University of Southampton

Gravitational wave production during reheating: From the inflaton to primordial black holes

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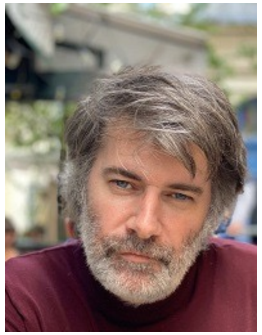
Based on: Phys.Rev.D 111 (2025) 3, 035020

Collaborators:



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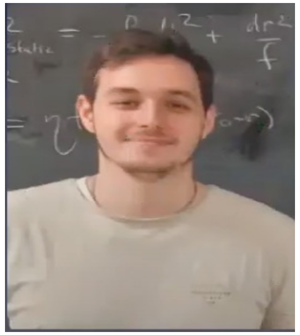

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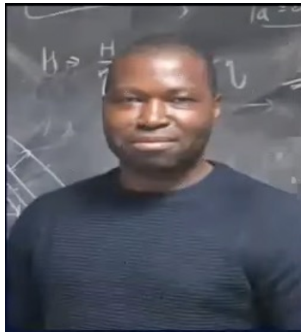


Essodjolo Kpatcha

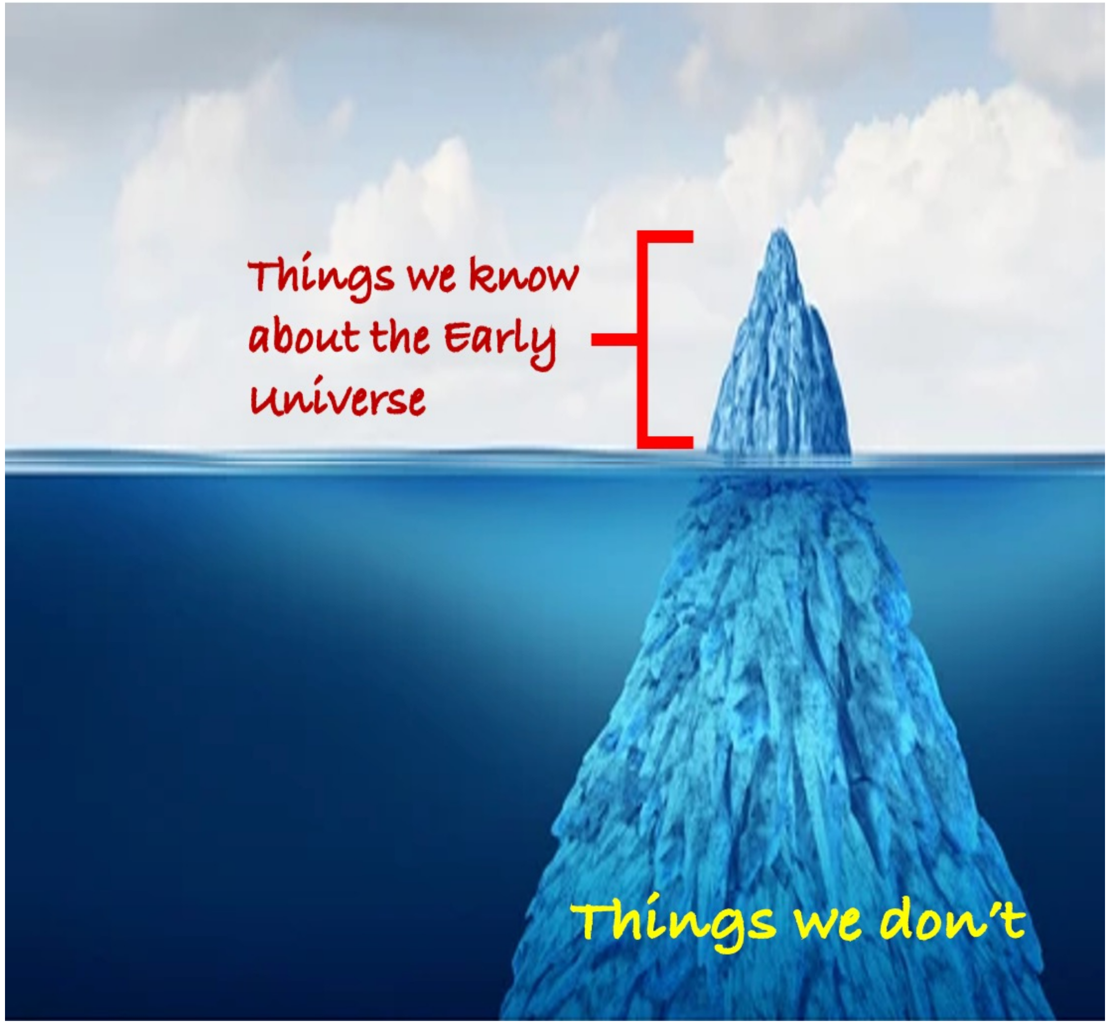
The Early Universe

Cosmological Puzzles

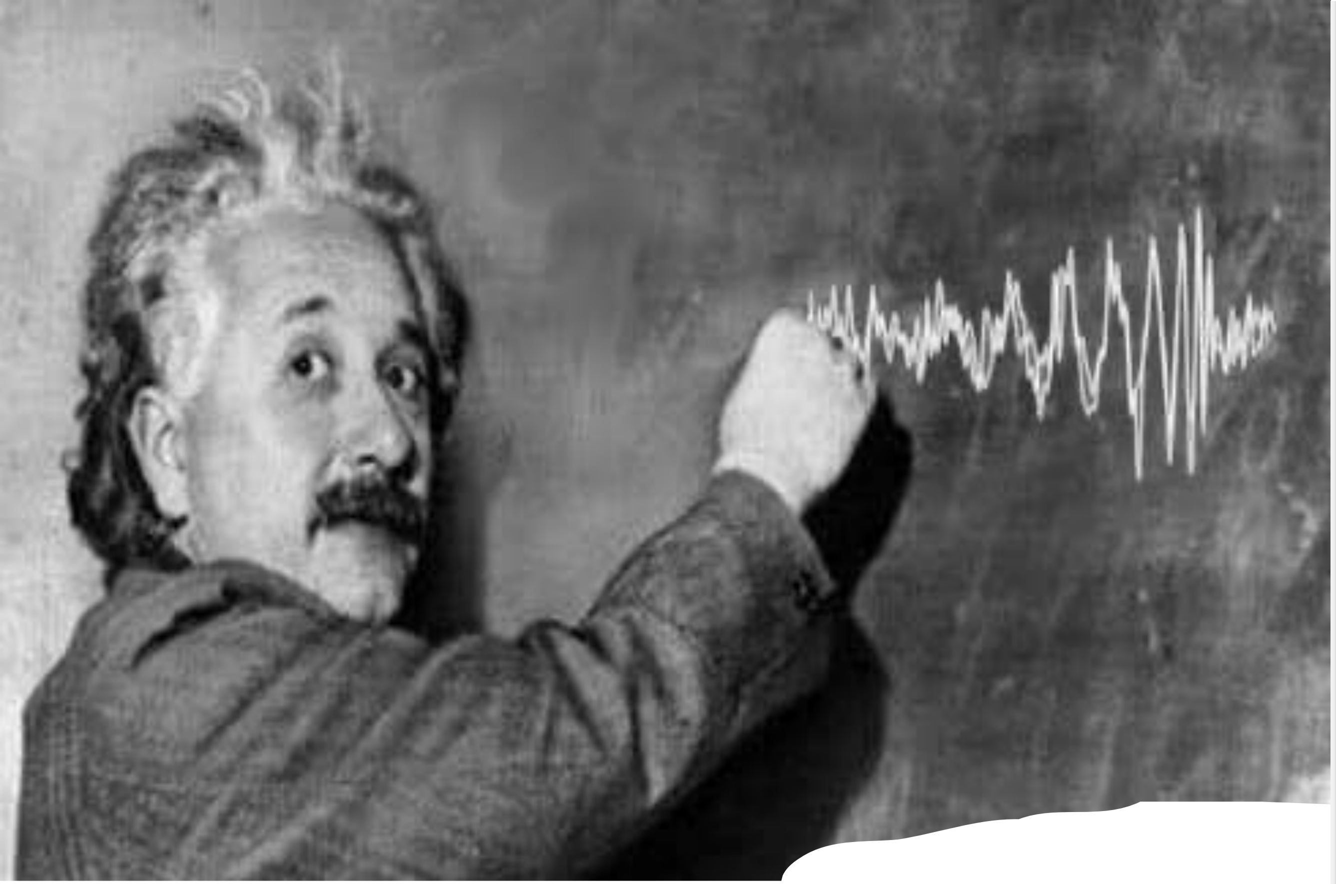
1. Dark Energy
2. Dark Matter
3. Matter-Antimatter asymmetry
4. Origin of large-scale structures

Some of the high scale physics

1. Inflation
2. Scale of Quantum Gravity
3. Primordial Black Holes




THE LIGHT AT THE END
OF THE TUNNEL
GW
IS JUST THE LIGHT OF AN
ONCOMING [redacted]



The Nobel Prize in Physics 2017

© Nobel Media. Ill. N. Elmehed
Rainer Weiss
Prize share: 1/2

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Barry C. Barish
Prize share: 1/4

