



LABORATÓRIO DE INSTRUMENTAÇÃO
E FÍSICA EXPERIMENTAL DE PARTÍCULAS
partículas e tecnologia



Fundação
para a Ciência
e a Tecnologia



Universidade do Minho
Escola de Ciências

Using Machine Learning to find new dark matter phenomenology in a scotogenic model

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Based on 2505.08862 - FAS, Miguel Crispim Romão, Nuno Filipe Castro and Werner Porod

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LIP - Minho

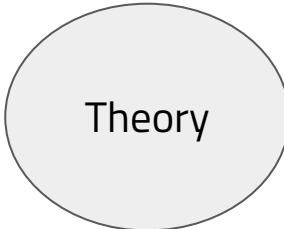
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supported by

Beyond Standard Model Scans

BSM Parameter Spaces Scans

$$\mathcal{L}(\theta)$$

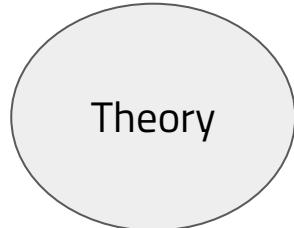


$$\boxed{1} \quad \begin{aligned} & -\frac{1}{2} \partial_\mu \partial_\nu \partial_\lambda \partial_\sigma u_{\mu\nu} C^\mu_a C^\nu_b C^\lambda_c C^\sigma_d - \frac{1}{2} \partial_\mu^2 \rho \partial_\nu \rho \partial_\lambda \partial_\sigma u_{\mu\nu} C^\mu_a C^\nu_b C^\lambda_c C^\sigma_d \\ & + \partial_\mu^2 H \partial_\nu^2 H \partial_\lambda^2 H \partial_\sigma^2 H - 2 \partial_\mu^2 H \partial_\nu^2 H \partial_\lambda \partial_\sigma u_{\mu\nu} C^\mu_a C^\nu_b C^\lambda_c C^\sigma_d - M \partial_\mu^2 \partial_\nu^2 \partial_\lambda \partial_\sigma u_{\mu\nu} C^\mu_a C^\nu_b C^\lambda_c C^\sigma_d \\ & - 2 \partial_\mu^2 H \partial_\nu^2 \partial_\lambda \partial_\sigma u_{\mu\nu} C^\mu_a C^\nu_b C^\lambda_c C^\sigma_d - M \partial_\mu^2 \partial_\nu^2 u_{\mu\nu} C^\mu_a C^\nu_b C^\lambda_c C^\sigma_d - \frac{1}{2} \partial_\mu^2 M \partial_\nu^2 \partial_\lambda \partial_\sigma u_{\mu\nu} C^\mu_a C^\nu_b C^\lambda_c C^\sigma_d + \\ & \frac{21}{2} H^2 + (H^2 \partial^2 \varphi + 2 \partial^2 \varphi \cdot \nabla^2 \varphi) + [\partial_\mu \partial_\nu \partial_\lambda \partial_\sigma u_{\mu\nu} C^\mu_a C^\nu_b C^\lambda_c C^\sigma_d] - \frac{1}{2} \partial_\mu [\partial_\sigma \partial_\lambda \partial_\nu H] \partial_\nu H - \\ & W_1^2 H^2 \partial_\nu^2 \partial_\lambda^2 H - Z_1^2 (W_1^2 A_1 W_2 + A_1 W_2 A_1^T) + Z_2^2 (W_2^2 A_1 W_1 + A_1 W_1 A_1^T) - \\ & (Z_1^2 W_1^2 A_1 W_2 + A_1 W_2 A_1^T) - \frac{1}{2} \partial_\mu^2 \partial_\nu^2 \partial_\lambda^2 H + 2 \partial_\mu^2 \partial_\nu^2 \partial_\lambda^2 \partial_\sigma^2 H - \\ & W_2^2 A_1 W_1^2 + A_1 W_1 A_1^T W_1^2 - \partial_\mu^2 \partial_\nu^2 \partial_\lambda^2 \partial_\sigma^2 H = \eta^2 [W_1^2 W_2^2 A_1^2 W_1^2 A_1^T - \\ & \frac{1}{2} (W_1^2 W_2^2 W_1^2 W_2^2 + g^2 (Z_1^2 W_1^2 Z_2^2 W_2^2 + Z_2^2 W_1^2 Z_1^2) + \\ & g^2 (Z_1^2 W_1^2 W_2^2 A_1^2 W_2^2 + Z_2^2 W_2^2 A_1^2 W_1^2)) + (\partial_\mu^2 H)^2 + (H^2 \partial^2 \varphi)^2 - 2 H^2 \partial_\mu^2 \varphi - \\ & W_1^2 W_2^2] - 2 A_1 Z_1^2 W_1^2 W_2^2 - \eta \partial_\mu H^2 \partial^2 \varphi - 2 H^2 \partial_\mu^2 \varphi + \\ & \frac{1}{2} g^2 \alpha_H (H^2 \partial^2 \varphi)^2 + (\partial_\mu^2 H)^2 + (H^2 \partial^2 \varphi)^2 - 2 \partial_\mu^2 H \partial_\mu^2 \varphi - 2 g^2 \alpha_\varphi (H^2 \partial^2 \varphi)^2 - \\ & \partial_\mu^2 \partial_\nu^2 \partial_\lambda^2 \partial_\sigma^2 H = \frac{1}{2} g^2 \alpha_H^2 (H \partial_\mu \partial_\nu \partial_\lambda \partial_\sigma H) - \frac{1}{2} g^2 \alpha_\varphi^2 (H \partial_\mu \partial_\nu \partial_\lambda \partial_\sigma \varphi) + \\ & \eta \partial_\mu M_A (W_1^2 \varphi - W_2^2 \varphi) - \eta \partial_\mu \bar{M}_A (W_2^2 \varphi - W_1^2 \varphi) + \\ & \eta \partial_\mu M_A (W_2^2 \varphi - W_1^2 \varphi) - \eta \partial_\mu \bar{M}_A (W_1^2 \varphi - W_2^2 \varphi) + \\ & \frac{1}{2} \partial_\mu^2 Z_1^2 Z_2^2 H^2 + (H^2 \partial^2 \varphi)^2 - 2 H^2 \partial_\mu^2 \varphi = - \frac{1}{2} g^2 \partial_\mu^2 \partial_\nu^2 \partial_\lambda^2 \partial_\sigma^2 H + \\ & W_2^2 \varphi^2 + \frac{1}{2} g^2 Z_2^2 W_1^2 (W_1^2 \varphi - W_2^2 \varphi) + i g^2 A_0 A_\mu (W_1^2 \varphi + \\ & W_2^2 \varphi) + i g^2 A_0 A_\mu (W_2^2 \varphi - W_1^2 \varphi) + \frac{1}{2} g^2 (2 d_2^2 - 1) Z_2^2 W_1^2 \varphi^2 - \\ & g^2 (g^2 \alpha_{A_0} A_\mu^2 \varphi^2 - \partial_\mu^2 \varphi^2) + g^2 (g^2 \alpha_{A_0} A_\mu^2 \varphi^2 + \partial_\mu^2 \varphi^2) - \frac{1}{2} g^2 (g^2 \alpha_{A_0} m_\mu^2 \varphi^2 - \\ & \frac{1}{2} g^2 Z_2^2 (W_2^2 \varphi + W_1^2 \varphi) + i g^2 (d_2^2 (4 z_2^2 - 1 - z_1^2) \varphi^2) + [\frac{1}{2} g^2 (1 + z_2^2)^2 - \\ & 1 - z_1^2] \eta^2 + (\partial_\mu^2 \eta^2) (1 - \frac{1}{2} z_2^2 - z_1^2) \eta^2 = \frac{1}{2} g^2 [(z_2^2 + 1 + z_1^2)^2 + \\ & (\partial_\mu^2 \eta^2)^2] + \frac{1}{2} g^2 \eta^2 [(1 - \frac{1}{2} z_2^2 - z_1^2)^2 + \eta^2 (\eta^2 + |\eta|^2)] + \\ & \frac{1}{2} \partial_\mu^2 (H \partial^2 \varphi) + i g^2 (W_1^2 \partial_\mu \partial_\nu X^A - W_2^2 \partial_\mu \partial_\nu X^A) + \frac{1}{2} g^2 (H^2 \partial^2 \varphi)^2 + X^A \partial_\mu^2 \varphi - \\ & \frac{1}{2} g^2 (H^2 \partial^2 \varphi)^2 + i g^2 (W_1^2 \partial_\mu \partial_\nu X^A - W_2^2 \partial_\mu \partial_\nu X^A) + \eta g^2 (C_\mu^A C_\nu^B + \\ & m_A^2 C_\mu^A C_\nu^B) + \frac{1}{2} g^2 \eta^2 (C_\mu^A C_\nu^B + C_\mu^B C_\nu^A) + [\eta g^2 (C_\mu^A C_\nu^B + C_\mu^B C_\nu^A) - m_A^2 \eta^2 C_\mu^A C_\nu^B] - \\ & \frac{3}{2} g^2 (H \partial_\mu^2 \eta^2) + X^A \partial_\mu^2 (X^B - M^B X^A + \bar{N}^B (\partial^2 - M^2) X^A + X^B \partial^2 - \\ & \frac{1}{2} g^2 (X^A \partial^2 X^B + X^B \partial^2 X^A) + i g^2 W_1^2 (A_0 X^A - A_\mu X^A + \partial_\mu X^A) + i g^2 W_2^2 (A_0 X^A - \\ & A_\mu X^A) - \frac{1}{2} g^2 (W_1^2 X^A \partial^2 X^B + X^B \partial^2 X^A) + i g^2 A_0 (A_\mu X^A - \partial_\mu X^A) X^B + \\ & \frac{1}{2} g^2 (A_\mu X^A \partial^2 X^B + X^B \partial^2 X^A) + i g^2 (W_1^2 X^A \partial^2 X^B + X^B \partial^2 X^A) + i g^2 M_A (X^A X^B \partial^2 - X^B X^A \partial^2) + \\ & i g^2 M_A (X^A X^B \partial^2 - X^B X^A \partial^2) + i g^2 M_A (X^A X^B \partial^2 - X^B X^A \partial^2) + i g^2 M_A (X^A X^B \partial^2 - X^B X^A \partial^2) \end{aligned}$$

