

Some novel and attractive features of Warm Inflation

Suratna Das
Ashoka University

In collaboration with Rudnei O. Ramos, Archana Santosh, Swagat Mishra, Varun Sahni

Cosmic Inflation

- Inflation is a very brief epoch when the universe expanded exponentially
- The dynamics is determined by Einstein's General Theory of Relativity :

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

Cosmic Inflation

- Inflation is a very brief epoch when the universe expanded exponentially
- The dynamics is determined by Einstein's General Theory of Relativity :

$$\boxed{G_{\mu\nu}} = 8\pi G T_{\mu\nu}$$

Background geometry :
FRW metric

$$ds^2 = -dt^2 + a^2(t)d\mathbf{x}^2$$

Cosmic Inflation

- Inflation is a very brief epoch when the universe expanded exponentially
- The dynamics is determined by Einstein's General Theory of Relativity :

$$\boxed{G_{\mu\nu}} = 8\pi G T_{\mu\nu}$$

Background geometry :
FRW metric

$$ds^2 = -dt^2 + a^2(t) d\mathbf{x}^2$$

scale factor

Cosmic Inflation

- Inflation is a very brief epoch when the universe expanded exponentially
- The dynamics is determined by Einstein's General Theory of Relativity :

$$\boxed{G_{\mu\nu}} = 8\pi G T_{\mu\nu}$$

Background geometry :
FRW metric

$$ds^2 = -dt^2 + a^2(t) d\mathbf{x}^2$$

scale factor

$$a(t) = e^{Ht}$$

H : Hubble parameter

Cosmic Inflation

- Inflation is a very brief epoch when the universe expanded exponentially
- The dynamics is determined by Einstein's General Theory of Relativity :

$$\boxed{G_{\mu\nu}} = 8\pi G \boxed{T_{\mu\nu}}$$

Background geometry :
FRW metric

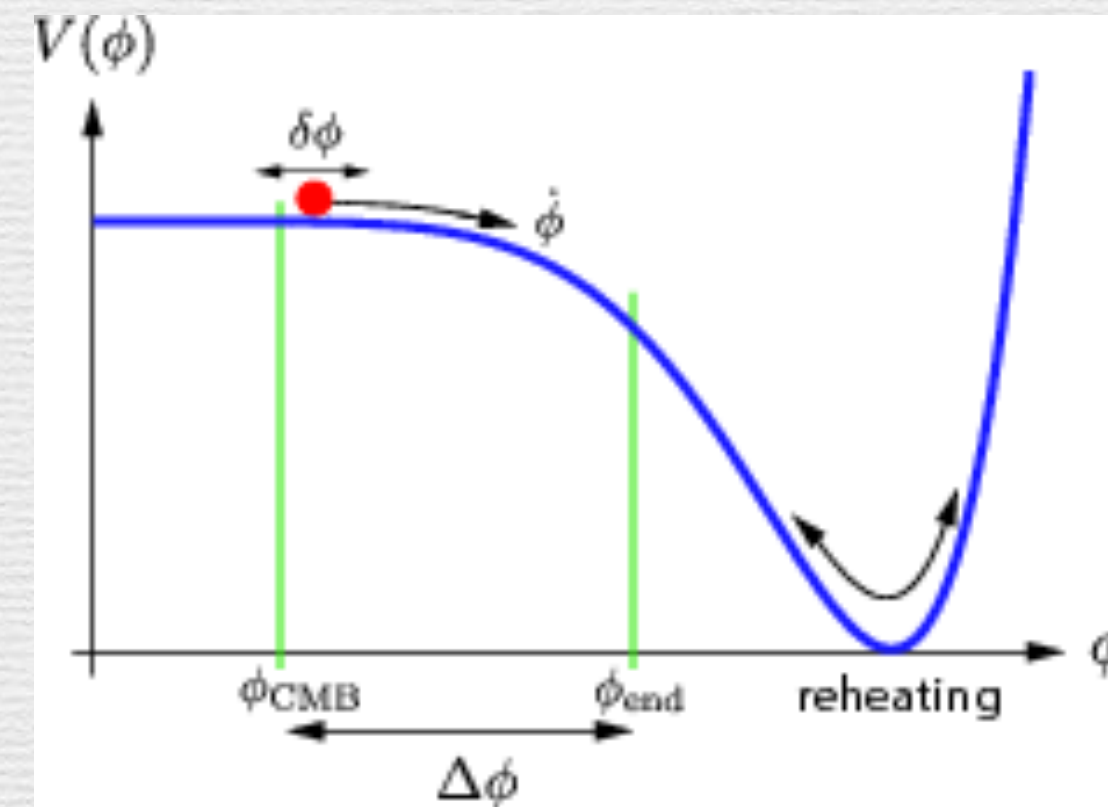
Slowly rolling scalar field :
Inflaton field $\phi(t)$

$$ds^2 = -dt^2 + a^2(t)d\mathbf{x}^2$$

scale factor

$$a(t) = e^{Ht}$$

H : Hubble parameter



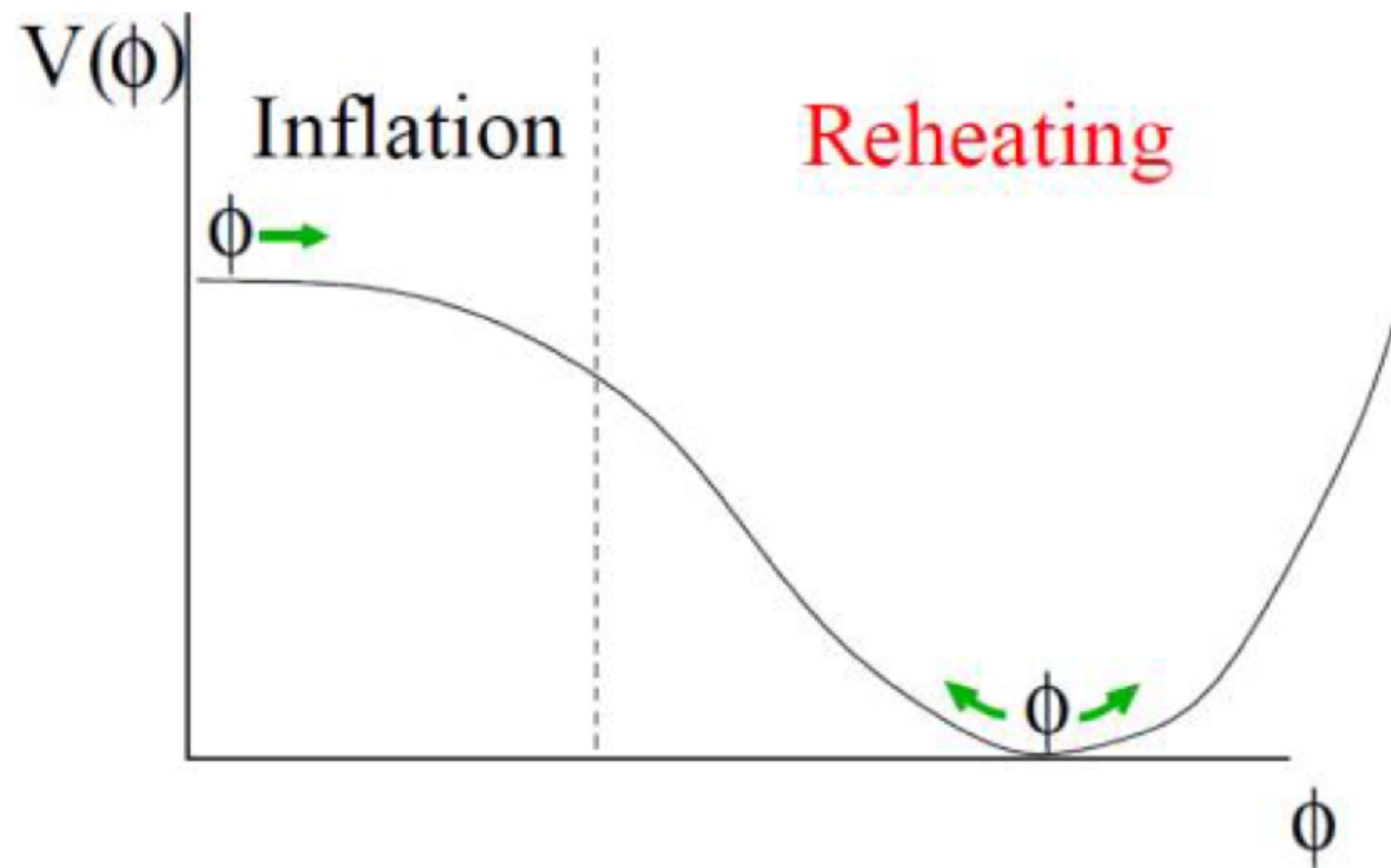
$$\epsilon_V = \frac{M_{\text{Pl}}^2}{2} \left(\frac{V_{,\phi}}{V} \right)^2$$

