Gravitational Wave
with
Domain Wall Domination

Sung Mook Lee

CERN

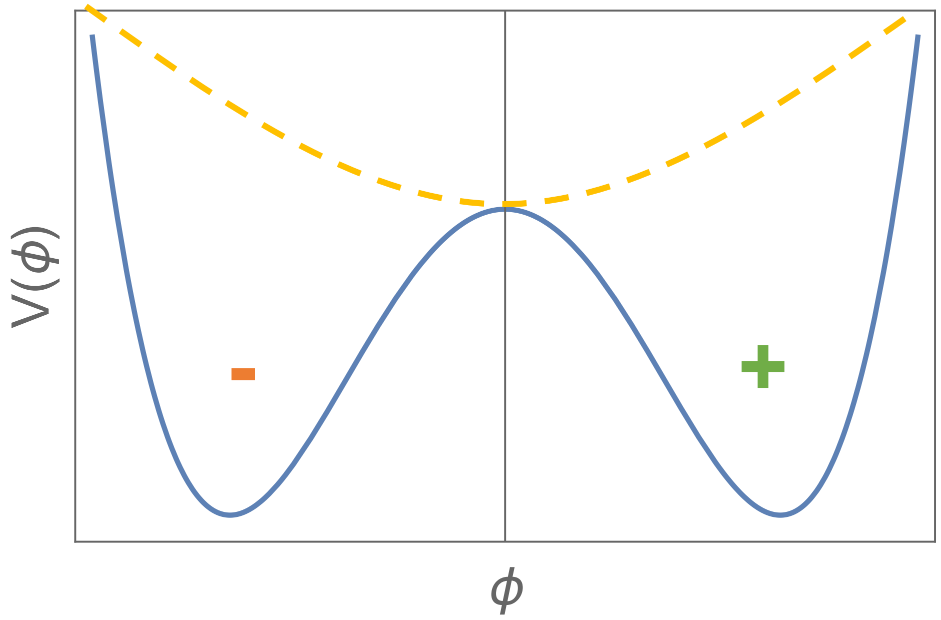
2504.02462 with Sungwoo Hong (KAIST) and Qiuyue Liang (IPMU)

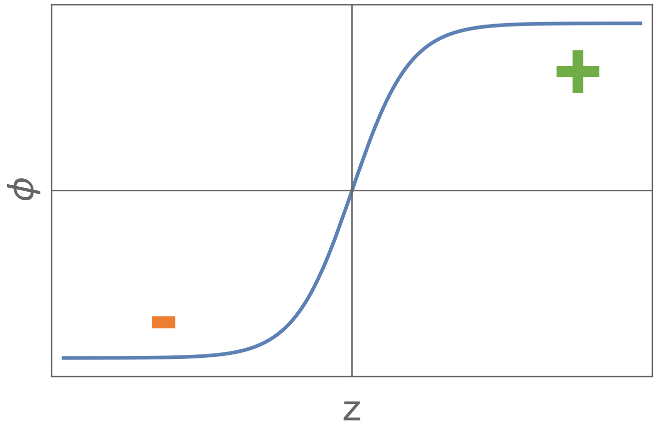
Domain Wall

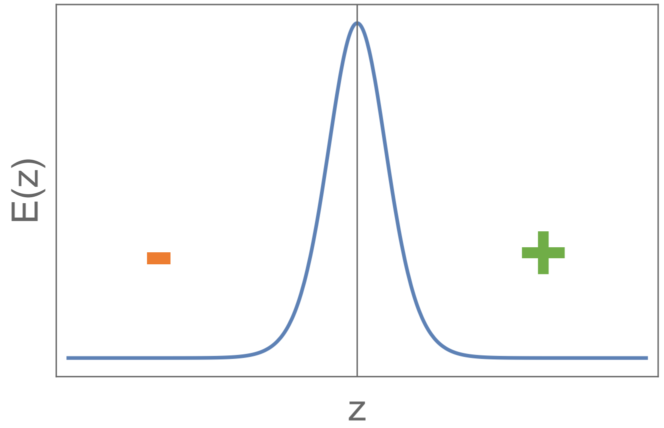
- *Topological Defect* from SSB of *discrete* symmetry

$$\phi(z) = v \tanh\left[\sqrt{\frac{\lambda}{2}} v z\right]$$

$$V(\phi) = \frac{\lambda}{4}\left(\phi^2 - v^2\right)^2$$

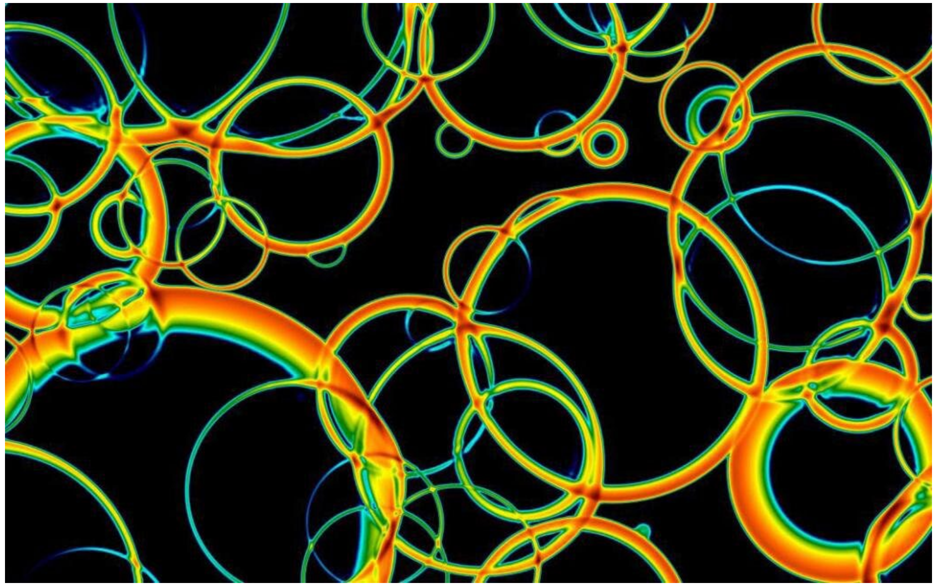


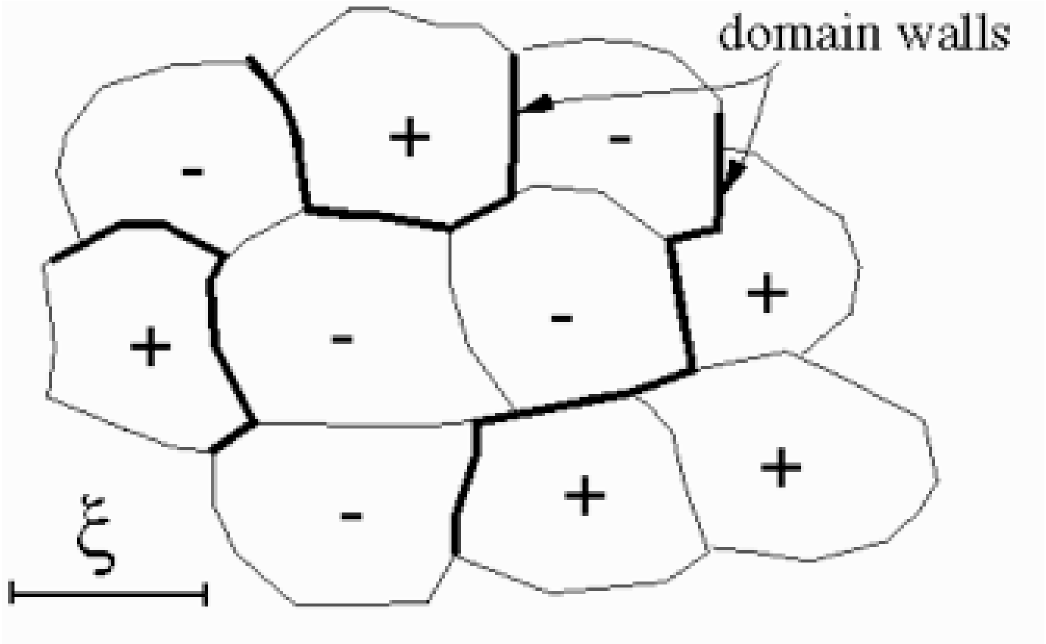




Domain Wall

Phase transition

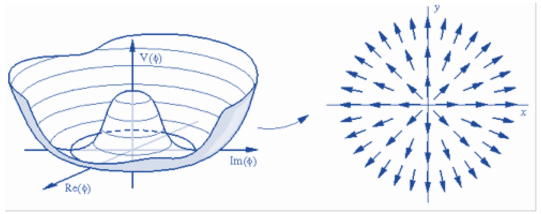


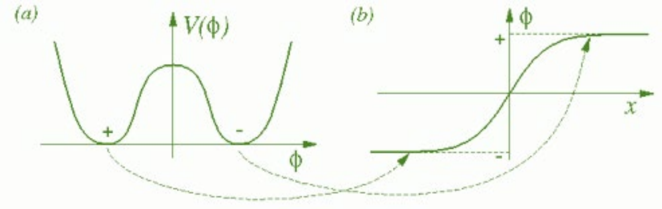


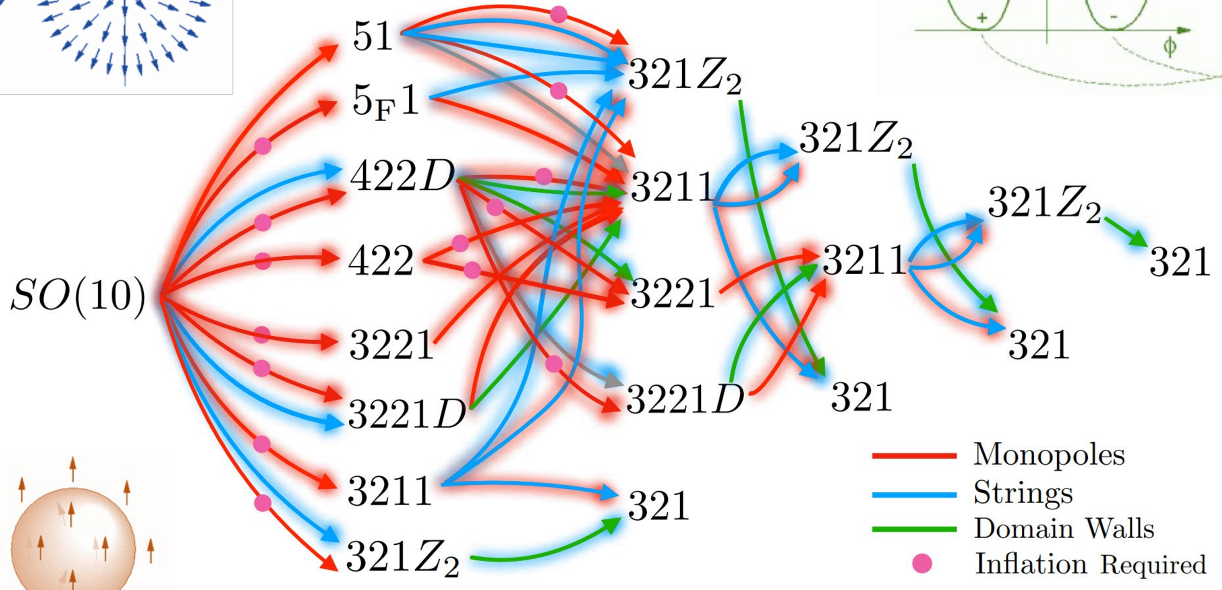
- **Surface tension** $\sigma \sim \sqrt{\lambda} v^3$

Topological Defects

- What about Cosmology?
  - Not present in SM, but prevalent in many BSM (e.g. GUT, Axion cosmic string)







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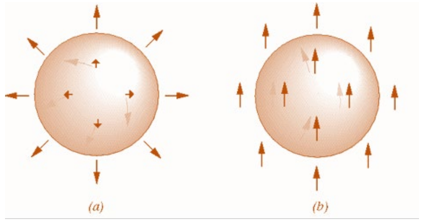


Dynamics of Domain Wall

DW Dynamics in Expanding Universe

- The universe is expanding

$$\rho_{\mathrm{DW}}^{(\mathrm{single})} \propto \frac{1}{a} \qquad \rho_{\mathrm{String}}^{(\mathrm{single})} \propto \frac{1}{a^2}$$

DW Dynamics in Expanding Universe

- The universe is expanding

$$\rho_{\mathrm{DW}}^{(\mathrm{single})} \propto \frac{1}{a} \qquad \rho_{\mathrm{String}}^{(\mathrm{single})} \propto \frac{1}{a^2}$$

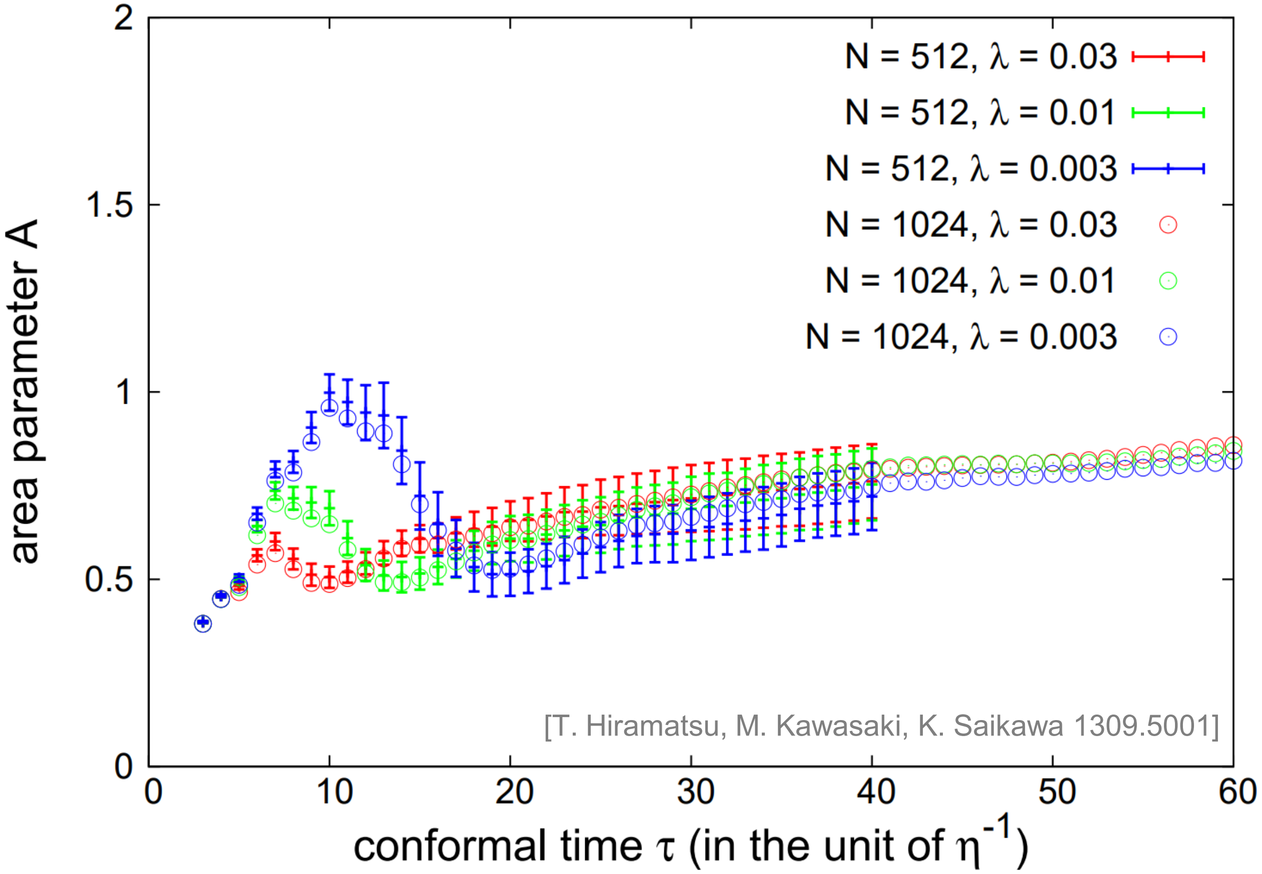
- Defects form ‘networks’
  - interact each other, and asymptotically fall into ‘*scaling regime*’

$$\rho_{\mathrm{DW}}^{(\mathrm{network})} \propto \frac{1}{t} \qquad \rho_{\mathrm{String}}^{(\mathrm{network})} \propto \frac{1}{t^2} \begin{cases} a^{-4} & (\mathrm{RD}) \\ a^{-3} & (\mathrm{MD}) \end{cases}$$