θ : Free parameters

BSM Parameter Spaces Scans

$$\mathcal{L}(\theta)$$



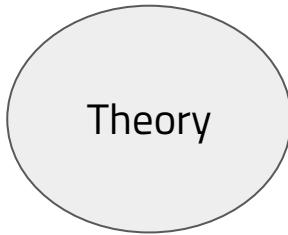
$$\{\mathcal{O}_i(\theta)\}$$

Predictions

$$\boxed{\begin{aligned} & -\frac{1}{2}\partial_\mu\phi_1\partial_\mu^\ast\phi_2 - g_{\mu\nu}\partial_\mu\phi_1\partial_\nu^\ast\phi_2 - \frac{1}{2}\partial_\mu^\ast\phi_1\partial_\mu^\ast\phi_2 - g_{\mu\nu}\partial_\mu\phi_1\partial_\nu^\ast\phi_2 \\ & + \log[H^2 - (\phi_1^2 + \phi_2^2)] - \log[\phi_1^2 + \phi_2^2] - i\phi_1 H^\dagger \partial_\mu^\ast\phi_2 A_\mu - i\phi_2 H^\dagger \partial_\mu^\ast\phi_1 A_\mu - \\ & - \log[H^2 - (\phi_1^2 + \phi_2^2)] - M\phi_1\phi_2 + i\phi_1 H^\dagger \partial_\mu^\ast\phi_2 A_\mu - \frac{1}{2}g_1 M\phi_1\phi_2 - iH^\dagger \partial_\mu^\ast\phi_1 A_\mu + \\ & + \frac{1}{2}g_2 H^\dagger \partial_\mu^\ast\phi_2 A_\mu + 2g_2\phi_1\phi_2] + \frac{i\partial_\mu^\ast\phi_1}{2}H^\dagger A_\mu - \frac{1}{2}g_2 [H^\dagger \partial_\mu^\ast\phi_1]^\dagger V_\mu^\dagger - \\ & - W_1^\dagger V_\mu^\dagger) - Z_1^\dagger W_2^\dagger A_1\phi_2 + W_2^\dagger A_1\phi_1) + Z_2^\dagger W_1^\dagger A_1\phi_1 + W_1^\dagger A_1\phi_1 + \\ & - [Z_2^\dagger W_1^\dagger A_1\phi_1 + Z_1^\dagger W_2^\dagger A_1\phi_2 + Z_2^\dagger W_1^\dagger A_2\phi_2 + Z_1^\dagger W_2^\dagger A_2\phi_1 - \\ & - W_2^\dagger A_2\phi_2] + A_1[W_2^\dagger A_1\phi_1 + W_1^\dagger A_1\phi_2] - \gamma_1^\dagger W_1^\dagger W_2^\dagger A_1\phi_1 + W_2^\dagger A_1\phi_2 + \\ & + \frac{1}{2}g_2^\dagger W_2^\dagger W_1^\dagger A_2\phi_2 + \frac{1}{2}g_2^\dagger Z_1^\dagger W_2^\dagger A_1\phi_2 - Z_2^\dagger W_1^\dagger A_1\phi_1 + \\ & - [Z_1^\dagger W_2^\dagger A_2\phi_2 + Z_2^\dagger W_1^\dagger A_1\phi_1] - \gamma_2^\dagger W_1^\dagger W_2^\dagger A_1\phi_1 - W_1^\dagger A_2\phi_2 - \\ & - W_2^\dagger A_1\phi_2 - 2A_1 Z_1^\dagger W_2^\dagger A_1\phi_1 - g_3(H^2 + H^\dagger \partial_\mu^\ast\phi_2) + 2H^\dagger \partial_\mu^\ast\phi_2 - 2g_2(H^2 + H^\dagger \partial_\mu^\ast\phi_2)^\dagger + \\ & - 2g_2^\dagger Z_1^\dagger W_2^\dagger A_1\phi_1 + (g_2^\dagger H^2 + H^\dagger \partial_\mu^\ast\phi_2 + 2H^\dagger \partial_\mu^\ast\phi_2 - 2g_2(H^2 + H^\dagger \partial_\mu^\ast\phi_2)^\dagger) - \\ & - 2g_2^\dagger Z_1^\dagger W_2^\dagger A_1\phi_1 + (g_2^\dagger H^2 + H^\dagger \partial_\mu^\ast\phi_2 + 2H^\dagger \partial_\mu^\ast\phi_2 - 2g_2(H^2 + H^\dagger \partial_\mu^\ast\phi_2)^\dagger) + \\ & + W_1^\dagger(\phi_1^\dagger\phi_1 - \phi_2^\dagger\phi_2) + \frac{1}{2}g_3^\dagger W_1^\dagger(H_1\phi_2^\dagger - \phi_2^\dagger H_1) - \frac{1}{2}g_3^\dagger W_1^\dagger(H_2\phi_2^\dagger - \\ & - \phi_2^\dagger H_2) + \frac{1}{2}g_3^\dagger (Z_1^\dagger)(H_2\phi_2^\dagger - \phi_2^\dagger H_2) - i\frac{1}{2}g_3^\dagger M_2^\dagger W_1^\dagger(\phi_2^\dagger - W_1^\dagger\phi_1) + \\ & + ig_5 M_2^\dagger (W_1^\dagger\phi_2^\dagger - W_1^\dagger\phi_1) - i(g_3^\dagger W_1^\dagger(\phi_2^\dagger - \phi_1^\dagger\phi_2) + \\ & + g_3^\dagger W_1^\dagger(\phi_1^\dagger\phi_2 - \phi_2^\dagger\phi_1) + (g_3^\dagger H^2 + H^\dagger \partial_\mu^\ast\phi_2 + 2H^\dagger \partial_\mu^\ast\phi_2 - 2g_2(H^2 + H^\dagger \partial_\mu^\ast\phi_2)^\dagger) - \\ & - 2g_2^\dagger Z_1^\dagger W_2^\dagger + (g_2^\dagger + 2g_2^\dagger - 1)\phi_2^\dagger\phi_1^\dagger - \frac{1}{2}g_2^\dagger Z_2^\dagger W_1^\dagger W_2^\dagger A_1\phi_1 - \\ & - W_2^\dagger A_2\phi_2^\dagger + \frac{1}{2}g_2^\dagger Z_1^\dagger W_2^\dagger (W_2^\dagger\phi_1^\dagger - W_1^\dagger\phi_2^\dagger) + \frac{1}{2}g_2^\dagger (2g_2^\dagger - 1)Z_2^\dagger W_1^\dagger\phi_2^\dagger - \\ & - W_1^\dagger\phi_1^\dagger + ig_5^\dagger A_1 H_1^\dagger W_2^\dagger (W_2^\dagger\phi_1^\dagger - W_1^\dagger\phi_2^\dagger) - g_3^\dagger(2g_2^\dagger - 1)Z_2^\dagger W_1^\dagger\phi_2^\dagger - \\ & - g_3^\dagger A_2 W_2^\dagger (W_2^\dagger\phi_1^\dagger - \phi_1^\dagger\phi_2) + g_3^\dagger(\phi_1^\dagger\phi_2 - \phi_2^\dagger\phi_1) - m_3^\dagger(\phi_1^\dagger\phi_2 - \phi_2^\dagger\phi_1) - \\ & - \frac{1}{2}g_2^\dagger Z_1^\dagger W_2^\dagger \phi_1^\dagger\phi_2 + (g_2^\dagger + 2g_2^\dagger - 1)\phi_2^\dagger\phi_1^\dagger + \frac{1}{2}g_2^\dagger (2g_2^\dagger - 1) - \\ & - 1 - \gamma_1^\dagger\phi_1^\dagger) + (g_3^\dagger + 1 - \frac{1}{2}g_2^\dagger - \gamma_2^\dagger)\phi_1^\dagger + \frac{1}{2}g_3^\dagger((\phi_1^\dagger + 1 - \gamma_1^\dagger)\phi_2^\dagger) + \\ & + (g_2^\dagger - 1 - \frac{1}{2}g_2^\dagger + \gamma_1^\dagger)\phi_2^\dagger + \frac{1}{2}g_3^\dagger((\phi_2^\dagger + 1 - \gamma_2^\dagger)\phi_1^\dagger) + (g_2^\dagger - 1 - \frac{1}{2}g_2^\dagger + \\ & + \frac{1}{2}g_3^\dagger) + \frac{1}{2}g_3^\dagger((\phi_1^\dagger + 1 - \gamma_1^\dagger)\phi_2^\dagger) + \frac{1}{2}g_3^\dagger((\phi_2^\dagger + 1 - \gamma_2^\dagger)\phi_1^\dagger) - \\ & - \frac{1}{2}g_2^\dagger (H(\phi_1^\dagger)^\dagger + g_2^\dagger(\phi_1^\dagger)^\dagger) + \frac{1}{2}g_2^\dagger (H(\phi_2^\dagger)^\dagger - m_2^\dagger(\phi_2^\dagger)) - m_2^\dagger C_2^\dagger(1 - \\ & - \gamma_2^\dagger)\phi_1^\dagger) - \frac{1}{2}g_2^\dagger (H(\phi_2^\dagger)^\dagger) + \frac{1}{2}g_2^\dagger (H(\phi_1^\dagger)^\dagger) + \frac{1}{2}g_3^\dagger((\phi_1^\dagger X^\dagger)^\dagger + X^\dagger(\phi_1^\dagger)^\dagger) - \\ & + \frac{1}{2}g_3^\dagger((\phi_2^\dagger X^\dagger)^\dagger + X^\dagger(\phi_2^\dagger)^\dagger) + g_3^\dagger X^\dagger(\phi_1^\dagger X^\dagger - M\phi_1^\dagger X^\dagger - X^\dagger(\phi_2^\dagger M^\dagger) + X^\dagger M^\dagger X^\dagger - \\ & - X^\dagger X^\dagger M^\dagger + g_3^\dagger X^\dagger(\phi_2^\dagger X^\dagger) - ig_5^\dagger A_1 C_2^\dagger(\phi_1^\dagger X^\dagger - \phi_2^\dagger X^\dagger) + ig_5^\dagger A_2 C_2^\dagger(\phi_1^\dagger X^\dagger - \phi_2^\dagger X^\dagger) - ig_5^\dagger X^\dagger A_1 C_2^\dagger(\phi_1^\dagger X^\dagger - \phi_2^\dagger X^\dagger) + ig_5^\dagger X^\dagger A_2 C_2^\dagger(\phi_1^\dagger X^\dagger - \phi_2^\dagger X^\dagger) + \\ & + \frac{1}{2}g_2^\dagger g_3^\dagger(X^\dagger X^\dagger\phi_1^\dagger - X^\dagger X^\dagger\phi_2^\dagger) + i(g_3^\dagger X^\dagger X^\dagger\phi_1^\dagger - X^\dagger X^\dagger\phi_2^\dagger) + \\ & + ig M_2^\dagger(\phi_1^\dagger X^\dagger\phi_1^\dagger - X^\dagger X^\dagger\phi_1^\dagger) + \frac{1}{2}g_2^\dagger M(\phi_1^\dagger X^\dagger\phi_1^\dagger - X^\dagger X^\dagger\phi_1^\dagger) + \end{aligned}} \boxed{\begin{aligned} & - ig M_2^\dagger(\phi_1^\dagger X^\dagger\phi_1^\dagger - X^\dagger X^\dagger\phi_1^\dagger) + \frac{1}{2}g_2^\dagger M(\phi_1^\dagger X^\dagger\phi_1^\dagger - X^\dagger X^\dagger\phi_1^\dagger) + \\ & + ig M_2^\dagger(\phi_1^\dagger X^\dagger\phi_1^\dagger - X^\dagger X^\dagger\phi_1^\dagger) + \frac{1}{2}g_2^\dagger M(\phi_1^\dagger X^\dagger\phi_1^\dagger - X^\dagger X^\dagger\phi_1^\dagger) + \end{aligned}}$$

- θ: Free parameters
- O: Observable

BSM Parameter Spaces Scans



$\{\mathcal{O}_i(\theta)\}$

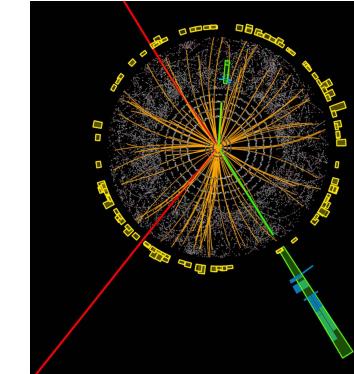
Predictions

$\{\mathcal{O}_{\text{LB}}^i \leq \mathcal{O}^i(\theta) \leq \mathcal{O}_{\text{UB}}^i\}$

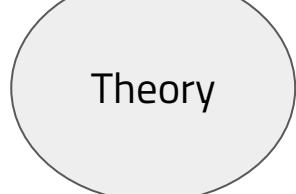
Agrees
with exp.
results?

