Gravitational Wave Duet by Resonating Binary Black Holes with Axion-Like Particles

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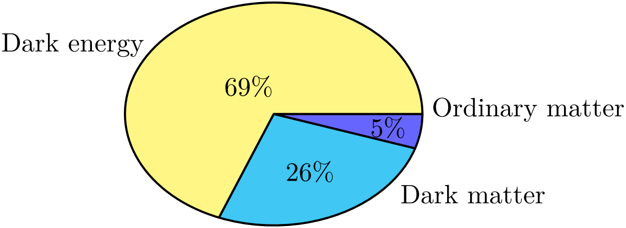
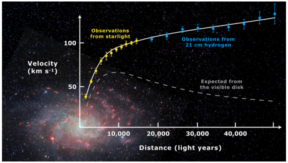
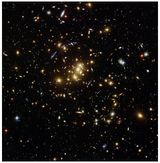
Dark Matter

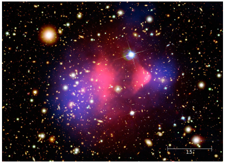




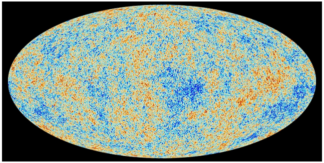
Galaxy Rotation Curves



Gravitational Lensing



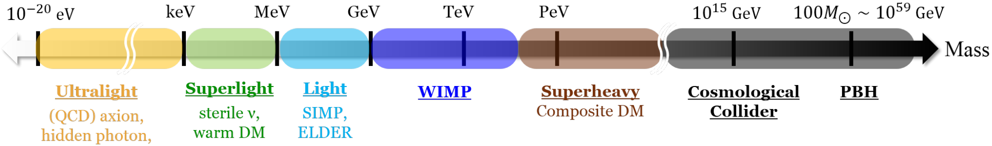
Bullet Cluster

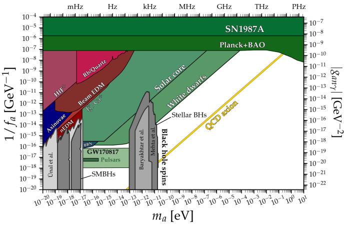


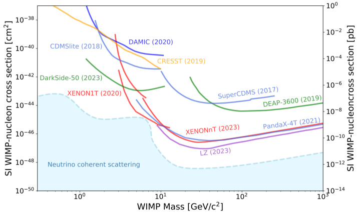
Cosmic Microwave Background

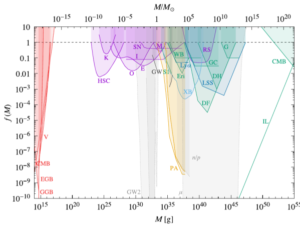
What is dark matter ?

Dark Matter Candidates









- Tight bounds are imposed on WIMP
- Next decade: A paradigm shift ?
  - Ultralight Dark Matter ?
  - Primordial Black Holes ?

Axion-Like Particles

Real scalar field ϕ

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{M_{\rm pl}^2}{2}R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right]$$

Potential of ALPs

$$V(\phi) = m^2 f_a^2\left(1 - \cos\frac{\phi}{f_a}\right) \approx \frac{1}{2}m^2\phi^2 - \frac{1}{4!}\frac{m^2}{f_a^2}\phi^4$$

- QCD axion $\rightarrow$ Strong CP problem.
- ALPs exhibit a broader spectrum of masses and coupling constants.
- Attractive self-interaction: $\lambda = -m^2/f_a^2 < 0$

In the non-relativistic limit

$$\phi(t,\boldsymbol{x}) = \frac{1}{\sqrt{2m}}\left(e^{-imt}\psi(t,\boldsymbol{x}) + e^{imt}\psi^*(t,\boldsymbol{x})\right)$$

Fast-varying phase

Slowly varying complex scalar (capturing the dynamics of ϕ)

ALP Soliton

Metric

$$ds^2 = -[1 + 2\Phi(t, \boldsymbol{x})]dt^2 + [1 - 2\Psi(t, \boldsymbol{x})]\delta_{ij}dx^i dx^j$$

