One-Loop Correction to the Higgs Mass

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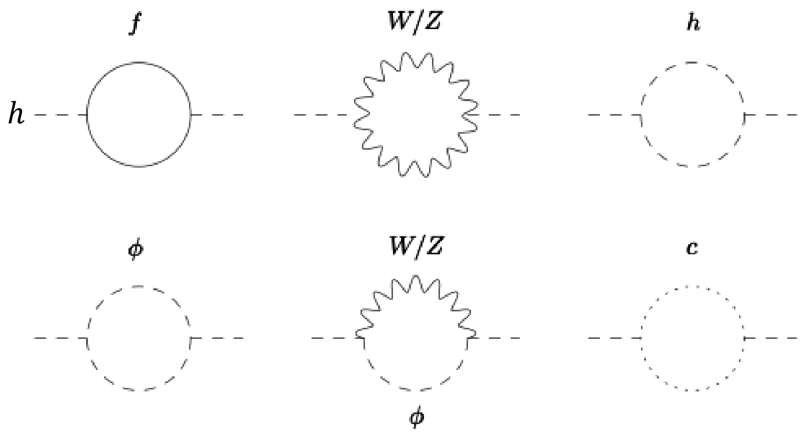
Summary

1. We calculate the one-loop correction to the Higgs mass within the Standard Model,

[KSC] 2310.00586, 2310.10004, 2506.18667



Naïve expectation:

$$\delta m_h^2(p^2) \sim y_t^2 \int_0^{\Lambda} d^4k \frac{1}{k^2 - M^2} \sim y_t^2 \Lambda^2.$$

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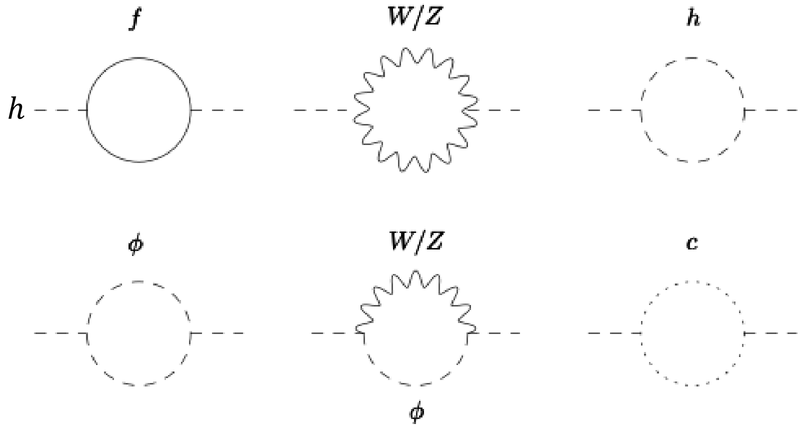
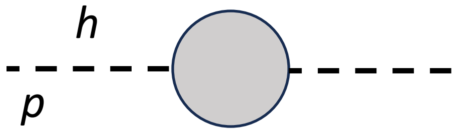
   We show it is finite if faithfully renormalized.

2. We show that heavy fields in the general loop decouple. *cf.* [Appelquist-Carazzone 75]

   [KSC] 2408.06406, 2410.21118





M somewhere
$M^2 \gg p^2$

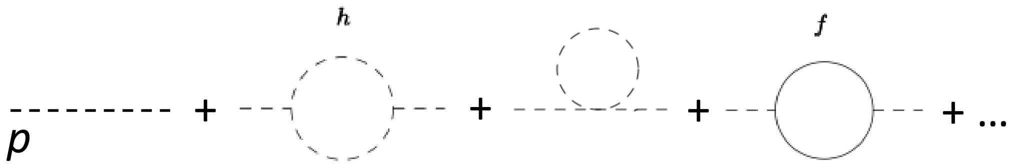
$$\sim \frac{(p^2 - m_h^2)^2}{M^2}$$

Observables

- EFT: Write down all the renormalizable interactions allowed by symmetry;

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m_B^2\phi^2 - \frac{6\lambda_B}{24}\phi^4 + \sum\frac{y_{Bf}}{\sqrt{2}}\phi\bar{f}f + \dots$$

- The bare mass m_B, the bare coupling λ_B define the theory but are not directly observable.
- Interactions always modify them.
- Loop corrections interfere, only the sum is observed.



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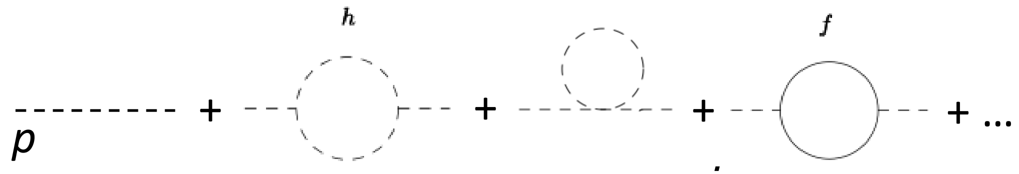


Old formulation:
Why is $\tilde{\Sigma}(p^2) \sim \Lambda^2$ large?

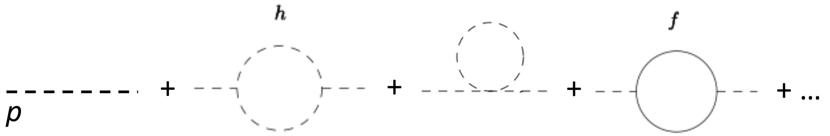
observables $\quad \langle\phi(p)\phi(-p)\rangle_{1PI} = \dfrac{i}{\underbrace{p^2 - m_B^2 - \tilde{\Sigma}(p^2)}}$

not separately observable

always the whole propagator appears in the *S*-matrix

Measuring parameters

- The bare params are not directly observable $\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m_B^2\phi^2 - \frac{6\lambda_B}{24}\phi^4 + \sum \frac{y_{Bf}}{\sqrt{2}}\phi\bar{f}f + \dots$
- Only the combination $\Gamma^{(2)}(p^2) = p^2 - m_B^2 - \tilde{\Sigma}(p^2)$ is observable.



