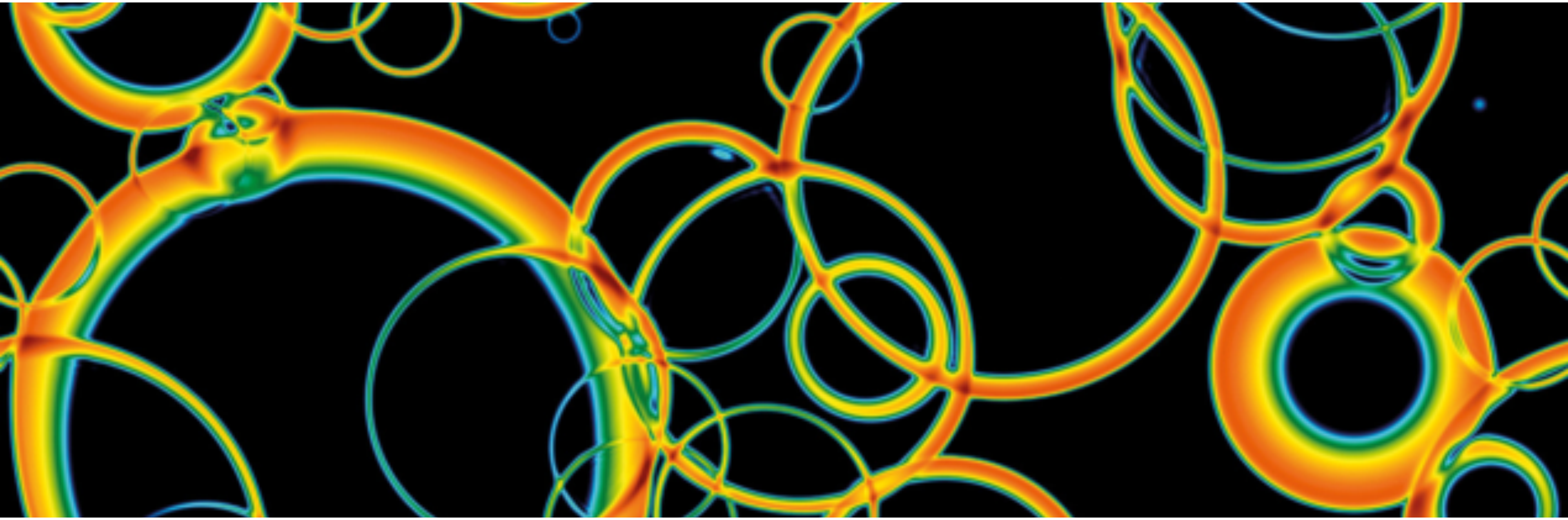
Perturbative aspects of the electroweak phase transitions in the cxSM

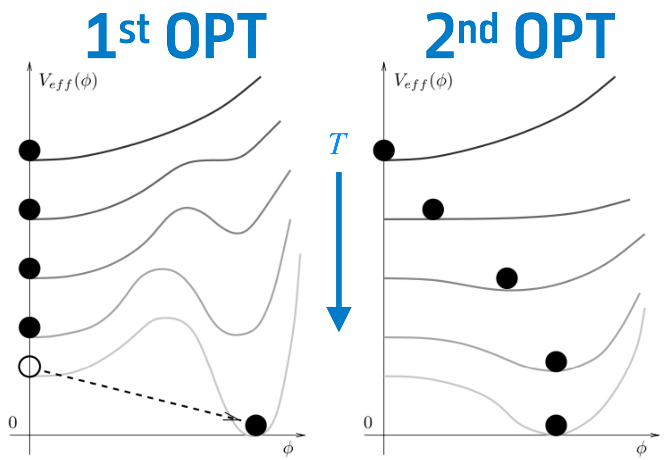


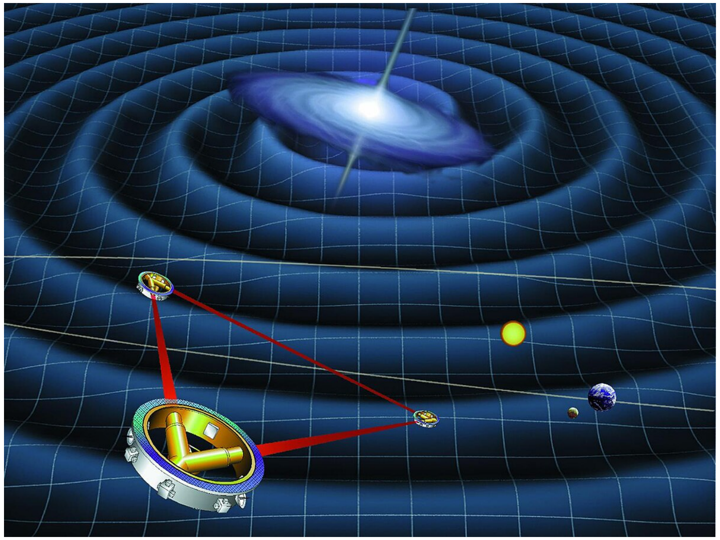
Thomas Biekötter, **Andrii Dashko**, Maximilian Löschner, Georg Weiglein

Durham, PASCOS-2025, 22/08/2025

Motivation: First Order Phase Transition (FOPT) in the Early Universe

- Mechanism to satisfy Sakharov's conditions for generation matter-antimatter asymmetry
- Perfect candidate for gravitation wave detection in close future experiments (LISA, etc)
- First order phase transitions = hope for perturbative description





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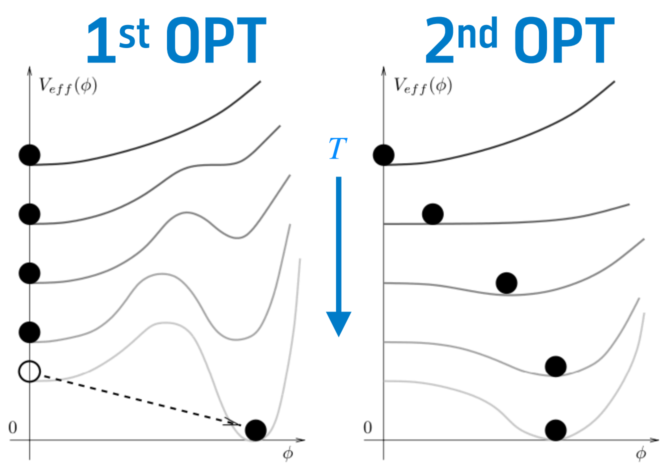
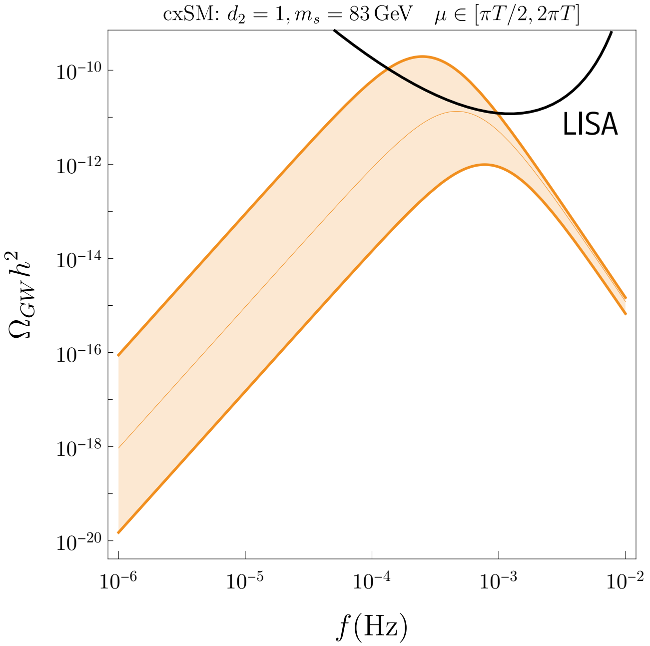
- **Large uncertainties in theoretical predictions**