Schrödinger-Poisson equations

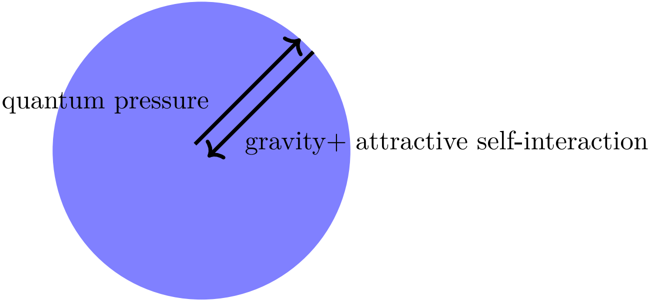
$$i\dot{\psi} = -\frac{\nabla^2\psi}{2m} + m\left(\Phi + \Phi_{\rm self}\right)\psi$$

$$\nabla^2\Phi = 4\pi G\rho$$

$$\Phi_{\rm self} \equiv \frac{\lambda|\psi|^2}{8m^3}$$

$$\rho = m|\psi|^2$$

Soliton: equilibrium solution



Metric Oscillation

General form of ALPs

$$\phi(t,\boldsymbol{x}) = \phi_0(\boldsymbol{x})\cos(\omega_a t + \Upsilon(\boldsymbol{x}))$$

- $\phi_0(\boldsymbol{x})$ and $\Upsilon(\boldsymbol{x})$ are functions that exhibit slow changes in positions
- Oscillation frequency of ALPs

$$\omega_a = m(1 + \frac{\lambda}{16m^2}\phi_0^2)$$

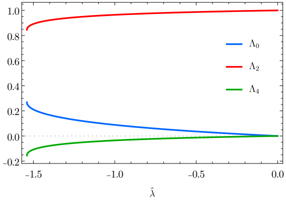
Metric oscillation

$$\phi \leftrightarrow T_{ab} \leftrightarrow G_{ab} \leftrightarrow \Psi$$

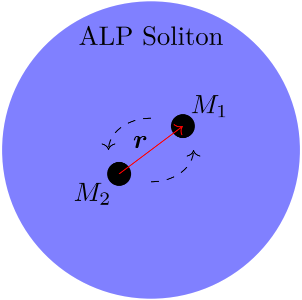
$$\ddot{\Psi} = -4\pi G\bar{\rho}_{\rm DM}\Big[\Lambda_2\cos(2\omega_a t + 2\Upsilon) + \Lambda_4\cos(4\omega_a t + 4\Upsilon)\Big]$$

Dimensionless parameters: $\Lambda_{2,4}$ and $\hat{\lambda}$

$\Lambda_{2,4}$: functions of $\hat{\lambda} \equiv \lambda\bar{\rho}_{\rm DM}/m^4$



Oscillating Force on Binary Black Holes



Geodesic deviation equation for the binary black holes

$$\ddot{r}^i = -R^i{}_{0j0} r^j = -\ddot{\Psi} r^i$$

Force on binary black holes

$$\ddot{\boldsymbol{r}} = -F_{\rm DM} \hat{\boldsymbol{r}}$$

- Oscillating force on binary black holes induced by ALPs

$$F_{\rm DM} = \ddot{\Psi} r = -4\pi G \bar{\rho}_{\rm DM}\, r \Big[\Lambda_2 \cos(2\omega_a t + 2\Upsilon) + \Lambda_4 \cos(4\omega_a t + 4\Upsilon) \Big]$$

Orbital Evolution of Binary Black Holes

Orbital evolution due to oscillating force induced by ALPs

Orbital evolution due to emission of GWs

$$\frac{\mathrm{d}a}{\mathrm{d}t} = -2\sqrt{\frac{a^3}{GM}}\frac{e}{\sqrt{1-e^2}}\sin(\varphi-\varphi_{\mathrm{p}})F_{\mathrm{DM}} \quad + \quad \left\langle\frac{\mathrm{d}a}{\mathrm{d}t}\right\rangle = -\frac{64G^3\mu M^2}{5c^5a^3(1-e^2)^{7/2}}\left(1+\frac{73}{24}e^2+\frac{37}{96}e^4\right)$$