$$\eta_V = M_{\text{Pl}} \left(\frac{V_{,\phi\phi}}{V} \right)$$

A variant Inflationary scenario:
Warm Inflation

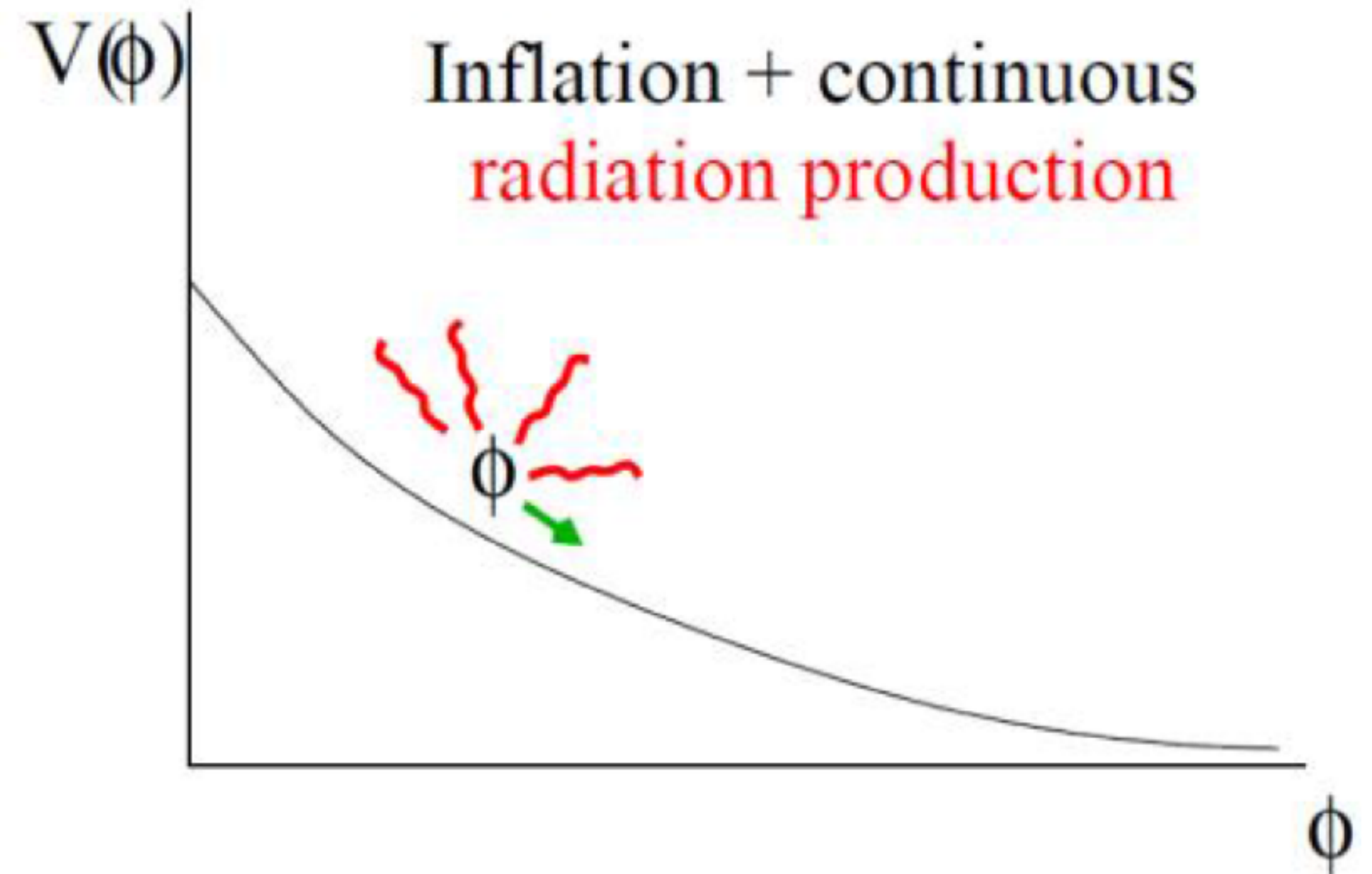
Cold Inflation vs Warm Inflation

Cold Inflation



$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = 0$$

Warm Inflation

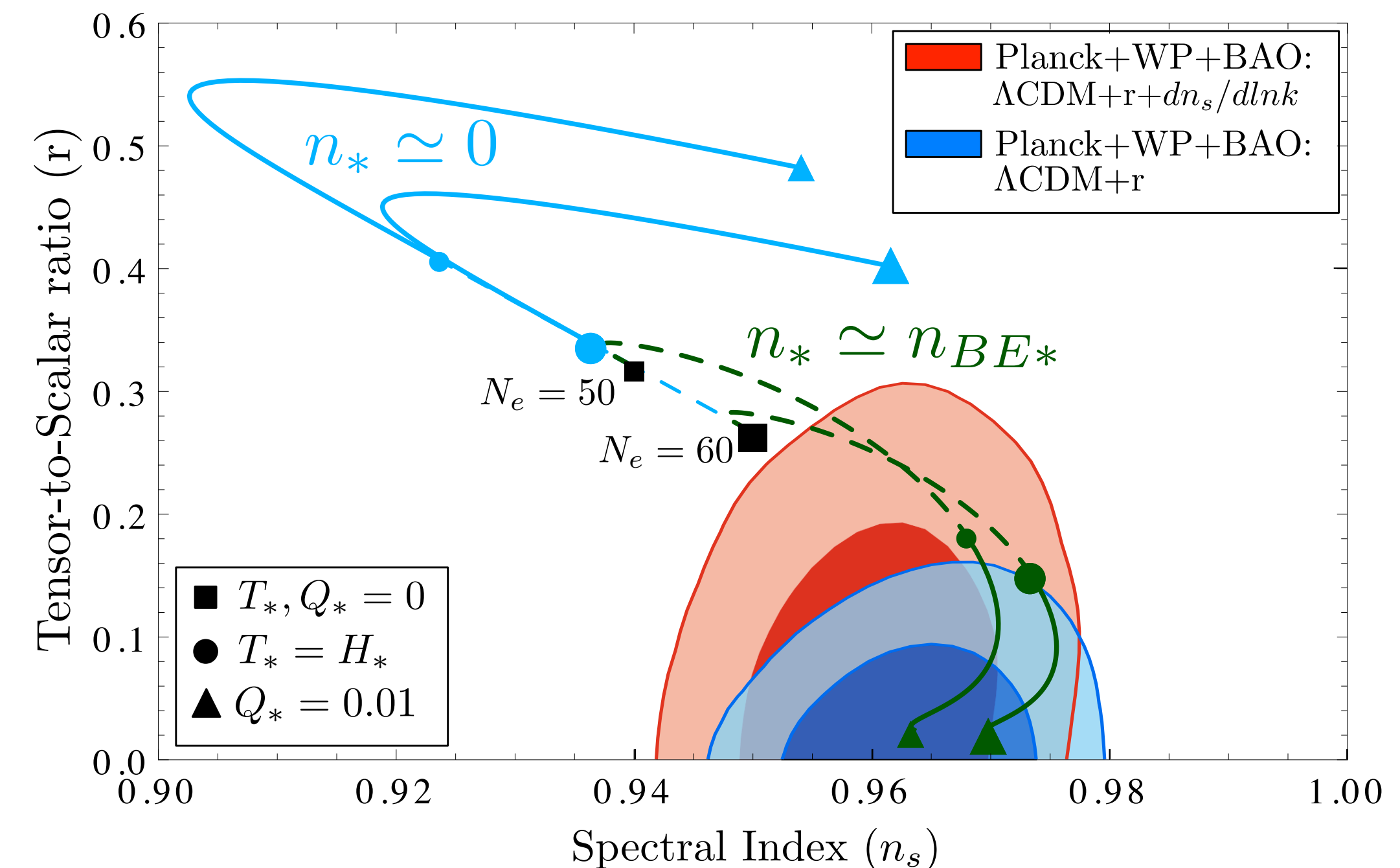
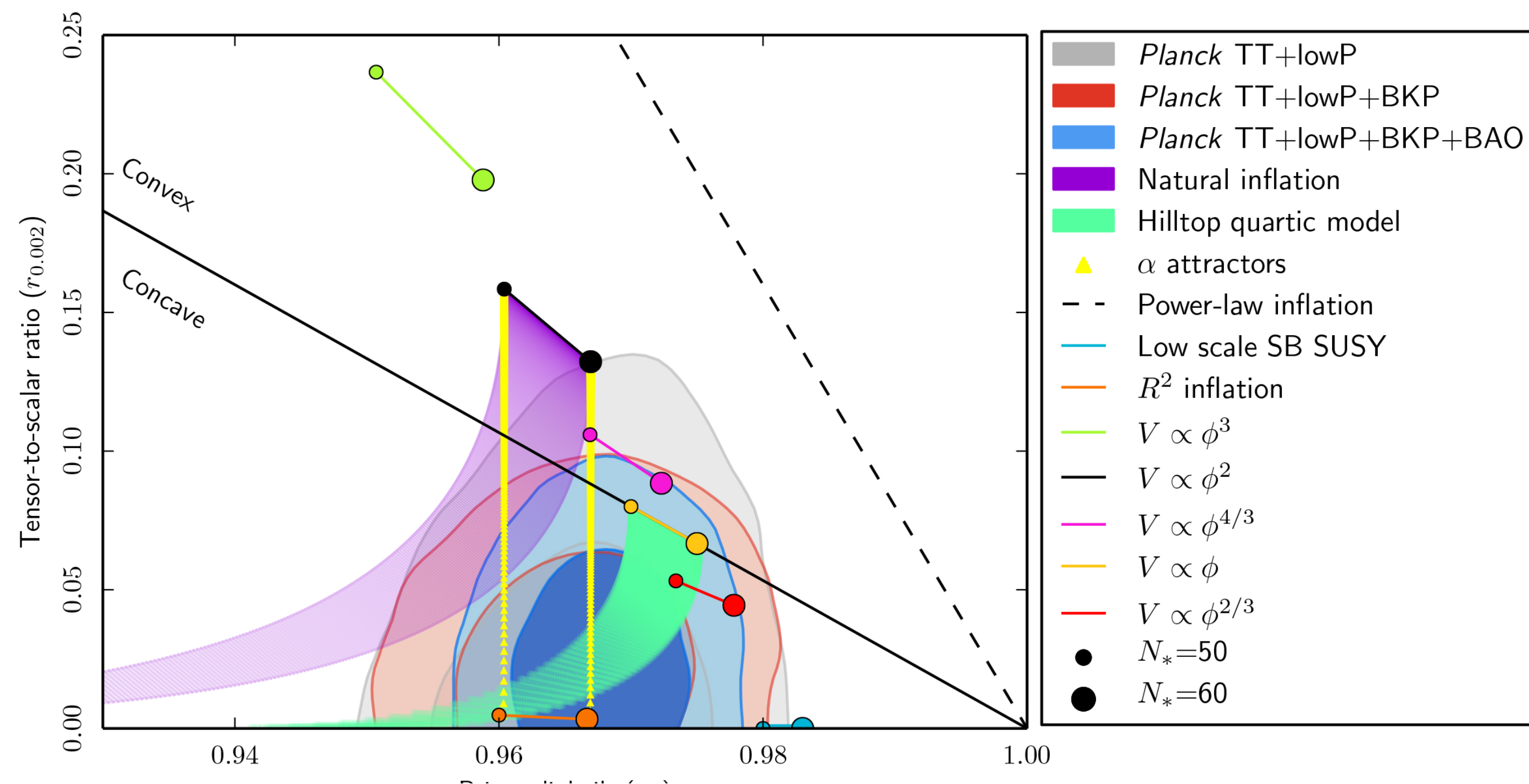


$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = -\Upsilon(T, \phi)\dot{\phi}$$

$$\dot{\rho}_R + 4H\rho_R = \Upsilon(T, \phi)\dot{\phi}^2$$

Why Warm Inflation?

- Warm Inflation does not require a subsequent “reheating” phase, physics of which is still unknown. WI smoothly ends in a radiation dominated phase
- WI has a more enhanced scalar power spectrum than CI. Thus yields a smaller tensor-to-scalar ratio. The potentials that are ruled out in CI for generating large tensor-to-scalar ratios can be accommodated in WI.



Why Warm Inflation?

- Strong dissipative ($Q \gg 1$) WI models prefer small-field inflation models whereas CI is often realised in large-field models. Present data prefers small-field models.
- WI significantly alleviates the eta-problem and the gravitino problem in the context of Supergravity.
- WI is favoured over CI if one considers the Swampland Conjectures arising in String Theory. PRD '19 (2 papers), Physics of the Dark Universe '20, PRD '20 (two papers)

The Inflationary Universe: A Possible Solution to the Horizon and Flatness Problems

Alan H. Guth (SLAC)

Jul, 1980

32 pages


Part of [QUANTUM COSMOLOGY](#), 139-148

Published in: *Phys.Rev.D* 23 (1981) 347-356, *Adv.Ser.Astrophys.Cosmol.* 3 (1987) 139-148 (reprint)

DOI: [10.1103/PhysRevD.23.347](#)

Report number: SLAC-PUB-2576

View in: [OSTI Information Bridge Server](#), [ADS Abstract Service](#), [KEK scanned document](#)

 pdf  links  cite  claim

 reference search  10,680 citations

Citations per year



Abstract: (APS)

The standard model of hot big-bang cosmology requires initial conditions which are problematic in two ways: (1) The early universe is assumed to be highly homogeneous, in spite of the fact that separated regions were causally disconnected (horizon problem); and (2) the initial value of the Hubble constant must be fine tuned to extraordinary accuracy to produce a universe as flat (i.e., near critical mass density) as the one we see today (flatness problem). These problems would disappear if, in its early history, the universe supercooled to temperatures 28 or more orders of magnitude below the critical temperature for some phase transition. A huge expansion factor would then result from a period of exponential growth, and the entropy of the universe would be multiplied by a huge factor when the latent heat is released. Such a scenario is completely natural in the context of grand unified models of elementary-particle interactions. In such models, the supercooling is also relevant to the problem of monopole suppression. Unfortunately, the scenario seems to lead to some unacceptable consequences, so modifications must be sought.

- astrophysics
- inflation
- thermodynamics
- critical phenomena
- Einstein equation: solution
- fundamental constant
- grand unified theory: SU(5)
- baryon number: violation
- CP: violation
- tensor: energy-momentum
- Show all (17)

The Inflationary Universe: A Possible Solution to the Horizon and Flatness Problems

Alan H. Guth (SLAC)

Jul, 1980

32 pages


Part of [QUANTUM COSMOLOGY](#), 139-148

Published in: *Phys.Rev.D* 23 (1981) 347-356, *Adv.Ser.Astrophys.Cosmol.* 3 (1987) 139-148 (reprint)

DOI: [10.1103/PhysRevD.23.347](#)

Report number: SLAC-PUB-2576

View in: [OSTI Information Bridge Server](#), [ADS Abstract Service](#), [KEK scanned document](#)

 pdf  links  cite  claim

 reference search  10,680 citations

Citations per year



Abstract: (APS)

The standard model of hot big-bang cosmology requires initial conditions which are problematic in two ways: (1) The early universe is assumed to be highly homogeneous, in spite of the fact that separated regions were causally disconnected (horizon problem); and (2) the initial value of the Hubble constant must be fine tuned to extraordinary accuracy to produce a universe as flat (i.e., near critical mass density) as the one we see today (flatness problem). These problems would disappear if, in its early history, the universe supercooled to temperatures 28 or more orders of magnitude below the critical temperature for some phase transition. A huge expansion factor would then result from a period of exponential growth, and the entropy of the universe would be multiplied by a huge factor when the latent heat is released. Such a scenario is completely natural in the context of grand unified models of elementary-particle interactions. In such models, the supercooling is also relevant to the problem of monopole suppression. Unfortunately, the scenario seems to lead to some unacceptable consequences, so modifications must be sought.

- astrophysics
- inflation
- thermodynamics
- critical phenomena
- Einstein equation: solution
- fundamental constant
- grand unified theory: SU(5)
- baryon number: violation
- CP: violation
- tensor: energy-momentum
- Show all (17)

The Inflationary Universe: A Possible Solution to the Horizon and Flatness Problems

Alan H. Guth (SLAC)

Jul, 1980

32 pages

Part of [QUANTUM COSMOLOGY](#), 139-148

Published in: *Phys.Rev.D* 23 (1981) 347-356, *Adv.Ser.Astrophys.Cosmol.* 3 (1987) 139-148 (reprint)

DOI: [10.1103/PhysRevD.23.347](#)

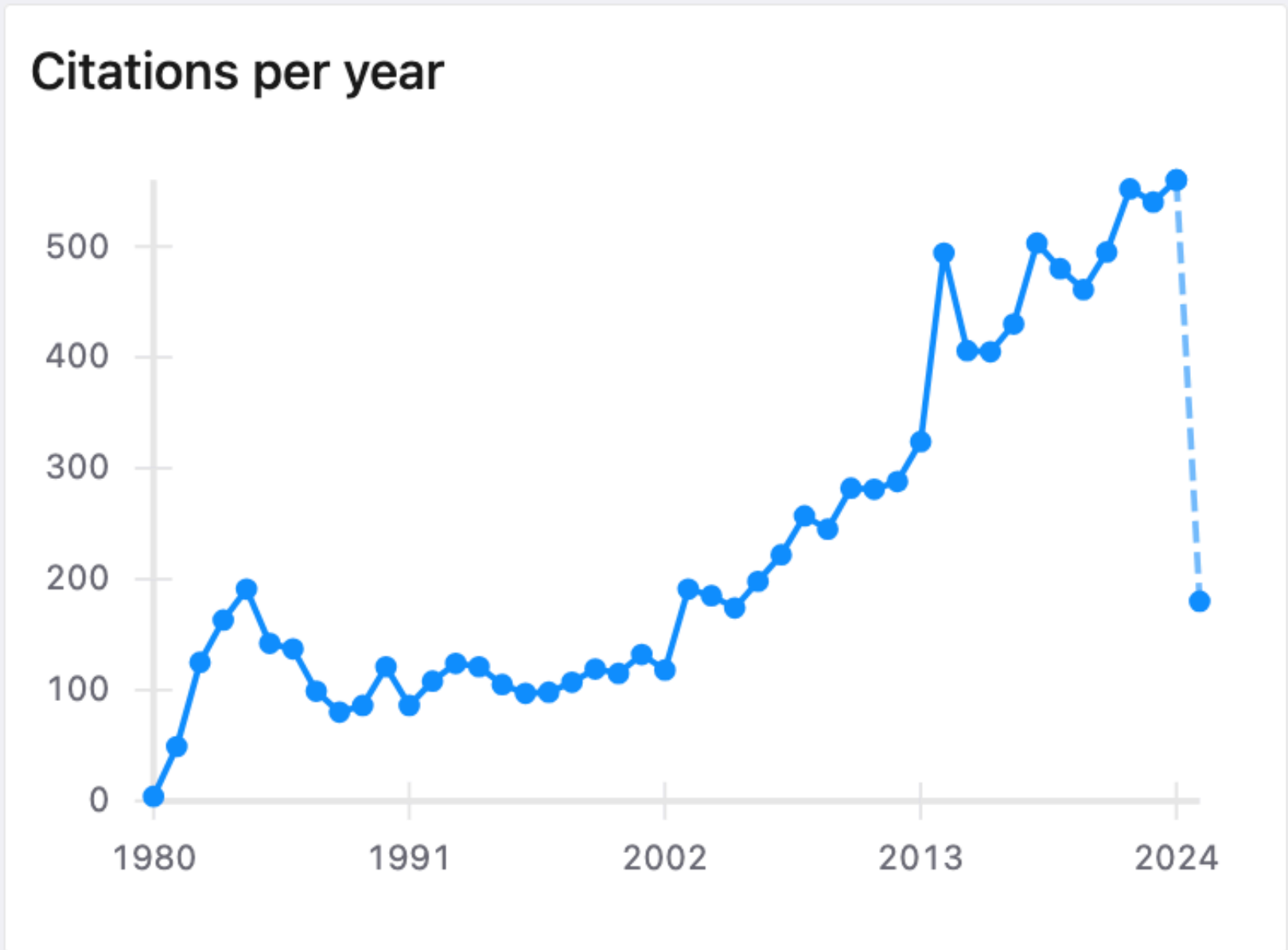
Report number: SLAC-PUB-2576

View in: [OSTI Information Bridge Server](#), [ADS Abstract Service](#), [KEK scanned document](#)

pdf links cite claim

reference search

10,680 citations



Abstract: (APS)

The standard model of hot big-bang cosmology requires initial conditions which are problematic in two ways: (1) The early universe is assumed to be highly homogeneous, in spite of the fact that separated regions were causally disconnected (horizon problem); and (2) the initial value of the Hubble constant must be fine tuned to extraordinary accuracy to produce a universe as flat (i.e., near critical mass density) as the one we see today (flatness problem). These problems would disappear if, in its early history, the universe supercooled to temperatures 28 or more orders of magnitude below the critical temperature for some phase transition. A huge expansion factor would then result from a period of exponential growth, and the entropy of the universe would be multiplied by a huge factor when the latent heat is released. Such a scenario is completely natural in the context of grand unified models of elementary-particle interactions. In such models, the supercooling is also relevant to the problem of monopole suppression. Unfortunately, the scenario seems to lead to some unacceptable consequences, so modifications must be sought.

- astrophysics
- inflation
- thermodynamics
- critical phenomena
- Einstein equation: solution
- fundamental constant
- grand unified theory: SU(5)
- baryon number: violation
- CP: violation
- tensor: energy-momentum
- Show all (17)

Warm inflation

Arjun Berera (Penn State U.)