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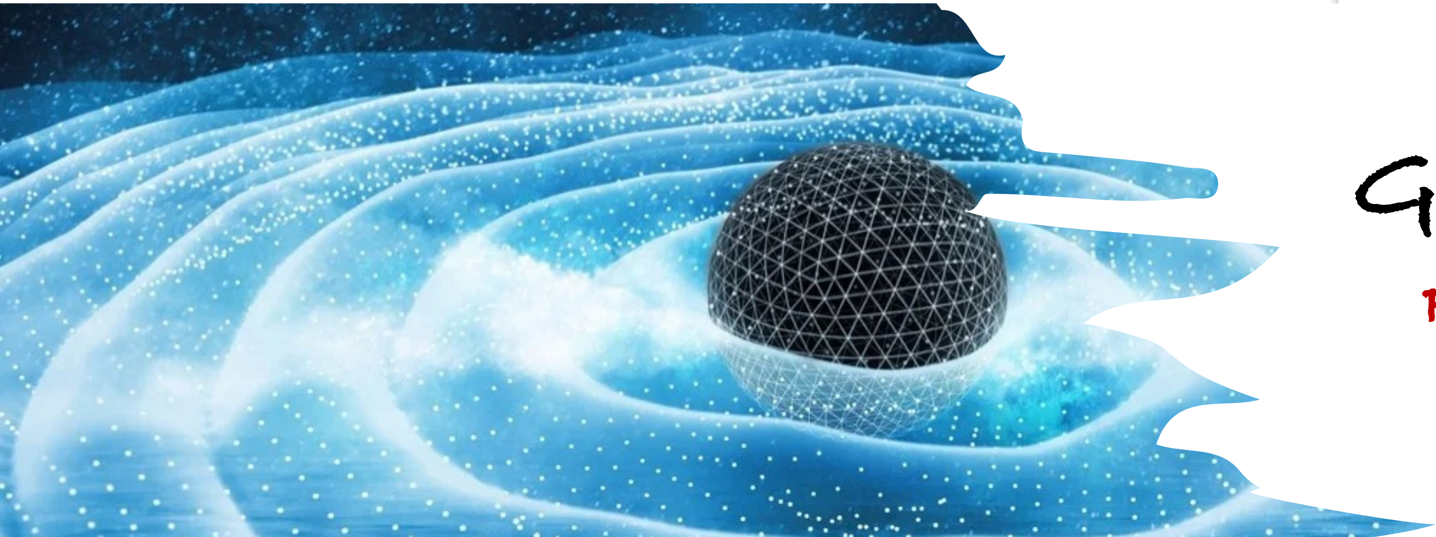
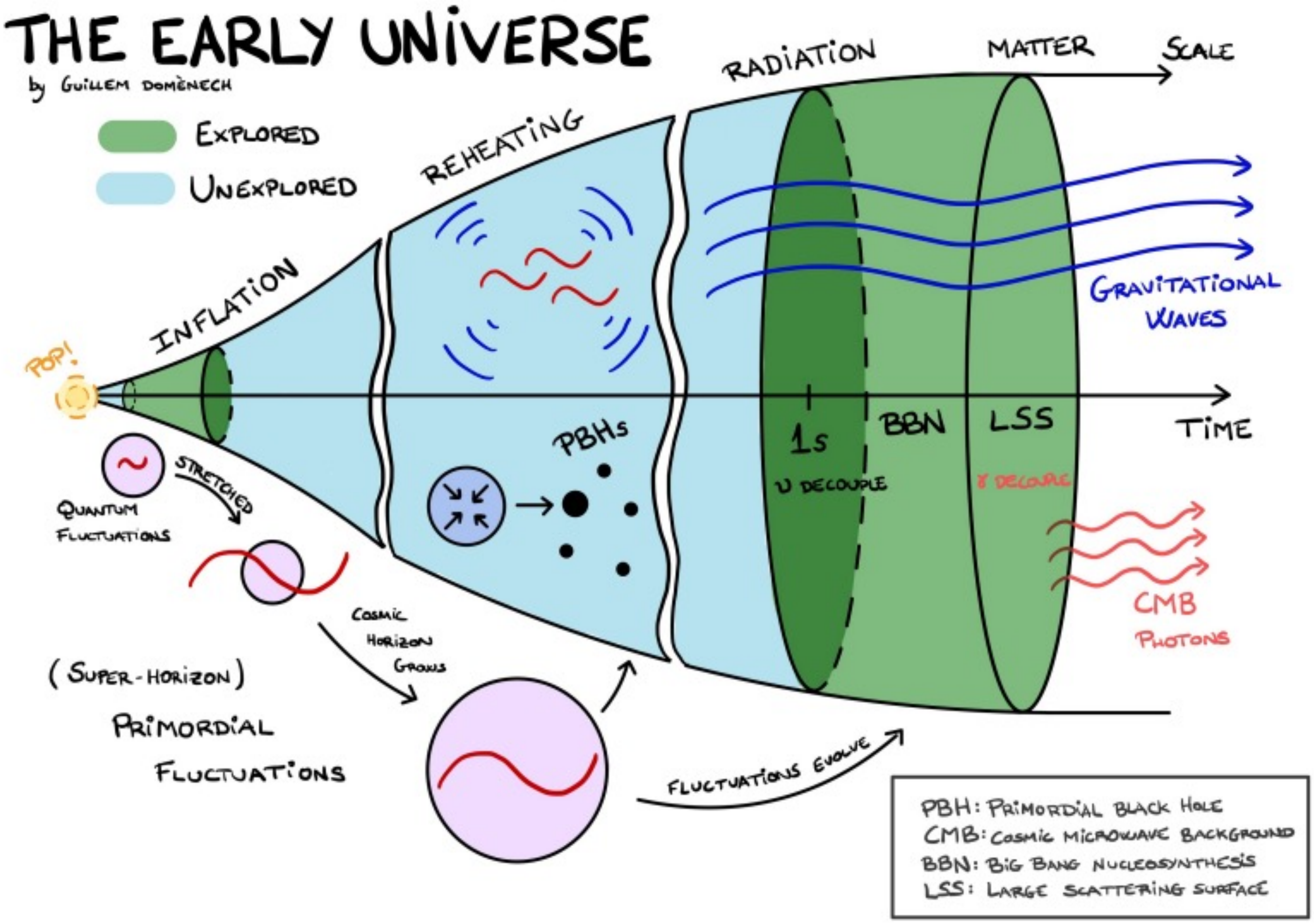
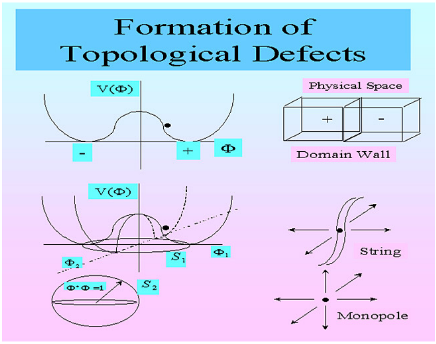


Gravitational Waves:

Ripples in the fabric of spacetime



1st order Phase Transition



Topological defect

Gravitational Waves: Theory

Einstein's Equation:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \mathcal{R} = 8\pi G T_{\mu\nu}$$

→ Space-time determines the trajectories of all object

⇅

Massive object curve space-time

Considering a small perturbation around the metric tensor:

$$g^{\mu\nu} = \eta^{\mu\nu} - \kappa h^{\mu\nu} + \kappa^2 h^{\mu\lambda} h_{\lambda}{}^{\nu} + \cdots$$ where $|h_{\mu\nu}| << 1$ & $\kappa \equiv \sqrt{32\pi G} = 2/M_P$

Small deviation in flat space-time

Propagation of GW in vacuum :

$$(\partial_t^2 - \partial_x^2) h_{\mu\nu} = 16\pi G T_{\mu\nu}$$

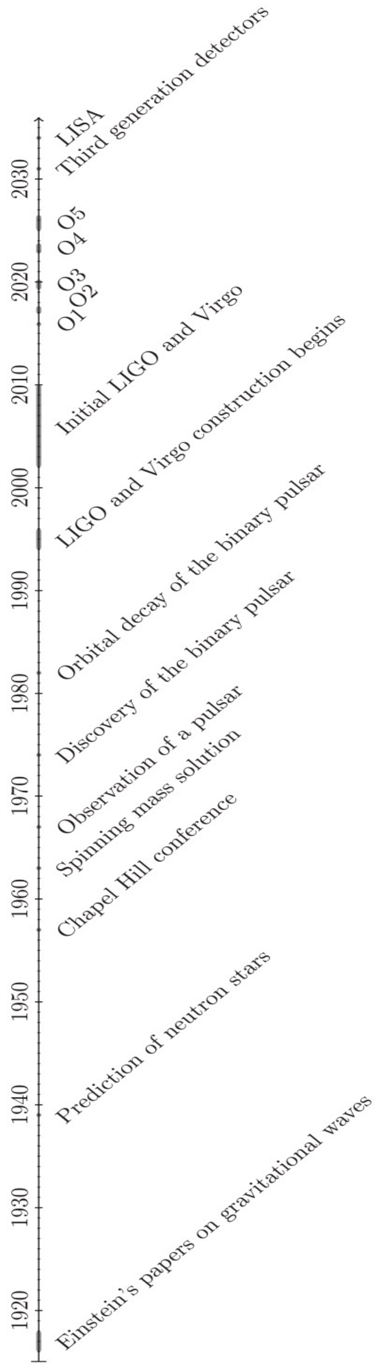
In the far-field regime, the amplitude can be approximated as,

$$h_{ij} \simeq \frac{2G}{r} \ddot{Q}_{ij}(t_{Ret})$$

- → Need a Quadrupole Moment
- → Derivative suggests that the source cannot be static
- → Decreases with the distance

