The Quasi-Palatini Formulation of Nonminimal Gravity

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Outline

- Metric vs Palatini
- Equivalence classes in scalar-tensor gravity
- Formulations vs actions vs theories vs models
- The formulation interpolation
- Quasi-Palatini inflation

Metric vs Palatini

- “Standard" GR encoded in Einstein–Hilbert action

$$S[g_{\mu\nu}] = \frac{1}{2}\int \mathrm{d}^4x\,\sqrt{-g}\,R$$

- Connection set by hand to be Levi–Civita, but why? Notions of distance and geodesics are *a priori* conceptually distinct
- Metric-affine approach: connection $\Gamma^{\rho}_{\mu\nu}$ is independent of metric $g_{\mu\nu}$

$$S[g_{\mu\nu},\Gamma^{\rho}_{\mu\nu}] = \frac{1}{2}\int \mathrm{d}^4x\,\sqrt{-g}\,\mathcal{R}$$

with

$$\mathcal{R}(g,\Gamma) \equiv g^{\mu\nu}\mathcal{R}_{\mu\nu}(\Gamma) \equiv g^{\mu\nu}\left(\Gamma^{\rho}_{\mu\nu,\rho} - \Gamma^{\rho}_{\nu\rho,\mu} + \Gamma^{\rho}_{\rho\lambda}\Gamma^{\lambda}_{\mu\nu} - \Gamma^{\rho}_{\mu\lambda}\Gamma^{\lambda}_{\nu\rho}\right)$$

- Turns out that equations of motions impose Levi–Civita form

Metric vs Palatini

- Einstein–Hilbert action amenable to extensions e.g. addition of scalar field:
$$S[g_{\mu\nu},\phi] = \frac{1}{2}\int \mathrm{d}^4x\,\sqrt{-g}\,f(\phi)R + S_m[g_{\mu\nu},\phi]$$

- Can use metric-affine approach: for $S_m = S_m[g_{\mu\nu},\phi]$, we are in the so-called Palatini formulation

$$S[g_{\mu\nu},\Gamma^{\rho}_{\mu\nu},\phi] = \frac{1}{2}\int \mathrm{d}^4x\,\sqrt{-g}\,f(\phi)\mathcal{R} + S_m[g_{\mu\nu},\phi]$$

- This time, equations of motion set ∇^{Γ} to be compatible with $f(\phi)g_{\mu\nu}$ instead, hence
$$\mathcal{R} = R - 6f^{-1/2}\nabla^2\sqrt{f}$$

- Can be used to recast Palatini action in metric form, but equations of motion will differ for $f'(\phi) \neq 0$ between metric and Palatini approach

Equivalence classes in scalar-tensor gravity

- Specialize to scalar-tensor theories; either metric or Palatini

$$S = \frac{1}{2}\int \mathrm{d}^4x\,\sqrt{-g}\,\left[f(\phi)\begin{Bmatrix} R \\ \mathcal{R} \end{Bmatrix} - k_{AB}(\phi)(\partial_\mu\phi^A)(\partial^\mu\phi^A) - 2V(\phi) + \mathcal{L}_m\right]$$

- Can eliminate nonminimal coupling by way of conformal transformation $g_{\mu\nu} \to f(\phi)g_{\mu\nu}$

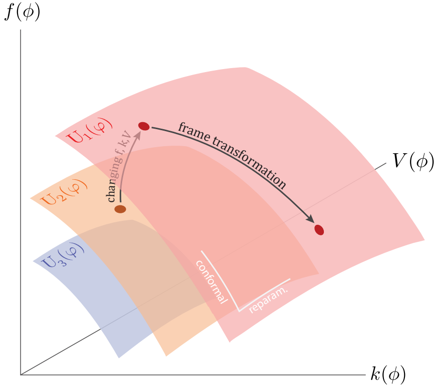
$$S = \frac{1}{2}\int \mathrm{d}^4x\,\sqrt{-g}\,\left[R - G_{AB}(\partial_\mu\phi^A)(\partial^\mu\phi^A) - 2U(\phi) + \widetilde{\mathcal{L}}_m\right]$$

- Field space metric:

$$G_{AB}(\phi) = \frac{k_{AB}}{f} + \frac{3\delta_m}{2}\frac{f_{,A}f_{,B}}{f^2}, \qquad \delta_m \equiv \begin{cases} 1 & (\text{metric}) \\ 0 & (\text{Palatini}) \end{cases}$$

