21/07/2024

No gauge cancellation at high energy in the five-vector R_ξ gauge

[Phys. Rev. D 111, 076031]

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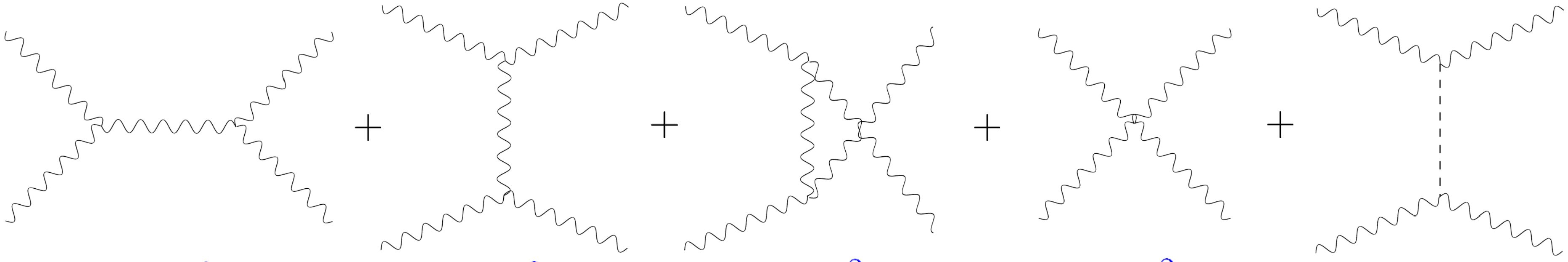
Motivation

$\boxed{W_L Z_L \to W_L Z_L}$

at high energy ($s \to \infty$)



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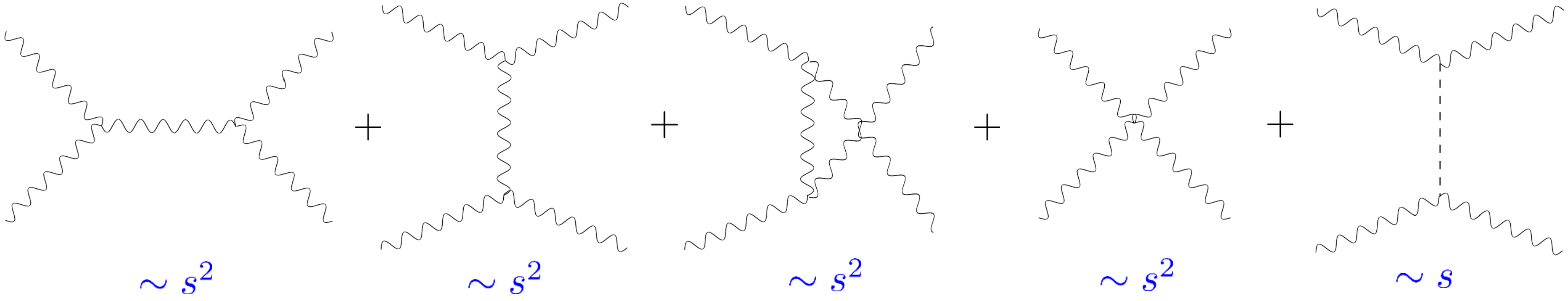

B.W. Lee, C. Quigg, H. B. Thacker. PRD 1977

$$\mathcal{M}_{\text{tot}}(W_L Z_L \to W_L Z_L) = \text{finite}$$

[gauge cancellation]

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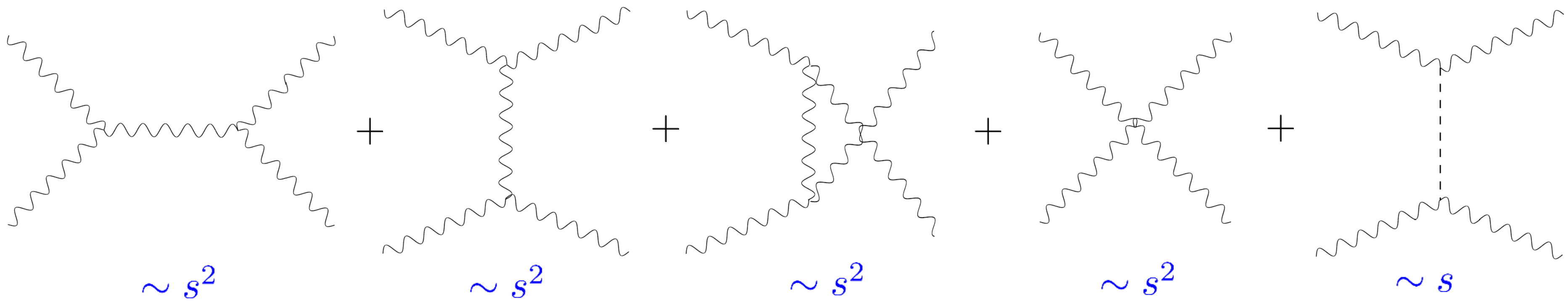

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[gauge cancellation]

Origin of high-energy divergences: $\epsilon^\mu_{0L} = \frac{1}{m}(p, 0, 0, E) \sim \frac{p^\mu}{m}$

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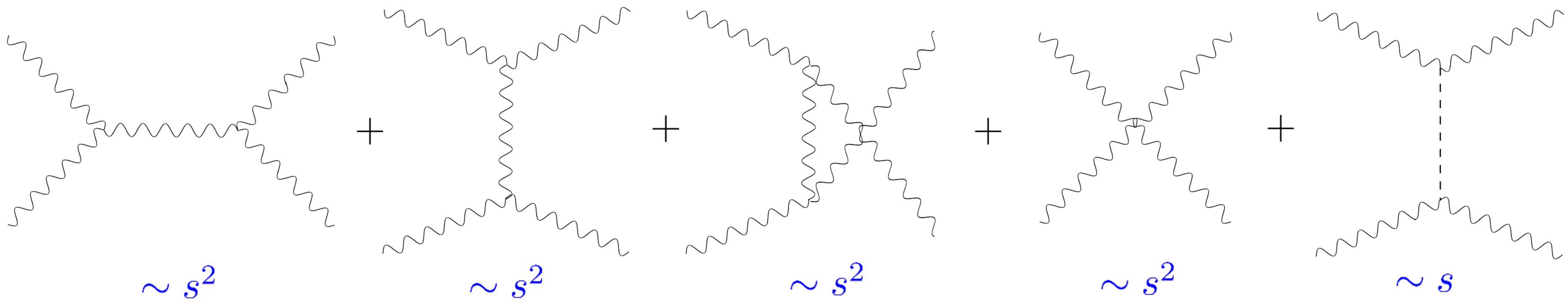

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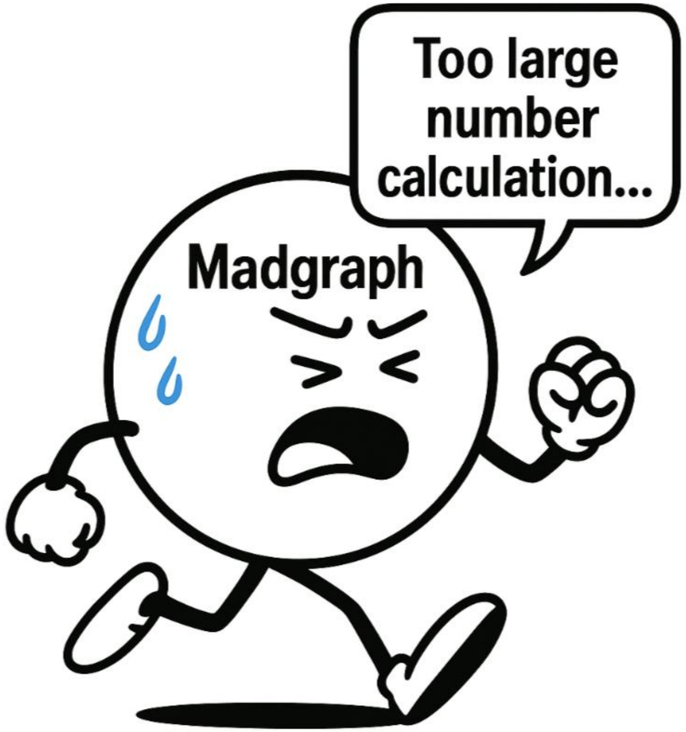