Power Emitted:

$$P_{GW} \simeq \frac{G}{45} \sum_{i,j} \langle \dddot{Q}_{ij} \dddot{Q}_{ij} \rangle$$

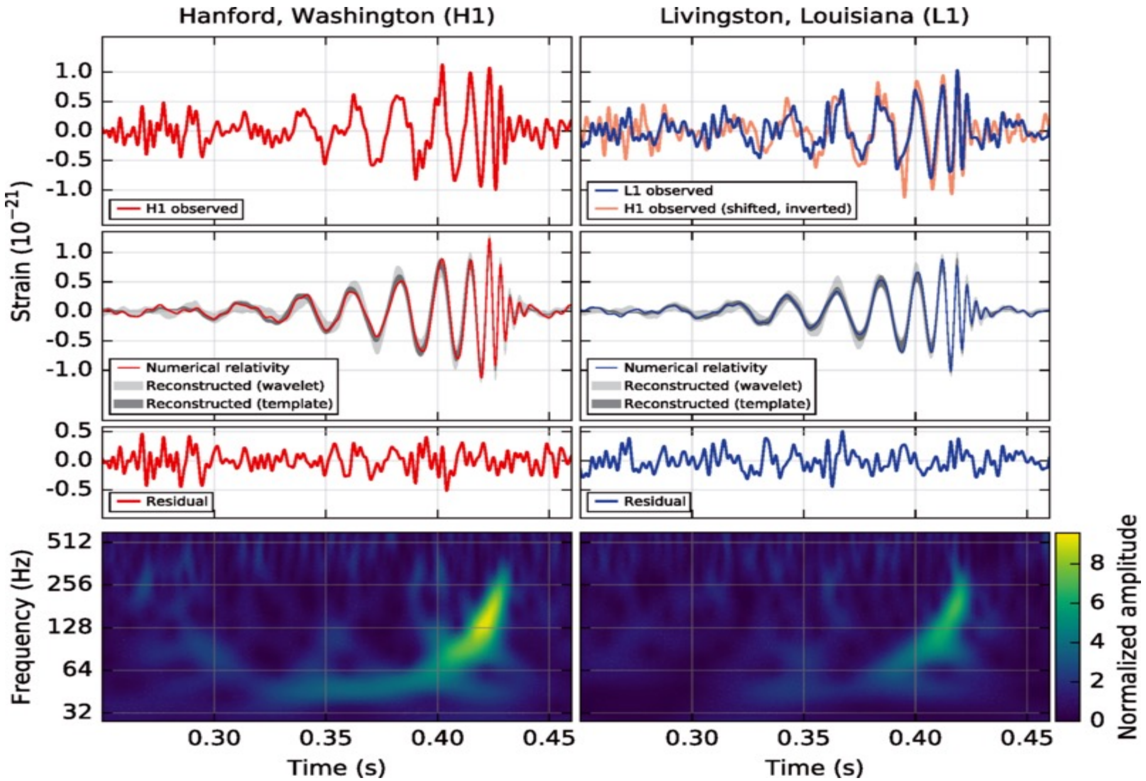


Timeline of significant events in the history of gravitational waves

RevModPhys.94.025001

Recent Discoveries

Discovery of GW by LIGO-VIRGO Col.



PRL 116, 061102 (2016)

Source of GW: Merging of pair of BHs at $z = 0.09$

Recent results reported by PTA projects

The New York Times

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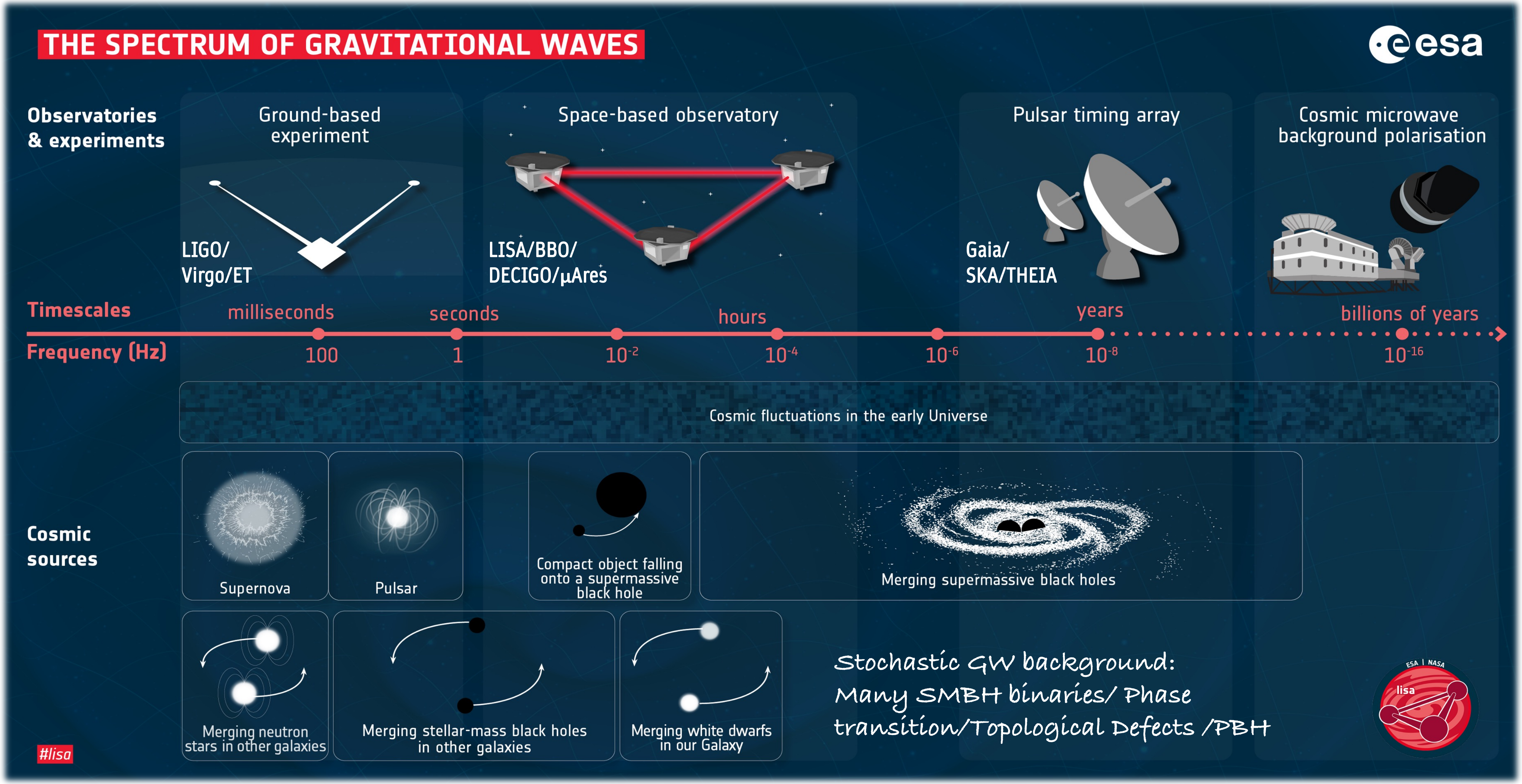
The Cosmos Is Thrumming With Gravitational Waves, Astronomers Find

Several PTA projects have reported positive evidence of a stochastic gravitational wave background.