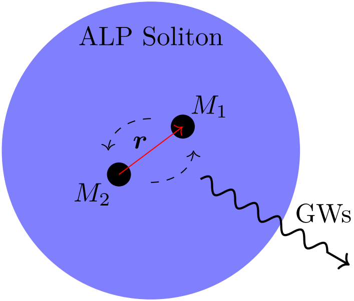
$$\frac{\mathrm{d}e}{\mathrm{d}t} = -\sqrt{\frac{a}{GM}}\sqrt{1-e^2}\sin(\varphi-\varphi_{\mathrm{p}})F_{\mathrm{DM}} \quad + \quad \left\langle\frac{\mathrm{d}e}{\mathrm{d}t}\right\rangle = -\frac{304G^3\mu M^2 e}{15c^5a^4(1-e^2)^{5/2}}\left(1+\frac{121}{304}e^2\right)$$

$$\frac{\mathrm{d}\varphi_{\mathrm{p}}}{\mathrm{d}t} = \sqrt{\frac{a}{GM}}\frac{\sqrt{1-e^2}}{e}\cos(\varphi-\varphi_{\mathrm{p}})F_{\mathrm{DM}}$$

$$\frac{\mathrm{d}\varphi}{\mathrm{d}t} = \sqrt{\frac{GM}{a^3}}\frac{[1+e\cos(\varphi-\varphi_{\mathrm{p}})]^2}{(1-e^2)^{3/2}}$$

$$M = M_1 + M_2$$
$$\mu = M_1 M_2/M$$
$$T = 2\pi/\omega, \quad \langle\cdots\rangle \equiv \int_0^T \frac{dt}{T}(\cdots)$$

Non-zero eccentricity



Orbital Evolution Equations

$$\left\langle \frac{\mathrm{d}\alpha}{\mathrm{d}\tau} \right\rangle = \zeta \alpha^{5/2} \frac{2e}{\sqrt{1-e^2}} \Big[\Lambda_2 \sin(\pi\nu + \gamma) \mathscr{S}(\nu, e) + \Lambda_4 \sin(2\pi\nu + 2\gamma) \mathscr{S}(2\nu, e) \Big] - \frac{64\eta}{5\alpha^3 (1-e^2)^{7/2}} \left(1 + \frac{73}{24} e^2 + \frac{37}{96} e^4 \right),$$

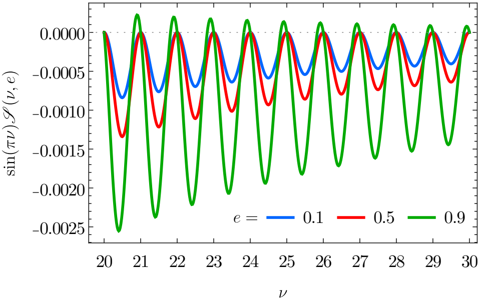
$$\left\langle \frac{\mathrm{d}e}{\mathrm{d}\tau} \right\rangle = \zeta \alpha^{3/2} \sqrt{1-e^2} \Big[\Lambda_2 \sin(\pi\nu + \gamma) \mathscr{S}(\nu, e) + \Lambda_4 \sin(2\pi\nu + 2\gamma) \mathscr{S}(2\nu, e) \Big] - \frac{304\eta e}{15\alpha^4 (1-e^2)^{5/2}} \left(1 + \frac{121}{304} e^2 \right),$$

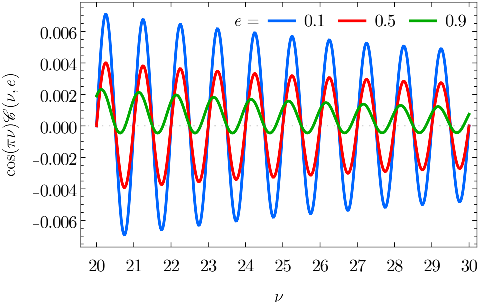
$$\left\langle \frac{\mathrm{d}\varphi_{\mathrm{p}}}{\mathrm{d}\tau} \right\rangle = -\zeta \alpha^{3/2} \frac{\sqrt{1-e^2}}{e} \Big[\Lambda_2 \cos(\pi\nu + \gamma) \mathscr{C}(\nu, e) + \Lambda_4 \cos(2\pi\nu + 2\gamma) \mathscr{C}(2\nu, e) \Big],$$

$$\left\langle \frac{\mathrm{d}\varphi}{\mathrm{d}\tau} \right\rangle = \alpha^{-3/2}.$$

$$\alpha \equiv a/R_*, \ R_* \equiv GM/c^2, \ \nu \equiv 2\omega_a/\omega$$

$$\tau \equiv \frac{tc}{R_*}, \quad \eta \equiv \frac{\mu}{M}, \quad \zeta \equiv \frac{4\pi G \bar{\rho}_{\mathrm{DM}} R_*^2}{c^2},$$