$\Rightarrow$

- Long simulation time
- Reduction of the simulation accuracy

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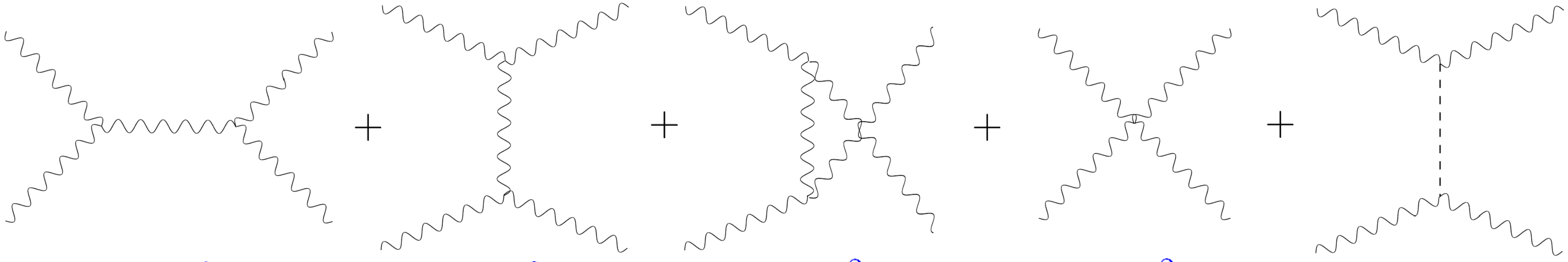

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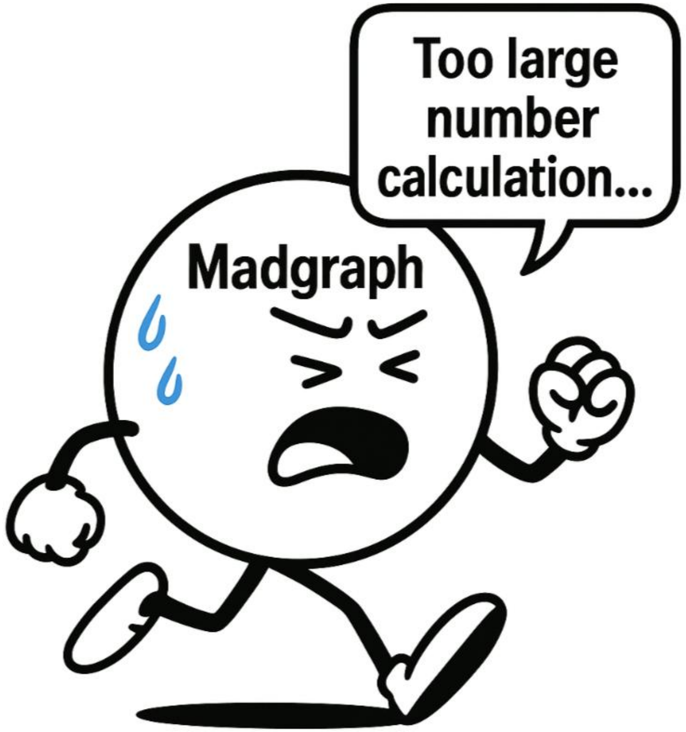

$\Rightarrow$

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Prescription?

Equivalent (EQ) gauge

A. Wulzer; NPB 2014,
G. Cuomo, L. Vecchi, A. Wulzer; SPP 2020

$$\epsilon_L^\mu = \frac{1}{m}(p-E,0,0,E-p) = \epsilon_{0L}^\mu - \frac{p^\mu}{m} \sim 0 \quad \text{at high } E$$

$$\epsilon_\pi = -i$$

$$\Rightarrow \epsilon_L^M = \frac{1}{m}(p-E,0,0,E-p,-i)$$

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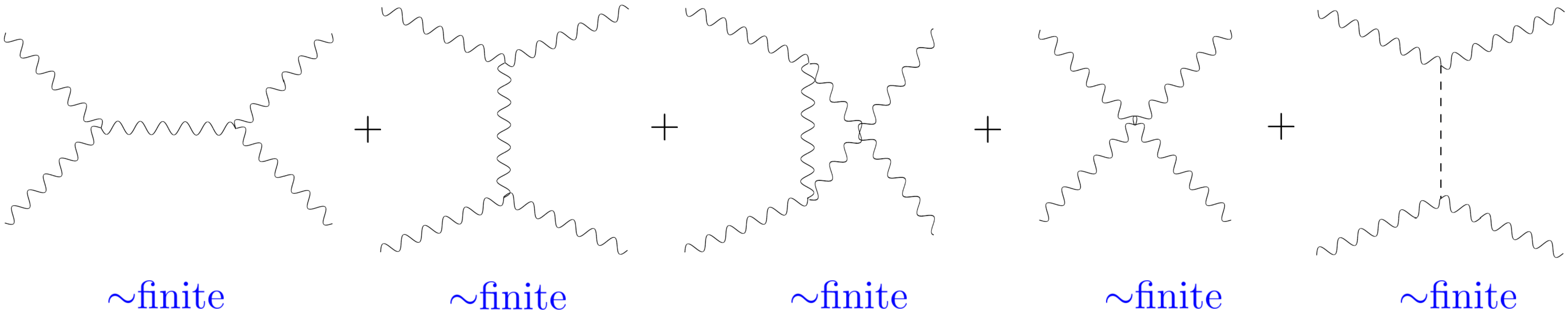

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Feynman-diagram (FD) gauge

J. Chen, K. et al; EPJP 2024
K. Hagiwara, et al; PRD 2024
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$$\mathcal{L}^{(0)} = -\frac{1}{4}(\partial^\mu Z^\nu - \partial^\nu Z^\mu)^2 + \frac{1}{2}m^2 Z^\mu Z_\mu + \frac{1}{2}(\partial^\mu \pi)^2 + m Z^\mu \partial_\mu \pi - \frac{1}{2\xi}(n^\mu Z_\mu)^2$$

$n^\mu = (\mathrm{sgn}(p^0), \vec{p}/|\vec{p}|)$ $\quad$ $n^\mu(\partial)$ in the Lagrangian?

Motivation

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$n^\mu = (\mathrm{sgn}(p^0), \vec{p}/|\vec{p}|)$ $\quad$ $n^\mu(\partial)$ in the Lagrangian?

Other gauges
- generating no high-energy divergences?
- realizable within the Lagrangian for an intuitive identification of the divergences?