NANOGrav, 2306.16219

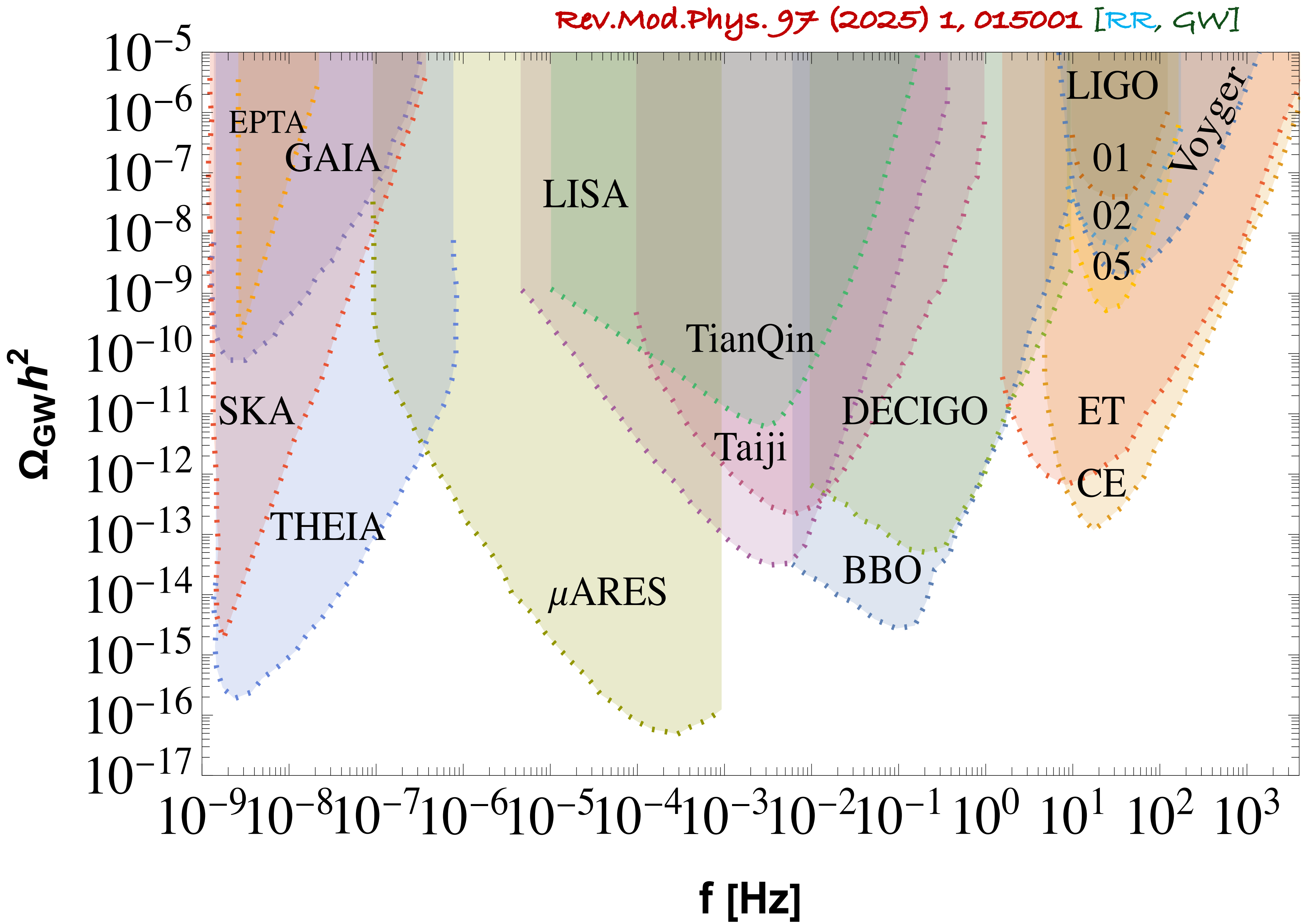
Source of SGWB: Merging of SMBH binaries/ Cosmological origin/combination of Both.

GW Detections



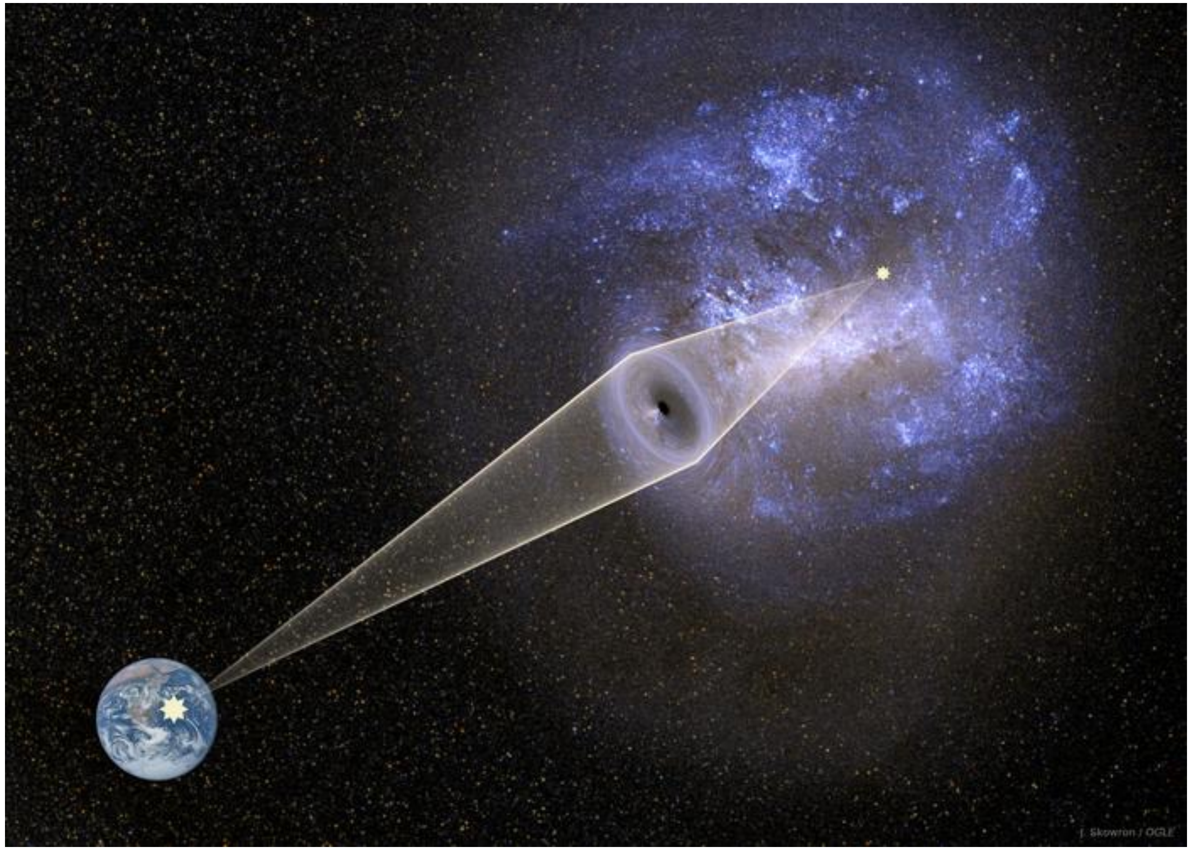
Stochastic GW background: Many SMBH binaries/ Phase transition/Topological Defects /PBH

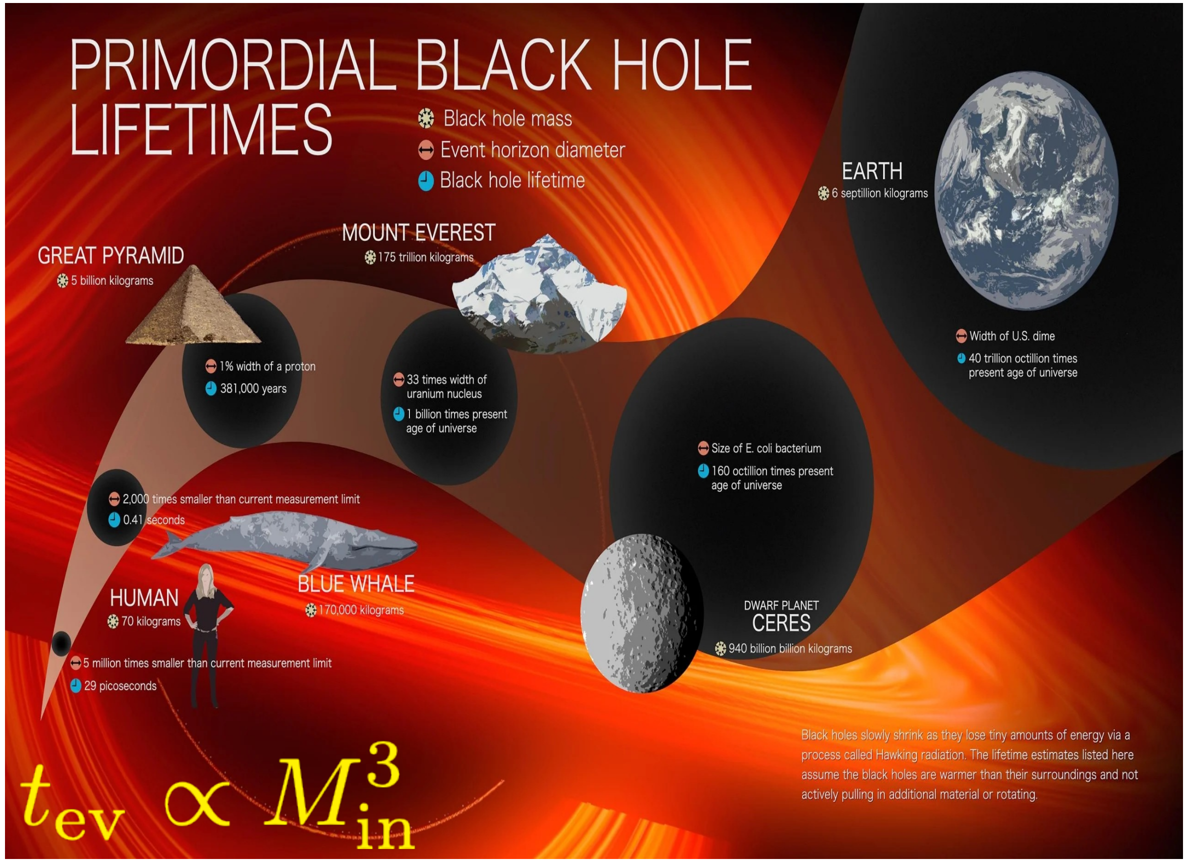
Credit to ESA



Rev.Mod.Phys. 97 (2025) 1, 015001 [RR, GW]
EPTA
GAIA
LISA
TianQin
SKA
Taiji
DECIGO
THEIA
BBO
μARES
LIGO
01
02
05
Voyger
ET
CE
$\Omega_{GW}h^2$
f [Hz]

