

Neutron star heating by DM in an ALP mediated LFV model



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21/07/2025

AGENDA

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01 MOTIVATION

04 KINETIC ENERGY
DEPOSITION AND
HEATING

02 NEUTRON STARS

05 RESULTS

03 CAPTURE RATE OF
DM

06 CONCLUSIONS

01

Motivation

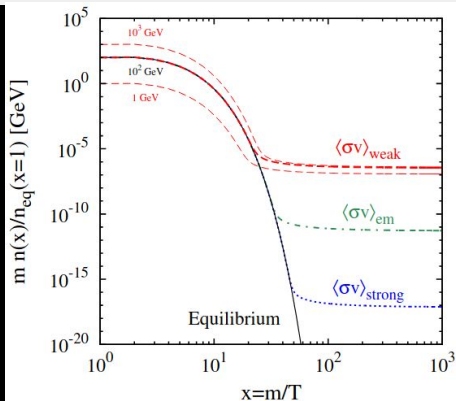
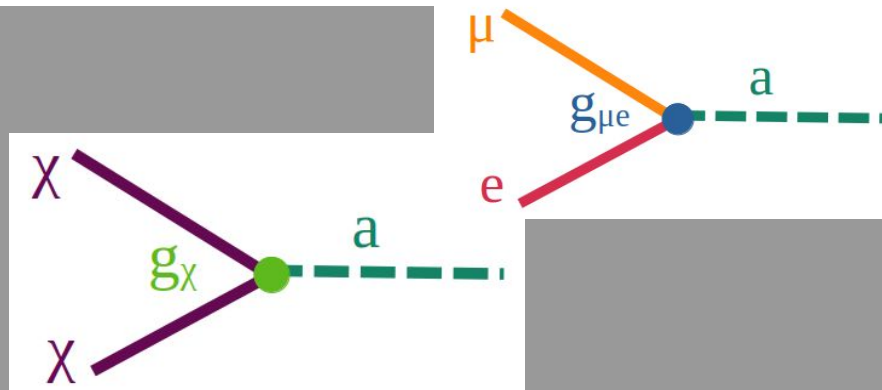
LFV Dark matter model (μe sector)

Challenges

$$\mathcal{L} \supset -ig_{e\mu} a \bar{e} [\sin \phi + \cos \phi \gamma^5] \mu + \text{h.c.}$$

$$\mathcal{L} \supset -ig_{\chi} a \bar{\chi} \gamma^5 \chi$$

1911.06279, 2310.05827, 2407.15942



Credit:
1204.3622

p-wave: $\chi\chi \leftrightarrow aa$ 2310.05827

$$\langle \sigma v \rangle_{aa} \simeq \frac{6}{x_{f.o.}} \frac{g_{\chi}^4}{24\pi} \frac{m_{\chi}^2 (m_{\chi}^2 - m_a^2)^2}{(2m_{\chi}^2 - m_a^2)^4} \left(1 - \frac{m_a^2}{m_{\chi}^2} \right)^{1/2}$$

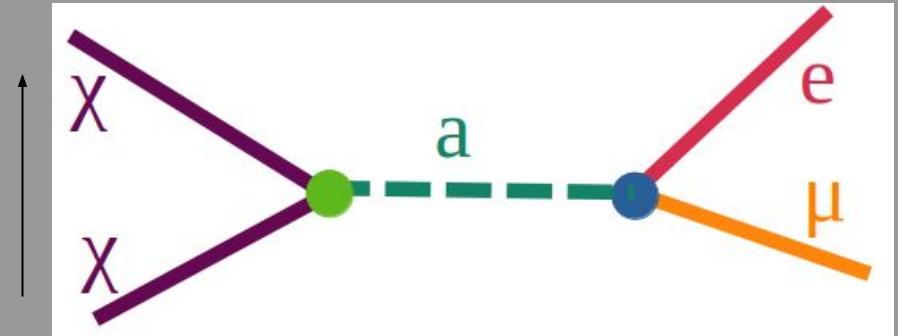
s-wave: $\chi\chi \leftrightarrow \mu e$ 2407.15942

$$\langle \sigma v \rangle_{\mu e} \simeq \frac{g_{\mu e}^2 g_{\chi}^2}{16\pi} \frac{(4m_{\chi}^2 - m_{\mu}^2)^2}{m_{\chi}^2 (4m_{\chi}^2 - m_a^2)^2}$$

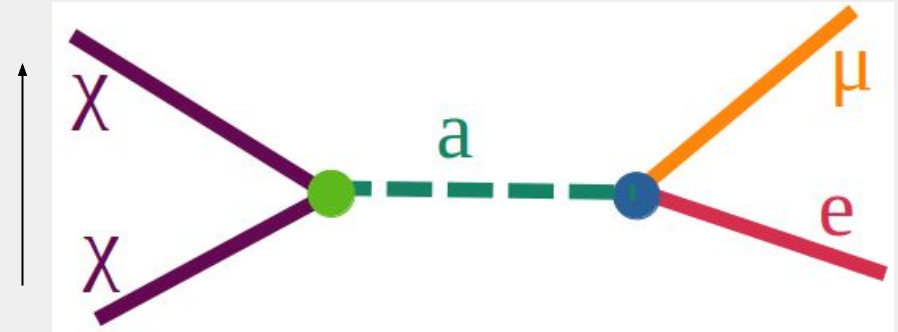
LFV Dark matter model (μe sector)

Difficulties in DM detection

1. No detectors made from muons



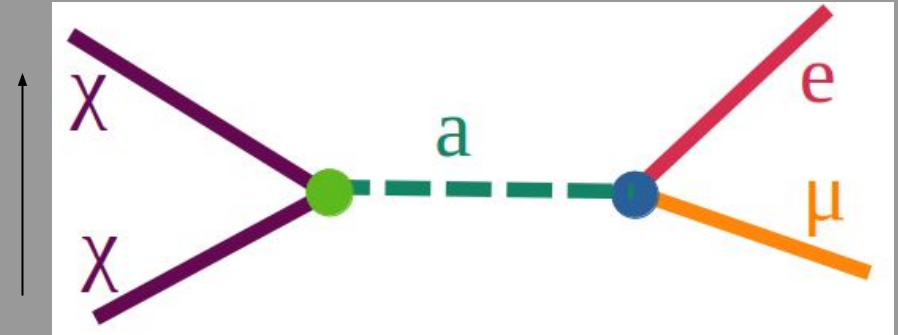
2. Halo DM is not sufficiently boosted to upscatter



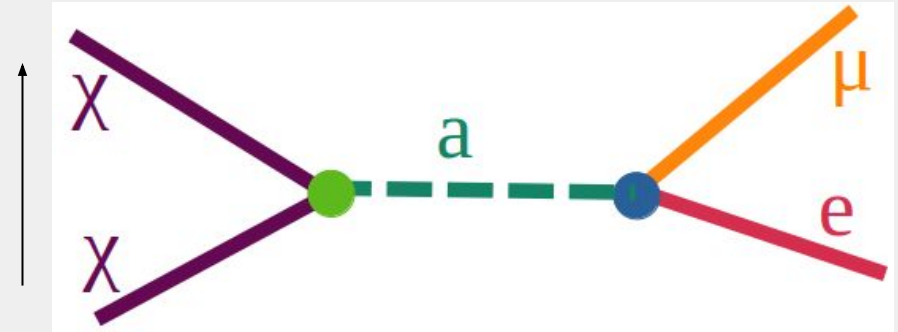
LFV Dark matter model (μe sector)

Advantages of NS as detectors

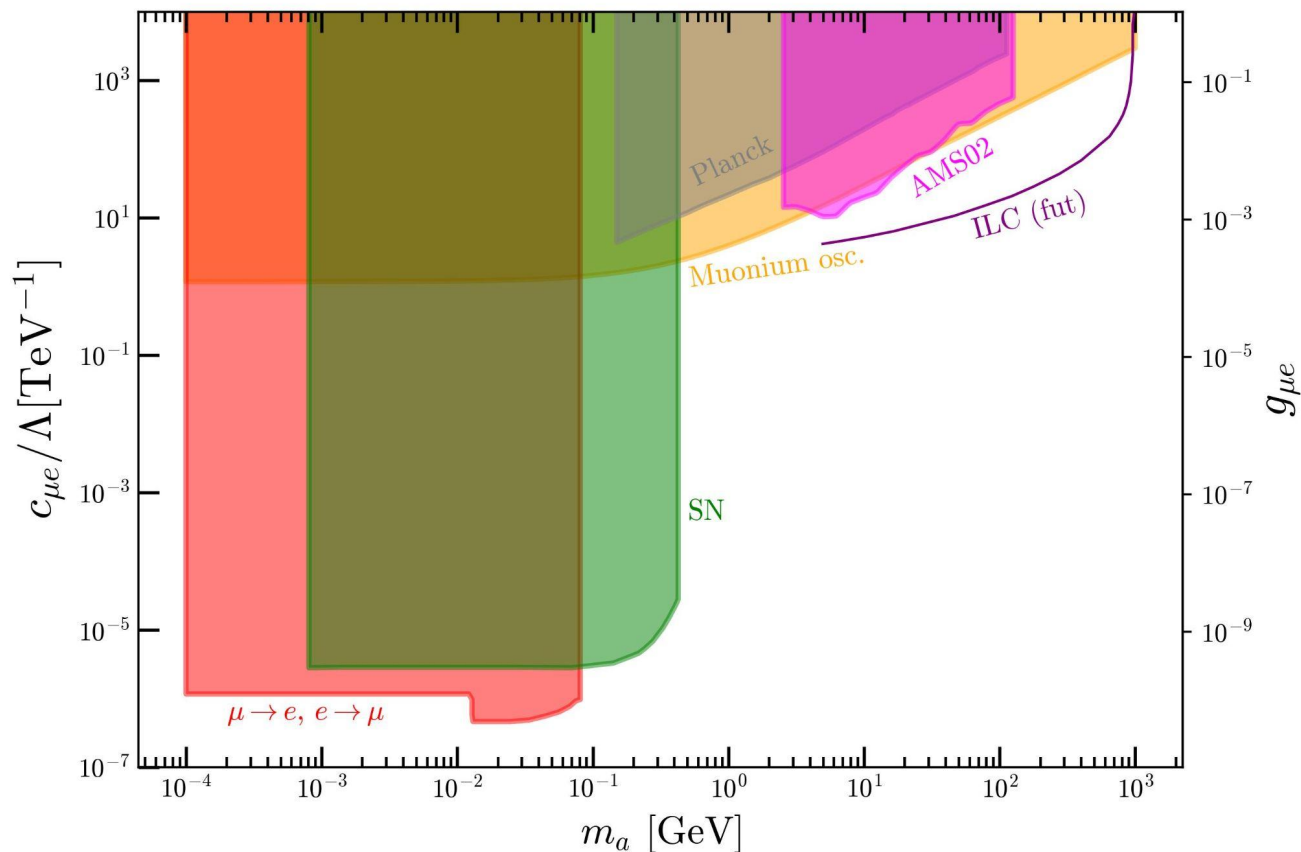
1. There are stable muons in NSs



2. Halo DM is boosted thanks to NS strong gravitational field



Current (and future) bounds



Planck: CMB [1807.06209]
(assumption: $\Omega h^2 = 0.12$)

