Multi-(super)field Inflation in Supergravity without Stabilizer Fields

in collaboration with **Jinn-Ouk Gong** and **Sergei V. Ketov**.

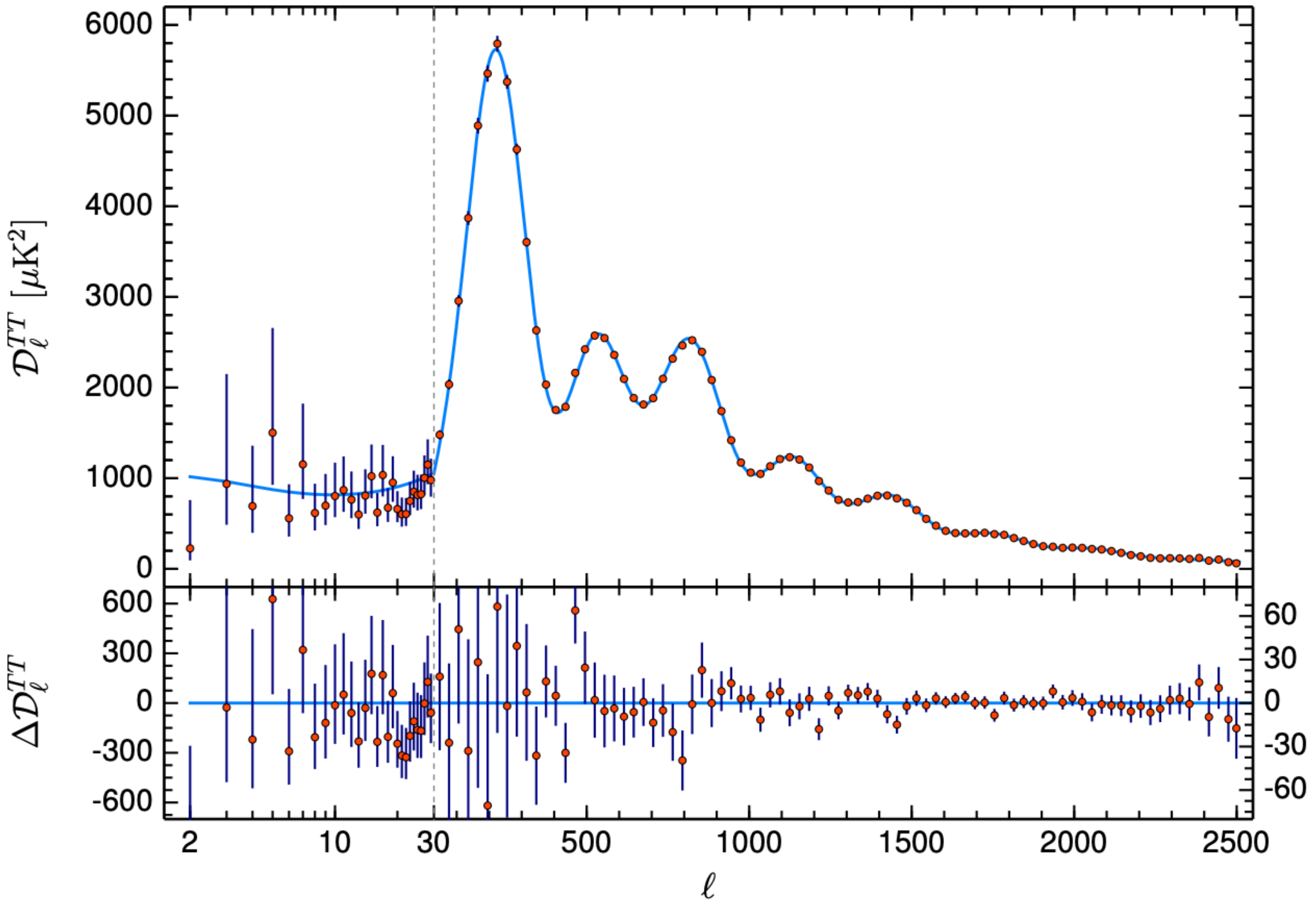
Takahiro Terada (KMI, Nagoya U), PASCOS 2025, Durham, July 22.

Basic Introduction

Cosmic Inflation

explains the homogeneity, flatness, and absence of unwanted relics,

and the origin of large-scale structures of the Universe.



[Akrami et al., Planck 2018, 1807.06211]

Phenomenologically successful.