Primordial Black Holes





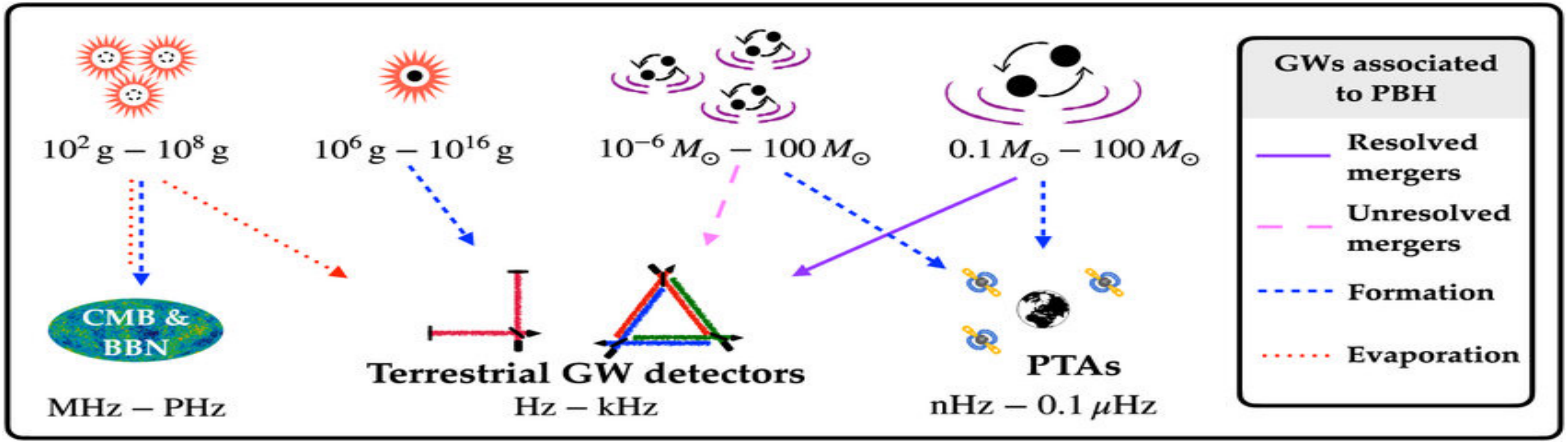
PBH → Stable (DM)

PBH → Unstable → DM

Any observational effects of such PBHs?

GW from PBH

Fig. taken from Class.Quant.Grav. 41 (2024) 14, 143001



Sources

1. PBH formation
2. PBH merger
3. Direct PBH evaporation
4. Inhomogeneity in PBH distribution

PBH in the presence of the inflaton

Reheating after inflation (A recap!)

Before BBN, the universe could be dominated by a field with general EOS: $P = w\rho$

Inflaton potential:

$$V(\phi) = \lambda M_P^4 \left(\frac{\phi}{M_P}\right)^k$$

Inflaton mass:

$$m_{\phi_{\rm end}} = \sqrt{k(k-1)\lambda}\left(\frac{\phi_{\rm end}}{M_p}\right)^{\frac{k-2}{2}} M_p$$

Equation of state, $P_\phi = w_\phi \rho_\phi$, with

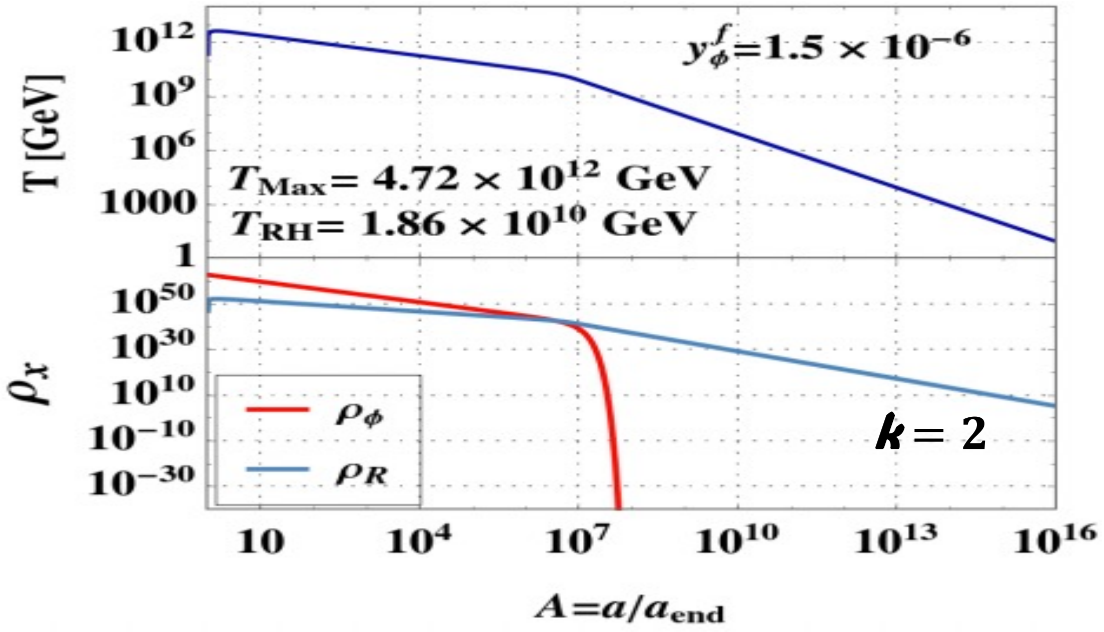
$$w_\phi = \frac{k-2}{k+2} \begin{cases} 0 \text{ if } k = 2 \\ 1/3 \text{ if } k = 4 \\ 1/2 \text{ if } k = 6 \\ 1 \text{ if } k \gg 1 \end{cases}$$

Boltzmann Equation:

$$\dot\rho_\phi + 3H(1+w_\phi)\rho_\phi = -(1+w_\phi)\Gamma_\phi \rho_\phi$$

$$\dot\rho_{\rm R} + 4H\rho_{\rm R} = (1+w_\phi)\Gamma_\phi \rho_\phi$$

$$\rho_\phi = \rho_{\rm end}\left(\frac{a_{\rm end}}{a}\right)^{3(1+w_\phi)}$$



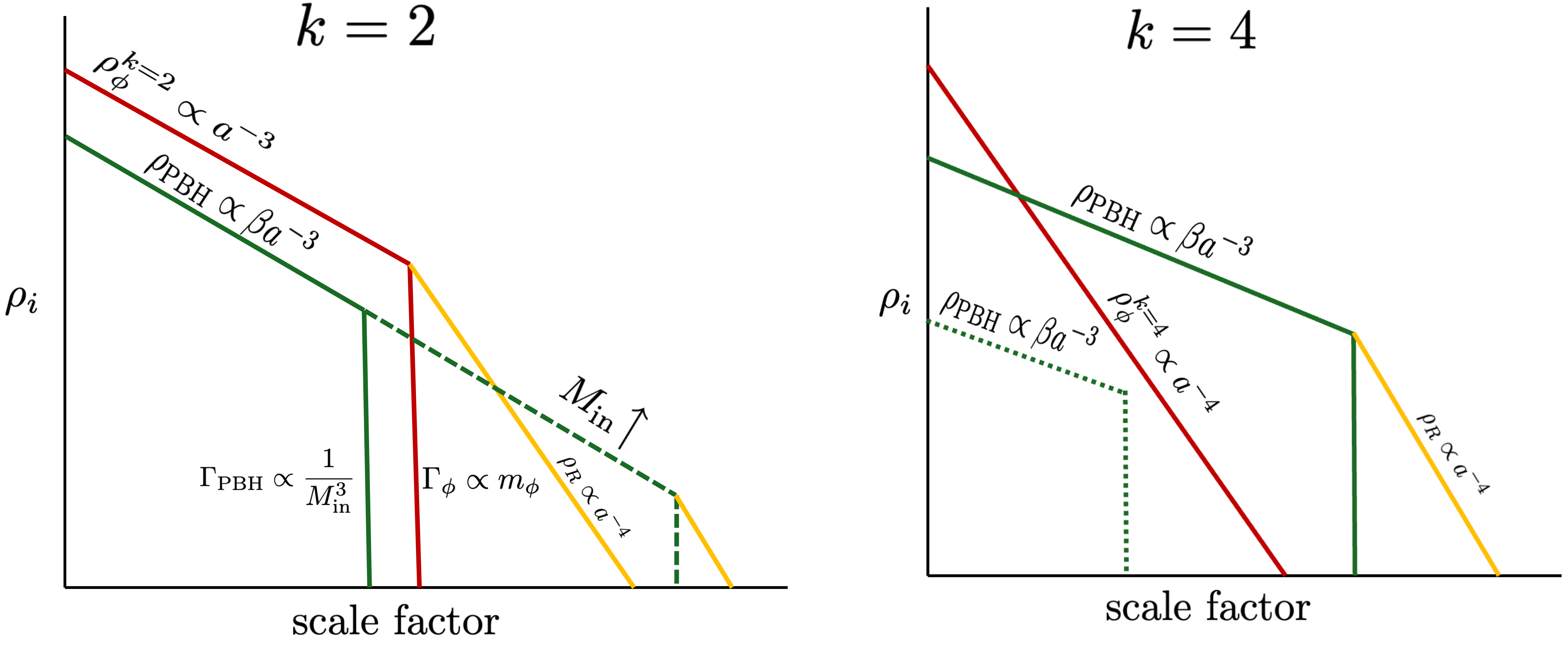
Phys.Rev.Lett. 132 (2024) 6, 061802 [AD, RR, AS]

PBH in the presence of inflaton condensate

Phys.Rev.D 111 (2025) 3, 035020 (MG,YM,EK,MOOR,RR)

$$\beta = \frac{\rho_{\rm PBH}}{\rho_\phi}$$

$$w_\phi = \frac{k-2}{k+2} \begin{cases} 0 \text{ if } k=2 \\ 1/3 \text{ if } k=4 \\ 1/2 \text{ if } k=6 \\ 1 \text{ if } k \gg 1 \end{cases}$$



