On geometrical destabilization during inflation

Tomasz Krajewski   Institute of Fundamental Technological Research

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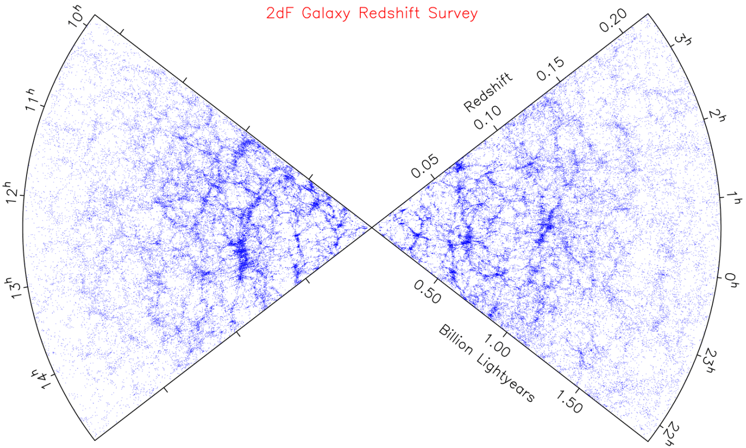
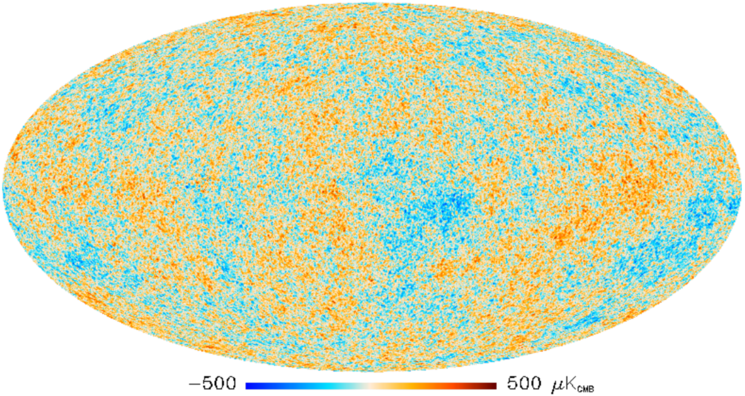
Large Scale Structure



Visualization of data from 2dF galaxy catalog.[1]

1. Colless, M. *et al. Mon. Not. Roy. Astron. Soc.* **328,** 1039. arXiv: astro-ph/0106498 [astro-ph] (2001).

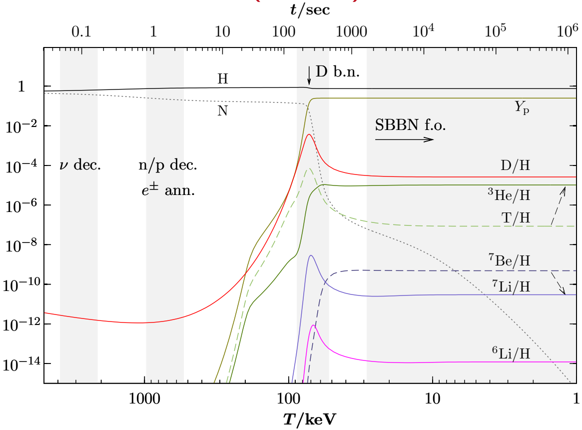
Cosmic Microwave Background (CMB) Radiation



CMB map reconstructed from Planck data.[2]

2. Ade, P. A. R. *et al. Astron. Astrophys.* **571,** A1. arXiv: 1303.5062 [astro-ph.CO] (2014).

Big Bang Nucleosynthesis (BBN)



Production of light elements in BBN.[3]

3. Pospelov, M. & Pradler, J. *Ann. Rev. Nucl. Part. Sci.* **60,** 539–568. arXiv: `1011.1054 [hep-ph]` (2010).

Conclusions from observational data

- The current Universe is homogeneous at scales larger than $100\mathrm{Mpc}$.
- The Universe was (nearly) isotropic during recombination. Relative fluctuations of the CMB radiation are of the order of:

$$\frac{\Delta T}{T} \sim 10^{-5}.$$

- The energy density of the Universe during BBN was dominated by radiation (particles moving with relativistic speeds).

Cosmological inflation

Cosmological inflation allows for simultaneous solution for many problems in cosmology:

- horizon problem,
- flatness problem,
- magnetic monopoles problem.

Moreover, it provides a very natural explanation of CMB inhomogeneities.