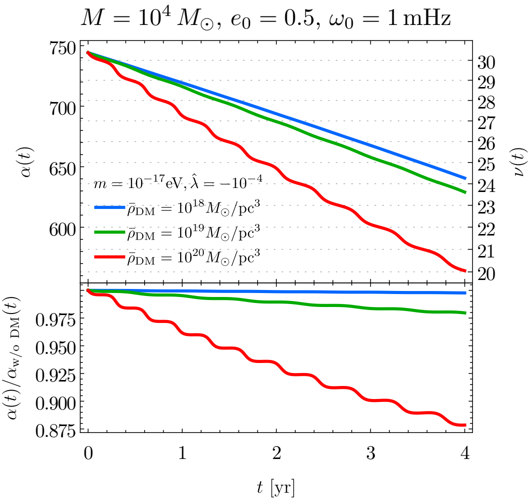




Evolution of Semi-major Axis

Dimensionless semi-major axis
$\alpha \equiv a/R_*$
$R_* \equiv GM/c^2$

Raito with respect to vacuum case



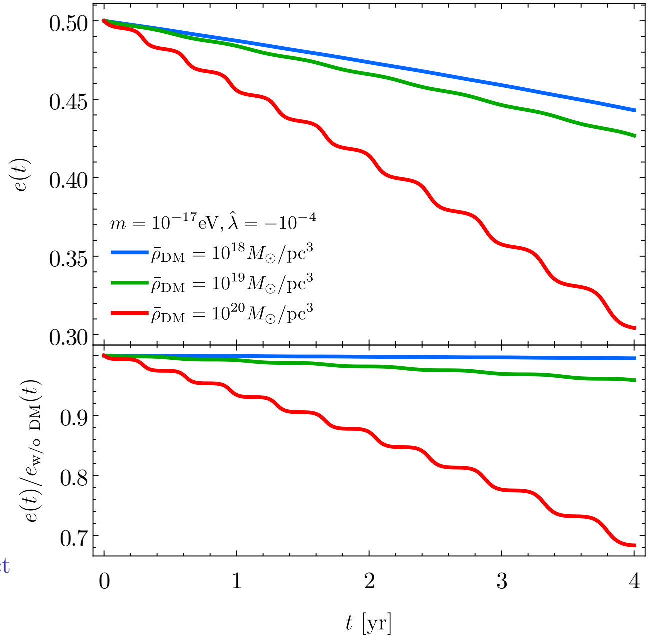
Frequency ratio
$\nu \equiv 2\omega_a/\omega$

ALPs frequency
$\omega_a = m(1 + \frac{\lambda}{16m^2}\phi_0^2)$

Binary frequency
$\omega = \sqrt{GM/a^3}$

- Distinctive oscillatory features in α, characterized by periodic dips occurring at specific intervals of ν.
- Highlighting the resonant interaction between the binary system and the surrounding ALP environment.