Numerical Simulations

$$\rho_{\rm DW}(t) = \mathcal{A}\frac{\sigma}{t}$$

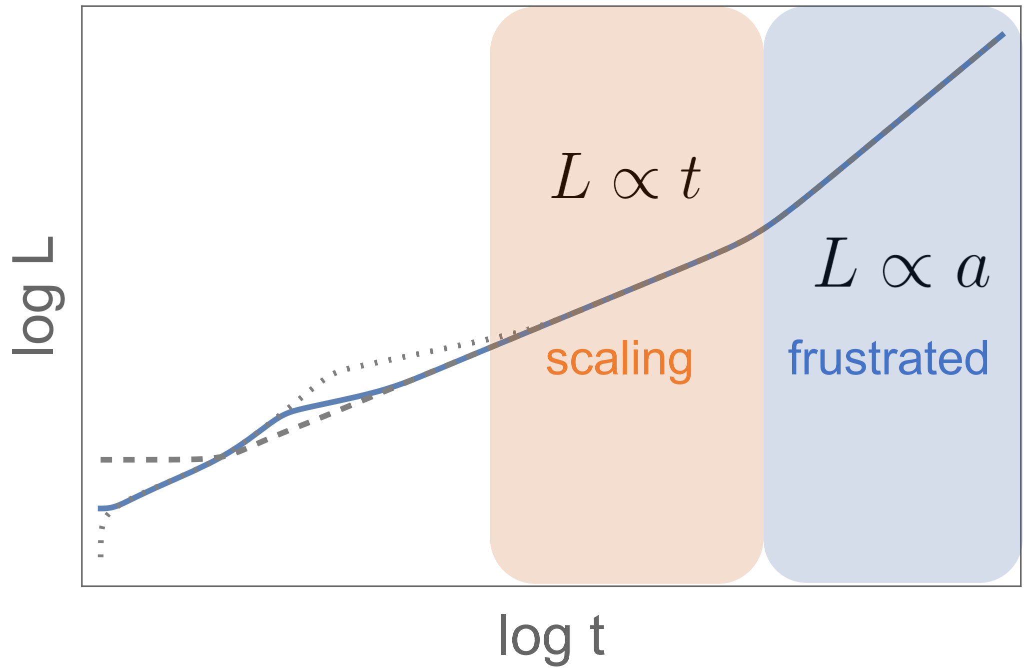
O(1) DW per Hubble Volume

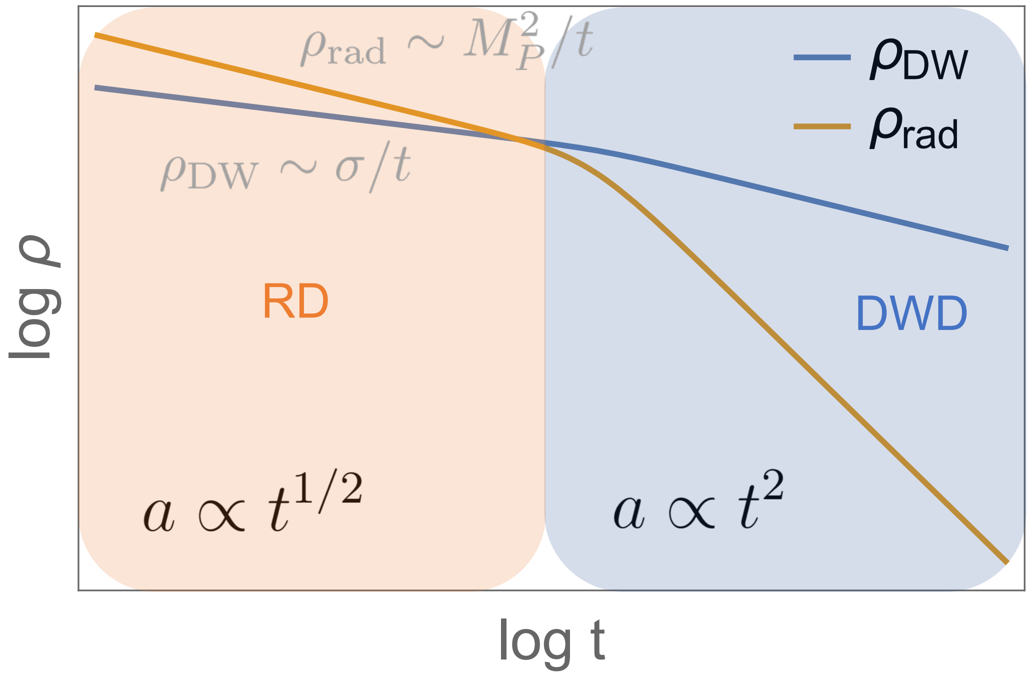


+ Analytically supported by VOS model

Scaling Solutions

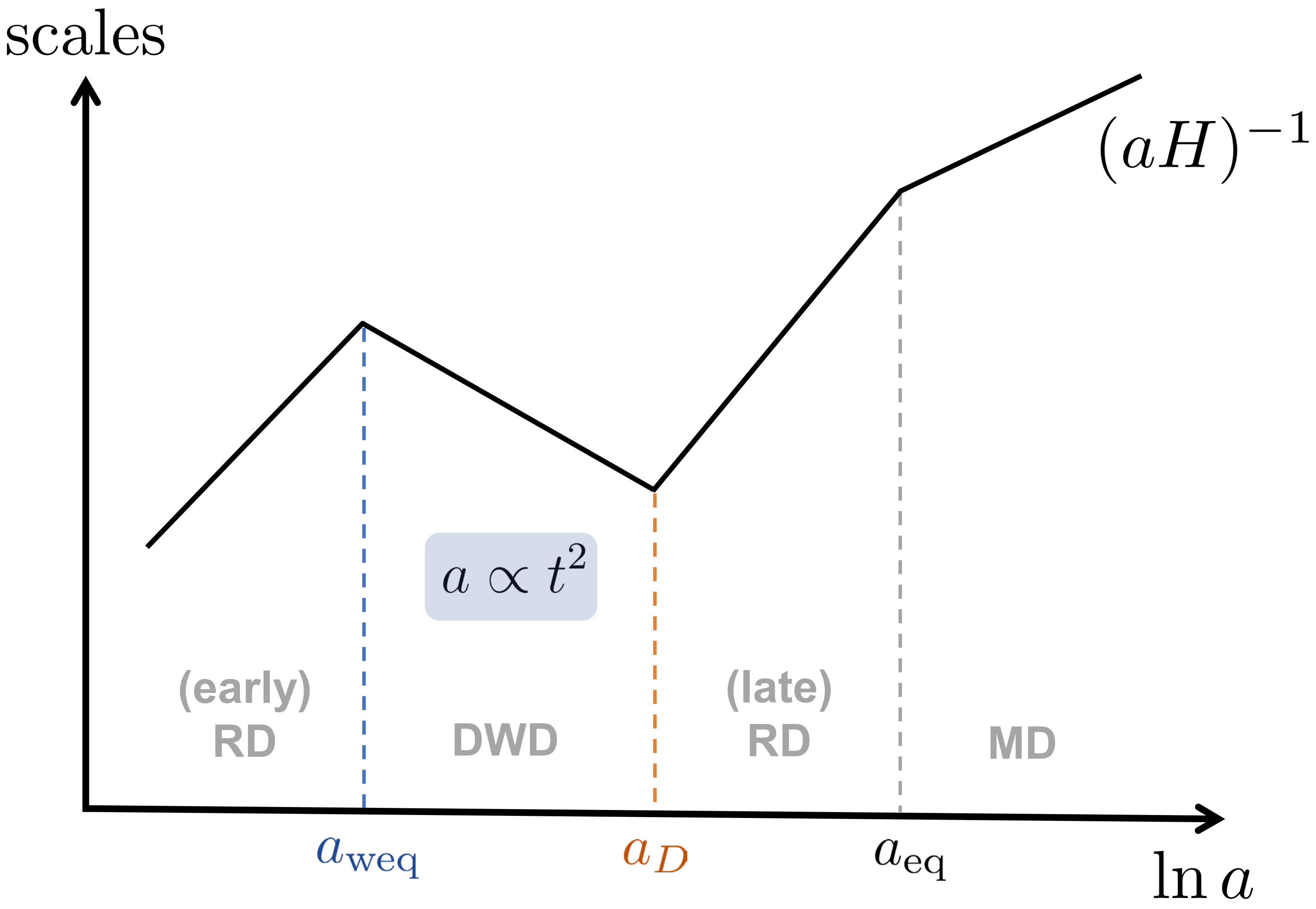
$$\rho_{\rm DW} = \frac{\sigma}{L}$$ ← characteristic length scale



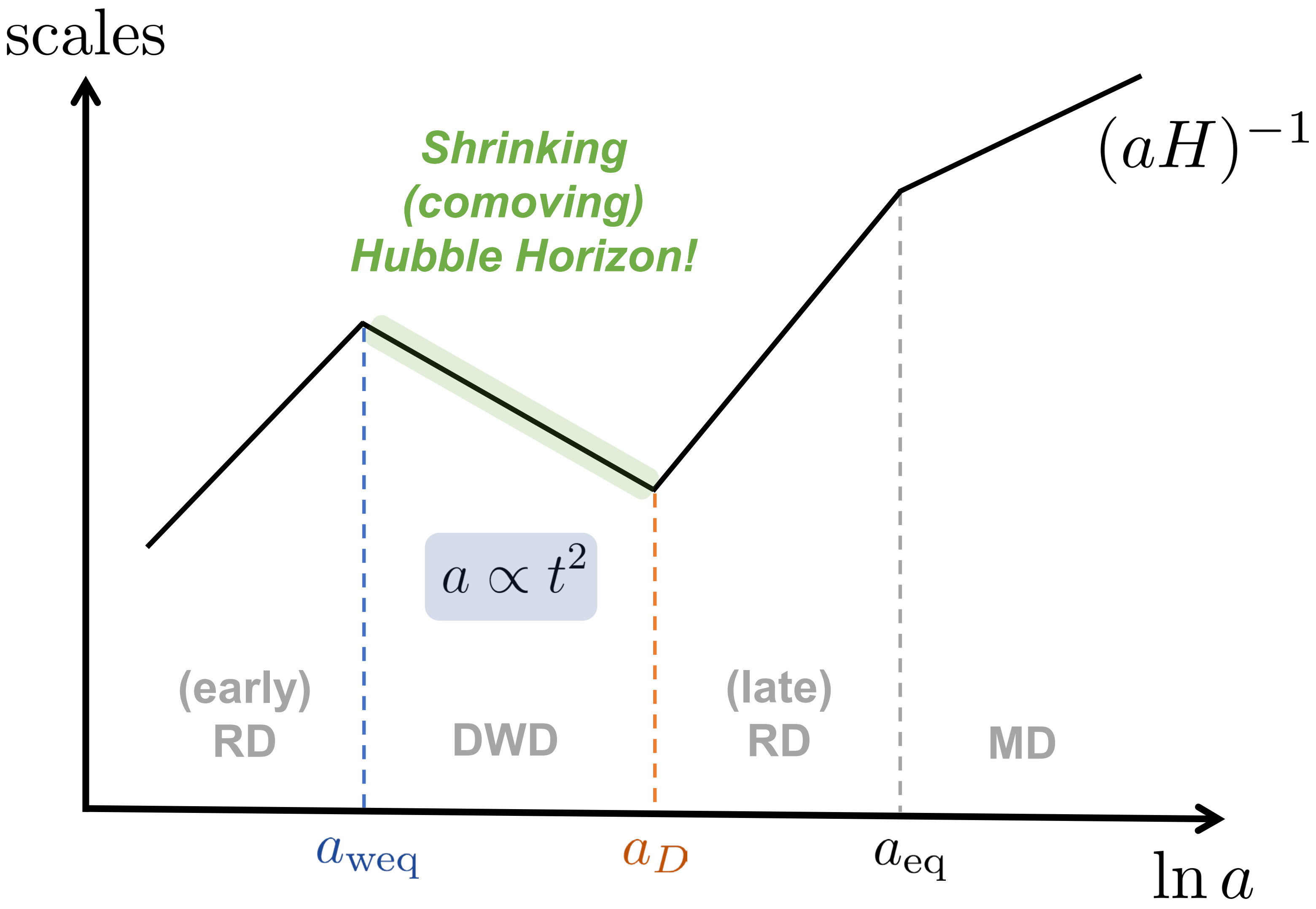


$$t_{\rm weq} \sim \frac{M_P^2}{\sigma}$$

Cosmological History with DWD



Cosmological History with DWD



GW Spectrum

General Formula

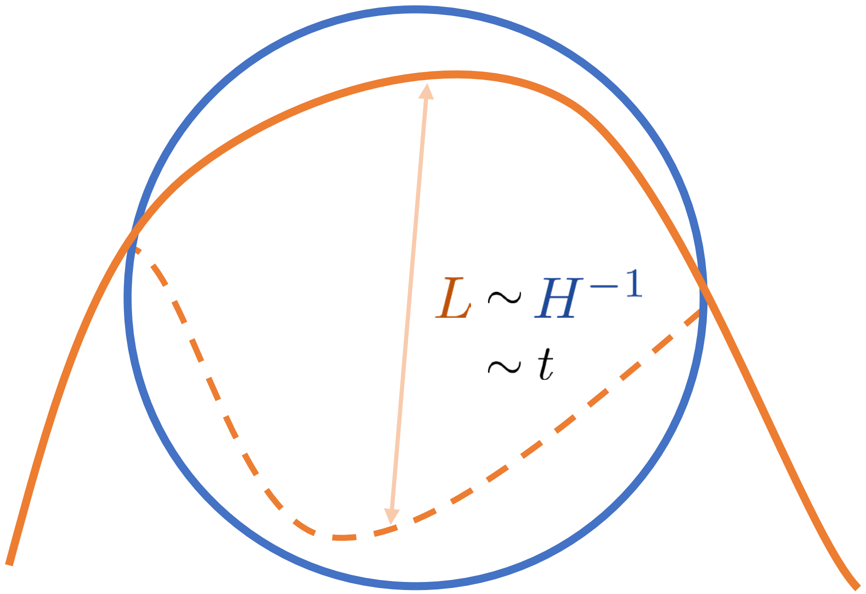
$$\left.\frac{d\rho_{\rm GW}}{dk}\right|_0 = \int_{t_{\rm PT}}^{t_0} dt\ a(t)\left|\mathcal{T}\left(a(t),k\right)\right|^2 p_{\rm GW}(t)\mathcal{P}\left(\frac{k}{a}\right)$$