1. Idea and derivation of the 5V R_ξ gauge
2. Identification of high-energy divergences within the Abelian Higgs model
3. Conclusion

Idea and derivation

BRST formalism $\longrightarrow$ Generalized Ward identity

$$p^\mu T_\mu(A) = imT(\pi)$$

C. Becchi, A. Rouet, and R. Stora; Annals Phys 1976.
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Main idea: Treat the massive polarizations "like" the massless polarizations in 5D

$$P^M = (p^\mu, im)$$

$$T_M(A) = \left(T_\mu(A), T(\pi)\right)$$

$$\Rightarrow \quad P^M T_M(A) = 0$$

P^M → Unphysical component of the five-vector gauge field

Idea and derivation

In the polar basis of the Higgs field $\Phi = \frac{1}{\sqrt{2}}(v+h)e^{i\pi/v}$

$M = 0, 1, 2, 3, 4$
$V_N = V^M g_{MN}$
$g_{MN} = \text{diag}(1, -1, -1, -1, -1)$

BRST formalism $\Longrightarrow$ $\mathcal{L}_{\text{gf+gh}} = -\frac{(\bar{\partial}_M A^M)^2}{2\xi} - \bar{c}(\bar{\partial}^M \partial_M)c$

5V R_ξ gauge-fixing term

$$A^M = (A^\mu, \pi)$$
$$\partial^M = (\partial^\mu, m)$$
$$\bar{\partial}^M = (\partial^\mu, -m)$$

$$m = ev$$

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Propagator

$$\mathcal{L}_{\text{free}} = -\frac{1}{4}F^{MN}F_{MN} - \frac{(\bar{\partial}_M A^M)^2}{2\xi}$$

$$-\frac{1}{4}F^{MN}F_{MN} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}m^2\left(A^\mu - \frac{1}{m}\partial^\mu\pi\right)^2$$

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$$\mathcal{L}_{\rm free} = -\frac{1}{4}F^{MN}F_{MN} - \frac{(\bar{\partial}_M A^M)^2}{2\xi} \quad \Longrightarrow \quad i\Pi_{MN} = \frac{i\left(-g_{MN} + (1-\xi)\frac{P_M P_N^*}{P^2}\right)}{P^2}$$

$$-\frac{1}{4}F^{MN}F_{MN} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}m^2\left(A^\mu - \frac{1}{m}\partial^\mu\pi\right)^2$$

$(P^2 \equiv P^M P_M^* = E^2 - p^2 - m^2)$

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$(P^2 \equiv P^M P_M^* = E^2 - p^2 - m^2)$

Notice

4V R_ξ gauge = 5V R_ξ gauge

$$-\frac{(\partial^\mu A_\mu + \xi m\pi)^2}{2\xi}\bigg|_{\xi=1} = -\frac{(\partial^\mu A_\mu + m\pi)^2}{2\xi}\bigg|_{\xi=1}$$

Implication of renormalizability in the 5V R_ξ gauge

Idea and derivation

In the polar basis of the Higgs field $\Phi = \frac{1}{\sqrt{2}}(v+h)e^{i\pi/v}$

$$\mathcal{L}(x) = -\frac{1}{4}(F^{\mu\nu})^2 + |D^\mu\Phi|^2 - \frac{\lambda}{4}\left(|\Phi|^2 - \frac{2\mu^2}{\lambda}\right)^2 \quad \Rightarrow$$

Gauge symmetric under

$$A^M \rightarrow A^M + \partial^M \Lambda$$

P^M: Unphysical component of A^M

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P^M: Unphysical component of A^M

Five space- and light-like vectors (based on P^M) in the 5V description

Space-like orthogonal

$$\epsilon^M(+) = -\frac{1}{\sqrt{2}}(0,-1,-i,0,0)$$

$$\epsilon^M(-) = -\frac{1}{\sqrt{2}}(0,+1,-i,0,0)$$

$$\epsilon^M(0) = \frac{1}{\sqrt{2}E}(0,0,0,m,ip)$$

Light-like

$$\epsilon^M(>) = \frac{1}{\sqrt{2}E}(E,0,0,p,im)$$

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Light-like

~~$\epsilon^M(>) = \frac{1}{\sqrt{2E}}(E,0,0,p,im)$~~

~~$\epsilon^M(<) = \frac{1}{\sqrt{2E}}(E,0,0,-p,-im)$~~

$a^\dagger_{>,<}(p)|\text{phys}\rangle$ are not in the cohomology ensured by the BRST formalism

Physical three transverse polarization modes

$$\epsilon^M(+) = -\frac{1}{\sqrt{2}}(0,-1,-i,0,0)$$
$$\epsilon^M(-) = -\frac{1}{\sqrt{2}}(0,+1,-i,0,0)$$
$$\epsilon^M(0) = \frac{1}{\sqrt{2}E}(0,0,0,m,ip)$$

$(E = m)$

$$\epsilon^M(0) = \frac{1}{\sqrt{2}}(0,0,0,1,0)$$

Vector (Spin 1)

$(E \to \infty)$

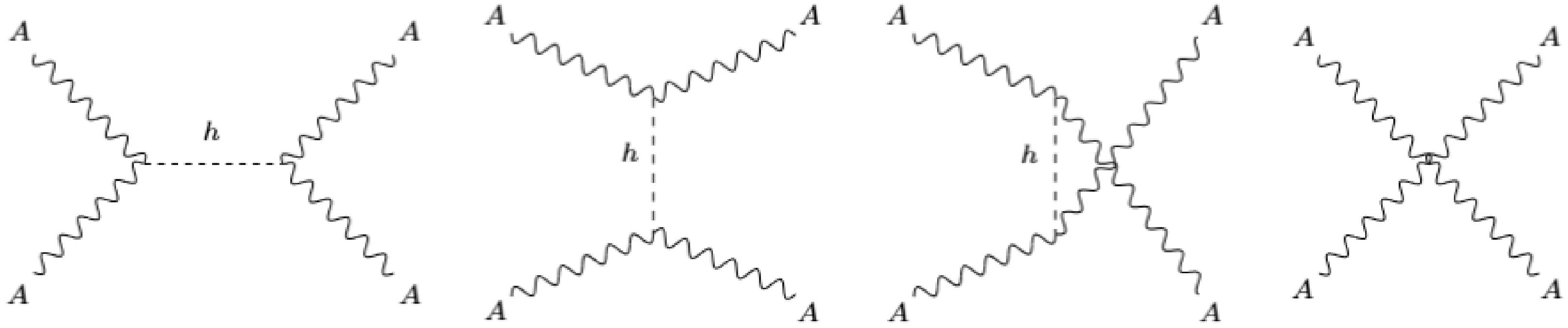
$$\epsilon^M(0) = \frac{1}{\sqrt{2}}(0,0,0,0,i)$$

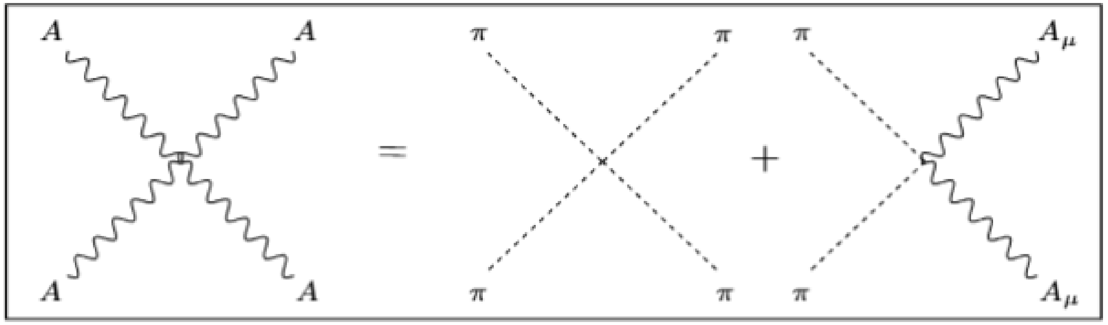
Scalar (Spin 0)