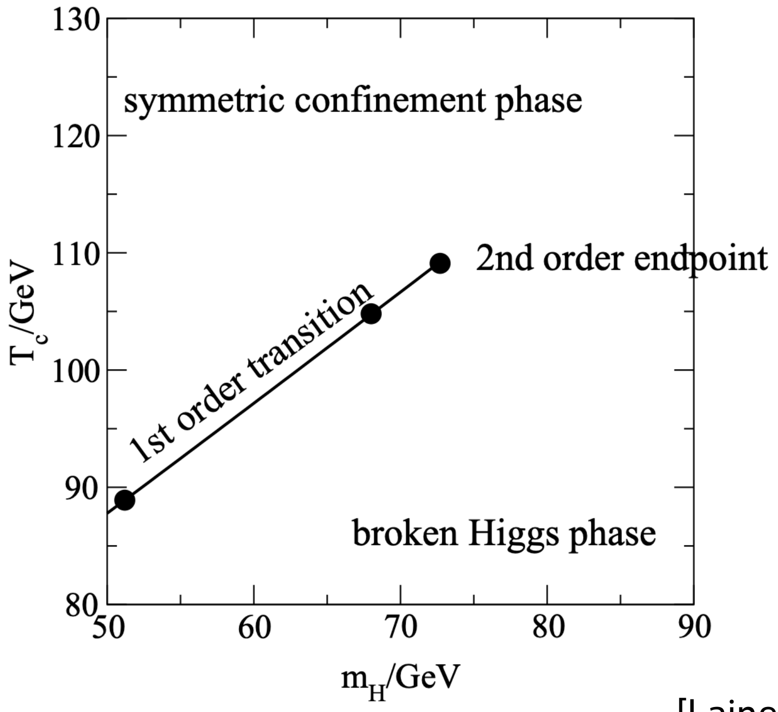




Motivation: Extended scalar sector

- Higgs is too light: no electroweak FOPT in the SM

- Naturally occurring in most of extensions of Higgs sectors (singlets, doublets, SUSY, SMEFT etc)



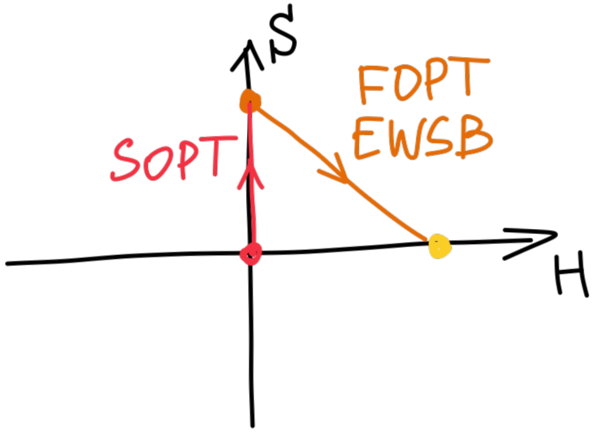
[Laine, 2000]

Benchmark model: cxSM

- U(1) symmetric complex singlet extension of the SM as a realistic toy model

$$\mathcal{L}_{cxSM} \supset D_\mu \Phi^\dagger D^\mu \Phi - \mu_h^2 \Phi^\dagger \Phi - \lambda_h (\Phi^\dagger \Phi)^2 + \\ \frac{1}{2}|\partial^\mu S|^2 - \frac{1}{2}\mu_s^2 |S|^2 - \frac{\lambda_s}{4}|S|^4 - \frac{1}{2}\lambda_{hs}|S|^2 \Phi^\dagger \Phi.$$

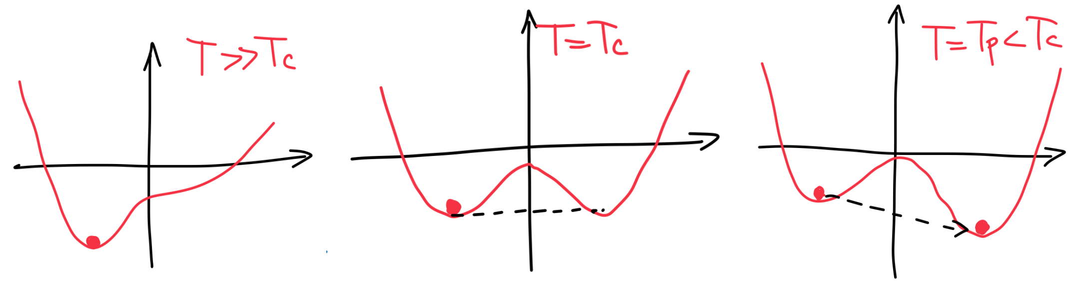
- Zero singlet vev at zero temperature (“flip-flop” transition)
- Collider “nightmare scenario”
- New scalars: possible Dark Matter candidates
- Extra input parameters: $m_S = m_A, \lambda_s, \lambda_{hs}$



Phase transition thermodynamics.

Free energy density = pressure = effective potential in the minimum

Critical temperature T_c



Nucleation temperature

$$\int_{t_c}^{t_n} dt \frac{\Gamma(t)}{H(t)^3} = \int_{T_n}^{T_c} \frac{dT}{T} \frac{\Gamma(T)}{H(T)^4} \sim 1,$$

Latent heat

$$L(T) = \Delta\rho = \Delta p + T\frac{d\Delta p}{dT}$$

$$L_c \equiv L(T_c) = \Delta\rho\Big|_{T=T_c} = T_c \frac{d\Delta p}{dT}\Bigg|_{T=T_c} = -T_c \frac{d\Delta V_{eff}}{dT}\Bigg|_{T=T_c}$$

Gravitational wave signal: working pipeline

Physical inputs

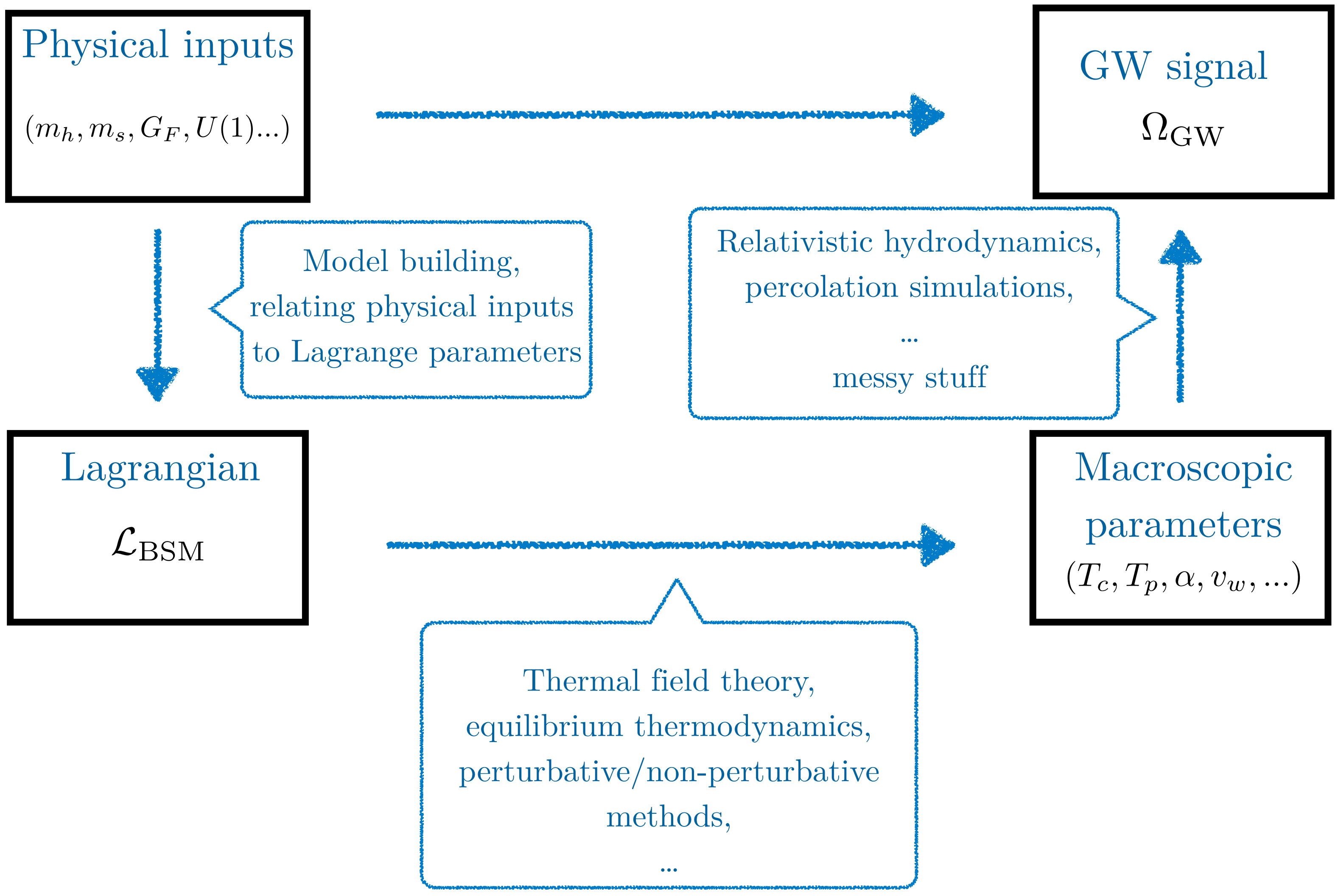
$(m_h, m_s, G_F, U(1)...)$

→

GW signal

Ω_{GW}

Gravitational wave signal: working pipeline



Effective potential

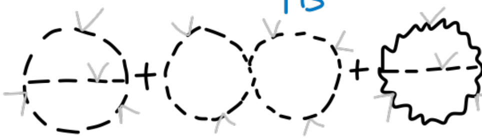
- Field-theoretic definition:

$$V_{eff}[\phi] = \frac{\Gamma[\phi]}{(vol)}$$

where $\Gamma[\phi] = W[J] - \int d^4x\, J(x)\phi(x)$ and $W[J]$ is the generating functional of connected Green's functions

$$V^{eff} = V_{tree} + V_{1\text{-}loop} + V_{2\text{-}loop} + \dots$$

$$V^{T=0}_{1\text{-}loop} = -\frac{i}{2}\int \frac{d^4k}{(2\pi)^4} \ln\left(k^2 - m_i^2\right) \underset{\overline{MS}}{=} \sum_p \frac{n_i \cdot m_i^4}{64\pi^2}\left(\log\left(\frac{m_i^2}{\mu^2}\right) - c_i\right)$$