Sep, 1995

11 pages

Published in: *Phys.Rev.Lett.* 75 (1995) 3218-3221


e-Print: [astro-ph/9509049](#) [astro-ph]

DOI: [10.1103/PhysRevLett.75.3218](#)

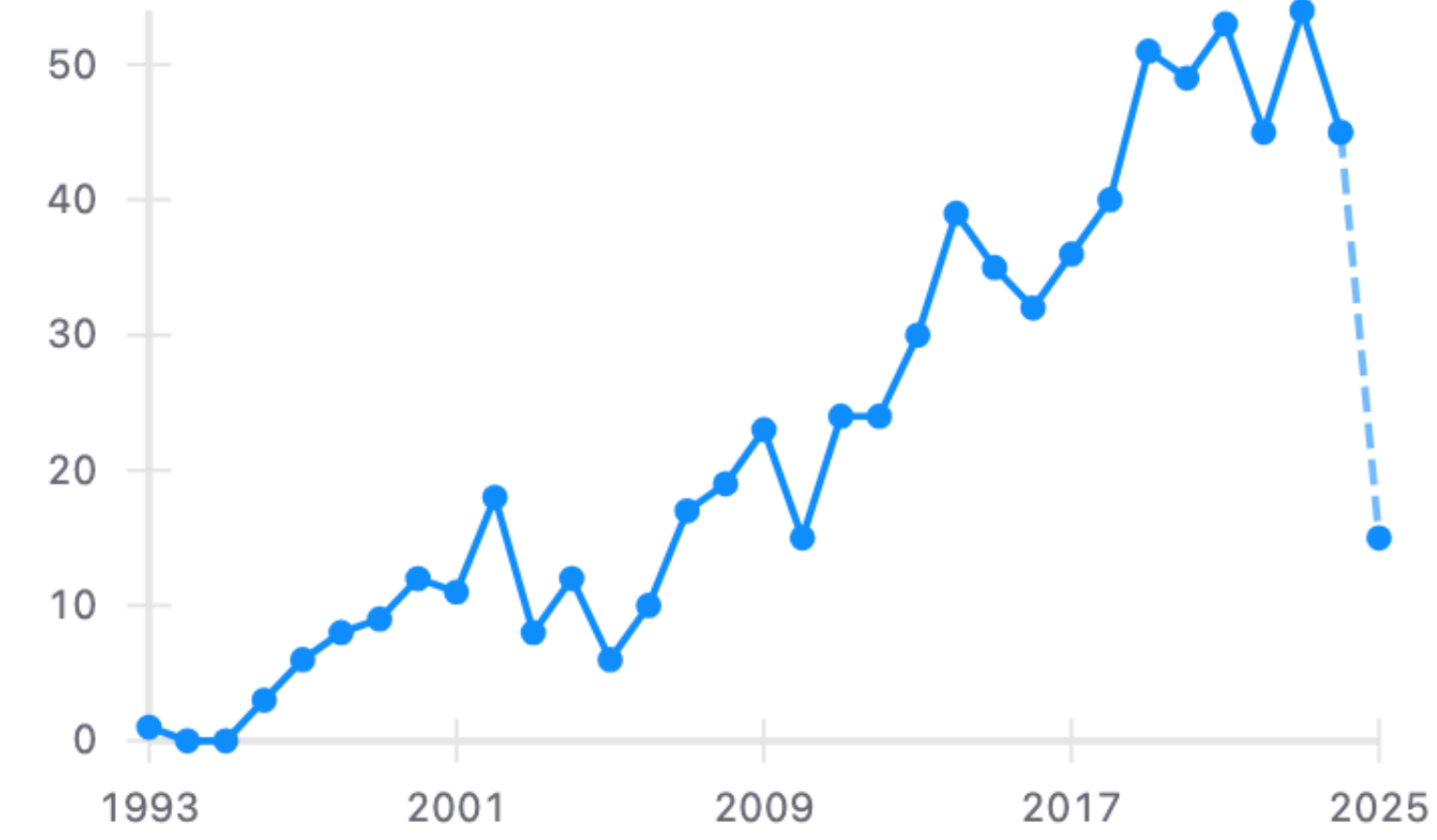
Report number: PSU-TH-159

View in: [OSTI Information Bridge Server](#), [ADS Abstract Service](#)

 pdf  cite  claim

 reference search  750 citations

Citations per year



Abstract: (arXiv)

Assuming a first order phase transition during inflation, a model scenario is described which does not require a tiny coupling constant. Thermal equilibrium is closely maintained as inflation commences. No large scale reheating is necessary. Solutions for $N > 70$ are found for any $\delta\rho/\rho$ in the range $10^{-5} - 10^{-3}$.


[inflation](#) [critical phenomena](#) [thermodynamics](#) [scaling](#) [coupling constant](#) [effective potential](#) [finite temperature](#) [numerical calculations](#)

Warm inflation

Arjun Berera (Penn State U.)
Sep, 1995

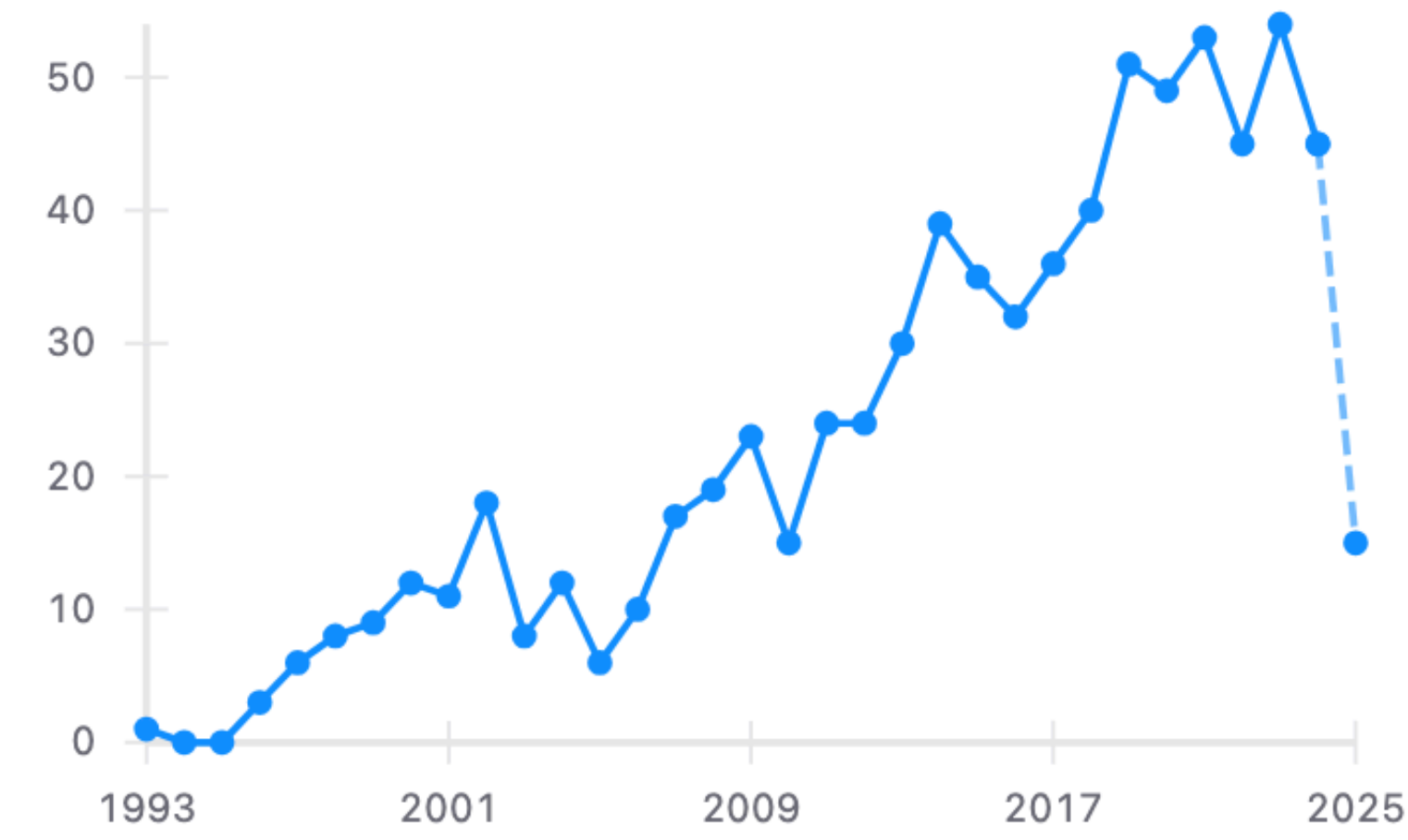
11 pages
Published in: *Phys.Rev.Lett.* 75 (1995) 3218-3221
e-Print: [astro-ph/9509049](#) [astro-ph]
DOI: [10.1103/PhysRevLett.75.3218](#)
Report number: PSU-TH-159
View in: [OSTI Information Bridge Server](#), [ADS Abstract Service](#)

 pdf  cite  claim

 reference search

 750 citations

Citations per year



Abstract: (arXiv)
Assuming a first order phase transition during inflation, a model scenario is described which does not require a tiny coupling constant. Thermal equilibrium is closely maintained as inflation commences. No large scale reheating is necessary. Solutions for $N > 70$ are found for any $\delta\rho/\rho$ in the range $10^{-5} - 10^{-3}$.

[inflation](#) [critical phenomena](#) [thermodynamics](#) [scaling](#) [coupling constant](#) [effective potential](#) [finite temperature](#) [numerical calculations](#)

Today's Topic: "How inflation ends?"
a.k.a. Graceful Exit

Graceful Exit?

- Inflation is a phase when the Universe has expanded exponentially (accelerated) for a very brief amount of time.
- Feeding in the FRW metric into the Einstein Field equations with the matter as perfect fluid $p=\omega\rho$, we get two Friedmann equations which tell us about the expansion history of the universe:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho, \quad \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(1 + 3\omega)\rho$$

- Thus $\omega=-1$ fluid can easily drive inflation, like a Cosmological Constant Λ
- The problem is with CC inflation will never end and hence no Graceful Exit

Graceful Exit?

- Inflation is a phase when the Universe has expanded exponentially (accelerated) for a very brief amount of time.
- Feeding in the FRW metric into the Einstein Field equations with the matter as perfect fluid $p=\omega\rho$, we get two Friedmann equations which tell us about the expansion history of the universe:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho, \quad \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(1+3\omega)\rho \quad || \quad \ddot{a} > 0 \Rightarrow \omega < -\frac{1}{3}$$

- Thus $\omega=-1$ fluid can easily drive inflation, like a Cosmological Constant Λ
- The problem is with CC inflation will never end and hence no Graceful Exit

The inflaton & its slow-roll

- A simple scalar field's energy density and pressure:

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi) + \frac{(\nabla\phi)^2}{2a^2}, \quad p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi) - \frac{(\nabla\phi)^2}{6a^2}$$

- Potential energy dominated scalar field, a.k.a. the inflaton field, can drive inflation
- The potential needs to be very flat, so that the kinetic term is negligible \longrightarrow slow-roll of the inflaton on its flat potential
- Slow-roll parameter $\epsilon_V \equiv \frac{M_{\text{Pl}}^2}{2} \left(\frac{V_{,\phi}}{V} \right)^2 \ll 1$

The inflaton & its slow-roll

- A simple scalar field's energy density and pressure:

$$\rho_\phi = \boxed{\frac{1}{2}\dot{\phi}^2} + V(\phi) + \frac{(\nabla\phi)^2}{2a^2}, \quad p_\phi = \boxed{\frac{1}{2}\dot{\phi}^2} - V(\phi) - \frac{(\nabla\phi)^2}{6a^2} \quad \boxed{\omega_\phi = 1}$$

- Potential energy dominated scalar field, a.k.a. the inflaton field, can drive inflation
- The potential needs to be very flat, so that the kinetic term is negligible \longrightarrow slow-roll of the inflaton on its flat potential
- Slow-roll parameter $\epsilon_V \equiv \frac{M_{\text{Pl}}^2}{2} \left(\frac{V_{,\phi}}{V} \right)^2 \ll 1$

The inflaton & its slow-roll

- A simple scalar field's energy density and pressure:

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi) + \frac{(\nabla\phi)^2}{2a^2}, \quad p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi) - \frac{(\nabla\phi)^2}{6a^2}$$

- Potential energy dominated scalar field, a.k.a. the inflaton field, can drive inflation
- The potential needs to be very flat, so that the kinetic term is negligible \longrightarrow slow-roll of the inflaton on its flat potential
- Slow-roll parameter $\epsilon_V \equiv \frac{M_{\text{Pl}}^2}{2} \left(\frac{V_{,\phi}}{V} \right)^2 \ll 1$

The inflaton & its slow-roll

- A simple scalar field's energy density and pressure:

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi) + \frac{(\nabla\phi)^2}{2a^2}, \quad p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi) - \frac{(\nabla\phi)^2}{6a^2} \quad \omega_\phi = -\frac{1}{3}$$

- Potential energy dominated scalar field, a.k.a. the inflaton field, can drive inflation
- The potential needs to be very flat, so that the kinetic term is negligible \longrightarrow slow-roll of the inflaton on its flat potential
- Slow-roll parameter $\epsilon_V \equiv \frac{M_{\text{Pl}}^2}{2} \left(\frac{V_{,\phi}}{V} \right)^2 \ll 1$

The inflaton & its slow-roll

- A simple scalar field's energy density and pressure:

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi) + \frac{(\nabla\phi)^2}{2a^2}, \quad p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi) - \frac{(\nabla\phi)^2}{6a^2}$$

- Potential energy dominated scalar field, a.k.a. the inflaton field, can drive inflation
- The potential needs to be very flat, so that the kinetic term is negligible \longrightarrow slow-roll of the inflaton on its flat potential
- Slow-roll parameter $\epsilon_V \equiv \frac{M_{\text{Pl}}^2}{2} \left(\frac{V_{,\phi}}{V} \right)^2 \ll 1$

The inflaton & its slow-roll

- A simple scalar field's energy density and pressure:

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + \boxed{V(\phi)} + \frac{(\nabla\phi)^2}{2a^2}, \quad p_\phi = \frac{1}{2}\dot{\phi}^2 - \boxed{V(\phi)} - \frac{(\nabla\phi)^2}{6a^2} \quad \boxed{\omega_\phi = -1}$$

- Potential energy dominated scalar field, a.k.a. the inflaton field, can drive inflation
- The potential needs to be very flat, so that the kinetic term is negligible \longrightarrow slow-roll of the inflaton on its flat potential
- Slow-roll parameter $\epsilon_V \equiv \frac{M_{\text{Pl}}^2}{2} \left(\frac{V_{,\phi}}{V} \right)^2 \ll 1$