Goldstone absorption clearly seen at the level of polarization vectors

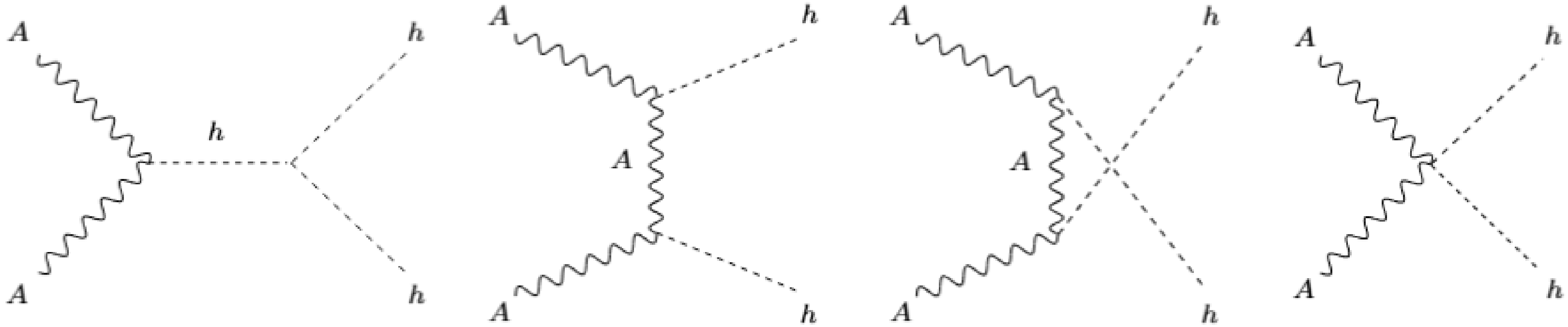
Identification of high-energy divergences

$[A_L A_L \to A_L A_L]$





$[A_L A_L \to hh]$



Identification of high-energy divergences

$A_L A_L \to A_L A_L$	Polar basis: $\frac{1}{\sqrt{2}} H e^{iA^4/v}$	
s channel	$-\dfrac{\lambda(s\beta^2+2m^2)^2}{2m_h^2(s-m_h^2)}$	$\to \infty$
t channel	$-\dfrac{\lambda(t+2m^2\cos\theta)^2}{2m_h^2(t-m_h^2)}$	$\to \infty$
u channel	$-\dfrac{\lambda(u-2m^2\cos\theta)^2}{2m_h^2(u-m_h^2)}$	$\to \infty$
contact	-	
sum in the limit $s\to\infty$	$-\frac{3}{2}\lambda$	

$A_L A_L \to hh$	Polar basis: $\frac{1}{\sqrt{2}} H e^{iA^4/v}$	
s channel	$\dfrac{3\lambda(s\beta^2+2m^2)}{2(s-m_h^2)}$	$\to \infty$
t channel	$-\dfrac{\lambda s(4t+s\beta^4-s\beta_h^2\cos^2\theta-4m_h^2)}{8m_h^2(t-m^2)}$	$\to \infty$
u channel	$-\dfrac{\lambda s(4u+s\beta^4-s\beta_h^2\cos^2\theta-4m_h^2)}{8m_h^2(u-m^2)}$	$\to \infty$
contact	$\dfrac{\lambda(s\beta^2+2m^2)}{2m_h^2}$	$\to \infty$
sum in the limit $s\to\infty$	$\dfrac{\lambda}{2}\left(1-\dfrac{2m^2(3+\cos^2\theta)}{m_h^2\sin^2\theta}\right)$	

in the limit $(s\to\infty)$ *or* $(v\to 0)$

$m = ev$

$m_h = \lambda^{1/2} v/\sqrt{2}$

Identification of high-energy divergences

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in the limit $(s\to\infty)$ or $(v\to 0)$

$m = ev$

$m_h = \lambda^{1/2} v/\sqrt{2}$

Divergences of "tree-level" $\mathcal{M}$'s in $v\to 0$ $\Rightarrow$ Trace clearly the origin in $\mathcal{L}$ (classical)

Identification of high-energy divergences

In the limit $v \to 0$ ($v \equiv \epsilon$)

$$\mathcal{L}(x) = -\frac{1}{4}F^{MN}F_{MN} + \frac{1}{2}(\partial^\mu h)^2 + \left(e^2\epsilon\, h + \frac{e^2}{2}h^2\right)\left(A^\mu - \frac{1}{e\epsilon}\partial^\mu A^4\right)^2 - \frac{\lambda}{4}\epsilon\, h^3 - \frac{\lambda}{16}h^4$$

First origin of divergences

Identification of high-energy divergences

How to cure the divergences

In the limit $v \to 0$ $(v \equiv \epsilon)$

$$\mathcal{L}(x) = -\frac{1}{4}F^{MN}F_{MN} + \frac{1}{2}(\partial^\mu h)^2$$

$$+\left(e^2\epsilon\, h + \frac{e^2}{2}h^2\right)\left(A^\mu - \frac{1}{e\epsilon}\partial^\mu A^4\right)^2 - \frac{\lambda}{4}\epsilon\, h^3 - \frac{\lambda}{16}h^4$$

The criminal of divergences

$\Rightarrow$ $A^4(x) \equiv e v \alpha(x)$

First origin of divergences

Identification of high-energy divergences

In the limit $v \to 0$ $(v \equiv \epsilon)$

$$\mathcal{L}(x) = -\frac{1}{4}F^{MN}F_{MN} + \frac{1}{2}(\partial^\mu h)^2$$
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The criminal of divergences

First origin of divergences

How to cure the divergences

$$\Rightarrow \quad A^4(x) \equiv ev\alpha(x)$$

Polar basis

$$\Phi = \frac{1}{\sqrt{2}}He^{iA^4/v}$$

Cartesian basis

$$\Phi = \frac{1}{\sqrt{2}}(\boldsymbol{H} + i\boldsymbol{A}^4)$$

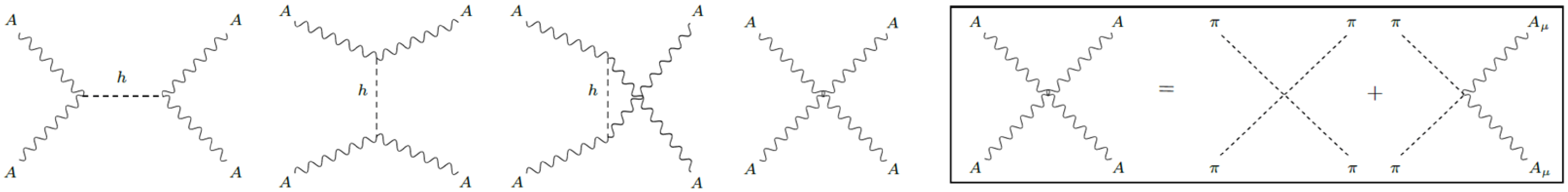
$$A^4 = v\sin^{-1}\left(\frac{\boldsymbol{A}^4}{\boldsymbol{H}^2 + (\boldsymbol{A}^4)^2}\right) = v\cos^{-1}\left(\frac{\boldsymbol{H}}{\boldsymbol{H}^2 + (\boldsymbol{A}^4)^2}\right),$$

$$H = \sqrt{\boldsymbol{H}^2 + (\boldsymbol{A}^4)^2},$$

Ensuring no divergences

Identification of high-energy divergences

$[A_L A_L \;\rightarrow\; A_L A_L]$



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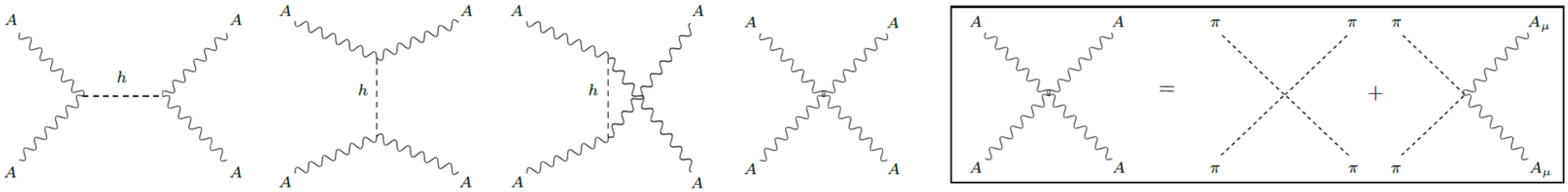