θ : Free parameters
 O : Observable

$$\begin{aligned}
 & -\frac{1}{2}\partial_{\mu}\partial_{\nu}\partial_{\rho}^2\partial_{\sigma}^2 - g_{\mu\rho}g_{\nu\sigma}\partial_{\mu}\partial_{\nu}\partial_{\rho}\partial_{\sigma}^2 - \frac{1}{2}\partial_{\mu}^2\partial_{\nu}^2\partial_{\rho}^2\partial_{\sigma}^2 + \\
 & \log(H^2) - \partial_{\mu}\partial_{\nu}\partial_{\rho}^2\partial_{\sigma}^2 - H^2\partial_{\mu}^2\partial_{\nu}^2\partial_{\rho}^2\partial_{\sigma}^2 - \partial_{\mu}H\partial_{\nu}H\partial_{\rho}^2\partial_{\sigma}^2 - \\
 & \log(H^2) - \partial_{\mu}\partial_{\nu}\partial_{\rho}^2\partial_{\sigma}^2 - H^2\partial_{\mu}^2\partial_{\nu}^2\partial_{\rho}^2\partial_{\sigma}^2 - \frac{1}{2}\partial_{\mu}^2\partial_{\nu}^2\partial_{\rho}^2\partial_{\sigma}^2 + \\
 & \frac{3H}{2}\partial_{\mu}^2 + (H^2 - \partial_{\rho}^2 + 2\rho^2)\partial_{\sigma}^2 + \frac{3H^2}{2}\partial_{\mu}^2\partial_{\nu}^2\partial_{\rho}^2\partial_{\sigma}^2 + \\
 & Z_1^2(W_1^2\Lambda_1 W_1^2 - W_1^2\Lambda_0 W_1^2) + Z_2^2(W_2^2\Lambda_1 W_2^2 - W_2^2\Lambda_0 W_2^2) + \\
 & W_1^2\partial_{\mu}^2\partial_{\nu}^2\partial_{\rho}^2\partial_{\sigma}^2 + W_2^2\partial_{\mu}^2\partial_{\nu}^2\partial_{\rho}^2\partial_{\sigma}^2 - W_1^2\partial_{\mu}^2W_1^2\Lambda_1 W_1^2 + \\
 & W_1^2\partial_{\mu}^2W_1^2\Lambda_0 W_1^2 + \partial_{\mu}^2W_1^2W_2^2\Lambda_1 W_2^2 - \partial_{\mu}^2W_1^2W_2^2\Lambda_0 W_2^2 + \\
 & \frac{1}{2}(W_1^2W_2^2W_1^2W_2^2 + g_{\mu\rho}^2)(Z_1^2W_1^2Z_2^2W_2^2(W_1^2W_2^2)) + \\
 & \frac{1}{2}(W_1^2W_2^2W_1^2W_2^2 - g_{\mu\rho}^2)(Z_1^2W_1^2Z_2^2W_2^2(W_1^2W_2^2)) + \\
 & W_1^2W_2^2 - 2\lambda_1 Z_1^2W_1^2W_2^2 - g_{\mu\rho}^2(H^2 + H\partial_{\rho}^2\partial_{\sigma}^2 - 2(\partial_{\rho}^2\partial_{\sigma}^2)^2) - \\
 & g_{\mu\rho}^2(H^2 + \partial_{\rho}^2\partial_{\sigma}^2) + \frac{1}{2}\partial_{\mu}^2(W_1^2H^2 - \rho_{\mu\rho}\partial_{\rho}^2H) - \frac{1}{2}\partial_{\nu}^2(W_1^2H^2 - \rho_{\nu\rho}\partial_{\rho}^2H) + \\
 & W_1^2\partial_{\mu}^2\partial_{\nu}^2\partial_{\rho}^2\partial_{\sigma}^2 + \frac{1}{2}\partial_{\mu}^2(W_1^2H^2 - \rho_{\mu\rho}\partial_{\rho}^2H) - \frac{1}{2}\partial_{\nu}^2(W_1^2H^2 - \rho_{\nu\rho}\partial_{\rho}^2H) + \\
 & g_{\mu\rho}^2M_A(W_1^2\partial_{\rho}^2 - W_1^2\partial_{\sigma}^2) - g_{\mu\rho}^2M_A(W_2^2\partial_{\rho}^2 - W_2^2\partial_{\sigma}^2) + \\
 & g_{\mu\rho}^2M_A(W_1^2\partial_{\rho}^2 + W_2^2\partial_{\rho}^2 - 2(\partial_{\rho}^2)^2 - 1^2\partial_{\sigma}^2) - \frac{1}{2}\partial_{\mu}^2\partial_{\nu}^2(W_1^2\partial_{\rho}^2 - \\
 & W_2^2\partial_{\rho}^2) - \frac{1}{2}\partial_{\mu}^2Z_1^2W_1^2(W_1^2\partial_{\rho}^2 - W_1^2\partial_{\sigma}^2) + \partial_{\mu}^2A_0(W_1^2\partial_{\rho}^2 + \\
 & W_1^2\partial_{\sigma}^2) + \partial_{\mu}^2A_0(W_2^2\partial_{\rho}^2 - W_2^2\partial_{\sigma}^2) - \frac{1}{2}\partial_{\mu}^2(\partial_{\nu}^2W_1^2\partial_{\rho}^2 - \\
 & g_{\mu\rho}^2A_0(W_1^2\partial_{\rho}^2 - \partial_{\rho}^2\partial_{\sigma}^2) + \partial_{\nu}^2W_1^2\partial_{\rho}^2 - m_W^2\partial_{\mu}^2\partial_{\nu}^2 - \\
 & \frac{1}{2}\partial_{\mu}^2Z_2^2(W_2^2\partial_{\rho}^2 - W_2^2\partial_{\sigma}^2) + \partial_{\mu}^2W_2^2\partial_{\rho}^2 - \frac{1}{2}\partial_{\nu}^2W_2^2\partial_{\rho}^2 + \\
 & 1 - \gamma^2\eta^2) + (\partial_{\rho}^2)^2(1 - \frac{1}{2}\gamma^2 - \gamma^2\eta^2) + \frac{3}{2}\partial_{\mu}^2((1 - \gamma^2\eta^2) + \\
 & (\partial_{\rho}^2)^2) + \partial_{\mu}^2(\partial_{\nu}^2W_1^2\partial_{\rho}^2 - \frac{1}{2}\partial_{\mu}^2W_1^2\partial_{\nu}^2) + \partial_{\mu}^2(\partial_{\nu}^2W_2^2\partial_{\rho}^2 - \\
 & \frac{1}{2}\partial_{\mu}^2W_2^2\partial_{\nu}^2) + \partial_{\mu}^2(W_1^2\partial_{\rho}^2 - M^2\partial_{\rho}^2) - X(\partial_{\rho}^2 - M^2)X + X^2\partial_{\rho}^2 - \\
 & \frac{3}{2}\partial_{\mu}^2(H(\partial_{\rho}^2) + \partial_{\mu}^2(\partial_{\rho}^2\partial_{\sigma}^2)) - \frac{3}{2}\partial_{\mu}^2H(\partial_{\rho}^2) - \frac{3}{2}\partial_{\mu}^2H(\partial_{\rho}^2 - \\
 & \partial_{\mu}^2X^2) + \partial_{\mu}^2Z_1^2(\partial_{\mu}^2X^2 - \partial_{\mu}^2X^2) + i\partial_{\mu}^2A_0(\partial_{\mu}^2X^2 - \\
 & \partial_{\mu}^2X^2) - \frac{1}{2}\partial_{\mu}^2(X^2X^2 + X^2X^2H - \frac{1}{2}X^2X^2V) + \\
 & \frac{1}{2}\partial_{\mu}^2(X^2X^2\partial_{\rho}^2 - X^2X^2\partial_{\sigma}^2) + \frac{1}{2}\partial_{\mu}^2X(X^2\partial_{\rho}^2 - X^2X^2\partial_{\sigma}^2) + \\
 & igM_{S_0}(X^2X^2\partial_{\rho}^2 - X^2X^2\partial_{\sigma}^2) + \frac{1}{2}\partial_{\mu}^2M(X^2X^2\partial_{\rho}^2 - X^2X^2\partial_{\sigma}^2)
 \end{aligned}$$



BSM Parameter Spaces Scans



$\{\mathcal{O}_i(\theta)\}$

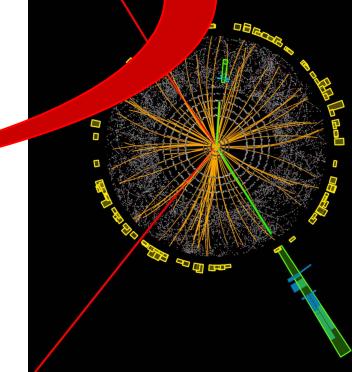
$\{\mathcal{O}_{\text{LB}}^i \leq \mathcal{O}^i(\theta) \leq \mathcal{O}_{\text{UB}}^i\}$

Predictions

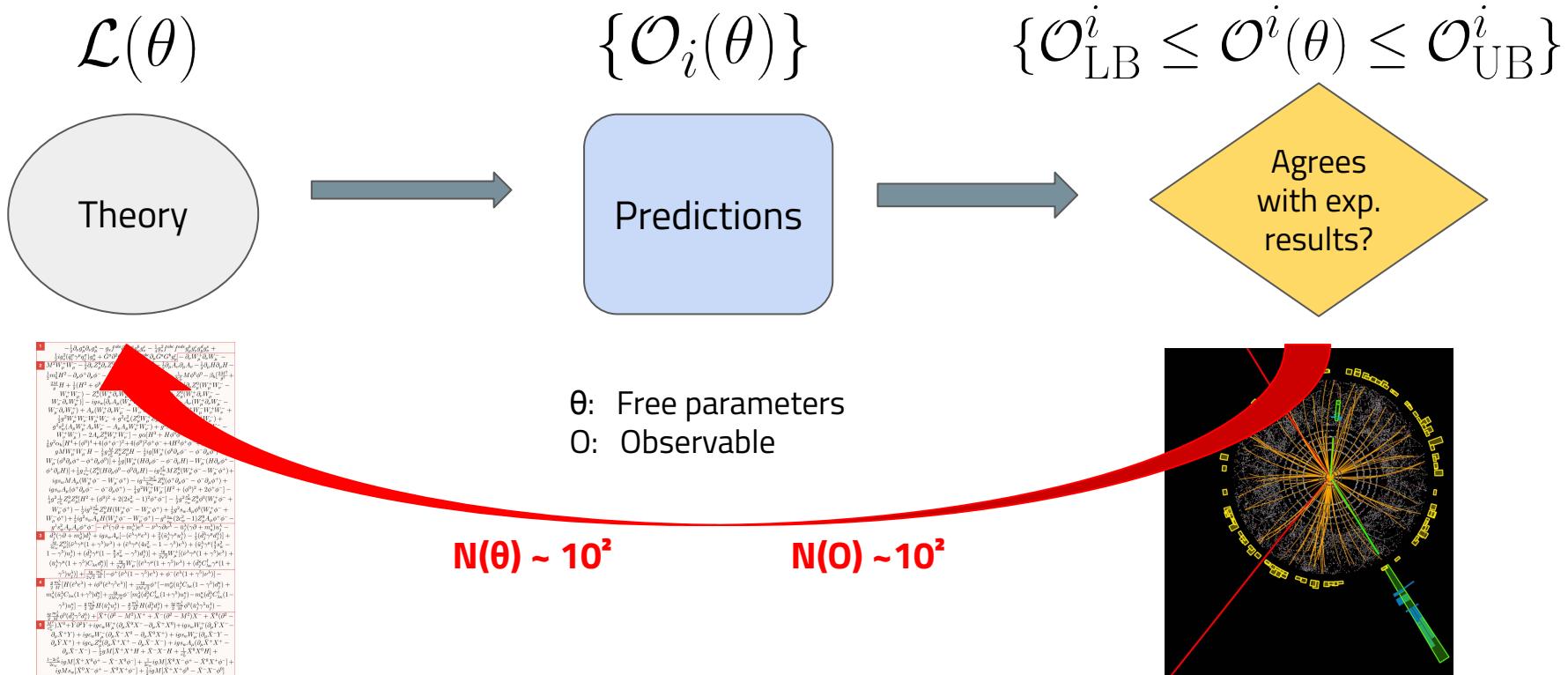
Agrees
with exp.
results?