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the external momentum,
not an artificial μ

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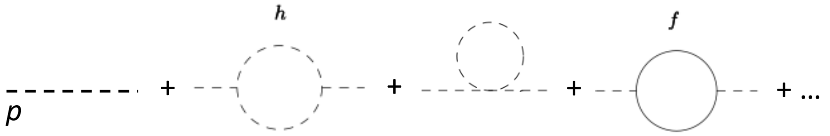

1. Physical quantity $\Gamma^{(2)}(p^2)$ is scale-*p*-dependent.
2. We compare them with the experiment “renormalization condition.”
   e.g. the pole mass $\Gamma^{(2)}(m_h^2) = 0$: $m_B^2 + \tilde{\Sigma}(m_h^2) = m_h^2$

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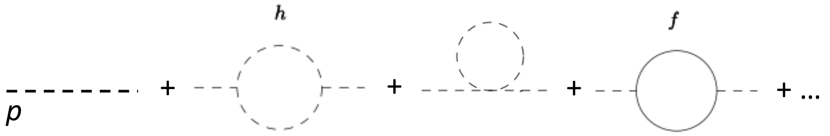

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$$\mathcal{L} = \frac{1}{2}\partial_\mu\partial^\mu\phi - \frac{1}{2}\left(m_h^2 - \tilde{\Sigma}(m_h^2)\right)\phi^2 - \frac{1}{4}\left(\lambda - W(s_0, t_0, u_0)\right)\phi^4 + \ldots$$

counterterms

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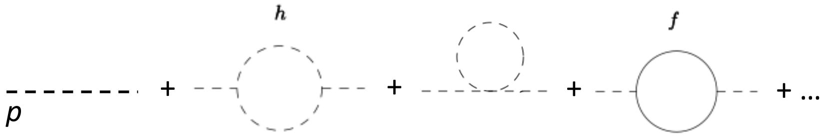

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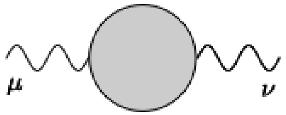
counterterms

New basis: $\Gamma^{(2)}(p^2) = p^2 - m_h^2 + \tilde{\Sigma}(m_h^2) - \tilde{\Sigma}(p^2)$

Before field-strength renormalization

The success of renormalization

- This combination is what we observe in the experiment.
- Vacuum polarization defines the electric charge



$$= e_B^2 \frac{-i g_{\mu\nu}}{p^2\left(1-\Pi(p^2)\right)}$$

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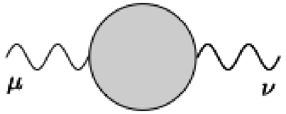

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- A reference value e at $p^2 = 0$:

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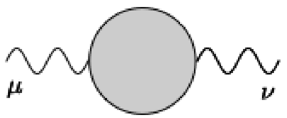

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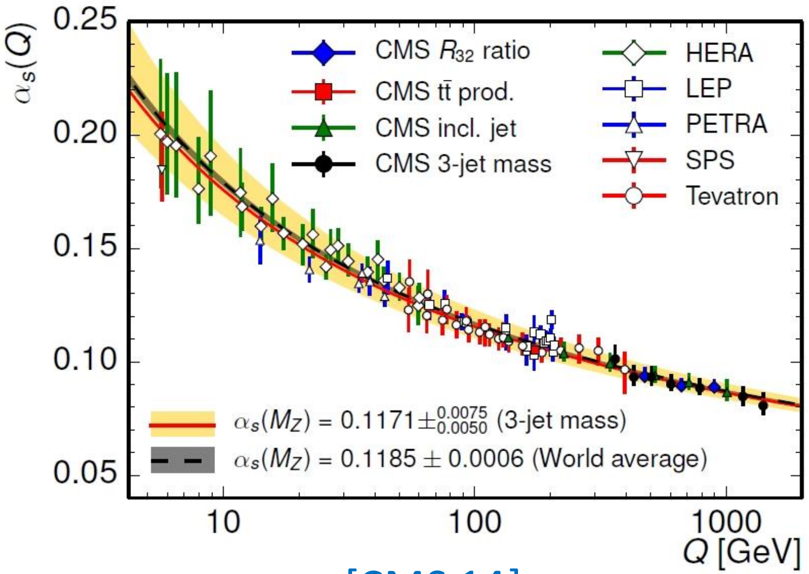
- Running, Lamb shift…



[CMS 14]

Running scalar mass

[Bogoliubov, Parasuik] [Hepp 66] [Zimmerman 69]
cf. [Coleman] [Cheng, Li]

- With modified mass, the resulting propagator has undesirable normalization.

$$p^2 \to m_h^2 : \quad \frac{i}{\underbrace{p^2 - m_h^2 + \tilde{\Sigma}(m_h^2) - \tilde{\Sigma}(p^2)}_{\Gamma^{(2)}(p^2)}} \to \frac{iZ}{p^2 - m_h^2} + \mathcal{O}((p^2 - m_h^2)^2)$$

- Re-normalize $\phi \to \sqrt{Z}\phi_r$ $\quad Z^{-1} = 1 - \frac{d\tilde{\Sigma}}{dp^2}(m^2).$ field-strength renormalization

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field-strength renormalization

$$\frac{i}{p^2 - \left(m_h^2 + \tilde{\Sigma}(p^2) - \tilde{\Sigma}(m_h^2) - (p^2 - m_h^2)\frac{d\tilde{\Sigma}}{dp^2}(m_h^2)\right)}$$

$\parallel$

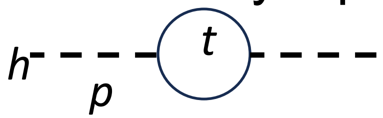
- The observed mass $m^2(p^2)$ in the *S*-matrix
- The value of $m^2(p^2)$ along the external momentum $\sqrt{p^2}$ is the prediction of QFT.

physical, effective, observed, measured

Regularization independence

[KSC] 2310.00586, 2310.10004
[Hepp 66] [Zimmerman 69]

- The one-loop correction by top quark



$$\tilde{\Sigma}_2\left(p^2\right) = -iN_c\left(-i\frac{y_t}{\sqrt{2}}\right)^2\int\frac{d^4k}{(2\pi)^4}\,\mathrm{tr}\,\frac{i}{\not{k}+p-m_t}\frac{i}{\not{k}-m_t}$$
$$= -2N_c y_t^2\int_0^1 dx\int\frac{d^4\ell}{(2\pi)^4}\frac{\ell^2-x(1-x)p^2+m_t^2}{\left(\ell^2+x(1-x)p^2-m_t^2\right)^2}.$$

$\Delta(p^2) = m_f^2 - x(1-x)p^2$

- Regularization

momentum cutoff $k_E^2 = \Lambda^2$

$$\int\frac{d^4k}{(2\pi)^4}\frac{1}{\left(k^2-\Delta\right)^2} = \frac{i}{16\pi^2}\left(\log\frac{\Lambda^2}{\Delta}-1\right)$$