$V^{T=0}_{2\text{-}loop} \supset$  $+ \dots$

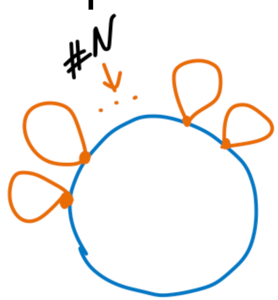
- Note: discussion for effective action follows same logic as presented here

Renormalization scale dependence & Daisy resummation

- Thermal loop corrections, which are dominant during the phase transition, introduce large renormalization scale dependence.

$$V_{therm} = \underbrace{V_{tree} + V_{1\text{-loop}}^{T=0}}_{\mu\text{-inv}} + \underbrace{V_{1\text{-loop}}^{T\neq 0}}_{\mu\text{-non-inv!}}$$

- The presence of hierarchy between hard ($\sim T$) and soft ($\sim gT$) scales requires resummation of the hard modes, which messes up with the loop order



$$\sim m^3 T \left(\frac{gT}{m}\right)^{2N}$$

* a daisy

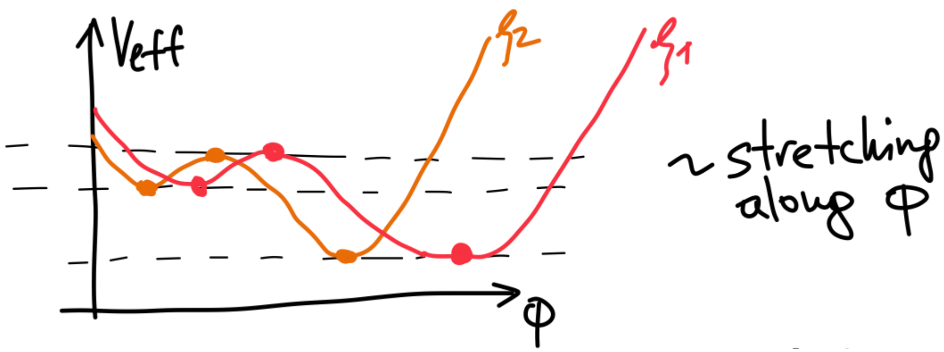
Gauge dependence

- The effective action itself is an intrinsically gauge dependent quantity, as it's defined for the non-zero source term, and the system has a non-dynamical background change.

- But, it's gauge dependent according to Nielsen identity:

$$\frac{\partial V_{eff}}{\partial \xi} = C_i(\varphi_i, \xi)\frac{\partial V_{eff}}{\partial \varphi_i}$$

- V_{eff} is gauge invariant at stationary point (extremums)



- Gauge invariant results can be obtained by systematic $\hbar$-expansion

[Nielsen, 1975]

[Patel, et.al., 2011]

Separating logarithms

- Large μ- dependence and need for resummations indicate large separation of scales

- Relevant scales:

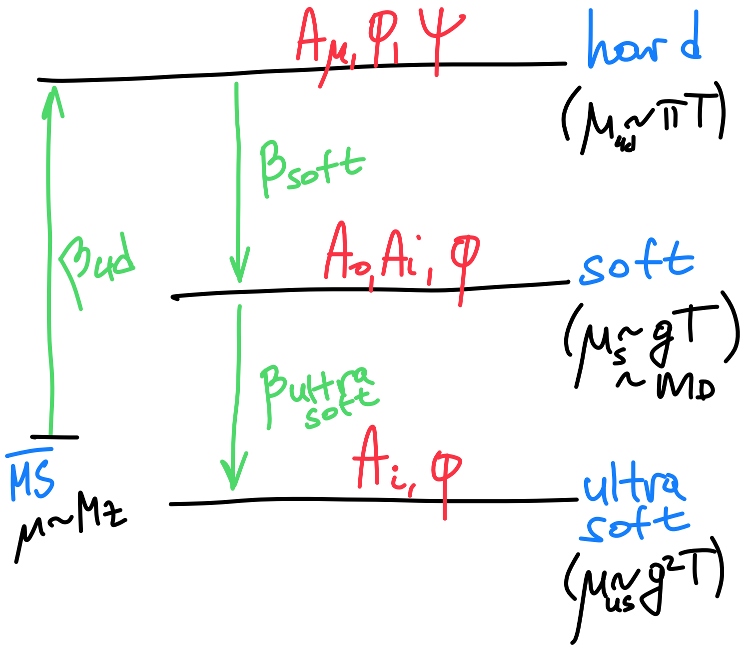
  - hard mode $\sim \pi T$
  - soft $\sim gT \sim m_D$
  - ultrasoft $\sim g^2 T$

- Basically, we have too many logarithms:

$$\log\left(\frac{\pi T}{\mu}\right), \log\left(\frac{gT}{\mu}\right), \log\left(\frac{g^2 T}{\mu}\right)$$

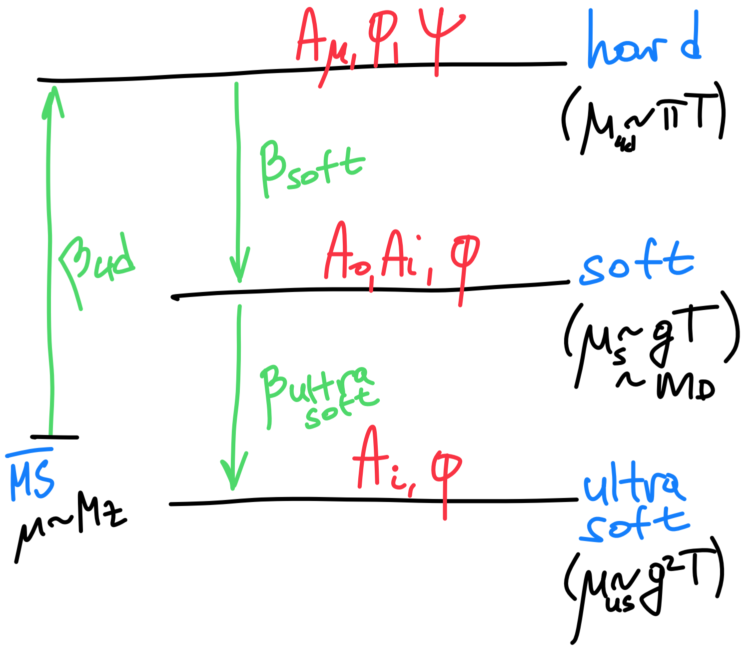
[Kajantie et al, 1994]

High-T EFT



A_μ, φ, ψ
hard
$(\mu_{4d} \sim \pi T)$
β_{soft}
β_{4d}
A_0, A_i, φ
soft
$(\mu_s \sim gT \sim m_D)$
$\beta_{ultrasoft}$
A_i, φ
ultra soft
$(\mu_{us} \sim g^2 T)$
$\overline{MS}$
$\mu \sim M_Z$