α-attractor T-model

α-attractor models of inflation origin from supergravity.
T-model of inflation is characterized by the superpotential:

$$W_H = \sqrt{\alpha}\mu S \left(\frac{T-1}{T+1} \right)^n ,$$

and by the Kähler potential:

$$K_H = -\frac{3\alpha}{2} \log \left(\frac{(T-\bar{T})^2}{4T\bar{T}} \right) + S\bar{S}.$$

Two-fields α-attractor T-model

The scalar sector can be expressed in terms of two real scalar fields ϕ and χ. The scalar Lagrangian takes particularly simple form:

$$\mathcal{L} = -\frac{1}{2}\Big(\partial_\mu \chi \partial^\mu \chi + e^{2b(\chi)} \partial_\mu \phi \partial^\mu \phi\Big) - V(\phi, \chi), \qquad b(\chi) := \log(\cosh(\beta\chi)),$$

with the potential:

$$V(\phi, \chi) = M^4 \left(\frac{\cosh(\beta\phi)\cosh(\beta\chi) - 1}{\cosh(\beta\phi)\cosh(\beta\chi) + 1}\right)^n \Big(\cosh(\beta\chi)\Big)^{2/\beta^2}, \qquad M^4 := \alpha\mu^2,$$

and

$$\beta := \sqrt{\frac{2}{3\alpha}}.$$

Field space metric

Field space in α-attractor T-model has non-trivial structure:

$$\mathcal{G} = \begin{pmatrix} 1 & 0 \\ 0 & e^{2b(\chi)} \end{pmatrix},$$

with negative curvature:

$$\mathbb{R} = -2\beta^2.$$

The so called geometrical destabilization is possible with two scenarios:

- during inflation leading to perturbation of inflationary trajectory or premature end of inflation,
- around the end of inflation leading to fast (p)reheating.

One-field α-attractor T-model

In the literature one-field simplification is considered with the potential of the form:

$$V(\phi, 0) = M^4 \tanh^{2n}\left(\frac{\beta|\phi|}{2}\right).$$

In order to find inflationary trajectory (at least the part along $\chi = 0$ direction) one need to solve set of coupled differential equations:

$$H^2 = \frac{1}{3}\left[\frac{1}{2}\dot{\phi}^2 + V(\phi, 0)\right], \qquad \ddot{\phi} + 3H\dot{\phi} + V_\phi(\phi, 0) = 0.$$

Perturbations in α-attractors

Linear perturbations are described in terms of gauge invariant Mukhanov-Sasaki variables:

$$Q_\phi := \delta\phi + \frac{\dot{\phi}}{H}\Psi, \qquad\qquad Q_\chi := \delta\chi + \frac{\dot{\chi}}{H}\Psi,$$

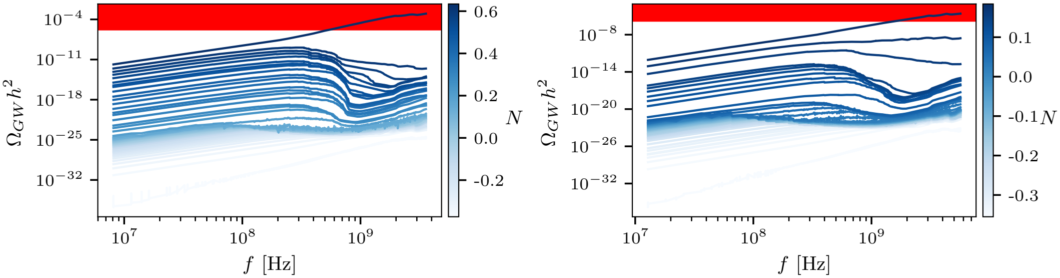
which obey the following equations of motion

$$\ddot{Q}_\varphi + 3H\dot{Q}_\varphi + \left(\frac{k^2}{a^2} + m_\varphi^2\right)Q_\phi = 0, \quad \text{for } \varphi = \phi, \chi,$$

where the effective masses m_φ^2 of perturbations are:

$$m_\phi^2 = V_{\phi\phi}(\phi, \chi), \qquad\qquad m_\chi^2 = V_{\chi\chi}(\phi, \chi) + \frac{1}{2}\dot{\phi}^2\mathbb{R} \quad \text{for } \mathbb{R} = -2\beta^2.$$

Spectrum of gravitational waves



Evolution of the spectrum of gravitational waves as a function of the number of e-folds N from the end of inflation for $n = 1.5$, $\alpha = 10^{-3}$ (left panel), $n = 1.5$, $\alpha = 10^{-4}$ (right panel).[4]