GW Emission

GW emission

$$\left.\frac{d\rho_{\rm GW}}{dk}\right|_0 = \int_{t_{\rm PT}}^{t_0} dt\; a(t)\left|\mathcal{T}\left(a(t), k\right)\right|^2 p_{\rm GW}(t)\mathcal{P}\left(\frac{k}{a}\right)$$

$$M_w \sim \sigma L^2 \qquad Q \sim M_w L^2$$



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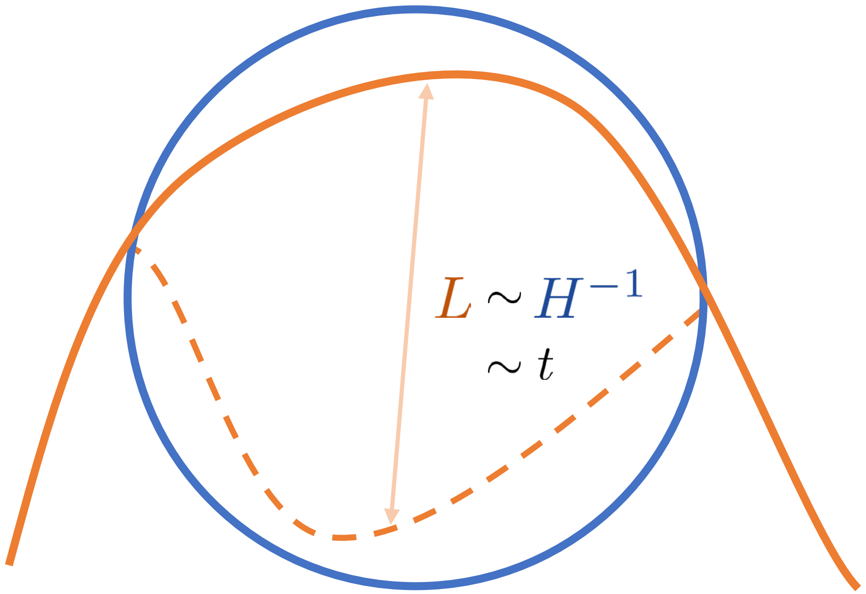
$$P_{\rm GW}^{\rm (scaling)} \sim G\dddot{Q}^2 \sim G\sigma^2 t^2$$



GW Emission

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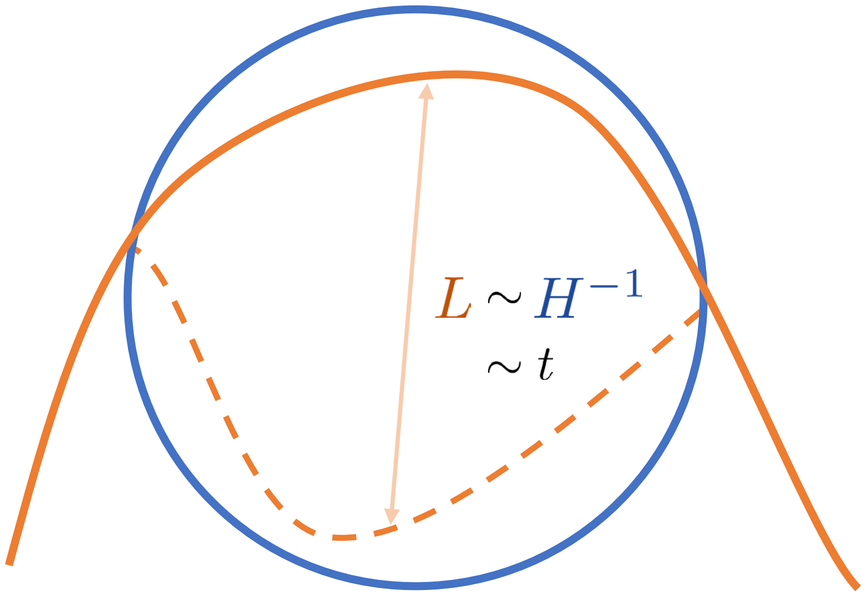
$$\left.\frac{d\rho_{\rm GW}}{dk}\right|_0 = \int_{t_{\rm PT}}^{t_0} dt\ a(t) \left|\mathcal{T}\left(a(t),k\right)\right|^2 p_{\rm GW}(t)\mathcal{P}\left(\frac{k}{a}\right)$$

$$M_w \sim \sigma L^2 \qquad Q \sim M_w L^2$$

$$P_{\rm GW}^{\rm (scaling)} \sim G\dddot{Q}^2 \sim G\sigma^2 t^2$$

$$p_{\rm GW}^{\rm (scaling)} \sim n_w P_{\rm GW}^{\rm (scaling)} \sim \frac{G\sigma^2}{t}$$

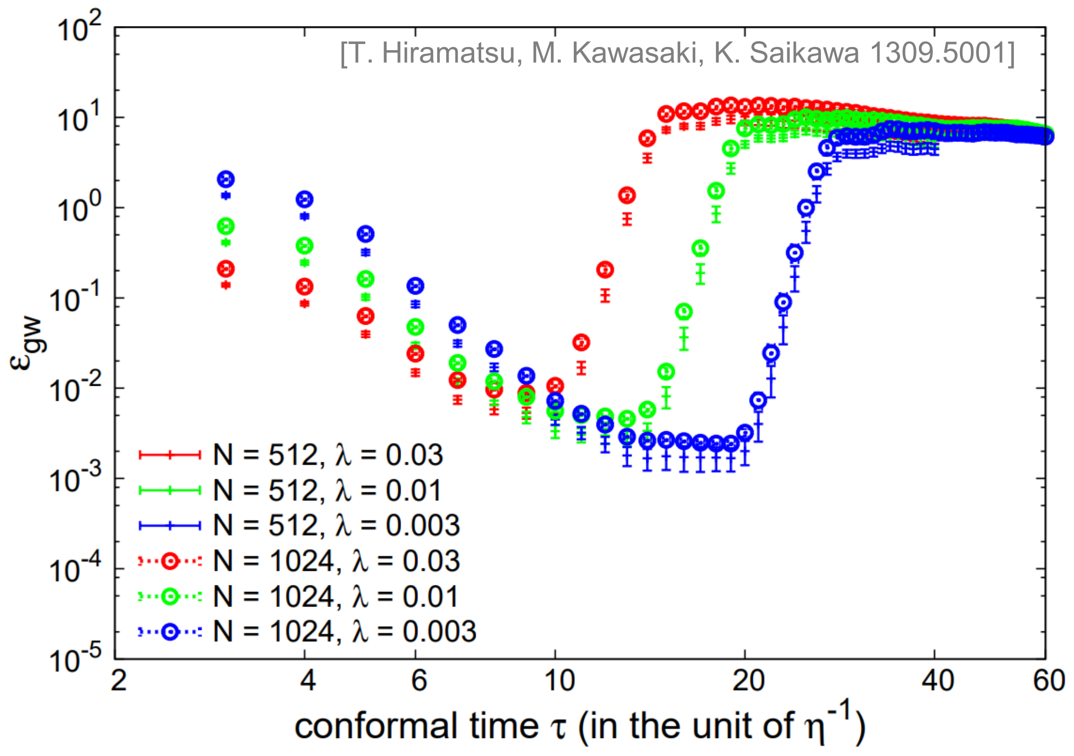
[T. Hiramatsu, M. Kawasaki, K. Saikawa 1309.5001]



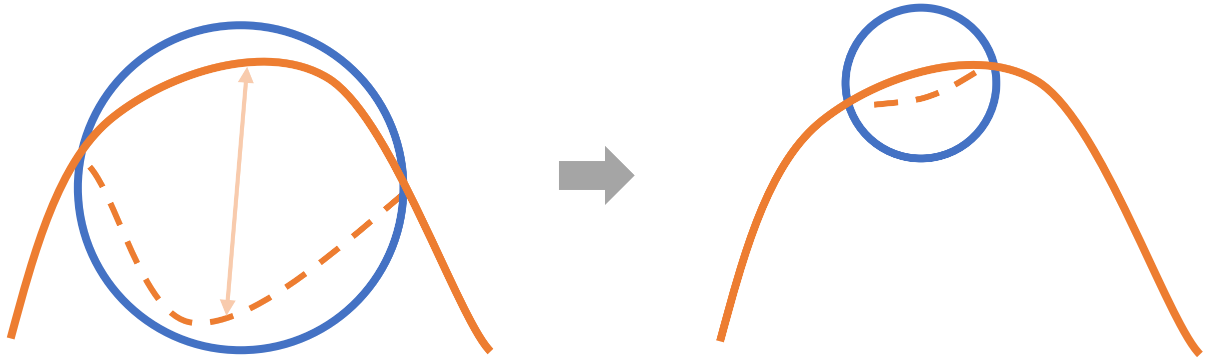
GW Emission

- Confirmed by simulations

$$\rho_{\rm GW} = \epsilon_{\rm GW} G \mathcal{A}^2 \sigma^2$$



- During DWD, GW emission is suppressed



Power Spectral Density (PSD)

$$\left.\frac{d\rho_{\rm GW}}{dk}\right|_0 = \int_{t_{\rm PT}}^{t_0} dt\ a(t)\left|\mathcal{T}\left(a(t),k\right)\right|^2 p_{\rm GW}(t)\mathcal{P}\left(\frac{k}{a}\right)$$

GW emission

Power Spectral Density (PSD)

- Spectral information is encoded in PSD

Power-law PSD

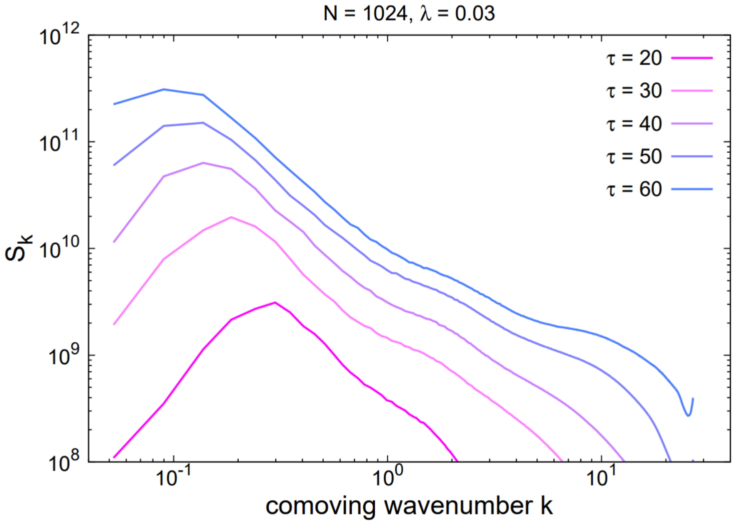
$$p(f_e) = -\frac{\nu+1}{f_{\min,e}^{\nu+1}} f_e^{\nu}\Theta(f_e - f_{\min}(t_e))$$

$$\int_{f_{\min,e}}^{\infty} df_e\ p(f_e) = 1$$

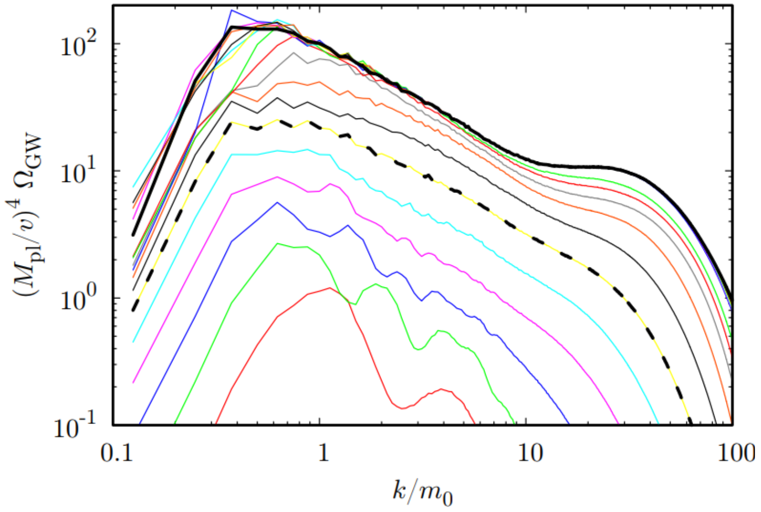
$$f_{\min,e} \sim \frac{H_e}{2\pi}$$

Power Spectral Density (PSD)

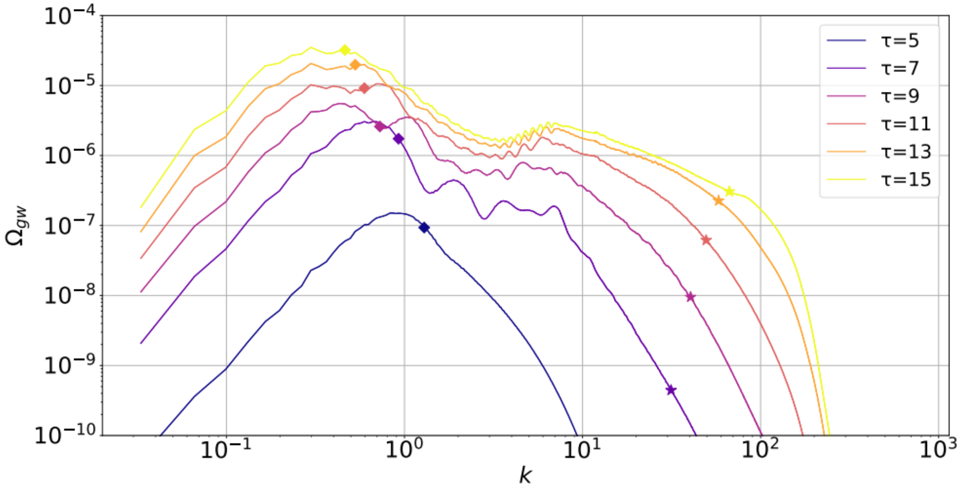
[T. Hiramatsu, M. Kawasaki, K. Saikawa 1309.5001]



[N. Kitajima, J. Lee, K. Murai, F. Takahashi, W.Yin 2306.17146]



[I. Dankovsky, E. Babichev, D. Gorbunov, S. Ramazanov, A. Vikman 2406.17053]



$$\Omega_{\rm gw} = \begin{cases} k^3 & \text{(IR)} \\ k^{-1} & \text{(UV)} \end{cases}$$

$$\Omega_{\rm gw} = \begin{cases} k^3 & \text{(IR)} \\ k^{-1} & \text{(UV)} \end{cases} + \text{Plateau}$$

$$\Omega_{\rm gw} = \begin{cases} k^3 & \text{(IR)} \\ k^{-1.7} & \text{(UV)} \end{cases} + \text{Plateau}$$