Evolution of Eccentricity

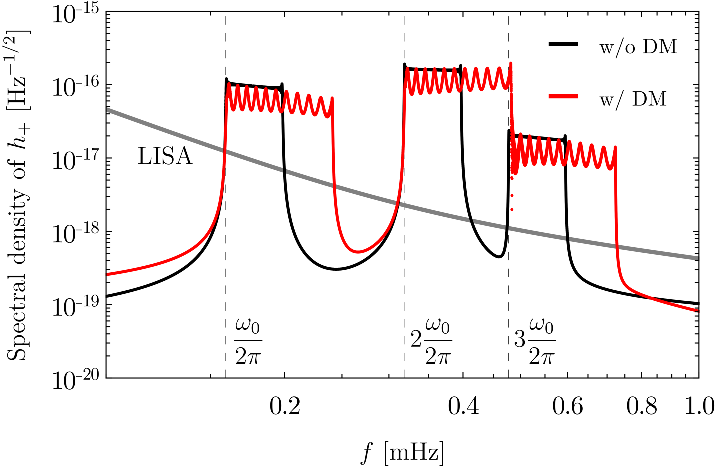


Eccentricity decreases in time

Raito with respect to vacuum case

Gravitational Waves

$$h_+(t) =_{\rm ret} \frac{1}{1-e^2}\frac{4(GM_c)^{5/3}\omega^{2/3}}{d_L c^4}\left\{\frac{1+\cos^2\iota}{2}\cos(2\varphi-2\beta)+\frac{e}{4}\sin^2\iota\left[\cos(\varphi-\varphi_{\rm p})+e\right]\right.$$
$$\left.+\frac{e}{8}(1+\cos^2\iota)\left[5\cos(\varphi-2\beta+\varphi_{\rm p})+\cos(3\varphi-2\beta-\varphi_{\rm p})+2e\cos(2\beta-2\varphi_{\rm p})\right]\right\}$$



Each broad peak corresponds to the n-th harmonic in Fourier decomposition of Keplerian motion.

$m = 10^{-17}\,\mathrm{eV}$
$\hat{\lambda} = -10^{-4}$
$\bar{\rho}_{\rm DM} = 10^{20}\,M_\odot/\,\mathrm{pc}^3$

$d_L = 0.1\,\mathrm{Gpc}$
$\{\iota, \beta\} = \pi/4$

- Identifying oscillatory patterns in GWs may indicate the existence of ALPs.

Fisher Matrix Analysis

Predict how well the experiment will be able to
constrain the model parameters before doing the experiment.

Fisher information matrix

$$\Gamma_{ij} = \left(\frac{\partial \boldsymbol{d}(f)}{\partial \theta_i}, \frac{\partial \boldsymbol{d}(f)}{\partial \theta_j} \right)_{\boldsymbol{\theta}=\hat{\boldsymbol{\theta}}}$$

$$\boldsymbol{d}(f) = \left[\frac{\tilde{h}_1(f)}{\sqrt{S_1(f)}}, \frac{\tilde{h}_2(f)}{\sqrt{S_2(f)}}, \ldots, \frac{\tilde{h}_N(f)}{\sqrt{S_N(f)}} \right]^{\mathrm{T}}$$

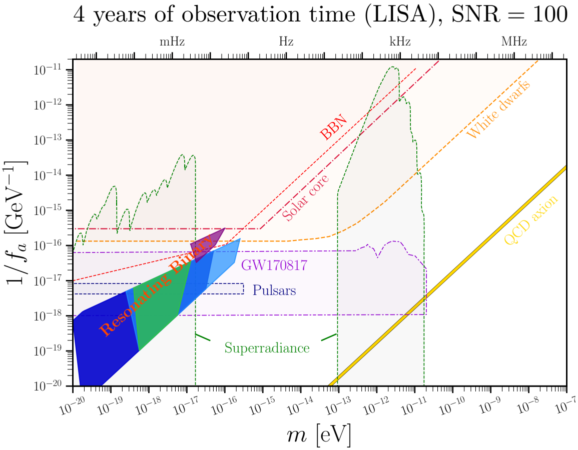
- $\boldsymbol{\theta}$ represents the vector of parameters with its true value denoted by $\hat{\boldsymbol{\theta}}$.

$$\boldsymbol{\theta} = \{M, \eta, \omega_0, e_0, \varphi_0, d_L, \iota, \beta, \theta_{\mathrm{s}}, \phi_{\mathrm{s}}, \chi; \bar{\rho}_{\mathrm{DM}}, m, \hat{\lambda}\}$$

Root-mean-squared errors for the parameters

$$\sigma_{\theta_i} = \sqrt{(\Gamma^{-1})_{ii}}$$

Detectable Regions of ALPs Parameter Space



$M = 10^2\, M_\odot$
$e_0 = 0.5$
$\bar{\rho}_{\rm DM} = 10^{18}\, M_\odot/\,{\rm pc}^3$