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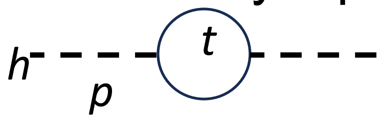

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$$= m^2 - \frac{y_f^2 N_c}{8\pi^2}\int_0^1 dx\left[3\Delta_f(p^2)\log\frac{\Delta_f(p^2)}{\Delta_f(m_h^2)} + 3(p^2 - m_h^2)x(1-x)\right] + \mathcal{O}\left(\frac{(p^2-m^2)^2}{\Lambda^2}\right)$$

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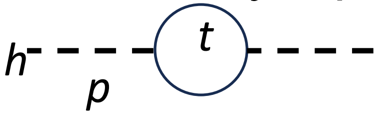

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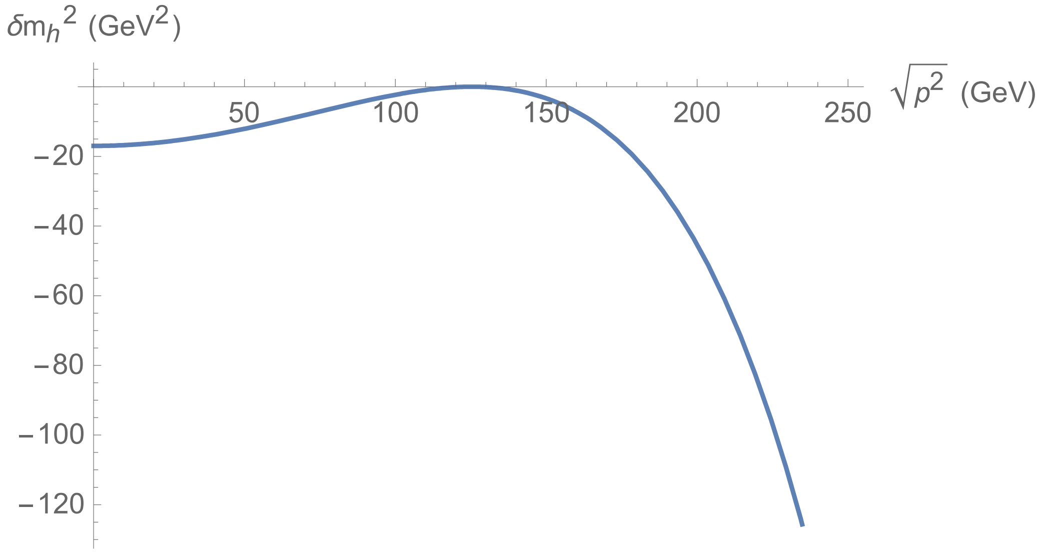

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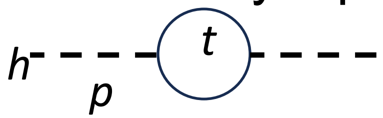

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dimensional regularization $d=4-\epsilon$ $\quad\Delta(p^2)=m_f^2-x(1-x)p^2$

$$\int\frac{d^dk}{(2\pi)^d}\frac{1}{(k^2-\Delta)^2}=\frac{i}{16\pi^2}\left(\frac{2}{\epsilon}-\log\Delta-\gamma+\log(4\pi)+\mathcal{O}(\epsilon)\right),$$
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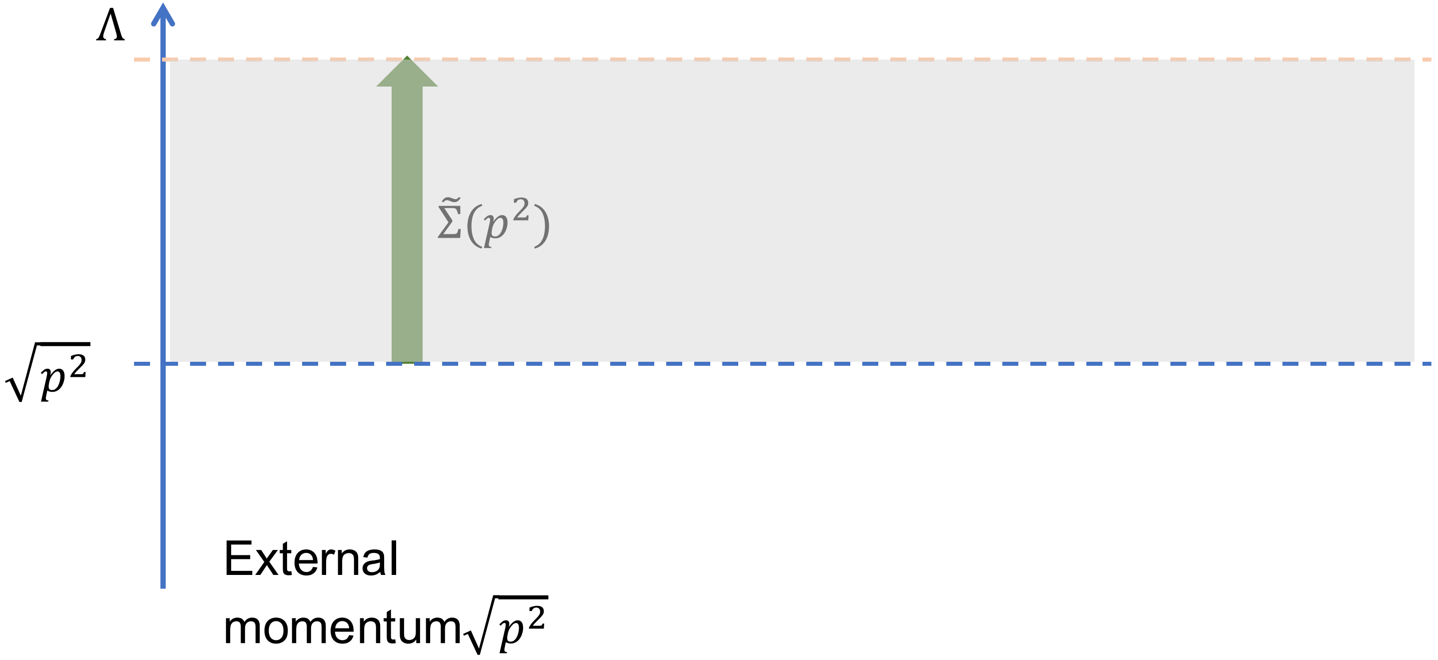
The same once we identify $\log\Lambda\leftrightarrow 1/\epsilon$ up to a polynomial in Δ