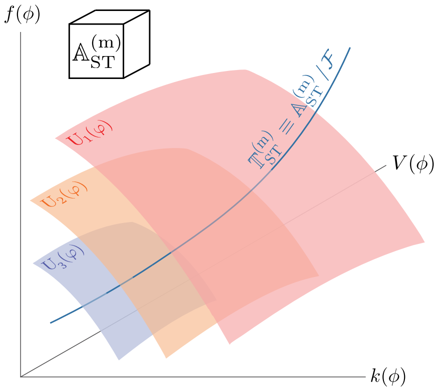
- Same conformally dimensionless (i.e. invariant) parameters $\to$ no difference in physics (Jarv et al. [1612.06863], Karamitsos et al. [1706.07011])

Equivalence classes in scalar-tensor gravity



theory = equivalence class of actions

Equivalence classes in scalar-tensor gravity



theory = equivalence class of actions

theory **space** = quotient space of action space
(up to frame transformations)

Formulations vs actions vs theories vs models

- "Actions to equations" is a many-to-one mapping: GR can be formulated in different ways as part of the geometric trinity of gravity

$$S_{\mathrm{GR}} = \frac{1}{2}\int \mathrm{d}^4x\sqrt{-g}\,\mathcal{R}, \quad S_{\mathrm{TEGR}} = -\frac{1}{2}\int \mathrm{d}^4x\sqrt{-g}\,\mathcal{T}, \quad S_{\mathrm{STEGR}} = -\frac{1}{2}\int \mathrm{d}^4x\sqrt{-g}\,\mathcal{Q}$$

- Switching formulations is therefore a formal procedure: given an action $S[R, g_{\mu\nu}, \phi]$:

$$R \to \begin{cases} \mathcal{R} & \text{to Palatini} \\ -\mathcal{T} & \text{to metric teleparallel} \\ -\mathcal{Q} & \text{to symmetric teleparallel} \end{cases}$$

- Contain same dynamics as standard metric Einstein–Hilbert action, therefore rightfully called "teleparallel equivalents" to GR
- Switching formulations in Einstein–Hilbert action does not change the physics
- Switching formulations in a nonminimal action does

Formulations vs actions vs theories vs models

- In practice, model is selected first, independent of formulation

Model	$f(\phi)$	$k(\phi)$	$V(\phi)$
Higgs inflation	$1+\xi\phi^2$	1	$\frac{\lambda}{4}(\phi^2-v^2)$
Induced gravity inflation	$\xi\phi^2$	1	$\frac{\lambda}{4}(\phi^2-v^2)$
nonminimal α-attractors	$1+\xi\phi^2$	$\frac{1}{(\phi^2-6\alpha)^2}$	$V(\phi)$
Brans–Dicke	ϕ	$\frac{\omega_{\mathrm{BD}}}{\phi}$	$V(\phi)$

- “Formulation" is a bit of a misnomer as it alters the physical content of a *model*; process of replacing $R \to \mathcal{R}$ has been called “naive Palatini" (Iglesias *et al.* [0708.1163]), but terminology persists

- Choice of formulation always applies at the level of the model (or equivalently the action)

Formulations vs actions vs theories vs models

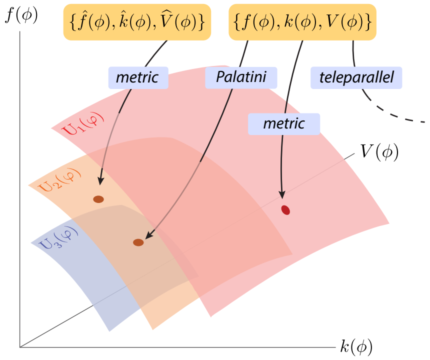
- Speaking of studying a theory in some formulation is a **category error**: we choose a model, express it in a formulation leading to an action which specifies a theory (but not uniquely

$$\text{Model} \xrightarrow{\text{formulation choice}} \text{Action} \xrightarrow{\text{invariant parameters}} \text{Theory}$$

- "Scalar-tensor gravity" formalized as a (concrete) category $\mathcal{C}^{\text{metric}}_{\text{ST}}$ in commutative diagram

$$\begin{tikzcd}
\mathbb{M}^{\text{metric}}_{\text{ST}} \arrow[loop left, "\mathrm{id}_{\mathbb{M}}^{\Omega, J^A{}_B}"] \arrow[r, "c_{\mathrm{m}}"] \arrow[dr, "b_{\mathrm{m}}"] & \mathbb{A}^{\text{metric}}_{\text{ST}} \arrow[loop right, "\mathrm{id}_{\mathbb{A}}^{\Omega, J^A{}_B}"] \arrow[d, "t_{\mathrm{m}}"] \\
 & \mathbb{T}^{\text{metric}}_{\text{ST}} \arrow[loop right, "\mathrm{id}_{\mathbb{T}}"]
\end{tikzcd}$$

Formulations vs actions vs theories vs models



- Can map Palatini action to metric action (not the case in teleparallel gravity due to boundary terms)
- Palatini scalar-tensor gravity not “richer" than metric scalar-tensor gravity; same number of functional degrees of freedom
- Palatini gravity $\cong$ metric gravity $\ncong$ teleparallel gravity

The formulation interpolation: hybrid metric-Palatini

- Choice of formulation is discrete: can we make it continuous?