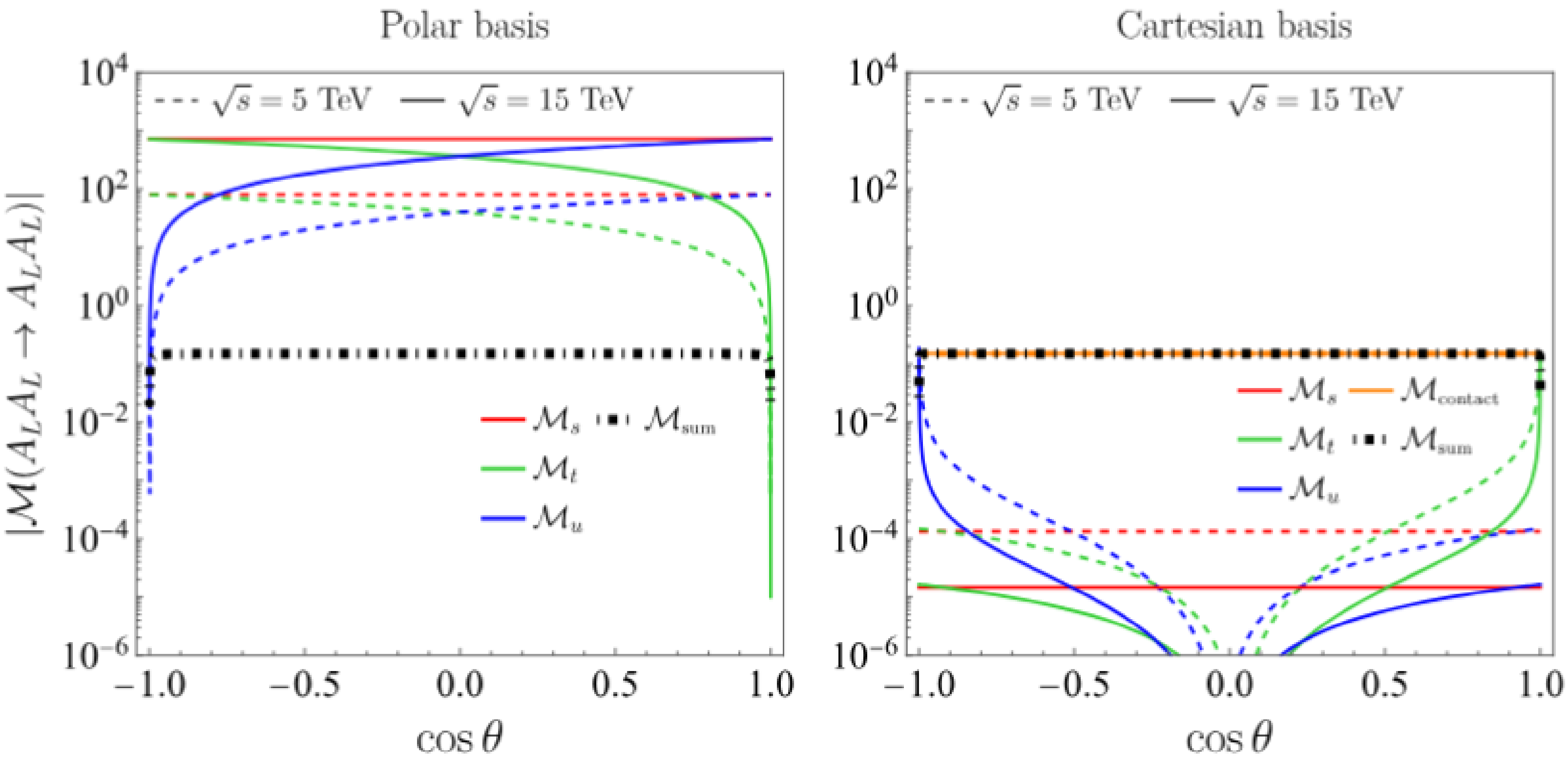

$\lambda = 0.1$

$m = 91\text{GeV}$

$m_h = 125\text{GeV}$

Identification of high-energy divergences

$$\text{5V } R_\xi \qquad \epsilon^M(p,0) = \frac{1}{E}(0,0,0,m,-ip),$$

$$\text{4V } R_\xi \qquad \epsilon^M_0(p,0) = \epsilon^M(p,0) + \frac{p}{m}\frac{P^M}{E} = \frac{1}{m}(p,0,0,E,0)$$

$$\text{FD or (EQ)} \qquad \epsilon^M_{\text{FD}}(p,0) = \epsilon^M(p,0) + \frac{(p-E)}{m}\frac{P^M}{E} = \frac{1}{m}(p-E,0,0,E-p,-im)$$

Identification of high-energy divergences

5V R_ξ $\quad \epsilon^M(p,0) = \frac{1}{E}(0,0,0,m,-ip),$

Second origin of divergences

4V R_ξ $\quad \epsilon_0^M(p,0) = \epsilon^M(p,0) + \frac{p}{m}\boxed{\frac{P^M}{E}} = \frac{1}{m}(p,0,0,E,0)$

FD or (EQ) $\quad \epsilon_{\rm FD}^M(p,0) = \epsilon^M(p,0) + \boxed{\frac{(p-E)}{m}}\boxed{\frac{P^M}{E}} = \frac{1}{m}(p-E,0,0,E-p,-im)$