What is the underlying model

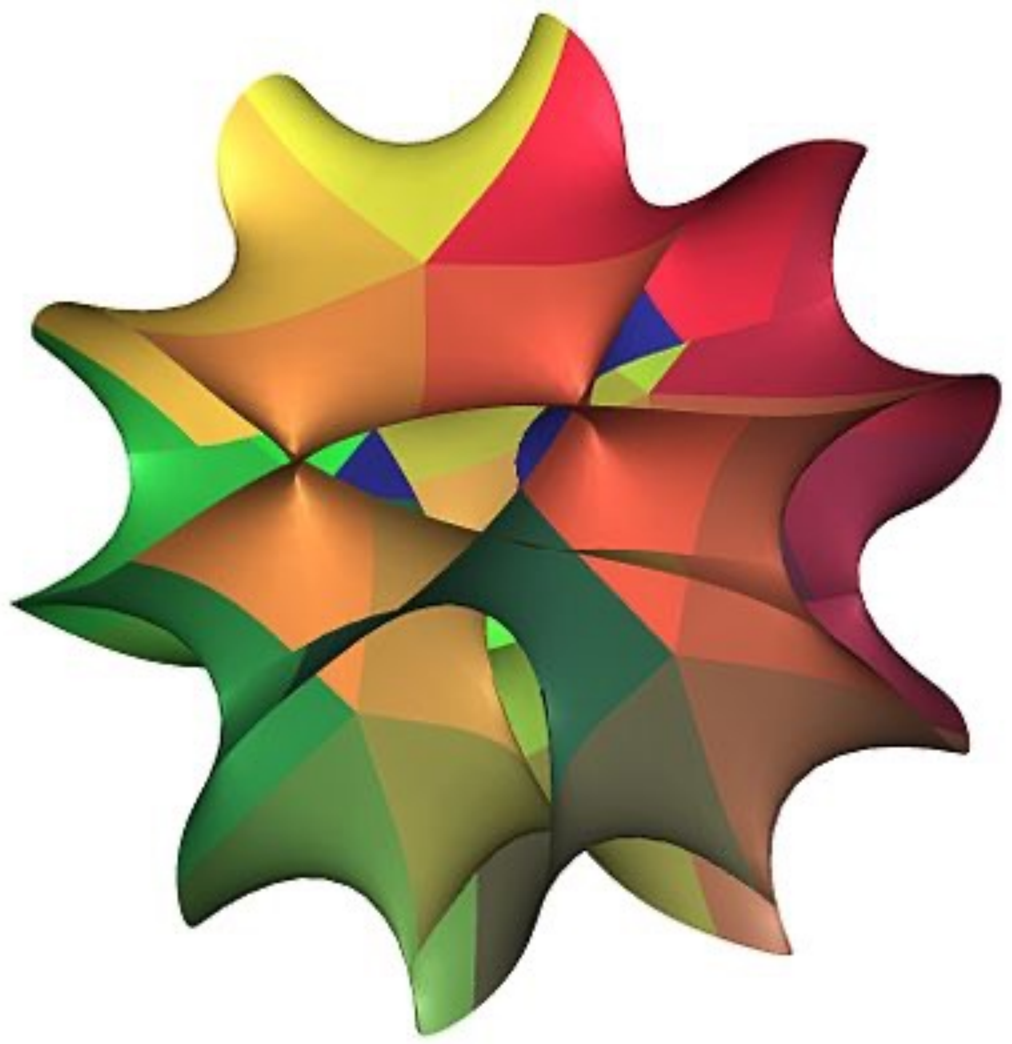
in, *e.g.*, supergravity or String Theory?

Multiple Fields

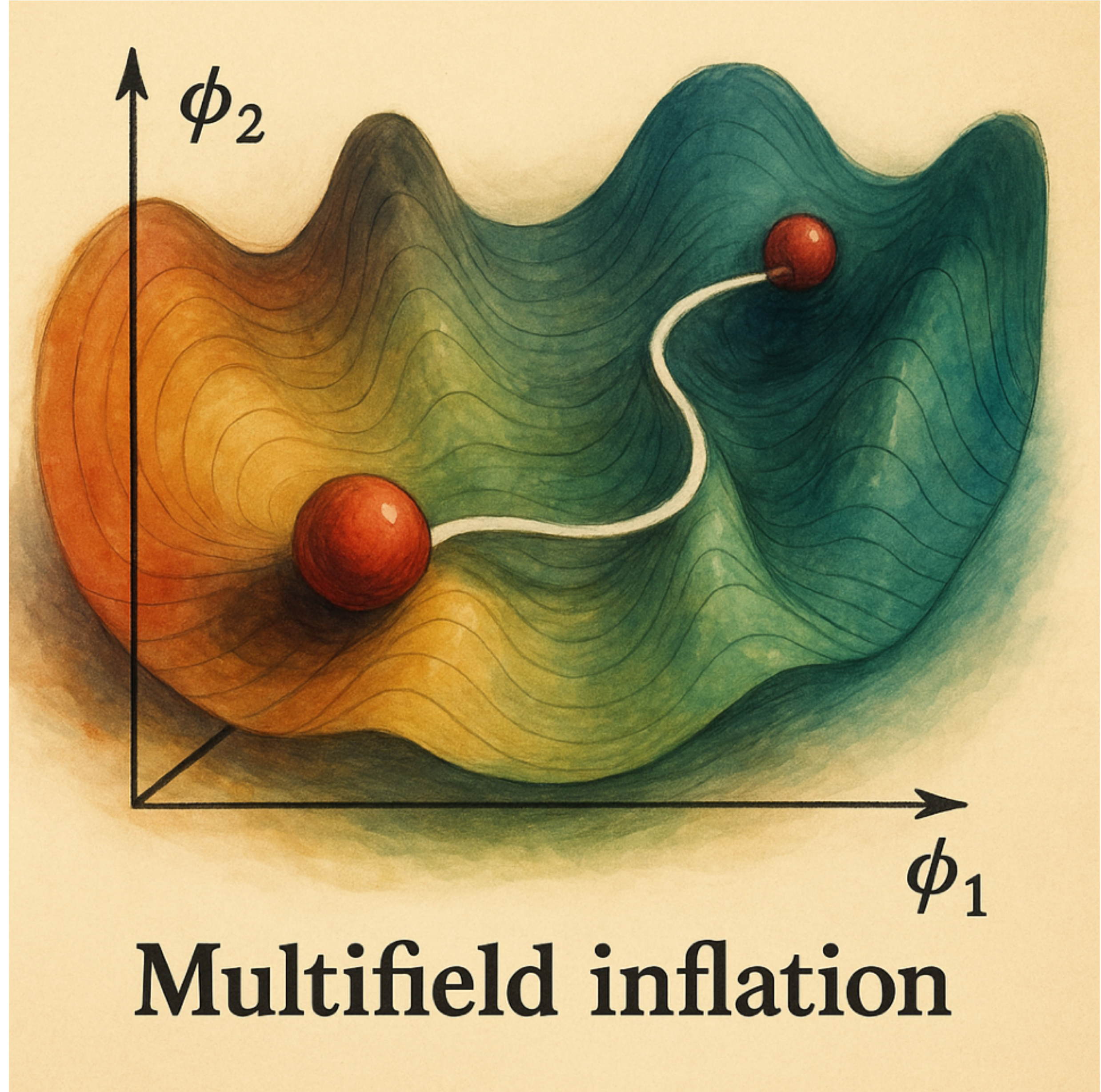
There are **many fields** in the SM of particle physics.

There are **many scalar fields** in the MSSM.

There are **many moduli** (approximately massless scalar fields) in String Theory.