$$\boxed{1} \begin{aligned} & -16g_1\phi_1\phi_2^2 - g_2\phi_2^2\phi_3^2 - 2g_2^2\phi_2^2 + \frac{1}{2}\phi_2^2\phi_3^2\mu_1^2\phi_1^2\phi_2^2\phi_3^2 + \\ & \log[\phi_2^2(\phi_1\phi_2^2\phi_3^2)^2 + G^2] + \frac{1}{2}\phi_2^2\phi_3^2\mu_1^2\phi_1^2\phi_2^2\phi_3^2 + \\ & \log[2H^2\phi_2^2\phi_3^2(\phi_1\phi_2^2\phi_3^2)^2 + 2H^2\phi_2^2\phi_3^2(\phi_1\phi_2^2\phi_3^2)^2 + \\ & 2H^2H + H^4 + 2Z^2W_1^2V_1^2 + Z_1^2W_1^2V_1^2 + \\ & W_1^2V_1^2 + Z_1^2W_1^2V_1^2 + Z_2^2W_2^2V_2^2 + Z_2^2W_2^2V_2^2 + \\ & W_2^2V_2^2 + Z_2^2W_2^2V_2^2 + 2(Z_1^2W_1^2V_1^2 + Z_2^2W_2^2V_2^2) + \\ & W_1^2\phi_1^2\phi_2^2\phi_3^2 + A_3(\phi_1\phi_2^2\phi_3^2)^2 - H^2\phi_2^2\phi_3^2 + \\ & \frac{1}{2}(W_1^2W_2^2W_1^2W_2^2 + g_2^2\phi_2^4)(2W_1^2V_1^2 + 2W_2^2V_2^2) - \\ & 2\phi_1H^2\phi_2^2\phi_3^2 - 2\phi_2^2Z^2W_1^2V_1^2 - g_3H^2\phi_2^2\phi_3^2 + Hg^2\phi_2^2\phi_3^2 + \\ & \frac{1}{2}\phi_2^2\phi_3^2(\phi_1H^2\phi_2^2\phi_3^2 + g_2^2\phi_2^4) + \frac{1}{2}\phi_2^2\phi_3^2(\phi_1H^2\phi_2^2\phi_3^2 + g_3^2\phi_3^4) + \\ & \phi_2^2\phi_3^2(\phi_1H^2\phi_2^2\phi_3^2 + g_2^2\phi_2^4) + M_2^2Z_1^2W_1^2\phi_1^2\phi_2^2\phi_3^2 + \\ & M_2^2Z_2^2W_2^2\phi_1^2\phi_2^2\phi_3^2 + \dots \end{aligned}$$

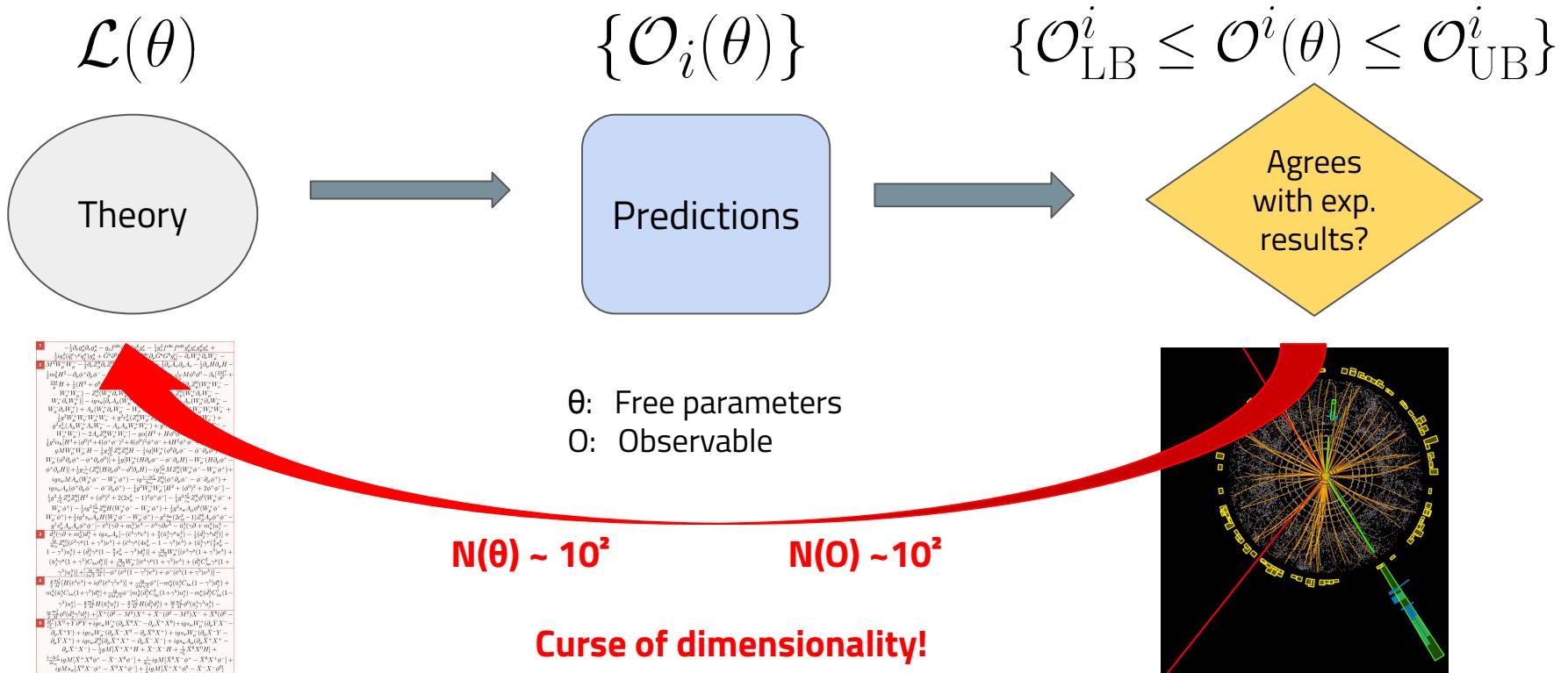
θ : Free parameters
 O : Observable



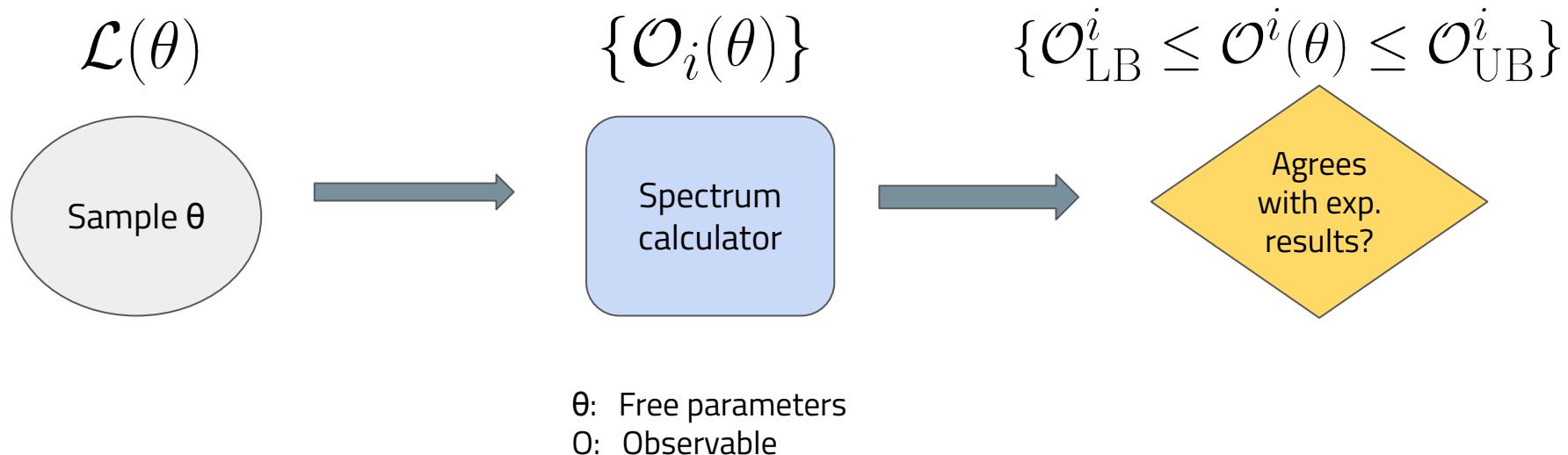
BSM Parameter Spaces Scans



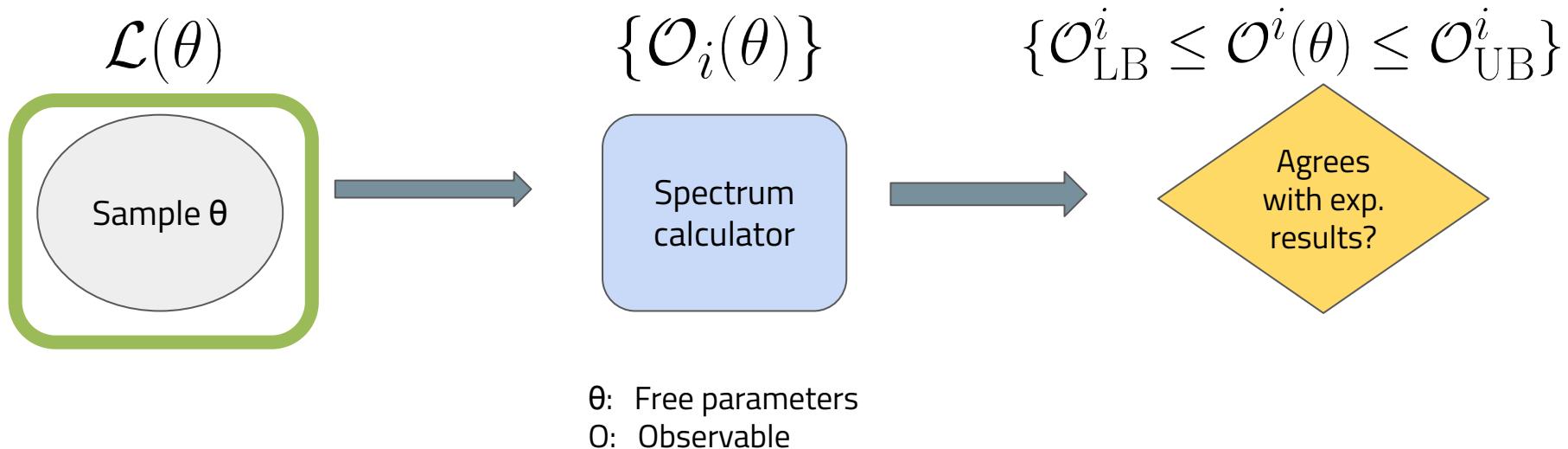
BSM Parameter Spaces Scans



BSM Parameter Spaces Scans



BSM Parameter Spaces Scans



Proposed solution: Increase efficiency by modifying sampling

Methodology

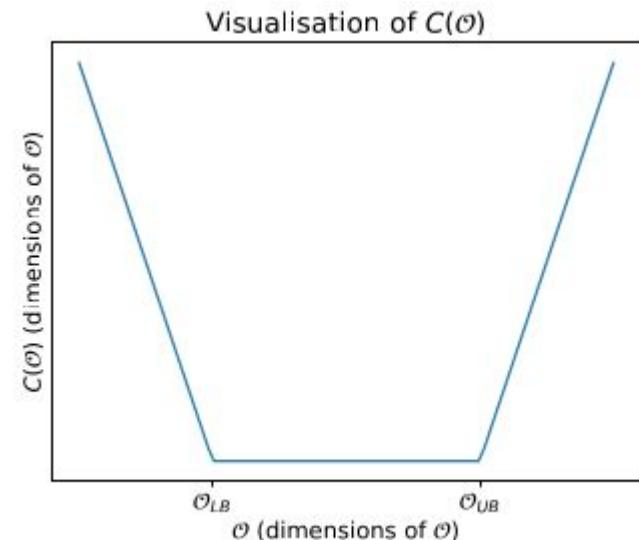
Parameter Spaces Scans Black Box Optimisation

FAS, MCR, NFC, MN and WP
Phys. Rev. D 107, 035004
arXiv 2206.09223

- **How far** is the point from being valid
 - Cost function $C(\mathcal{O})$
- To find valid points is to minimise $C(\mathcal{O})$



Optimisation problem!

