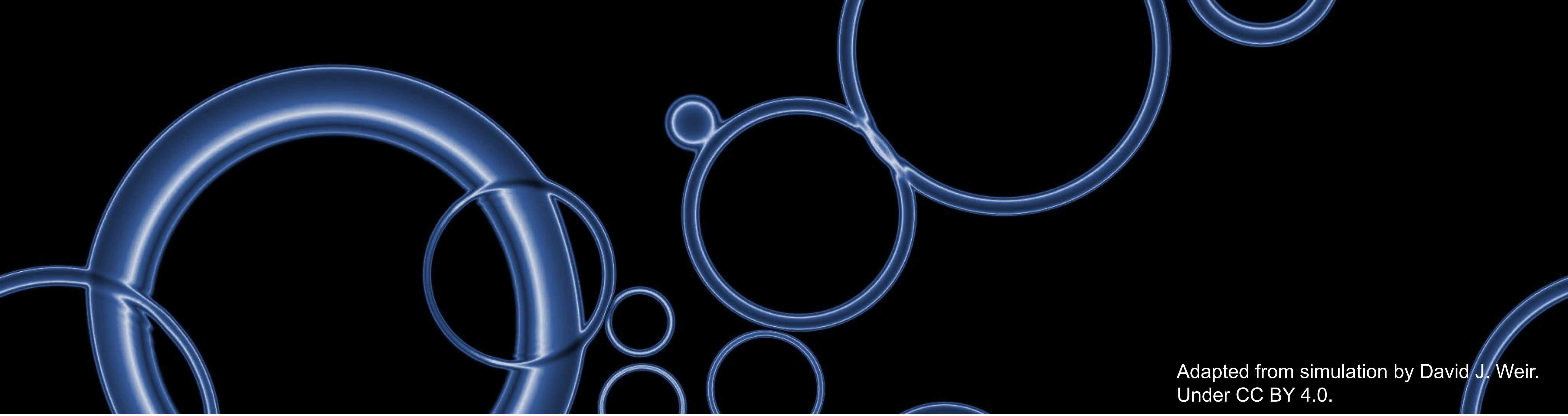


Higher-order-operator corrections to phase transitions in dimensionally reduced EFTs

PASCOS 2025 · IPPP, Durham University | July 21st 2025

Luis Gil (Universidad de Granada)

Based on:

M. Chala, LG and Z. Ren [2505.14335]

M. Chala, J. C. Criado, LG , J. López Miras [2406.02667]

High Energy Theory

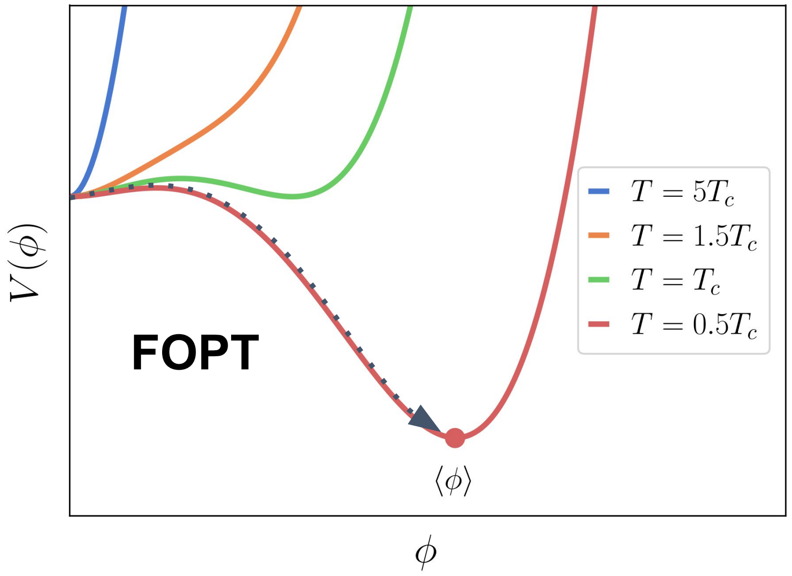
Junta de Andalucía

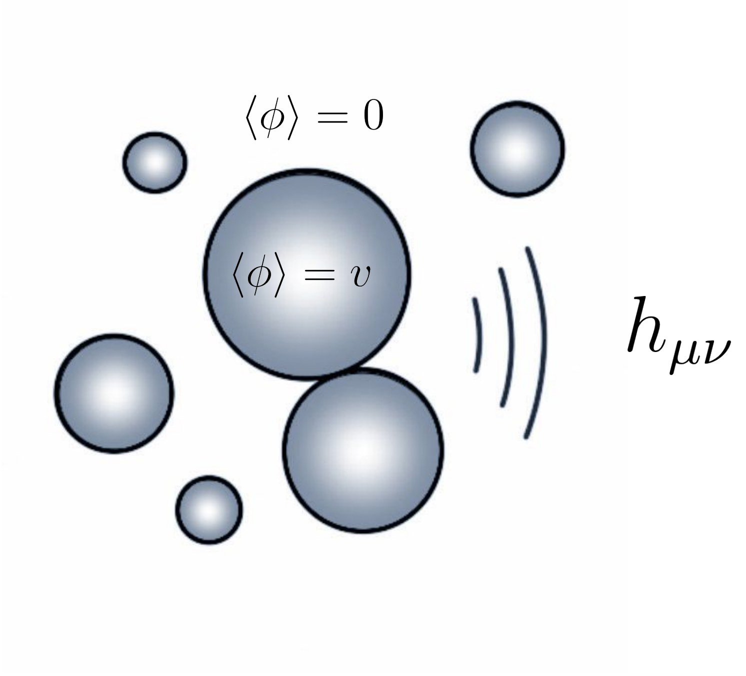
GOBIERNO DE ESPAÑA

MINISTERIO DE CIENCIA, INNOVACIÓN Y UNIVERSIDADES

Motivation

From bubbles to gravitational waves

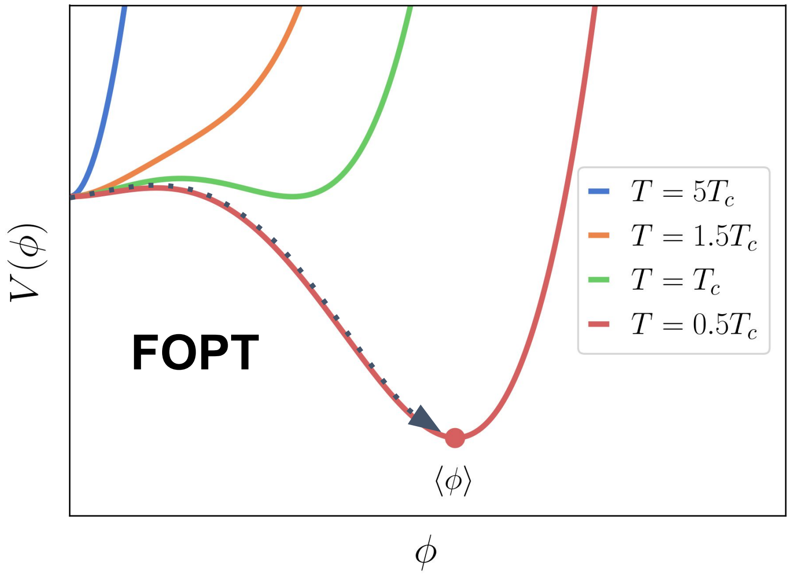


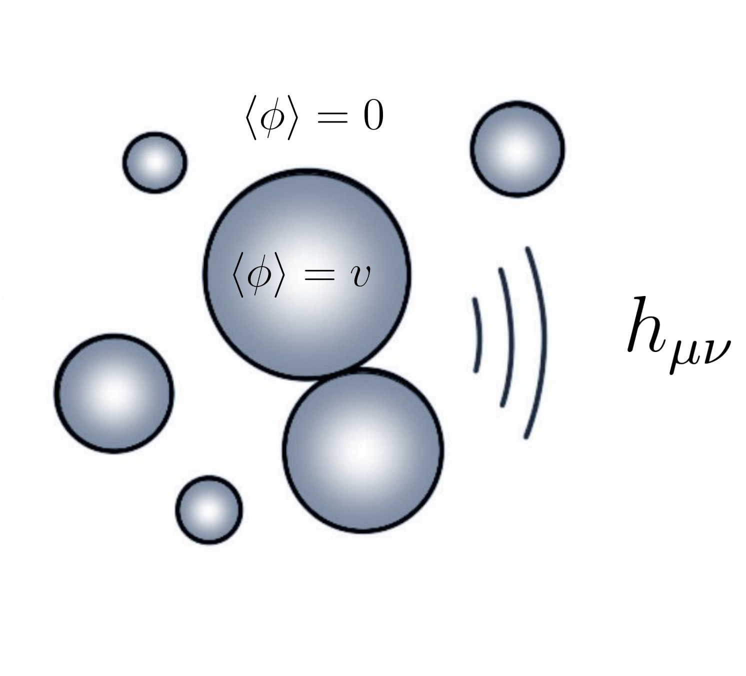


Motivation

From bubbles to gravitational waves

See talk by Maria Cristina on Thursday!





Need to precisely compute phase transition (PT) parameters: $T_*, \alpha, \beta/H_*$

then…

Must determine effective potential at finite temperature beyond leading order

[Croon et al. - 2009.10080]

Thermal field theory

Outline of the Matsubara formalism

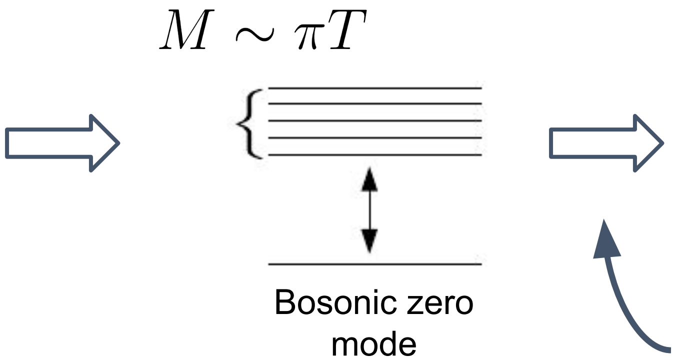
- **Generating functional** in thermal field theory (= Euclidean QFT with periodic time):

$$\mathcal{Z}_{\rm th} = \mathrm{Tr}\left(e^{-\beta\mathcal{H}}\right) = \sum_q \langle q\ 0|e^{-\beta\mathcal{H}}|q\ 0\rangle = \mathcal{N}\int_{q(0)=q(-i\beta)} \mathcal{D}q\,\exp\left(-S_E\right)$$

- Fields decompose in tower of 3D **Matsubara modes** (~ Kaluza-Klein) with thermal masses:

Bosons: $\omega_n = 2\pi n T$

Fermions: $\omega'_n = 2\pi\left(n+\frac{1}{2}\right)T$



3D EFT for bosonic zero modes

Expansion in **m/T**

What about effective operators?

[Chala, Criado, LG, López Miras - 2406.02667]

Previous work: real scalar + fermion

In dimensional reduction, **3D effective operators** have often been neglected because:

- Require **higher-order matching** (not implemented in existing tools)
- Harder to include in PT computations (perturbative or lattice)

However, they should be relevant in **strong transitions** $\frac{1}{\sqrt{\lambda}}\frac{m}{T} \gtrsim 1$

The only we might be able to observe!
[Caprini et al. - 1910.13125]

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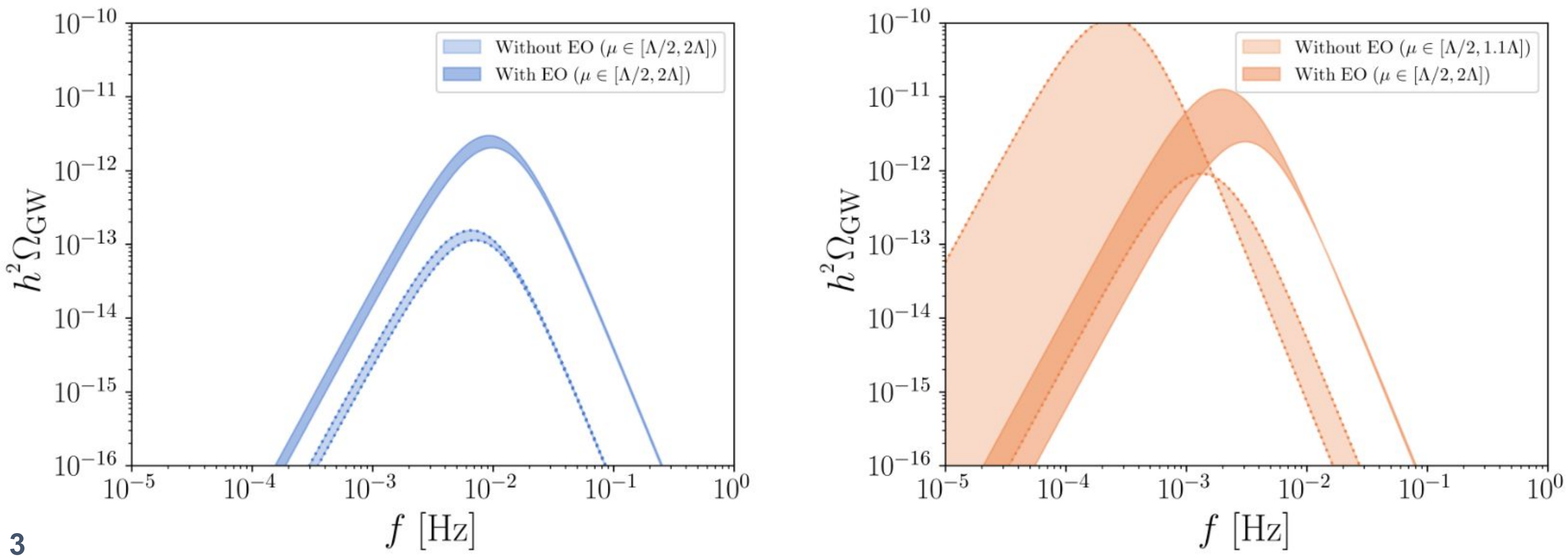