AMS02: annihilation into
positrons [2107.10261]
(assumption: $\Omega h^2 = 0.12$)

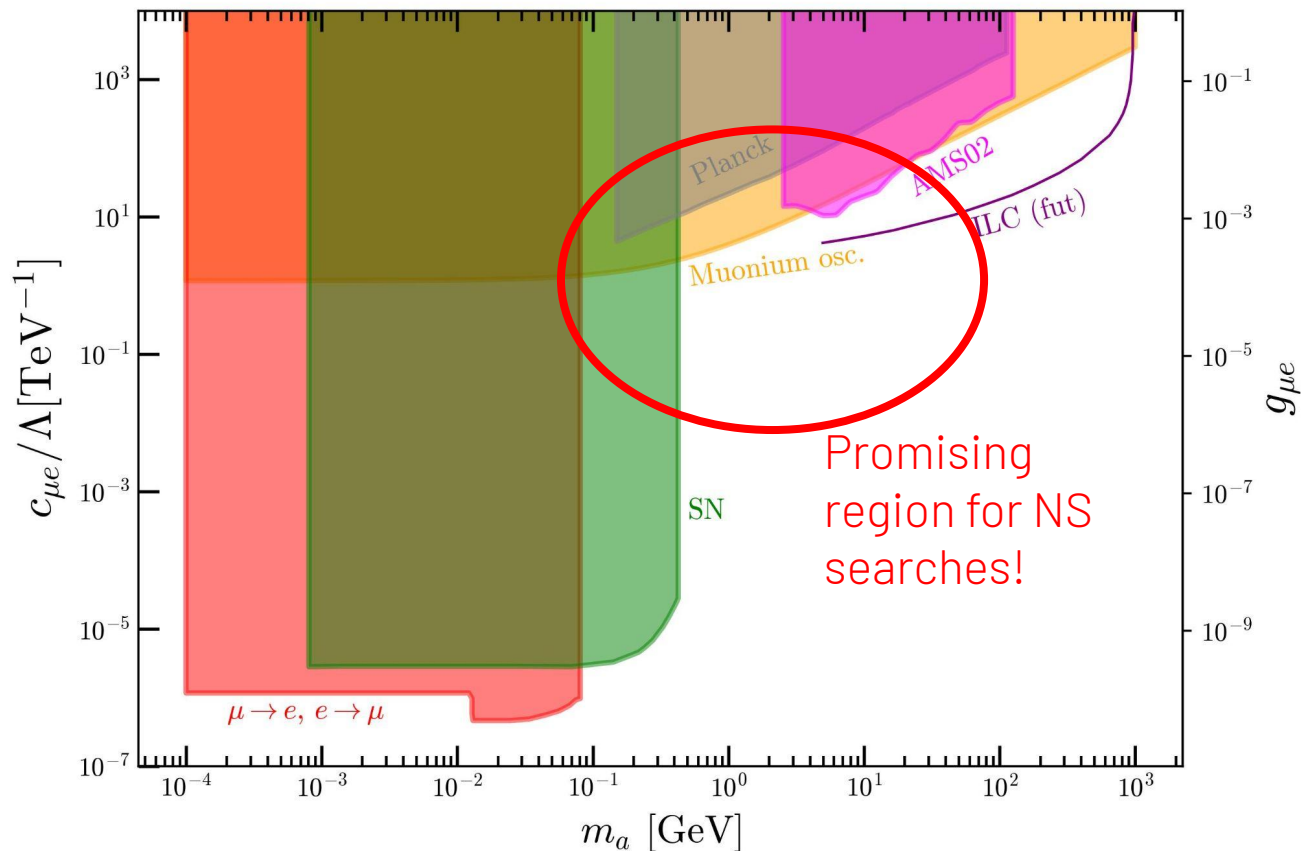
Muonium oscillation:
especially from MACS ($\mu^- e^+ \leftrightarrow \mu^+ e^-$) [1711.08430]

μ to e + inv.: from TWIST,
SINDRUM, MEG...
[1908.00008]

Supernovae: axion emission
[2309.03889]

ILC [1711.08430]