A 2D slice of a 6D CY quintic manifold.
[Andrew J. Hanson, from Wikipedia]

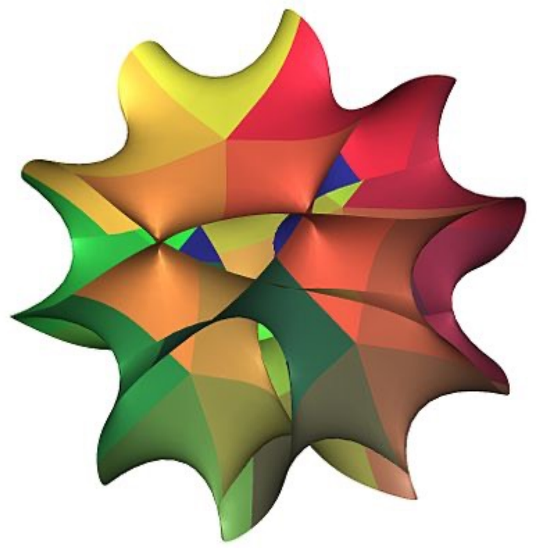


[Produced by ChatGPT]

Motivation

Eventually want to derive it from String Theory,
but beyond the scope of this work

Top-down
Fundamental

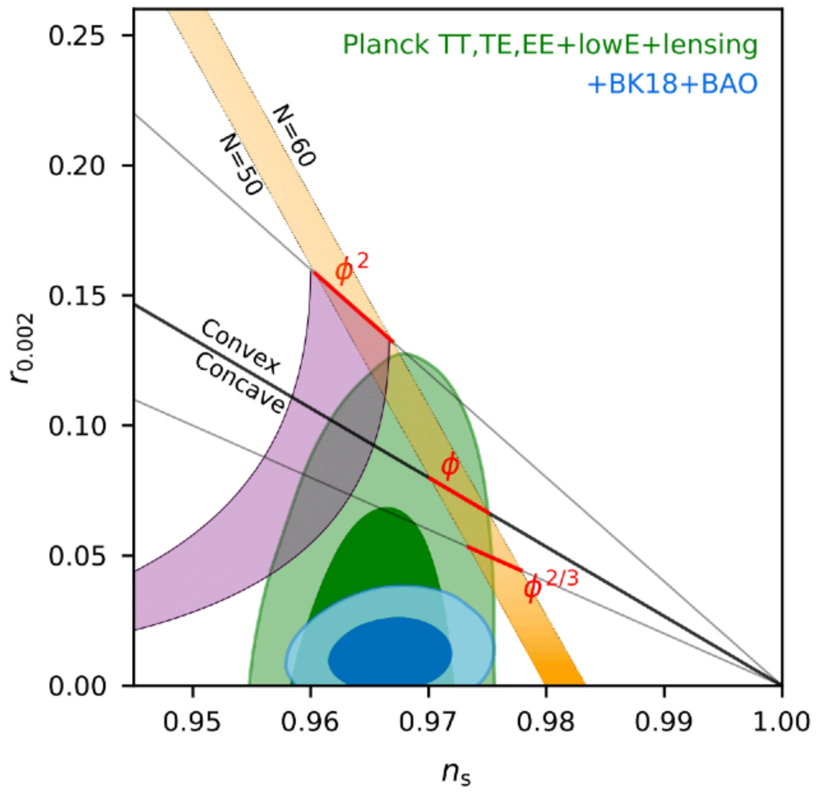


stepping stone to find
new/viable String inflation models

Inflation in Supergravity

Prediction of observables

Bottom-up
Phenomenological



[BICEP/Keck, 2110.00483]

Embed generic inflationary models
Only several generic mechanisms are known.

Difficulties of Inflation in Supergravity and Their Solutions

Potential tends to be Steep or Negative

We need **flat** and **positive** potential for slow-roll inflation. However…

$$V = e^K \left(K^{i\bar{j}} D_i W \bar{D}_{\bar{j}} \bar{W} - 3|W|^2 \right) \qquad \text{where } D_i W \equiv W_i + K_i W.$$

exponentially steep (the η problem) — applies to e^K

negative — applies to $-3|W|^2$

One typically impose a(n approximate) **shift symmetry** on the Kähler potential.

$$K(\Phi, \bar{\Phi}, \cdots) = K(\Phi - \bar{\Phi}, \cdots) \qquad \text{invariant under } \Phi \to \Phi + r \quad (r \in \mathbb{R}).$$

This kills the exponential dependence, but the potential tends to be negative

$$V \sim -3e^K |W|^2$$

in the large-field region where $|W_i| \ll |W|$ for generic polynomial superpotentials.

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Typical Solution: Introducing a Stabilizer Field

$$K(\Phi, \bar{\Phi}, S, \bar{S}) = K(\Phi - \bar{\Phi}, S, \bar{S})$$

$$W(\Phi, S) = S f(\Phi)$$

Φ: inflaton superfield

S: stabilizer superfield

Arrange the model so that $S = 0$ and hence $W = 0$ during inflation.

$$V = e^K K^{S\bar{S}} |f(\Phi)|^2$$

Alternative Solution: Non-minimal Self-Interaction

$W(\Phi)$ can be generic, while the essential form of K in the canonical basis is

$$K(\Phi, \bar{\Phi}) = i\kappa(\Phi - \bar{\Phi}) - \frac{1}{2}(\Phi - \bar{\Phi})^2 - \frac{\xi}{12}(\Phi - \bar{\Phi})^4 + \cdots$$

giving positive contribution to V

canonical kinetic term

SUSY-breaking mass term stabilizing the sinflaton

$$V = e^K K^{\Phi\bar{\Phi}} \left((\kappa^2 - 3K_{\Phi\bar{\Phi}})\,|\,W\,|^2 + 2\kappa\,\mathrm{Im}(\bar{W} W_\Phi) + |\,W_\Phi\,|^2 \right)$$

Positive and flat potential can be obtained for $K^{\Phi\bar{\Phi}}\kappa^2 \gtrsim 3$.

Multi-(super)field Extension

Proposed Setup

$K(\Phi^i, \bar{\Phi}^{\bar{i}}) = K(\Phi^i - \bar{\Phi}^{\bar{i}})$ (shift-symmetric; $i = 1,2,3,\cdots,N$)

$W(\Phi^i)$ can be generic.