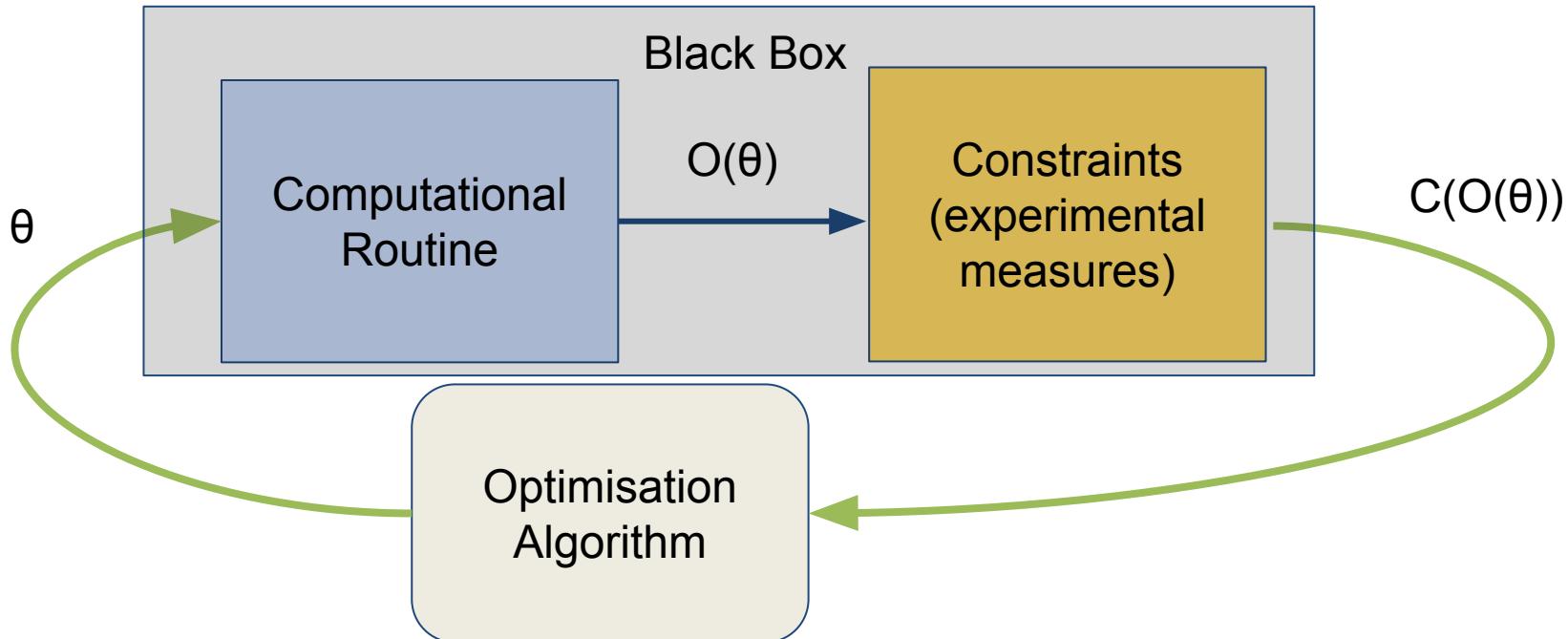


$$C(\mathcal{O}) = \max(0, -\mathcal{O} + \mathcal{O}_{LB}, \mathcal{O} - \mathcal{O}_{UB})$$

Point is valid when **$C(\mathbf{0}) = 0$**

Parameter Spaces Scans Black Box Optimisation

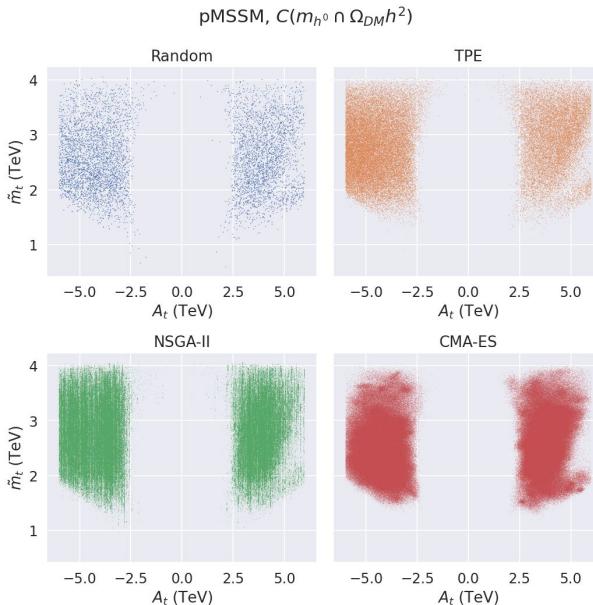
FAS, MCR, NFC, MN and WP
Phys. Rev. D 107, 035004
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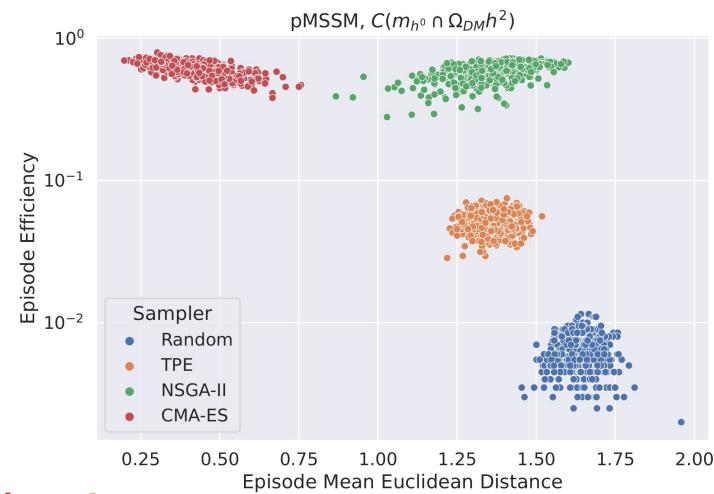
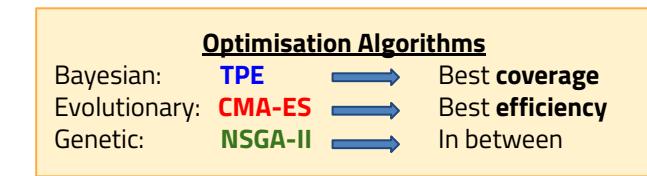
Black Box Optimization Scans

First case study

- Physics cases: **Supersymmetry** constrained by **Higgs mass** and **Dark Matter Relic Density**
 - cMSSM:** 4 free parameters
 - pMSSM:** 19 free parameters



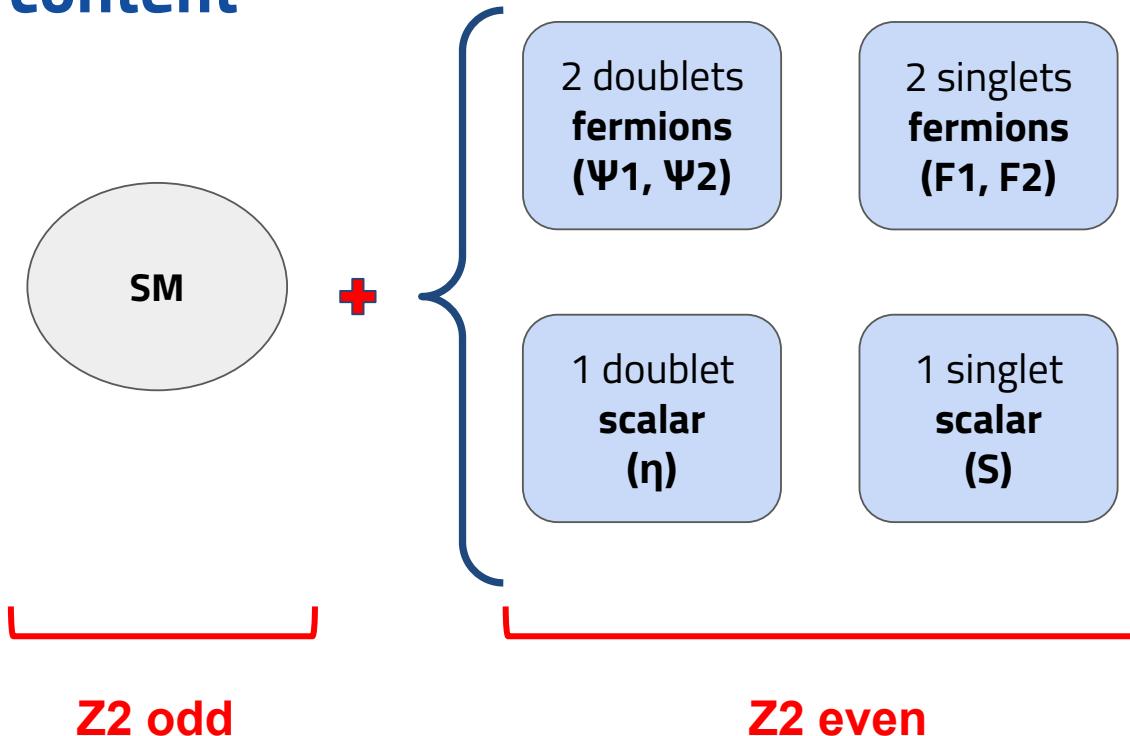
Gain of ~ 100x in efficiency!



Scotogenic Model

Scotogenic Model

Particle content



Scotogenic Model

Particle content

	Fermions			Scalars		
	Ψ_1	Ψ_2	F_1	F_2	η	S
$SU(2)_L$	2	2	1	1	2	1
$U(1)_Y$	-1	1	0	0	1	0

- 46 free **parameters**
- 31 (experimental / theoretical) **constraints**

- **Neutrino masses at 1-loop level**
- **DM candidates**
- **LFV**
- **(g - 2) μ**

Scotogenic Model

Fermion sector

$$\begin{aligned}\mathcal{L}_{\text{fermion}} \supset & -\frac{1}{2}M_{F_{ij}}F_iF_j - M_\Psi\Psi_1\Psi_2 \\ & - y_{1i}\Psi_1HF_i - y_{2i}\Psi_2\tilde{H}F_i \\ & - g_\Psi^k\Psi_2L_kS - g_{F_j}^k\eta L_kF_j - g_R^ke_k^c\tilde{\eta}\Psi_1 + \text{h.c.}\end{aligned}$$

Neutrino masses at 1-loop level

Scotogenic Model

Neutrino masses

$$\nu_i \rightarrow \overset{\phi_n^0}{\bullet} \leftarrow \overset{\chi_k^0}{\bullet} \nu_j \equiv \overline{\nu_j^c} (\mathcal{M}_\nu)_{ji} \nu_i$$

$$\mathcal{G} = \begin{pmatrix} g_\Psi^1 & g_\Psi^2 & g_\Psi^3 \\ g_{F_1}^1 & g_{F_1}^2 & g_{F_1}^3 \\ g_{F_2}^1 & g_{F_2}^2 & g_{F_2}^3 \end{pmatrix}$$

$$\mathcal{M}_\nu = \mathcal{G}^T M_L \mathcal{G}$$

Scotogenic Model

Neutrino masses

$$\nu_i \rightarrow \overset{\phi_n^0}{\bullet} \xrightarrow{\chi_k^0} \nu_j \quad \equiv \quad \overline{\nu_j^c} (\mathcal{M}_\nu)_{ji} \nu_i$$

$$\mathcal{G} = \begin{pmatrix} g_\Psi^1 & g_\Psi^2 & g_\Psi^3 \\ g_{F_1}^1 & g_{F_1}^2 & g_{F_1}^3 \\ g_{F_2}^1 & g_{F_2}^2 & g_{F_2}^3 \end{pmatrix}$$

$$\mathcal{M}_\nu = \mathcal{G}^T M_L \mathcal{G}$$

$$|U_{\text{PMNS}}^{ij}| \sim 1 \quad \xrightarrow{\hspace{1cm}} \quad \left\{ \begin{array}{l} |g_\Psi^1| \sim |g_\Psi^2| \sim |g_\Psi^3| \\ |g_{F_k}^1| \sim |g_{F_k}^2| \sim |g_{F_k}^3| \end{array} \right.$$