- For example, during RD $\nu = -2 \quad \Rightarrow \quad \Omega_{\rm gw} \propto f^{-1}$

GW Evolution

GW emission

$$\left.\frac{d\rho_{\rm GW}}{dk}\right|_0 = \int_{t_{\rm PT}}^{t_0} dt\; a(t)\left|\mathcal{T}\left(a(t),k\right)\right|^2 p_{\rm GW}(t)\,\mathcal{P}\left(\frac{k}{a}\right)$$

evolution

Power Spectral Density (PSD)

$$h'' + 2\mathcal{H}h' - \nabla^2 h = a^2\frac{32\pi G\rho}{3}\Pi^{\rm TT}$$

GW Evolution

$$\left.\frac{d\rho_{\rm GW}}{dk}\right|_0 = \int_{t_{\rm PT}}^{t_0} dt\, a(t)\left|\mathcal{T}\left(a(t),k\right)\right|^2 p_{\rm GW}(t)\,\mathcal{P}\left(\frac{k}{a}\right)$$

GW emission

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Power Spectral Density (PSD)

$$h'' + 2\mathcal{H}h' - \nabla^2 h = a^2\frac{32\pi G\rho}{3}\Pi^{\rm TT}$$

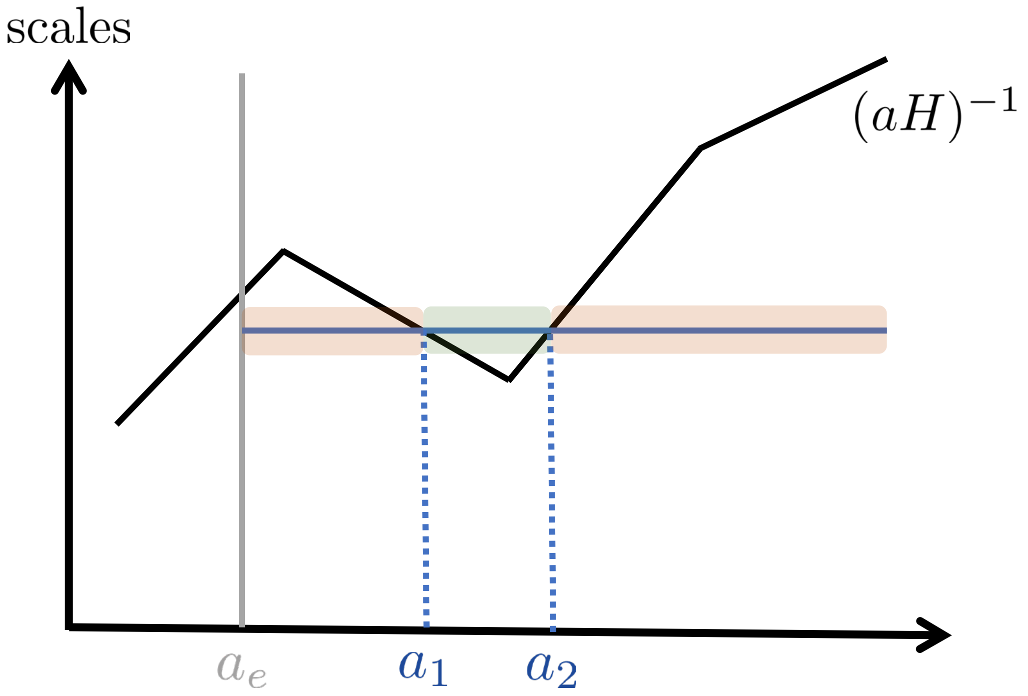
Subhorizon $h(a(\eta),k) = \frac{1}{a(\eta)}\left[c_1 e^{ik\eta} + c_2 e^{-ik\eta}\right] \propto a^{-1}$

Superhorizon $h(a(\eta),k) = c_1 + c_2\int\frac{1}{a^2(\eta)} \propto a^0$

GW Evolution

- Evolution history is encoded in the *Transfer function*

$$h(a_0, k) \equiv \mathcal{T}(a_e.k)h(a_e, k)$$



$$\mathcal{T} = \left(\frac{a_2}{a_0}\right)^1 \left(\frac{a_1}{a_2}\right)^0 \left(\frac{a_e}{a_1}\right)^1 \propto k^{-3}$$

Causality Tail

[C. Caprini, R. Durrer, T. Konstandin, G. Servant, 0901.1661]
[R-G. Cai, S. Pi, M. Sasaki, 1909.13728]
[A. Hook, G. Marques-Tavares, D. Racco, 2010.03568]

- Normally (RD), $\Omega_{\rm GW} \propto k^3$

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- Normally (RD), $\Omega_{\rm GW} \propto k^3$

- Generalized to arbitrary equation of state:
  - Decelerating universe $\Omega_{\rm GW} \propto k^{\frac{15w+1}{3w+1}} \propto \begin{cases} k^3 & (w=1/3) \\ k^1 & (w=0) \end{cases}$

[A. Hook, G. Marques-Tavares, D. Racco, 2010.03568]

Causality Tail

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    [A. Hook, G. Marques-Tavares, D. Racco, 2010.03568]
  - Accelerating universe + RD $\Omega_{\rm GW} \propto k^{\frac{5+3w}{1+3w}}$

    [S. Hong, **SML**, Q. Liang, 2504.02462]

- DWD universe: $w = -\frac{2}{3}$ $\Omega_{\rm GW} \propto k^{-3}$

Final Spectrum

$$\left.\frac{d\rho_{\mathrm{GW}}}{dk}\right|_0 = \int_{t_{\mathrm{PT}}}^{t_0} dt\, a(t)\left|\mathcal{T}\left(a(t), k\right)\right|^2 p_{\mathrm{GW}}(t)\mathcal{P}\left(\frac{k}{a}\right)$$

Final Spectrum

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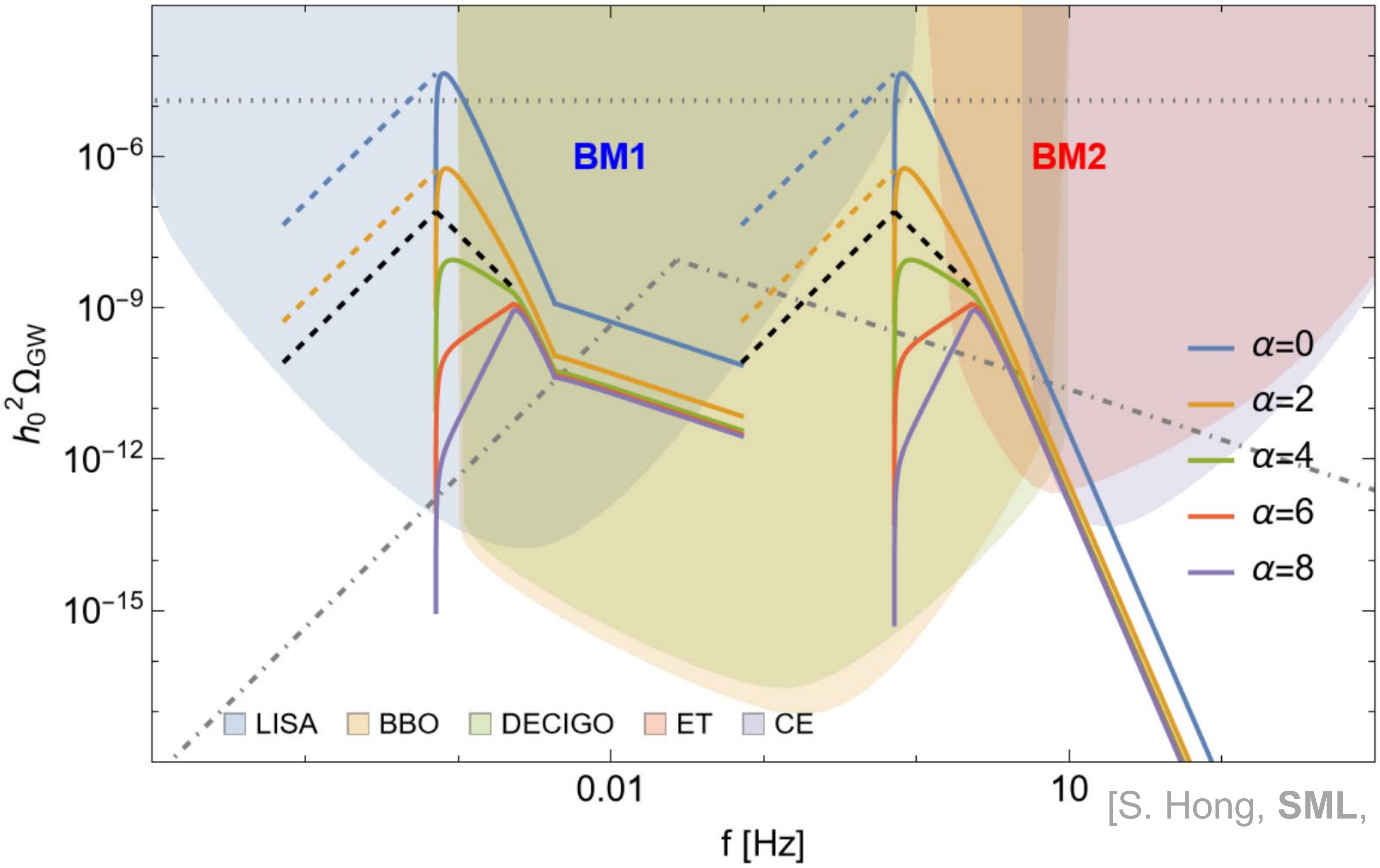
$$h^2\Omega_{\rm GW}(k) \simeq \begin{cases} \dfrac{2r^\alpha(\nu-1)}{5-2\alpha-\nu}\mathcal{A}\left[\left(\dfrac{k}{k_{\rm weq}}\right)^{-5-\nu} - \left(\dfrac{k}{k_{\rm weq}}\right)^{-10+2\alpha}\right] & (k_{\rm weq} < k < k_r) \\ \dfrac{2r^5(\nu-1)}{5-\nu}\mathcal{A}\left[\dfrac{\left(r^{\alpha+\frac{\nu}{2}}(5-\nu) - 2r^{\frac{5}{2}}\alpha\right)}{r^{\frac{5}{2}}(5-2\alpha-\nu)}\left(\dfrac{k}{k_r}\right)^{-5-\nu} - \left(\dfrac{k}{k_r}\right)^{-10}\right] & (k_r < k < k_D) \\ \dfrac{2(\nu-1)}{5-\nu}\mathcal{A}\left[\dfrac{\left(r^\alpha(5-\nu) - 2r^{\frac{5}{2}-\frac{\nu}{2}}\alpha\right)}{(5-2\alpha-\nu)}\left(\dfrac{k_D}{k_{\rm weq}}\right)^{-5-\nu}\left(\dfrac{k}{k_D}\right)^{1-\nu} - \left(\dfrac{k_D}{k_{\rm weq}}\right)^{-10}\left(\dfrac{k}{k_D}\right)^{-4}\right] & (k > k_D) \end{cases}$$

$$\mathcal{A} = 8\pi h^2 \eta_{\rm GW} a_{\rm eq}^4 H_{\rm eq}^2/(3H_0^2)$$

[S. Hong, **SML**, Q. Liang 2504.02462]