We assume that there exists a point in the field space where the cubic terms are negligible.

Around the point, after canonical normalization of the quadratic term, we can expand K as follows

$$K = i\kappa_i I^i - \frac{1}{2}\delta_{ij}I^iI^j - \frac{\xi_{ijkl}}{12}I^iI^jI^kI^l + \cdots$$

where $I^i \equiv \Phi^i - \bar{\Phi}^{\bar{i}}$

giving positive contributions to V | canonical kinetic terms | SUSY-breaking mass terms stabilizing the imaginary parts

$$V = e^K\left((K^{i\bar{j}}\kappa_i\kappa_j - 3)\,|W|^2 + \mathrm{Im}\left(K^{i\bar{j}}\kappa_j\bar{W}W_i\right) + K^{i\bar{j}}W_i\bar{W}_{\bar{j}}\right)$$

Stabilization of Imaginary Parts

The SUSY breaking mass squared matrix: $12(H^2 + m_{3/2}^2)\xi_{ij\mathscr{G}\mathscr{G}}$

Field expectation value: $|\chi^i| \lesssim \dfrac{(\xi^{-1})_{ij\mathscr{G}\mathscr{G}} \left| \sqrt{2}\kappa_j(3H^2 + 2m_{3/2}^2) + \sqrt{2}i\left(\widetilde{W}\overline{\widetilde{W}}_{\bar{j}} - \overline{\widetilde{W}}\widetilde{W}_j\right) \right|}{12(H^2 + m_{3/2}^2)}$

where $\mathscr{G} = \langle G_i \Phi^i / \sqrt{G^j G_j} \rangle$ is the (time-dependent) sGoldstino

with $G \equiv K + \ln |W|^2$ being the total Kähler potential.

Imaginary parts can be stabilized by sufficiently large positive ξ.

Some (Optional) **Simplifications**

For a superpotential with real coefficients,

$$V = e^K \left((K^{i\bar{j}}\kappa_i\kappa_j - 3)|W|^2 + \mathrm{Im}\left(K^{i\bar{j}}\kappa_j\bar{W}W_i\right) + K^{i\bar{j}}W_i\bar{W}_{\bar{j}} \right).$$

If we take $K^{i\bar{j}}\kappa_i\kappa_j = 3$, the first term also vanishes

$$V = e^K K^{i\bar{j}}W_i\bar{W}_{\bar{j}} \simeq \sum_i |W_i|^2 .$$

This is essentially the global SUSY-breaking *F* term.

Example Models

(Quasi-)de Sitter from (Nearly) Constant W

$$W = W_0 = \text{const}\,.$$

$$V = e^{K} e^{i\kappa_k(\Phi^k - \bar{\Phi}^{\bar{k}})} \left(K^{\bar{j}i} \kappa_i \kappa_{\bar{j}} - 3 \right) |\, W_0 \,|^2\,.$$

De Sitter can be realized. Let's modify it to realize **slow-roll inflation.**

$$W = W_0 \left(1 - a_\Phi e^{-b_\Phi \Phi} - a_\Psi e^{-b_\Psi \Psi} - a_{\Phi\Psi} e^{-b'_\Phi \Phi - b'_\Psi \Psi} \right)$$

Multi-field exponentially flat potential

$$\frac{V}{|\, W_0 \,|^2} = \left| a_\Phi b_\Phi e^{-b_\Phi \Phi} + a_{\Phi\Psi} b'_\Phi e^{-b'_\Phi \Phi - b'_\Psi \Psi} \right|^2 + \left| a_\Psi b_\Psi e^{-b_\Psi \Psi} + a_{\Phi\Psi} b'_\Psi e^{-b'_\Phi \Phi - b'_\Psi \Psi} \right|^2$$

$$+ (\kappa^2 - 3) \left| 1 - a_\Phi e^{-b_\Phi \Phi} - a_\Psi e^{-b_\Psi \Psi} - a_{\Phi\Psi} e^{-b'_\Psi \Phi - b'_\Psi \Psi} \right|^2.$$

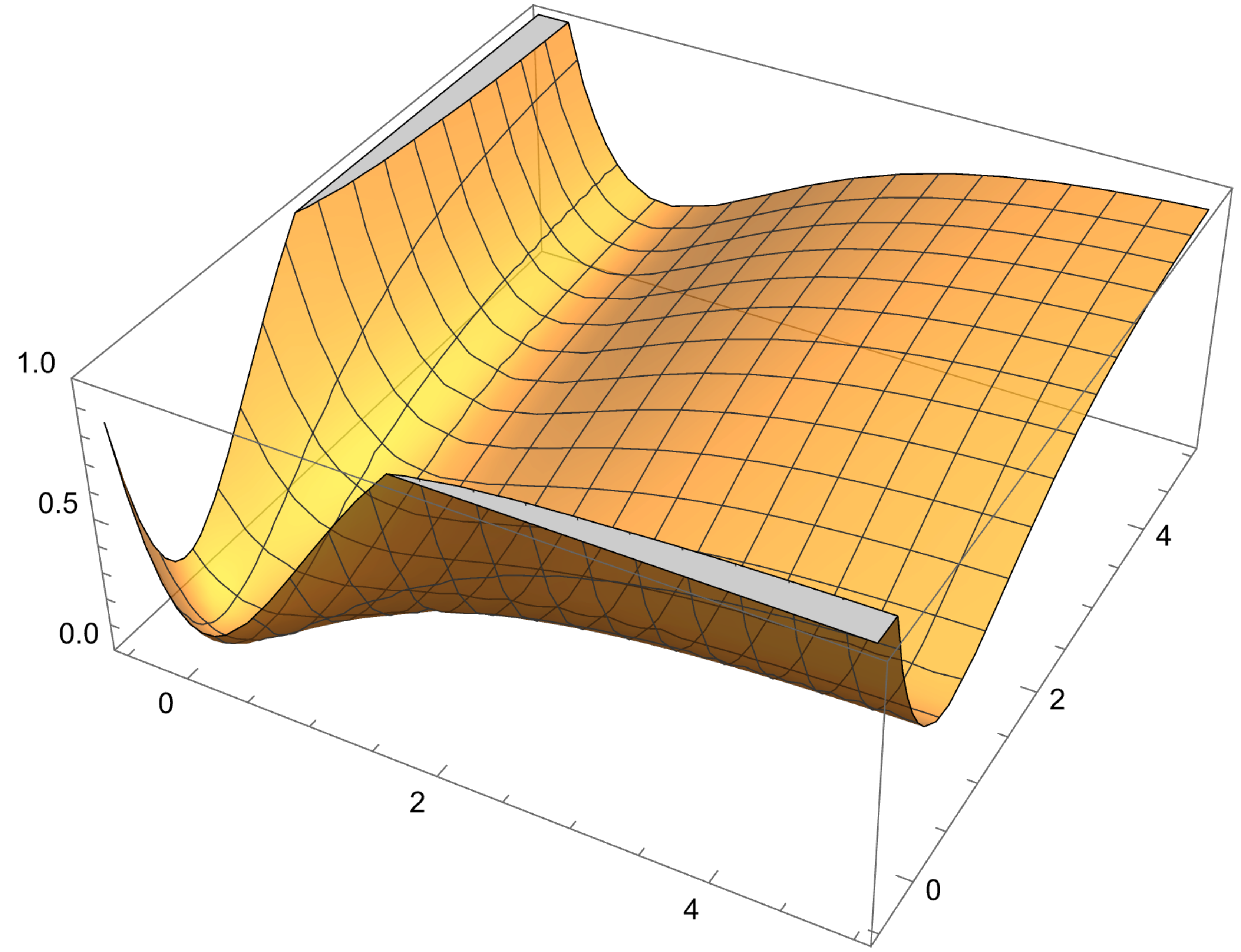
$$\kappa^2 \equiv \kappa_\Phi^2 + \kappa_\Psi^2$$