- The observed mass is finite and independent of the regularization.

$$m^2(p^2)=m^2+\tilde{\Sigma}(p^2)-\tilde{\Sigma}(m^2)-(p^2-m^2)\frac{d\tilde{\Sigma}}{dp^2}(m^2).$$
$$=m^2-\frac{y_f^2N_c}{8\pi^2}\int_0^1dx\left[3\Delta_f(p^2)\log\frac{\Delta_f(p^2)}{\Delta_f(m_h^2)}+3(p^2-m_h^2)x(1-x)\right]+\mathcal{O}\left(\frac{(p^2-m^2)^2}{\Lambda^2}\right)$$

Finite renormalization, relative coupling

- Loop correction $\tilde{\Sigma}_1^{f\bar{f}}(p^2) = -2y_f^2 \int_0^1 dx \int_0^{\Lambda} \frac{d^4 l_E}{(2\pi)^4} \frac{l_E^2 - \Delta(p^2)}{\left(l_E^2 + \Delta(p^2)\right)^2}$
- Renormalized parameters have no reference to $\Lambda^2 \gg (p^2 - m_h^2)$



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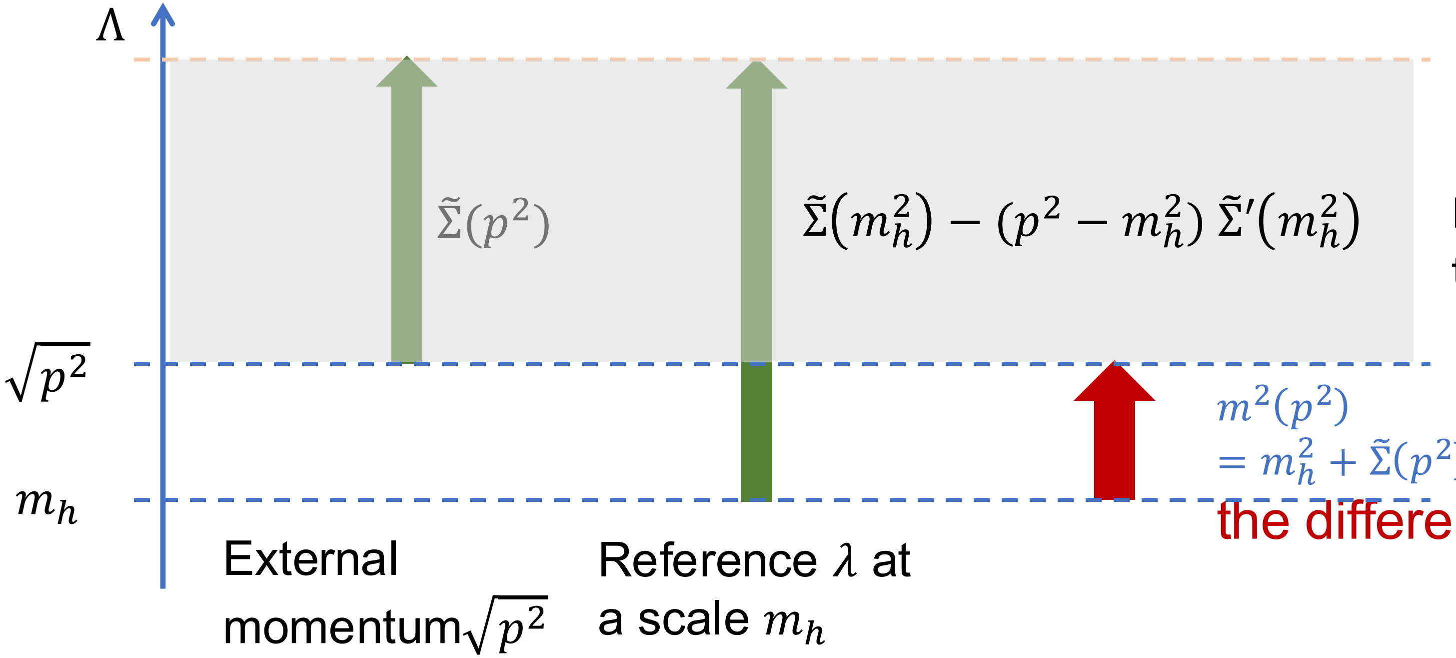


In the large momentum region two integrals cancel

$m^2(p^2)$
$= m_h^2 + \tilde{\Sigma}(p^2) - \tilde{\Sigma}(m_h^2) - (p^2 - m_h^2)\,\tilde{\Sigma}'(m_h^2)$

the difference of the same function

Scale dependence

- The essence of renormalization:
  physical parameters are scale-dependent function of the external momenta.

$$m^2(p^2) = m_h^2 + \tilde{\Sigma}(p^2) - \tilde{\Sigma}(m_h^2) - (p^2 - m_h^2)\tilde{\Sigma}'(m_h^2)$$

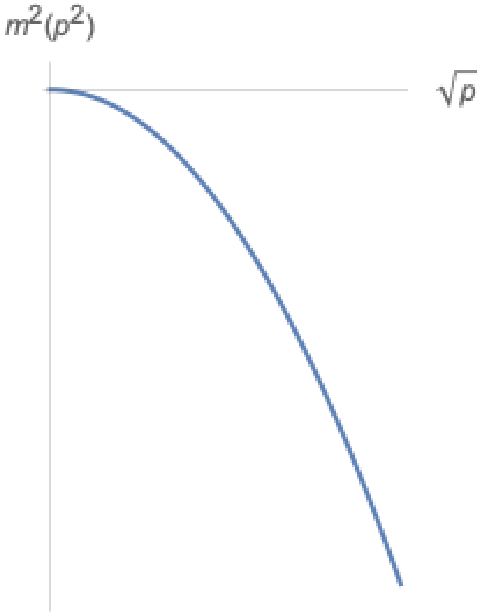
- This is the solution of the 1st order DE [Callan 70] [Symanzik 70,71]

$$p^2 \frac{dm^2(p^2)}{dp^2} = p^2 \frac{d\tilde{\Sigma}(p^2)}{dp^2} - p^2 \frac{d(p^2 - m_h^2)}{dp^2} \tilde{\Sigma}'(m_h^2) = \beta(p^2)$$