GW from direct PBH Evaporation

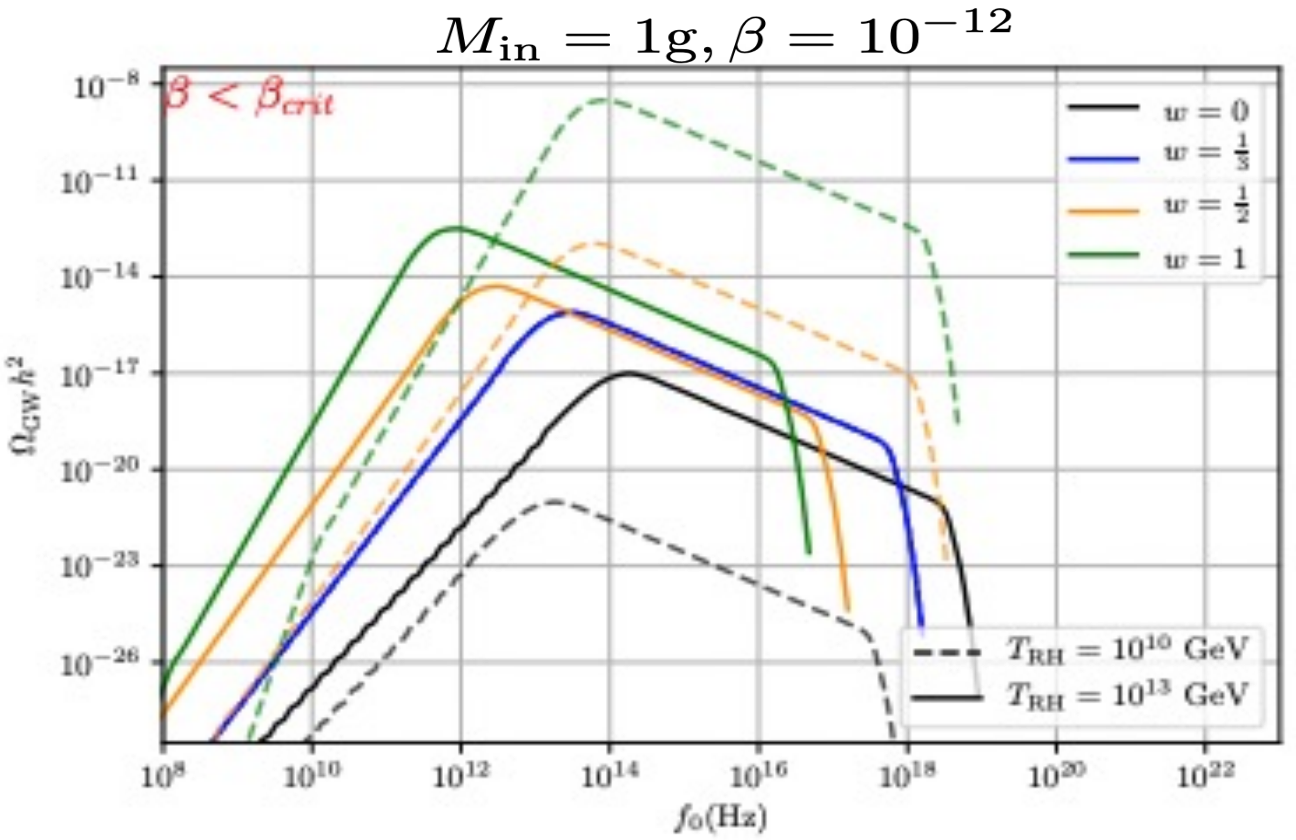
Phys.Rev.D 111 (2025) 3, 035020 [MG,YM,EK,MOOR,RR]

(i) GW from PBH evaporation ($\beta < \beta_c$)

$$w_\phi = \frac{k-2}{k+2}\begin{cases} 0 \text{ if } k=2 \\ 1/3 \text{ if } k=4 \\ 1/2 \text{ if } k=6 \\ 1 \text{ if } k \gg 1 \end{cases}$$

$$k_0^{\rm peak} \simeq 2.8\,\frac{M_P^2}{M_{\rm in}}\frac{a_{\rm ev}}{a_0} \simeq 2.8 T_0\,\frac{g_0^{\frac{1}{3}}}{g_{RH}^{\frac{1}{3}}}\left(\frac{M_{\rm in}}{M_P}\right)^{\frac{1-w}{1+w}} \times \left(\frac{M_P}{T_{\rm RH}}\right)^{\frac{3w-1}{3w+3}}\left(\frac{320\sqrt{3}(1+w)\sqrt{\alpha}}{27 g_*(T_{\rm BH})\pi}\right)^{\frac{2}{3+3w}},$$

$$\Omega_{\rm GW}^{\rm BH}(f_0^{\rm peak}) \propto \beta\,\frac{M_{\rm in}^{\frac{2-2w}{1+w}}}{T_{\rm RH}^{\frac{12w-4}{3w+3}}}$$

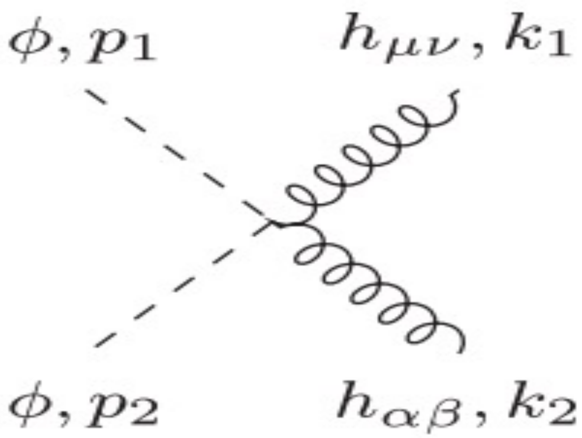


The **true end state** of PBH evaporation is **unknown**

GWs from inflaton scattering

$$\dot{\rho}^{\phi}_{\rm GW} + 4H\rho^{\phi}_{\rm GW} = (1+w_\phi)\Gamma^{\rm GW}_{\phi}\rho_\phi \quad \text{where} \quad (1+w_\phi)\Gamma_h\rho_\phi = \frac{\rho_\phi^2\omega}{4\pi M_P^4}\Sigma^k,$$

$$\text{and} \quad \Sigma^k = \sum_{n=1}^{\infty} n|(\mathcal{P}^k)_n|^2.$$



$$\Omega^{\phi}_{\rm GW} \sim T_{\rm RH}^{\frac{3k-12}{k(k-4)}} f_0^{\frac{4k-7}{k-4}}, \qquad f_0(a) = \frac{\omega(a)}{2\pi}\frac{a}{a_0} = \frac{\gamma_k}{2\pi}M_P\left(\frac{\rho_{\rm end}}{M_P^4}\right)^{\frac{k-2}{2k}}\frac{a_{\rm end}^{\frac{3k-6}{k+2}}}{a_0}a^{\frac{8-2k}{k+2}}.$$

$$f_0^{\rm peak} = f_0^{\rm min} \text{ for } k=2, \text{ and } f_0^{\rm max} \text{ otherwise.}$$

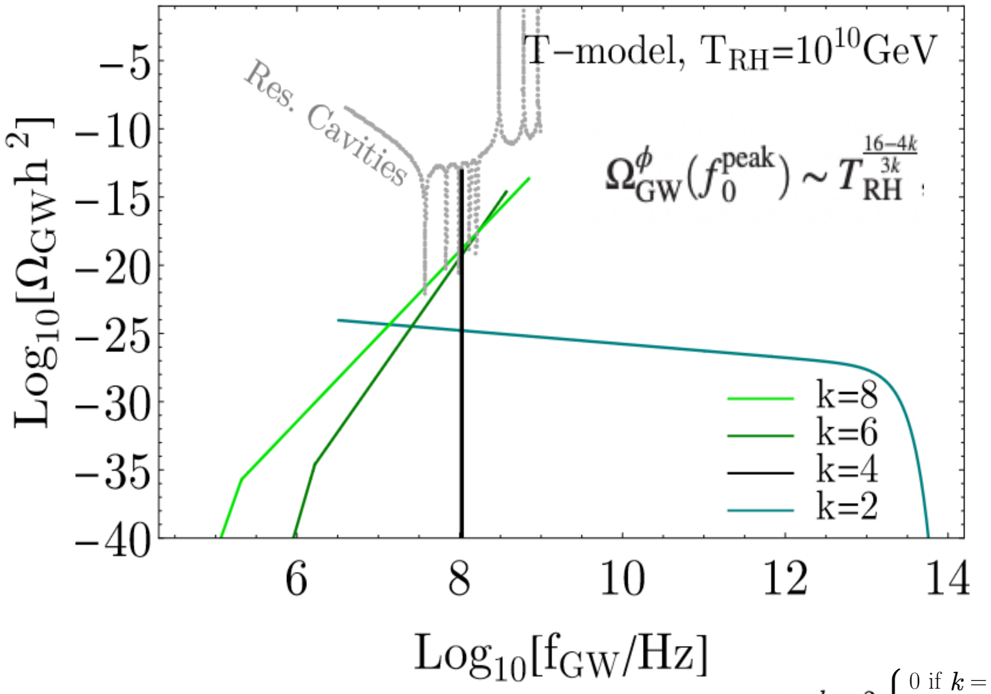
$$f_0^{\rm min}|_{k=2} = \frac{\omega_{\rm end}}{2\pi}\frac{a_{\rm end}}{a_{\rm RH}}\frac{a_{\rm RH}}{a_0} \simeq 4.9\times10^6\left(\frac{T_{\rm RH}}{10^{10}\,\text{GeV}}\right)^{\frac{1}{3}}\,\text{Hz}$$

