SymTFTs for $U(1)$ symmetries from descent

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Based on [2411.15126] with Iñaki García Etxebarria

Outline

1 Anomaly descent

2 Symmetry descent

Anomaly descent

Topological operators

- Suppose a theory has conserved $(p+1)$-form current

$$\partial_\mu j^{\mu\nu_1\dots\nu_p} = 0 \tag{1}$$

- Can define a charge operator

$$Q(\Sigma_{d-p-1}) = \int *j_{p+1} \tag{2}$$

that measures 'units of charge' within Σ_{d-p-1}.

- Can define topological operators that generate a p**-form symmetry** $G^{[p]}$:

$$\mathcal{U}_\alpha(\Sigma_{d-p-1}) := e^{i\alpha Q(\Sigma)} \tag{3}$$

that acts on p-dimensional charged defects.

't Hooft Anomalies

- Consider QFT $\mathcal{Z}$ in d-dim with a p-form symmetry $G^{[p]}$.
- Gauging $G^{[p]} \rightsquigarrow$ add gauge field $B \in H^{p+1}(\mathcal{M}_d; G)$:

$$\mathcal{Z}^{\text{gauged}} = \sum_B \mathcal{Z}[B] = \sum_B \mathcal{Z} \cdot e^{i \int B \wedge J} \tag{4}$$

- Symmetry has an **'t Hooft anomaly** if

$$\mathcal{Z}[B + d\lambda] = e^{i \int_{\mathcal{M}_d} \mathcal{A}[B,\lambda]} \mathcal{Z}[B] \tag{5}$$

- Trying to gauge symmetry with 't Hooft anomaly kills the path integral:

$$\boxed{\mathcal{Z}^{\text{gauged}} = 0} \tag{6}$$

The anomaly theory

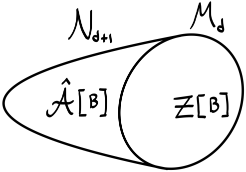
- Can cancel anomaly by defining the **anomaly theory**: $(d+1)$-dim TQFT $\widehat{\mathcal{A}}$ on $\mathcal{N}_{d+1}$ with $\partial\mathcal{N}_{d+1} = \mathcal{M}_d$:

$$\int_{\mathcal{N}} \widehat{\mathcal{A}}[B + d\lambda] = \int_{\mathcal{N}} \widehat{\mathcal{A}}[B] - \int_{\mathcal{M}} \mathcal{A}[B, \lambda] \tag{7}$$

- Constructs anomaly-free path integral:

$$\widehat{\mathcal{Z}}[B] := e^{i\int_{\mathcal{N}} \widehat{\mathcal{A}}[B]} \cdot \mathcal{Z}[B] \tag{8}$$

which gives $\widehat{\mathcal{Z}}^{\text{gauged}} \neq 0$.

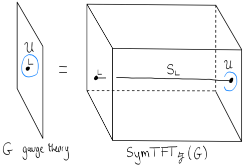


SymTFTs

- Extend idea to the **Symmetry TFT (SymTFT)** ξ [Pulmann, Severa, Valach '19; Ji, Wen '19; Gaiotto, Kulp '20; Apruzzi, Bonetti, García-Etxebarria, Hosseini, Schäfer-Nameki '21; Freed, Moore, Teleman '22;...], by gauging $\widehat{\mathcal{A}}[B]$ and adding 'kinetic'/BF terms:

$$\xi := \sum_B \int B \wedge d\tilde{B} + \widehat{\mathcal{A}}[B] \tag{9}$$

- We choose $\mathcal{N}_{d+1} = \mathcal{M}_d \times [0, L]$, so there are 2 boundaries:



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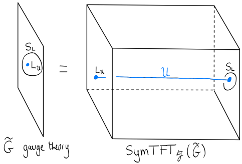


- Imposing different boundary conditions on the second boundary changes the *global form* of the boundary theory.