High-T EFT



A_μ, φ, ψ
hard
$(\mu_{4d} \sim \pi T)$
β_{soft}
β_{4d}
A_0, A_i, φ
soft
$(\mu_s \sim gT \sim m_D)$
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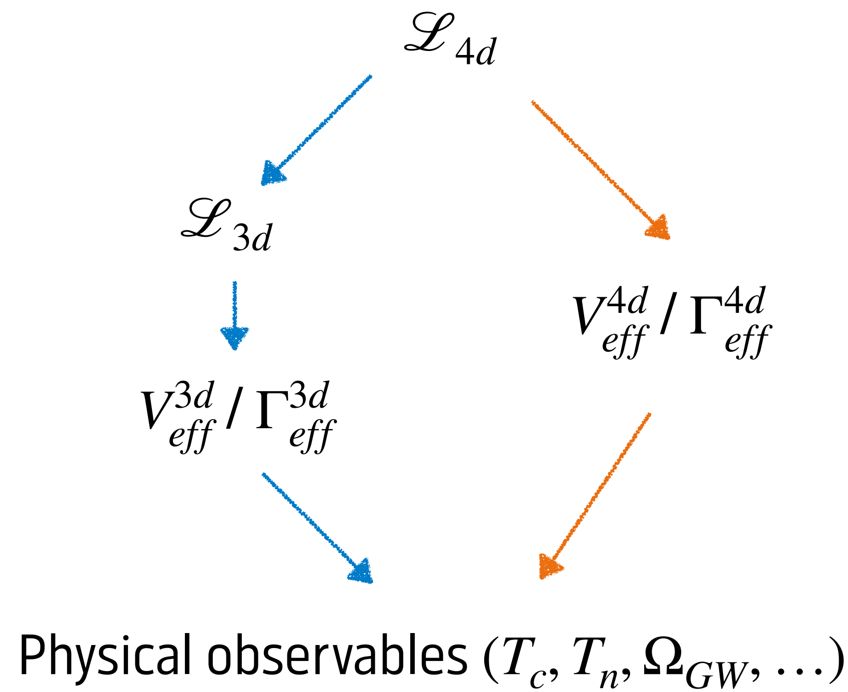
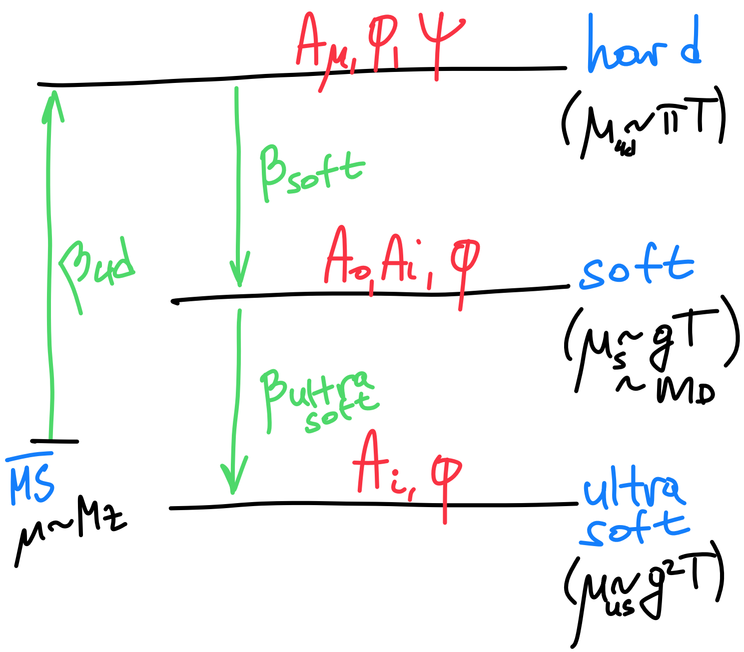
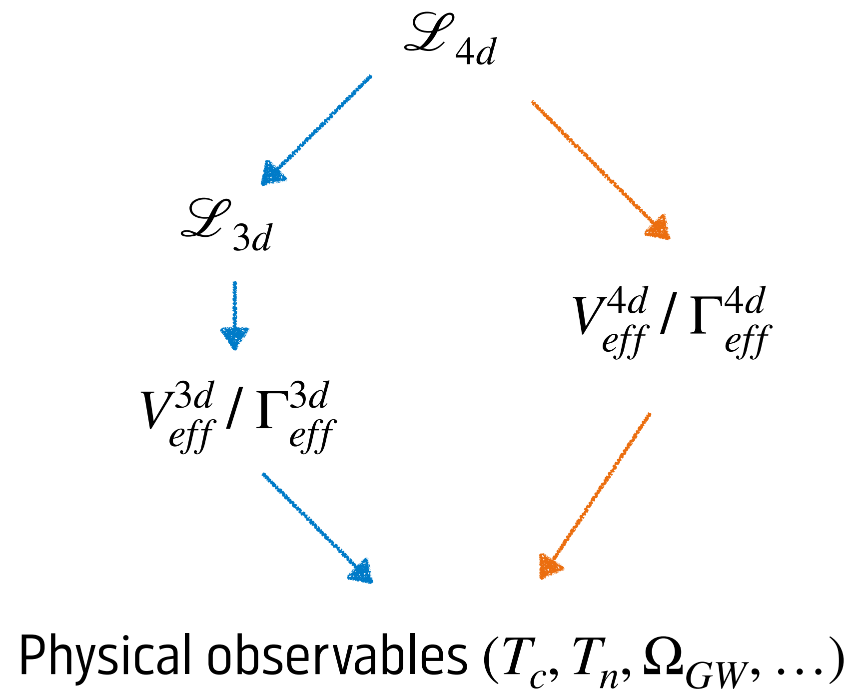


$\mathscr{L}_{4d}$
$\mathscr{L}_{3d}$
V_{eff}^{4d} / Γ_{eff}^{4d}
V_{eff}^{3d} / Γ_{eff}^{3d}
Physical observables $(T_c, T_n, \Omega_{GW}, \ldots)$

High-T EFT





High-T EFT lives in 3D:

- Simpler integrals
- Superrenormalizable theory*

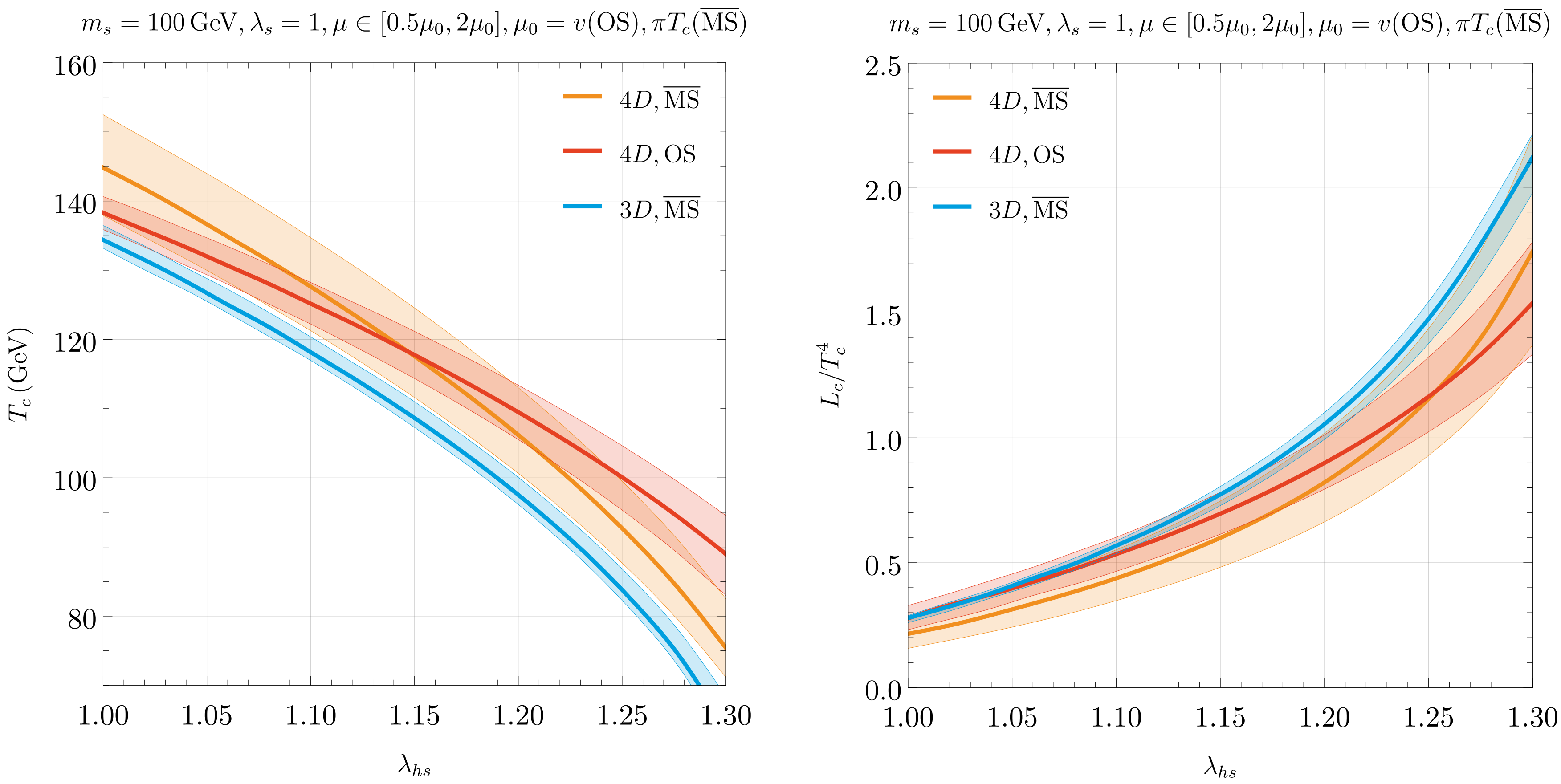
Validity of EFT truncation can be studied by higher dimensional operators in the EFT

(see talks of L. Gil and C. Fiore)

Temperature is integrated out

- Use $T = 0$ QFT framework
- Resummations (daisies, etc) are already included
- Gauge invariance is straightforward