4. Krajewski, T. & Turzyński, K. *JCAP* **10,** 005. arXiv: `2204.12909 [astro-ph.CO]` (2022).

Geometrical destabilization of inflation

- Geometrical destabilization may happen not only after inflation, during preheating when it leads to efficient reheating, but also during inflation.
- Geometrical destabilization during inflation was seen as a threat to the inflationary model, since it causes deviation from inflationary trajectory with possibility of premature end of inflation.
- Our numerical simulations have shown that this is not the case. Produced fluctations of a spectator field backreact on the trajectory of the inflaton and prevents field fluctuations from growing indefinitely.

A minimal realization

For our numerical simulations, we use a model of slow-roll inflation driven by a scalar field ϕ with Starobinsky potential

$$V(\phi) = \Lambda^4 \left(1 - e^{-\sqrt{\frac{2}{3}}\frac{\phi}{M_{Pl}}}\right)^2 .$$

The potential of the spectator field χ is quadratic one with mass m.
The model is supplemented with the dimension six operator

$$-(\partial\phi)^2\chi^2/M^2$$

leading to Lagrangian density:

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2\left(1 + 2\frac{\chi^2}{M^2}\right) - V(\phi) - \frac{1}{2}(\partial\chi)^2 - \frac{1}{2}m^2\chi^2,$$

where M is a scale of new physics that lies well above the Hubble scale, $M \gg H$.

Perturbations in minimal model

The dimension six operator generates a curved field space whose Ricci scalar is negative and reads

$$\mathbb{R} = -\frac{4}{M^2(1+2\chi^2/M^2)^2}.$$

Along the inflationary valley $\chi = 0$, Q_s coincides with the fluctuation of χ and has effective mass:

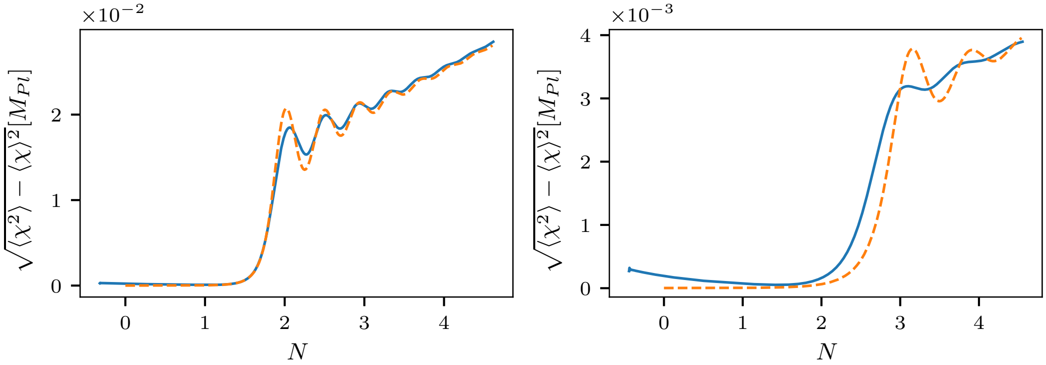
$$m^2_{s(\mathrm{eff})} = m^2 - 4\epsilon H^2 \left(M_{Pl}/M\right)^2 .$$

As ϵH^2 grows during inflation, at the critical point such that

$$\epsilon_{\mathrm{c}} = \frac{1}{4}\left(\frac{m}{H_{\mathrm{c}}}\right)^2 \left(\frac{M}{M_{Pl}}\right)^2 ,$$

the effective mass becomes negative, which triggers geometrical destabilization.

Standard deviation of spectator field



Evolution of the standard deviation of χ for $\left(\frac{m}{H_c}\right)^2 = 10^2$, and $\left(\frac{M}{M_{Pl}}\right)^2 = 10^{-3}$ (left panel), 10^{-4} (right panel).[5]

5. Krajewski, T. & Turzyński, K. *JCAP* **10,** 064. arXiv: `2205.13487 [astro-ph.CO]` (2022).

Kinematical backreaction[6]