- With the boundary condition

$$m^2(m_h^2) = m_h^2$$

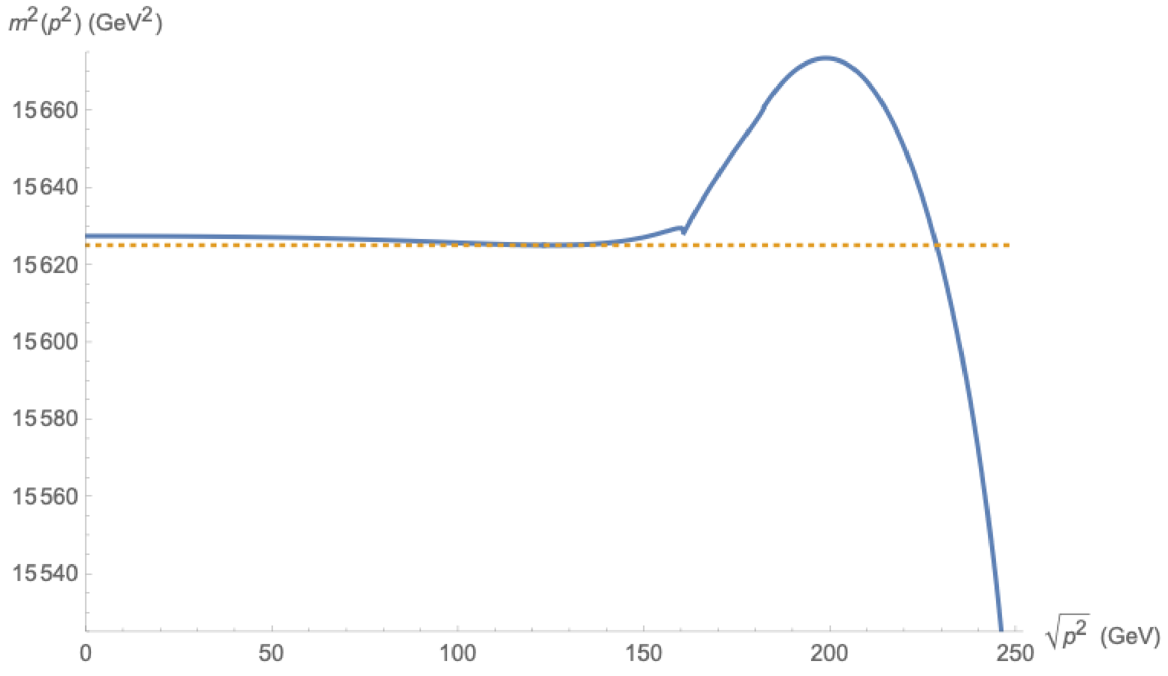
- We can see only the difference, that is, the relative value btwn p^2 and m_h^2 without reference to UV scale Λ.