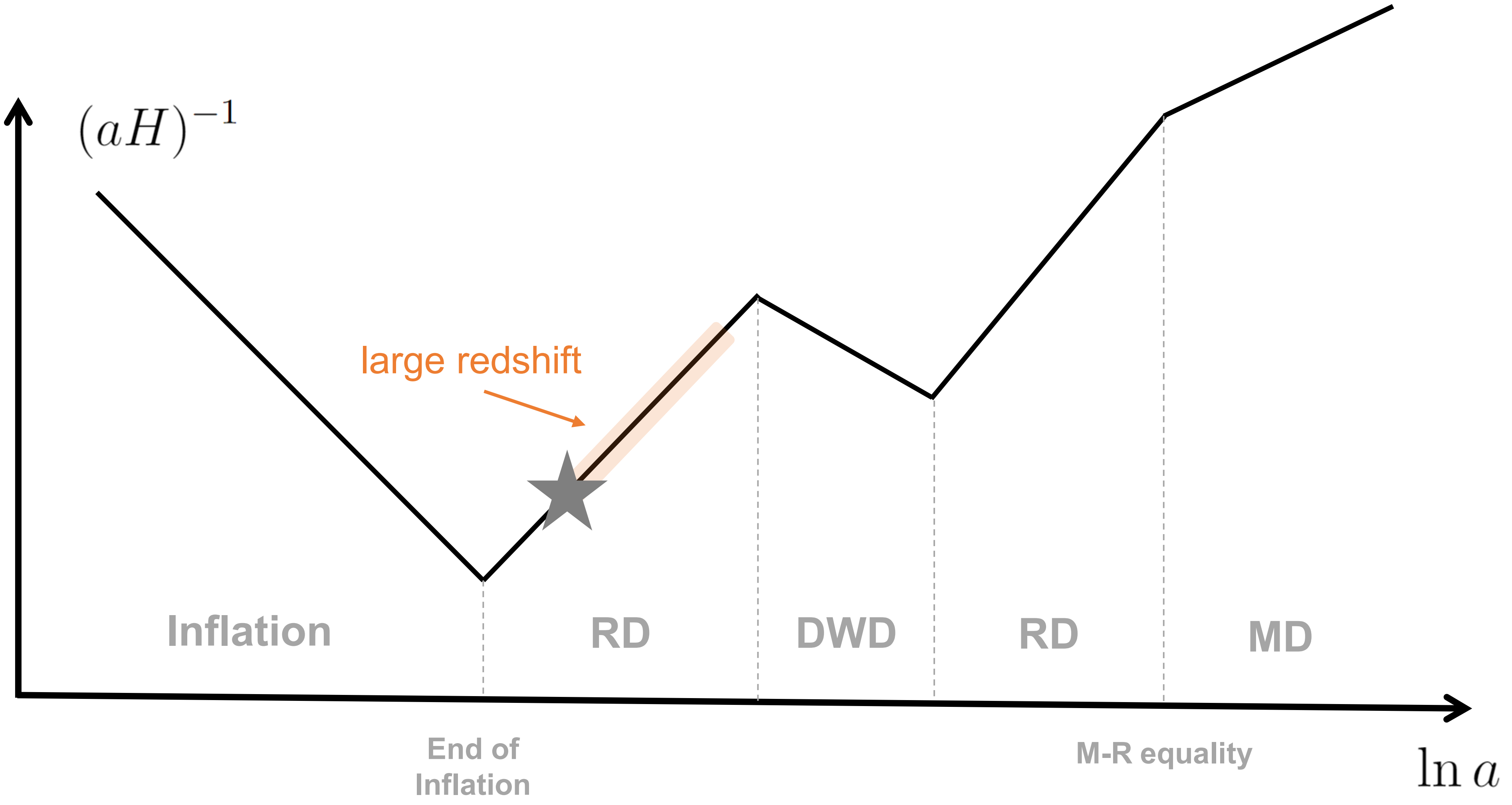
GW Spectrum with DWD



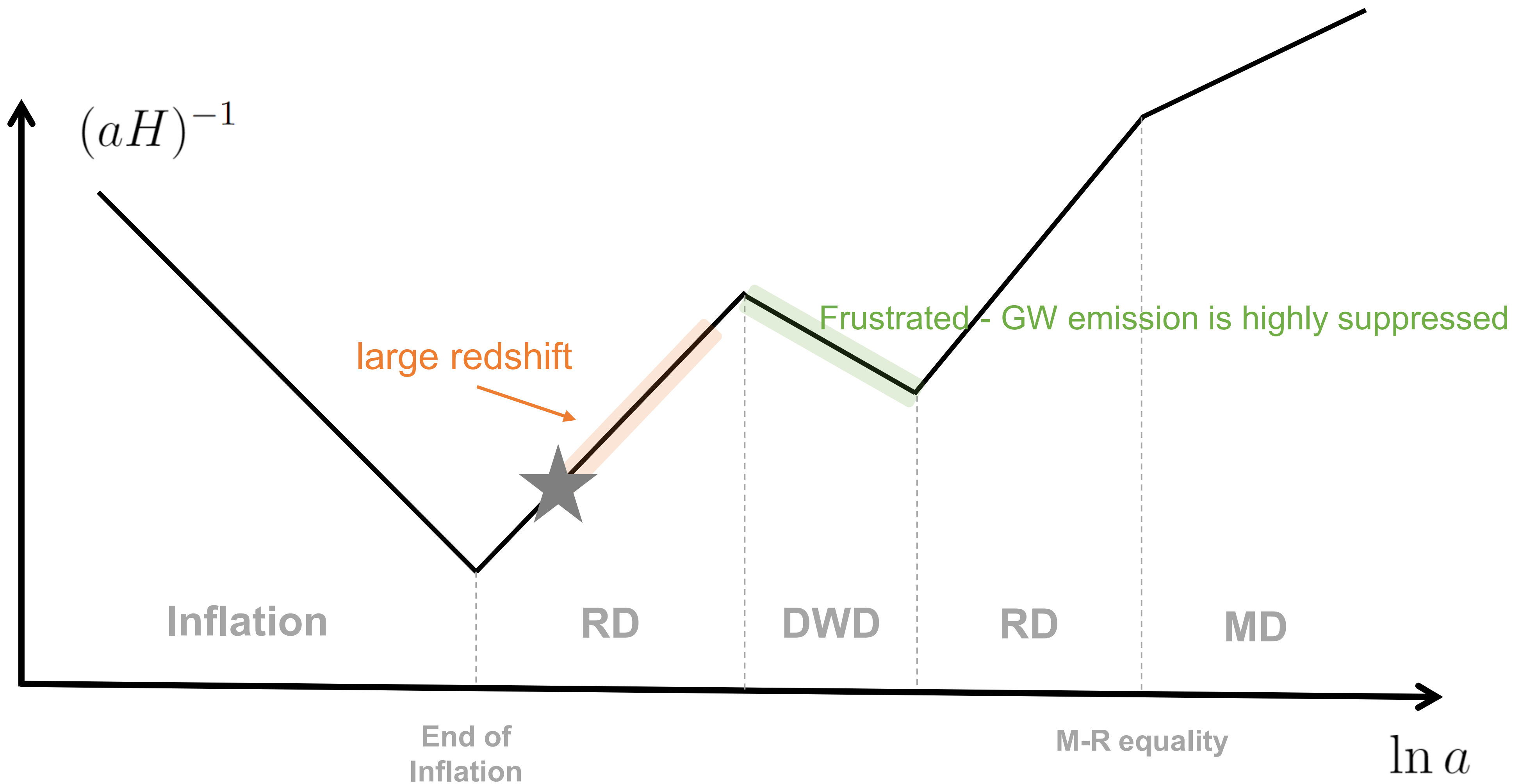
[S. Hong, **SML**, Q. Liang 2504.02462]

Intuitive Picture

GW Spectrum with DW Dominant Era



GW Spectrum with DW Dominant Era



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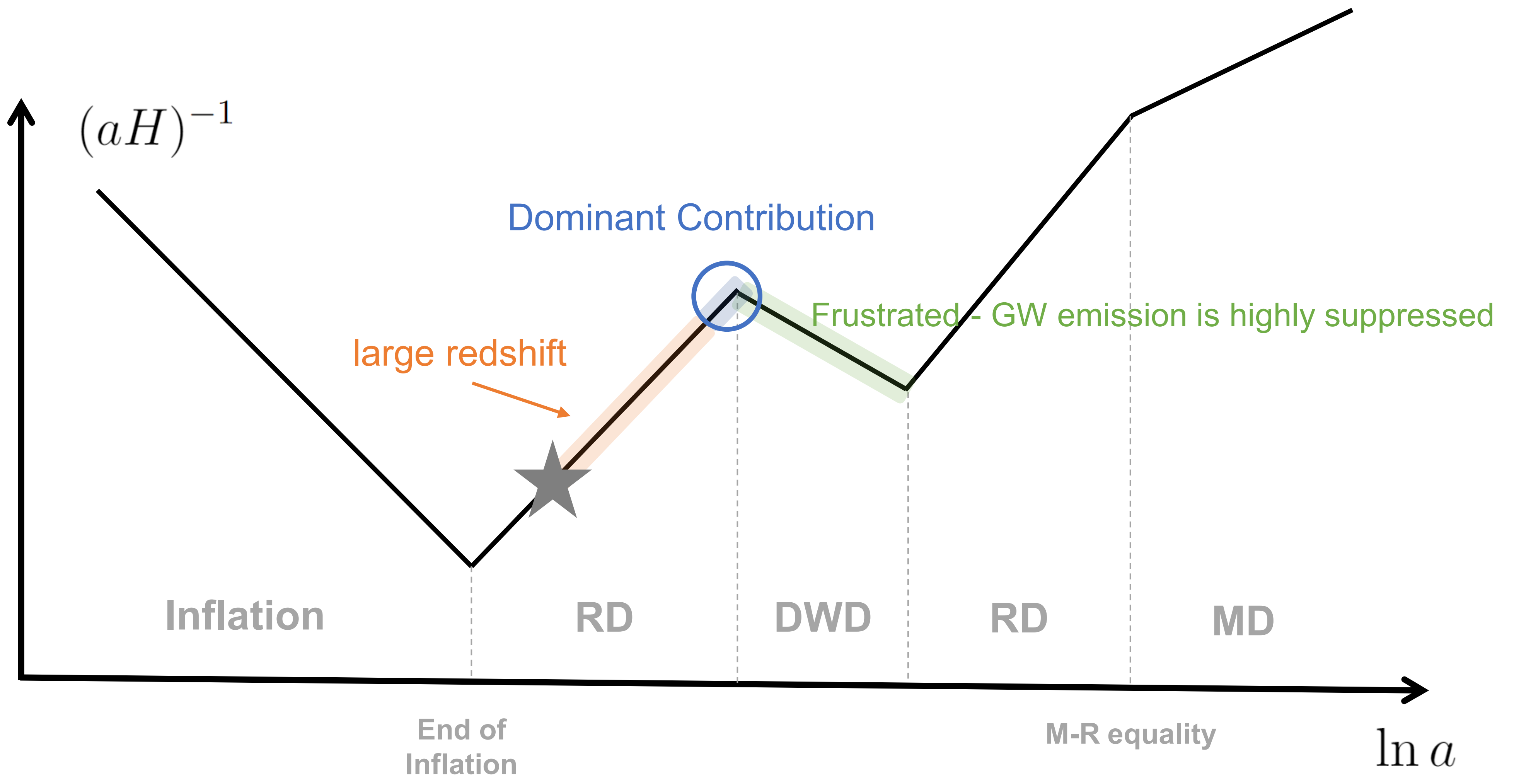

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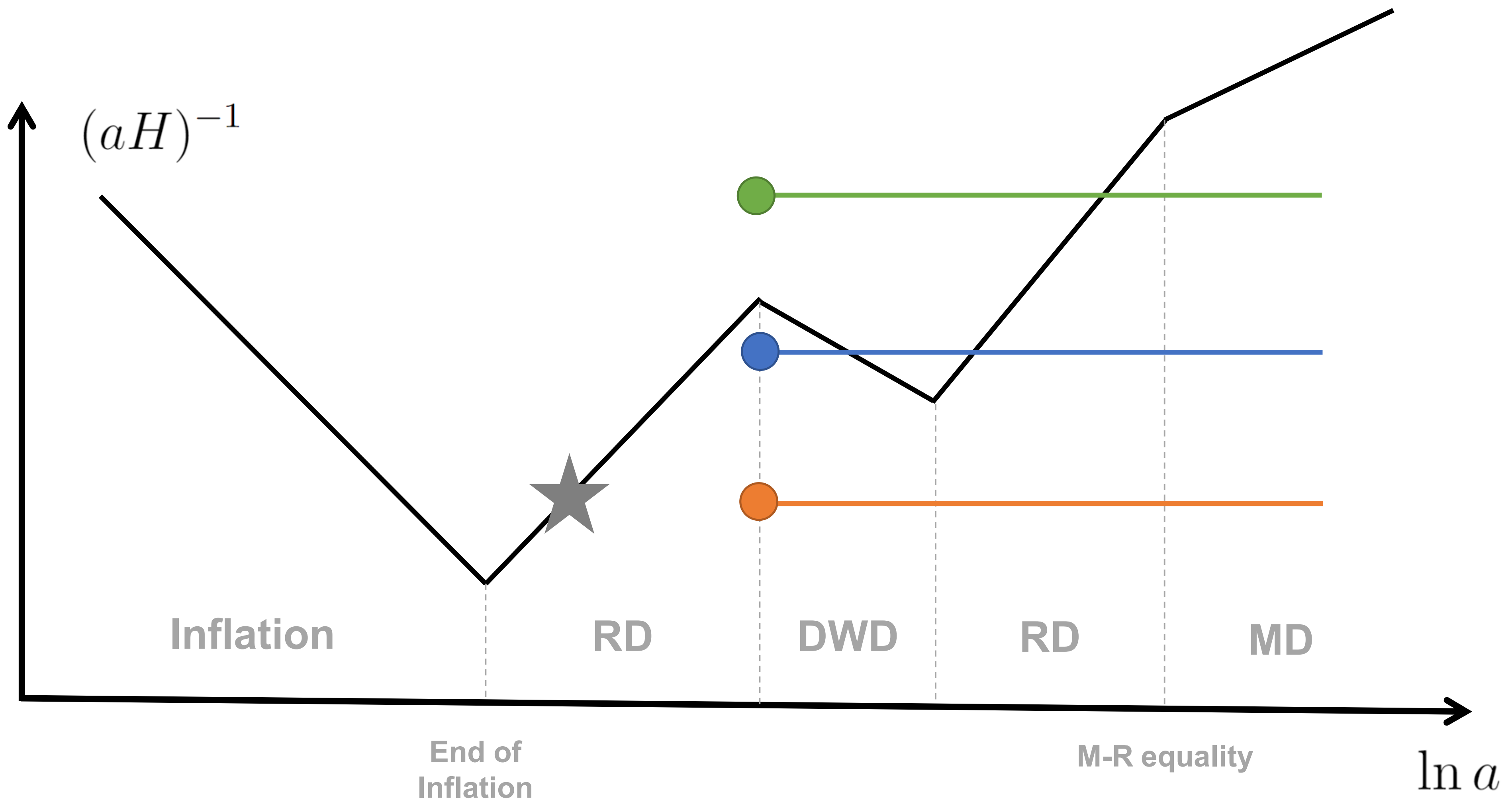