Red box : Unphysical contribution

Blue box : Eliminating the contribution at high energy

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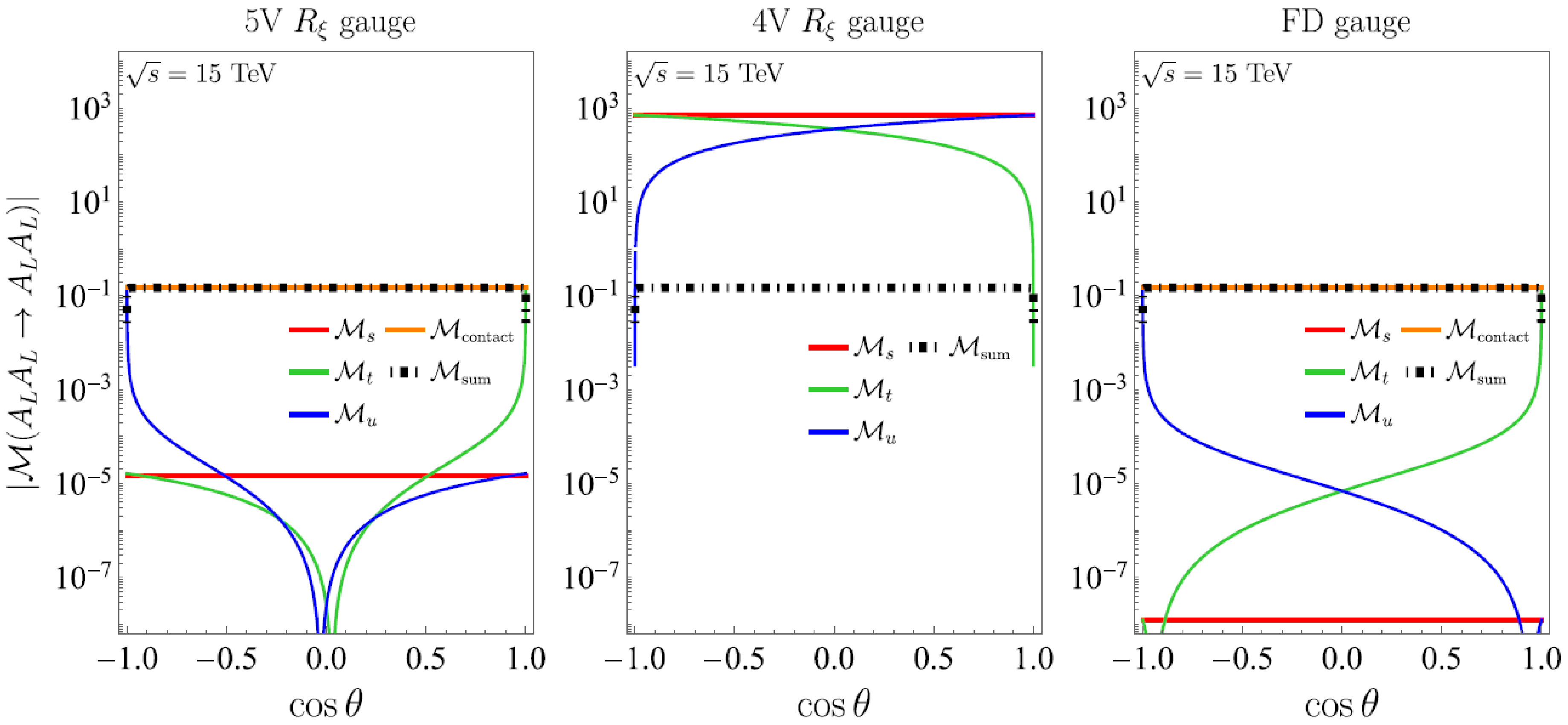

$$\epsilon^M(p,0) = \frac{1}{E}(0,0,0,m,-ip),$$
$$\epsilon_0^M(p,0) = \epsilon^M(p,0) + \frac{p}{m}\frac{P^M}{E} = \frac{1}{m}(p,0,0,E,0)$$
$$\epsilon_{\rm FD}^M(p,0) = \epsilon^M(p,0) + \frac{(p-E)}{m}\frac{P^M}{E} = \frac{1}{m}(p-E,0,0,E-p,-im)$$

Summary

1. 5V R_ξ gauge
2. Tree-level high-energy divergences
3. Advantages
   × Realizable within the Lagrangian ⇒ Intuitive derivation of the propagator
   × Massless-like structure ($P^M T_M(A) = 0$)
   ⇒ Criterion for assessing the degree of divergence from other gauges
   ⇒ Clear illustration of the GBET at the level of $\epsilon^M(P,0)$
   × No gauge cancellation ⇒ Improving numerical calculations
   × Partial equivalence with the 4V R_ξ gauge ⇒ Implications for renormalizability
4. Extention to the non-Abelian case
5. Generalization (suggested by Prof. A. Wulzer)

$$\epsilon_{\rm general}^M(p,0) = \epsilon^M(p,0) + \mathcal{O}(P)\frac{P^M}{m} \quad \text{and} \quad \epsilon_{\rm general}^M(p,\pm) = ???$$

Thank you!

Backup

Connecting the 5V R_ξ gauge with other gauges

[Polar basis]

$$\mathcal{L}(x) = -\frac{1}{4}F^{MN}F_{MN} + \frac{1}{2}(\partial^\mu h)^2 - \frac{1}{2}m_h^2 h^2 + \left(\frac{\lambda^{1/2}m^2}{\sqrt{2}m_h}h + \frac{\lambda m^2}{4m_h^2}h^2\right)\left(A^\mu - \frac{1}{m}\partial^\mu A^4\right)^2 - \frac{\lambda^{1/2}m_h}{2\sqrt{2}}h^3 - \frac{\lambda}{16}h^4 - \frac{(\partial_M A^M)^2}{2\xi} - \bar{c}(\partial^M\partial_M)c$$

Vertices

$$\Gamma^{[AAh]}_{M_1M_2}(P_1,P_2) = \frac{\sqrt{2}\lambda m^2}{m_h P_{1,4}P_{2,4}}\begin{pmatrix} P_{1,4}P_{2,4}\,g_{\mu_1\mu_2} & P_{1,4}P_{2\mu_1} \\ P_{2,4}P_{1\mu_2} & P_1^\mu P_{2\mu}\end{pmatrix},$$

$$\Gamma^{[AAhh]}_{M_1M_2}(P_1,P_2) = \frac{\lambda m^2}{m_h^2 P_{1,4}P_{2,4}}\begin{pmatrix} P_{1,4}P_{2,4}\,g_{\mu_1\mu_2} & P_{1,4}P_{2\mu_1} \\ P_{2,4}P_{1\mu_2} & P_1^\mu P_{2\mu}\end{pmatrix},$$

$$\Gamma^{[hhh]} = -\frac{3\lambda^{1/2}m_h}{\sqrt{2}},$$

$$\Gamma^{[hhhh]} = -\frac{3\lambda}{2},$$

$$\left(\begin{array}{c|c} 4\times 4 & 4\times 1 \\ \hline 1\times 4 & 1\times 1 \end{array}\right)$$

$$\begin{aligned}
\mathbf{s}A^M &= [iQ_B, A^M] = \partial^M c,\\
\mathbf{s}\,h &= [iQ_B, h] = 0,\\
\mathbf{s}B &= [iQ_B, B] = 0,\\
\mathbf{s}\,\bar{c} &= \{iQ_B, \bar{c}\} = B,\\
\mathbf{s}\,c &= \{iQ_B, c\} = 0,
\end{aligned}$$

5V Ward identities

$$P_1^{M_1}\Gamma_{M_1M_2}(P_1,P_2) = P_2^{M_2}\Gamma_{M_1M_2}(P_1,P_2) = 0 \qquad \text{for incoming } A\text{'s},$$

$$P_1^{M_1*}\Gamma_{M_1M_2}(-P_1^*,-P_2^*) = P_2^{M_2*}\Gamma_{M_1M_2}(-P_1^*,-P_2^*) = 0 \qquad \text{for outgoing } A\text{'s},$$

Connecting the 5V R_ξ gauge with other gauges

[Polar basis]

Amplitudes from 5V to 4V gauges

$(Q^M : T^M \text{ or } U^M)$

$$\Gamma^{[AAh]}_{\mu_2 4}(P_2,Q)\,(-g^{44})\,\Gamma^{[AAh]}_{4\mu_1}(-Q^*,P_1)$$
$$= \Gamma^{[AAh]}_{\mu_2\sigma}(P_2,Q)\left(\frac{Q^\sigma Q^{\rho *}}{m^2}\right)\Gamma^{[AAh]}_{\rho\mu_1}(-Q^*,P_1),$$

$$\Gamma^{[AAh]}_{\mu_2 S}(P_2,Q)\left(\frac{-g^{SR}}{Q^2}\right)\Gamma^{[AAh]}_{R\mu_1}(-Q^*,P_1)$$
$$= \Gamma^{[AAh]}_{\mu_2\sigma}(P_2,Q)\left(\frac{-g_{\sigma\rho}+\frac{(1-\xi)Q^\sigma Q^{\rho *}}{Q^2+(1-\xi)m^2}}{Q^2}\right)\Gamma^{[AAh]}_{\rho\mu_1}(-Q^*,P_1)$$
$$+\,\Gamma^{[AAh]}_{\mu_2 4}(P_2,Q)\left(\frac{1}{Q^2+(1-\xi)m^2}\right)\Gamma^{[AAh]}_{4\mu_1}(-Q^*,P_1)$$