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02

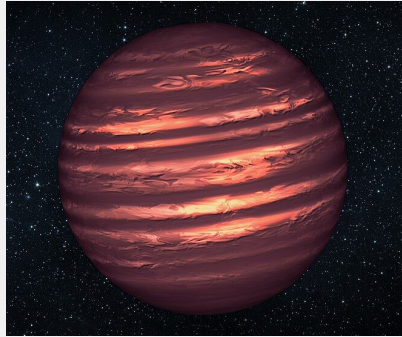
Neutron stars

End of life of stars

10

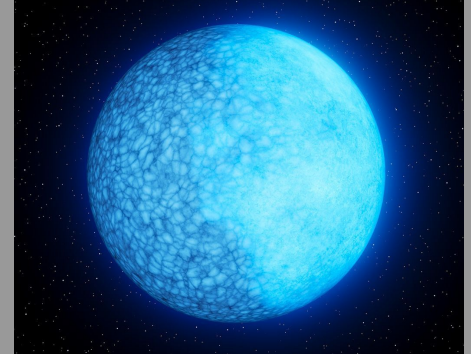
Brown dwarf

13 - 80 M_J



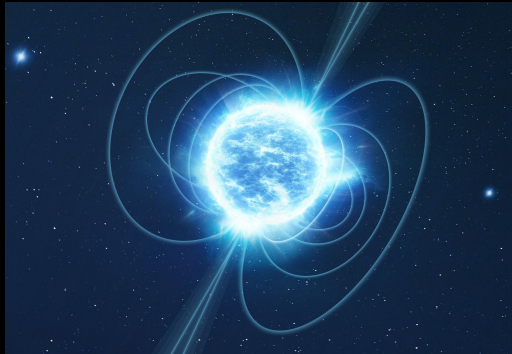
White dwarf

0.17 - 1.33 M_{\odot}

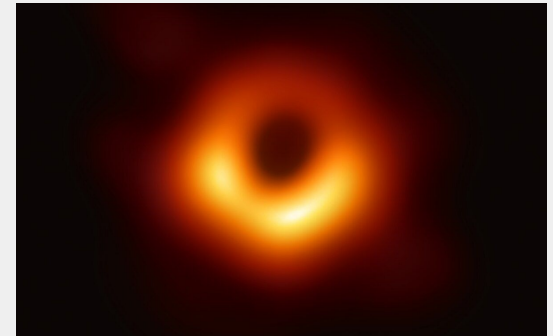


Neutron star

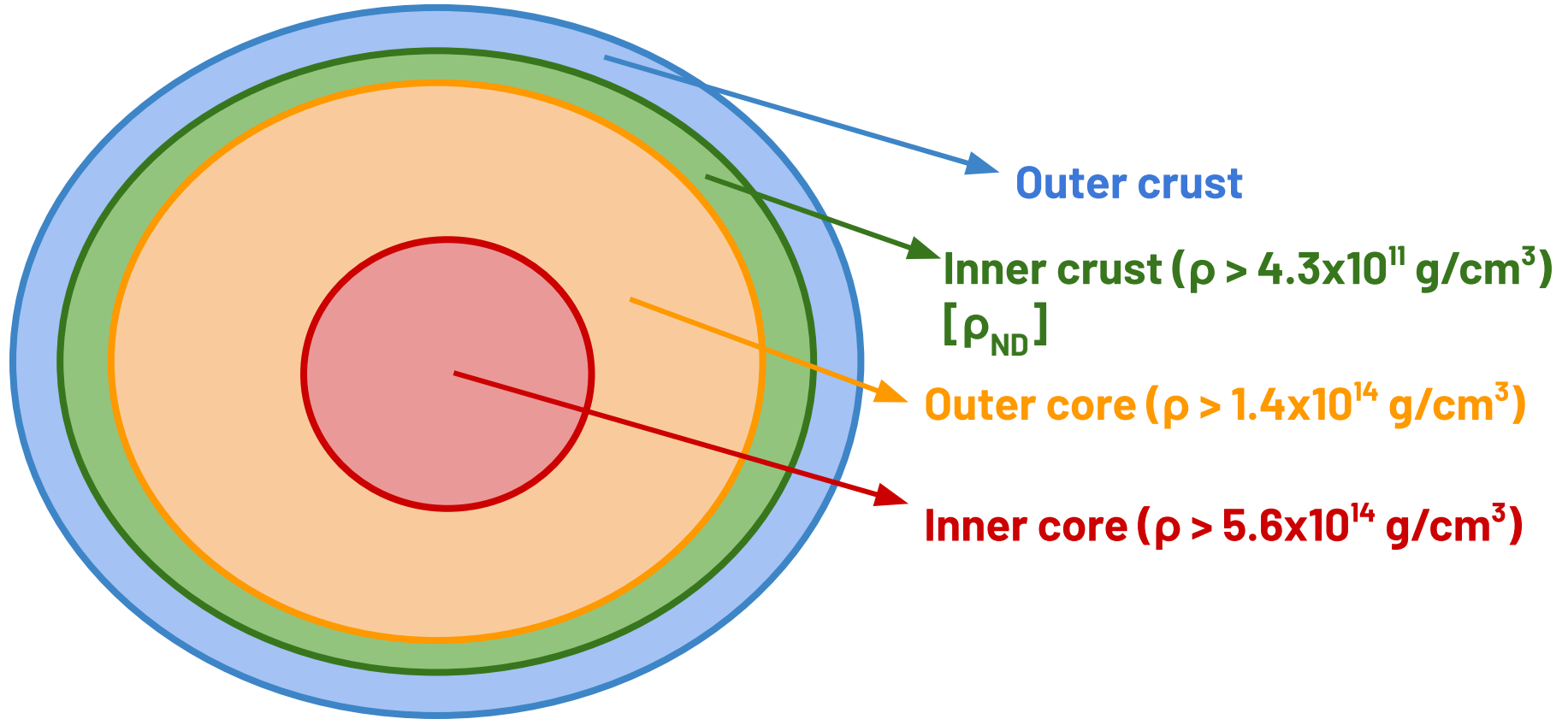
1.1 - 2.3 M_{\odot}



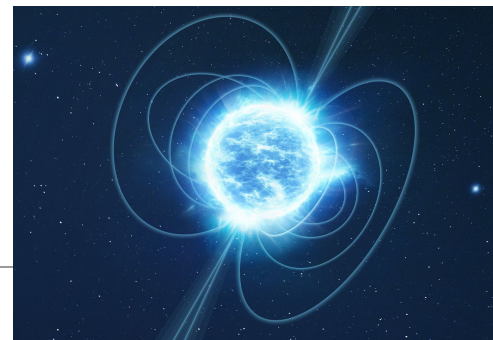
Black hole



Parts of a Neutron Star



Main characteristics of NS



Density

- Up to 10^{15} – 10^{16} g/cm³.
- Core made of $npe\mu$ or exotic matter

Mass

- 1.1 – 2.3 M_{\odot}

How to model

- EoS + **TOV equations**
(Tolman-Oppenheimer-Volkoff) / Hartle and Thorne / Komatsu-Eriguchi-Hachisu

EoS

- *Ab initio*
 - Pheno: BMF (Brussels-Montreal Functionals, BSk), RMF (Relativistic Mean Field)
 - ...
-

Brussels-Montreal Functionals

BSk-N EoS

Effective forces

1. Skyrme form (16 parameters)
2. Pairing force
3. Wigner terms (esp. for small A)
4. Collective energy correction

Fitting parameters

To nuclear masses (2353, Atomic Mass Evaluation, AME 2012) using the Hartree-Fock-Bogoliubov method (multistate as Slater determinant + pairing correlations)

Neutron matter (NeuM) is modelled a (NS-core) EoS (usually using 3 body-forces).

$$\begin{aligned} v_{ij} = & t_0(1 + x_0 P_\sigma) \delta(\mathbf{r}_{ij}) + \frac{1}{2} t_1(1 + x_1 P_\sigma) \frac{1}{\hbar^2} [p_{ij}^2 \delta(\mathbf{r}_{ij}) + \delta(\mathbf{r}_{ij}) p_{ij}^2] + t_2(1 + x_2 P_\sigma) \frac{1}{\hbar^2} \mathbf{p}_{ij} \cdot \delta(\mathbf{r}_{ij}) \mathbf{p}_{ij} \\ & + \frac{1}{6} t_3(1 + x_3 P_\sigma) n(\mathbf{r})^\alpha \delta(\mathbf{r}_{ij}) + \frac{1}{2} t_4(1 + x_4 P_\sigma) \frac{1}{\hbar^2} [p_{ij}^2 n(\mathbf{r})^\beta \delta(\mathbf{r}_{ij}) + \delta(\mathbf{r}_{ij}) n(\mathbf{r})^\beta p_{ij}^2] \\ & + t_5(1 + x_5 P_\sigma) \frac{1}{\hbar^2} \mathbf{p}_{ij} \cdot n(\mathbf{r})^\gamma \delta(\mathbf{r}_{ij}) \mathbf{p}_{ij} + \frac{i}{\hbar^2} W_0(\boldsymbol{\sigma}_i + \boldsymbol{\sigma}_j) \cdot \mathbf{p}_{ij} \times \delta(\mathbf{r}_{ij}) \mathbf{p}_{ij}, \end{aligned}$$

Brussels-Montreal Functionals

BSk-N EoS

Effective forces

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Symmetry coefficient

$J = E_{\text{sym}}(n_0 = 0.16 \text{ fm}^{-3}) \Rightarrow n_0$: nuclear saturation density, $\sim 30 \text{ MeV}$

$$\delta = \frac{N - P}{A}$$

$$E_{\text{sym}} = \left. \frac{1}{2} \frac{\partial^2 E(n, d)}{\partial \delta^2} \right|_{\delta=0}$$

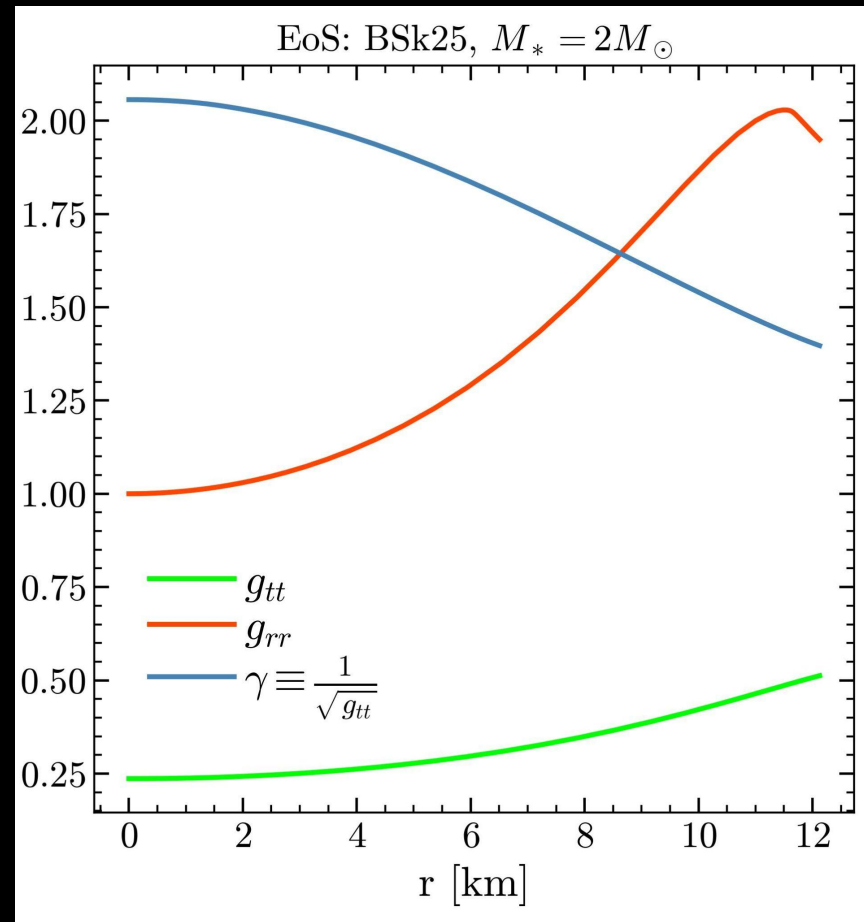
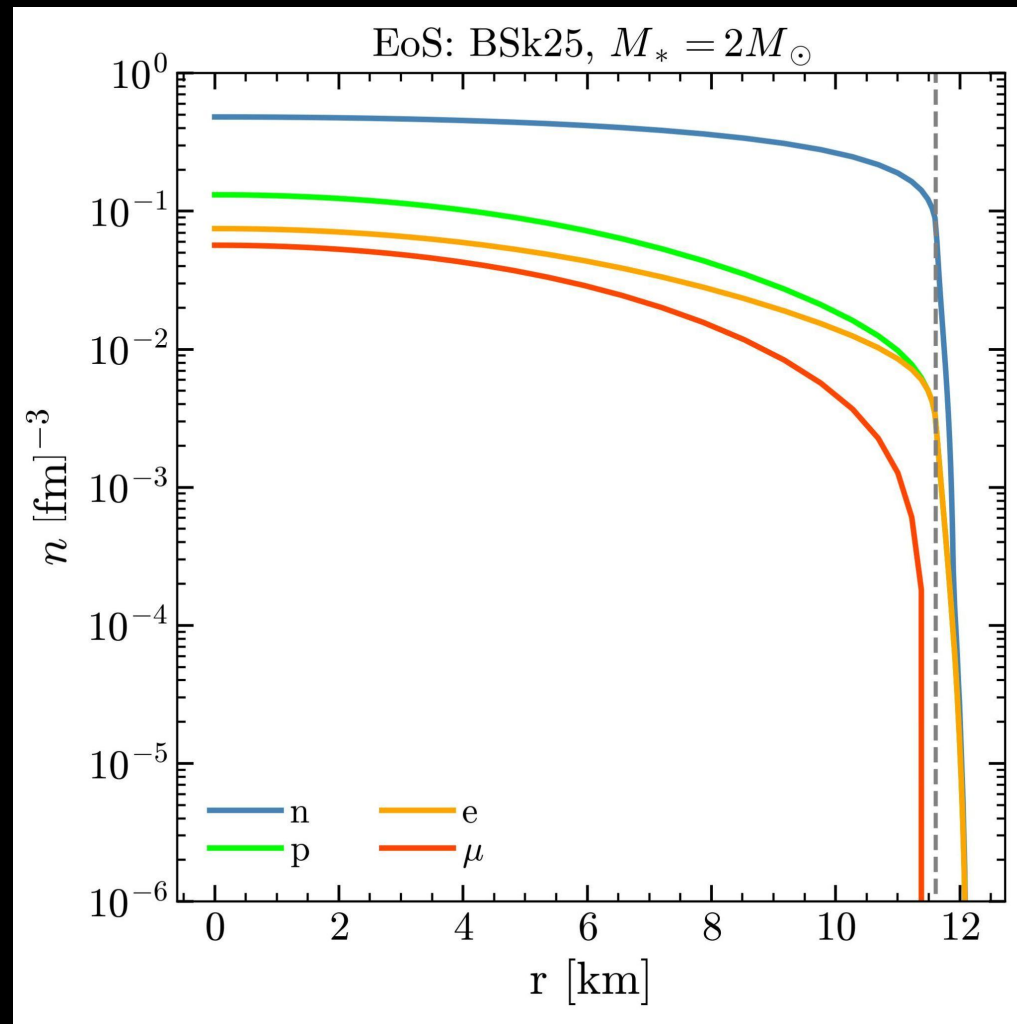
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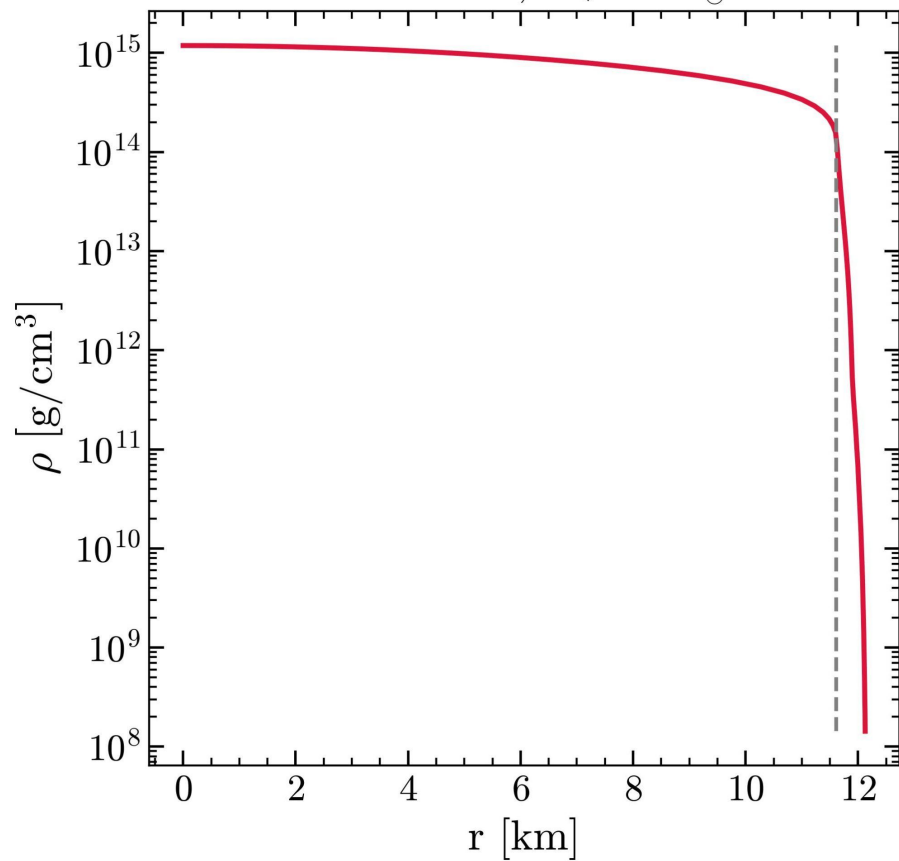
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Other assumptions