$$f_0^{\rm max}|_{k=2} = m_\phi\frac{a_{\rm RH}}{a_0} \simeq 5.6\times10^{13}\frac{10^{10}}{T_{\rm RH}}\,\text{Hz}.$$

k=4: $$nf_0 \sim n\times0.27\lambda^{\frac14}\frac{\rho_{\rm end}^{\frac14}}{M_P}\frac{a_{\rm end}}{a_0} \simeq 3n\times10^8\,\text{Hz}.$$

$$f_0^{\rm max}|_{k>4} = \frac{\omega_{\rm end}}{2\pi}\frac{a_{\rm end}}{a_{\rm RH}}\frac{a_{\rm RH}}{a_0} = \alpha^{\frac{k+2}{6k}}\frac{g_0^{\frac13}}{g_{\rm RH}^{\frac13}}\frac{T_0}{\rho_{\rm end}^{\frac{k+2}{6k}}}T_{\rm RH}^{\frac{4-k}{3k}}$$

Phys.Rev.D 109 (2024) 8, 083516

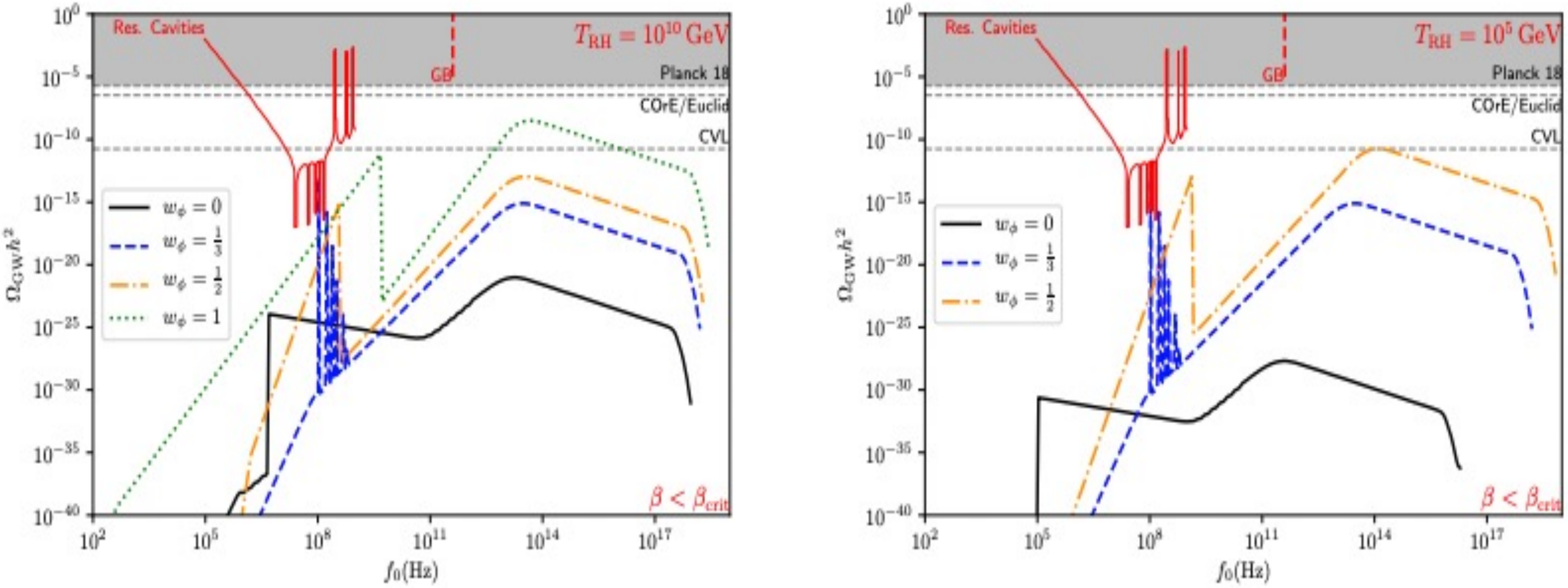


$$w_\phi = \frac{k-2}{k+2}\begin{cases}0 \text{ if } k=2\\ 1/3 \text{ if } k=4\\ 1/2 \text{ if } k=6\\ 1 \text{ if } k\gg1\end{cases}$$

GWs: PBH existing in non-standard cosmology

Phys.Rev.D 111 (2025) 3, 035020 [MG,YM,EK,MOOR,RR]

(ii) GW from PBH evaporation + inflaton scattering



$$M_{\rm in} = 1{\rm g}, \beta = 10^{-12}$$

$$w_\phi = \frac{k-2}{k+2} \begin{cases} 0 \text{ if } k = 2 \\ 1/3 \text{ if } k = 4 \\ 1/2 \text{ if } k = 6 \\ 1 \text{ if } k \gg 1 \end{cases}$$

PBH domination
$(\beta > \beta_c)$

$$w_\phi = \frac{k-2}{k+2} \begin{cases} 0 \text{ if } k=2 \\ 1/3 \text{ if } k=4 \\ 1/2 \text{ if } k=6 \\ 1 \text{ if } k \gg 1 \end{cases}$$

Our focus:

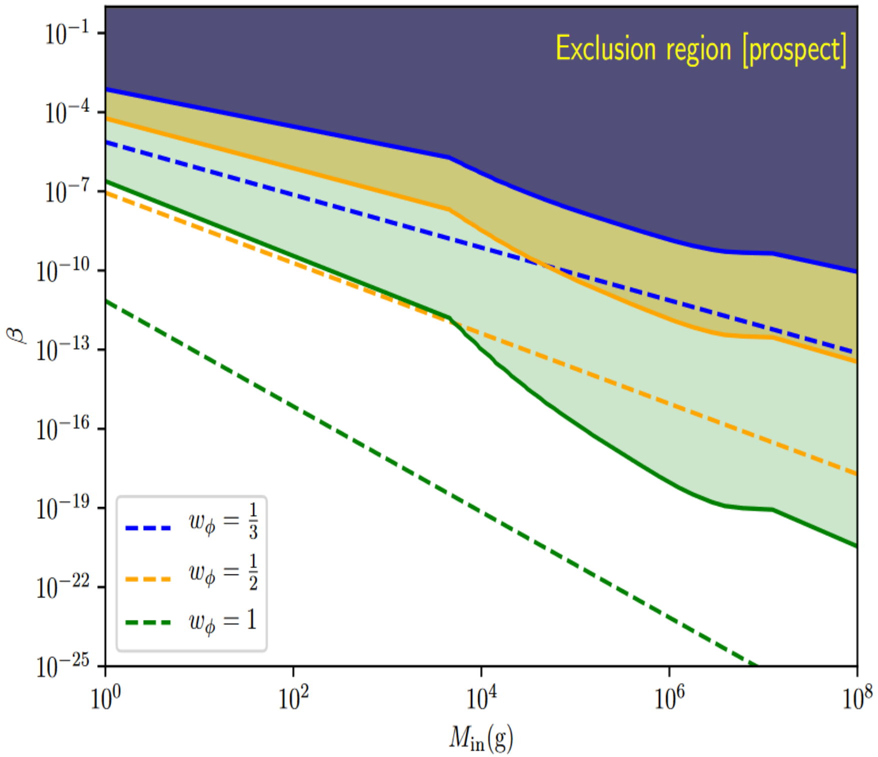
Scenarios where PBH domination happens due to the rapid redshifting of the inflaton energy density

Note that:

$$\rho_{\rm PBH} \propto a^{-3}$$

Only possible when: $\rho_\phi \propto a^{-3(1+w_\phi)}$ with $w_\phi > 0$

Just a reminder!