5V R_ξ $\quad \epsilon^M(p,0) = \frac{1}{E}(0,0,0,m,-ip),$

4V R_ξ $\quad \epsilon_0^M(p,0) = \epsilon^M(p,0) + \frac{p}{m}\frac{P^M}{E} = \frac{1}{m}(p,0,0,E,0)$

FD or (EQ) $\quad \epsilon_{\rm FD}^M(p,0) = \epsilon^M(p,0) + \frac{(p-E)}{m}\frac{P^M}{E} = \frac{1}{m}(p-E,0,0,E-p,-im)$

$$\Gamma^{[AAh]}_{M_1M_2}(P_1,P_2) = \frac{\sqrt{2\lambda}m^2}{m_h P_{1,4}P_{2,4}}\begin{pmatrix} P_{1,4}P_{2,4}\,g_{\mu_1\mu_2} & P_{1,4}P_{2\mu_1} \\ P_{2,4}P_{1\mu_2} & P_1^\mu P_{2\mu}\end{pmatrix},$$

$$\Gamma^{[AAhh]}_{M_1M_2}(P_1,P_2) = \frac{\lambda m^2}{m_h^2 P_{1,4}P_{2,4}}\begin{pmatrix} P_{1,4}P_{2,4}\,g_{\mu_1\mu_2} & P_{1,4}P_{2\mu_1} \\ P_{2,4}P_{1\mu_2} & P_1^\mu P_{2\mu}\end{pmatrix},$$

$$\Gamma^{[hhh]} = -\frac{3\lambda^{1/2}m_h}{\sqrt{2}},$$
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5V Ward identities

$P_1^{M_1}\Gamma_{M_1M_2}(P_1,P_2) = P_2^{M_2}\Gamma_{M_1M_2}(P_1,P_2) = 0$ for incoming A's,

$P_1^{M_1 *}\Gamma_{M_1M_2}(-P_1^*,-P_2^*) = P_2^{M_2 *}\Gamma_{M_1M_2}(-P_1^*,-P_2^*) = 0$ for outgoing A's,

$$S^M = ((P_1+P_2)^\mu, im_h)$$
$$T^M = ((P_1-P_3)^\mu, im_h)$$
$$U^M = ((P_1-P_4)^\mu, im_h)$$

Connecting the 5V R_ξ gauge with other gauges

[Polar basis]

Amplitudes from 5V to FD (EQ) gauges

$(Q^M : T^M \text{ or } U^M)$

$$\Gamma^{[AAh]}_{\mu_2 S}(P_2, Q)\,(-g^{SR})\,\Gamma^{[AAh]}_{R\mu_1}(-Q^*, P_1)$$
$$= \Gamma^{[AAh]}_{\mu_2 S}(P_2, Q)\left(-g^{SR} + \frac{\tilde{\epsilon}^S(q,0)Q^{R*} + Q^S\tilde{\epsilon}^{R*}(q,0)}{|\sqrt{Q^\mu Q^*_\mu}|}\right)$$
$$\times\,\Gamma^{[AAh]}_{R\mu_1}(-Q^*, P_1),$$

5V R_ξ $\quad \epsilon^M(p,0) = \frac{1}{E}(0,0,0,m,-ip),$

4V R_ξ $\quad \epsilon^M_0(p,0) = \epsilon^M(p,0) + \frac{p}{m}\frac{P^M}{E} = \frac{1}{m}(p,0,0,E,0)$

FD or (EQ) $\quad \epsilon^M_{\rm FD}(p,0) = \epsilon^M(p,0) + \frac{(p-E)}{m}\frac{P^M}{E} = \frac{1}{m}(p-E,0,0,E-p,-im)$

$$\Gamma^{[AAh]}_{M_1M_2}(P_1,P_2) = \frac{\sqrt{2\lambda}m^2}{m_h P_{1,4}P_{2,4}}\begin{pmatrix} P_{1,4}P_{2,4}\,g_{\mu_1\mu_2} & P_{1,4}P_{2\mu_1} \\ P_{2,4}P_{1\mu_2} & P_1^\mu P_{2\mu}\end{pmatrix},$$

$$\Gamma^{[AAhh]}_{M_1M_2}(P_1,P_2) = \frac{\lambda m^2}{m_h^2 P_{1,4}P_{2,4}}\begin{pmatrix} P_{1,4}P_{2,4}\,g_{\mu_1\mu_2} & P_{1,4}P_{2\mu_1} \\ P_{2,4}P_{1\mu_2} & P_1^\mu P_{2\mu}\end{pmatrix},$$

$$\Gamma^{[hhh]} = -\frac{3\lambda^{1/2}m_h}{\sqrt{2}},$$
$$\Gamma^{[hhhh]} = -\frac{3\lambda}{2},$$

5V Ward identities

$$P_1^{M_1}\Gamma_{M_1M_2}(P_1,P_2) = P_2^{M_2}\Gamma_{M_1M_2}(P_1,P_2) = 0 \qquad \text{for incoming } A\text{'s},$$
$$P_1^{M_1*}\Gamma_{M_1M_2}(-P_1^*,-P_2^*) = P_2^{M_2*}\Gamma_{M_1M_2}(-P_1^*,-P_2^*) = 0 \qquad \text{for outgoing } A\text{'s},$$

$$S^M = ((P_1+P_2)^\mu, im_h)$$
$$T^M = ((P_1-P_3)^\mu, im_h)$$
$$U^M = ((P_1-P_4)^\mu, im_h)$$

Identification of the divergences

[Cartesian basis]

$$\Gamma^{[\boldsymbol{AAh}]}_{M_1M_2}(P_1,P_2)=\frac{\lambda^{1/2}}{\sqrt{2}m_h}\begin{pmatrix}2m^2g_{\mu_1\mu_2} & im(2P_{2\mu_1}+P_{1\mu_1})\\ im(2P_{1\mu_2}+P_{2\mu_2}) & -m_h^2\end{pmatrix},$$

$$\Gamma^{[\boldsymbol{AAhh}]}_{MN}=\frac{\lambda}{2}\begin{pmatrix}\frac{2m^2}{m_h^2}g_{\mu\nu} & 0\\ 0 & -1\end{pmatrix}$$

$$\begin{aligned}\Gamma^{[\boldsymbol{AAAA}]}_{MNRS}=&-\frac{\lambda m^2}{m_h^2}(g_{+MN}g_{-RS}+g_{+MR}g_{-NS}+g_{+MS}g_{+NR}\\&+g_{+NR}g_{-MS}+g_{+NS}g_{-MR}+g_{+RS}g_{+MN})\\&-\frac{3\lambda}{2}g_{-MN}g_{-RS},\end{aligned}$$