$M = 10^4\, M_\odot$

$e_0 = 0.3$

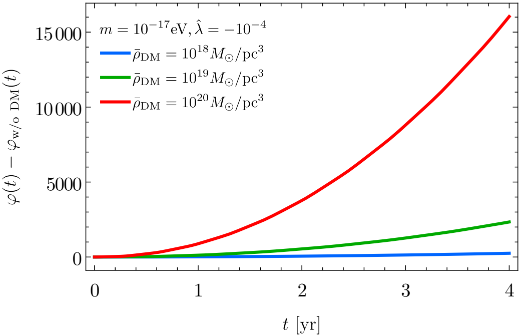
$\bar{\rho}_{\rm DM} = 10^{16}\, M_\odot/\,{\rm pc}^3$

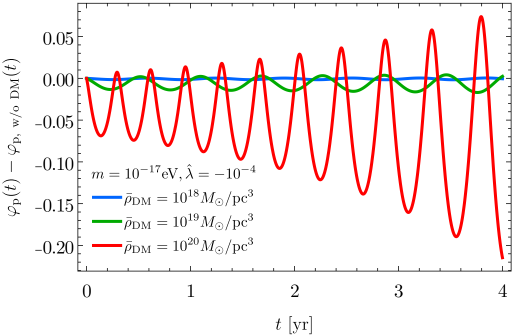
- This result does not rely on presupposed ALPs interaction with photons or nucleons, highlighting potential of GWs to detect ALPs solely through their gravitational effects.
- This method stands as one of solutions in the “nightmare scenario” for dark matter detection, when ALPs exhibit non-existent couplings to the Standard Model particles.

Summary

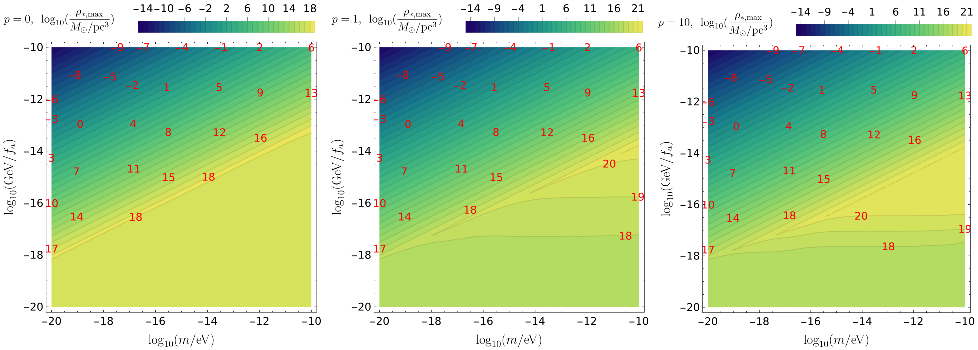
- Axion-like particle is one of the most attractive dark matter candidate.
- We explored the resonant interactions between axion-like particles and binary black hole mergers, mediated via spacetime metric perturbations from the oscillating wave-like nature of ALPs.
- This unique interaction leaves distinctive oscillatory features in gravitational waveforms, detectable by observatories like LISA.
- We performed a rigorous Fisher matrix analysis to delineate the detectable ALP parameter space using these features.

Backup


$m = 10^{-17}\mathrm{eV}, \hat{\lambda} = -10^{-4}$
$\bar{\rho}_{\mathrm{DM}} = 10^{18} M_\odot/\mathrm{pc}^3$
$\bar{\rho}_{\mathrm{DM}} = 10^{19} M_\odot/\mathrm{pc}^3$
$\bar{\rho}_{\mathrm{DM}} = 10^{20} M_\odot/\mathrm{pc}^3$
$\varphi(t) - \varphi_{\mathrm{w/o\ DM}}(t)$
15 000
10 000
5000
0
1
2
3
4
t [yr]


$m = 10^{-17}\mathrm{eV}, \hat{\lambda} = -10^{-4}$
$\bar{\rho}_{\mathrm{DM}} = 10^{18} M_\odot/\mathrm{pc}^3$
$\bar{\rho}_{\mathrm{DM}} = 10^{19} M_\odot/\mathrm{pc}^3$
$\bar{\rho}_{\mathrm{DM}} = 10^{20} M_\odot/\mathrm{pc}^3$
$\varphi_{\mathrm{P}}(t) - \varphi_{\mathrm{P,\ w/o\ DM}}(t)$
0.05
0.00
-0.05
-0.10
-0.15
-0.20
0
1
2
3
4
t [yr]

Soliton in Axion Minicluster



Maximum achievable central densities of soliton cores as a function of m and f_a.