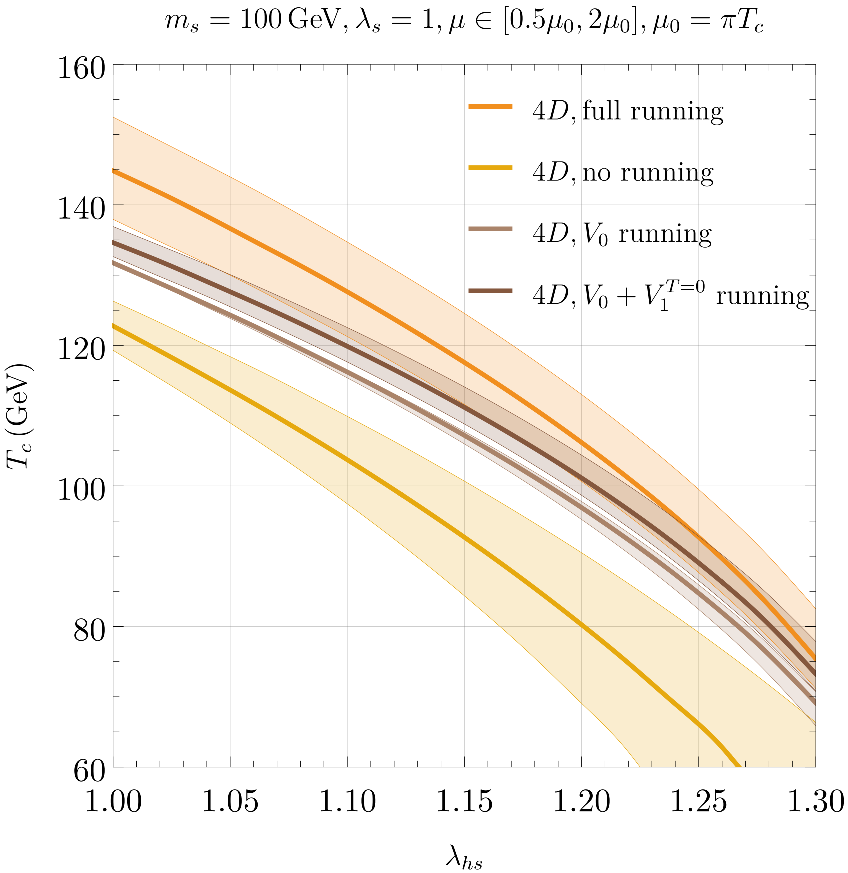
Renormalization scale dependence

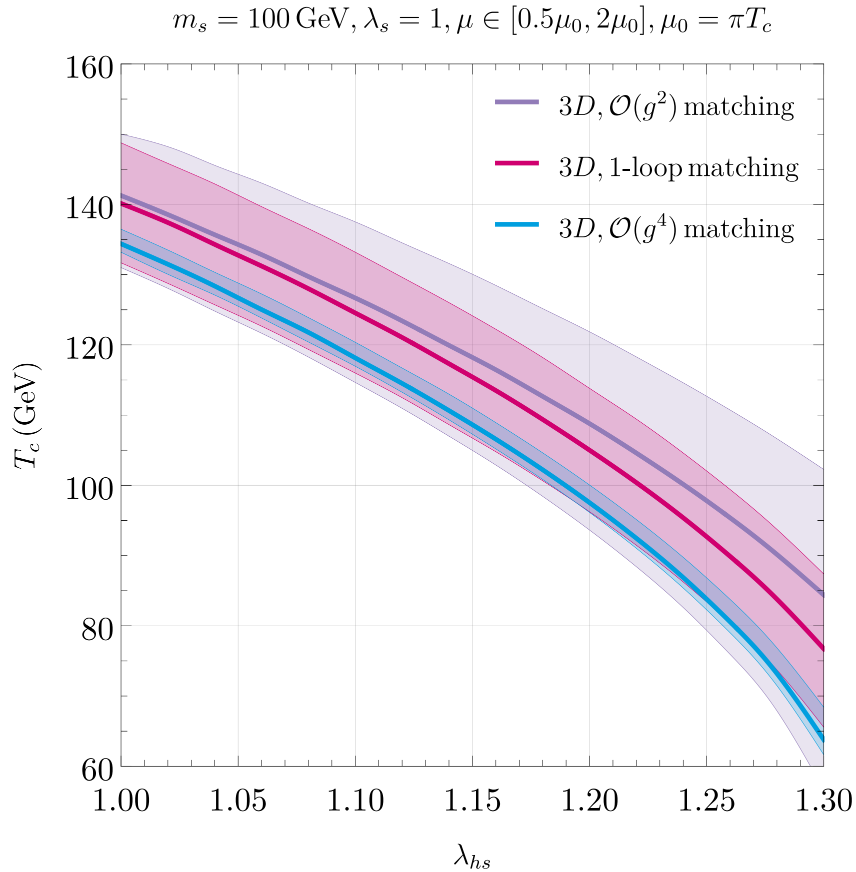


1-loop potential
+ AE-daisy resummation

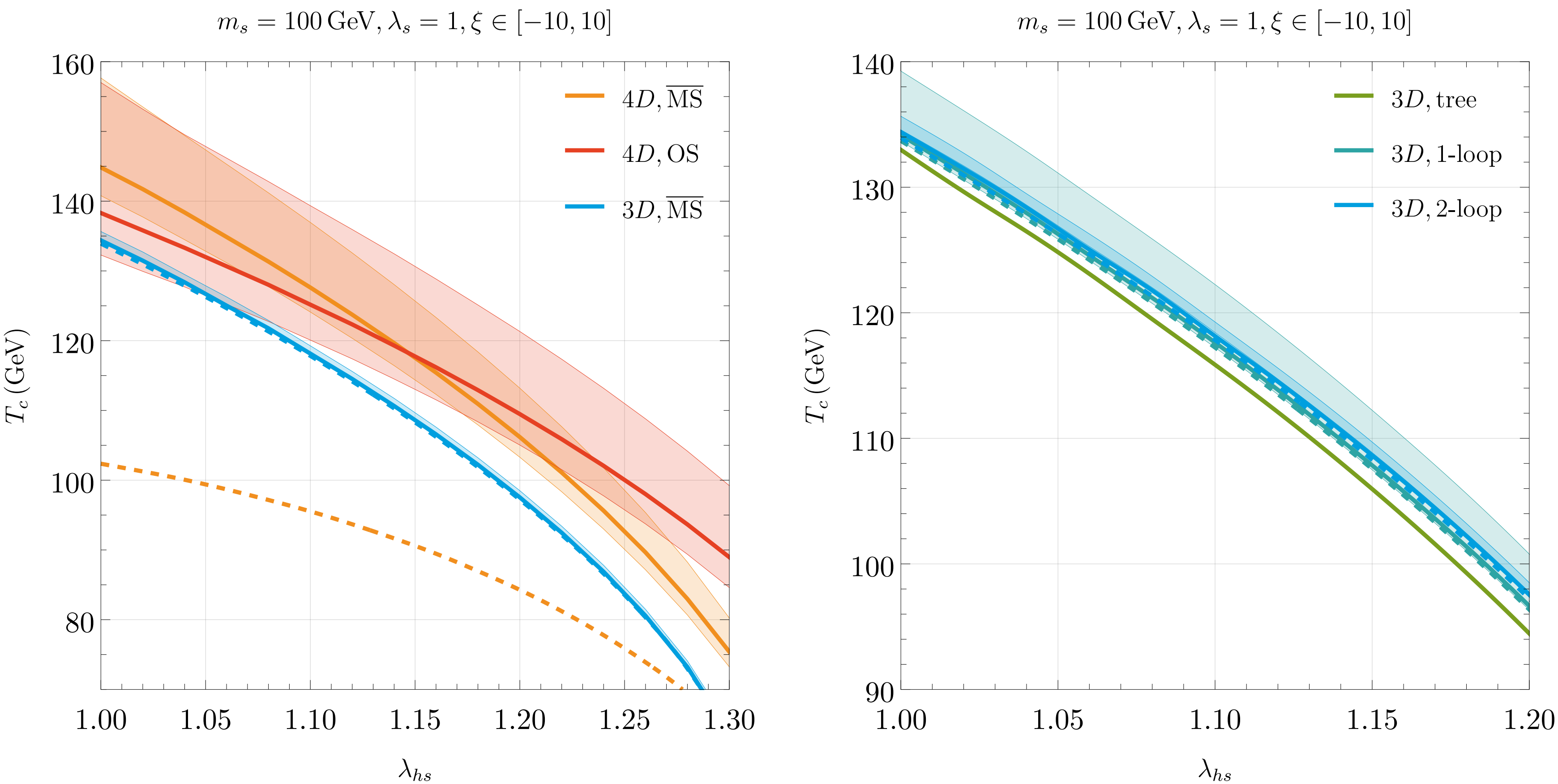
2-loop potential
+ $\mathcal{O}(g^4)$-matching