Scotogenic Model

Mass basis

- Scalar sector

$$\left(\phi_1^0, \phi_2^0, A^0 \right)^T = U_\phi \left(S, \eta^0, A^0 \right)^T$$

- Fermion sector

$$\left(\chi_1^0, \chi_2^0, \chi_3^0, \chi_4^0 \right)^T = U_\chi \left(F_1, F_2, \Psi_1^0, \Psi_2^0 \right)^T$$

Scotogenic Model

Mass basis

- Scalar sector

$$\left(\phi_1^0, \phi_2^0, A^0 \right)^T = U_\phi \left(S, \eta^0, A^0 \right)^T$$

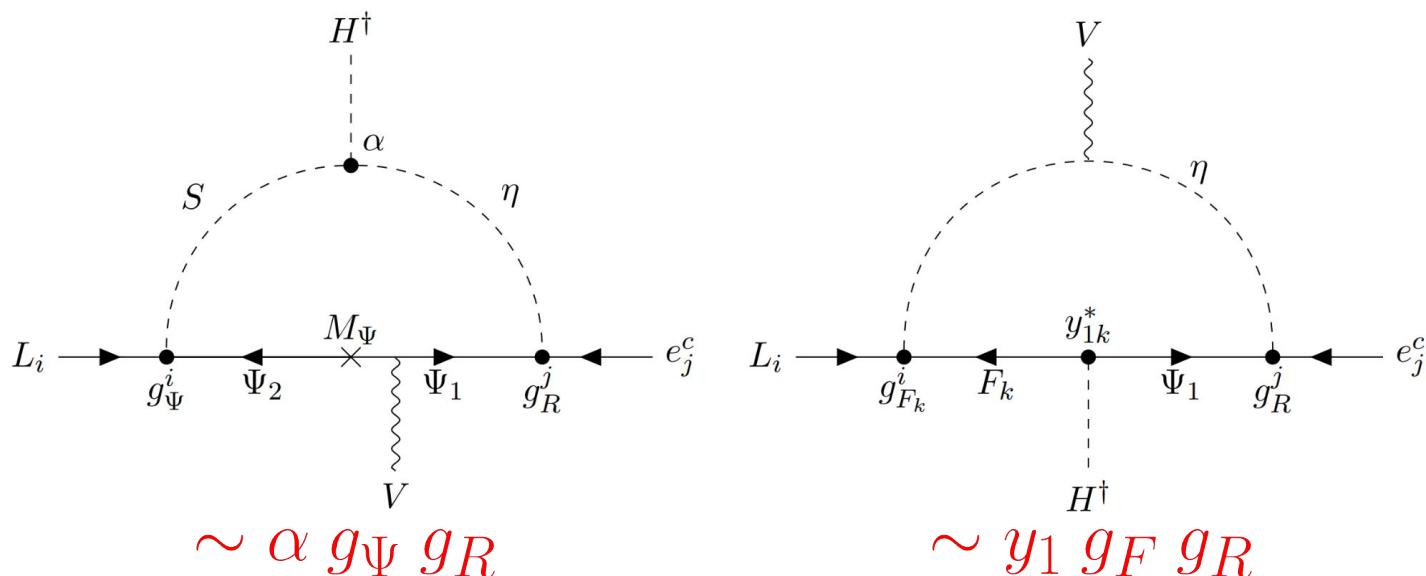
- Fermion sector

DM candidates

$$\left(\chi_1^0, \chi_2^0, \chi_3^0, \chi_4^0 \right)^T = U_\chi \left(F_1, F_2, \Psi_1^0, \Psi_2^0 \right)^T$$

Scotogenic Model

LFV and $(g-2)\mu$

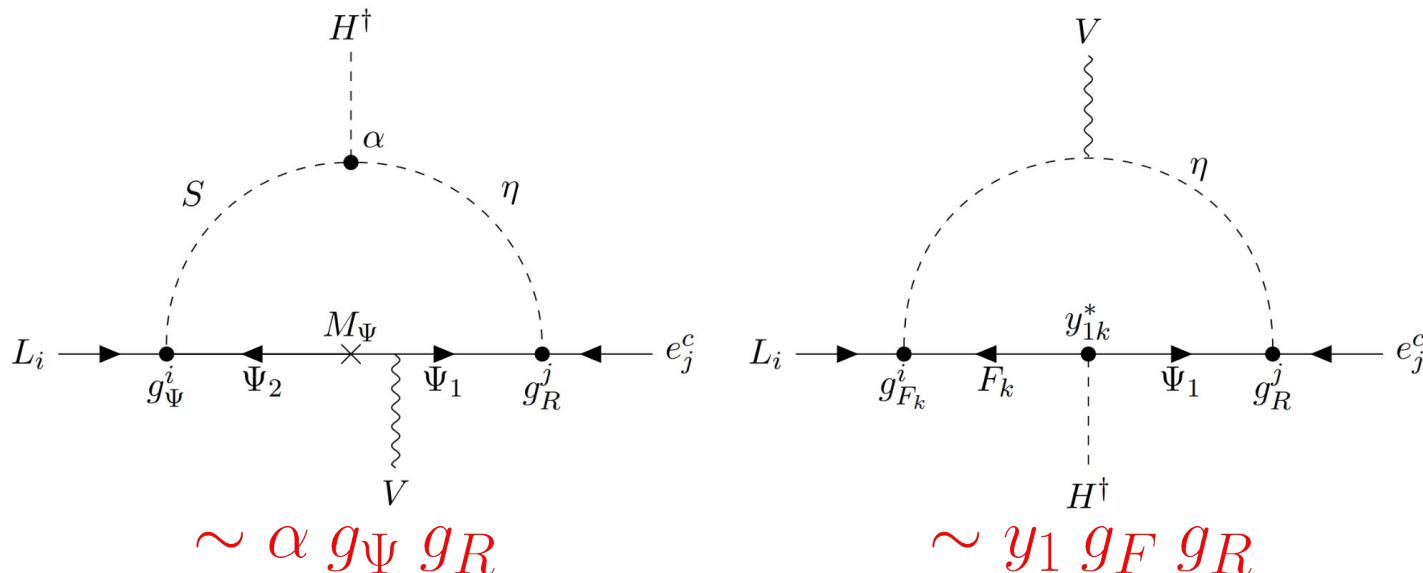


Scotogenic Model

LFV and $(g-2)\mu$

$$\text{BR}(\mu \rightarrow e\gamma) < 4.2 \times 10^{-13} \rightarrow |g_F^2|, |g_\Psi^2|, |g_R^2| \text{ small}$$

$$a_\mu^{\text{BSM}} = (251 \pm 59) \times 10^{-11} \rightarrow |g_F^2|, |g_\Psi^2|, |g_R^2| \text{ large}$$

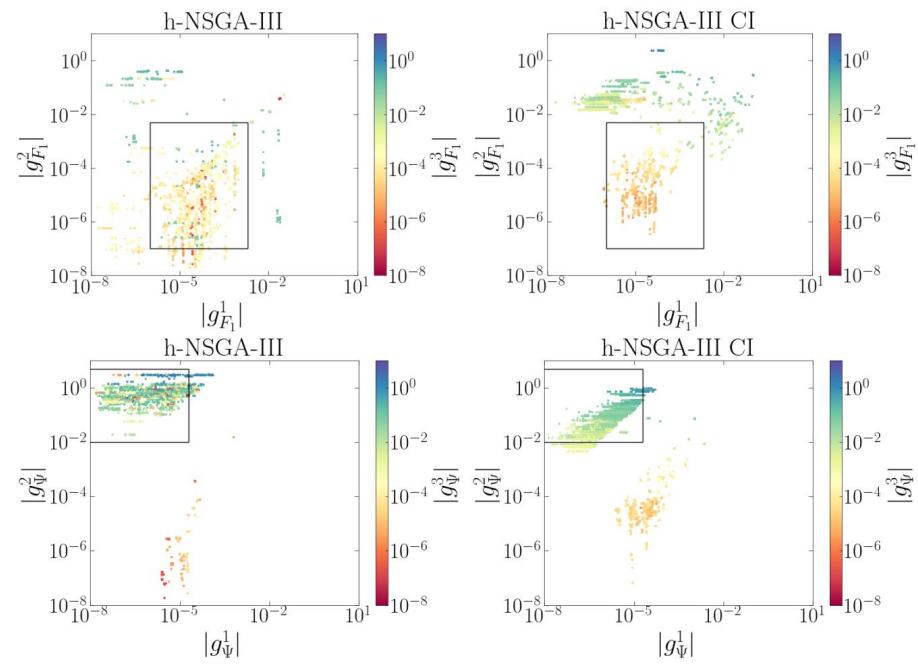
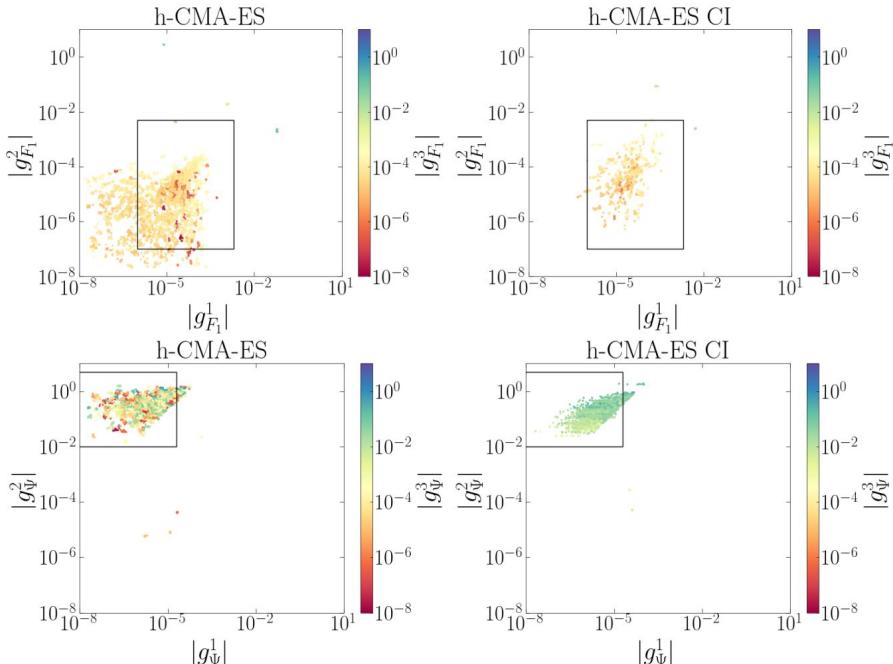