Symmetry descent

QFTs from strings

- Instead of using string theory on $\mathcal{M}_d \times X_{10-d}$ to study d-dim quantum gravity, where X_{10-d} is a compact Calabi-Yau, we can make X_{10-d} *non-compact* - can then study d-dim QFTs.
- $\partial X_{10-d} = L_{9-d}$ determines the possible symmetries of the QFT
- Wrapping p-branes of string theory around cycles $D \in H_k(L_{9-d}; \mathbb{Z})$ give $\widehat{D}^{[p-k]}$ symmetries, where

$$\widehat{\mathbb{Z}} = U(1),\ \widehat{\mathbb{Z}}_N = \mathbb{Z}_N \tag{10}$$

SymTFTs from strings

- Can obtain the anomaly theory of the QFT by dimensionally reducing the CS terms of string theory on L_{9-d}: [Apruzzi, Bonetti, García-Etxebarria, Hosseini, Schäfer-Nameki '21]

$$\int_{\mathcal{N}_{d+1}\times L_{9-d}} C\wedge H\wedge F = \sum_{B,\dots}\int_{\mathcal{N}_{d+1}} \widehat{\mathcal{A}}[B,\dots] \tag{11}$$

- Obtaining the BF terms of the SymTFT had to be inferred separately:

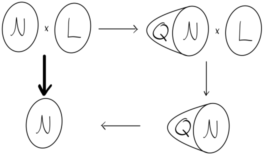
$$\int_{\mathcal{N}_{d+1}\times L_{9-d}} F\wedge *F \neq \sum_{B,\dots}\int_{\mathcal{N}_{d+1}} B\wedge d\tilde{B}+\dots \tag{12}$$

- How do we obtain all terms in one procedure?

The symmetry descent mechanism

- Solution was posed in [García-Etxebarria, Hosseini '24]: write the **string theory action as a boundary theory** of an 11d TQFT on $Q_{d+2} \times L_{9-d}$, where $\partial Q_{d+2} = \mathcal{N}_{d+1}$:

$$\int_{Q\times L} F \wedge d\tilde{F} + F \wedge H \wedge G = \sum_{B,\dots} \int_Q dB \wedge d\tilde{B} + d\widehat{\mathcal{A}}[B,\dots] \quad (13)$$



- Going to the boundary $\partial Q_{d+2} = \mathcal{N}_{d+1}$, we obtain the SymTFT Lagrangian:

$$\xi = \sum_{B,\dots} \int_{\mathcal{N}} B \wedge d\tilde{B} + \widehat{\mathcal{A}}[B,\dots] \quad (14)$$

- [García-Etxebarria, Hosseini '24] assumed $G = \mathbb{Z}_N$. [**FG**, García-Etxebarria '24] extended to $G = U(1)$, found SymTFT agreed with [Antinucci, Benini '24; Brennan, Sun '24].
- ◇ SymTFTs we obtained had **bulk EM duality** → boundary QFTs can have different local dynamics!
- ? **Interesting problem**: SymTFT of [Antinucci, Benini '24; Brennan, Sun '24] suggests certain boundary conditions for $U(1)$ SymTFT give $\mathbb{R}$ symmetries - **we did not find these**:
- ? Swampland conjectures forbid non-compact gauge groups in quantum gravity; **does geometric engineering then forbid global $\mathbb{R}$ symmetries in this class of QFTs?**

Thanks for listening!

References I

- Andrea Antinucci and Francesco Benini, *Anomalies and gauging of U(1) symmetries*, Phys. Rev. B **111** (2025), no. 2, 024110.
- Fabio Apruzzi, Federico Bonetti, Iñaki García Etxebarria, Saghar S. Hosseini, and Sakura Schafer-Nameki, *Symmetry TFTs from String Theory*, Commun. Math. Phys. **402** (2023), no. 1, 895–949.
- T. Daniel Brennan and Zhengdi Sun, *A SymTFT for continuous symmetries*, JHEP **12** (2024), 100.
- Daniel S. Freed, Gregory W. Moore, and Constantin Teleman, *Topological symmetry in quantum field theory.*

References II

- Davide Gaiotto and Justin Kulp, *Orbifold groupoids*, JHEP **02** (2021), 132.
- Wenjie Ji and Xiao-Gang Wen, *Categorical symmetry and noninvertible anomaly in symmetry-breaking and topological phase transitions*, Phys. Rev. Res. **2** (2020), no. 3, 033417.
- Ján Pulmann, Pavol Ševera, and Fridrich Valach, *A nonabelian duality for (higher) gauge theories*, Adv. Theor. Math. Phys. **25** (2021), no. 1, 241–274.