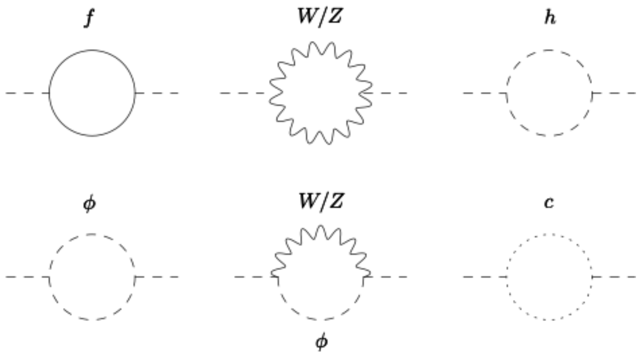


The complete one-loop correction

[KSC] 2506.18667

- Finite and power running

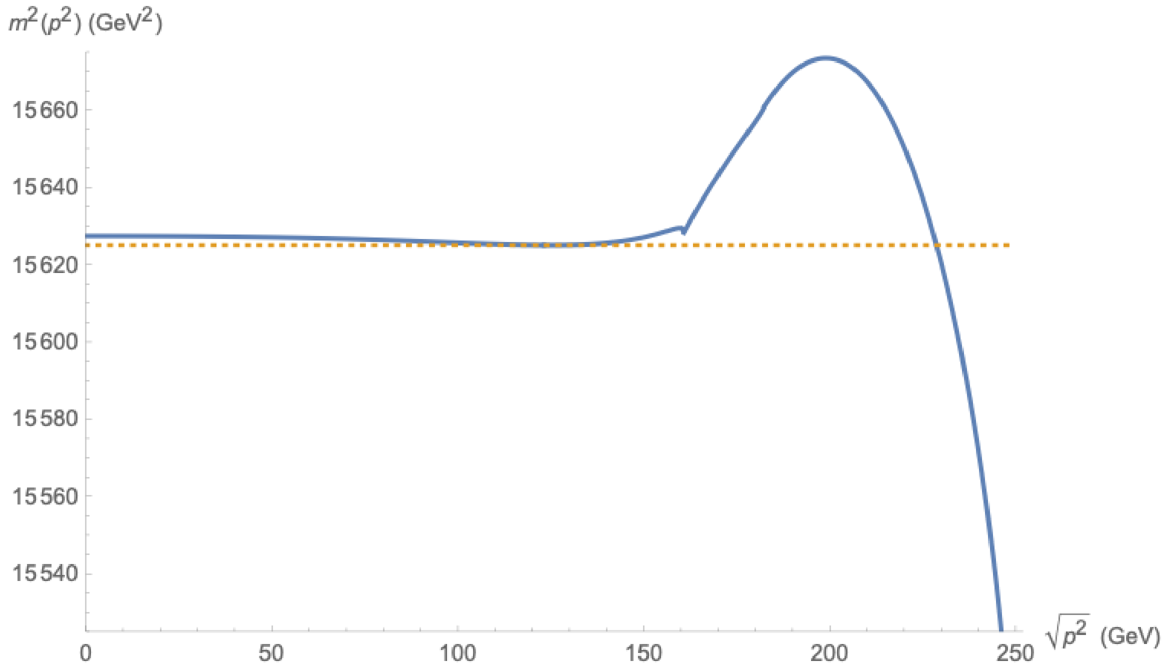


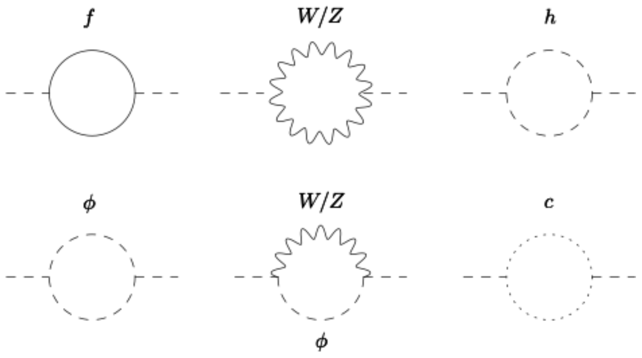


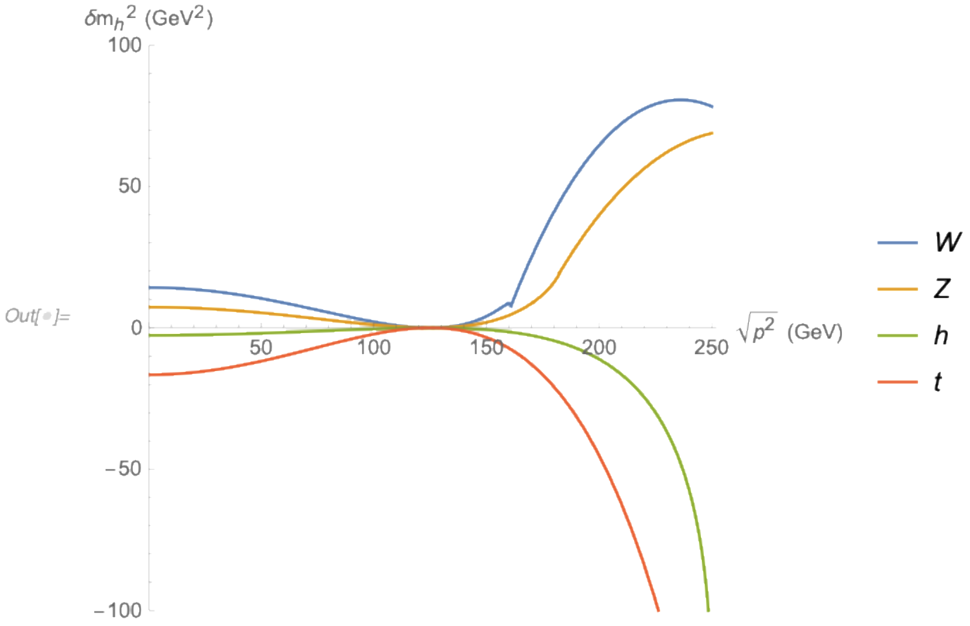
The complete one-loop correction

[KSC] 2506.18667

- Finite and power running

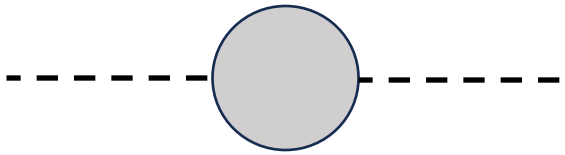




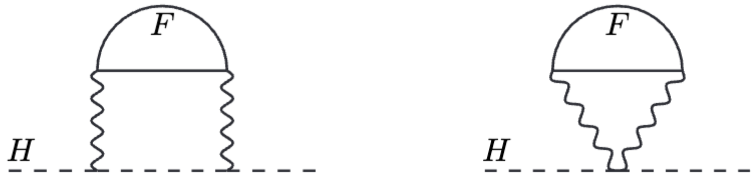


The (technical) hierarchy problem

- Loop correction to the scalar (mass)^2



$$m_B^2 + \sum_n \tilde{\Sigma}_n\left(p^2\right)$$



From S. Martin

- Suppose a heavy field with mass M ~ the scale of new physics.
- New formulation

$$\tilde{\Sigma}_n\left(p^2\right) = \mathcal{O}\left(M^2 \log M^2\right)$$

- Large M dominant - (unknown) UV sensitivity

naturalness ['t Hooft 80]

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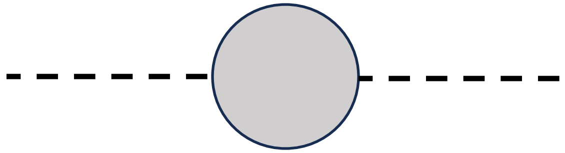

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From S. Martin 97

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naturalness ['t Hooft 80]

- At every order. Direct or indirect - Tuning at a certain order ruined.
- Real problem:

Is $m^2\left(p^2\right) = m^2 + \sum_{l=1}^{\infty}\left[\tilde{\Sigma}_l\left(p^2\right) - \tilde{\Sigma}_l\left(m^2\right) - \left(p^2 - m^2\right)\frac{d\tilde{\Sigma}_l}{dp^2}\left(m^2\right)\right]$ small?

Decoupling in the scalar mass

[KSC] 2310.00586, 2310.10004

- EW top quark $m_f = y_f \, v/\sqrt{2}$.
- Imagine another superheavy fermion with M_f.

$$\tilde{\Sigma}_2^{\rm ren}(p^2) = -\frac{3y^2 N_c}{16\pi^2}\left[p^2 - m^2 + 6\int_0^1 dx\, \Delta(p^2) \log\frac{\Delta(p^2)}{\Delta(m^2)}\right] + \mathcal{O}(y^3)$$

$$\Delta(p^2) = M_f^2 - x(1-x)p^2$$

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$$\Delta(p^2) = M_f^2 - x(1-x)p^2$$

- In the decoupling limit $M_f^2 \gg p^2$,

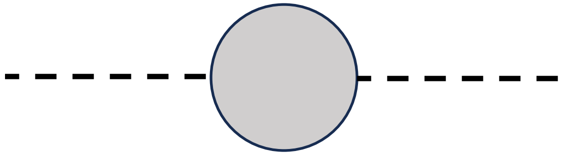
$$\tilde{\Sigma}_2^{\rm ren}(p^2) = -\frac{3y^2 N_c (p^2 - m^2)^2}{16\pi^2 M_f^2}\left[\frac{1}{10} + \frac{p^2 + 2m^2}{140 M_f^2} + \ldots\right].$$

- Suppressed as $\frac{(p^2-m^2)^2}{M_f^2}$; Insensitve to UV.

Decoupling in general amplitude

[KSC] 2410.21118

- Q: Does a general *L*-loop amplitude involving general fields with arbitrary spin behaves the same?