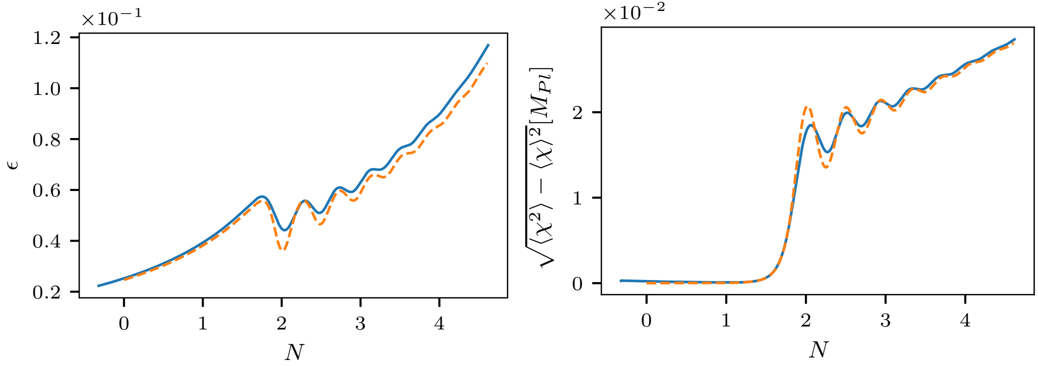
Destabilization of the inflationary trajectory in the χ direction can significantly affect the motion of the inflaton field ϕ.
The EOM of inflaton takes the form:

$$\ddot{\phi} + 3H\dot{\phi} + 2b'\dot{\phi}\dot{\chi} + e^{-2b}\frac{\partial V}{\partial \phi} = 0,$$

where $e^{2b} = 1 + 2\frac{\chi^2}{M^2}$ and $b' = 2\frac{\chi}{M^2}e^{-2b}$ in minimal realization.
χ increasing during geometrical destabilization effectively reduce the slope of the potential (through the e^{-2b} factor).
This slows down the field ϕ so that the slow-roll parameter ϵ is reduced and the instability condition is no longer satisfied.

6. Grocholski, O. *et al.* *JCAP* **1905,** 008. arXiv: `1901.10468 [astro-ph.CO]` (2019).

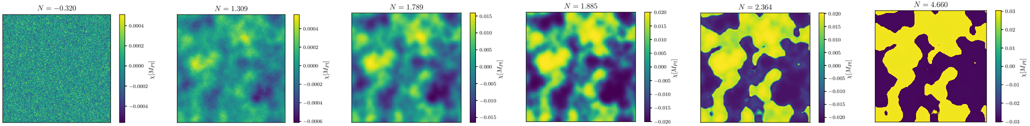
Evolution of slow roll parameter



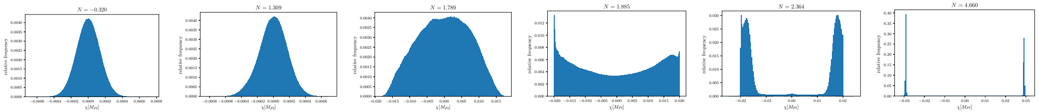
Evolution of the slow roll parameter ϵ (left panel) and of the standard deviation of χ (right panel).[5]

5. Krajewski, T. & Turzyński, K. *JCAP* **10,** 064. arXiv: `2205.13487 [astro-ph.CO]` (2022).

Evolution of spectator field in space



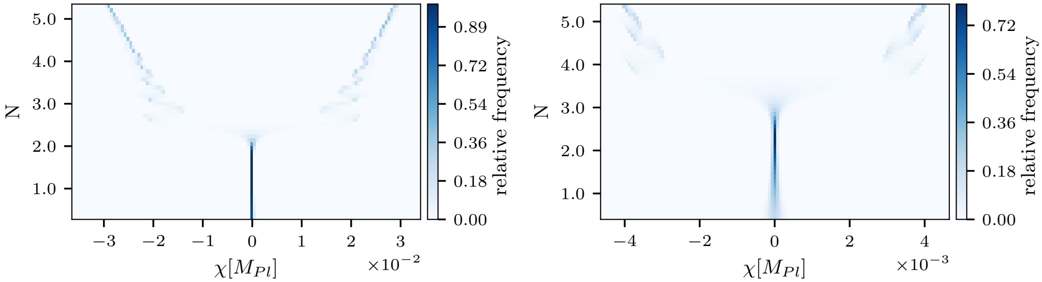
Snapshots of the spatial distribution of the spectator field χ. The plots are order in increasing time from left to right.[5]



Histograms of relative frequency of different values of the spectator field χ. The plots are ordered in increasing time from left to right.[5]