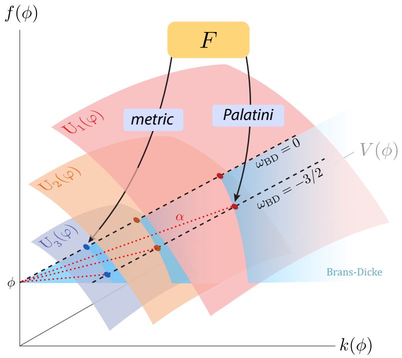
- Hybrid metric-Palatini models? (Capozziello et al. [1508.04641])

$$S_\alpha = \frac{1}{2}\int \mathrm{d}^4x\sqrt{-g}\,\left[\alpha R + F(\mathcal{R})\right] + S_m,$$

- Brans–Dicke parameter $\omega_{\mathrm{BD}}(\phi) = -\frac{3\phi}{2(\alpha+\phi)}$ interpolates between the classes of metric $F(R)$ ($\omega_{\mathrm{BD}} = 0$) and Palatini $F(R)$ ($\omega_{\mathrm{BD}} = -3/2$)

- Effectively an $F(X)$ deformation of GR for $X \equiv F'(\mathcal{R})\mathcal{R} - 2F(\mathcal{R})$

The formulation interpolation: hybrid metric-Palatini



- Hybrid metric-Palatini inflation absolutely useful: just not an *interpolation* between different formulations of the same model (parameter α interpolates between *classes* instead)
- Given a fixed model, we instead wish for a way to interpolate between the metric and Palatini interpretations of it instead, giving rise to a continuous family of actions

The formulation interpolation: quasi-Palatini

- Since “metric to Palatini" is achieved by $R \to \mathcal{R}$, we propose the quasi-Palatini formulation by weighting R and $\mathcal{R}$

$$R \to R_{\delta_P} \equiv (1-\delta_P)R + \delta_P \mathcal{R}$$

- $0 \leq \delta_P \leq 1$ encodes “Palatininess" of resulting action: does not need to depend on ϕ (that would correspond to a more complex trajectory between metric and Palatini actions in model space)

- Can even interpolate between any two actions by weighting them, regardless of whether they correspond to formulations (“external" interpolation as opposed to the previous “internal" interpolation)

$$S_{\delta_S} = (1-\delta_S)S_1 + \delta_S S_2.$$

The formulation interpolation: quasi-Palatini

- For scalar-tensor $f(\phi)R$ models, internal and external interpolations match; now $\delta_m = 1 - \delta_P$ is continuous

$$S_{\delta_P} = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{R}{2} - \left(\frac{k(\phi)}{f(\phi)} + \frac{3\delta_m}{2}\frac{f'(\phi)^2}{f(\phi)^2}\right)(\partial\phi)^2 - U(\phi)\right]$$

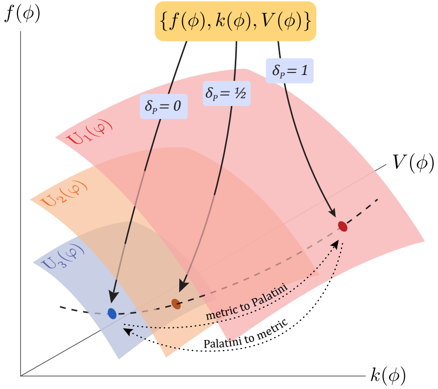
- For $F(R)$ models, $F(\delta_m R + \delta_P \mathcal{R})$ belongs to generalized hybrid metric-Palatini gravity, but internal interpolation (weighting R and $\mathcal{R}$) returns a single-field scalar-tensor theory, since Hessian of F vanishes

$$S^{\mathrm{int}}_{\delta_P} = \frac{1}{2}\int \mathrm{d}^4x\sqrt{-g}\left[\phi R + \frac{3\delta_P}{2\phi}(\partial\phi)^2 - 2V(\phi)\right]$$