(Quasi-)de Sitter from (Nearly) Constant *W*

$$W = W_0 = \text{const.} \quad \Rightarrow \quad V = e^{K} e^{i\kappa_k(\Phi^k - \bar{\Phi}^{\bar{k}})} \left(K^{\bar{j}i} \kappa_i \kappa_{\bar{j}} - 3 \right) |W_0|^2 .$$

De Sitter can be realized. Let's modify it to realize **slow-roll inflation**.

$$W = W_0 \left(1 - a_\Phi e^{-b_\Phi \Phi} - a_\Psi e^{-b_\Psi \Psi} - a_{\Phi\Psi} e^{-b'_\Phi \Phi - b'_\Psi \Psi}\right)$$



Multi-field exponentially flat potential

$$\frac{V}{|W_0|^2} = \left| a_\Phi b_\Phi e^{-b_\Phi \Phi} + a_{\Phi\Psi} b'_\Phi e^{-b'_\Phi \Phi - b'_\Psi \Psi} \right|^2 + \left| a_\Psi b_\Psi e^{-b_\Psi \Psi} + a_{\Phi\Psi} b'_\Psi e^{-b'_\Phi \Phi - b'_\Psi \Psi} \right|^2$$

$$+(\kappa^2 - 3) \left| 1 - a_\Phi e^{-b_\Phi \Phi} - a_\Psi e^{-b_\Psi \Psi} - a_{\Phi\Psi} e^{-b'_\Psi \Phi - b'_\Psi \Psi} \right|^2 .$$