$$\Gamma^{[\boldsymbol{hhh}]}=-\frac{3\lambda^{1/2}m_h}{\sqrt{2}},$$

$$\Gamma^{[\boldsymbol{hhhh}]}=-\frac{3}{2}\lambda,$$

$$g_{+MN}=\mathrm{diag}(1,-1,-1,-1,0)$$
$$g_{-MN}=\mathrm{diag}(0,0,0,0,-1)$$

$$\begin{aligned}\mathcal{L}(x)=&-\frac{1}{4}(\boldsymbol{A}^{MN})^2+\frac{1}{2}(\partial^\mu\boldsymbol{h})^2-\frac{1}{2}m_h^2\boldsymbol{h}^2\\&+\frac{\lambda^{1/2}}{\sqrt{2}m_h}\boldsymbol{h}\left(m^2\boldsymbol{A}^\mu\boldsymbol{A}_\mu-m(2\boldsymbol{A}^\mu\partial_\mu\boldsymbol{A}^4+\boldsymbol{A}^4\partial_\mu\boldsymbol{A}^\mu)-\frac{m_h^2}{2}(\boldsymbol{A}^4)^2\right)\\&+\frac{\lambda}{8}\boldsymbol{h}^2\left(\frac{2m^2}{m_h^2}\boldsymbol{A}^\mu\boldsymbol{A}_\mu-(\boldsymbol{A}^4)^2\right)-\frac{\lambda^{1/2}m_h}{2\sqrt{2}}\boldsymbol{h}^3-\frac{\lambda}{16}\boldsymbol{h}^4\\&+\frac{\lambda}{16}(\boldsymbol{A}^4)^2\left(\frac{4m^2}{m_h^2}\boldsymbol{A}^\mu\boldsymbol{A}_\mu-(\boldsymbol{A}^4)^2\right),\end{aligned}$$

$$\begin{aligned}\mathbf{s}\boldsymbol{A}^\mu&=[iQ_B,\boldsymbol{A}^\mu]=\partial^\mu c,\\ \mathbf{s}\boldsymbol{A}^4&=[iQ_B,\boldsymbol{A}^4]=e(v+\boldsymbol{h})c,\\ \mathbf{s}\,\boldsymbol{h}&=[iQ_B,\boldsymbol{h}]=-e\boldsymbol{A}^4c,\\ \mathbf{s}B&=[iQ_B,B]=0,\\ \mathbf{s}\,\bar{c}&=\{iQ_B,\bar{c}\}=B,\\ \mathbf{s}\,c&=\{iQ_B,c\}=0,\end{aligned}$$

Identification of the divergences

[Cartesian basis]

$$\Gamma^{[\boldsymbol{AAh}]}_{\mu_2 4}(P_2,Q)\,(-g^{44})\,\Gamma^{[\boldsymbol{AAh}]}_{4\mu_1}(-Q^*,P_1)$$
$$= \Gamma^{[\boldsymbol{AAh}]}_{\mu_2\sigma}(P_2,Q)\left(\frac{(Q^\sigma + P_2^\sigma/2)(Q^{\rho *} - P_1^{\rho *}/2)}{m^2}\right)\Gamma^{[\boldsymbol{AAh}]}_{\rho\mu_1}(-Q^*,P_1),$$

$$\Gamma^{[\boldsymbol{AAh}]}_{\mu_2 4}(P_2,Q)\,(-g^{44})\,\Gamma^{[\boldsymbol{AAh}]}_{4\mu_1}(-Q^*,P_1)$$
$$= \Gamma^{[\boldsymbol{AAh}]}_{\mu_2\sigma}(P_2,Q)\left(\frac{(Q^\sigma + P_2^\sigma/2)(Q^{\rho *} - P_1^{\rho *}/2)}{m^2}\right)\Gamma^{[\boldsymbol{AAh}]}_{\rho\mu_1}(-Q^*,P_1),$$

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$$\Gamma^{[\boldsymbol{AAh}]}_{M_1M_2}(P_1,P_2) = \frac{\lambda^{1/2}}{\sqrt{2}m_h}\begin{pmatrix} 2m^2 g_{\mu_1\mu_2} & im(2P_{2\mu_1}+P_{1\mu_1}) \\ im(2P_{1\mu_2}+P_{2\mu_2}) & -m_h^2 \end{pmatrix},$$

$$\Gamma^{[\boldsymbol{AAhh}]}_{MN} = \frac{\lambda}{2}\begin{pmatrix} \frac{2m^2}{m_h^2}g_{\mu\nu} & 0 \\ 0 & -1 \end{pmatrix}$$

$$\Gamma^{[\boldsymbol{AAAA}]}_{MNRS} = -\frac{\lambda m^2}{m_h^2}(g_{+MN}g_{-RS} + g_{+MR}g_{-NS} + g_{+MS}g_{+NR}$$
$$+ g_{+NR}g_{-MS} + g_{+NS}g_{-MR} + g_{+RS}g_{+MN})$$
$$- \frac{3\lambda}{2}g_{-MN}g_{-RS},$$

$$\Gamma^{[\boldsymbol{hhh}]} = -\frac{3\lambda^{1/2}m_h}{\sqrt{2}},$$

$$\Gamma^{[\boldsymbol{hhhh}]} = -\frac{3}{2}\lambda,$$

Nonconserved 5V Ward identities

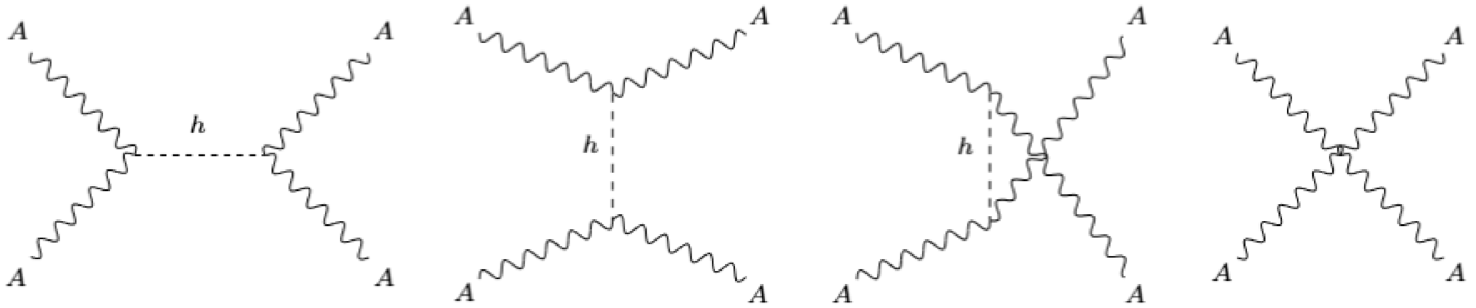
$$Q^{R*}\Gamma^{[\boldsymbol{AAh}]}_{RM_1}(-Q^*,P_1) = \frac{2\sqrt{2}\eta m^2}{m_h}P^*_{1M_1},$$

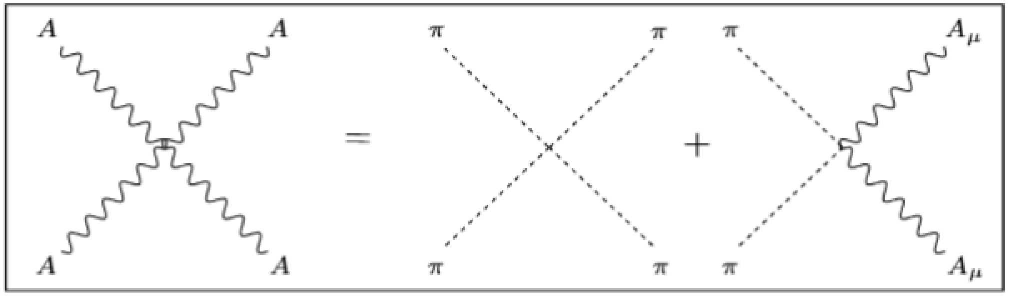
$$Q^S\Gamma^{[\boldsymbol{AAh}]}_{M_2S}(P_2,Q) = \frac{2\sqrt{2}\eta m^2}{m_h}P^*_{2M_2},$$

$$S^M = ((P_1+P_2)^\mu, im_h)$$
$$T^M = ((P_1-P_3)^\mu, im_h)$$
$$U^M = ((P_1-P_4)^\mu, im_h)$$

$Diagrams$

$[A_L A_L \rightarrow A_L A_L]$





$[A_L A_L \rightarrow hh]$