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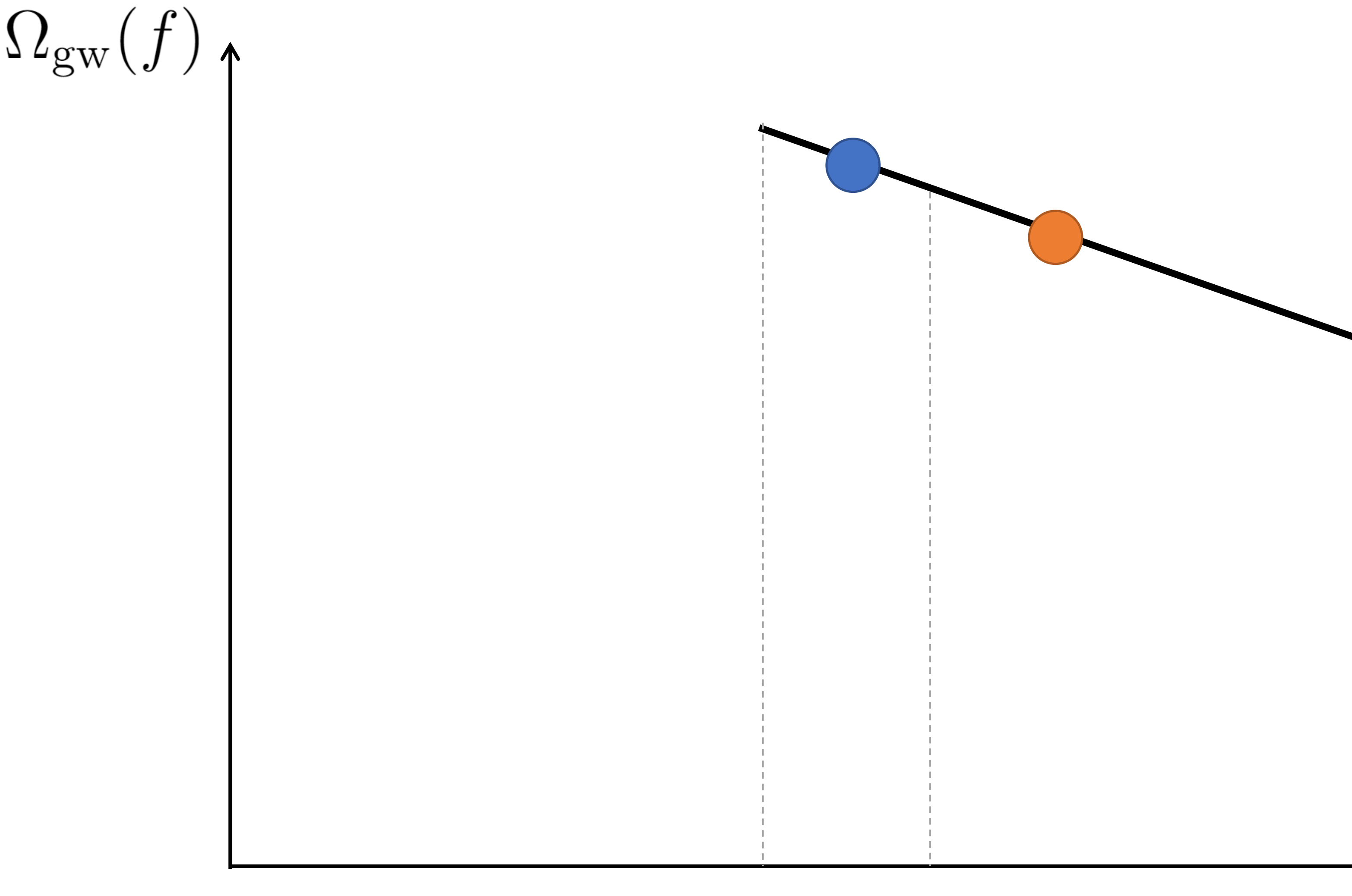
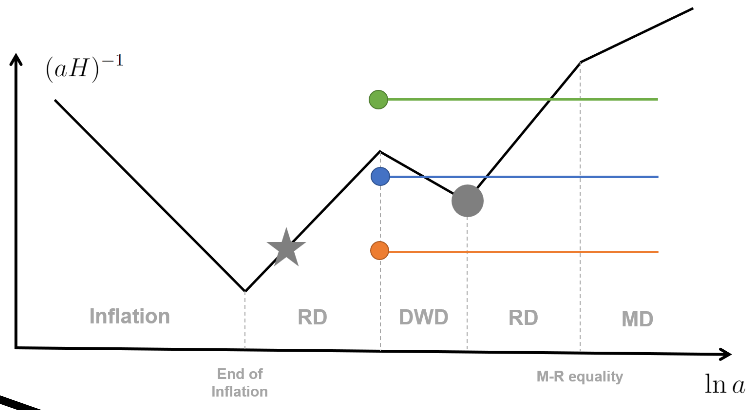
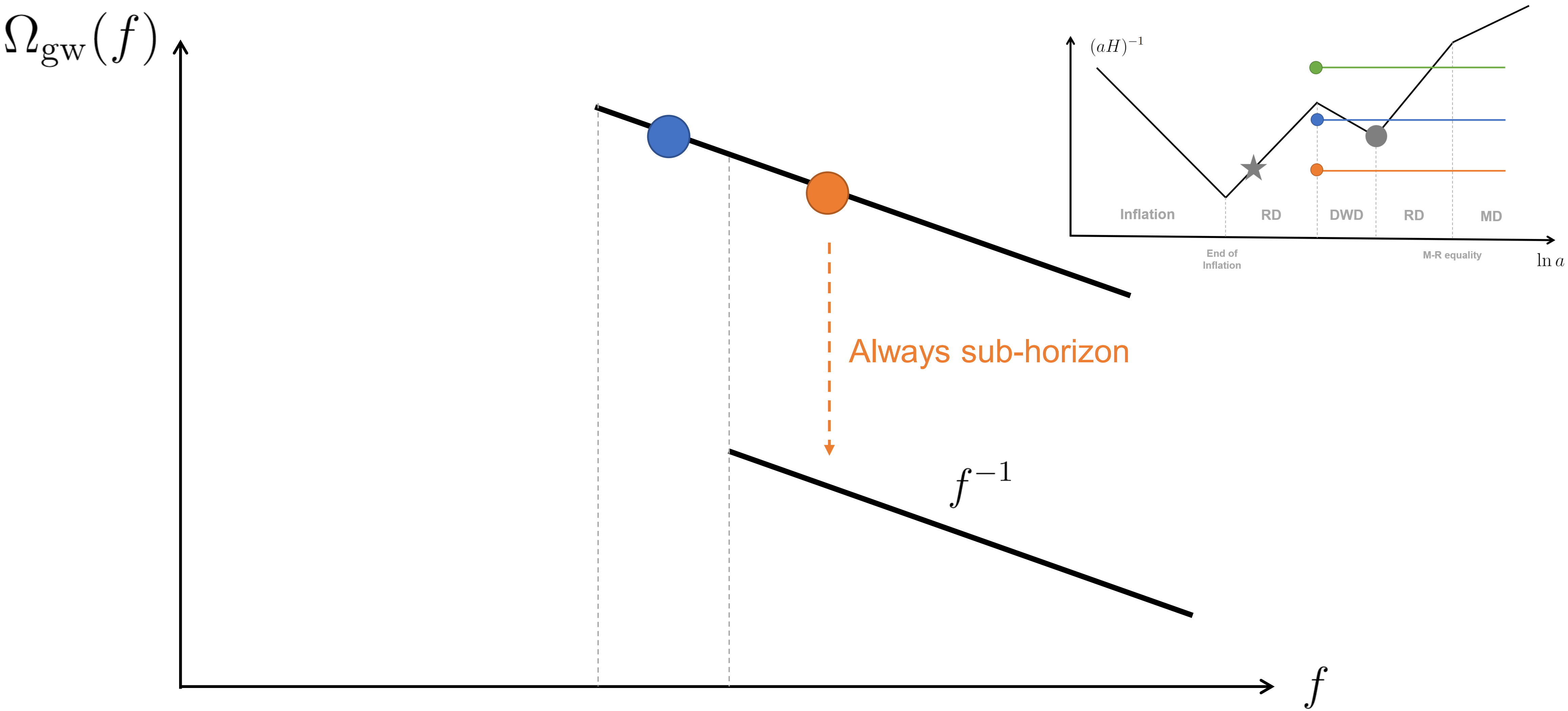
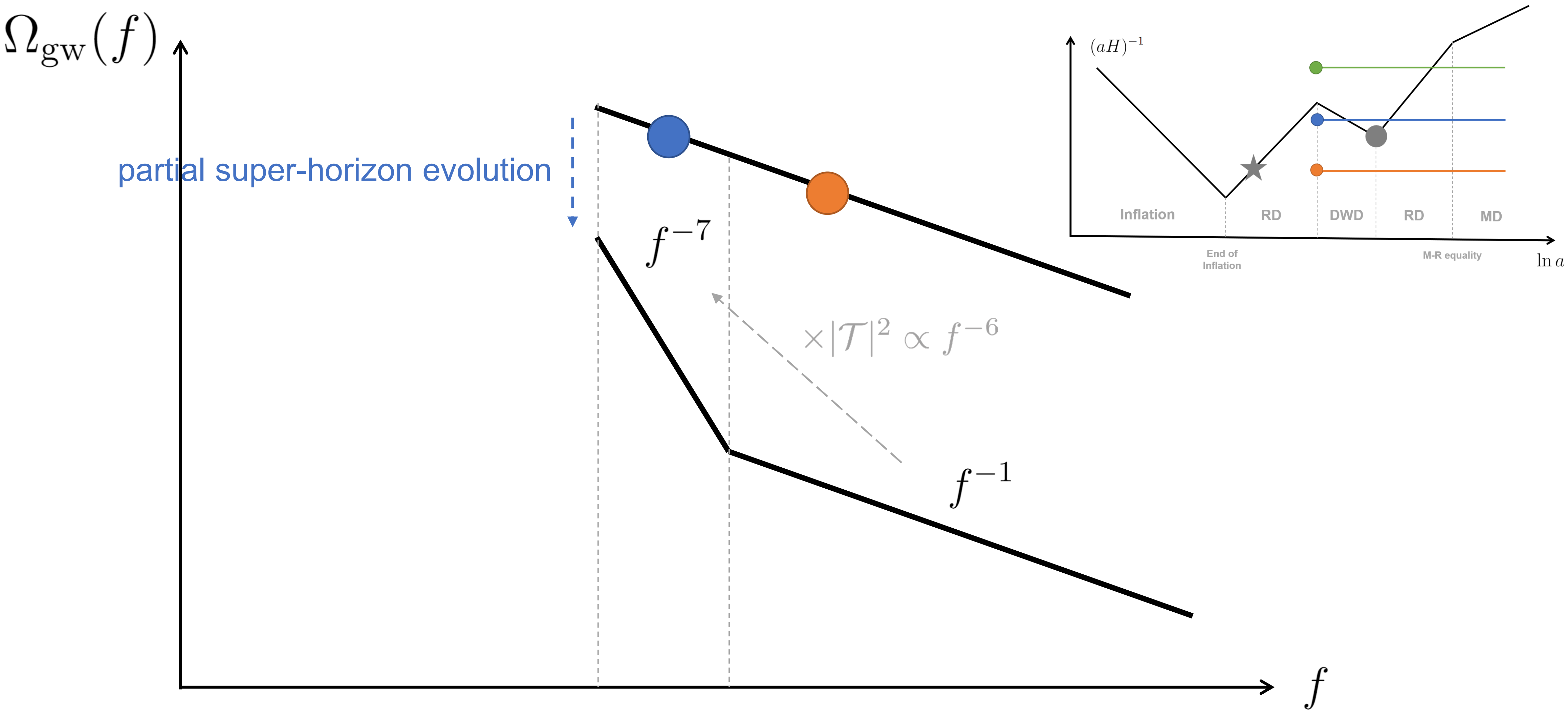




