Truly confining supersymmetric gauge theory

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Outline

- Motivation

  Confinement can be understood as dual Meissner effect.

  We want to understand more deeply about confinement!

- We studied toy model using technology of susy.

- we obtained some non-perturbative results.

Today's contents

- What is SUSY?
- What is QCD and its phase?
- SUSY QCD with non-trivial center symmetry

What is SUSY?

Bosonic symmetry

- Poincare symmetry + **fermionic** symmetry

$$Q|boson\rangle = |fermion\rangle$$

$$Q|fermion\rangle = |boson\rangle$$

- Theory has **same number** of boson and fermion state

squark, slepton, gaugino, Higgsino …

What is advantages of SUSY?

- Holomorphy of superpotential: W

$$\mathcal{L} \supset \int d^2\theta \, W_{eff}(\Phi, \Lambda, g)$$

$$\frac{\partial W_{eff}}{\partial \bar{\Phi}} = \frac{\partial W_{eff}}{\partial \bar{\Lambda}} = \frac{\partial W_{eff}}{\partial \bar{g}} = 0$$

Ex) non-renormalization theorem

Ex) ADS potential

$$W_{eff} = W_{tree} + W_{NP}$$

ADS potential

$$W_{NP} \propto \left(\frac{\Lambda^{3N}}{detM}\right)^{\frac{1}{N-F}}$$

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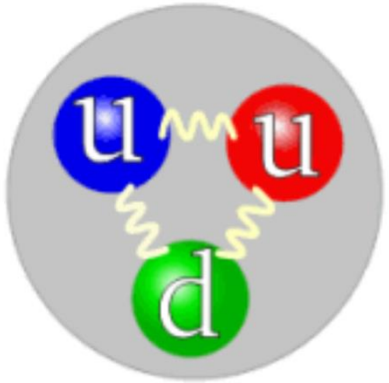
What is QCD?

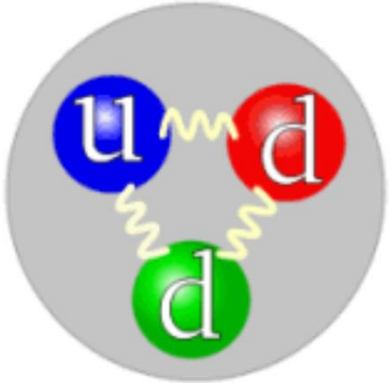
- Yang-Mills theory with matters

$$\mathcal{L}_{\mathrm{QCD}} = \sum_{\psi} \left(i\bar{\psi}^j \gamma^\mu (\mathcal{D}_\mu \psi)_j - m_\psi \bar{\psi}^j \psi_j \right) - \frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu}$$