Conclusions
• dim-6 ops. can drastically change PTs • EFT expansion still valid

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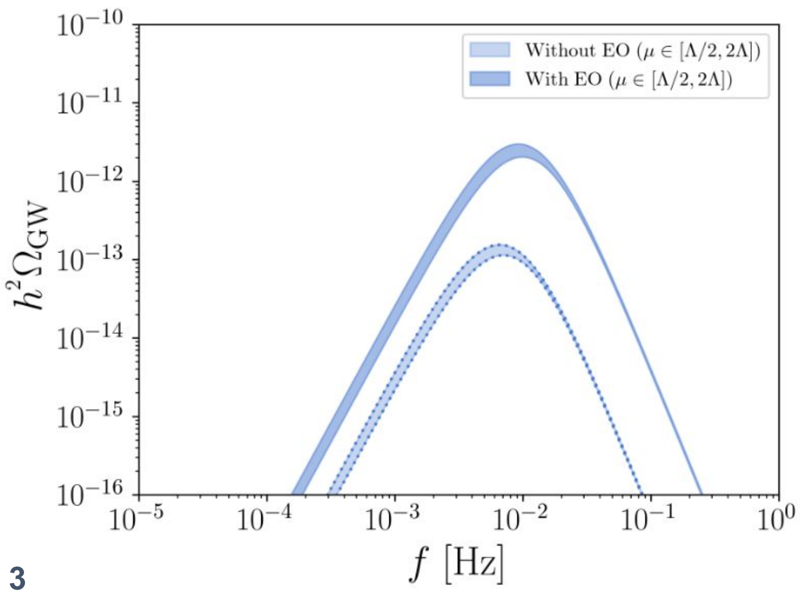
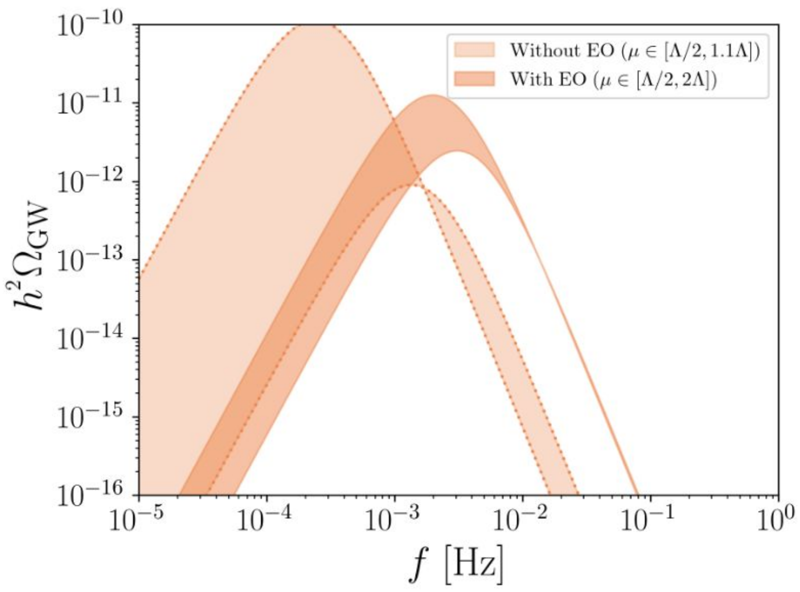
Similar findings in Abelian Higgs model:

[Bernardo, Klose, Schicho, Tenkanen - 2503.18904]





Conclusions

- dim-6 ops. can drastically change PTs
- EFT expansion still valid

What about effective operators?

[Chala, LG, Ren - 2505.14335]

This work: (Toy) Higgs sector of SMEFT + fermions

$$\mathscr{L}_4 = \partial_\mu \phi^\dagger \partial^\mu \phi - m^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2 - \frac{c_{\phi 6}}{\Lambda^2} (\phi^\dagger \phi)^3$$
$$+ i\overline{\psi_L} \not{\partial} \psi_L + i\overline{\psi_R} \not{\partial} \psi_R - y(\phi \overline{\psi_L} \psi_R + \text{h.c.})$$

Power counting
$(m/T)^2 \sim y \sim \lambda,\ c_{\phi^6} \sim \lambda^2$

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Power counting
$(m/T)^2 \sim y \sim \lambda,\ c_{\phi^6} \sim \lambda^2$

Goal: Compute full **O(λ³)** 3D EFT and compare contribution to PT, at the **same order in λ** from:

a) effective terms **b)** renormalizable terms

Are 1-loop contributions from effective operators **(often neglected)** comparable* to higher-loop contributions from renormalizable operators **(usually included)**?