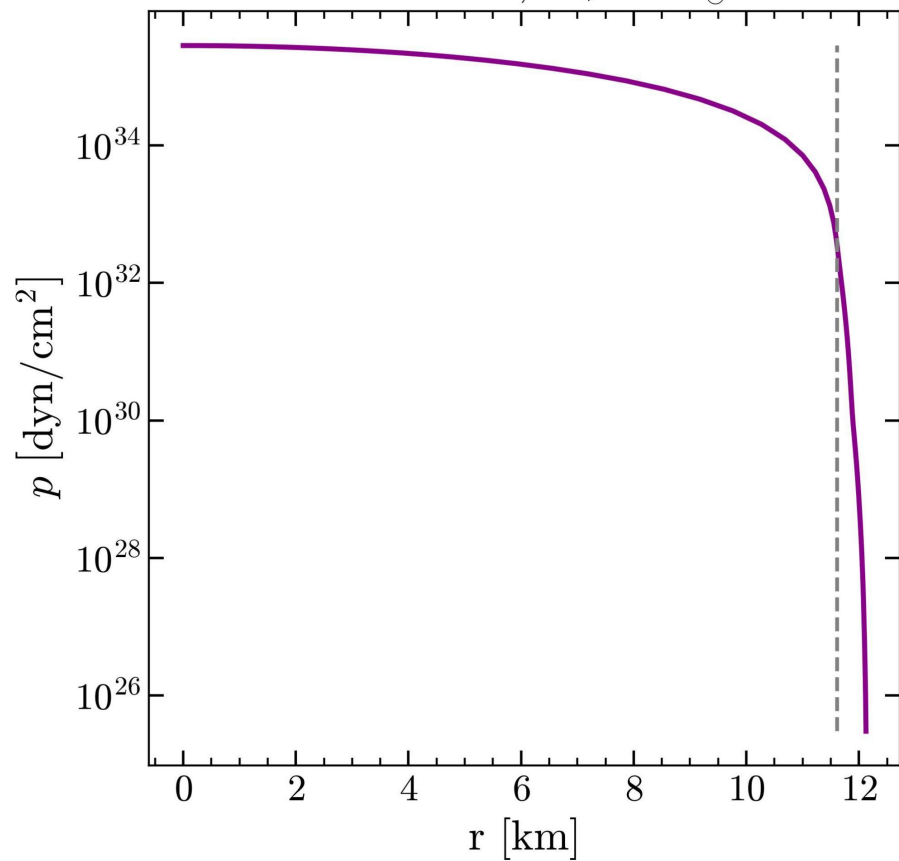
1. Equal filling approximation (EFA) for pairing force \Rightarrow if A is odd:
 $|k(t)\rangle \rightarrow (|k(t)\rangle + |k(-t)\rangle) / 2$
2. Coulomb exchange term for protons is dropped (fits are better).