- Describing hadron physics

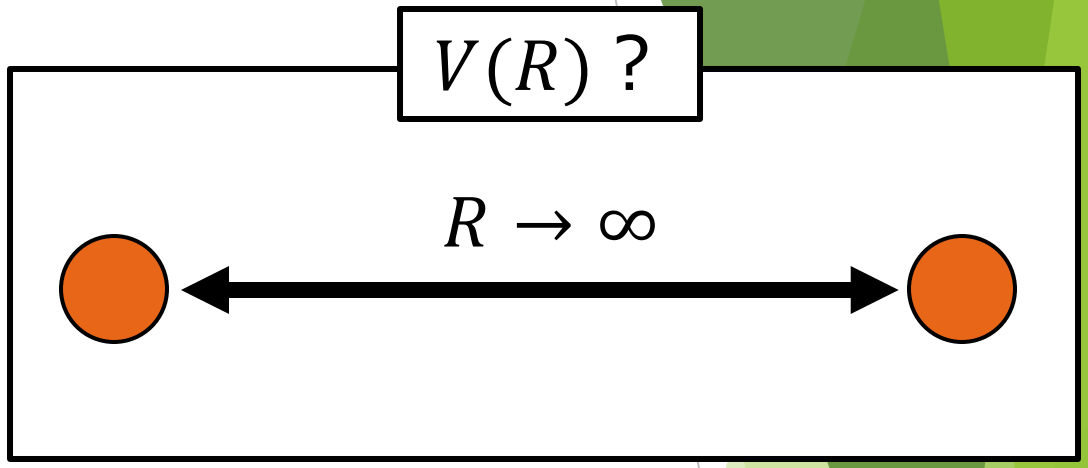


proton



neutron

Phase of QCD



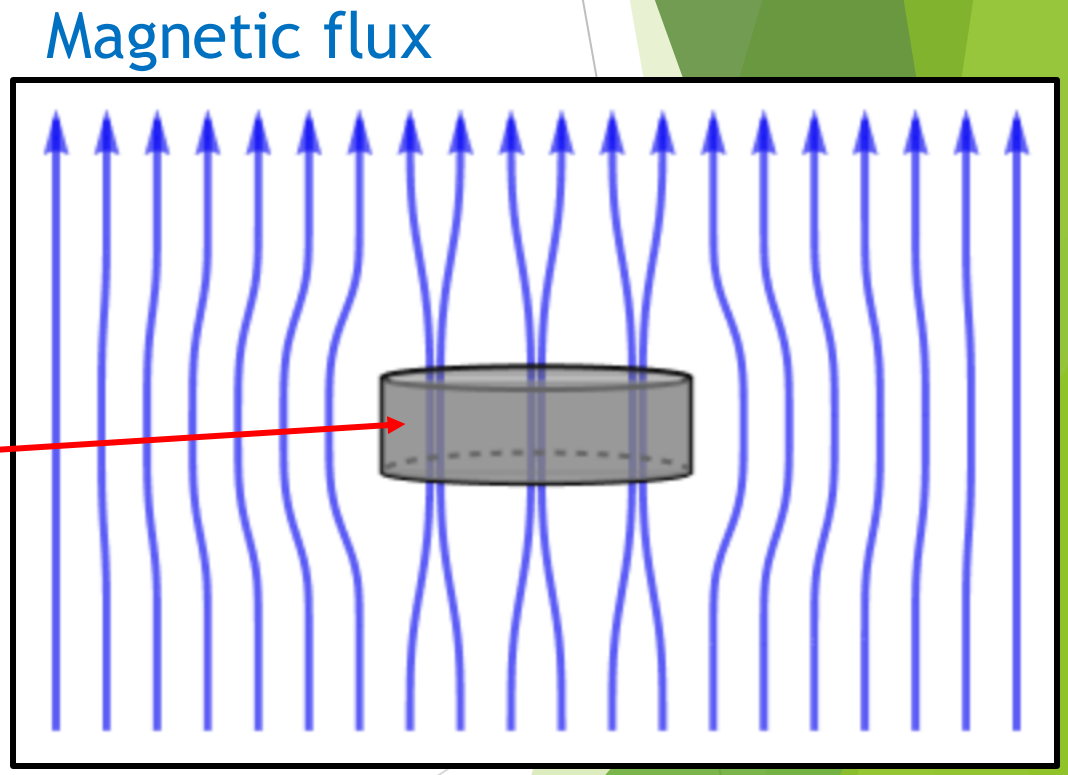
- Coulomb phase: $V(R) \sim 1/R$
- Free electric phase: $V(R) \sim 1/R\log(R)$
- Free magnetic phase: $V(R) \sim \log(R)/R$
- Higgs phase: $V(R) \sim$ const.
- Confinement phase: $V(R) \sim R$

} Condensation of charge

Meissner effect

- Condensate **electric** charge
- Screening of electromagnetic field
- **Magnetic** flux is allowed

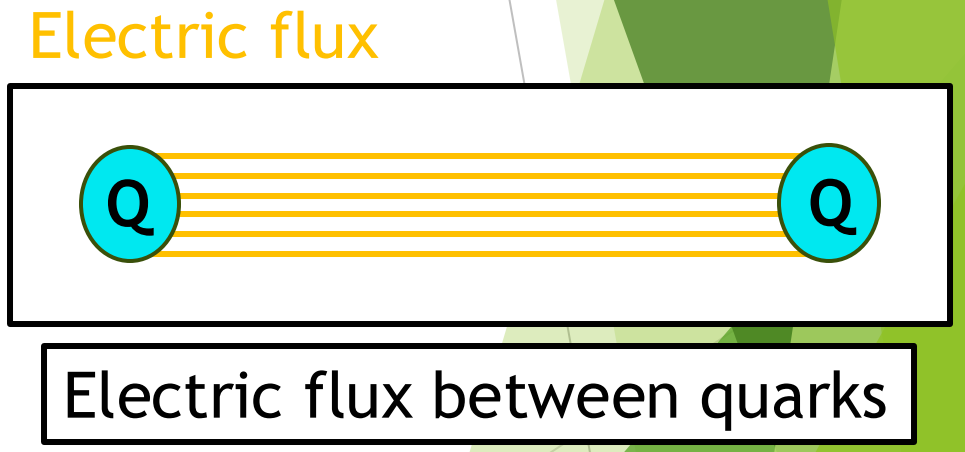
What about electromagnetic dual?



Dual Meissner effect

- Condensate **magnetic** charge
- Screening of electromagnetic field
- **Electric** flux is allowed

Explain confinement!



Electromagnetic duality

Coulomb phase ⟷ Coulomb phase

Free electric phase ⟷ Free magnetic phase

Confinement phase ⟷ **Higgs** phase

Dual Meissner effect

Superconductor (Meissner effect)

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What is center?