(*) Within the regime of validity of the EFT expansion (we always ensure dim-8 << dim-6)

Crafting a 3D EFT

Matching in dimensional reduction

[Li et al. - 2201.04639]

1) Build off-shell basis of 3D operators involving the Higgs zero mode at dim-6 (**ABC4EFT**):

$$\mathcal{L}_{\text{EFT}} = K_3 \partial_\mu \varphi^\dagger \partial^\mu \varphi + m_3^2 \varphi^\dagger \varphi + \lambda_3 (\varphi^\dagger \varphi)^2 + c_{\varphi^6} (\varphi^\dagger \varphi)^3 + c^{(1)}_{\partial^2 \varphi^4} (\varphi^\dagger \varphi)(\partial_\mu \varphi^\dagger \partial^\mu \varphi)$$
$$+ r^{(2)}_{\partial^2 \varphi^4} \left[(\varphi^\dagger \varphi)(\partial^2 \varphi^\dagger \varphi) + \text{h.c.}\right] + r^{(3)}_{\partial^2 \varphi^4} \left[i(\varphi^\dagger \varphi)(\partial^2 \varphi^\dagger \varphi) + \text{h.c.}\right] + r_{\partial^4 \varphi^2} \varphi^\dagger \partial^4 \varphi$$

2) Match the hard-region of renormalized Green's functions with the **Higgs zero mode** in external legs up to **O(λ³)**

$$P = (0, \mathbf{p})$$

Power counting
$(m/T)^2 \sim y \sim \lambda,\ c_{\phi^6} \sim \lambda^2$

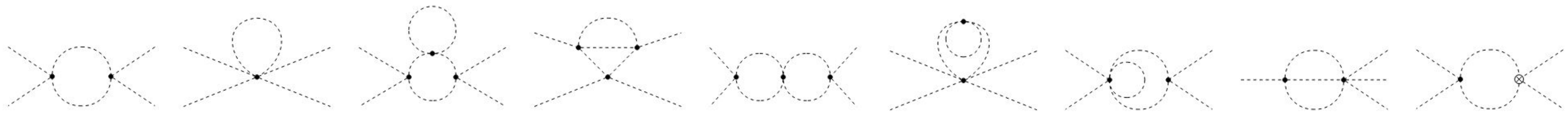
- 3-loop for effective mass
- 2-loop for quartic and kinetic
- 1-loop for dim-6

[Kajantie et al. - hep-ph/9508379]

Crafting a 3D EFT

Example: Matching the quartic term

In the 4D theory

$$\Gamma_{\phi_0\phi_0\phi_0\phi_0} =$$ 

a) **Sum-integrals:** Loop integrals are replaced by loop **sum-integrals** (bosonic, fermionic or mixed)**:**

$$\SumInt_{Q} \text{ or } \{Q\} \equiv T \sum_{n=-\infty}^{\infty} \int_q \qquad \text{where} \qquad \int_q \equiv \tilde{\mu}^{2\epsilon} \int \frac{d^{3-2\epsilon}q}{(2\pi)^{3-2\epsilon}}$$

Not vanishing in dim. reg. after hard-region expansion (heavy T scale remains):

$$\frac{1}{(Q+P)^2+m^2} = \frac{1}{Q^2} + \cdots = \frac{1}{\mathbf{q}^2+\omega_n^2} + \ldots$$

Only a few (**bosonic)** are known at 3-loops!

(needed for eff. mass)

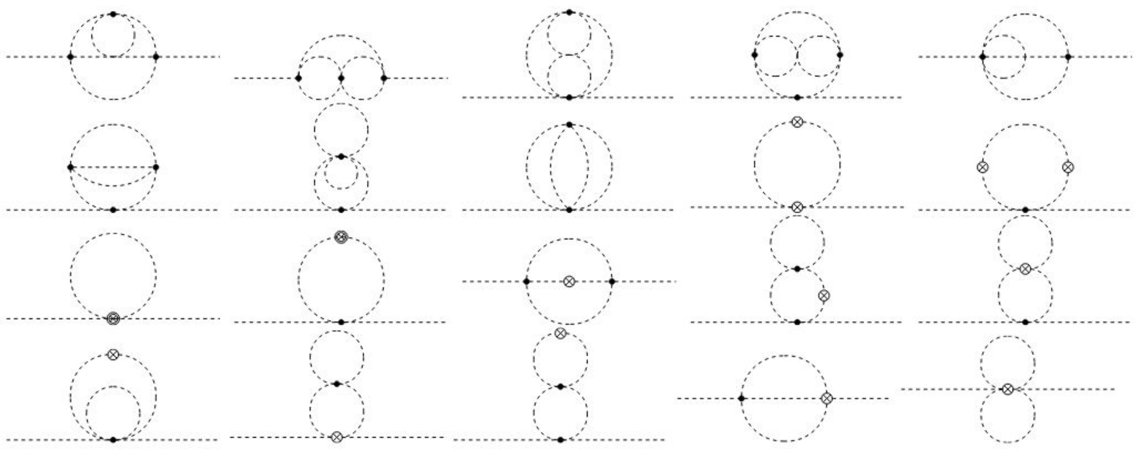
Crafting a 3D EFT

[Chala, LG, Ren - 2505.14335]

*A brief aside about 3-loop sum-integrals

In the 4D theory

The effective mass at O(λ³) requires the computation of **3-loop diagrams**:



Only a few bosonic, mass dimension 2 cases are known in the literature!

$$\sum\!\!\!\!\!\!\int_{QRH} \frac{1}{Q^4 H^2 (Q-R)^2 (R-H)^2} = \tilde{\mu}^{6\epsilon}\left\{ \frac{T^2(4\pi T^2)^{-3\epsilon}}{8(4\pi)^4\epsilon^2}\left[1 + b_{21}\epsilon + b_{22}\epsilon^2 + \mathcal{O}(\epsilon^3)\right]\right\}$$

$$b_{21} = \frac{17}{6} + \gamma_E + 2\frac{\zeta'(-1)}{\zeta(-1)}$$

$$b_{22} = \frac{131}{12} + \frac{31\pi^2}{36} + 8\log 2\pi - \frac{9\gamma_E}{2} - \frac{15\gamma_E^2}{2} + (5 + 2\gamma_E)\frac{\zeta'(-1)}{\zeta(-1)} + 2\frac{\zeta''(-1)}{\zeta(-1)} - 16\gamma_1 + \frac{4\zeta(3)}{9} + C_b\,,$$

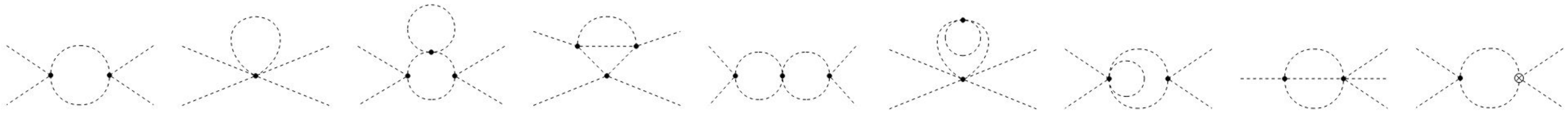
Most (bosonic) 3-loop sum-integrals that are known were computed for hot QCD thermodynamics.

Crafting a 3D EFT

Example: Matching the quartic term

In the 4D theory

$$\Gamma_{\phi_0\phi_0\phi_0\phi_0} =$$



b) **4D Renormalization:** The **4D UV divergences are insensitive to the T scale**, therefore we can compute counterterms (CT) in regular QFT and insert them in thermal diagrams.