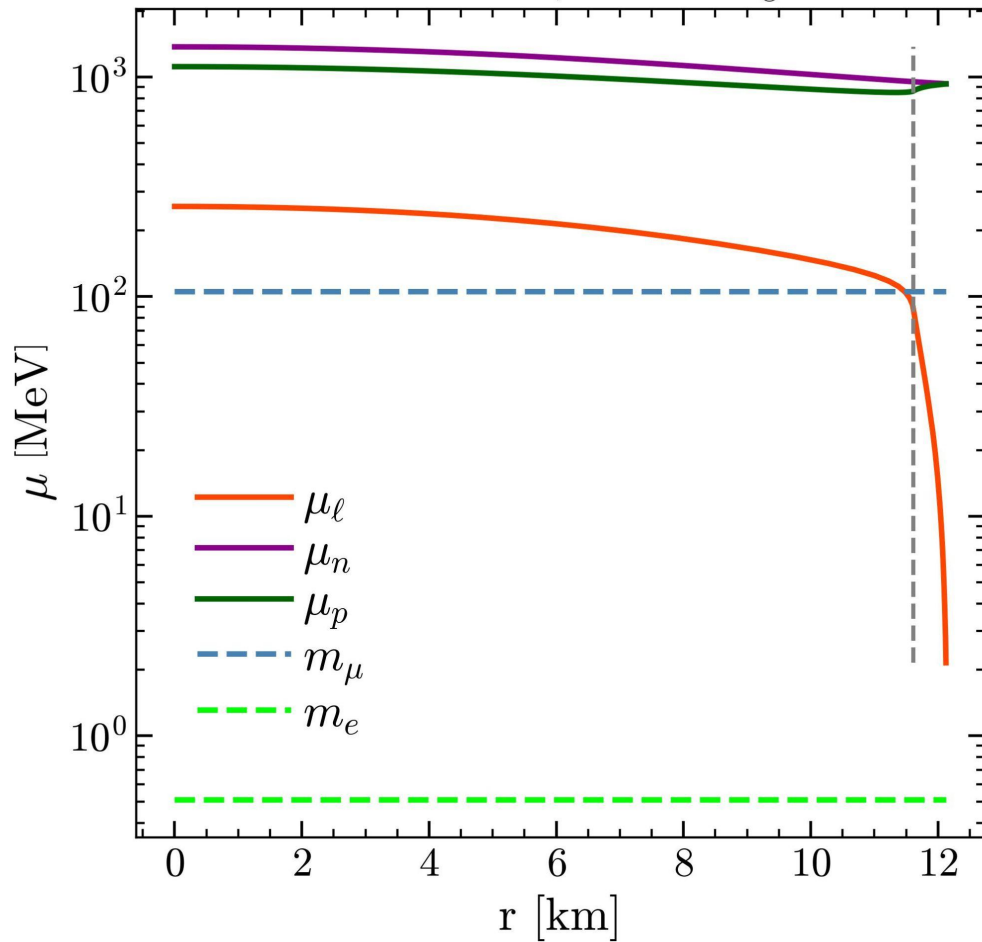
EoS: BSk25, $M_* = 2M_\odot$



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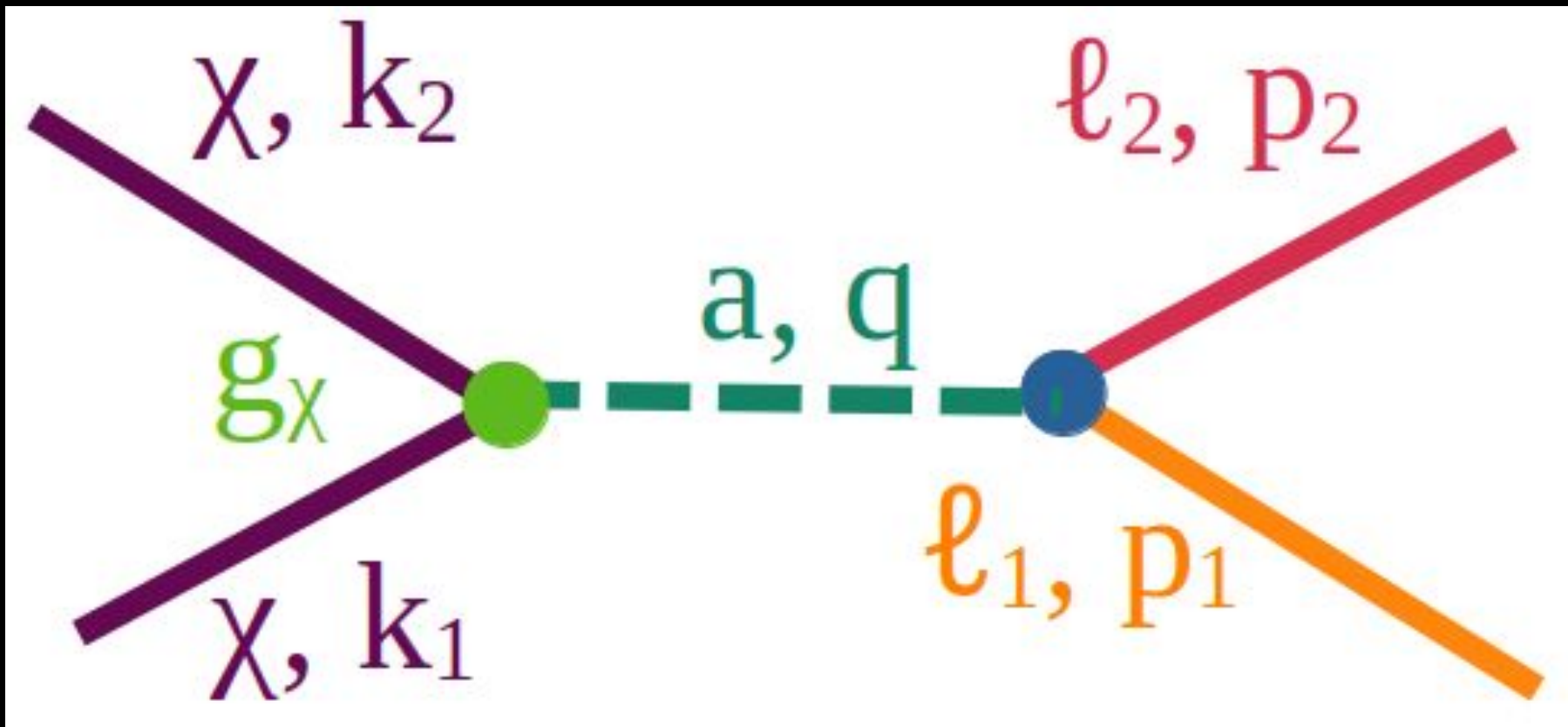


EoS: BSk25, $M_* = 2M_\odot$



03

Capture of DM by NSs



$$\chi e \rightarrow \chi \mu$$

$$\chi \mu \rightarrow \chi e$$

Faulkner and Gilliland (1985)

It started with the SOLAR COSMION (WIMP) to solve the solar neutrino problem and the missing mass problem (DM), due to their efficiency in energy transport.

Gaisser et al. (1986)
and DM

part of their argument that sky interacting massive particles (WIMPs) could explain both the "dark matter problem" and the neutrino problem." Press and Spergel (1985) gave an estimate of the capture rate by a massive body of WIMP particles from the Galactic halo assuming a Maxwellian distribution in phase space and a Boltzmann-Gibbs distribution in the Galactic halo or Galactic disk. Their argument made admittedly crude assumptions about the interaction cross-section between WIMPs and ordinary matter, but it was sufficient to show that the flux of WIMPs in the IP phase space which they hoped would explain the solar neutrino deficit had a character of their argument. The argument used WIMP calculations was equally applicable to the solar neutrino deficit. The probability of a given WIMP interacting with the body was of order unity, and when it was much less than unity, this was a variant feature for Press and Spergel because, to solve the solar neutrino problem, it is best to have WIMPs with small weak interaction cross sections.

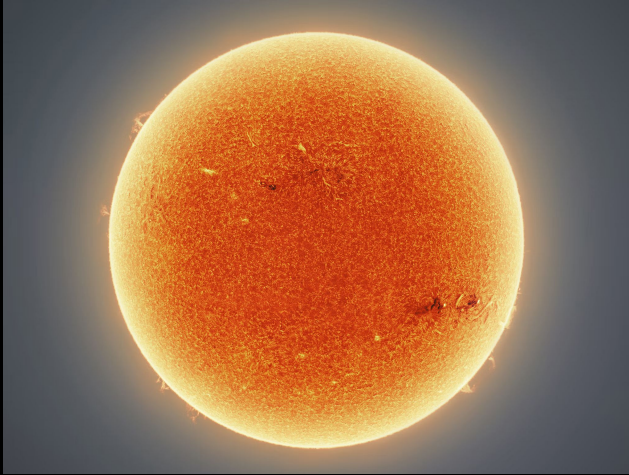
The authors also realized that if WIMPs and anti-WIMPs were both present in the Galactic halo, they would annihilate each other. They noted that the annihilation products would tend to collect in the Sun (Silk, Olive, and Srednicki 1983; Gaisser, Steigman, and Tilav 1986; Srednicki, Olive, and Silk 1987), and that this might provide a source of energy for the Sun.

Gould (1987)
and capture by the Earth (+Sun)



What to measure?

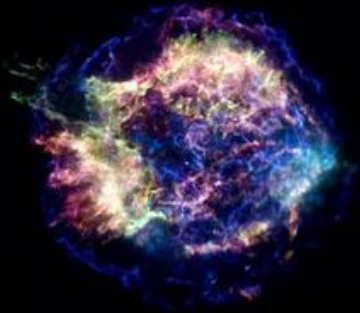
01



Mass accumulation

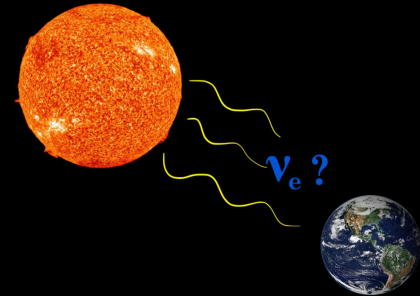
01a

Gravitational
effects



01b

Emission of SM
particles



02

Heating of star



CAPTURE MECHANISM

22

Sources for capture rates in compact stars: 1703.07784, 2010.00601, 2108.02525, 1807.02840, 1904.09803, 2004.14888, 2010.13257, 2012.08918, 2212.09785, 2312.11892, 2206.06667, 2408.03759, 2408.00594, 2404.16272, 2410.13908, 0709.1485, 1004.0586, 1001.2737, 1309.1721, 1703.04043, 1906.10145, 1812.08773, 2307.14435, 2404.10039, 1704.01577, 2208.07770...

What happens?



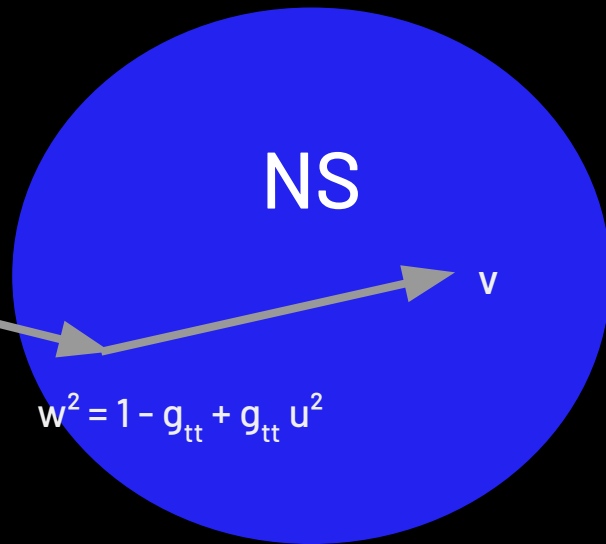
$u \sim 0$

Gravitational force

Interaction rate

$$\Omega^-(w) \equiv \int_0^v dv R^-(w \rightarrow v)$$

:: probability / time ::



$$w^2 = 1 - g_{tt} + g_{tt} u^2$$

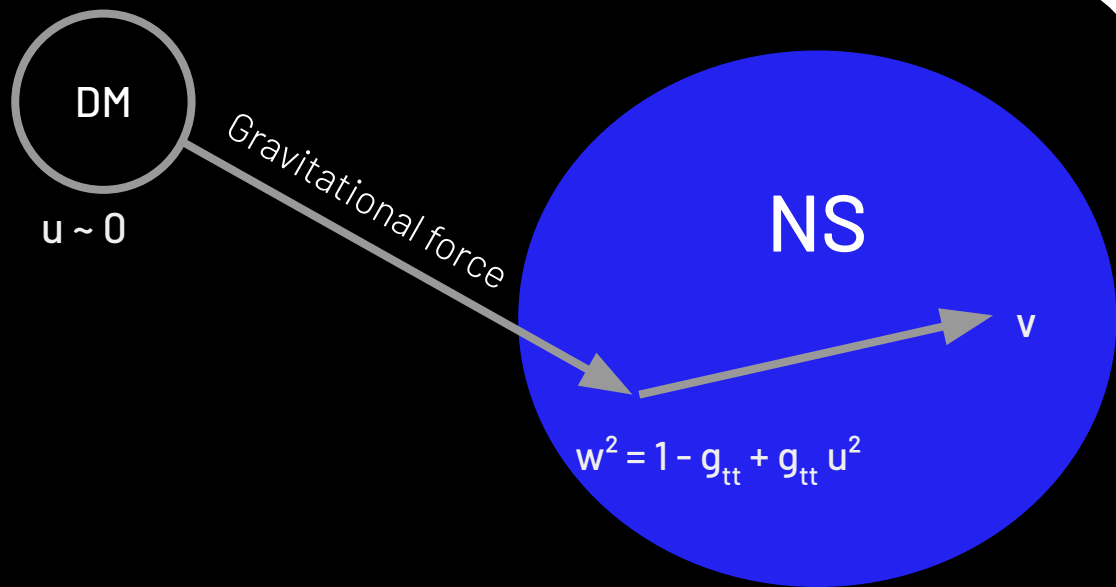
CAPTURE MECHANISM

23

What happens?