Slow-roll & Graceful Exit

- Slow-roll EoM of inflaton:

$$3H\dot{\phi} \approx -V_{,\phi}$$

- It turns out $\epsilon \ll 1$ implies

$$\frac{1}{2}\dot{\phi}^2 \ll V(\phi)$$

- Thus inflation ends when $\epsilon \sim 1$

Slow-roll & Graceful Exit

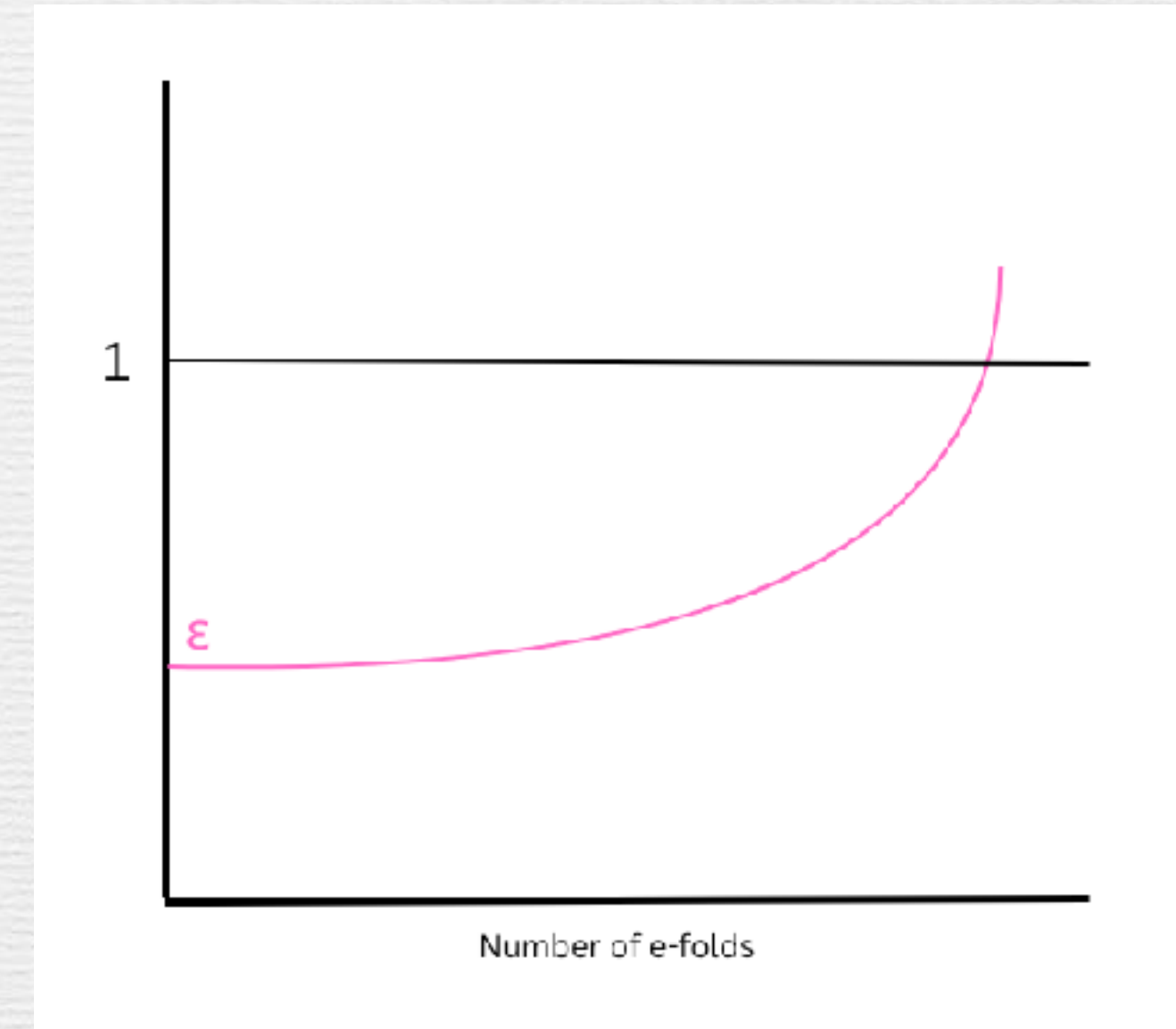
- Slow-roll EoM of inflaton:

$$3H\dot{\phi} \approx -V_{,\phi}$$

- It turns out $\epsilon \ll 1$ implies

$$\frac{1}{2}\dot{\phi}^2 \ll V(\phi)$$

- Thus inflation ends when $\epsilon \sim 1$



Slow-roll & Graceful Exit

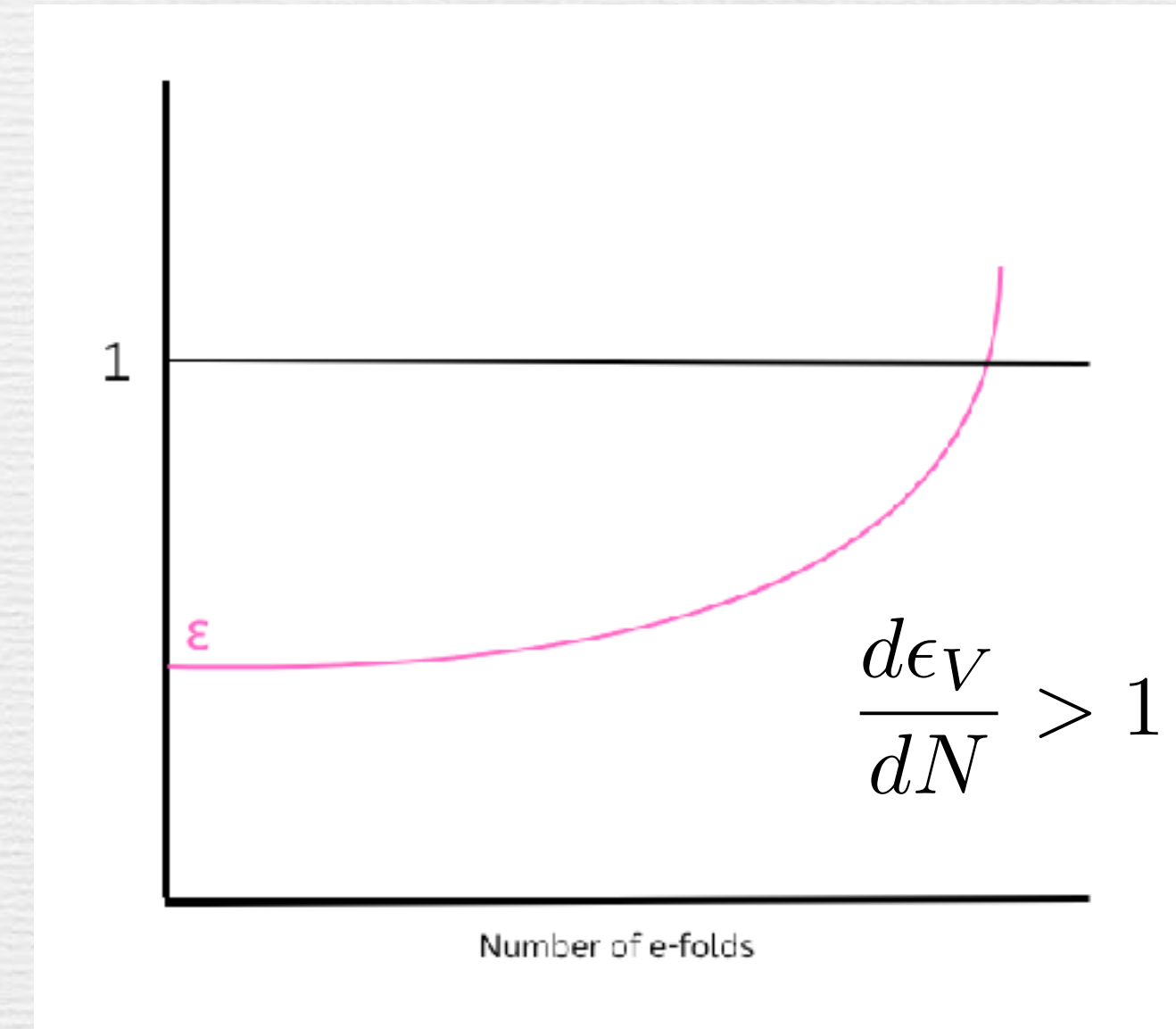
- Slow-roll EoM of inflaton:

$$3H\dot{\phi} \approx -V_{,\phi}$$

- It turns out $\epsilon \ll 1$ implies

$$\frac{1}{2}\dot{\phi}^2 \ll V(\phi)$$

- Thus inflation ends when $\epsilon \sim 1$



Slow-roll & Graceful Exit

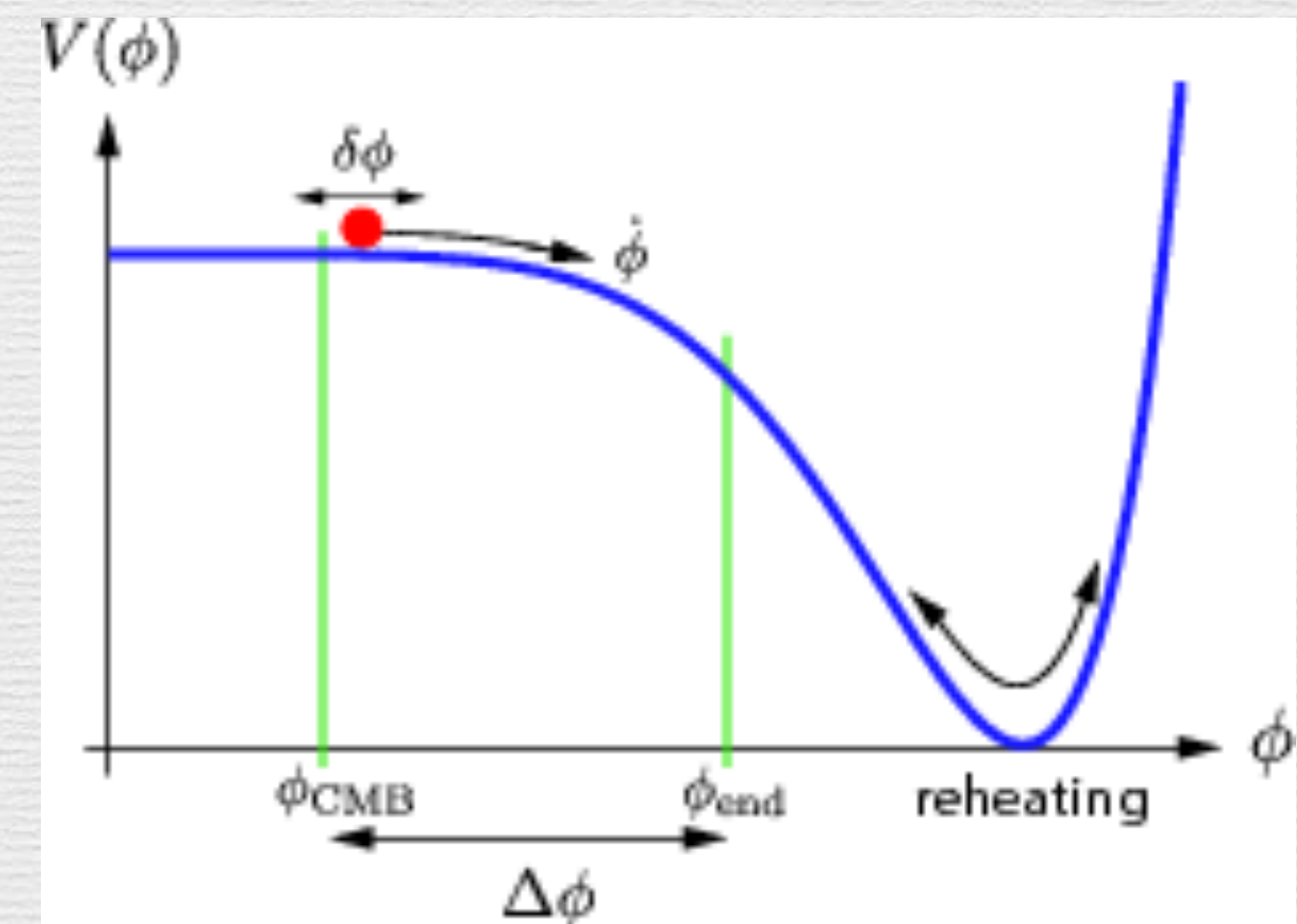
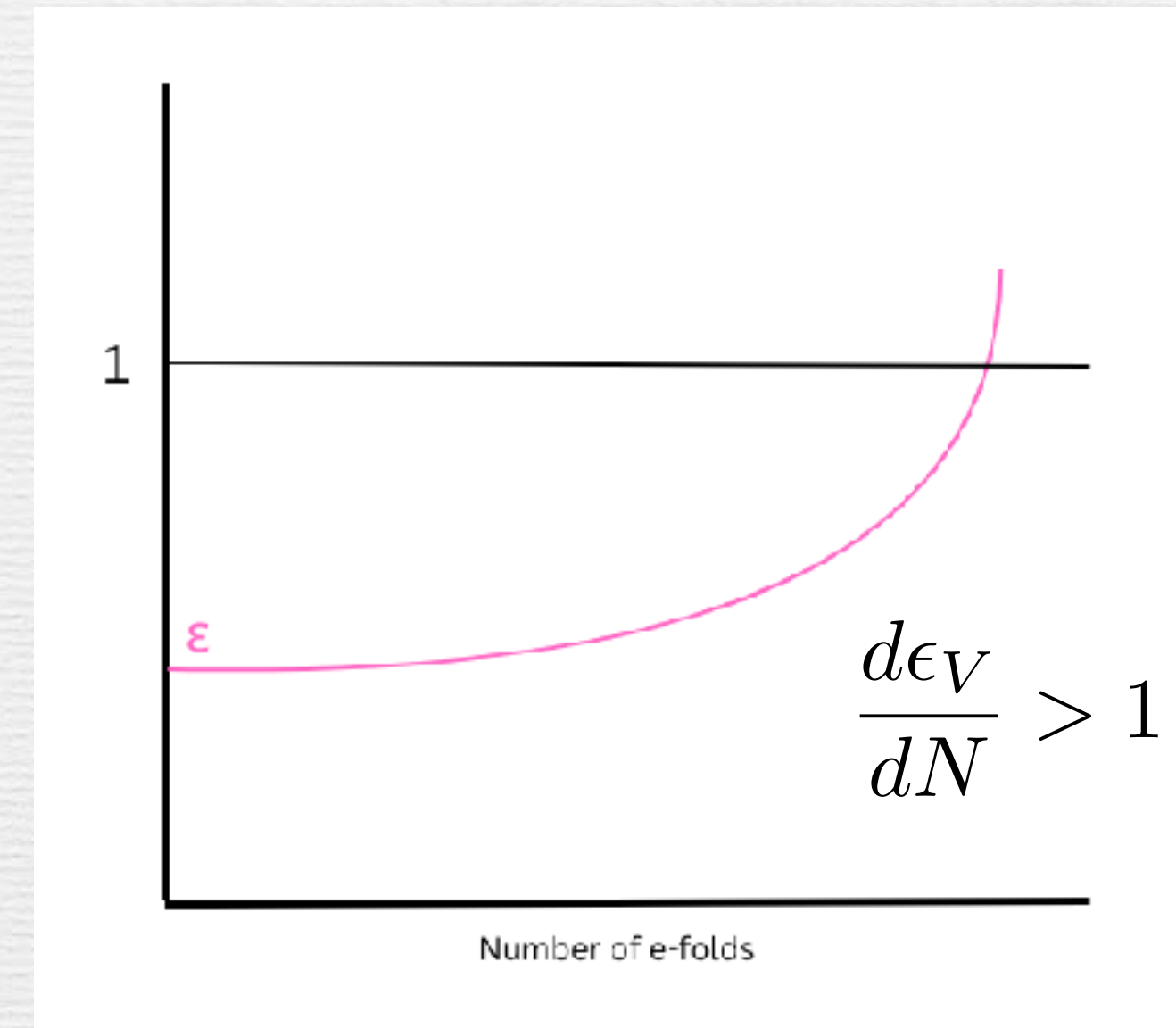
- Slow-roll EoM of inflaton:

$$3H\dot{\phi} \approx -V_{,\phi}$$

- It turns out $\epsilon \ll 1$ implies

$$\frac{1}{2}\dot{\phi}^2 \ll V(\phi)$$

- Thus inflation ends when $\epsilon \sim 1$



Graceful Exit in Warm Inflation

- The slow-roll EoM of the inflaton field in Warm Inflation

$$3H(1 + Q) \approx -V_{,\phi} \quad || \quad Q \equiv \frac{\Upsilon}{3H}$$

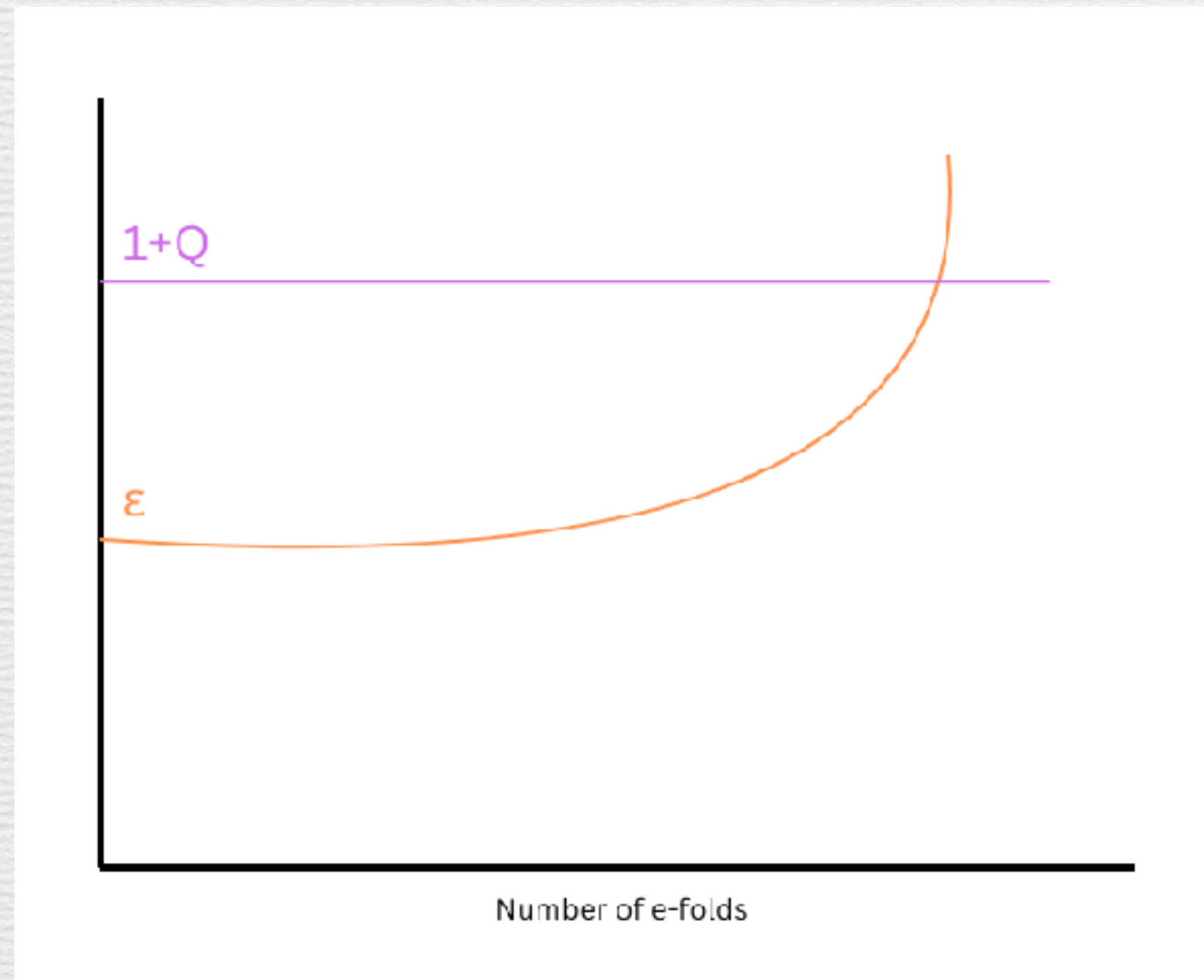
- Thus inflation happens when $\epsilon_V \ll 1 + Q$ and ends when $\epsilon_V \sim 1 + Q$
- Graceful exit depends on the evolution of both ϵ and Q
- Assuming a simple form of the dissipative coefficient $\Upsilon \propto T^p \phi^c$

$$\frac{d \ln \epsilon_V}{dN} = \frac{4\epsilon_V - 2\eta_V}{1 + Q}, \quad \frac{d \ln Q}{dN} = \frac{(2p + 4)\epsilon_V - 2p\eta_V - 4c\kappa_V}{4 - p + (4 + p)Q}$$

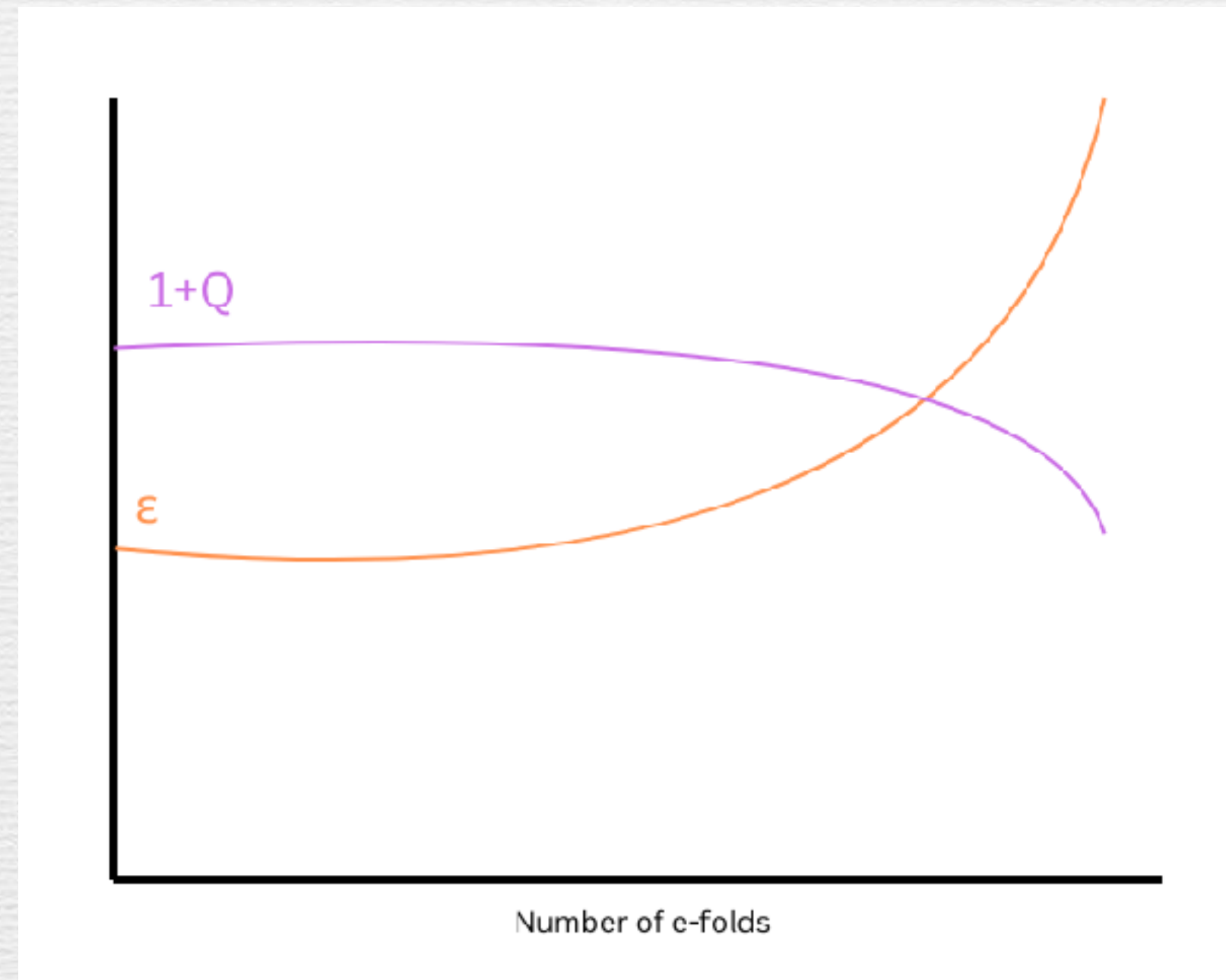
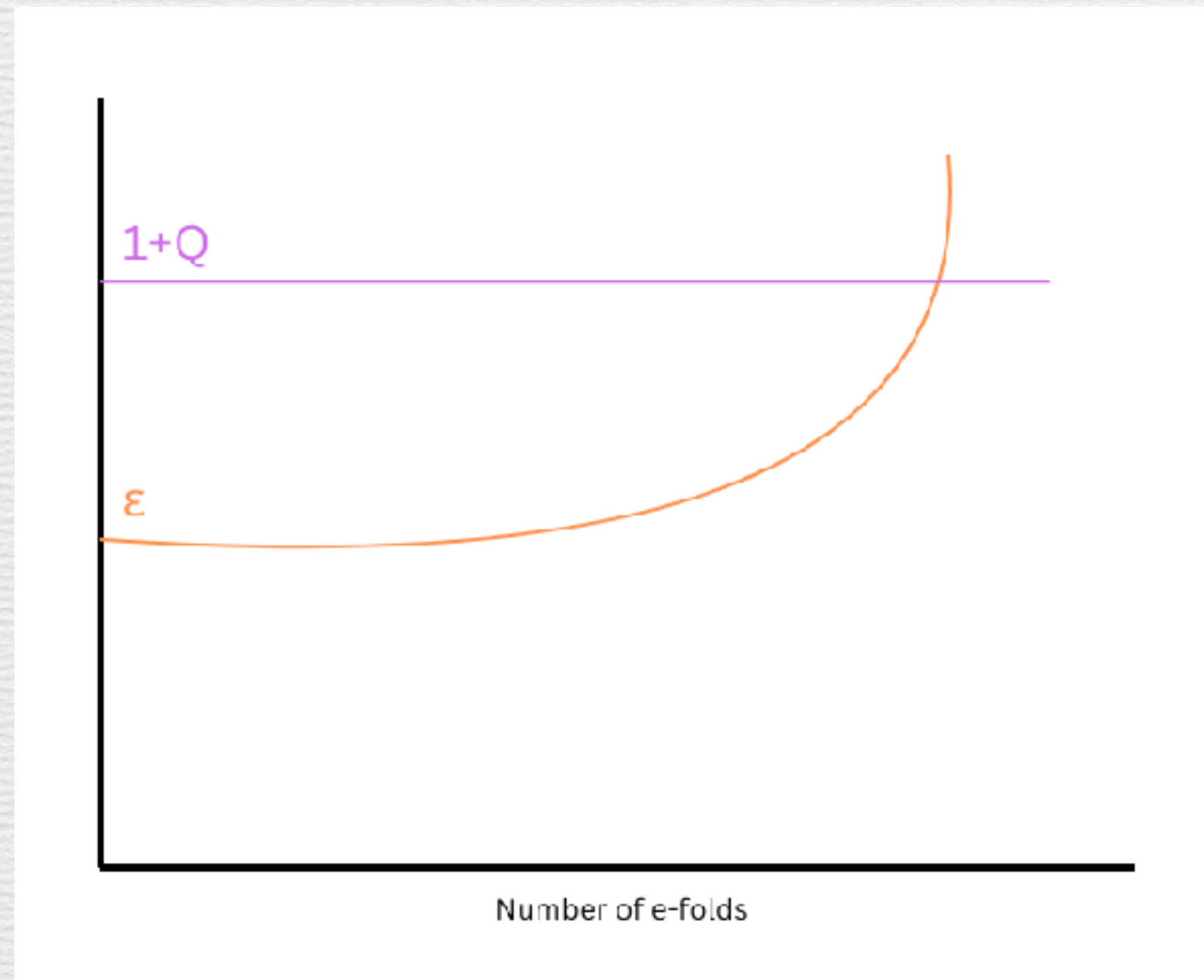
How many ways Warm Inflation can
Gracefully exit?
i.e. meeting the condition $\varepsilon=1+Q$