Renormalization scale dependence





Gauge dependence



Nucleation: bounce revisited

$$\Gamma(T) \approx A e^{-S_B/T}$$

$$S_B \equiv S\left[\phi_i^B\right] = 4\pi \int_0^\infty d\rho \rho^2 \left[\frac{1}{2}\left(\frac{d\phi_i^B(\rho)}{d\rho}\right)^2 + V_0(\phi_i^B(\rho))\right]$$

$$V_0 \rightarrow V_{eff}$$

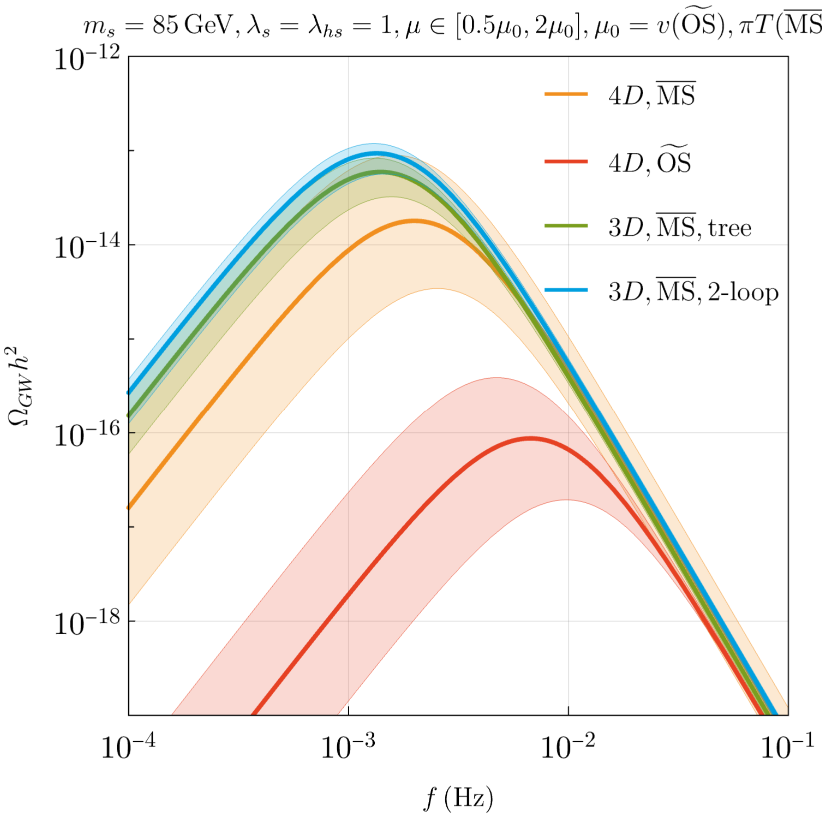
- In 4D, bounce may not even exist (again, because quantum/ thermal corrections significantly change the vacuum structure)

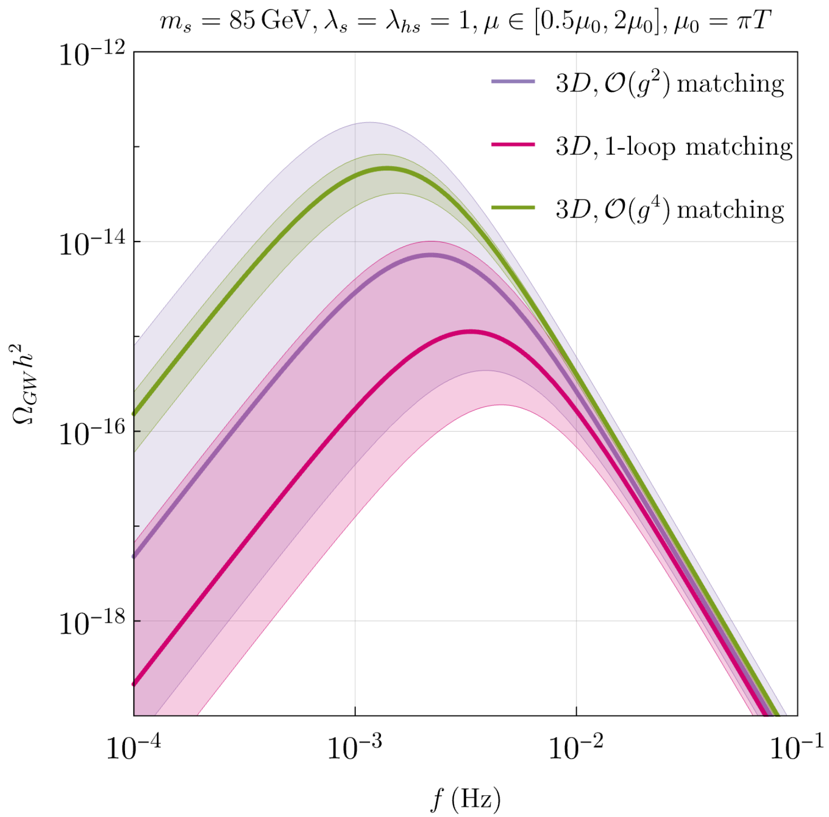
- Double counting of fluctuations
- Not a strict perturbative expansion in any parameter
- Gauge dependent

Separate **UV/IR** physics!