These will remove all UV poles arising in the hard-region expansion of sum-integrals.

$$\delta\lambda = \frac{1}{8\pi^2\epsilon}\left(5\lambda^2 + \frac{9}{2}m^2\frac{c_{\phi^6}}{\Lambda^2}\right) + \frac{1}{64\pi^4}\left(-\frac{1}{\epsilon}16\lambda^3 + \frac{1}{\epsilon^2}25\lambda^3\right)$$

Crafting a 3D EFT

Example: Matching the quartic term

In the 3D theory

$$\Gamma_{\varphi\varphi\varphi\varphi} = -(\lambda_3 + \delta\lambda_3)$$

(!) Regular 3D loop integrals vanish in dim. reg. in hard-region expansion

c) **3D renormalization:** UV divergences in the 3D EFT must **cancel any leftover pole** in the matching.

$$\lambda_3 = \lambda T + \frac{c_{\phi 6}}{\Lambda^2}\left(\frac{3}{4}T^3 - \frac{9}{16\pi^2}m^2 T L_b\right) - \frac{5}{8\pi^2}\lambda^2\left[T L_b - \frac{\zeta(3)}{4\pi^2 T}m^2\right] - \frac{3}{8\pi^2}\lambda y^2 T L_f$$
$$+ \frac{9}{8\pi^2}\lambda\frac{c_{\phi 6}}{\Lambda^2}T^3\left[2\log 2\pi - \frac{12\zeta'(2)}{\pi^2} + \frac{1}{\epsilon}\right] + \frac{\lambda^3 T}{128\pi^4}\left[50L_b^2 + 60L_b + \frac{269}{3} + \frac{20\zeta(3)}{3}\right]$$

$$\delta\lambda_3 = \frac{9}{8\pi^2\epsilon}\lambda\frac{c_{\phi 6}}{\Lambda^2}T^3$$

Powerful consistency check at any loop order!

Corrected typo in 2-loop formula [Davydychev et al. - 2312.17367]!

Crafting a 3D EFT

Example: Matching the quartic term

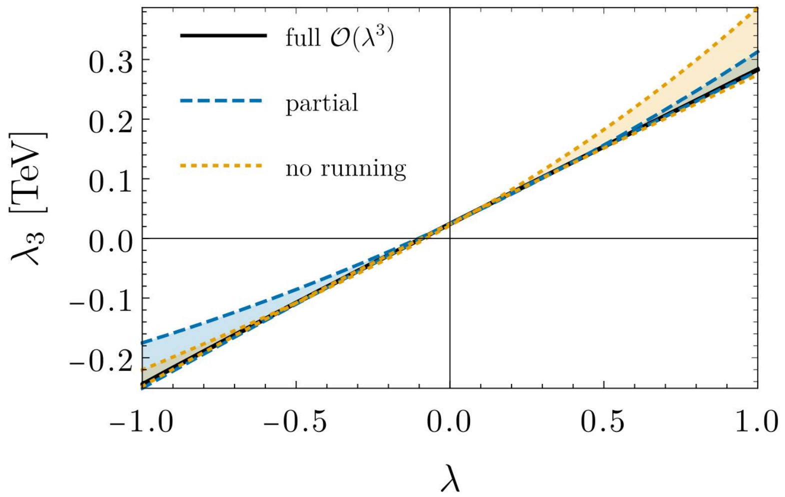
In the 3D theory

$$\Gamma_{\varphi\varphi\varphi\varphi} = -(\lambda_3 + \delta\lambda_3)$$

d) **Matching scale indep.:** Achieved after including **inserting the 4D RGEs** and the **3D Coleman-Weinberg potential** in the matching equations:

$$V_{\text{eff}}^{\text{1-loop}} = -\frac{1}{12\pi} m_{\text{eff}}^3 \,, \quad m_{\text{eff}}^2 = 2\left[m_3^2 + 4\lambda_3 \varphi^\dagger\varphi + 9 c_{\varphi^6}(\varphi^\dagger\varphi)^2\right]$$

$$V_{\text{eff}}^{\text{2-loop}} = \frac{1}{4\pi^2}\left\{\left[\frac{9}{2} m_3^2 c_{\varphi^6} - \left(1 + 2\log\frac{\mu}{3m_3}\right)\lambda_3^2\right]\varphi^\dagger\varphi - 9\left(1 + 2\log\frac{\mu}{3m_3}\right) c_{\varphi^6}\lambda_3(\varphi^\dagger\varphi)^2\right\}$$



Crafting a 3D EFT

Example: Matching the quartic term

Our 3D EFT is now fully determined to **O(λ³)**!

In the 3D theory

$$\Gamma_{\varphi\varphi\varphi\varphi} = -(\lambda_3 + \delta\lambda_3)$$

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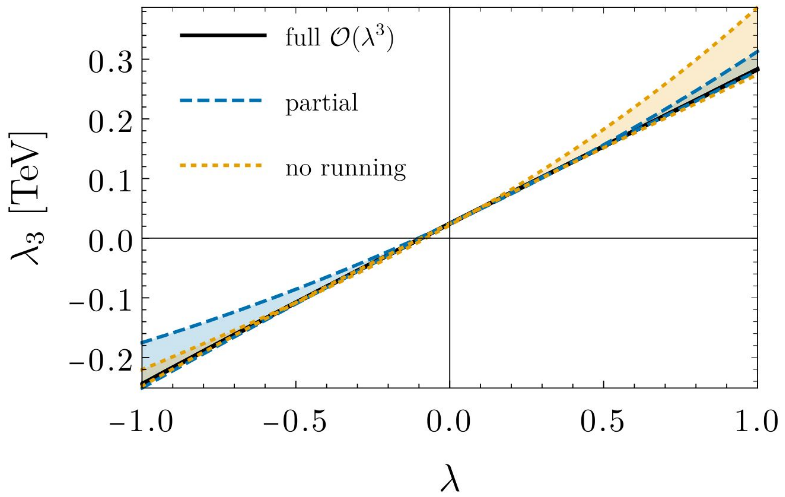


GW power spectrum

Effective vs. renormalizable operators

[Chala, LG, Ren - 2505.14335]

We see how 3D effective operators change the **stochastic GW background** produced during a FOPT:

a) only O(λ²) (dashed red) b) O(λ²) + renormalizable O(λ³) (dashed blue) c) O(λ²) + effective O(λ³) (**solid black**)