$\kappa^2 \equiv \kappa_\Phi^2 + \kappa_\Psi^2$

Hybrid Inflation

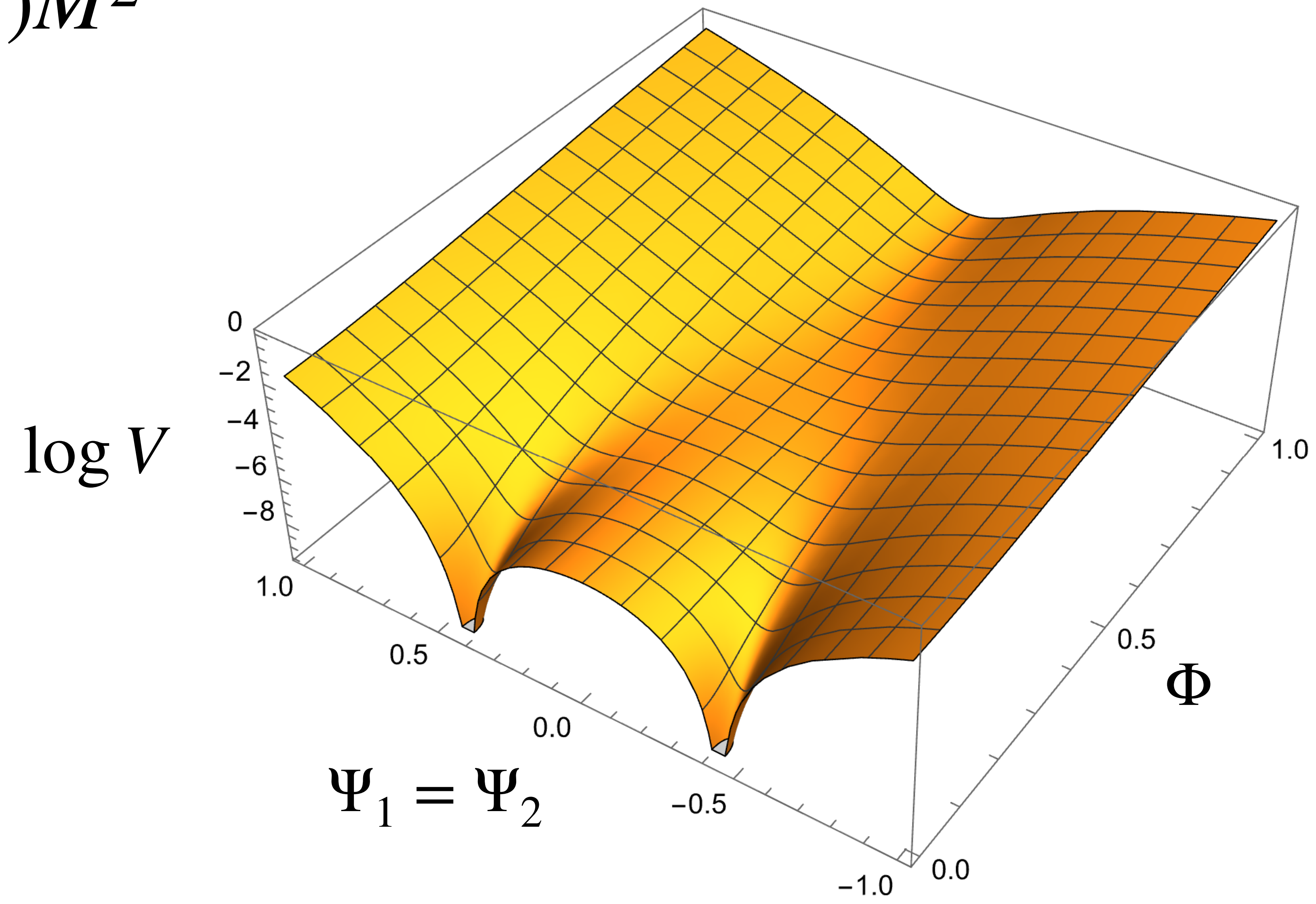
$$W = \lambda\Phi(M^2 - \Psi_1\Psi_2)$$

$$\kappa^2 \equiv \kappa_\Phi^2 + \kappa_{\Psi_1}^2 + \kappa_{\Psi_2}^2$$

$$\frac{V}{\lambda^2} = |M^2 - \Psi_1\Psi_2|^2 + |\Phi|^2(|\Psi_1|^2 + |\Psi_2|^2) + (\kappa^2 - 3)|\Phi|^2|M^2 - \Psi_1\Psi_2|^2$$

During inflation, *i.e.*, for $|\Phi| > |\Phi_c| \equiv \frac{M}{\sqrt{1-(\kappa^2-3)M^2}}$, we have $\Psi_1 = \Psi_2 = 0$,

$$\frac{V}{\lambda^2|M|^4} = 1 + (\kappa^2 - 3)|\Phi|^2$$



Polynomial Chaotic Inflation

2-field model

$$W = \frac{1}{2} m_\Phi \Phi^2 + \frac{1}{2} m_\Psi \Psi^2$$

$$\begin{aligned} V = {} & |m_\Phi|^2 |\Phi|^2 + |m_\Psi|^2 |\Psi|^2 \\ & + \frac{\kappa_\Phi}{2} \mathrm{Im}(m_\Phi \overline{m_\Psi} \Phi \bar{\Psi}^2) + \frac{\kappa_\Psi}{2} \mathrm{Im}(\overline{m_\Phi} m_\Psi \bar{\Phi}^2 \Psi) \\ & + \frac{1}{4} (\kappa_\Phi^2 + \kappa_\Psi^2 - 3) \left| m_\Phi \Phi^2 + m_\Psi \Psi^2 \right|^2 \end{aligned}$$

N-flation

$$W = \frac{m}{2} \sum_{i=1}^{N} (\Phi^i)^2$$

$$V = m^2 |\Phi^i|^2 \left(1 + \frac{\kappa^2 - 3}{4} |\Phi^j|^2 \right)$$

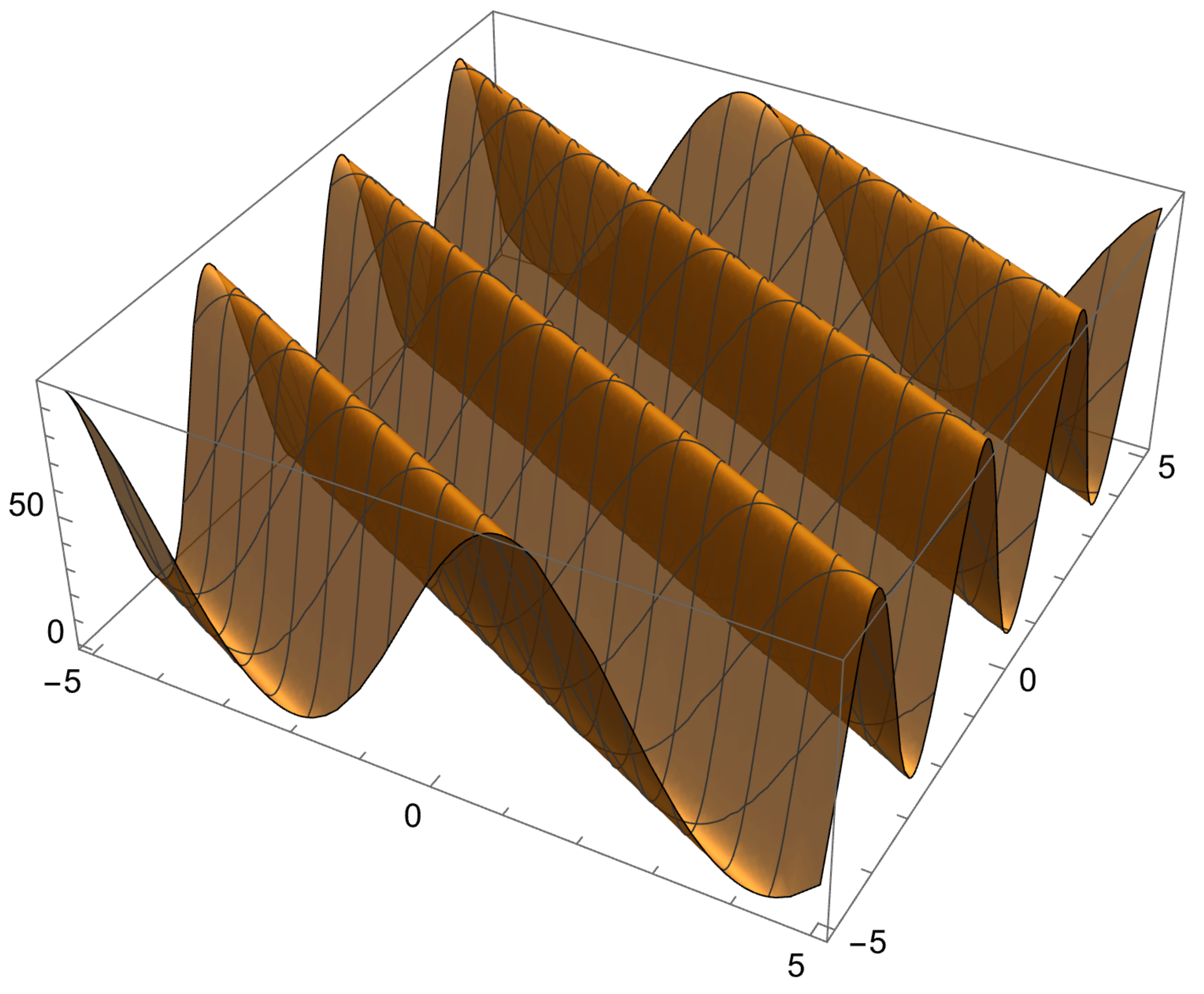
(after stabilization of the imaginary parts)