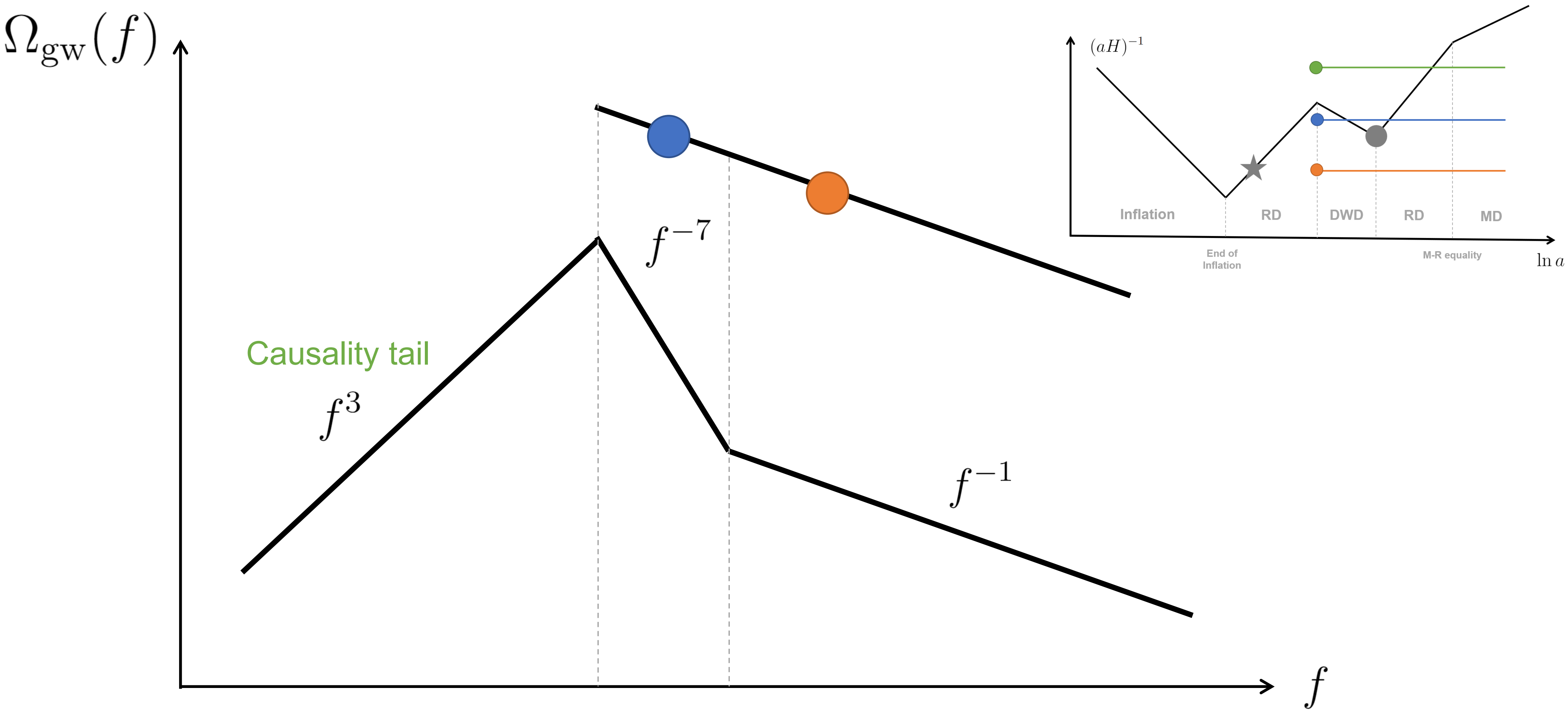
GW Spectrum with DW Dominant Era



GW Spectrum with DW Dominant Era



GW Spectrum with DW Dominant Era



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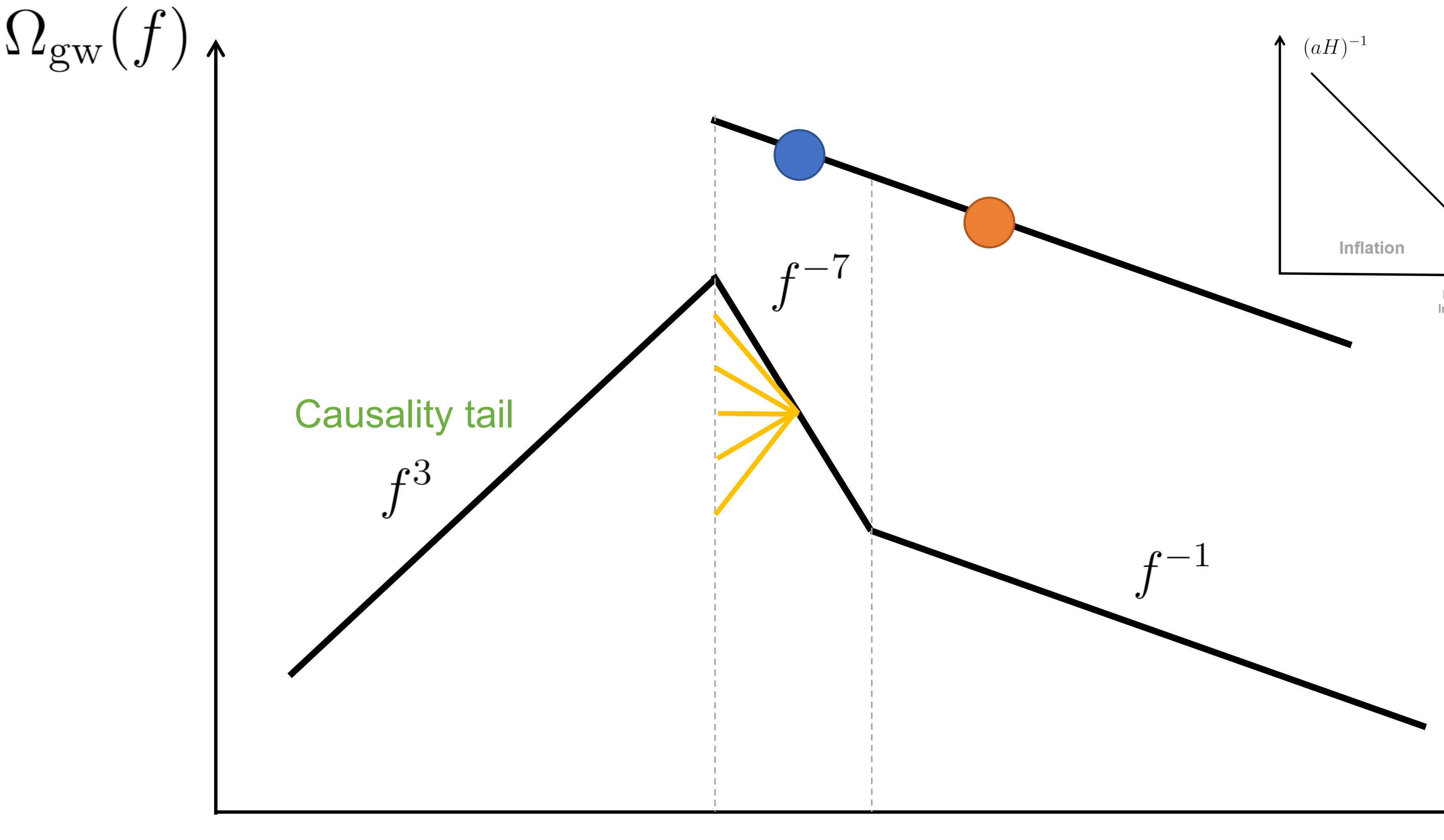
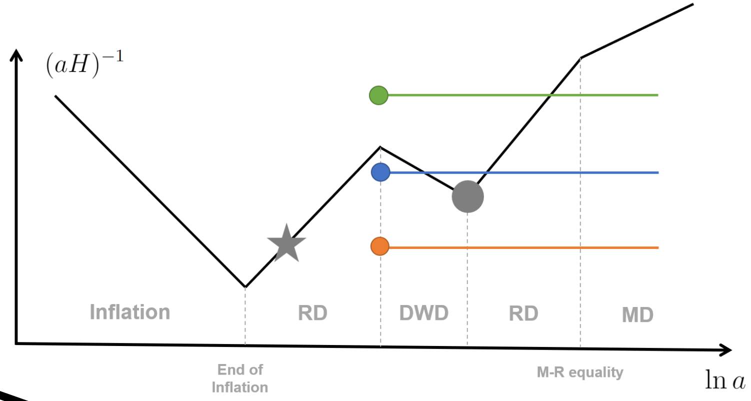
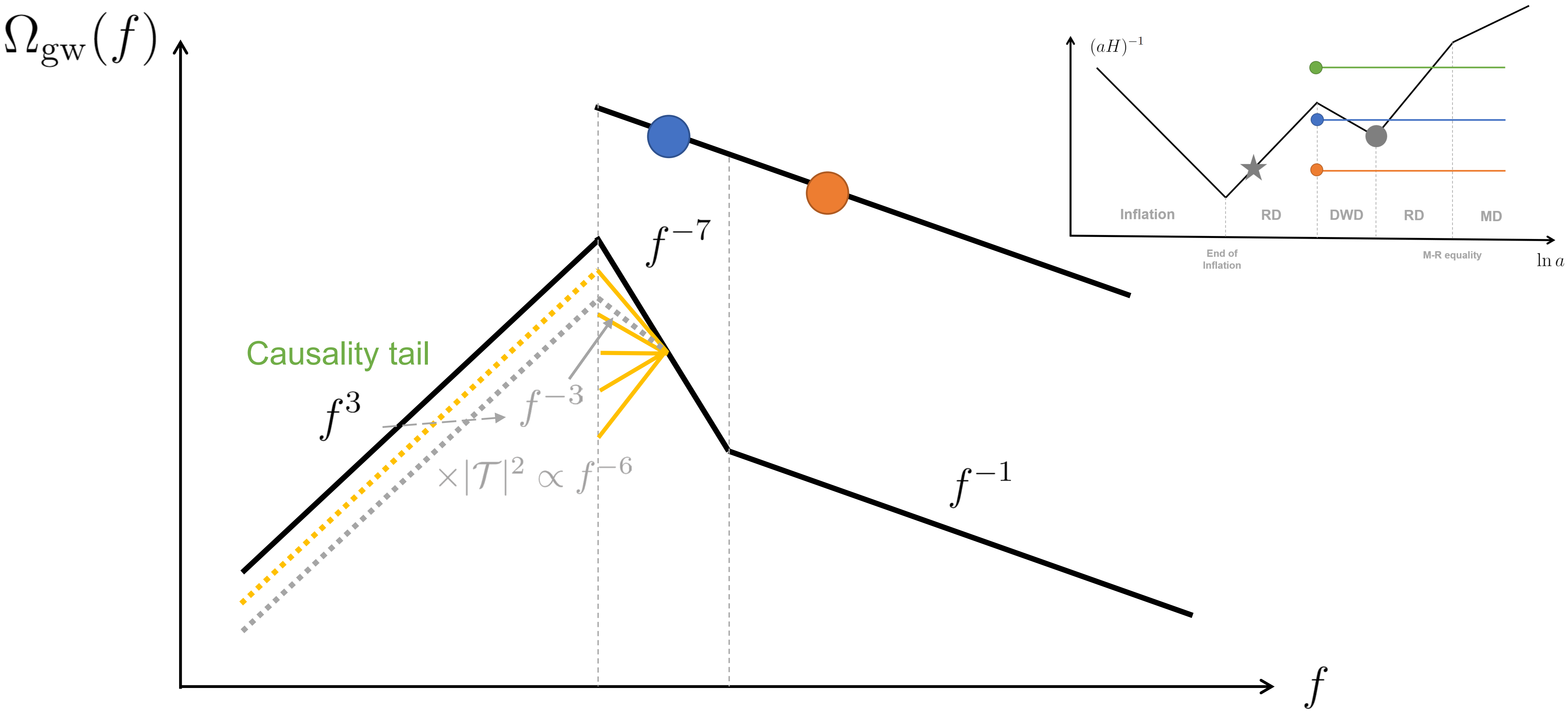
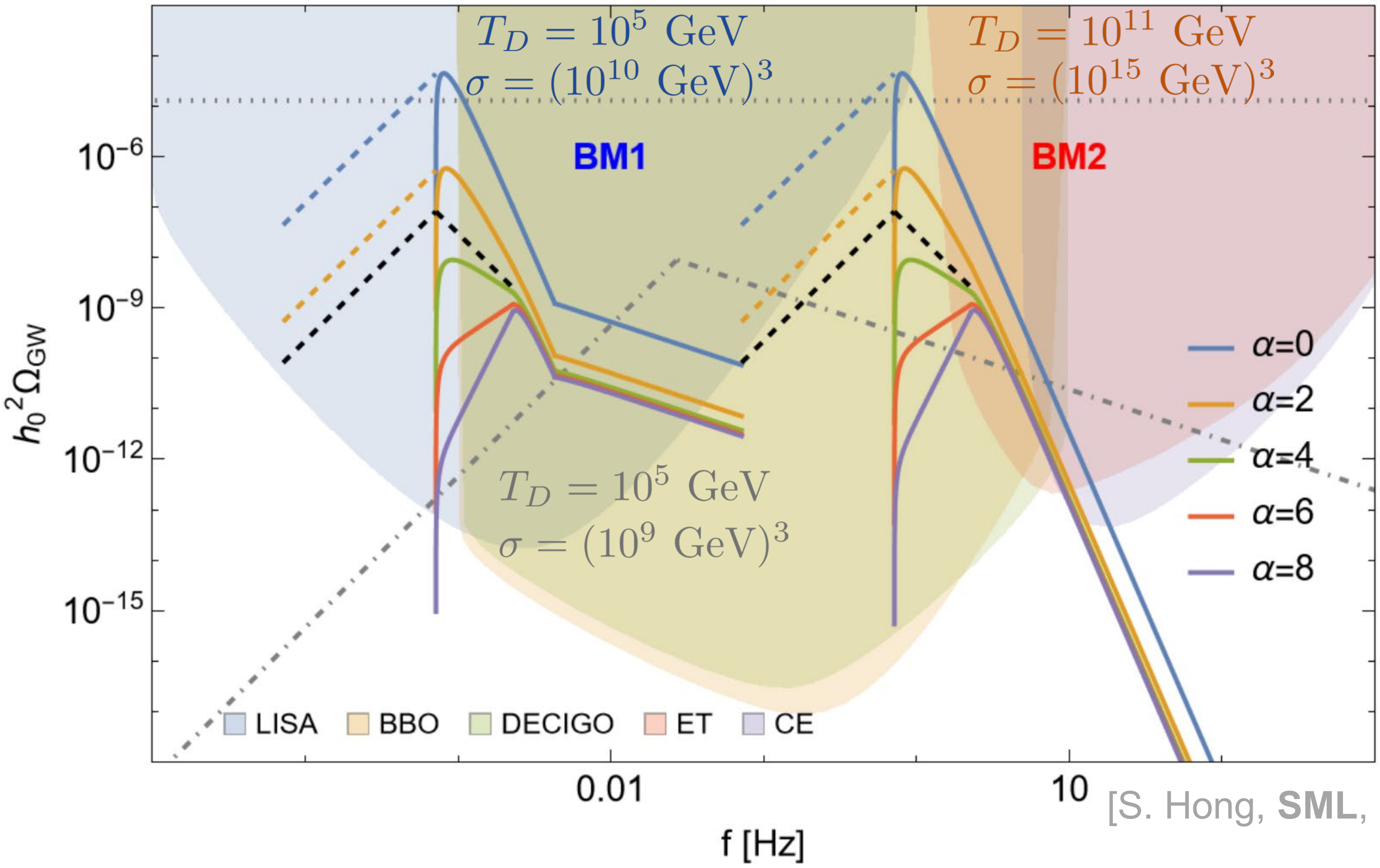




GW Spectrum with DW Dominant Era



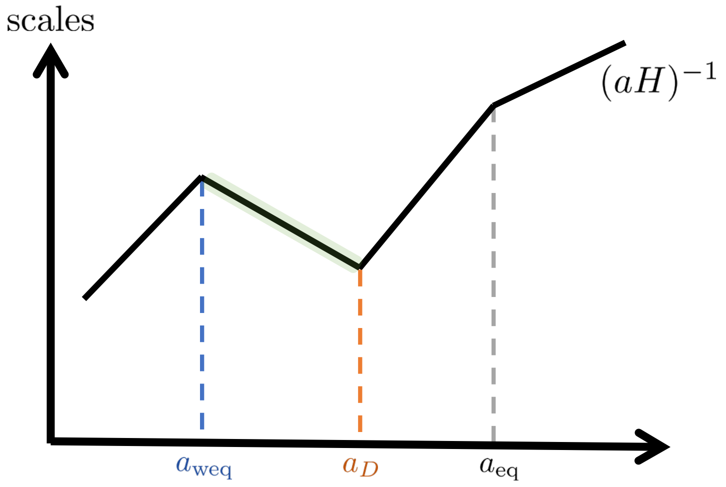
GW Spectrum with DWD



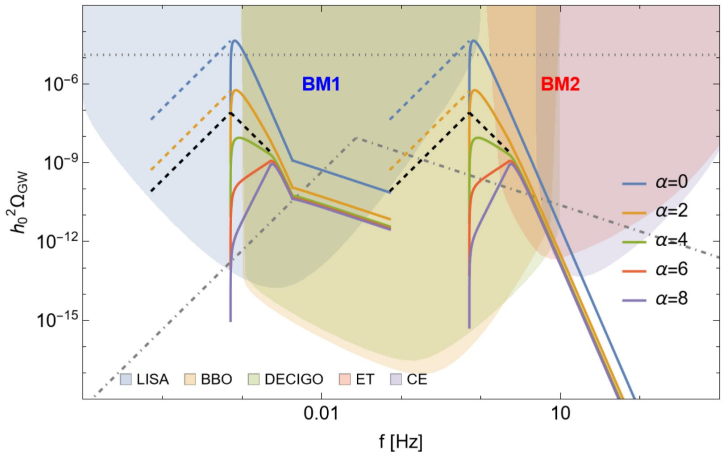
[S. Hong, **SML**, Q. Liang 2504.02462]

Conclusion

- Long lived DWs lead to DW dominant phase.



- It may leave distinctive SGWB spectrum.

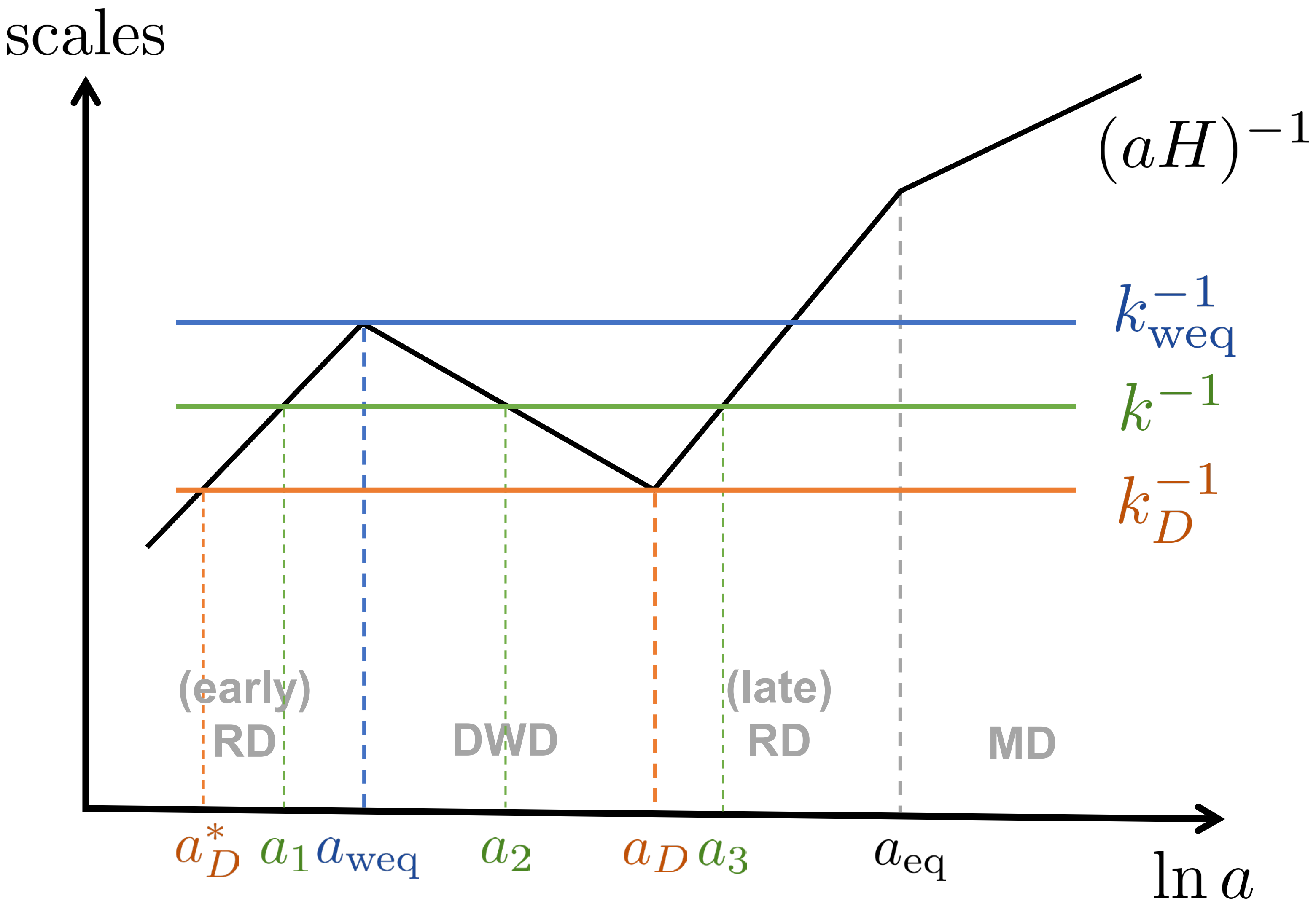


- More analytical/numerical understandings are still required.