When ε grows

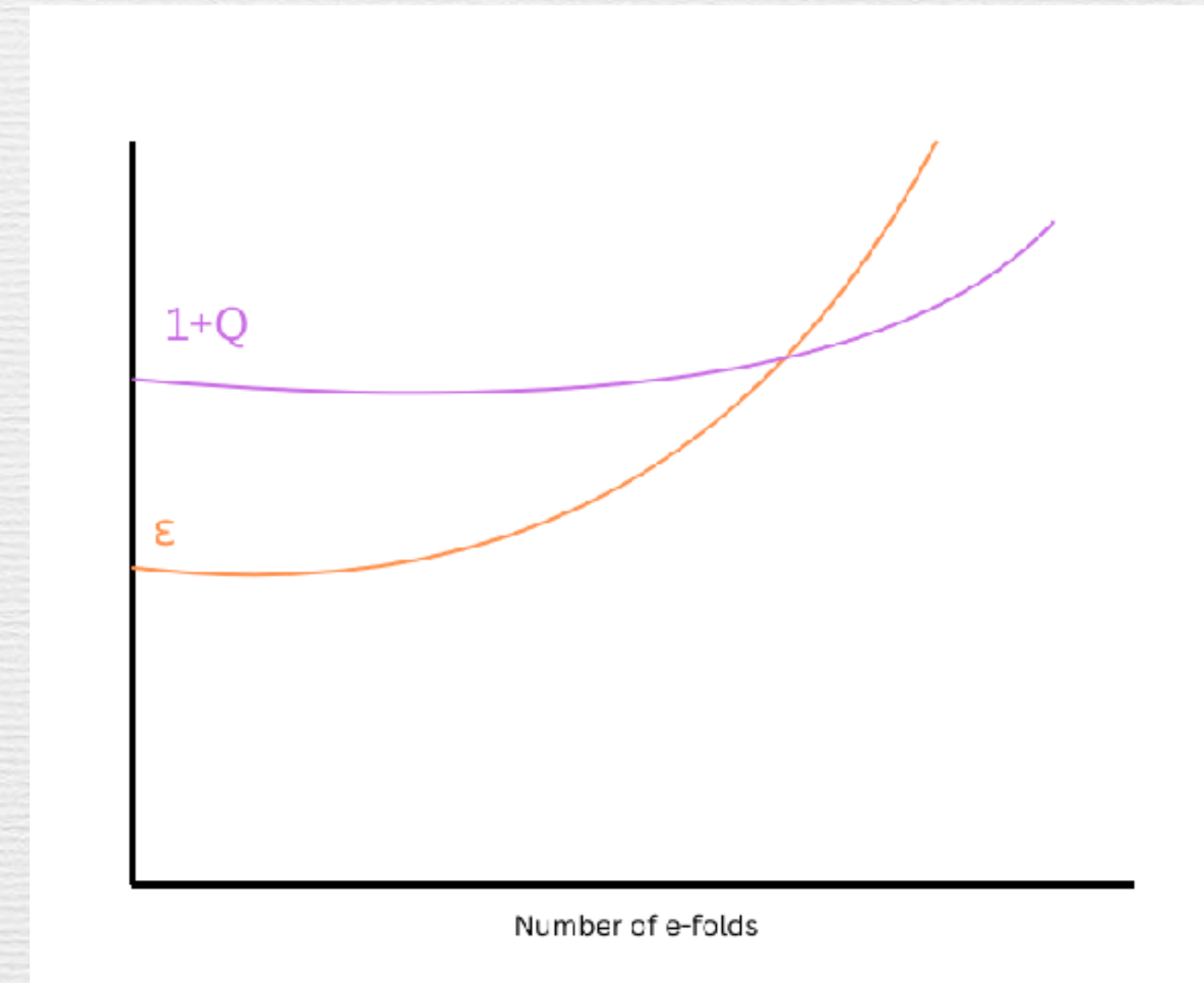
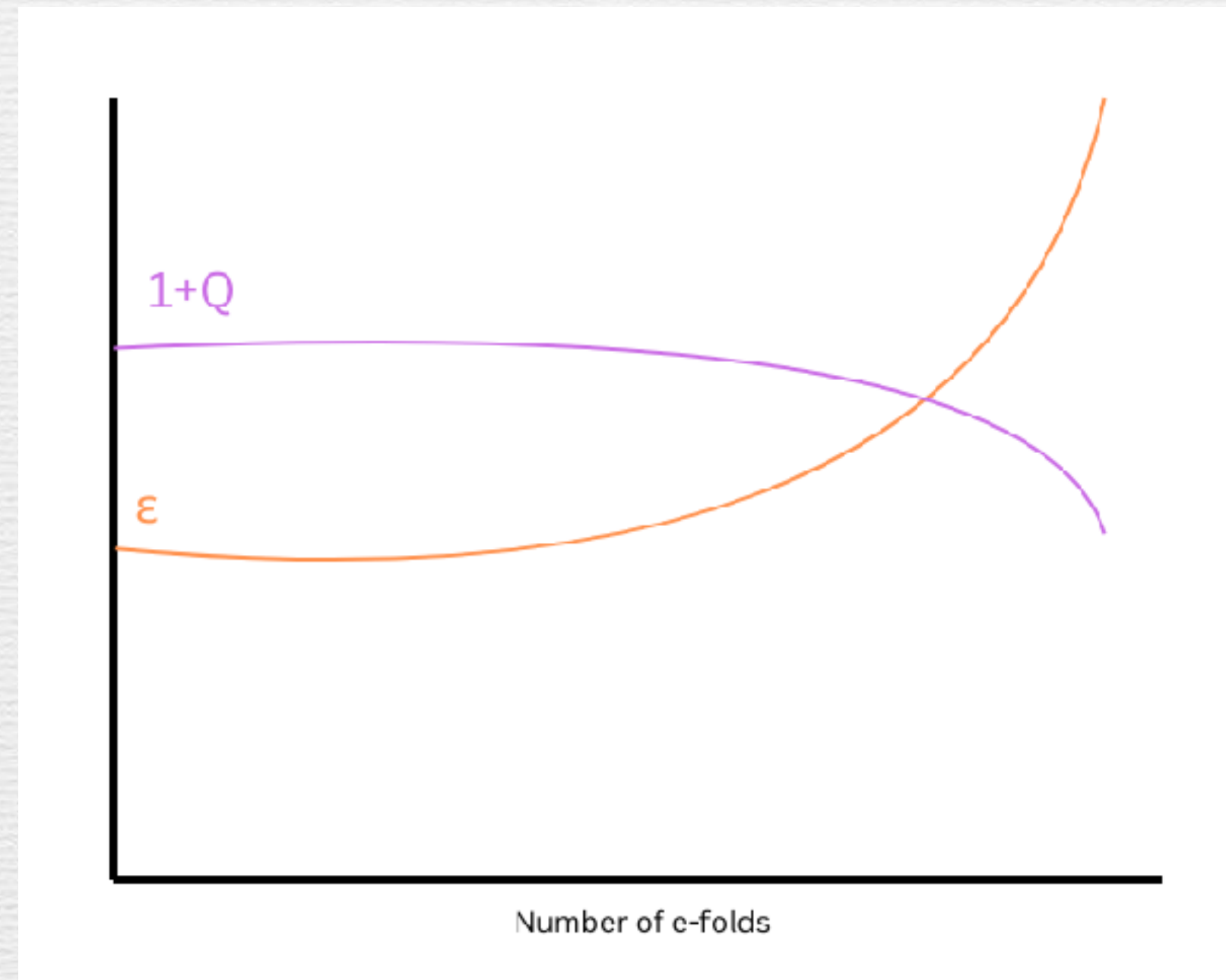
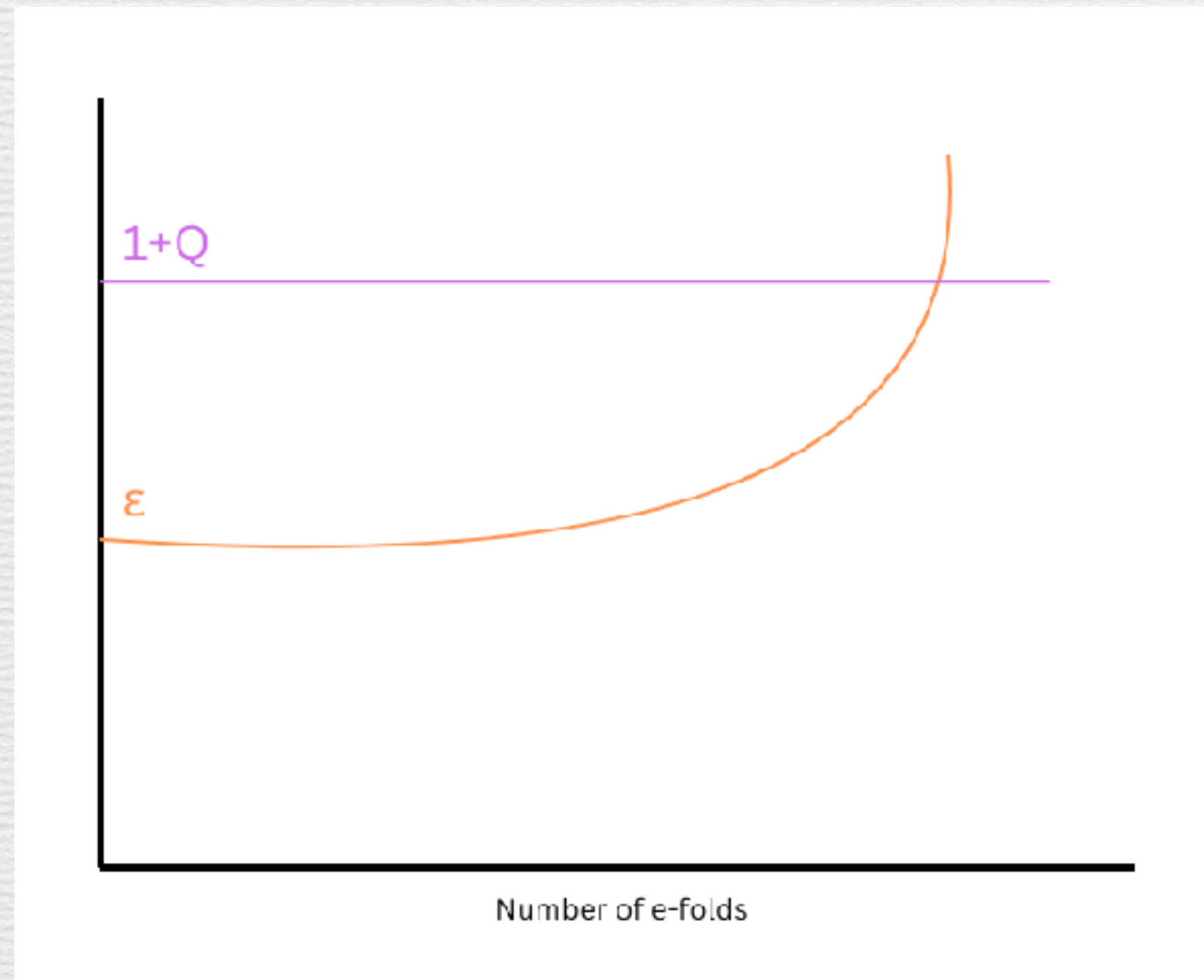
When ε grows



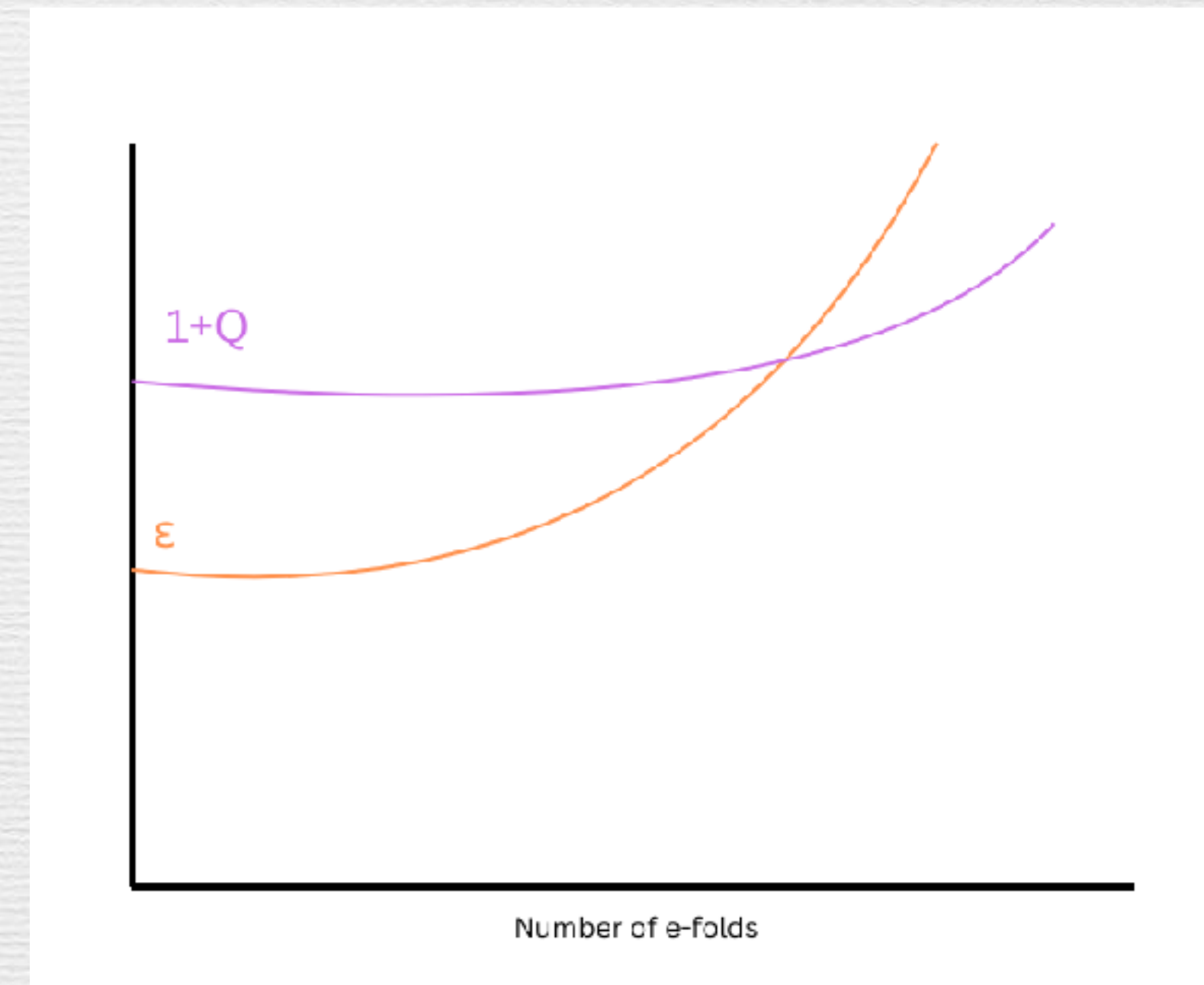
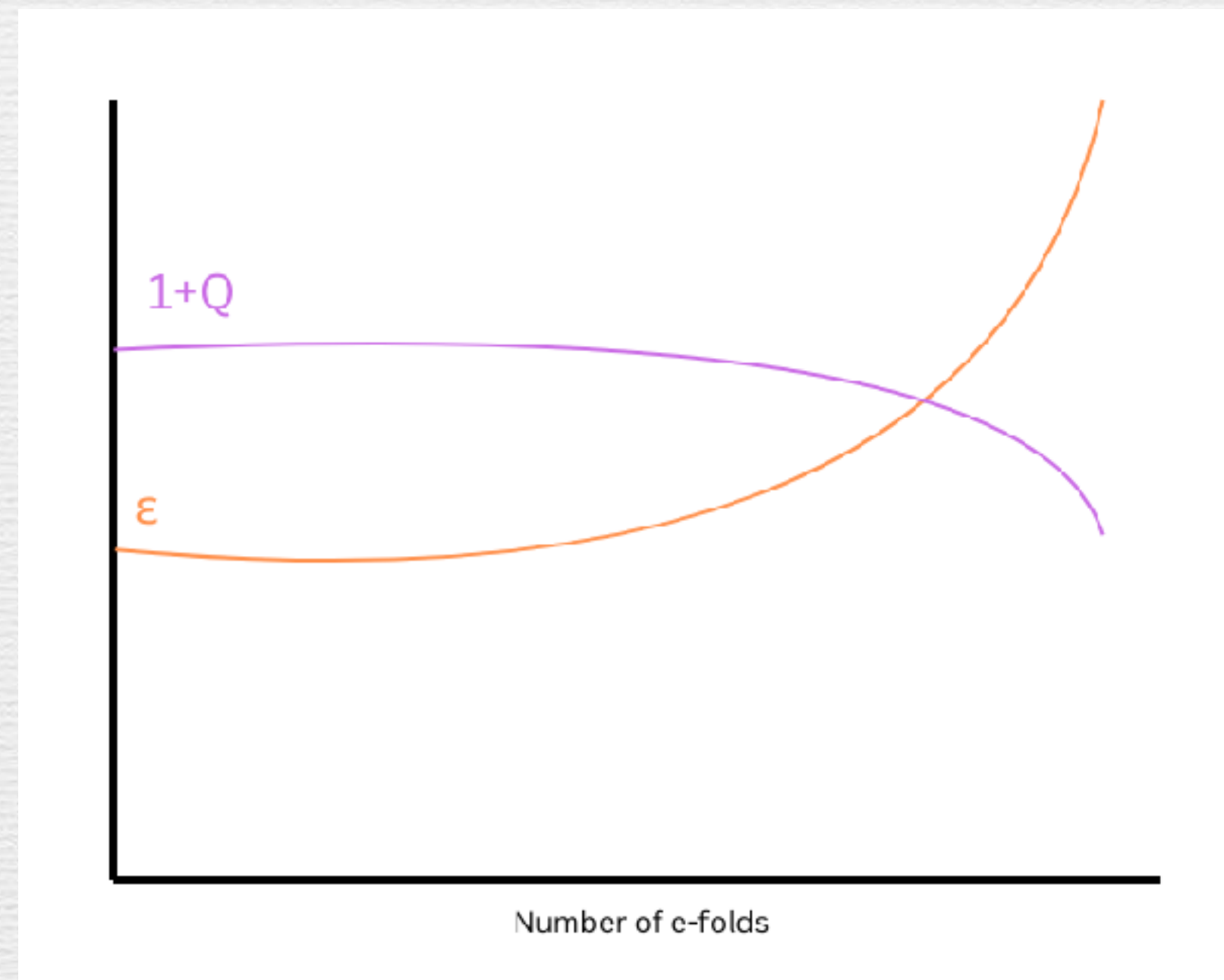
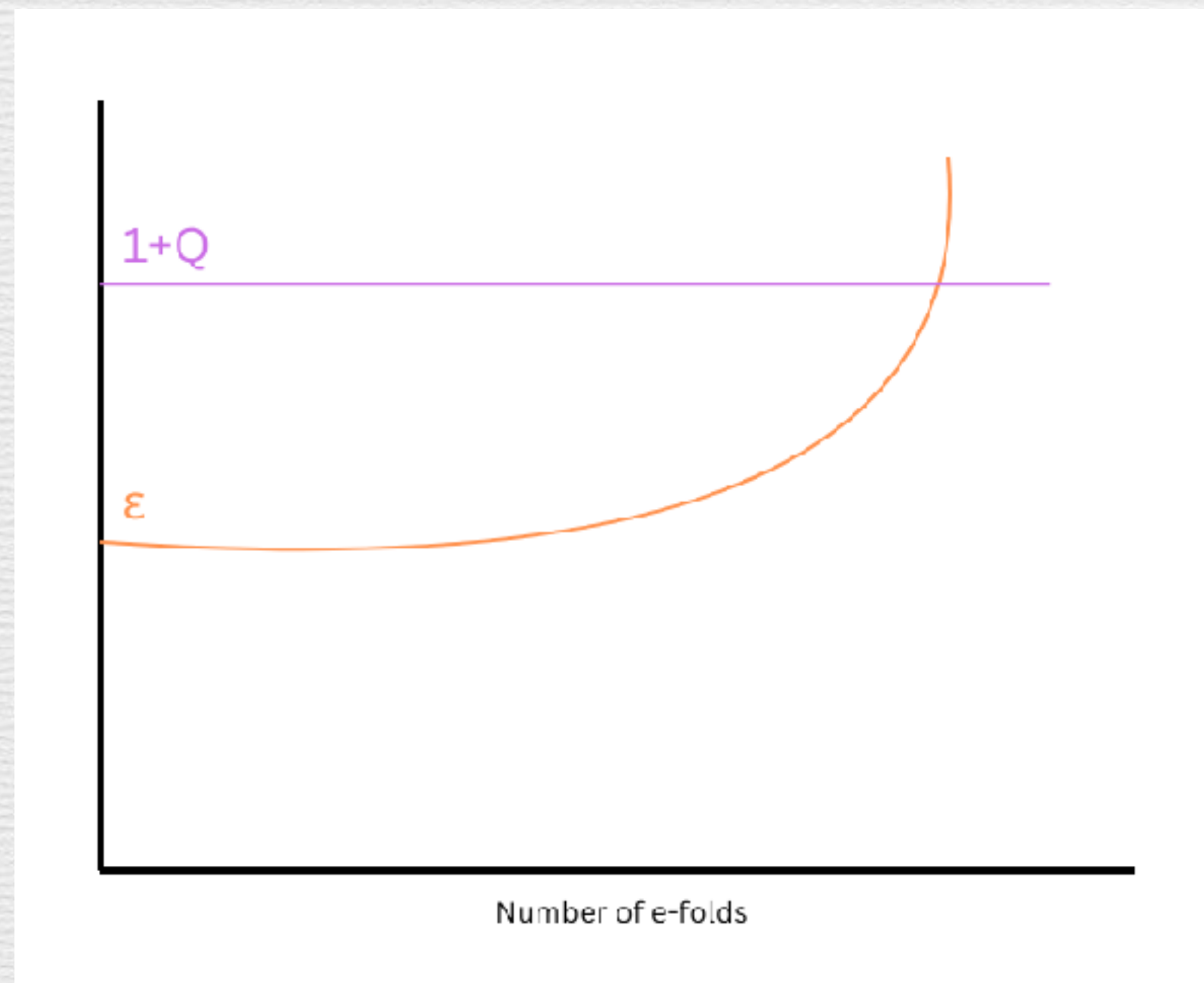
When ε grows