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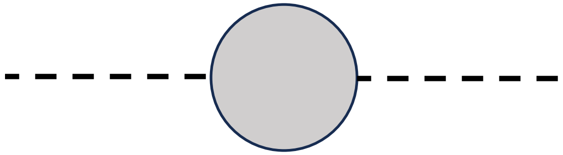

- A: Yes.
- Idea: The general 1PI amplitude has the same structure.
- Feynman parametrization makes the denominator quadratic in the internal and external momenta$\Delta(p^2) = M^2 - fp^2$

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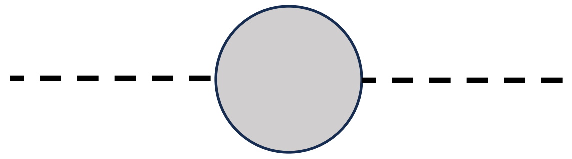


- A: Yes.
- Idea: The general 1PI amplitude has the same structure.
- Feynman parametrization makes the denominator quadratic in the internal and external momenta $\Delta(p^2) = M^2 - fp^2$
- After internal momentum integrations,

$$\tilde{\Sigma}_n(p^2) = \int d\mathbf{x} \left[A + c\Delta(p^2) + \left(d + e\Delta(p^2)\right) \log \frac{B}{\Delta(p^2)} \right],$$

$$m^2\left(p^2\right) = m^2 + \sum_{l=1}^{\infty} \left[\tilde{\Sigma}_l\left(p^2\right) - \tilde{\Sigma}_l\left(m^2\right) - \left(p^2 - m^2\right) \frac{d\tilde{\Sigma}_l}{dp^2}\left(m^2\right) \right]$$

$$\int d\mathbf{x} e \left[\Delta(p^2) \log \frac{\Delta(m^2)}{\Delta(p^2)} - (p^2 - m^2) f \right] \boxed{= m^2 + \mathcal{O}\left(\frac{(p^2 - m^2)^2}{M^2} \right).}$$

Conclusions

- Focus on physical observables; the couplings of the effective action.
    - The scalar mass runs as a function of the external momentum.

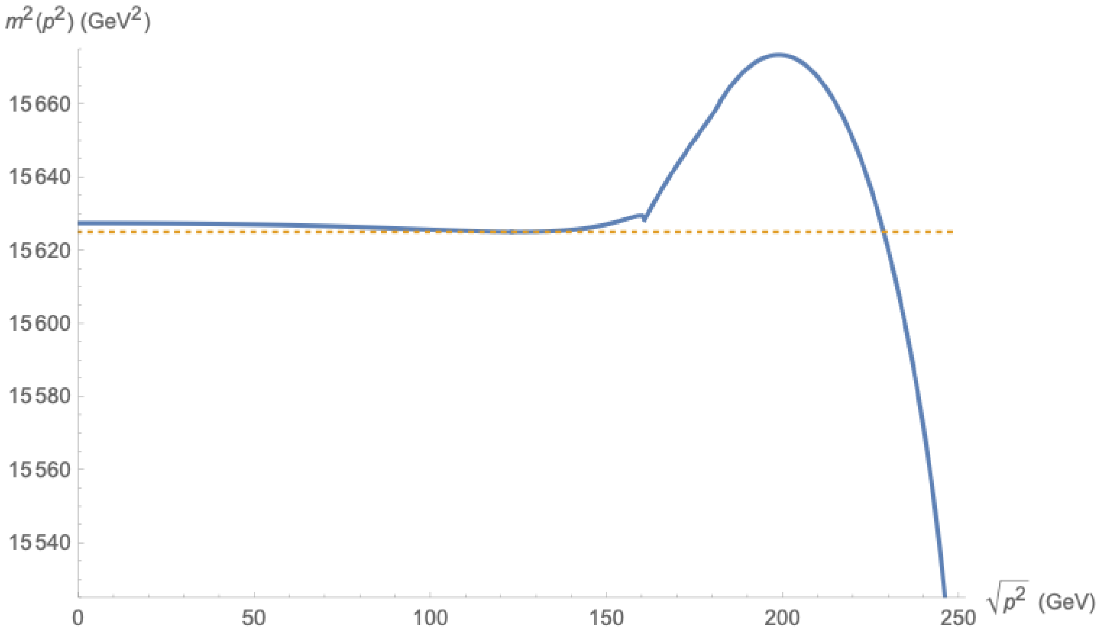
$m_B, \quad \tilde{\Sigma}(m^2)$

$m^2 + \tilde{\Sigma}(p^2) - \tilde{\Sigma}(m^2) - (p^2 - m^2)\frac{d\tilde{\Sigma}}{dp^2}(m^2)$

- Well-defined perturbation - predicts more precise momentum dep.
    - Subtraction structure; No scheme dependence; finite running.

- Stability against loop correction in the scalar mass.
    - Heavy fields decouple; insensitive to unknown UV physics.

- All is what we have expected.

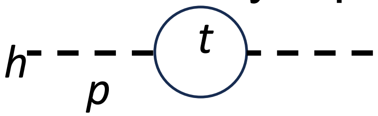


Supplementary

Regularization

[KSC] 2310.00586, 2310.10004

- The one-loop correction by top quark



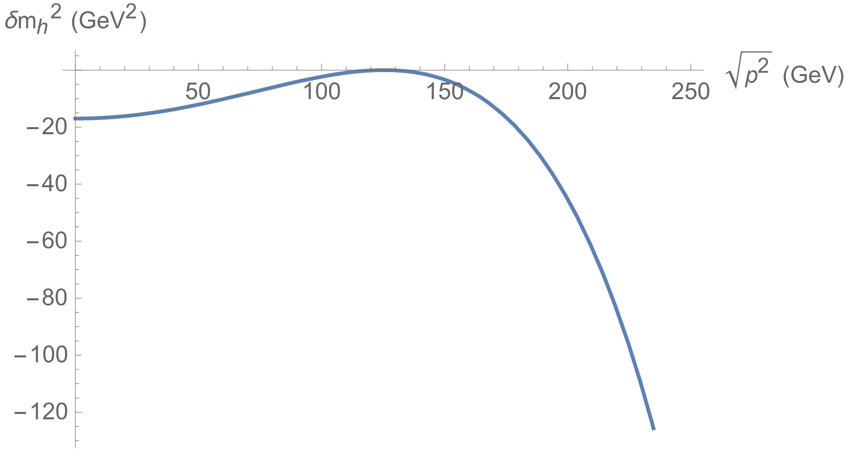
colors

$$\tilde{\Sigma}_2\left(p^2\right) = -iN_c\left(-i\frac{y_t}{\sqrt{2}}\right)^2\int\frac{d^4k}{(2\pi)^4}\,\mathrm{tr}\,\frac{i}{k+p-m_t}\,\frac{i}{k-m_t}$$
$$= -2N_c y_t^2\int_0^1 dx\int\frac{d^4\ell}{(2\pi)^4}\,\frac{\ell^2-x(1-x)p^2+m_t^2}{\left(\ell^2+x(1-x)p^2-m_t^2\right)^2}.$$