with $\kappa_i = \frac{\kappa}{\sqrt{N}}$

Axionic Models

$$W = W_0\left[1 - \alpha e^{i(\beta_\Phi \Phi + \beta_\Psi \Psi)} - \alpha' e^{i(\beta'_\Phi \Phi + \beta'_\Psi \Psi)}\right]$$

$$\begin{aligned} V = \kappa^2 - 3 + \alpha^2(\kappa^2 - 3 + \beta_\Phi^2 + \beta_\Psi^2) + \alpha'^2(\kappa^2 - 3 + {\beta'_\Phi}^2 + {\beta'_\Psi}^2) \\ -2\alpha(\kappa^2 - 3)\cos(\beta_\Phi \Phi + \beta_\Psi \Psi) - 2\alpha'(\kappa^2 - 3)\cos(\beta'_\Phi \Phi + \beta'_\Psi \Psi) \\ +2\alpha\alpha'(\kappa^2 - 3 + \beta_\Phi \beta'_\Phi + \beta_\Psi \beta'_\Psi)\cos\left[(\beta'_\Phi - \beta_\Phi)\Phi + (\beta'_\Psi - \beta_\Psi)\Psi\right] \end{aligned}$$



We utilized the Kim-Nilles-Peloso alignment mechanism.

R-axion models

$\mathrm{Re}\,\Psi$: R-axion $\quad \kappa_\Psi = \kappa$

$\mathrm{Im}\,\Psi$: R-saxion $\quad \kappa_\Phi = 0$

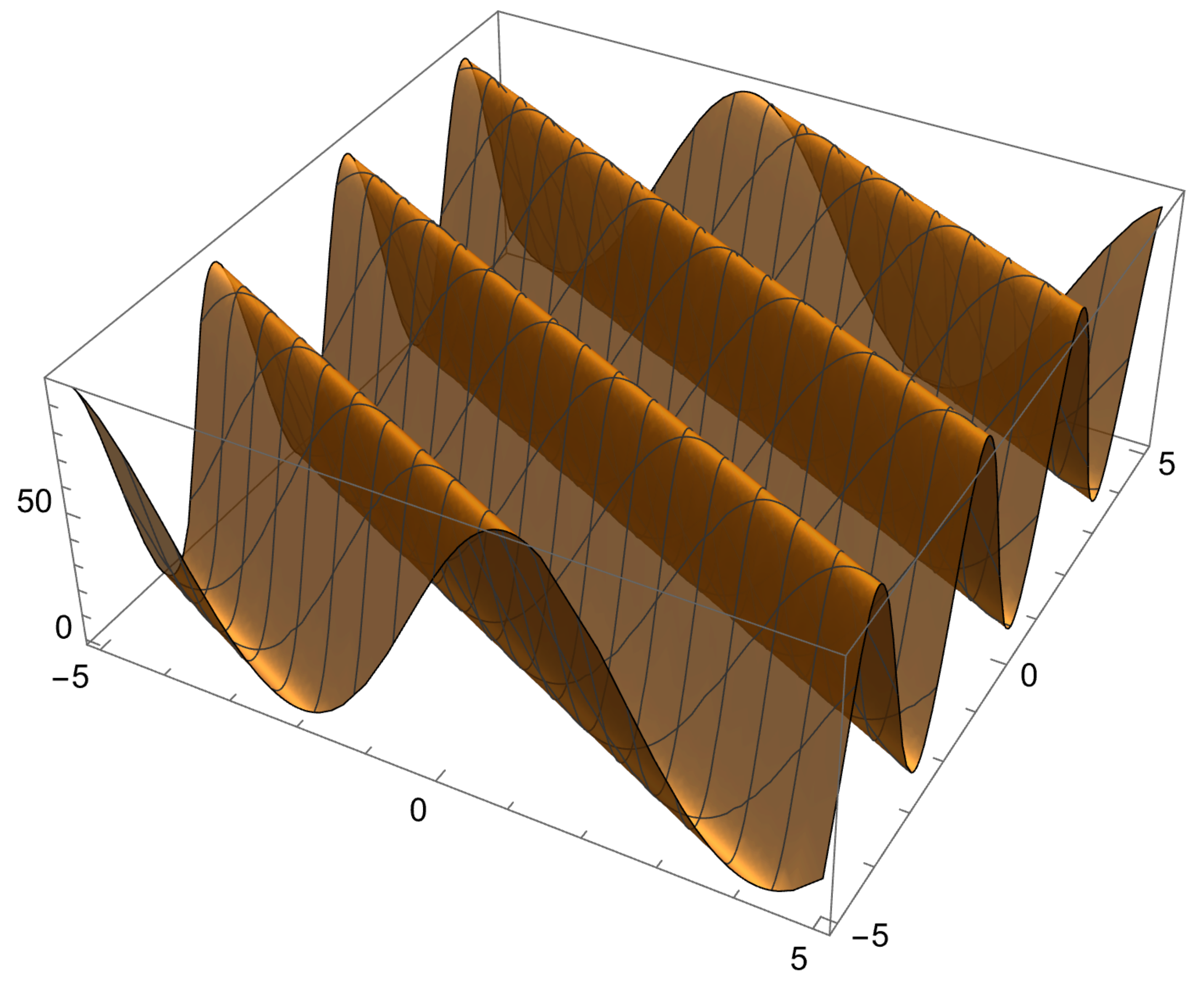
Φ : Inflaton $\quad W = W(\Phi)$

$$V = e^K e^{i\kappa(\Psi - \bar{\Psi})}\left[(\kappa^2 - 3)|W|^2 + K^{\Phi\bar{\Phi}}|W_\Phi + K_\Phi W|^2\right]$$