5. Krajewski, T. & Turzyński, K. *JCAP* **10,** 064. arXiv: `2205.13487 [astro-ph.CO]` (2022).

Evolution of spectator field perturbations



Time evolution of the distribution of the amplitude of χ in nodes of the lattice for $\left(\frac{m}{H_c}\right)^2 = 10^2$, and $\left(\frac{M}{M_{Pl}}\right)^2 = 10^{-3}$ (left panel), 10^{-4} (right panel). Field values from the range displayed in the plots are binned and the shade of the bin corresponds to the proportion of nodes at which the field value correspond to a given bin.[5]

5. Krajewski, T. & Turzyński, K. *JCAP* **10,** 064. arXiv: `2205.13487 [astro-ph.CO]` (2022).

Summary

1. Geometrical destabilization may take place in multi-field inflationary models with negative curvature of the field space.
2. Non-canonical kinetic terms can take their origin from UV completion or be included in effective field theory approach.
3. Lattice simulations proofed that short wavelength fluctuations of the inflaton field are produced by non-linear interactions from spectator ones.
4. Kinematical backreaction has been recently confirmed in numerical lattice simulations.
5. Geometrical destabilization during inflation leads to so called 'side tracked' inflation.

Summary

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Thank you for your attention.

Multi-field models of inflation

Geometrical destabilization may take place when action for scalar fields contain non-canonical kinetic terms.
Let us concentrate on non-linear sigma models with action given by:

$$S = \int d^4x\sqrt{-g}\left[M_{Pl}^{-2}R - \frac{1}{2}G_{IJ}\left(\phi^K\right)\partial_\mu\phi^I\partial^\mu\phi^J - V\left(\phi^K\right)\right] .$$

Non-canonical kinetic terms can be introduced directly into an inflationary model (as in the case of supergravity) or can come from quantum corrections in effective theory approach.

$$\mathcal{L}_{\rm eff}\left(\phi^I\right) = \mathcal{L}_\ell\left(\phi^I\right) + \sum_i c_i \frac{\mathcal{O}_i\left(\phi^I, \partial\phi^I, \ldots\right)}{\Lambda^{\delta_i - 4}}$$

Inflationary trajectory

The inflationary trajectory is solution of following set of equations:

$$3H^2 M_{Pl}^2 = \frac{1}{2}\dot{\sigma}^2 + V\,,$$
$$\dot{H} M_P^2 = -\frac{1}{2}\dot{\sigma}^2\,,$$
$$\mathcal{D}_t\dot{\phi}^I + 3H\dot{\phi}^I + G^{IJ}V_{,J} = 0\,.$$

where $\frac{1}{2}\dot{\sigma}^2 \equiv \frac{1}{2}G_{IJ}\dot{\phi}^I\dot{\phi}^J$ is the kinetic energy of the fields, $\mathcal{D}_t A^I \equiv \dot{A}^I + \Gamma^I{}_{JK}\dot{\phi}^J A^K$ is the covariant derivative in the field space and $H := \dot{a}/a$ is the Hubble parameter with a being the scale factor of the FRW metric.

Linear perturbations

The behavior of linear fluctuations around inflationary trajectory is described by the second-order action:

$$S_{(2)} = \int dt\, d^3x\, a^3 \left(G_{IJ}\mathcal{D}_t Q^I \mathcal{D}_t Q^J - \frac{1}{a^2} G_{IJ}\partial_i Q^I \partial^i Q^J - M_{IJ} Q^I Q^J \right),$$

where $Q^I := \delta\phi^I + \frac{\dot{\phi}^I}{H}\Psi$'s are so-called Mukhanov-Sasaki variables and M_{IJ} is a mass matrix:

$$M^I{}_J = V^{,I}{}_{;J} - \frac{1}{a^3 M_{Pl}^2}\mathcal{D}_t\left(\frac{a^3}{H}\dot{\phi}^I\dot{\phi}_J\right) - \mathcal{R}^I{}_{KLJ}\dot{\phi}^K\dot{\phi}^L.$$

Equations of motion read:

$$\mathcal{D}_t\mathcal{D}_t Q^I + 3H\mathcal{D}_t Q^I + \frac{k^2}{a^2}Q^I + M^I{}_J Q^J = 0.$$

Effective mass

We can rewrite EOMs in the adiabatic-entropic basis (e^I_σ, e^I_s) where $e^I_\sigma := \dot{\phi}^I/\dot{\sigma}$ is tangent to inflationary trajectory and e^I_s is orthonormal to e^I_σ.
The EOM for superhorizon modes of the entropic fluctuations simplifies to

$$\ddot{Q}_s + 3H\dot{Q}_s + m^2_{s(\mathrm{eff})} Q_s = 0\,,$$

with the effective entropic mass

$$\frac{m^2_{s(\mathrm{eff})}}{H^2} = \frac{V_{;ss}}{H^2} + 3\eta_\perp^2 + \epsilon\,\mathbb{R} M_{Pl}^2,$$

where $\eta_\perp \equiv -\frac{V_{,s}}{H\dot{\sigma}}$, $\mathbb{R}$ is the field-space Ricci scalar and ϵ is the slow-roll parameter.