EFTs

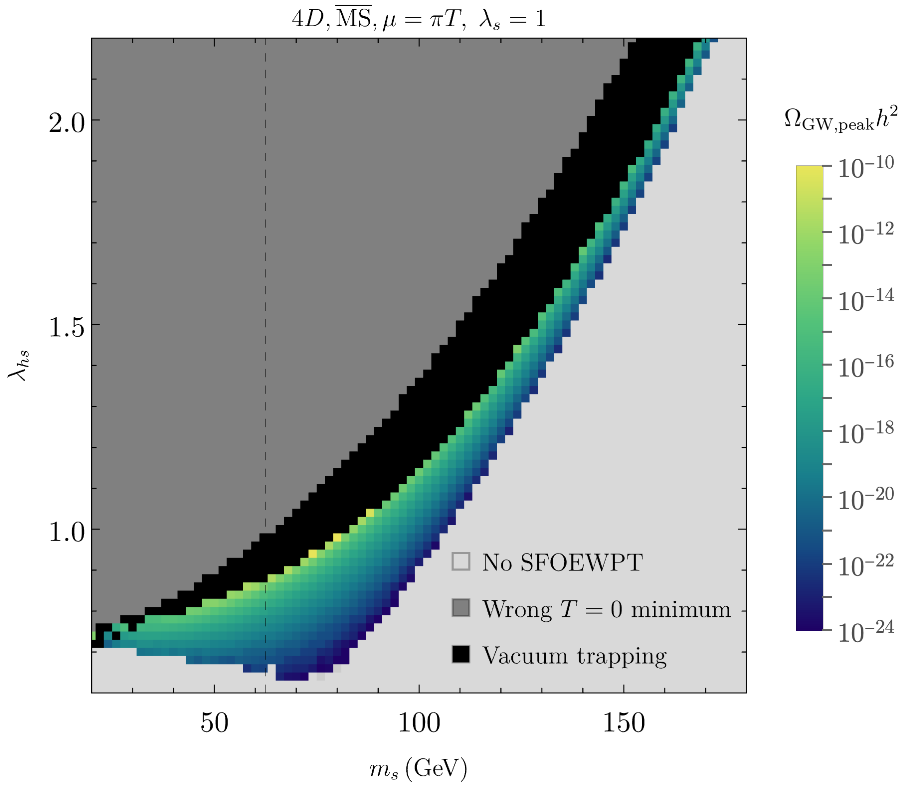
Predicted gravitational wave signal

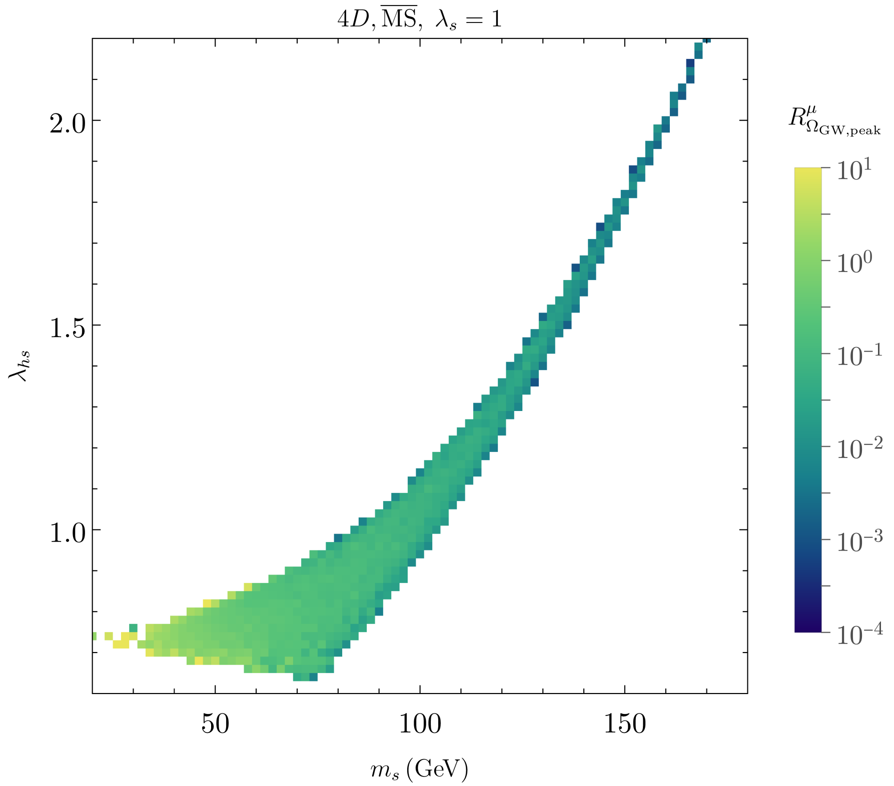




$$\Omega_{\text{GW}} = \Omega_{\text{GW}}^{\text{sound waves}}$$

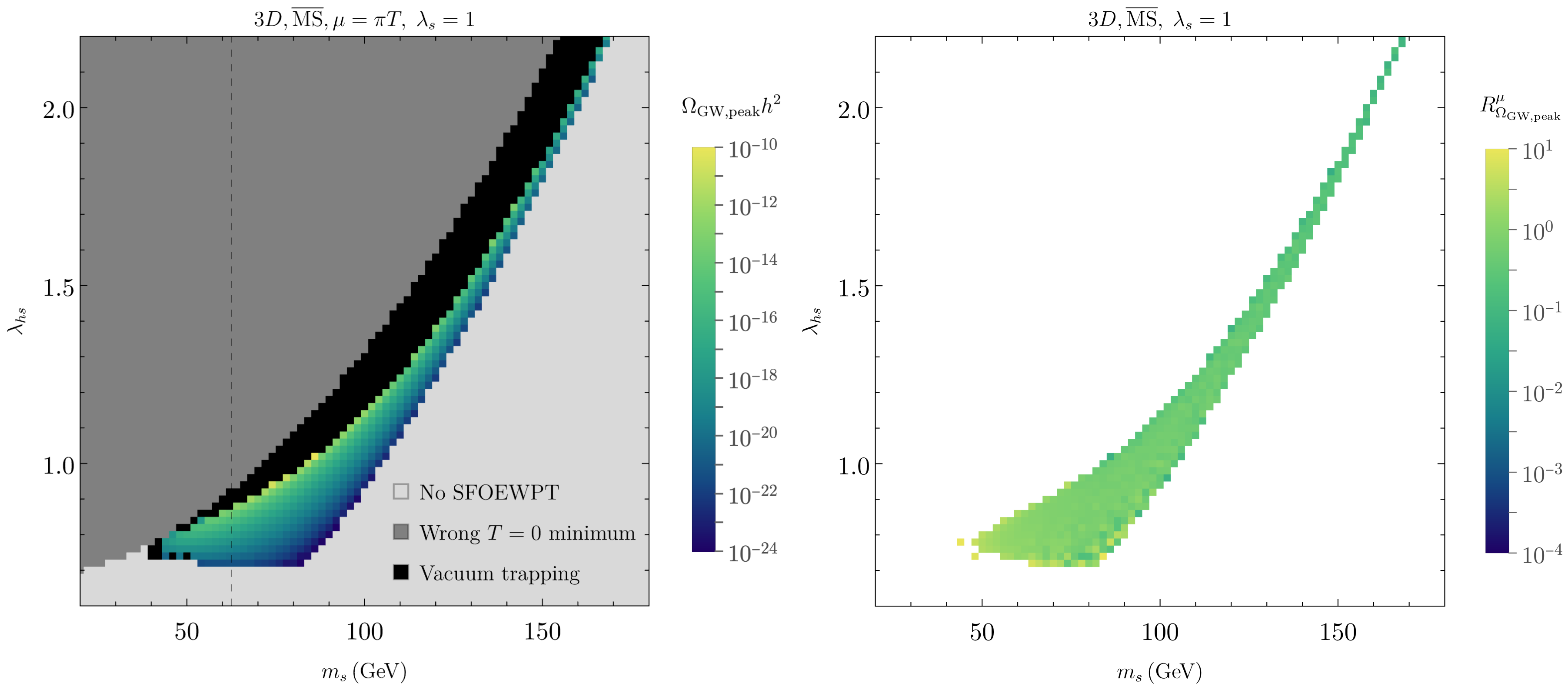
Model parameter scans



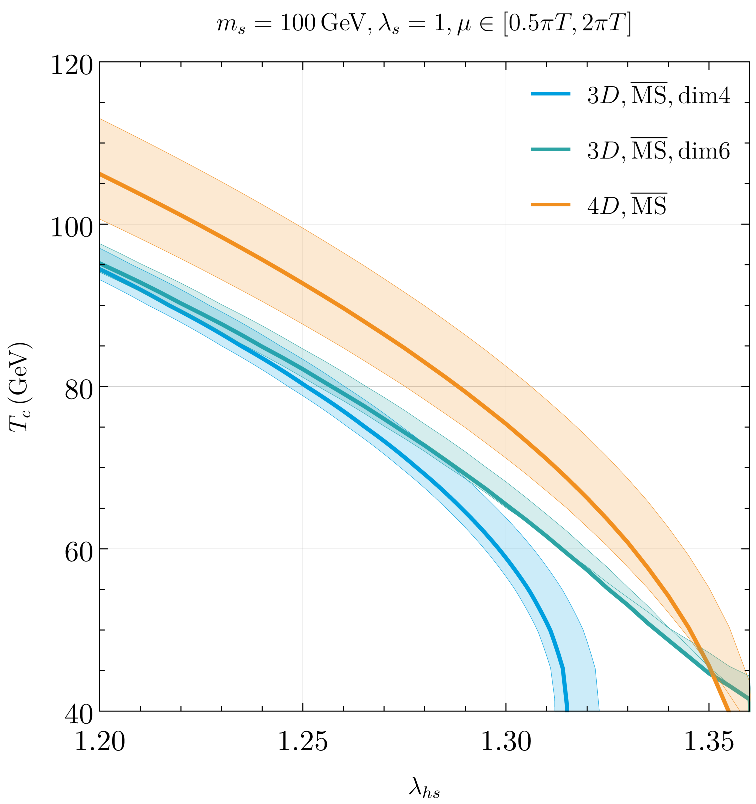


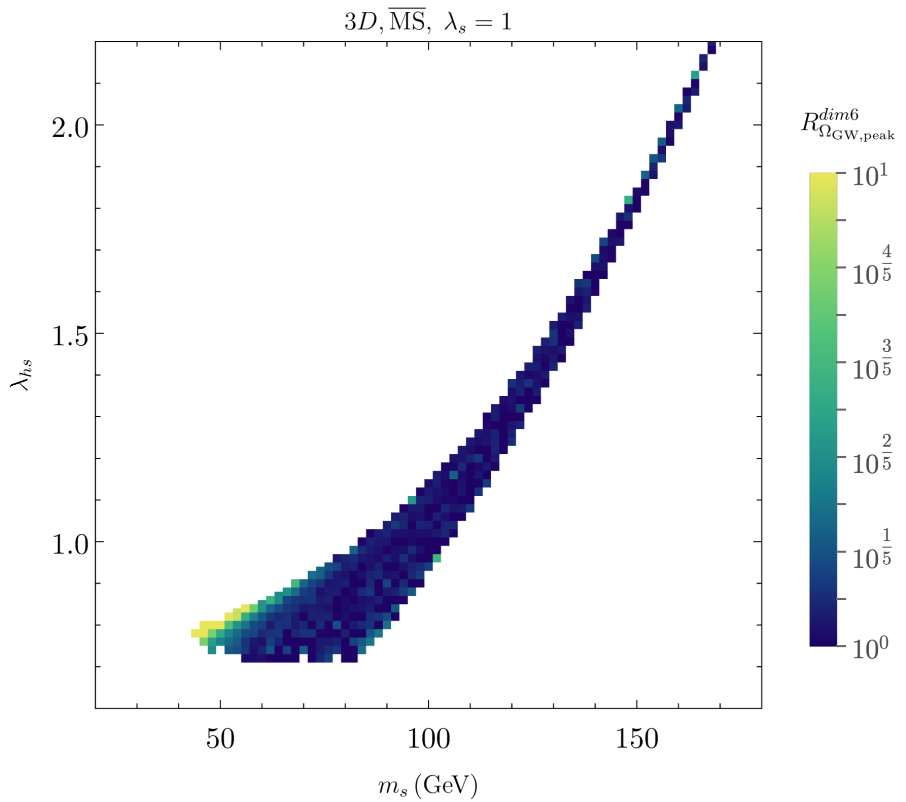
Quiet a lot of fine-tuning for strong FOPT:
usually the case for realistic BSM models

Model parameter scans



Impact of higher order operators





Higher dimensional operators in 3D-EFT approach are relevant in region of strong PT.