- Some elements in a group

  which **commute with all elements** in the group

Ex) consider $SU(N)$ Lie group

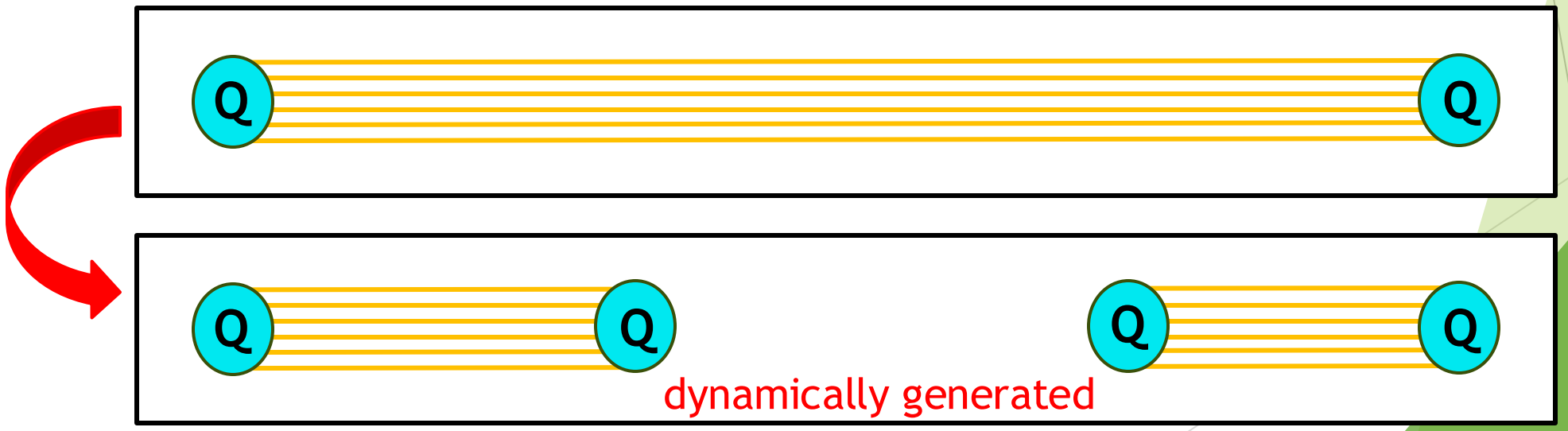
Center is $\mathbb{Z}_N$ (group of N-th root of one)

$$\omega_N g - g\omega_N = 0, \qquad g \in SU(N)$$

Screening

- Electric flux can be screened

when there exist **appropriate dynamical charge**.



Wilson loop

- Wilson loop sit on rep. R

$$W_R = Tr_R(Pe^{\int A}) \longrightarrow \underline{\textbf{Weight lattice}}\ \Lambda_w$$

- Gluon is on adjoint rep.

then, screen the loops on adjoint rep. $\longrightarrow$ **Root lattice** Λ_r

$V(R)\sim R$

- **unscreened loop (confinement phase)** $\longrightarrow \Lambda_w/\Lambda_r$ = center

Non-trivial center symmetry

- MODEL : SU(2k) + an **antisymmetric** quark

$$\int d^4\theta \{\mathrm{tr}(A(e^V)^\top A^\dagger e^V) + \mathrm{tr}(\tilde{A}e^{-V}\tilde{A}^\dagger(e^{-V})^\top)\}$$

This theory confines with **unscreened** Wilson loop.

Non-trivial center symmetry

- We obtain ADS superpotential

$$W_{ADS} = \sum_i \varepsilon_i \left(\frac{\Lambda^{4k+2}}{\prod_{j \neq i} \left(a_i \tilde{a}_i - a_j \tilde{a}_j \right)^2} \right)^{\frac{1}{2}}$$

- We finished some consistency check.

  symmetry/ holomorphy/ Higgs mechanism/
  discrete anomaly matching conditions/ …

- We showed **condensation of magnetic object**.

Other models

- We listed all confining theories with unscreened loops.

1. pure Yang-Mills
2. SU(6)+an rank 3 antisymmetric tensor
3. Sp(2k)+an antisymmetric tensor
4. SO(k)+(k-3) vectors
5. SO(k)+(k-4) vectors
6. Spin(12)+two spinors

- **All theories have condensation of magnetic objects**.

Today's summary

- What is **SUSY** and **advantage of SUSY**?
- What is **QCD**, **phases** and **electromagnetic duality**?
- What is **center**, **Wilson loop** and **screening**?
- We showed that **confining theories with unscreened loops** have **condensing magnetic objects**.