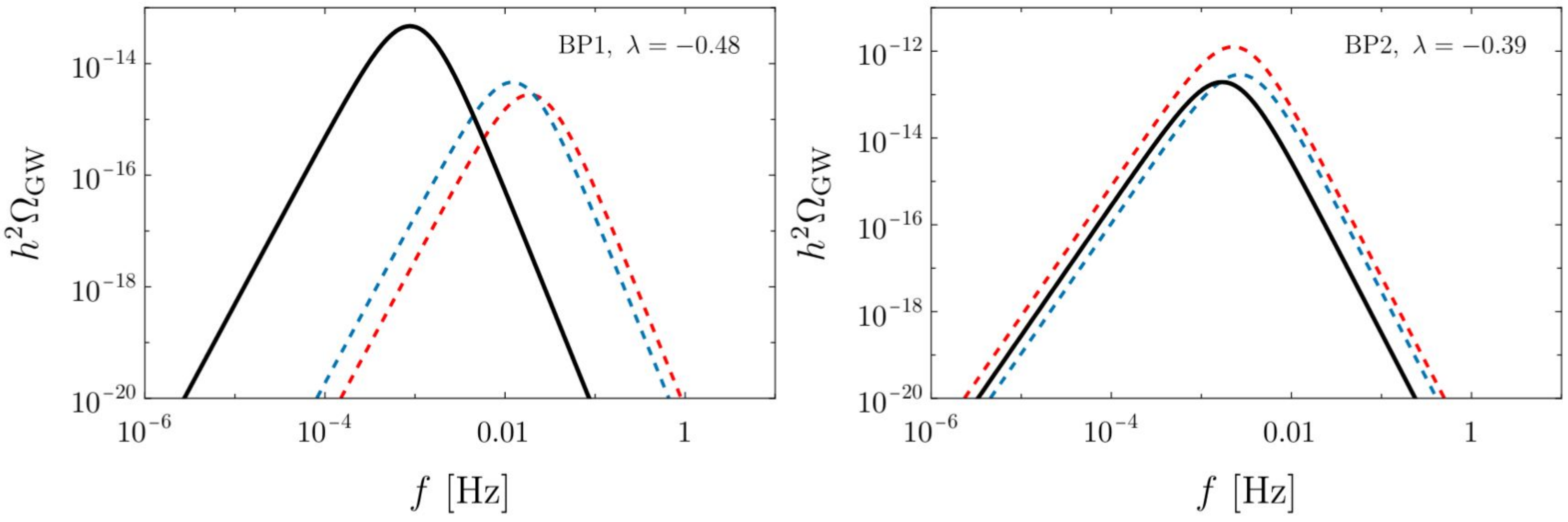


Fig.: Different contributions to the GW spectrum in two benchmark points at fixed λ

(*) Generated with **PTPlot**
[Caprini *et al.* - 1910.13125]

Take home messages

- Computed, for the first time, a full $O(\lambda^3)$ 3D EFT including dim-6 effective operators.
- Proved that 3D effective operators can be relevant when compared to higher-loop corrections to renormalizable operators in strong first-order phase transitions.
- Few works have carefully addressed higher-dimensional operators in 3D EFTs so far, e.g.: **[Laine, Schicho, Schröder '18], [Chala, Criado, LG, López Miras '24], [Bernardo, Klose, Schicho, Tenkanen '25], [Chala, Guedes '25], [Chala, LG, Ren '25]**

What's left to do? 🔍

Automated dimensional reduction with effective operators, unknown 3-loop sum-integrals, nucleation rate…

Thank you for your attention!

¡Gracias por vuestra atención!

Thermal field theory

Phase transition parameters

[Caprini et al. - 1910.13125]

The bounce™ (φ_c)

Transition rate: $\Gamma \sim e^{-S_3[\varphi_c]}$

Depends on static, non-homogeneous solutions to the EoMs:

$$\left.\frac{\delta S_3}{\delta \varphi}\right|_{\varphi=\varphi_c} = 0$$

These are the so-called **bounce solutions.**

[Coleman - PhysRevD.15.2929]

Nucleation temperature

$$\mathscr{P} \sim e^{-S_3[\varphi_c]}\Big|_{T=T_*} \sim 1$$

Strength parameter

$$\alpha \sim \frac{V_3(\varphi_T)}{T_*^3}$$

Inverse duration

$$\frac{\beta}{H_*} = T\left.\frac{dS_3[\varphi_c]}{dT}\right|_{T=T_*}$$

Bubble wall velocity

A bit more involved…

[Laurent, Cline - 2204.13120]

Results

Backup

[Chala, LG, Ren - 2505.14335]