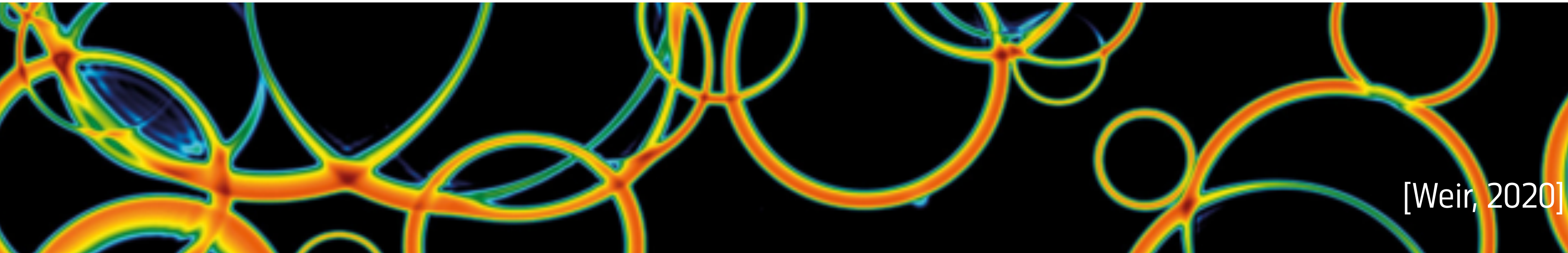
Conclusions.

- Phase transitions come with a lot of theoretical uncertainties: there's hope for perturbative treatment → lot's of things to do for physicists.

- Thermally driven phase transitions introduce new scaling relations, which require modification of the usual perturbation theory. Large scale separations and related resummations can be rigorously taken into account with the help of EFT techniques (dimensional reduction) — within which, higher dimensional operators could be relevant for strong PTs.

- Gauge dependence is also a good indicator of applicability of perturbative expansion for phase transitions: gauge invariance rely on perturbative expansion.

- To achieve similar to equilibrium thermodynamics renormalization scale independence for non-equilibrium quantities, the bounce action has to be calculated at higher orders.



Back-up: look at your own risk

Lagrange parameters determination

Another possible source of uncertainties

Physical inputs (m_h, m_s, m_z, G_F)

n-loop $\overline{MS}$ relations

$$m_h = M_h + \Pi_h(p^2 = m_h^2)$$

$$V_{eff}^{4d,T=0}$$

OS-like renormalization

$$\partial_h V_{tree} = \partial_h V_{eff} + \partial_h V_{c.t.}$$

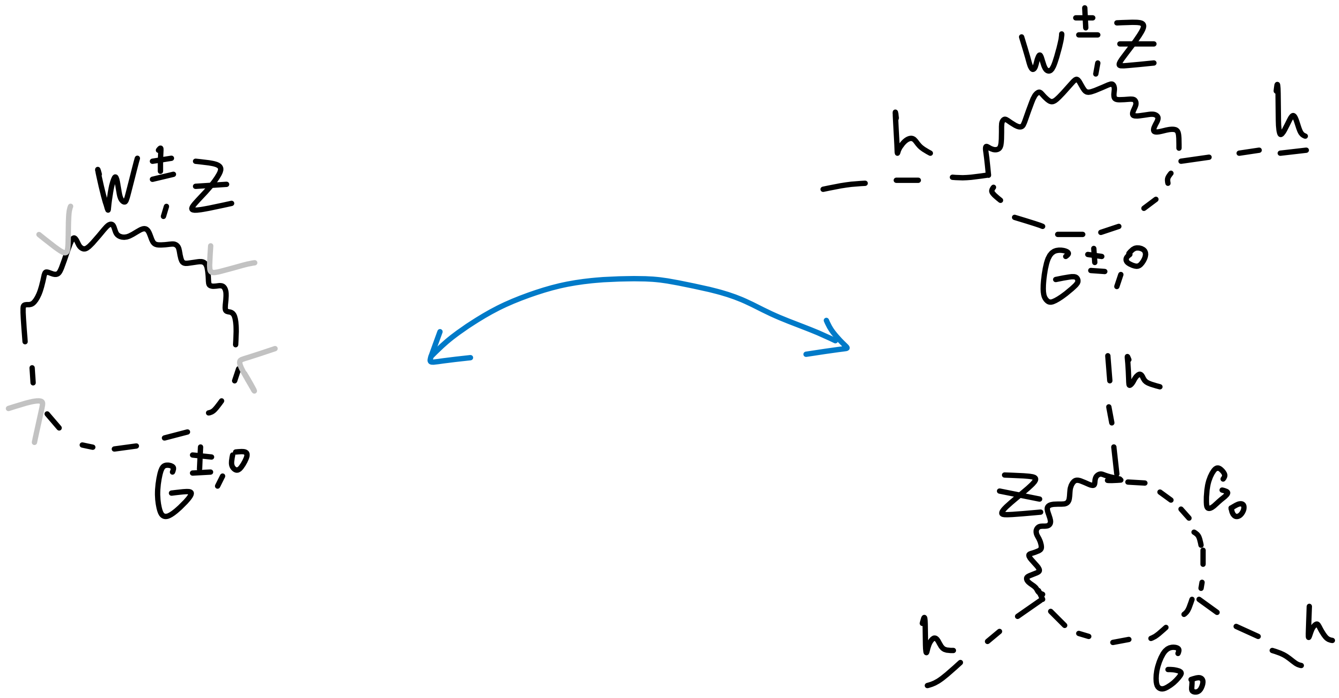
$$\partial_h^2 V_{tree} = \partial_h^2 V_{eff} + \partial_h^2 V_{c.t.}$$

$$(\sim \Pi_h(p^2 = 0))$$

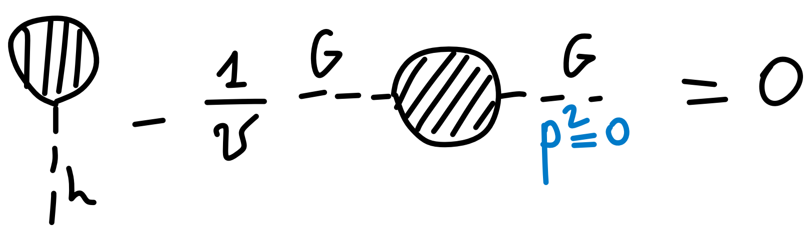
Lagrange parameters $(\mu_h, \lambda, g, \ldots)$

- Missing momentum contribution
- Gauge dependent
- Taking derivatives must be handled carefully*
- Goldtone catastrophe in Landau gauge
- Only scalar potential couplings are renormalised

Background R_ξ gauges: missing parts



- Only with the correct mixing, we get the relation right:



Fermi, background and standard R_ξ gauges.

$$\mathcal{L}_{GF} = -\frac{1}{2\xi_1^a}\left(\partial^\mu A_\mu^a + i\xi_2^a g_a t_{ik}^a \tilde{\phi}_k \phi_i^{GF}\right)^2$$

→ $\phi_i^{GF} = 0$ (Fermi gauge)

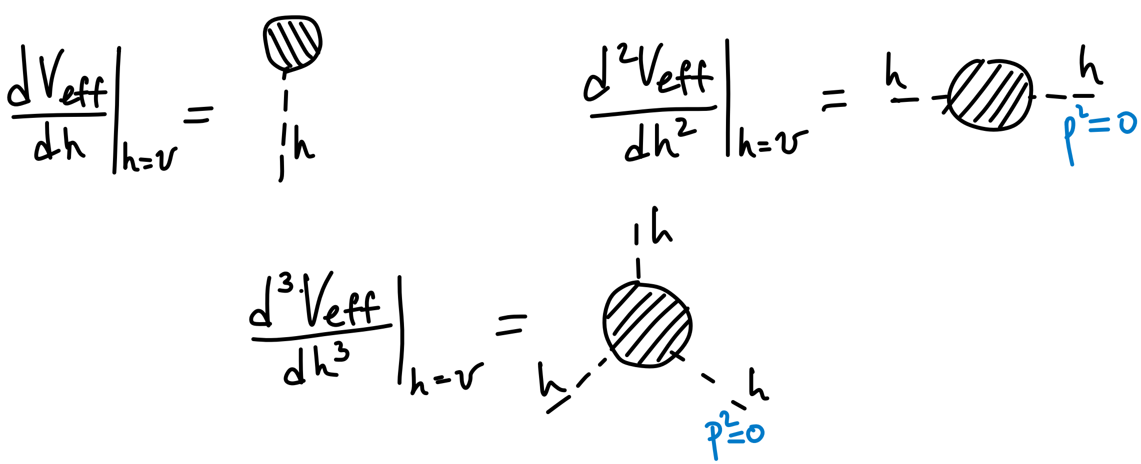
Uncanceled kinetic mixing
IR divergences

$\phi_i^{GF} = \phi_i$ background field

Different gauge fixing for each field value?

$\phi_i^{GF} = \phi_i^{min}$ minimal field configuration

Uncanceled kinetic mixing



3d EFT loop convergence