$$R^- \propto d\sigma/dv$$
$$\Omega^- \propto \sigma(v < v_e)$$

$$v_e = (1 - g_{tt})^{0.5}$$



CAPTURE MECHANISM

$$v_{\text{NS}} = 230 \text{ km/s}$$

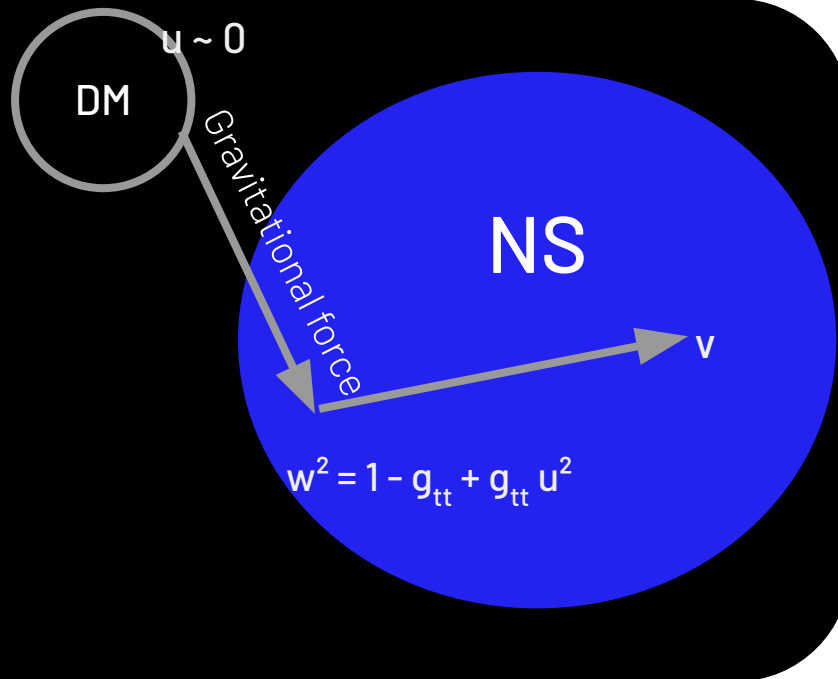
$$v_d = 270 \text{ km/s}$$

$$\rho_\chi = 0.4 \text{ GeV/cm}^3$$

What happens?

$$\begin{aligned} \Omega_{\alpha \rightarrow \beta}^- &= \frac{\zeta}{32\pi^3 m_\chi} \sqrt{\frac{g_{tt}}{1 - g_{tt}}} \\ &\times \int_{m_\alpha}^{\mu_l} dE_{p_1} E_{p_1} \int_{s_{\min}}^{s_{\max}} \frac{s ds}{\gamma_s(m_\alpha) \beta_s} \\ &\times \int_{t_{\min}}^{t_{\max}} dt \langle |\mathcal{M}|_{\text{cm}}^2 \rangle_{\alpha \rightarrow \beta}(t) \Theta(E_{p_2} - \mu_l) \end{aligned}$$

$$\begin{aligned} C &= \frac{4\pi}{v_{\text{NS}}} \frac{\rho_\chi}{m_\chi} \text{Erf} \left(\sqrt{\frac{3}{2}} \frac{v_{\text{NS}}}{v_d} \right) \\ &\times \int_0^{R_{\text{NS}}} r^2 \frac{\sqrt{1 - g_{tt}(r)}}{g_{tt}(r)} \Omega^-(r) \eta(r) dr \end{aligned}$$



CAPTURE MECHANISM

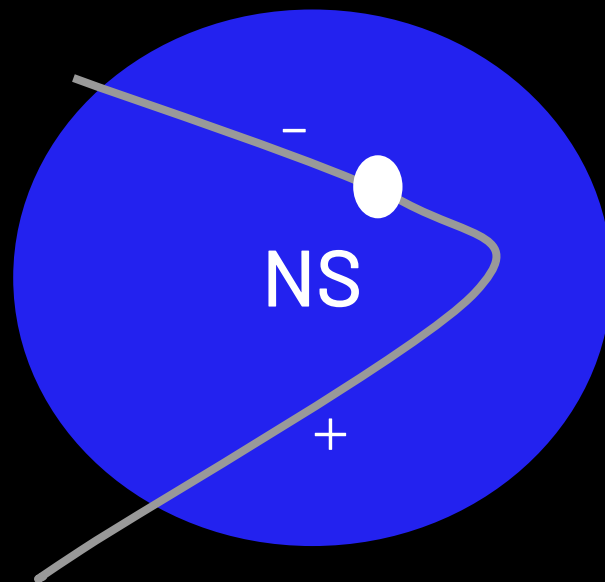
Optical factor

What happens?

$$\eta(r) = \frac{1}{2} \int_0^1 \frac{y \, dy}{\sqrt{1-y^2}} \left(e^{-\tau_{\chi}^{-}(r,y)} + e^{-\tau_{\chi}^{+}(r,y)} \right)$$

$$\tau_{\chi}^{-}(r,y) = \int_r^{R_{\text{NS}}} dr' \frac{\Omega^{-}(r')}{\sqrt{1-g_{tt}(r')} \sqrt{1-y^2 \frac{J_{\text{max}}^2(r)}{J_{\text{max}}^2(r')}}},$$

$$\begin{aligned} \tau_{\chi}^{+}(r,y) &= \tau_{\chi}^{-}(r,J) \\ &+ 2 \int_{r_{\text{min}}}^r dr' \frac{\Omega^{-}(r')}{\sqrt{1-g_{tt}(r')} \sqrt{1-y^2 \frac{J_{\text{max}}^2(r)}{J_{\text{max}}^2(r')}}}. \end{aligned}$$



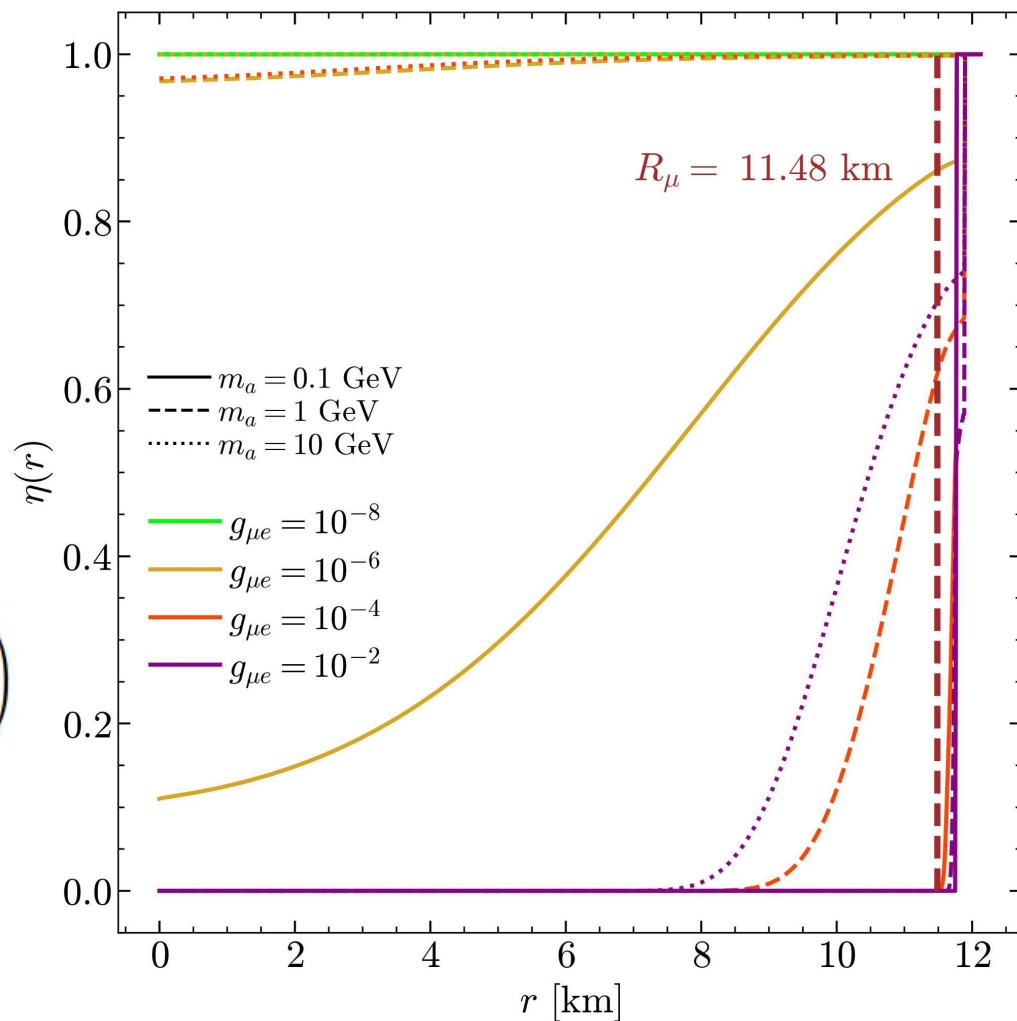
Optical factor

Case: $m_\chi = 2 m_a$

Limits:

- Saturation limit
- Geometric limit

$$C_{\text{NS}} = \frac{\pi R_{\text{NS}}^2 (1 - B(R_{\text{NS}}))}{v_{\text{NS}} B(R_{\text{NS}})} \frac{\rho_\chi}{m_\chi} \text{Erf} \left(\sqrt{\frac{3}{2}} \frac{v_{\text{NS}}}{v_d} \right)$$



04

Kinetic energy deposition
and heating

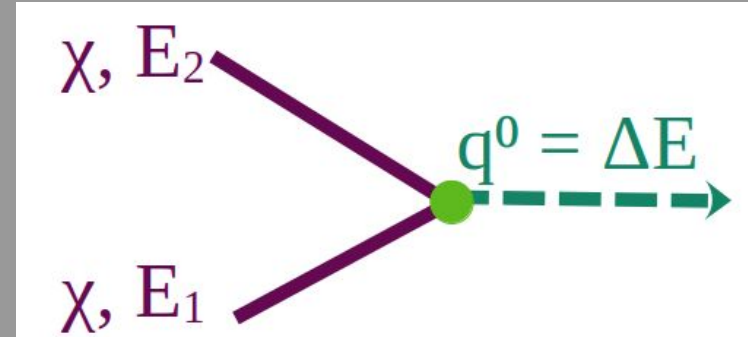
Kinetic energy deposition: heating NSs

Expected T of NSs: 10^3 K (20 Myr) and ~ 100 K (1 Gyr)

Sensitivities in the future: 2000-4000 K (JWST, TMT, E-ELT) [2403.07496]

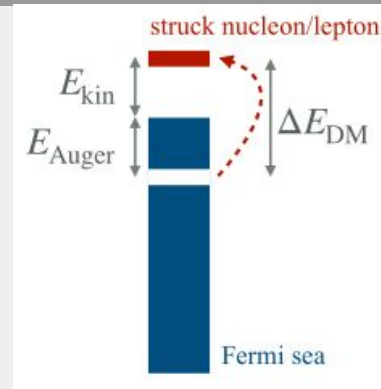
What to do?