Results

Scotogenic Model Results - parameters

Optimisation Algorithms

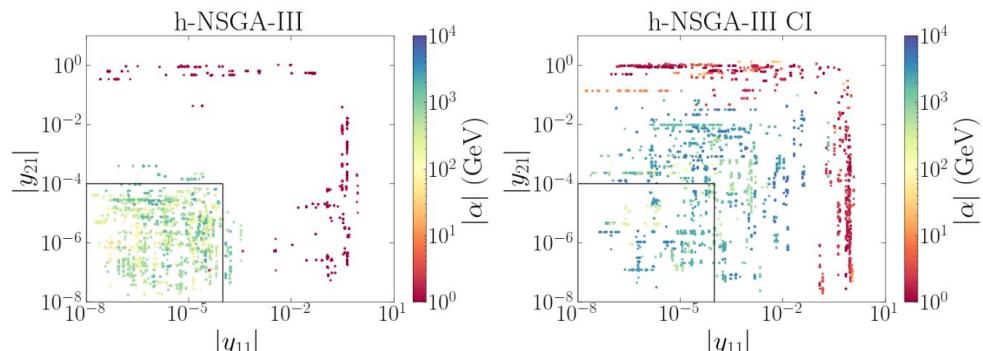
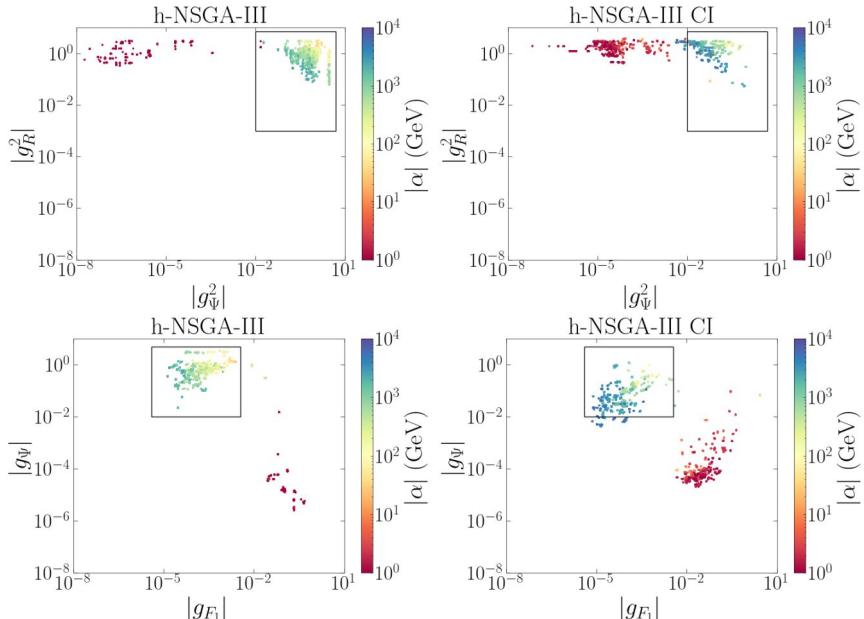
Single-objective: **CMA-ES** → local exploration
Multi-objective: **NSGA-III** → global exploration



boxes show range of points found with MCMC in 2301.08485

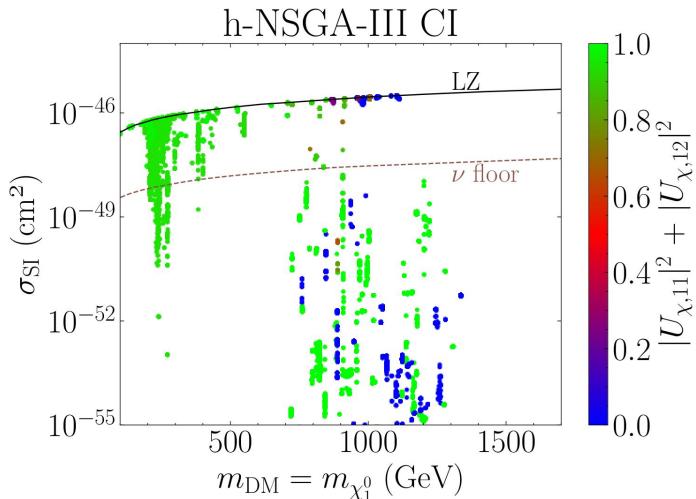
Scotogenic Model Results - parameters

Optimisation Algorithms	
Single-objective:	CMA-ES
Multi-objective:	NSGA-III
	local exploration
	global exploration

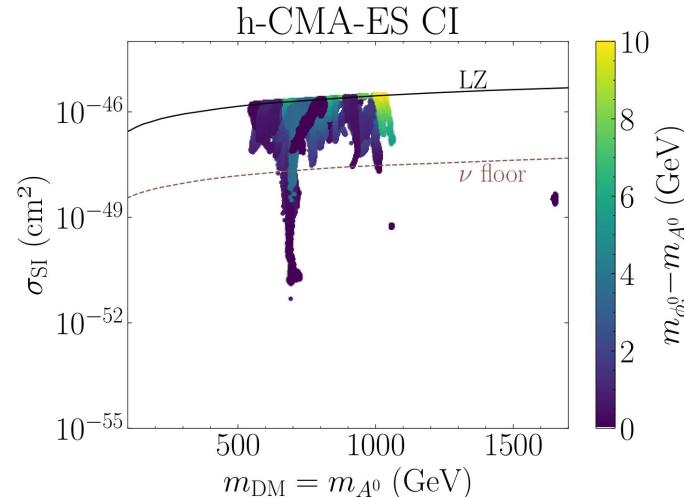


boxes show range of points found with MCMC in 2301.08485

Parameter Spaces Scan Results - Dark Matter



<u>Optimisation Algorithms</u>		
Single-objective:	CMA-ES	→ local exploration
Multi-objective:	NSGA-III	→ global exploration



New phenomenology:

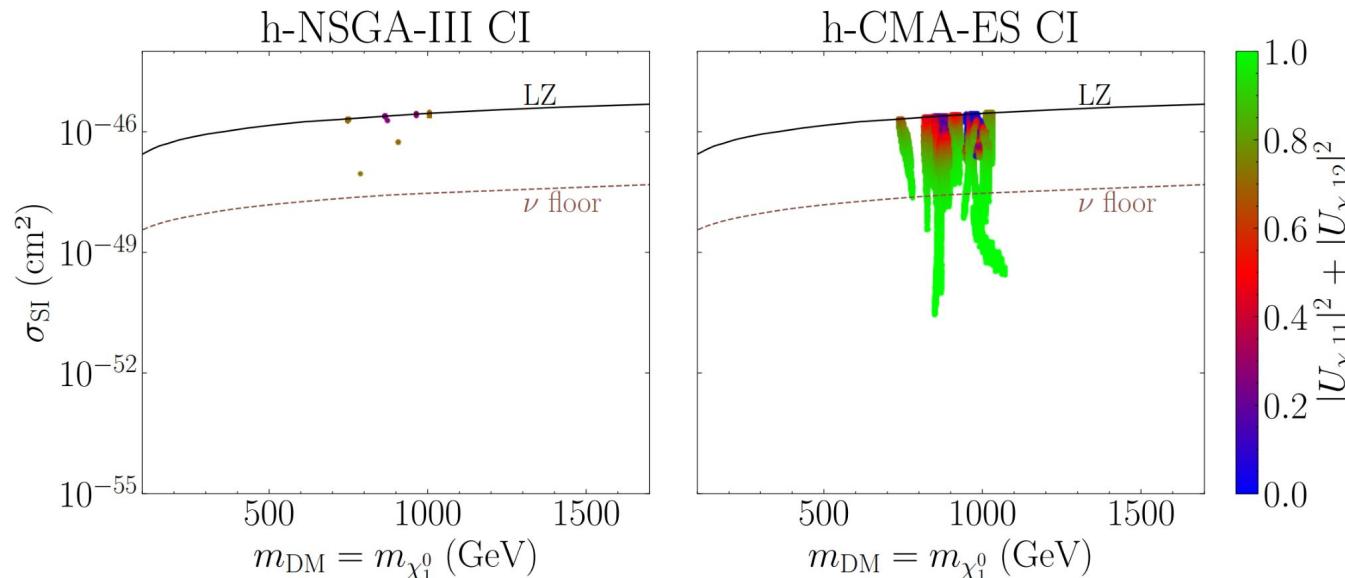
- Fermionic dark matter above neutrino floor
- Axial dark matter

Parameter Spaces Scan

Novelty detection

- Exploration is enhanced with a **novelty detection** (ND) reward.
- ND can be applied to **parameter space** and/or **observable space**.
- ND in **observable space** can explore **new phenomenology**.

JCR, MCR
arXiv 2240.07661
Phys. Rev. D 109, 095040

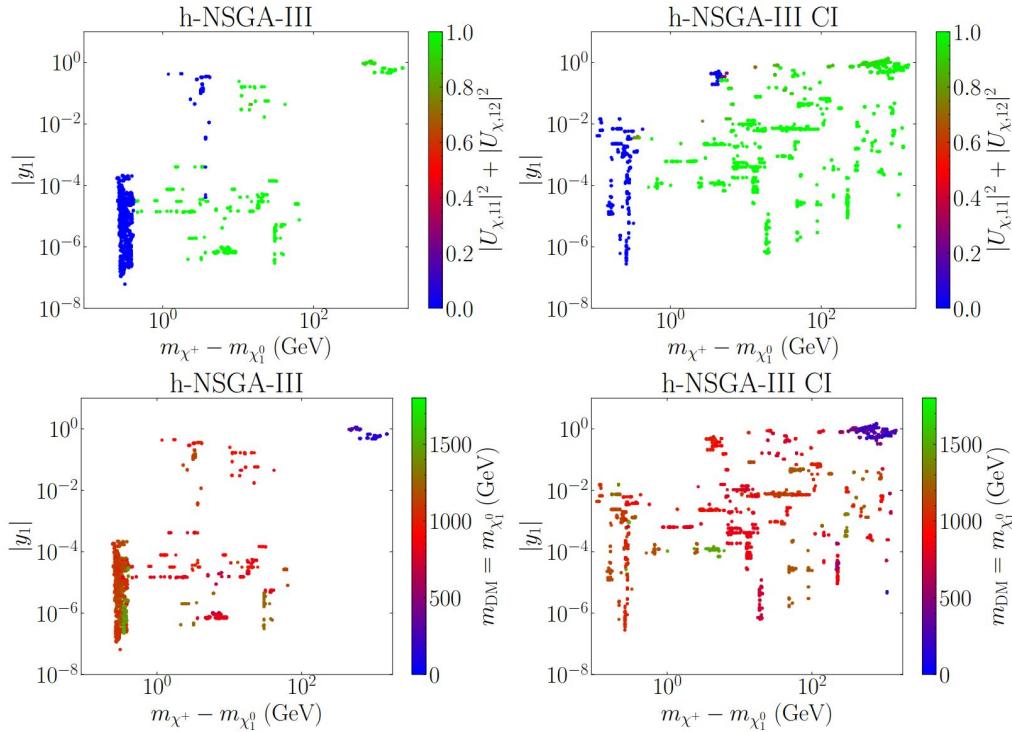


Conclusions

- The methodology allows for efficiently exploration of **parameter** and **phenomenological spaces** without the need of simplifying assumptions
- Algorithms show an **exploration-exploitation trade-off** which can be combined for a more powerful search strategy
- Use of **Multi-objective optimization** and the introduction of **hierarchy** in **single-objective optimization** show promising results
- Scotogenic new phenomenology:
 - **Fermionic DM above neutrino floor**
 - **Axial DM**

Backup

Parameter Spaces Scan Results - Dark Matter



Optimisation Algorithms		
Single-objective:	CMA-ES	→ local exploration
Multi-objective:	NSGA-III	→ global exploration