Thank you!

Back-Ups

Numerical Simulations : GW



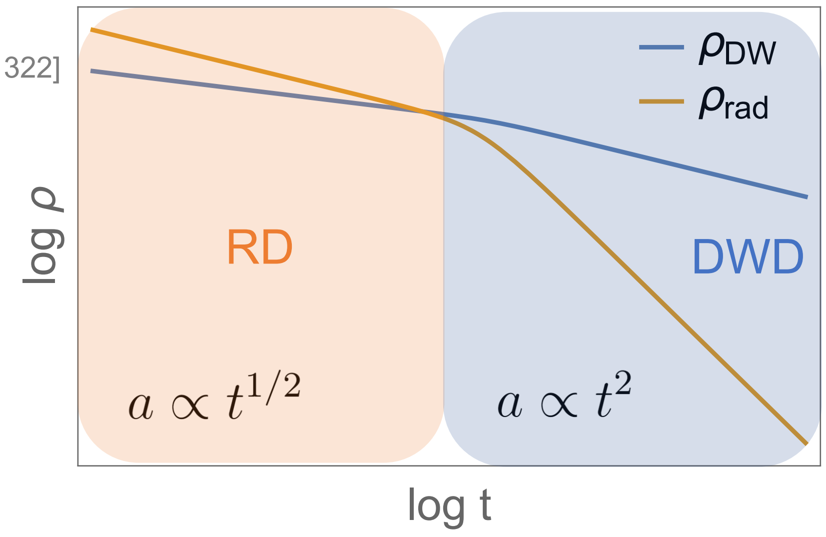
Analytic Understanding : VOS Model

- **Velocity-dependent One Scale**

[C. J. A. P. Martins, I. Yu. Rybak, A. Avgoustidis, E. P. S. Shellard 1602.01322]
[D. Gruber, L. Sousa, P.P. Avelino 2403.09816]

$$\frac{dL}{dt} = (1+3\bar{v}^2)HL + \tilde{c}\bar{v},$$
$$\frac{d\bar{v}}{dt} = (1-\bar{v}^2)\left(\frac{k_w}{L} - 3H\bar{v}\right)$$

$$\rho_{\rm DW} = \frac{\sigma}{L}$$



- **Stable DW**

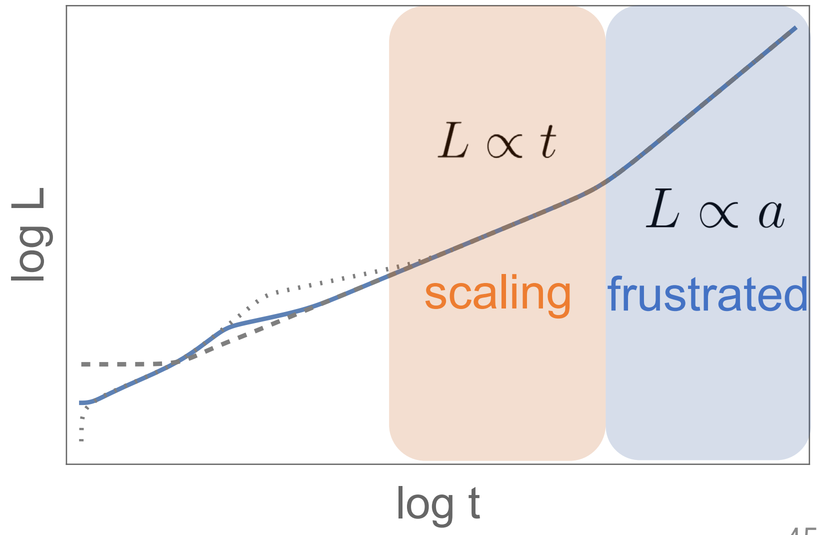
$$\rho_{\rm DW} = \mathcal{A}\frac{\sigma}{t}$$

  - $t_{\rm DWD} = \frac{3M_P^2}{4\mathcal{A}\sigma}$

$$\rho_{\rm rad} = 3M_P^2H^2 = \frac{3M_P^2}{4t^2}$$

  - ZKO bound : $\sigma^{1/3} \lesssim \mathcal{O}(\mathrm{MeV})$

[Ya.B. Zeldovich, I.Yu. Kobzarev, L.B. Okun '74]

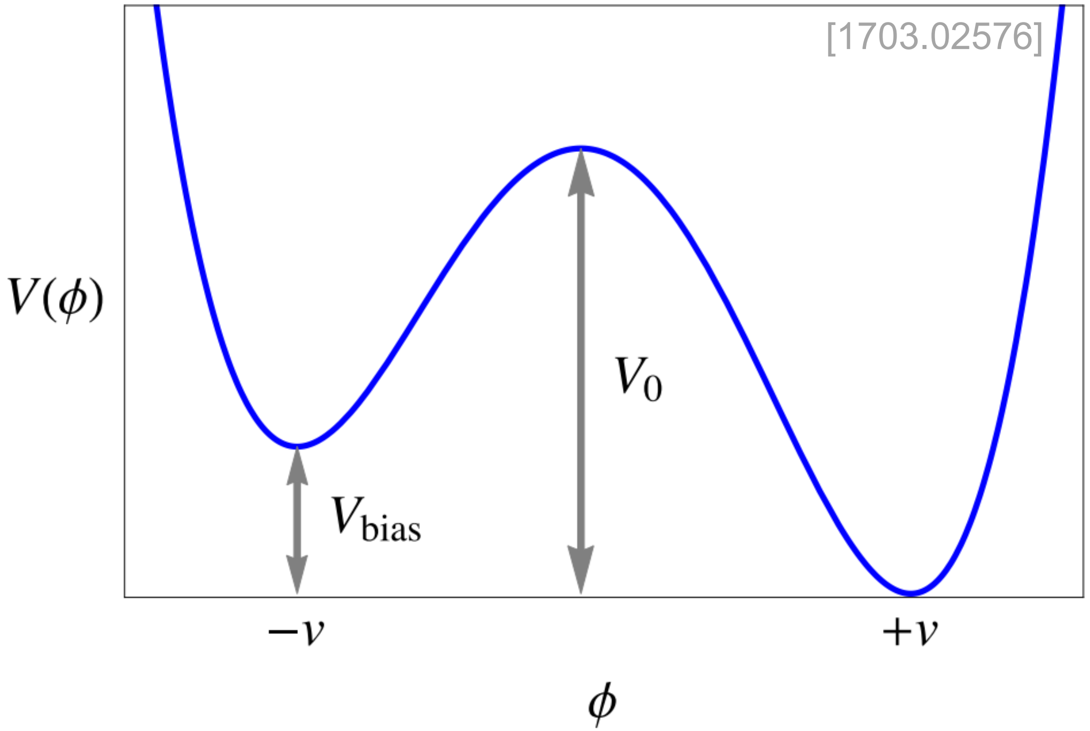


Unstable DW

▪ Potential Bias

[A. Vilenkin '81]
[G. B. Gelmini, M. Gleiser and E. W. Kolb, '89]
[S. E. Larsson, S. Sarkar and P. L. White '97]

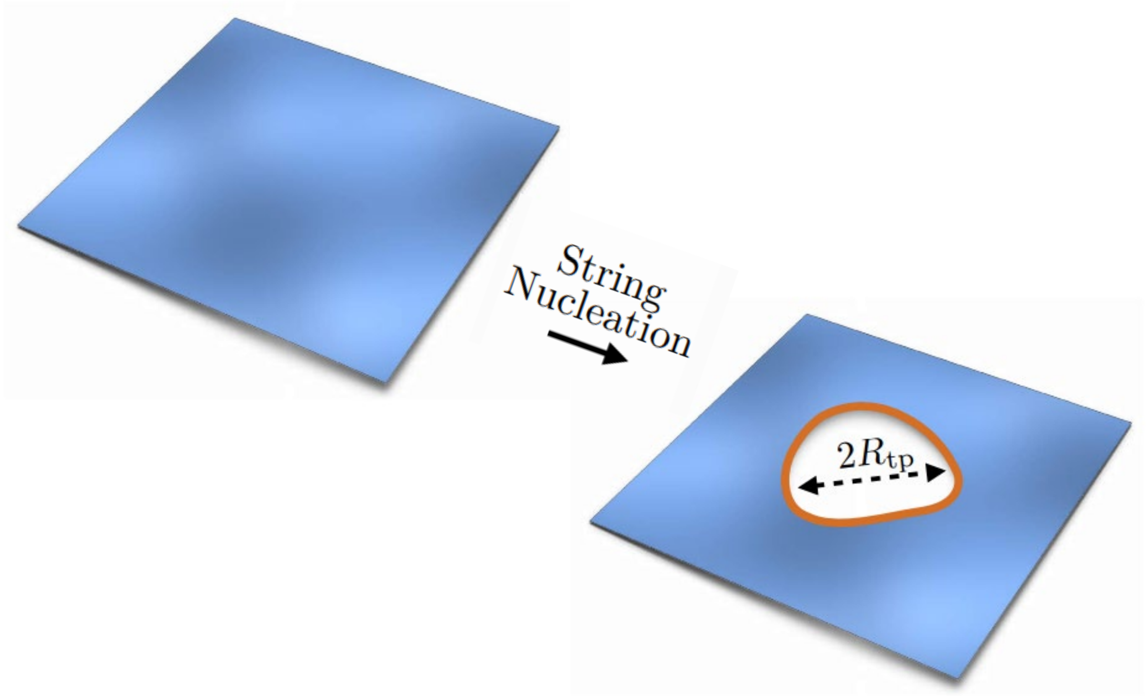
- Pressure catalyzes the collapse
- May not work for DW dominance



▪ String Nucleation

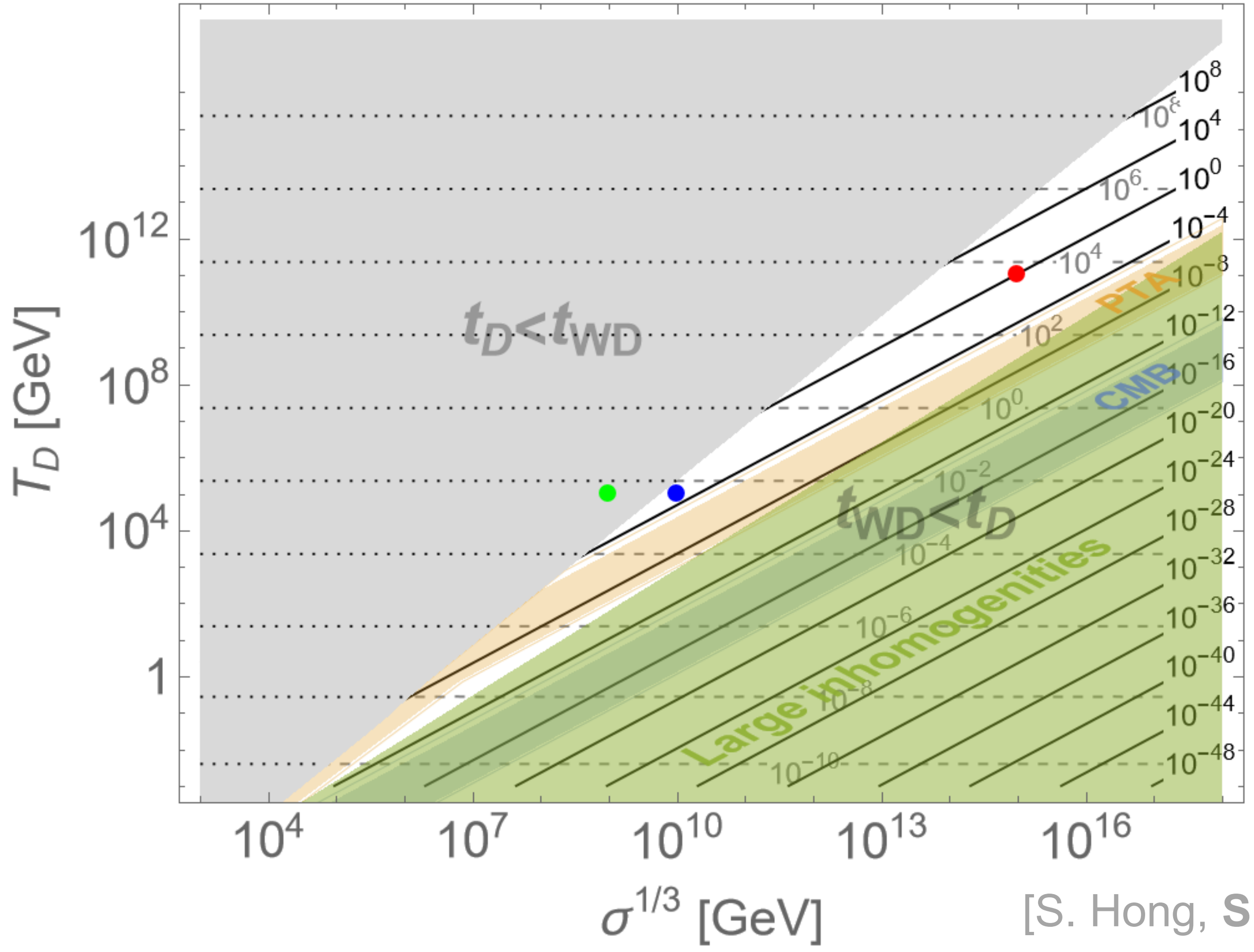
[T. W. B. Kibble, G. Lazarides, and Q. Shafi '82]
[J. Preskill, A. Vilenkin '92]

- Spontaneous decay of DW
- Still works for DW dominance if $\Gamma_{\rm DW} > H$



[2111.08750]

Large Inhomogenities



[S. Hong, **SML**, Q. Liang 2504.02462]

Causality Tail

[A. Hook, G. Marques-Tavares, D. Racco 2010.03568]

$$h'' + 2\mathcal{H}h' + k^2 h = J_* \delta(\tau - \tau_*)$$

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$$h'' + 2\mathcal{H}h' + k^2 h = J_* \delta(\tau - \tau_*)$$

- **Subhorizon** $\quad h \simeq \frac{a(\tau_*)}{a(\tau)} \frac{J_*}{k} \sin k(\tau - \tau_*) \propto k^{-1}$

- **Superhorizon** $\quad h \simeq \frac{a(\tau_k)}{a(\tau)} \frac{J_*}{\mathcal{H}_*} \sin k\tau \propto \begin{cases} k^{-1} & \text{(RD)} \\ k^{-2} & \text{(MD)} \end{cases}$

$$\frac{d\Omega_{\rm GW}}{d\ln k} \equiv \frac{1}{\rho_c} \frac{d\rho_{\rm GW}}{d\ln k} \propto \frac{k^5}{a^2} P_h(k,\tau)$$

$$\rho_{\rm GW}(\mathbf{x},\tau) \sim \frac{1}{32\pi G a^2} \langle h'(\mathbf{x},\tau) h'(\mathbf{x},\tau) \rangle$$

$$\langle h(k,\tau) h(k',\tau) \rangle = (2\pi)^3 \delta^3(k - k') P_h(k,\tau)$$

Causality Tail

[A. Hook, G. Marques-Tavares, D. Racco 2010.03568]