When ε grows



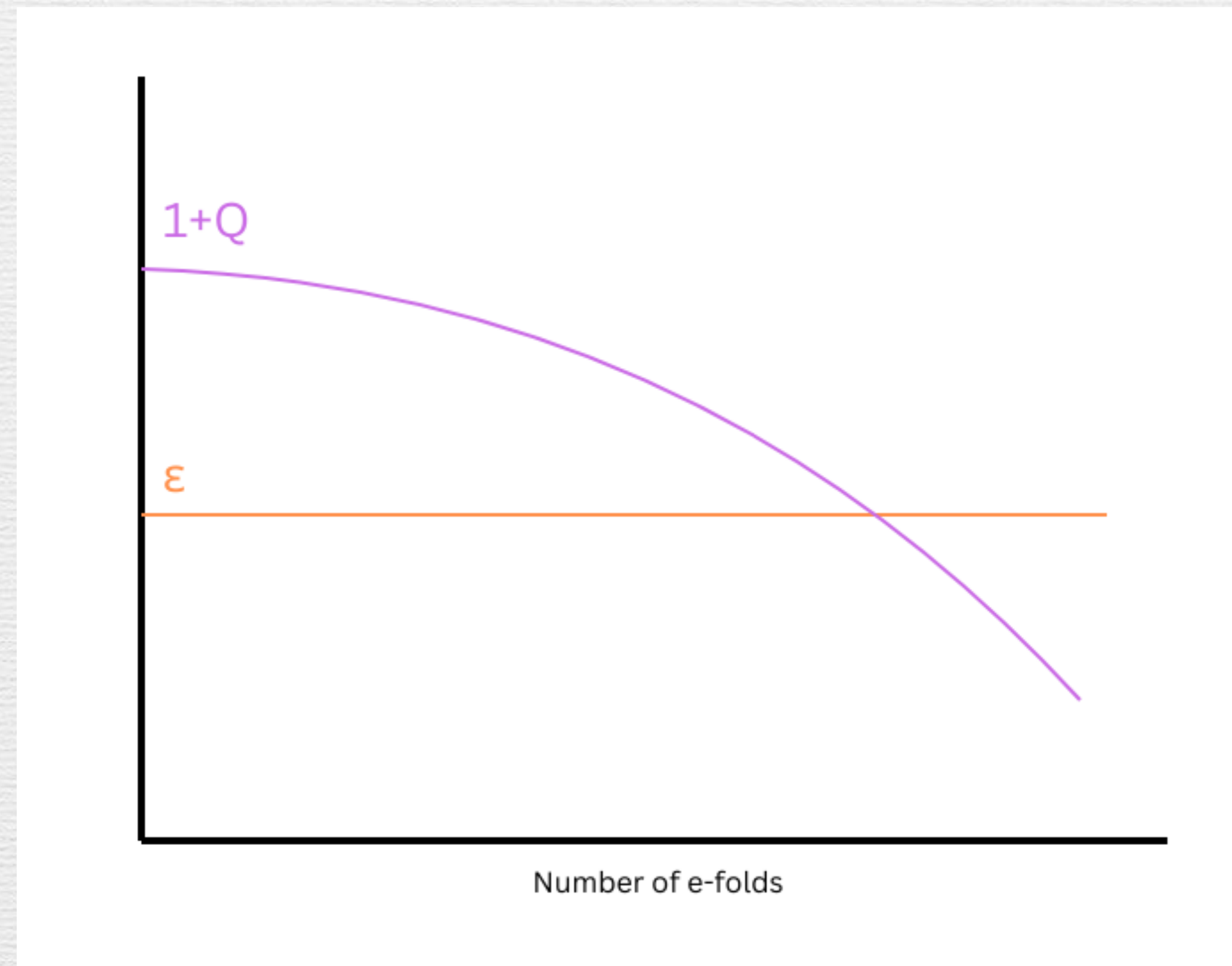
When ε grows



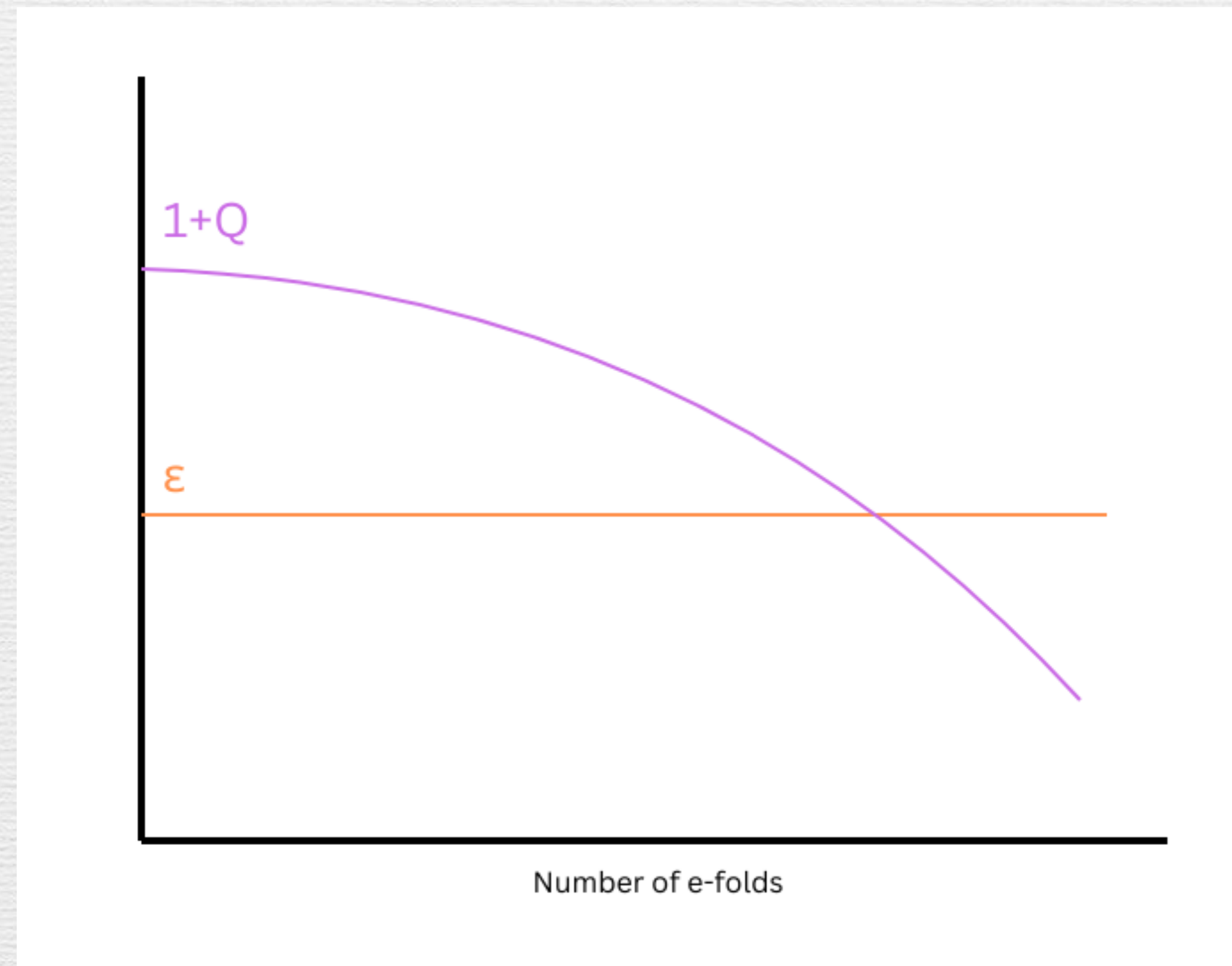
If Q increases faster than ε then there would be no graceful exit!!

When ε remains constant

When ε remains constant

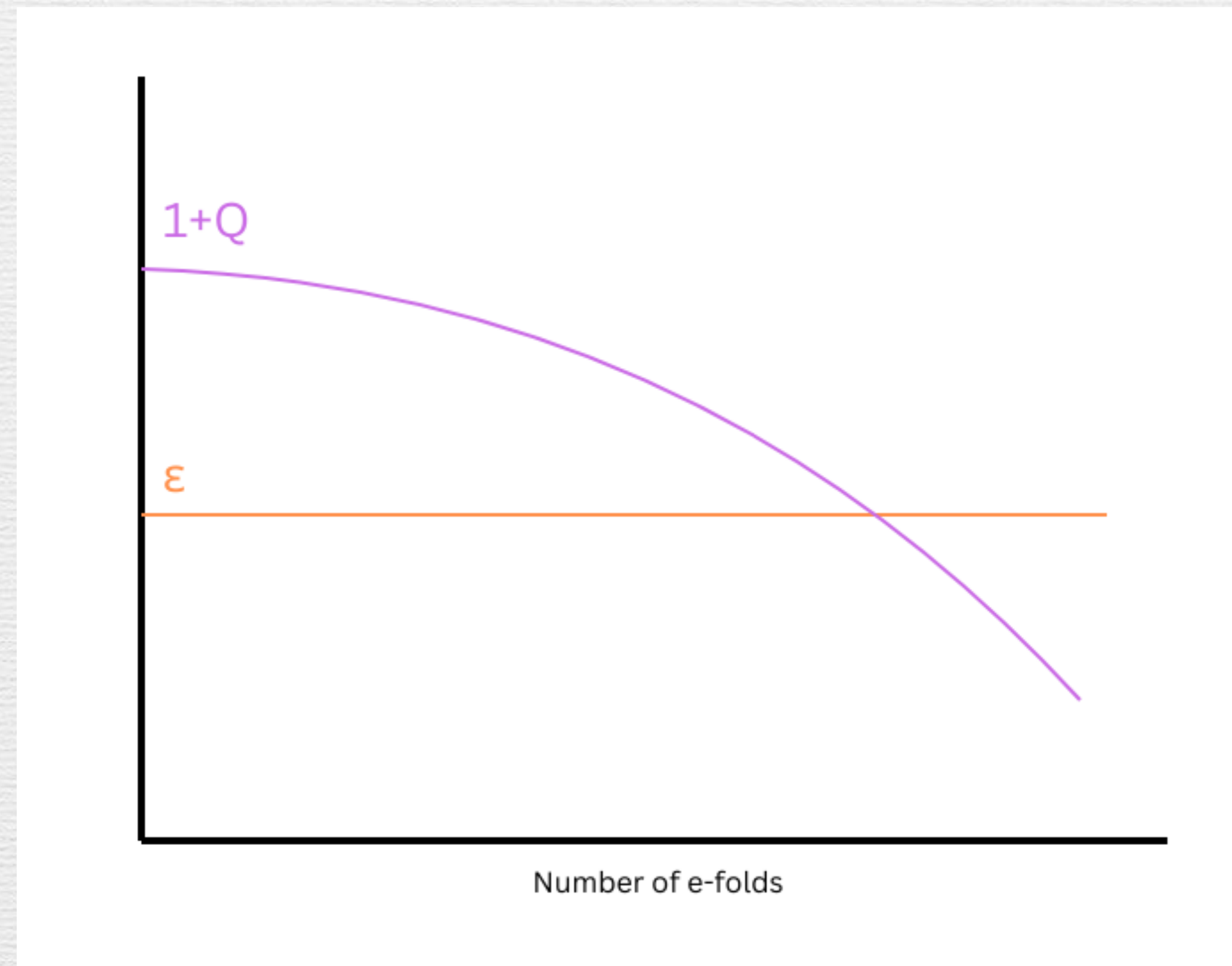


When ε remains constant



Graceful Exit in Power-law inflation possible in WI !!