Parameter	Interval	Parametrisation	Mapping
λ_H	[0.1, 0.3]	Both	linear
$\lambda_{4S}, \lambda_{4\eta}$	[10^{-8} , 1]	Both	log
$\lambda_{S\eta}, \lambda_S, \lambda_\eta, \lambda'_\eta, \lambda''_\eta$	$\pm[10^{-8}, 1]$	Both	symlog
α (GeV)	$\pm[1, 10^4]$	Both	symlog
M_S^2, M_η^2 (GeV 2)	$5 \times [10^5, 10^6]$	Both	linear
M_1, M_2 (GeV)	[100, 2000]	Both	linear
M_Ψ (GeV)	[700, 2000]	Both	linear
$\Re(y_{ij}), (i, j = 1, 2)$	$\pm[10^{-8}, 3]$	Both	symlog
$\Im(y_{ij}), (i, j = 1, 2)$	$\pm[10^{-8}, 3]$	Both	symlog
$\Re(g_R^k), (k = 1, 2, 3)$	$\pm[10^{-8}, 3]$	Both	symlog
$\Im(g_R^k), (k = 1, 2, 3)$	$\pm[10^{-8}, 3]$	Both	symlog
$\Re(g_\Psi^k), (k = 1, 2, 3)$	$\pm[10^{-8}, 3]$	Non CI	symlog
$\Im(g_\Psi^k), (k = 1, 2, 3)$	$\pm[10^{-8}, 3]$	Non CI	symlog
$\Re(g_{F_j}^k),, (k = 1, 2, 3), (j = 1, 2)$	$\pm[10^{-8}, 3]$	Non CI	symlog
$\Im(g_{F_j}^k), (k = 1, 2, 3), (j = 1, 2)$	$\pm[10^{-8}, 3]$	Non CI	symlog
m_{ν_1}	[$10^{-16}, 10^{-10}$]	CI	log
m_{ν_2}	[$\sqrt{m_{\nu_1}^2 + 6.82 \times 10^{-23}}, \sqrt{m_{\nu_1}^2 + 8.04 \times 10^{-23}}$]	CI	linear
m_{ν_3}	[$\sqrt{m_{\nu_1}^2 + 2.435 \times 10^{-21}}, \sqrt{m_{\nu_1}^2 + 2.598 \times 10^{-21}}$]	CI	linear
$\theta_{12}^{\text{PMNS}}$	[31.27, 35.86] $\frac{\pi}{180}$	CI	linear
$\theta_{13}^{\text{PMNS}}$	[8.20, 8.93] $\frac{\pi}{180}$	CI	linear
$\theta_{23}^{\text{PMNS}}$	[40.1, 51.7] $\frac{\pi}{180}$	CI	linear
δ_{CP}	[120, 369] $\frac{\pi}{180}$	CI	linear
$\Re(\theta_k^R), (k = 1, 2, 3)$	[0, 2π]	CI	linear
$\Im(\theta_k^R), (k = 1, 2, 3)$	$\pm[10^{-8}, 3]$	CI	symlog

Observable	Allowed Values	Parametrisation
m_h	[124.25, 126.25] GeV	Both
a_μ^{BSM}	[74, 428] $\times 10^{-11}$	Both
$\Omega_{\text{DM}} h^2$	[0.11, 0.13]	Both
$\text{BR}(\mu^- \rightarrow e^- \gamma)$	$< 4.2 \times 10^{-13}$	Both
$\text{BR}(\tau^- \rightarrow e^- \gamma)$	$< 3.3 \times 10^{-8}$	Both
$\text{BR}(\tau^- \rightarrow \mu^- \gamma)$	$< 4.2 \times 10^{-8}$	Both
$\text{BR}(\mu^- \rightarrow e^- e^+ e^-)$	$< 1.0 \times 10^{-12}$	Both
$\text{BR}(\tau^- \rightarrow e^- e^+ e^-)$	$< 2.7 \times 10^{-8}$	Both
$\text{BR}(\tau^- \rightarrow \mu^- \mu^+ \mu^-)$	$< 2.1 \times 10^{-8}$	Both
$\text{BR}(\tau^- \rightarrow e^- \mu^+ \mu^-)$	$< 2.7 \times 10^{-8}$	Both
$\text{BR}(\tau^- \rightarrow \mu^- e^+ e^-)$	$< 1.8 \times 10^{-8}$	Both
$\text{BR}(\tau^- \rightarrow \mu^- e^+ \mu^-)$	$< 1.7 \times 10^{-8}$	Both
$\text{BR}(\tau^- \rightarrow \mu^+ e^- e^-)$	$< 1.5 \times 10^{-8}$	Both
$\text{BR}(\tau^- \rightarrow e^- \pi)$	$< 8.0 \times 10^{-8}$	Both
$\text{BR}(\tau^- \rightarrow e^- \eta)$	$< 9.2 \times 10^{-8}$	Both
$\text{BR}(\tau^- \rightarrow e^- \eta')$	$< 1.6 \times 10^{-7}$	Both
$\text{BR}(\tau^- \rightarrow \mu^- \pi)$	$< 1.1 \times 10^{-7}$	Both
$\text{BR}(\tau^- \rightarrow \mu^- \eta)$	$< 6.5 \times 10^{-8}$	Both
$\text{BR}(\tau^- \rightarrow \mu^- \eta')$	$< 1.3 \times 10^{-7}$	Both
$\text{CR}_{\mu \rightarrow e}(\text{Ti})$	$< 4.3 \times 10^{-12}$	Both
$\text{CR}_{\mu \rightarrow e}(\text{Pb})$	$< 4.3 \times 10^{-11}$	Both
$\text{CR}_{\mu \rightarrow e}(\text{Au})$	$< 7.0 \times 10^{-13}$	Both
$\text{BR}(Z^0 \rightarrow e^\pm \mu^\mp)$	$< 7.5 \times 10^{-7}$	Both
$\text{BR}(Z^0 \rightarrow e^\pm \tau^\mp)$	$< 5.0 \times 10^{-6}$	Both
$\text{BR}(Z^0 \rightarrow \mu^\pm \tau^\mp)$	$< 6.5 \times 10^{-6}$	Both
$\sigma_{\text{SI}}^{\text{DM}}$	LZ bounds	Both
Δm_{21}^2	$[6.82, 8.04] \times 10^{-5} \text{eV}^2$	Non CI
Δm_{31}^2	$[2.435, 2.598] \times 10^{-3} \text{eV}^2$	Non CI
$\sin^2(\theta_{12})$	[0.269, 0.343]	Non CI
$\sin^2(\theta_{13})$	[0.02032, 0.02410]	Non CI
$\sin^2(\theta_{23})$	[0.415, 0.616]	Non CI
δ_{CP}	[120, 369] $\times \frac{2\pi}{360}$	Non CI
$ g_{F_j}^k , (k = 1, 2, 3), (j = 1, 2)$	[0, 4π]	CI
$ g_\Psi^k , (k = 1, 2, 3)$	[0, 4π]	CI

Algorithms

Covariance matrix adaptation evolution strategy (CMA-ES)

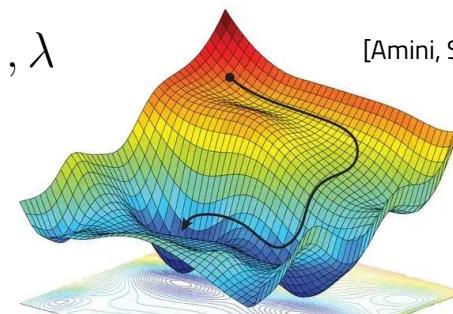
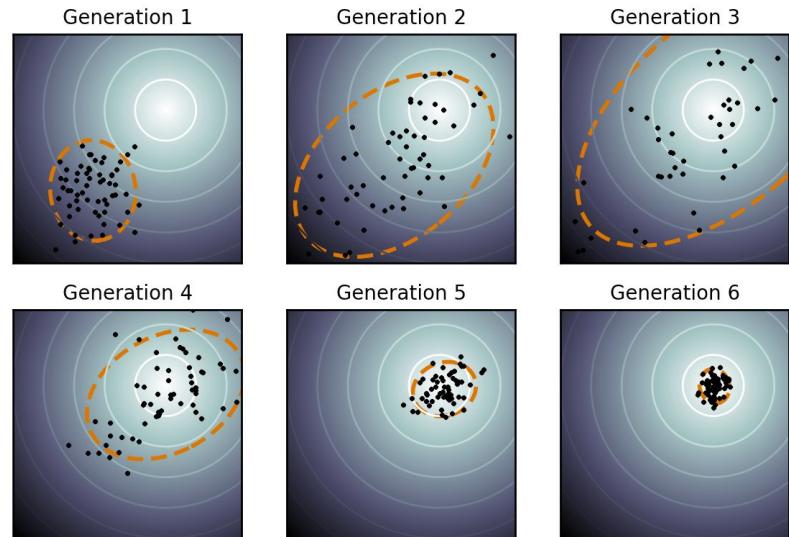
- Non genetic evolutionary algorithm
- Normal multivariate distribution
 - Mean: evolution direction
 - Covariance matrix: adapted iteratively

$$\theta_i^{(g+1)} \sim m^{(g)} + \sigma^{(g)} \mathcal{N}(0, \mathcal{C}^{(g)}) \quad \text{for } i = 1, \dots, \lambda$$

- Similar to a *gradient descent*

$$\mathbf{x}_{n+1} = \mathbf{x}_n - \gamma \nabla F(\mathbf{x}_n)$$

[<https://en.wikipedia.org/wiki/CMA-ES>]

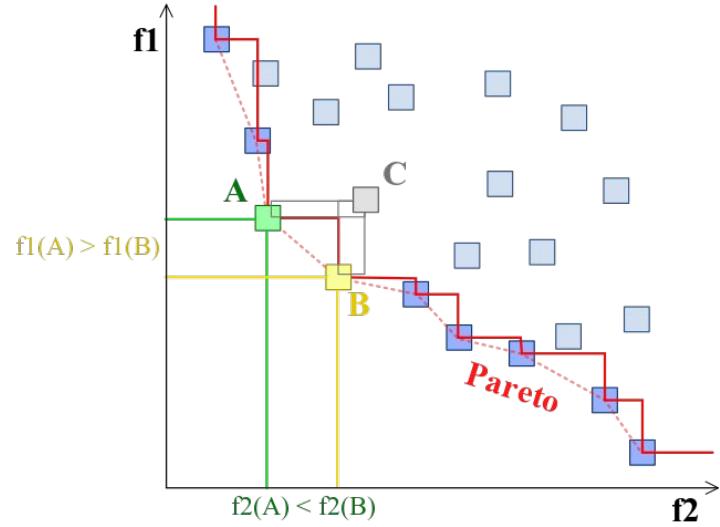
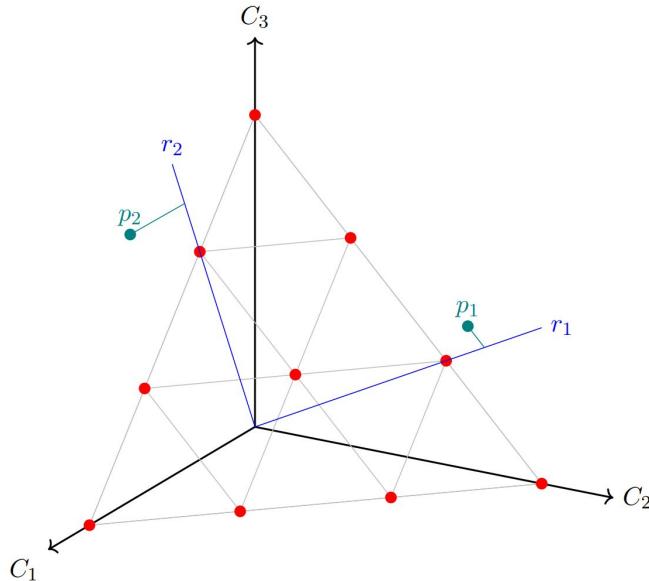


[Amini, Soleimany, Karaman, Rus, 2019]

Algorithms

Non-dominated Sorting Genetic Algorithm - III (NSGA-III)

- Multi-objective genetic algorithm
- Uses reference lines to increase diversity of solutions



[https://en.wikipedia.org/wiki/Pareto_front]

Algorithms

Non-dominated Sorting Genetic Algorithm - III (NSGA-III)

Hyperparameter	Value	Description	Algorithm
$\sigma^{(0)}$	1.0	Initial step size	CMA-ES
N	1000	Maximum number of generations	CMA-ES
N	2000	Maximum number of generations	NSGA-III
μ	400	Population size	NSGA-III
P_{cx}	0.9	Crossover probability	NSGA-III
η_{cx}	30	Crossover crowding factor	NSGA-III
P_{mut}	0.5	Mutation probability	NSGA-III
η_{mut}	40	Mutation crowding factor	NSGA-III
P_{ind}	$\frac{4}{N_{\text{dim}}}$	Independent mutation probability	NSGA-III