- External interpolation (weighting the actions) does return the more general biscalar scalar-tensor theory

$$S^{\mathrm{ext}}_{\delta_P} = \frac{1}{2}\int \mathrm{d}^4x\sqrt{-g}\left\{\Big[(1-\delta_P)\phi + \delta_P\psi\Big]R - \frac{3\delta_P}{2\psi}(\partial\psi)^2 - 2[(1-\delta_P)V(\phi) + \delta_P V(\psi)]\right\}$$

The formulation interpolation: quasi-Palatini



- Different values of δ_P give rise to physically different actions, i.e. actions corresponding to different theory equivalence classes

The formulation interpolation: Palatini discontinuity

- The parametrization $\delta_P \in [0,1]$ harbors a discontinuity: “jump” from no dynamical scalar degree of freedom at $\delta_P = 1$ (“pure Palatini”) to a dynamical field at $\delta_P < 1$

- Can switch parametrization, but then we lose the “x% Palatininess” description (anyway not a problem if we steer clear of pure Palatini)

- Discontinuity also apparent in conformal coupling

$$S = \frac{1}{2}\int \mathrm{d}^D x \sqrt{-g}\left\{\partial_\mu\phi\partial^\mu\phi - \xi\phi^2[(1-\delta_P)R + \delta_P\mathcal{R}]\right\}$$

- Conformal coupling is

$$\xi_{\delta_P} = \frac{D-2}{4(1-\delta_P)(D-1)}$$

Quasi-Palatini inflation

- Effects of quasi-Palatini can be all absorbed into the reparametrization equation for inflaton

$$\varphi(\phi) = \int d\phi \sqrt{\frac{k(\phi)}{f(\phi)} + \frac{3\delta_m}{2}\left(\frac{f'(\phi)}{f(\phi)}\right)^2}$$

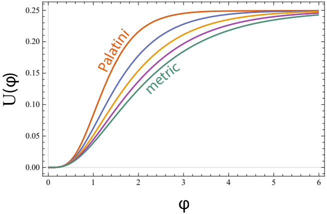
- This leads to a class of invariant potentials indexed by Palatininess δ_P

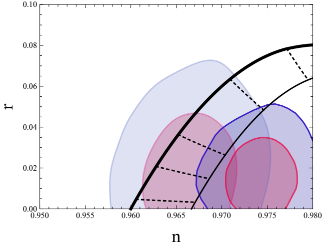
$$U_{\delta_P}(\varphi) = \frac{V(\phi_{\delta_P}(\varphi))}{f(\phi_{\delta_P}(\varphi))^2}$$

- The recipe: take our favorite scalar-tensor model, canonicalize the inflaton in quasi-Palatini, and calculate observables

$$\begin{aligned} n_s &= 1 - 2\epsilon + \eta, \\ r &= 16\epsilon, \\ A_s &= \frac{2}{3}\frac{U}{\pi^2\epsilon} \end{aligned}$$

Quasi-Palatini inflation: Higgs inflation





- Higgs inflation:

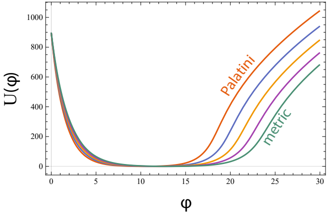
$$n_s = 1 - \frac{2}{N} + \frac{1 + 6\delta_m \xi}{4\xi N^2} + \mathcal{O}(N^{-3})$$

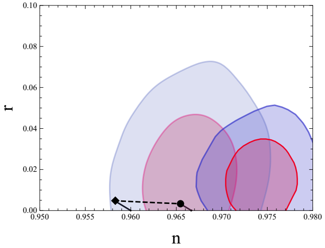
$$r = \frac{2(1 + 6\delta_m \xi)}{\xi N^2} + \mathcal{O}(N^{-3})$$

- Nonminimal coupliing ξ interpolated between $\mathcal{O}(10^4)$ for metric and $\mathcal{O}(10^9)$ for Palatini through normalization $A_s^* = (2.1 \pm 0.0589) \times 10^{-9}$

$$A_s = \frac{\lambda N^2}{72\pi^2 \delta_m \xi^2 + 12\pi^2 \xi}$$

Quasi-Palatini inflation: induced gravity inflation





- Induced gravity inflation:

$$n_s = 1 - \frac{2}{N} - \frac{3(1+6\delta_m\xi)}{4\xi N^2} + \mathcal{O}(N^{-3})$$

$$r = \frac{2(1+6\delta_m\xi)}{\xi N^2} + \mathcal{O}(N^{-3})$$

- Similar normalization equation

$$A_s = \frac{(1+6\delta_m\xi - 8\xi N)^4}{12288\pi^2\xi^5 N^2(1+6\delta_m\xi)}$$