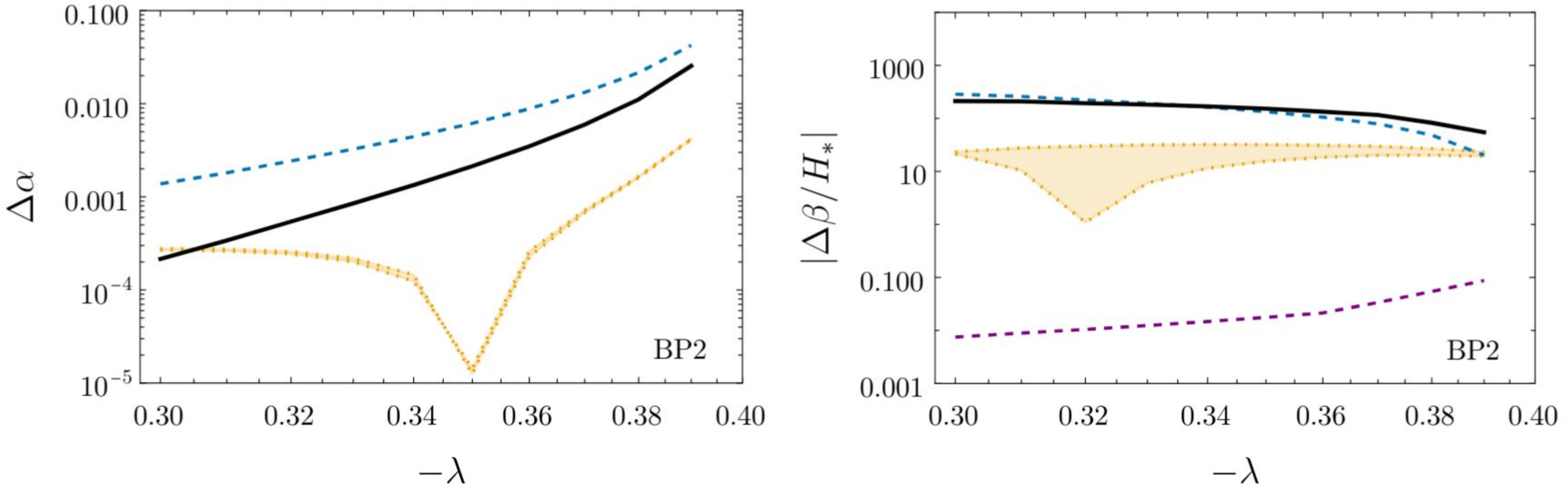


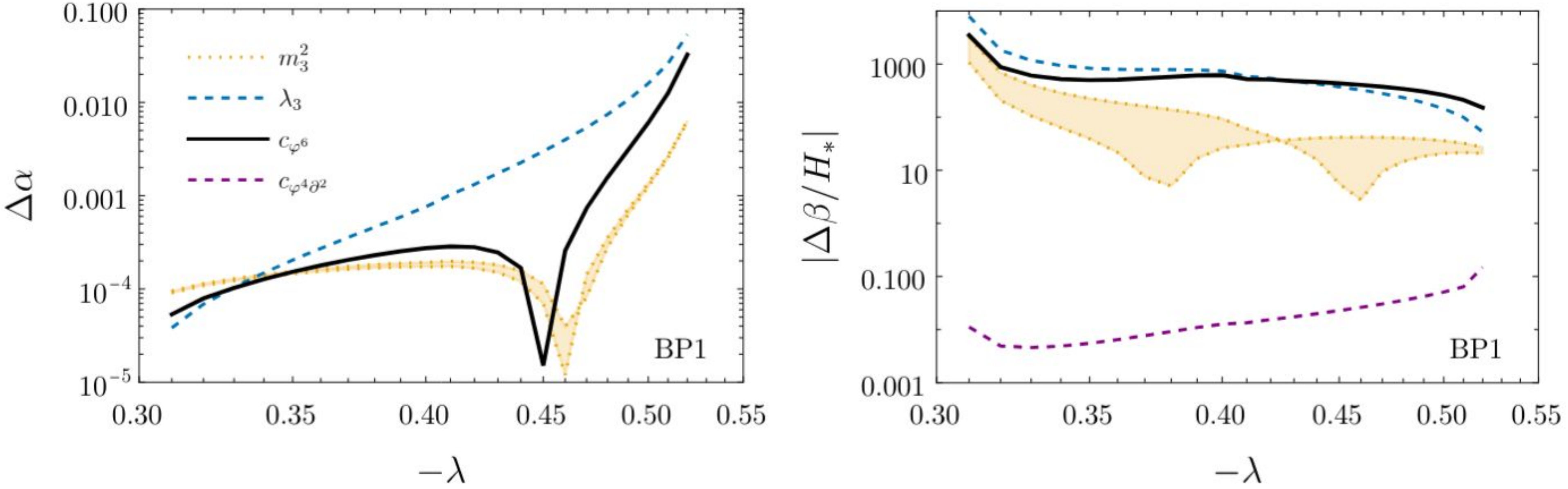
Fig. X: Different corrections to strength parameter (left) and inverse duration (right) at $O(\lambda^3)$ in BP2

Phase transition parameters

Effective vs. renormalizable operators

[Chala, LG, Ren - 2505.14335]

We compare **O(λ³)** contributions from **each operator separately** to PT parameters in two benchmark points (**BP**) for which we find strong FOPTs:



BP

(v_P, m_P^2)

$y = 0.9$

Fig. X: Different corrections to strength parameter (left) and inverse duration (right) at O(λ³) in BP1

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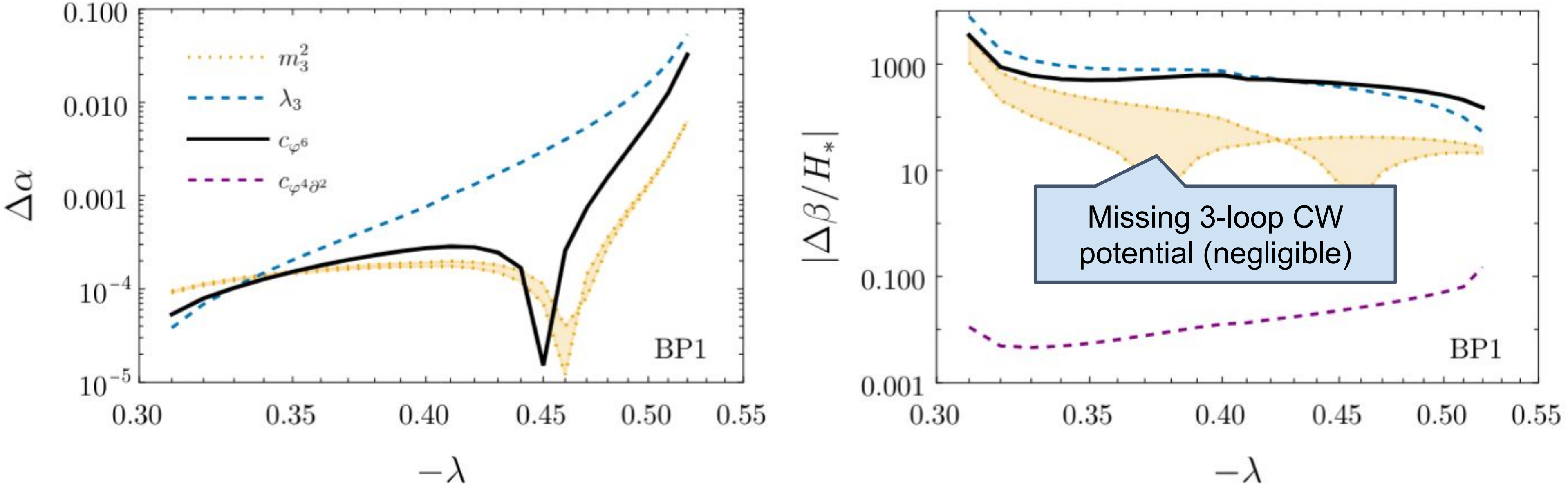