We need to add the kinetic energy lost for each point in parameter space to the innermost integrand of the capture rate to find the energy rate deposited into the NS.



What happens to the leptons?

The target lepton is removed from the thermal distribution. The outgoing lepton is produced with an energy, $E > \mu$, and loses its extra energy reconverts into the original flavor, respecting the thermal distributions.



Credits:
2307.14435

Kinetic energy deposition

Approaches

Just first interaction

$$E_k = E_{k_1} - E_{k_2}$$

Thermal first interaction

$$E_k = E_{k_1} - m_\chi$$

All energy lost on the surface

$$E_k = \left(\frac{1}{\sqrt{g_{tt}(R_{\text{NS}})}} - 1 \right) m_\chi$$

1704.01577

All energy lost at the center of NS

$$E_k = \left(\frac{1}{\sqrt{g_{tt}(0)}} - 1 \right) m_\chi$$

2312.11892

Is the last expression valid?

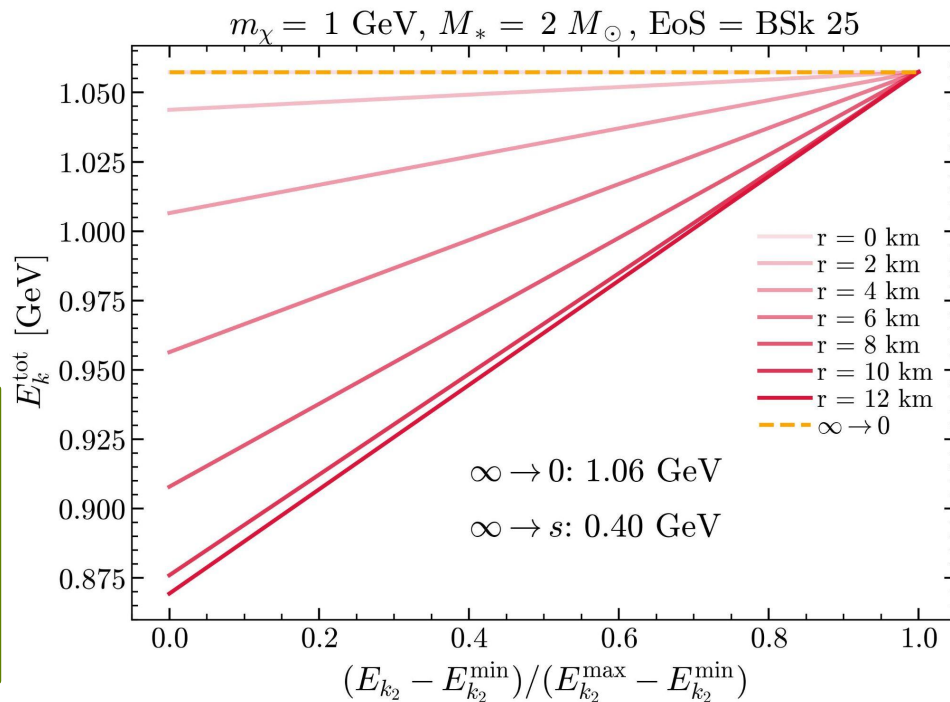
$$E_k = \left(\frac{1}{\sqrt{g_{tt}(0)}} - 1 \right) m_\chi$$

$$E_a = \sqrt{\frac{g_{tt}^b}{g_{tt}^a}} E_b$$

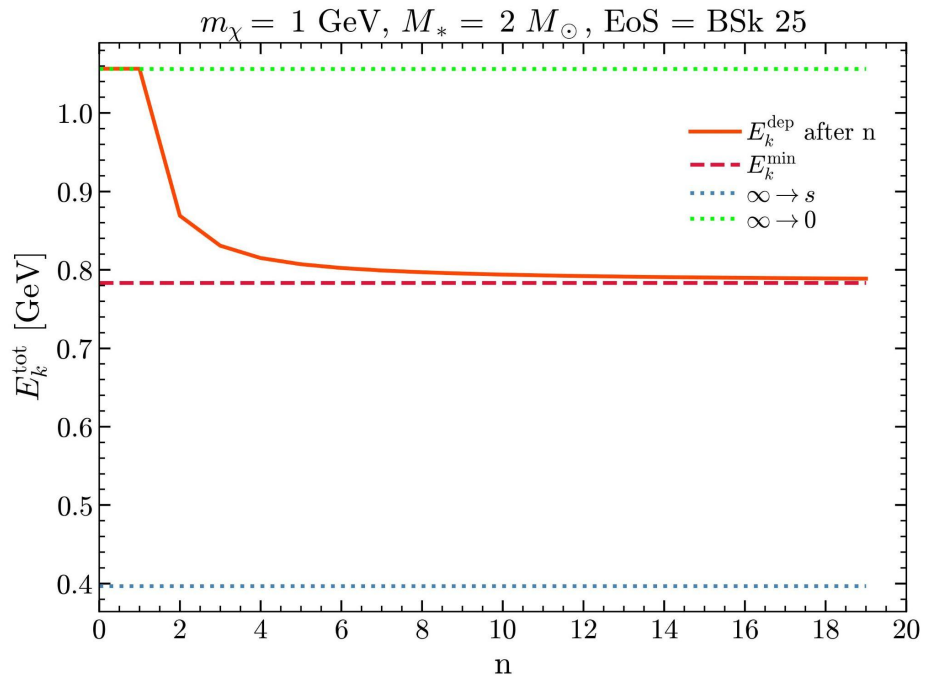
Local energy is NOT conserved!

Strategy for lower bound:

1. Large number of interactions
2. Deposit all the energy on each.



Lower bound: Infinite interactions losing all E_k

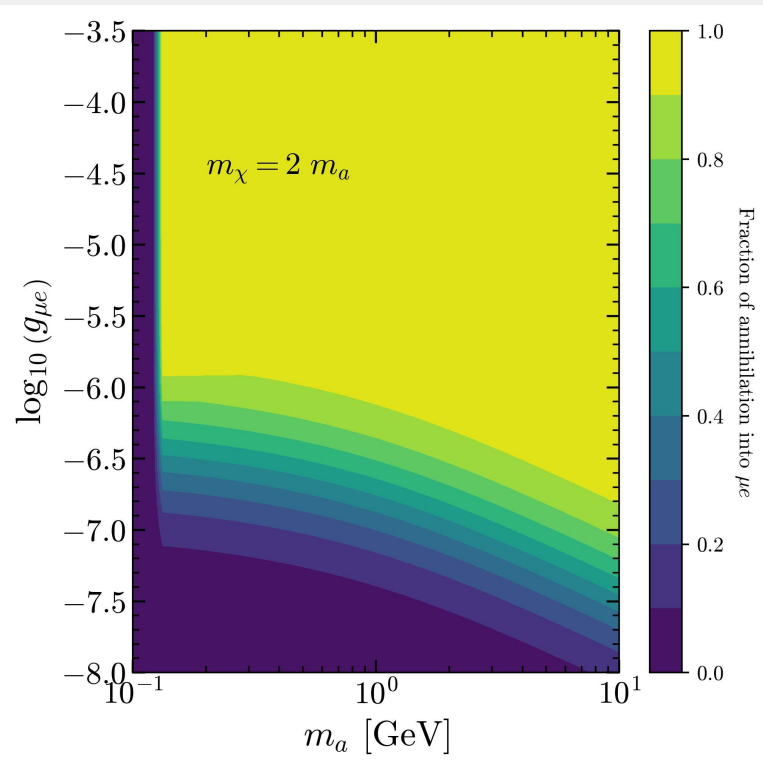


$$\begin{aligned}
 E_k^{\text{min}} &= E_s + \lim_{n \rightarrow \infty} \sum_{i=0}^n (E_i - m_\chi), \\
 &= m_\chi(\gamma_s - 1) + \lim_{n \rightarrow \infty} \sum_{i=1}^n m_\chi \left(\frac{\gamma(r_i)}{\gamma(r_{i-1})} - 1 \right), \\
 &= m_\chi(\gamma_s - 1) + \int m_\chi \frac{\gamma(r_i) - \gamma(r_{i-1})}{\gamma(r_{i-1})}, \\
 &= m_\chi(\gamma_s - 1) + m_\chi \int_{\gamma_s}^{\gamma_0} \frac{d\gamma}{\gamma}, \\
 &= m_\chi \left(\gamma_s - 1 + \ln \frac{\gamma_0}{\gamma_s} \right),
 \end{aligned}$$

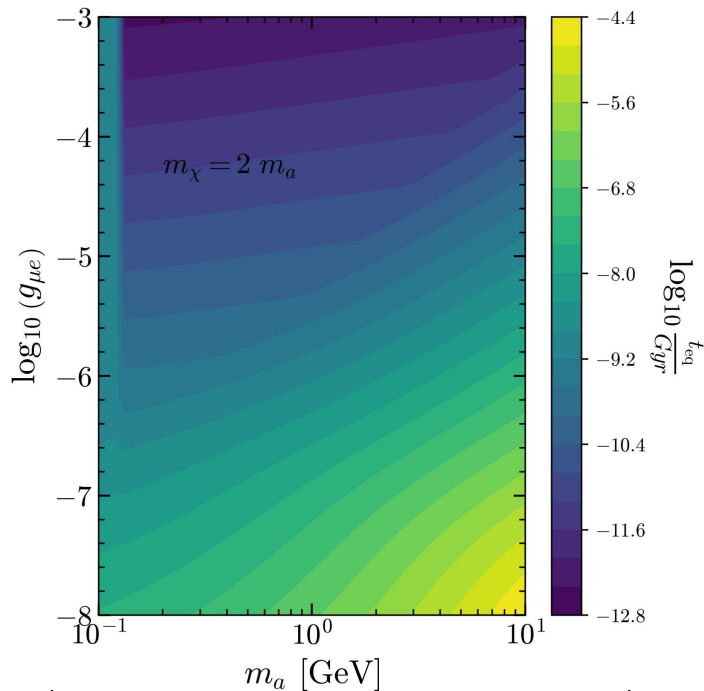
Annihilation energy deposition

Channels

$$\chi\bar{\chi} \rightarrow aa \quad \chi\bar{\chi} \rightarrow \mu^{\pm}e^{\mp}$$