Quasi-Palatini inflation: Starobinsky inflation

- Starobinsky inflation $F(R) = R + \beta R^2$: quasi-Palatini invariant potential

$$U(\varphi) = \frac{1}{2\beta}\left(1 - e^{-\sqrt{\frac{2}{3\delta_m}}\varphi}\right)^2$$

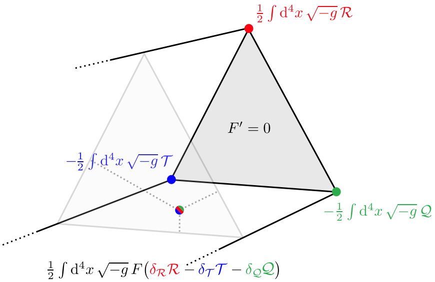
- Matches with α-attractor predictions (to be expected since δ_m appears as residue of pole)

$$n_s = 1 - \frac{2}{N} - \frac{9\delta_m}{2N^2}$$
$$r = \frac{12\delta_m}{N^2}.$$

- Tensor to scalar ratio can be driven down arbitrarily

- Discontinuity at $\delta_m = 0$ is artifact of interpolation parameter corresponding to vanishing of dynamics

BONUS: quasi-teleparallel prism of gravity



- Beyond scalar-tensor: $R \to \delta_{\mathcal{R}}\mathcal{R} - \delta_{\mathcal{T}}\mathcal{T} + \delta_{\mathcal{Q}}\mathcal{Q}$
- Base of prism is the trinity of gravity (Koivisto et al. [1903.06830]): different formulations (but same physics) of minimal models (GR + matter) at edgepoints; anywhere else, new physics
- But why stop there?

BONUS: extended formulations of gravity

- Extended trinity of gravity [Capozziello et al. (2503.08167)] includes boundary terms to ensure that formulations *do* return same physics

$$R \to \mathcal{R}$$
$$R \to \mathcal{T} - B_{\mathcal{T}}$$
$$R \to \mathcal{Q} - B_{\mathcal{Q}}$$

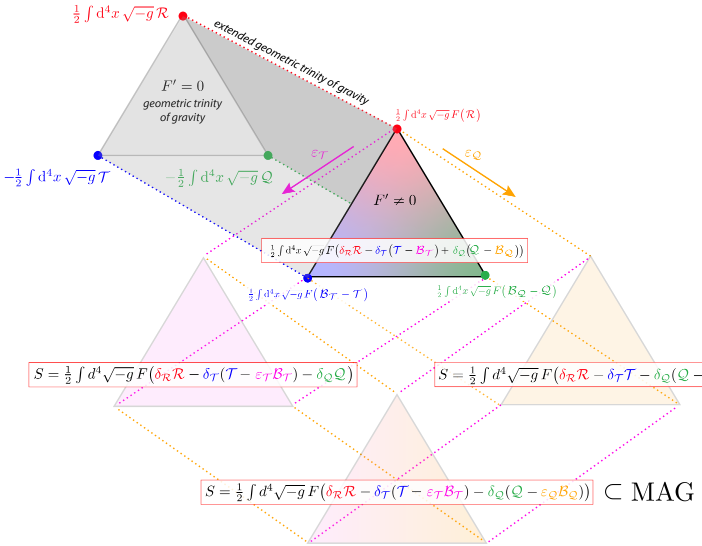
- Such replacement ensures $f(R)$ theory is formulated in a physically equivalent way

- Why not tune the boundary terms?

$$R \to \delta_{\mathcal{R}}\mathcal{R} - \delta_{\mathcal{T}}(\mathcal{T} - \epsilon_{\mathcal{T}}\mathcal{B}_{\mathcal{T}}) - \delta_{\mathcal{Q}}(\mathcal{Q} - \epsilon_{\mathcal{Q}}\mathcal{B}_{\mathcal{Q}})$$

- Scalar-tensor-torsion-nonmetricity theories???

BONUS: hyperprism of gravity



Conclusions

- A *new tool* for interpolating between metric and Palatini realizations of established models
- Motivates a continuous class of models that can be tested against observations
- Can rescue previously ruled out models minimally by adding “just enough" Palatini with no added model functions
- Can be used to tune the pole structure of resultant attractor-like theories
- Application to teleparallel formulations can motivate novel scalar-torsion/scalar-nonmetricity models