When ε remains constant

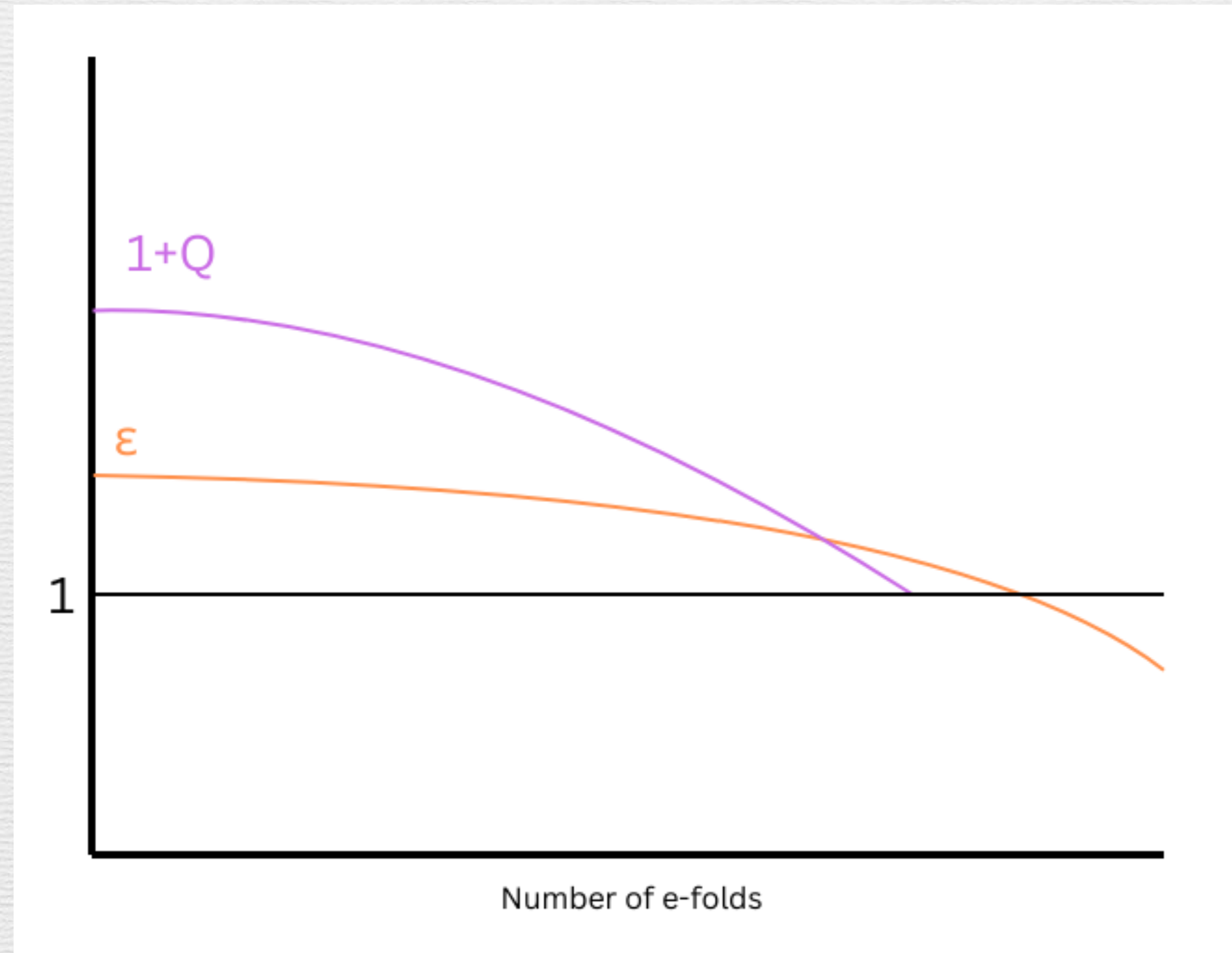


$$\Upsilon \propto T^3 \quad \text{or} \quad T^3 / \phi^2$$

Graceful Exit in Power-law inflation possible in WI !!

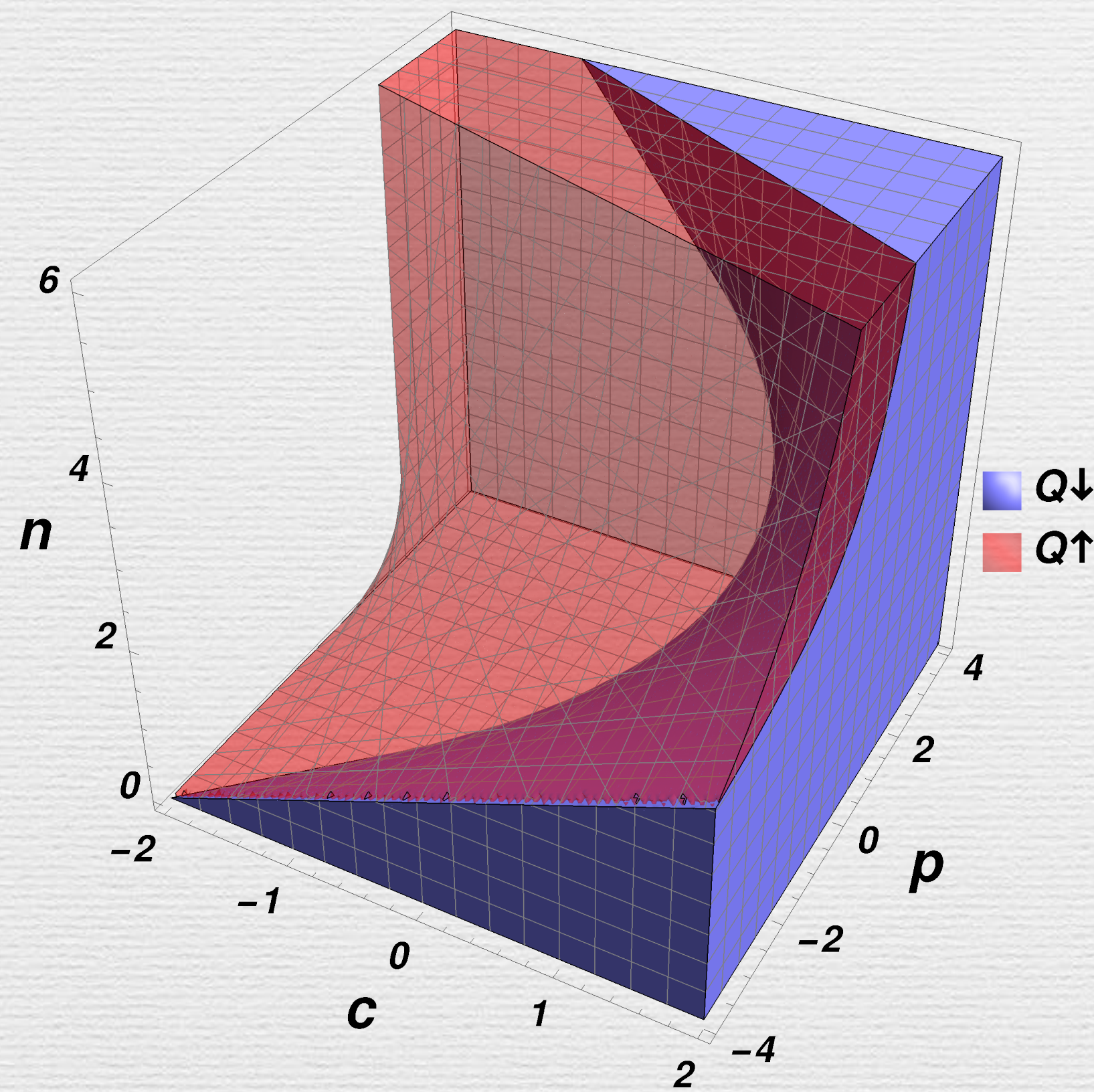
When ε decreases

When ε decreases

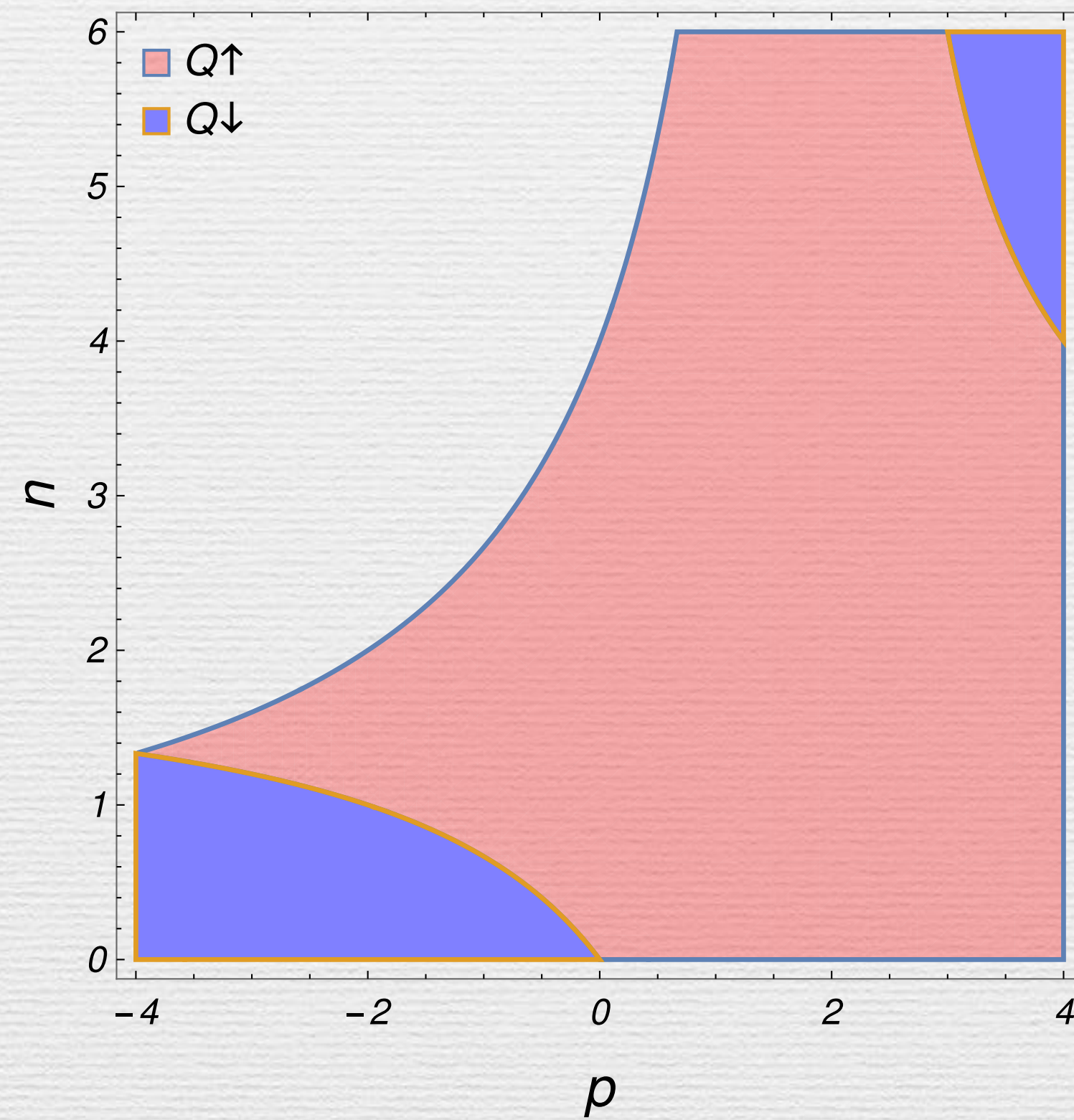


These Graceful Exit conditions put constraint
on the combination of the inflaton potential
and the form of the dissipative coefficient

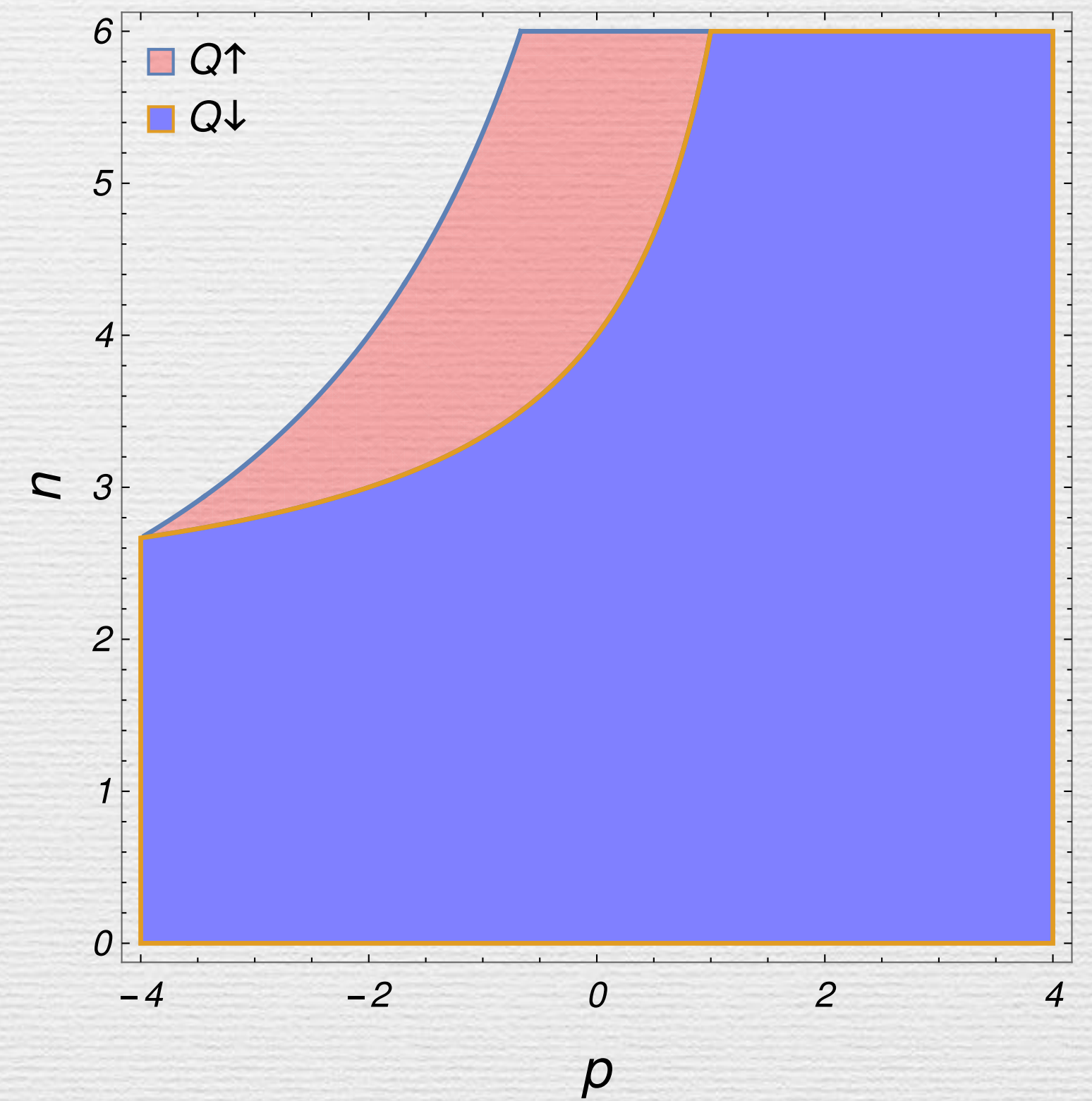
$$V(\phi) = \lambda \phi^n, \quad \Upsilon \propto T^p \phi^c$$



(a) 3d parameter space (c, p, n)



(b) Plane $c = 0$



(c) Plane $c = 2$

Does Warm Inflation always gracefully exit
to a radiation dominated Universe?

- The EoMs of Warm Inflation

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = -\Upsilon(T, \phi)\dot{\phi} \qquad \dot{\rho}_R + 4H\rho_R = \Upsilon(T, \phi)\dot{\phi}^2$$

- When a constant radiation bath is maintained

$$\rho_R \approx \frac{3}{2}Q\rho_{\text{K.E.}}$$

- In models where Q decreases, radiation energy density falls below kinetic energy density
- Warm Inflation can gracefully exit in a kination period in those models!
- Such kination period can have potential Gravitational Wave signatures!

- The EoMs of Warm Inflation

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = -\Upsilon(T, \phi)\dot{\phi} \quad \cancel{\dot{\rho}_R} + 4H\rho_R = \Upsilon(T, \phi)\dot{\phi}^2$$

- When a constant radiation bath is maintained

$$\rho_R \approx \frac{3}{2}Q\rho_{\text{K.E.}}$$

- In models where Q decreases, radiation energy density falls below kinetic energy density
- Warm Inflation can gracefully exit in a kination period in those models!
- Such kination period can have potential Gravitational Wave signatures!

Background Analysis of Warm inflation is not a
mundane task!

It reveals the nature of the Warm Inflation model

Thank you