Stochastic inflation approach

Classically, if χ is stabilized for a long period prior to destabilization, its vacuum expectation value rolls down to tiny values.
However, as soon as it becomes light, quantum fluctuations source its large-scale component and provide the main contribution to its mean displacement:[6]

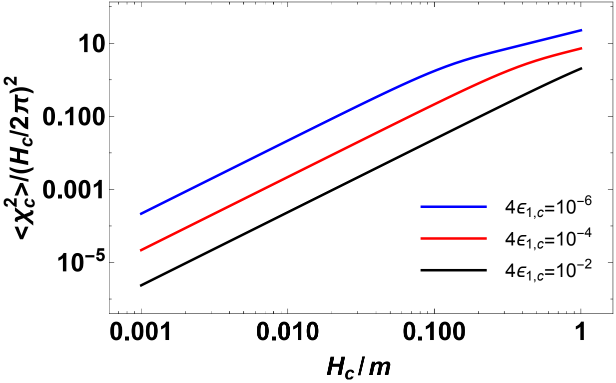
$$\langle \chi_{\rm c}^2 \rangle \simeq \left(\frac{H_{\rm c}}{2\pi}\right)^2 \left\{ \frac{1}{2}\sqrt{3\pi \Delta N_{\ell,\rm c}}\,\mathrm{erf}\left[\sqrt{\Delta N_{\ell,\rm c}/3}\right] - 3\epsilon_{\rm c}\Delta N_{\ell,\rm c}\left[e^{-\Delta N_{\ell,\rm c}/3} - 1\right] \right\},$$

where $\Delta N_{\ell,\rm c}$ is the number of e-folds elapsed in the light but stabilized phase

$$\Delta N_{\ell,\rm c} \equiv N_{\rm c} - N_\ell \simeq \left(\frac{H_{\rm c}}{m}\right)^2 \frac{1}{\eta_{\rm c} - 2\epsilon_{\rm c}}.$$

6. Grocholski, O. *et al.* *JCAP* **1905,** 008. arXiv: `1901.10468 [astro-ph.CO]` (2019).

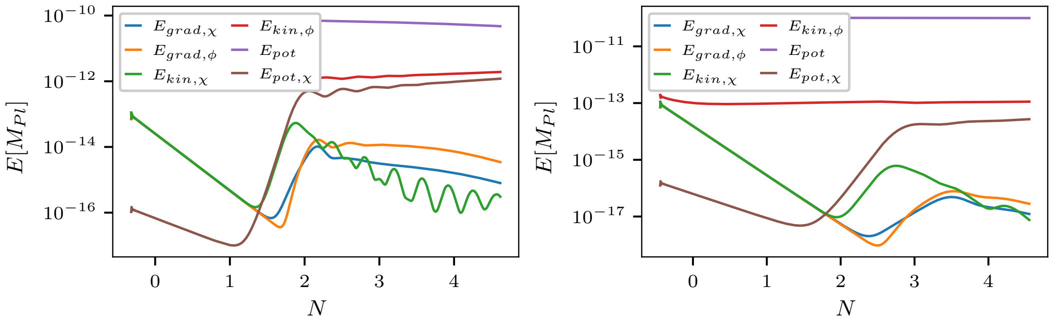
Spectator variance



Spectator variance $\langle\chi_c^2\rangle$ at the beginning of geometrical destabilization.[6]

6. Grocholski, O. *et al. JCAP* **1905,** 008. arXiv: 1901.10468 [astro-ph.CO] (2019).

Energy components



Evolution of various components of the energy density for $\left(\frac{m}{H_c}\right)^2 = 10^2$, and $\left(\frac{M}{M_{Pl}}\right)^2 = 10^{-3}$ (left panel), 10^{-4} (right panel).[5]

5. Krajewski, T. & Turzyński, K. *JCAP* **10,** 064. arXiv: `2205.13487 [astro-ph.CO]` (2022).