- Regularization

momentum cutoff $k_E^2 = \Lambda^2$

$$\int\frac{d^4k}{(2\pi)^4}\,\frac{1}{\left(k^2-\Delta\right)^2} = \frac{i}{16\pi^2}\left(\log\frac{\Lambda^2}{\Delta}-1\right)$$
$$\int\frac{d^4k}{(2\pi)^4}\,\frac{1}{k^2-\Delta} = \frac{-i}{16\pi^2}\left(\Lambda^2-\Delta\log\frac{\Lambda^2+\Delta}{\Delta}\right)$$

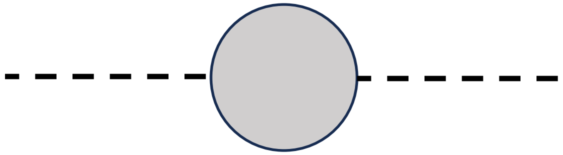


- The difference is finite

$$m^2(p^2) = m^2+\tilde{\Sigma}(p^2)-\tilde{\Sigma}(m^2)-(p^2-m^2)\frac{d\tilde{\Sigma}}{dp^2}(m^2).$$
$$= -\frac{y_f^2N_c}{8\pi^2}\int_0^1 dx\left[3\Delta_f(p^2)\log\left(\frac{\Lambda^2+\Delta_f(m_h^2)}{\Delta_f(m_h^2)}\,\frac{\Delta_f(p^2)}{\Lambda^2+\Delta_f(p^2)}\right)+\frac{2\Delta_f(m_h^2)\Lambda^2}{\Delta_f(m_h^2)+\Lambda^2}-\frac{2\Delta_f(p^2)\Lambda^2}{\Delta_f(p^2)+\Lambda^2}\right.$$
$$= \left.-(p^2-m_h^2)x(1-x)\left(\frac{2\Lambda^4}{(\Delta_f(m^2)+\Lambda^2)^2}-\frac{3\Lambda^2}{\Delta_f(m^2)+\Lambda^2}\right)\right]$$
$$-\frac{y_f^2N_c}{8\pi^2}\int_0^1 dx\left[3\Delta_f(p^2)\log\frac{\Delta_f(p^2)}{\Delta_f(m_h^2)}+3(p^2-m_h^2)x(1-x)\right]+\mathcal{O}\left(\frac{(p^2-m^2)^2}{\Lambda^2}\right)$$

General amplitude

- Q: Does a general L-loop amplitude involving general fields with arbitrary spin behaves the same?



- After evaluating the spin-dependent parts (trace, …)

$$\tilde{\Sigma}_n(p^2) = \int d^4k_1 \dots d^4k_n P(p, k_1, \dots, k_n) \prod_{j=1}^{l} \frac{1}{L_j(p, k_1, \dots, k_n) - m_j^2},$$

Polynomial in the momentum
X coupling

Quadratic form of the momentum

General amplitude

- Feynman parameterization $\frac{1}{ab} = \int_0^1 dx \frac{1}{[xa + (1-x)b]^2}$

$$\tilde{\Sigma}_n(p^2) = \int d\mathbf{x} \int d^4k_1 \dots d^4k_n \frac{\tilde{P}(k_1, \dots, k_n, p^2)}{[Q(k_1, \dots, k_n) + \Delta_n(p^2)]^l}$$

after taking the trace

quadratic in all int momenta

- Completing the square of the int mom, we define $\Delta(p^2)$ and the average mass

$$\Delta(p^2) \equiv -fp^2 + \sum_i g_i m_i^2,$$
$$\equiv -fp^2 + gM^2,$$

  - Or dominated by the largest mass
- After the mom integration $\tilde{\Sigma}(p^2)$ is the function of $\Delta(\mathrm{p}^2)$.

Free of divergence

- After the mom integration $\tilde{\Sigma}(p^2)$ is the function of $\Delta(\mathrm{p}^2)$

Cf. sim and delta dim 2

$$\tilde{\Sigma}_n(p^2) = \int d\mathbf{x} \left[A + c\Delta(p^2) + \big(d + e\Delta(p^2)\big) \log \frac{B}{\Delta(p^2)} \right],$$

Quadratic div

Log div

- The renormalized mass(d=0)

$$\begin{aligned}\tilde{\Sigma}_n^{\mathrm{ren}}(p^2) &= \tilde{\Sigma}_n(p^2) - \tilde{\Sigma}_n(m^2) - (p^2 - m^2)\frac{d\tilde{\Sigma}_n}{d(p^2)}(m^2) \\ &= \int d\mathbf{x} e \left[\Delta(p^2) \log \frac{\Delta(m^2)}{\Delta(p^2)} - (p^2 - m^2) f \right].\end{aligned}$$

- No dependence on *A, B* and *c*.

Decoupling in the general scalar mass

[KSC] 2408.06406, 2410.21118

- In the decoupling limit $M^2 \gg p^2$

$$\log \Delta(p^2) = \log gM^2 - \frac{fp^2}{gM^2} - \frac{f^2p^4}{2g^2M^4} + \mathcal{O}\left(\frac{p^6}{M^6}\right),$$

- The renormalized loop correction

$$\begin{aligned}\tilde{\Sigma}_n^{\rm ren}(p^2) &= \int d\mathbf{x} e\left[(gM^2 - fp^2)\left(\frac{fp^2}{gM^2} + \frac{f^2p^4}{2g^2M^4} + \mathcal{O}\left(\frac{p^6}{M^6}\right) - \frac{fm^2}{gM^2} - \frac{f^2m^4}{2g^2M^4} + \mathcal{O}\left(\frac{m^6}{M^6}\right)\right) - (p^2 - m^2)f\right] \\ &= \int d\mathbf{x} \frac{ef^2}{g}\left[-\frac{(p^2-m^2)^2}{2M^2} + \mathcal{O}\left(\frac{p^6}{M^4}\right) + \mathcal{O}\left(\frac{m^6}{M^4}\right)\right].\end{aligned}$$

- Decoupling – Insensitive to (unknown) UV physics.
- The *d*-term contrib. is subleading.