component case (Profumo:2004).
- In a two-component scenario, both active and inert higgsinos can contribute.
- Main annihilation channel: neutralinochargino coannihilation via SM gauge bosons.
- The two components freeze out almost independently; relic density increases nearly linearly with mass.
- Scans confirm viable regions where:

$$m(\tilde{H}^0) + m(\tilde{H}^{0,I}) \approx 1.53 \pm 0.03 \text{ TeV}$$

Relic Density and Higgsino Masses

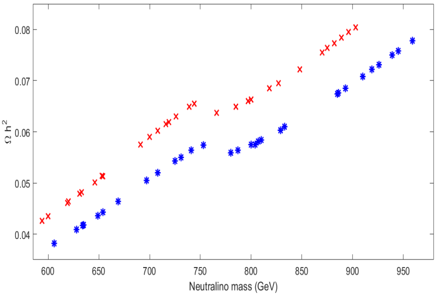


- The data points that give a relic density of $\Omega_{CDM} h^2 = 0.120 \pm 0.002$.
- The color indicates the percentage of the active higgsino component of the total relic density.
- We pick three of the data points, indicated by the arrows, for studying direct and indirect detection in more detail.

Benchmarks: Active and Inert Higgsinos

- Three representative benchmark points (BPs) analyzed:

Benchmark	Active mass (GeV)	Inert mass (GeV)	$\Omega^{A} h^2$	$\Omega^{I} h^2$
BP66	903	606	0.0804	0.0382
BP69	619	926	0.0461	0.0731
BP72	766	753	0.0637	0.0575



- Relic density of the individual higgsino components (red for active higgsino, blue for inert higgsino) for points satisfy the relic density constraint.

Annihilation Channels and Detection Prospects

- Dominant channels:
  - $\tilde{\chi}^0\tilde{\chi}^0 \to Z \to f\bar{f}$
  - $\tilde{\chi}^0\tilde{\chi}^\pm \to W^\pm \to f\bar{f}'$
- Higgs-mediated annihilation negligible due to small singlino component and $\lambda \sim \mathcal{O}(0.1)$.
- Spin-independent cross section:
  - Active higgsino: $\sim$1 order below Xenon1T bounds.
  - Inert higgsino: 12 orders smaller than active.
- Spin-dependent cross section also dominated by active higgsino.

Summary

- We studied a two-component DM scenario in the string-inspired E_6SSM.
- The DM candidates are:
  - **Active higgsino-like neutralino**: couples to SM fermions.
  - **Inert higgsino-like neutralino**: no direct coupling to SM fermions, but interacts via $SU(2)_L$ gauge bosons.
- Stability ensured by R-parity and Z_2^H symmetry.
- The relic density constraint $\Omega h^2 = 0.12 \pm 0.002$ implies:

$$m_{\chi_{\text{active}}} + m_{\chi_{\text{inert}}} \approx 1.5 \text{ TeV}$$

- Three benchmark cases studied: active heavier, inert heavier, and degenerate mass.
- **Direct Detection**:
  - Active higgsino detectable in future (Xenon-nT, Darwin).
  - Inert higgsino has suppressed cross section, likely undetectable.