If equilibrium is reached (C - A)



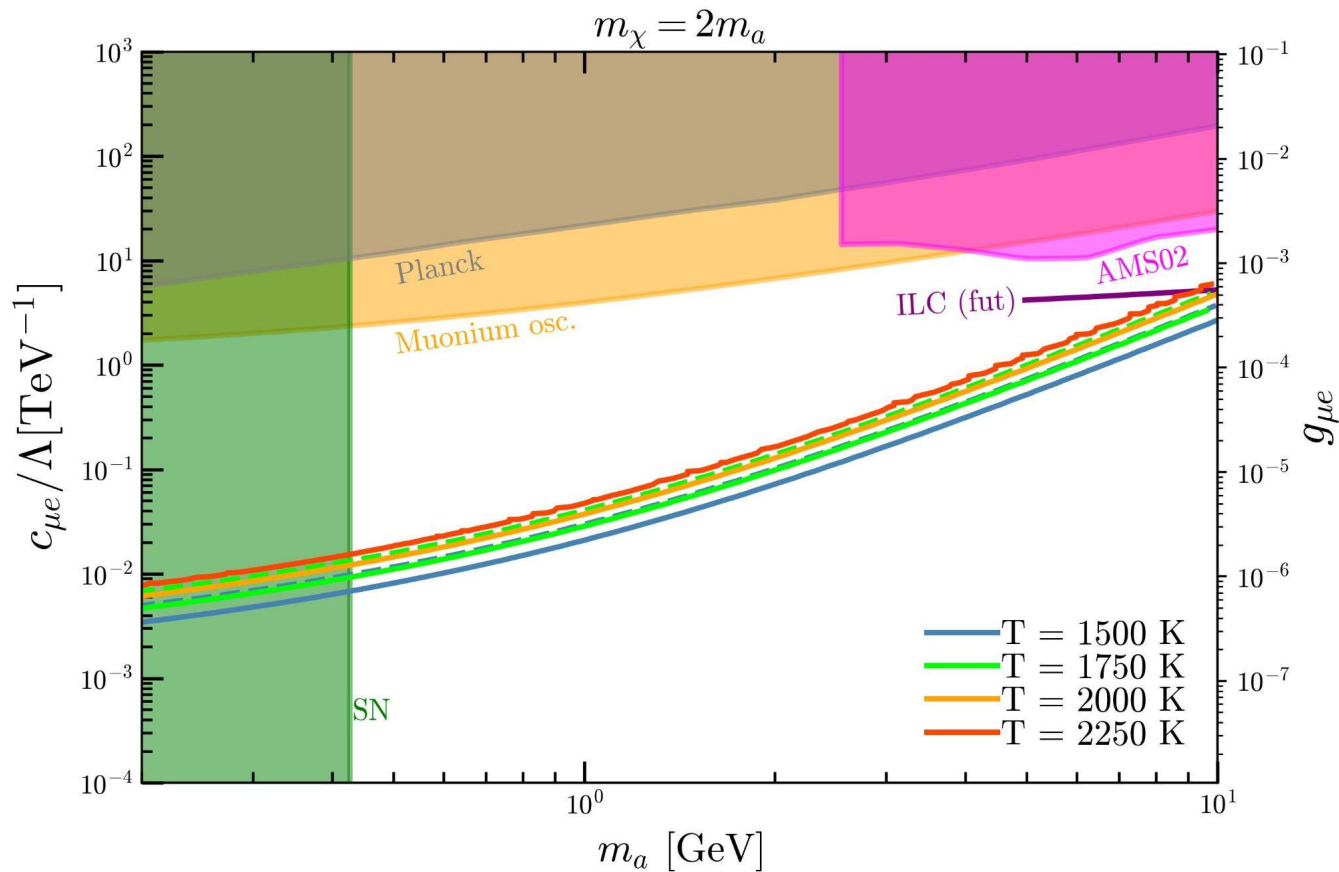
$$E_k = \left(\frac{1}{\sqrt{g_{tt}(R_{\text{NS}})}} + \frac{1}{2} \ln \frac{g_{tt}(R_{\text{NS}})}{g_{tt}(0)} \right) m_{\chi}$$

05

Results

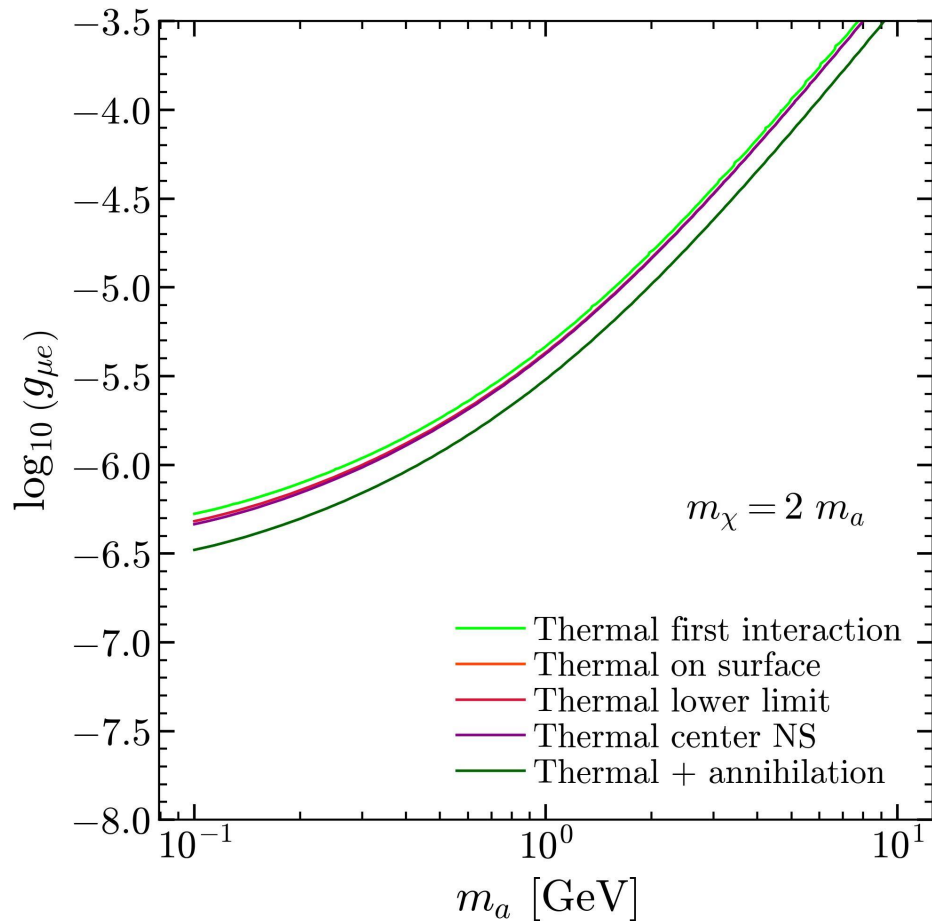
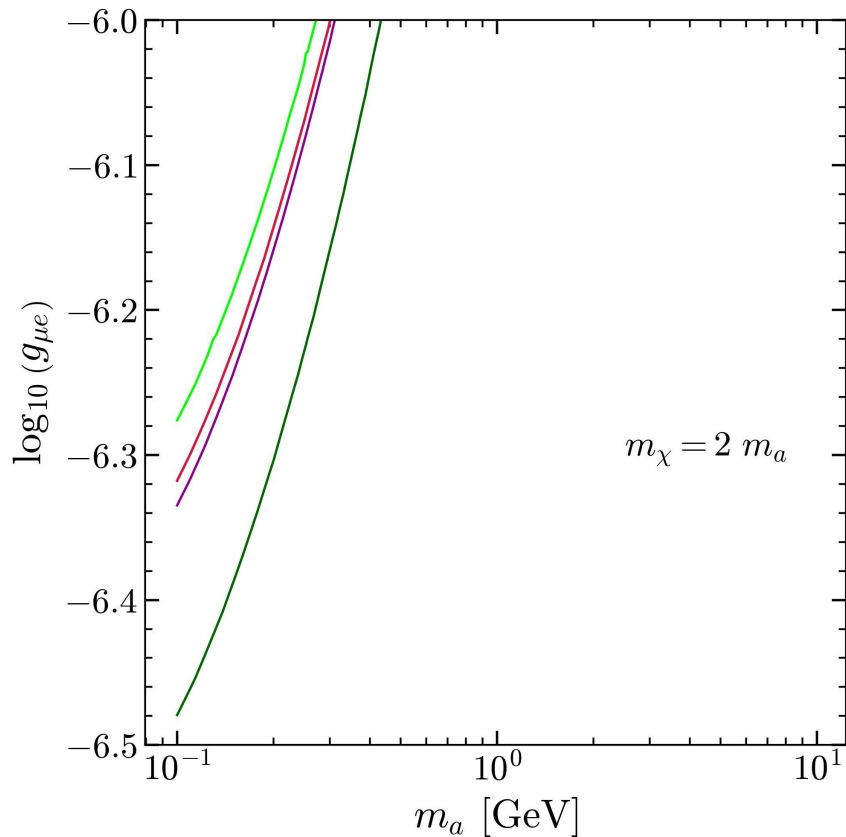
Thermal case

$$g_\chi = 0.097 \times (m_a / \text{GeV})^{1/2}$$



$$c_{\mu e} / \Lambda \equiv \frac{g_{\mu e}}{m_\mu [\text{TeV}]}$$

Different approaches: 1750 K



06

Conclusions

What was shown

- Mechanism of capture rate of dark particles in NSs.
- Different approaches to compute the heating of a NS.
- **Context:** LFV ALP model.

What we found

- We can set limits based on sensitivities of infrared telescopes in the near future (hopefully), including the Thirty Meter Telescope (TMT), and the European Extremely Large Telescope (E-ELT) and also the James Webb Space Telescope (JWST).

Future prospects

- Study the thermalization time, in order to know whether the thermalization limits can be used and where in parameter space.
 - Analyze the possibility of emissions from the star.
-

THANK YOU