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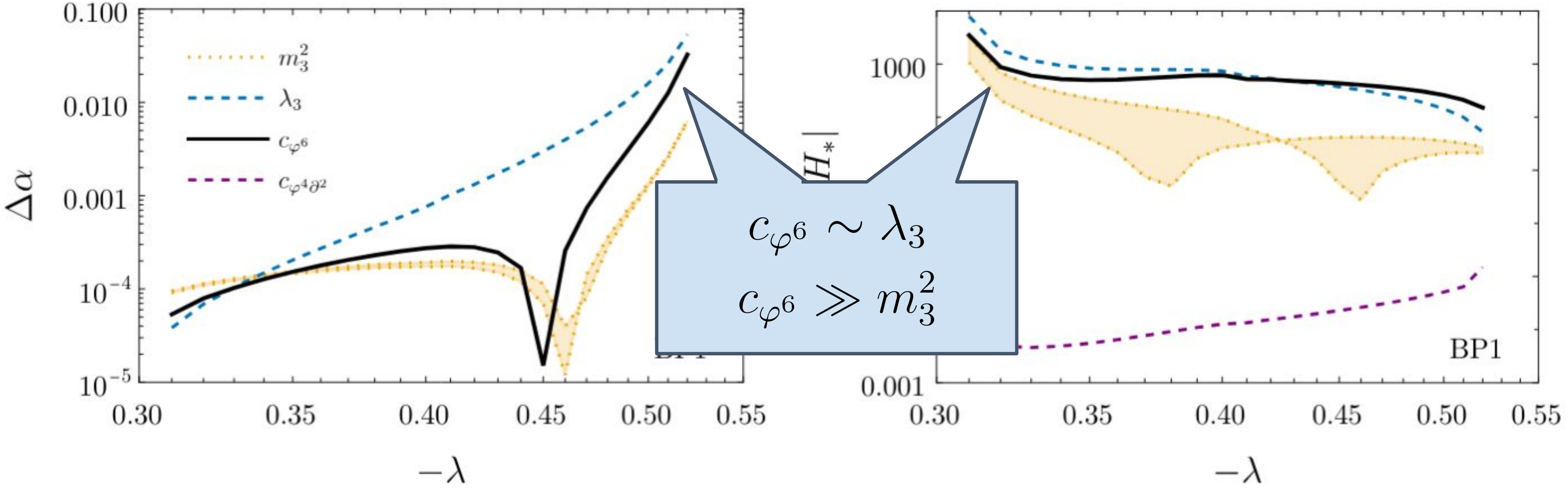

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Results

Backup

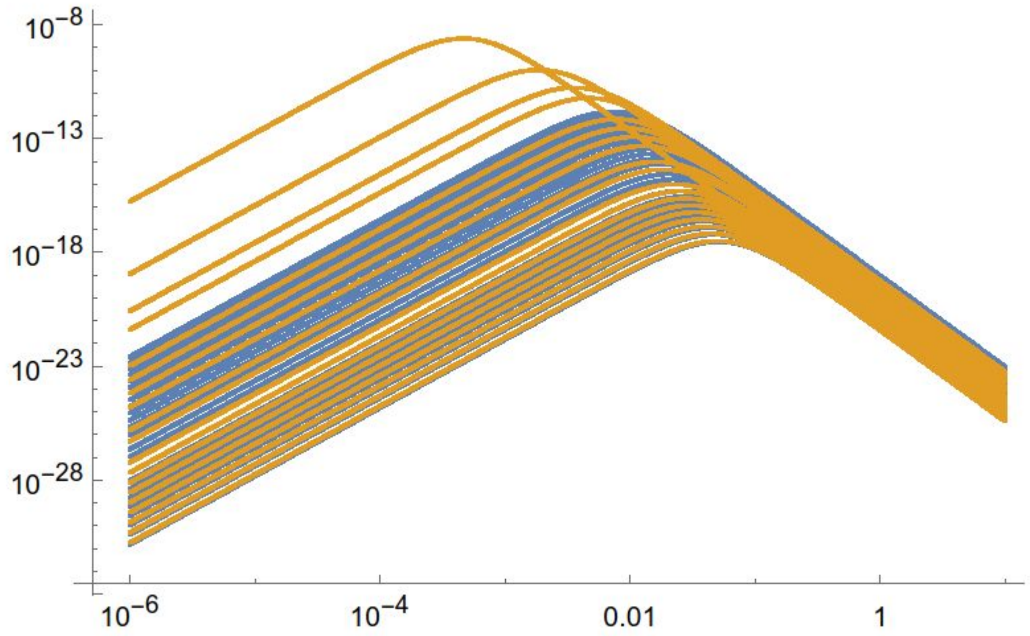

Fig. X: Discriminant power of EO (orange) vs renormalizable-only (blue) for different points in parameter space in the real scalar model.

Results

Backup

$$
\begin{aligned}
m_3^2 = m^2 &+ \lambda\left[\frac{1}{3}T^2 - \frac{1}{4\pi^2}m^2 L_b + \frac{\zeta(3)}{32\pi^4 T^2}m^4\right] + y^2\left(\frac{1}{4}T^2 - \frac{3}{16\pi^2}m^2 L_f\right) \\
&+ \frac{c_{\phi 6}}{\Lambda^2}\left(\frac{1}{8}T^4 - \frac{3}{16\pi^2}m^2 T^2 L_b\right) - \frac{1}{32\pi^2}\lambda y^2 T^2 \left(3L_b + L_f\right) \\
&+ \frac{1}{16\pi^2}\lambda^2\left[T^2\left(L_f - \frac{1}{3}L_b + 4\log\pi - \frac{24\zeta'(2)}{\pi^2} + \frac{2}{\epsilon}\right) + \frac{1}{4\pi^2}m^2\left(7L_b^2 + 5L_b + \frac{89}{12} + \frac{4\zeta(3)}{3}\right)\right] \\
&+ \frac{1}{16\pi^2}\lambda\frac{c_{\phi 6}}{\Lambda^2}T^4\left[\frac{3}{2}\left(L_b + L_f\right) + \frac{29}{10} - \frac{36\zeta'(2)}{\pi^2} + 360\zeta'(-3) - 3\gamma + 6\log\pi + \frac{3}{\epsilon}\right] \\
&+ \frac{1}{128\pi^4}\lambda^3 T^2\left[2C_b - 10C_s - \frac{85}{3}L_b^2 - 5L_f^2 + L_b\left(\frac{89}{3} + \frac{240\zeta'(2)}{\pi^2} - \frac{80\gamma}{3} - \frac{20}{\epsilon}\right)\right. \\
&- L_f\left(\frac{29}{3} - \frac{80\gamma}{3} + 40\log\pi\right) - \frac{1}{9}\left(313\pi^2 + 509\right) + \frac{4\zeta(3)}{3} + \left(41 - 20\gamma\right)\frac{8\zeta'(2)}{\pi^2} \\
&\left.- 160\zeta''(-1) + 8\gamma\left(19\gamma - 2\right) + \frac{992\gamma_1}{3} + \frac{4}{3}\left(-29 + 80\gamma - 60\log\pi\right)\log\pi\right], \qquad (24)
\end{aligned}
$$