Axionic Models

$$W = W_0\left[1 - \alpha e^{i(\beta_\Phi \Phi + \beta_\Psi \Psi)} - \alpha' e^{i(\beta'_\Phi \Phi + \beta'_\Psi \Psi)}\right]$$

$$\begin{aligned} V = {} & \kappa^2 - 3 + \alpha^2(\kappa^2 - 3 + \beta_\Phi^2 + \beta_\Psi^2) + \alpha'^2(\kappa^2 - 3 + {\beta'_\Phi}^2 + {\beta'_\Psi}^2) \\ & -2\alpha(\kappa^2 - 3)\cos(\beta_\Phi \Phi + \beta_\Psi \Psi) - 2\alpha'(\kappa^2 - 3)\cos(\beta'_\Phi \Phi + \beta'_\Psi \Psi) \\ & +2\alpha\alpha'(\kappa^2 - 3 + \beta_\Phi \beta'_\Phi + \beta_\Psi \beta'_\Psi)\cos\left[(\beta'_\Phi - \beta_\Phi)\Phi + (\beta'_\Psi - \beta_\Psi)\Psi\right] \end{aligned}$$



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$\mathrm{Re}\,\Psi$: *R*-axion

$\mathrm{Im}\,\Psi$: *R*-saxion

Φ : Inflaton

$\kappa_\Psi = \kappa$

$\kappa_\Phi = 0$

$W = W(\Phi)$

$$V = e^K e^{i\kappa(\Psi - \bar{\Psi})}\left[(\kappa^2 - 3)\,|\,W\,|^2 + K^{\Phi\bar{\Phi}}\,|\,W_\Phi + K_\Phi W\,|^2\right]$$

Summary

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Realizing (or embedding) **Inflation in supergravity** is nontrivial.

We have extended our previous proposal **without the stabilizer fields** into **multi-superfield** setups.

$$K = i\kappa_i I^i - \frac{1}{2}\delta_{ij} I^i I^j - \frac{\xi_{ijkl}}{12} I^i I^j I^k I^l + \cdots \quad \text{where } I^i \equiv \Phi^i - \bar{\Phi}^{\bar{i}} \qquad W: \text{ generic.}$$

This is not a particular model but a **mechanism** to embed **a large class of models**.

Inflationary observables such as isocurvature perturbations and non-Gaussianity are to be studied.

Appendix

On Kähler Geometry

Holomorphic sectional curvature (along the Goldstino direction)

$$\mathscr{K}(f) = -\frac{R_{i\bar{j}k\bar{l}} f^i f^{\bar{j}} f^k f^{\bar{l}}}{(g_{i\bar{j}} f^i f^{\bar{j}})^2} \simeq 2\xi_{ijkl} f^i f^{\bar{j}} f^k f^{\bar{l}} \equiv 2\xi_{\mathscr{G}\mathscr{G}\mathscr{G}\mathscr{G}} > 0$$

where $f^i \equiv G^i/\sqrt{G^j G_j}$ is the Goldstino direction with $G = K + \ln|W|^2$ being the total Kähler potential.

The sGoldstino is $\mathscr{G} \equiv f_i \Phi^i$.

The sign is opposite to the case of hyperbolic geometry (e.g., α-attractor).

On the Absence of Cubic Terms

It is like a requirement of the existence of the vacuum.

$$K = i\kappa_i I^i - \frac{1}{2}\delta_{ij} I^i I^j + i\frac{\tau_{ijk}}{6} I^i I^j I^k - \frac{\xi_{ijkl}}{12} I^i I^j I^k I^l + \cdots$$

Then,

$$V \sim e^K K^{i\bar{j}} D_i W \bar{D}_{\bar{j}} \bar{W} + \cdots \qquad \chi^i \equiv \sqrt{2}\,\mathrm{Im}\,\Phi^i = -iI^i/\sqrt{2}$$

$$\sim 3\left(1 - \sqrt{2}(\kappa_i + \tau_{i\mathscr{G}\mathscr{G}})\chi^i + 2\xi_{ij\mathscr{G}\mathscr{G}}\chi^i\chi^j + \cdots\right)(H^2 + m_{3/2}^2) + \cdots$$

Assuming this is the dominant part of the potential for the imaginary parts, the linear term $\propto (\kappa_i + \tau_{i\mathscr{G}\mathscr{G}})$ should be negligible at the minimum.

Other Choices of Kähler Potential

For example, one may consider a logarithmic Kähler potential

$$K = -\sum_i 3\alpha_i \log\left[1 - \frac{i}{\sqrt{3\alpha_i}}(\Phi^i - \bar{\Phi}^{\bar{i}}) + \xi_i(\Phi^i - \bar{\Phi}^{\bar{i}})^4 + \cdots\right].$$

After the stabilization of the imaginary parts, it is essentially same as the previous examples.

$$\frac{\mathscr{L}_{\rm kin}}{\sqrt{-g}} = -\delta_{i\bar{j}}\partial^\mu\bar{\Phi}^{\bar{j}}\partial_\mu\Phi^i$$

$$V = \delta^{\bar{j}i}\left(W_i\overline{W}_{\bar{j}} + i\alpha_i W\overline{W}_{\bar{j}} - i\alpha_j\overline{W}W_i + \alpha_i\alpha_j|W|^2\right) - 3|W|^2$$