Other possibility:

PBH domination due to inflaton decay before PBH evaporation

(We do not consider this possibility)

GWs during PBH domination

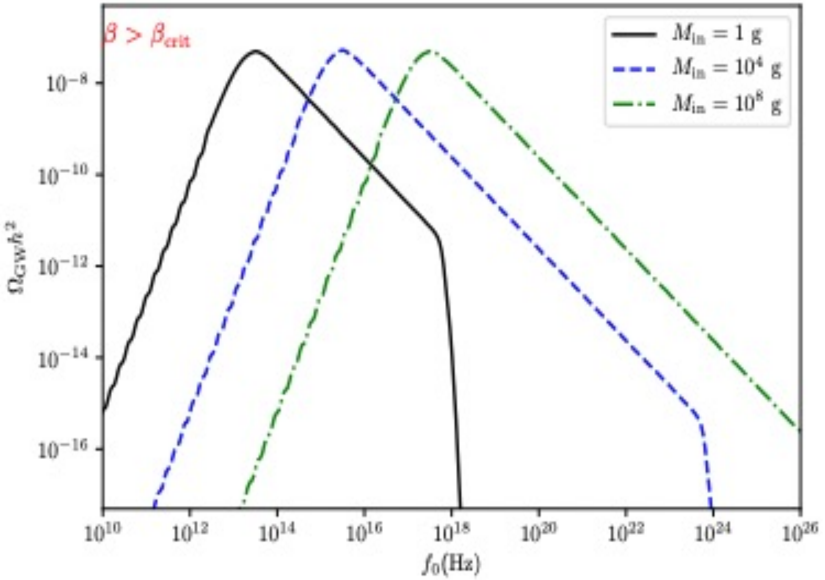
$$w_\phi = \frac{k-2}{k+2}\begin{cases} 0 \text{ if } k=2 \\ 1/3 \text{ if } k=4 \\ 1/2 \text{ if } k=6 \\ 1 \text{ if } k \gg 1 \end{cases}$$

GW from direct PBH evaporation

$$k_0^{\text{peak}} \simeq 2.8 \times T_{\text{BH}} \frac{a_{\text{RH}}}{a_0} \Rightarrow f_0^{\text{peak}} \simeq 3 \times 10^{13} \sqrt{\frac{M_{\text{in}}}{1\,\text{g}}}\,\text{Hz}$$

$$\left.\frac{d\Omega_{\text{GW}}^{\text{BH}}}{d\ln k_0}\right|_{\beta>\beta_c}^{\text{peak}} = \frac{27}{64\pi^3}\frac{\alpha}{\epsilon}\left(\frac{g_0}{g_{\text{RH}}}\right)^{\frac{4}{3}}\frac{T_0^4}{3M_P^2H_0^2} \times I_0^{\beta_c} \simeq 8.5 \times 10^{-8}$$ where $$I_0^{\beta_c} = \int_{\frac{k_0 M_{\text{in}}}{M_P^2}\frac{a_0}{a_{\text{ev}}}}^{\frac{k_0 M_{\text{in}}}{M_P^2}\frac{a_0}{a_{\text{BH}}}} \frac{\sqrt{Y}}{e^Y - 1}\,dY$$

(spectrum is independent of $w_\phi, \beta, M_{\text{in}}$ at the peak)

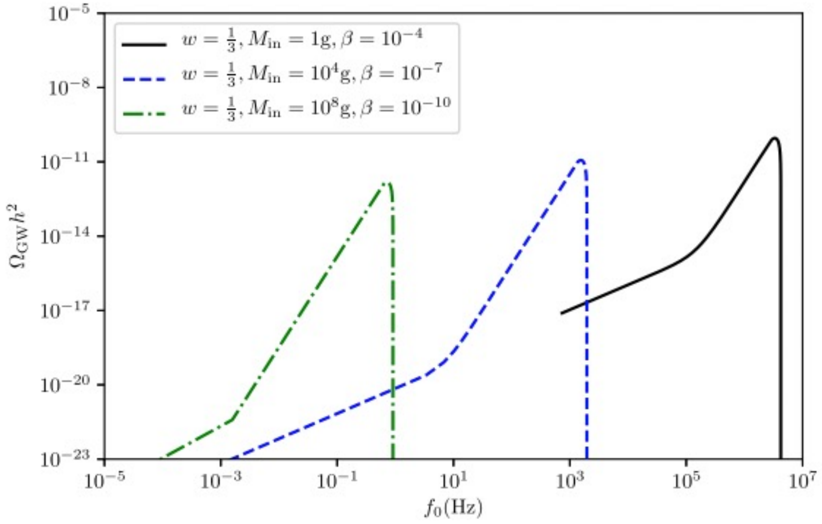


GW from inhomogeneous PBH distribution

$$f_0^{\text{peak}} = f_{\text{in}} \times \left(\frac{a_{\text{in}}}{a_{\text{ev}}}\right)\left(\frac{a_{\text{ev}}}{a_0}\right) \simeq 2.5 \times 10^3 \left(\frac{10^4}{M_{\text{in}}}\right)^{\frac{5}{6}}\text{Hz}$$ where $$f_{\text{in}} = \frac{d_{\text{in}}^{-1}}{2\pi}$$ and $$d_{\text{in}} = \left(\frac{\gamma}{\beta}\right)^{\frac{1}{3}} H_{\text{in}}^{-1}$$

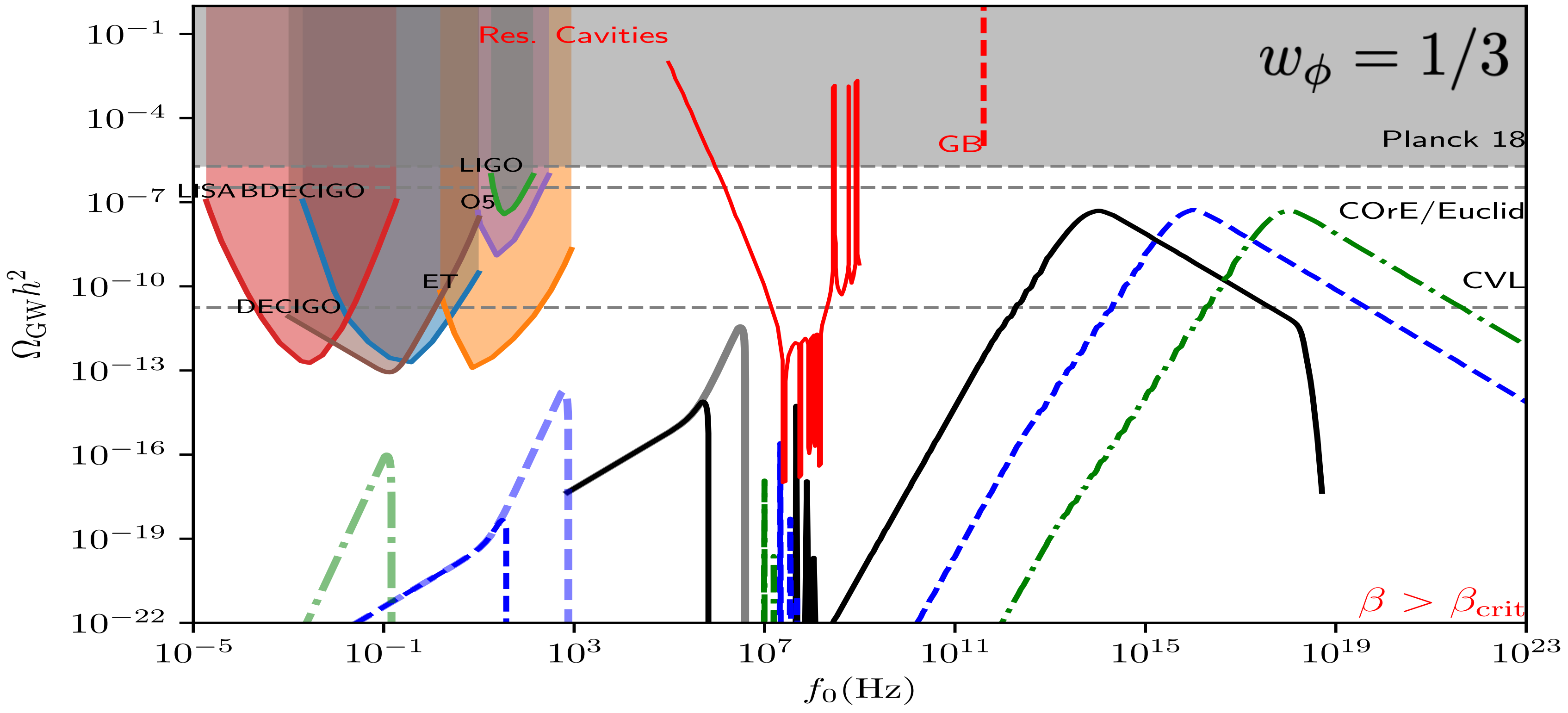
$$\Omega_{\text{GW}} \simeq \Omega_{\text{GW}}^{\text{peak}}\left(\frac{f_0}{f_0^{\text{peak}}}\right)^{11/3}\Theta(f_0^{\text{peak}} - f_0)$$

$$\Omega_{\text{GW}}^{\text{peak}} \approx 9.58 \times 10^{30} C^4(w)\beta^{\frac{4(1+w)}{3w}}\left(\frac{g_H}{108}\right)^{-17/9} \times \left(\frac{\gamma}{0.2}\right)^{8/3}\left(\frac{M_{\text{in}}}{10^4\ \text{g}}\right)^{34/9},$$



A complete GW spectra

(iii) GW from PBH evaporation + inflaton scattering + inhomogeneous PBH distribution



$M_{\rm in} = 1$ g (black), 10^4 g (blue) and 10^8 g (green) for $\beta = 10^{-4}$, 10^{-7} and 10^{-10}

$$w_\phi = \frac{k-2}{k+2} \begin{cases} 0 \text{ if } k = 2 \\ 1/3 \text{ if } k = 4 \\ 1/2 \text{ if } k = 6 \\ 1 \text{ if } k \gg 1 \end{cases}$$

Summary

1. We calculated the GW spectrum produced by the PBHs in the presence of the inflaton condensate

2. While considering a general EOS, we computed the GW spectra from:

   (a) Direct PBH evaporation
   (b) inflaton scattering
   (c) density fluctuations due to the inhomogeneous distribution of PBHs

3. For the first time, we present a complete, coherent analysis of the spectrum, revealing three peaks, one for each source.

4. A large range of frequency was covered: kHz to PHz

Final Remark

Gravitational wave cosmology offers an auspicious path to uncovering the physics of the early universe

Future Directions

1. Bridge the gap between the communities working on PBH formation and evaporation.

2. Both the formation and evaporation of PBH are effective sources of Gravitational Waves

Thank you!