# Geometric Dark Matter, and Black Hole Hair


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# Recent works: *intersection of particle physics and gravitation*

- Verbin, Y. , **P. B.**, Övgün, A., Huang, H. (2025), "New black hole solutions of second and first order formulations of nonlinear electrodynamics", *Physical Review D,* 111 (2025) 8, 084061
- Ghorani, E.; Matri, S., Rayimbaev, J., **P. B.**, Atamurotov, F.; Abdujabbarov, A.; Demir, D. (2024), "Constraints on metric-Palatini gravity from QPO Data", *European Physical Journal C,* 84 (2024) 10, 1022
- **P. B.**; Pantig, R. C.; Övgün, A. and Demir, D., (2024), "Asymptotically-Flat Black Hole Solutions in Symmergent Gravity", *Fortschritte der Physik,* 72 (2024) 4, 2300138
- **P. B.**; Pantig, R. C.; Övgün, A. and Demir, D., (2023)," Constraints on charged Symmergent black hole from shadow and lensing ", *Classical and Quantum Gravity*, 40 195003
- Ghorani, E.; **P. B.**; Atamurotov, F.; Rayimbaev, J.; Abdujabbarov, A.; Demir, D., (2023), "Probing Geometric Proca with Black Hole Shadow and Photon Motion", *European Physical Journal C,* 83, 318 (2023)
- Demir, D. and **P. B.** (2022), "Geometric Proca with matter in metric-Palatini gravity", *European Physical Journal C*, 82 (2022) 11, 996
- **P. B.** (2021), "A Family-nonuniversal U (1)' Model for Excited Beryllium Decays", *Chinese Journal of Physics*, 71 (2021) 506-517
- Demir, D. and **P. B.** (2020), "Geometric Dark Matter", *Journal of Cosmology and Astroparticle Physics*, 04 (2020) 051

# Recent Research

- The first focus point is **the fifth force** mediated by a **geometric massive Z’ vector boson** mainly with dark matter and black hole applications

- The second focus point is **symmergent gravity** recently with its applications to black holes

**Experiment:**

- Heavy Z' bosons seem to weigh very heavy (CMS and ATLAS $M_{Z'} \geq 5$ TeV ).
- Light Z' bosons might exist with mass near the WISP domain (Atomki: $M_{Z'} \approx 17$ MeV) .

A. J. Krasznahorkay et al. Phys.Rev.Lett. 116 (2016) 4

**Theory:**

- Z' bosons are often associated with some U(1)' symmetry beyond the SM.

BP, Chin. J. Phys. 71 (2021) 506

➢ Z' bosons can arise also from **beyond-the-General relativity (GR) geometry** (non-metricity vector in Palatini gravity).

**Geometric Z' – Geometric fifth force**

# General Relativity

It is defined by the Einstein-Hilbert action

$$\boxed{S\,[g] = \int d^4x\,\sqrt{-g}\,\frac{M_{Pl}^2}{2}\,g^{\mu\nu}\mathbb{R}_{\mu\nu}({}^g\Gamma)}$$

- its geometry is based on the metric tensor

- the Levi-Civita connection

$${}^g\Gamma^\lambda_{\mu\nu} = \frac{1}{2}g^{\lambda\rho}\left(\partial_\mu g_{\nu\rho} + \partial_\nu g_{\rho\mu} - \partial_\rho g_{\mu\nu}\right)$$

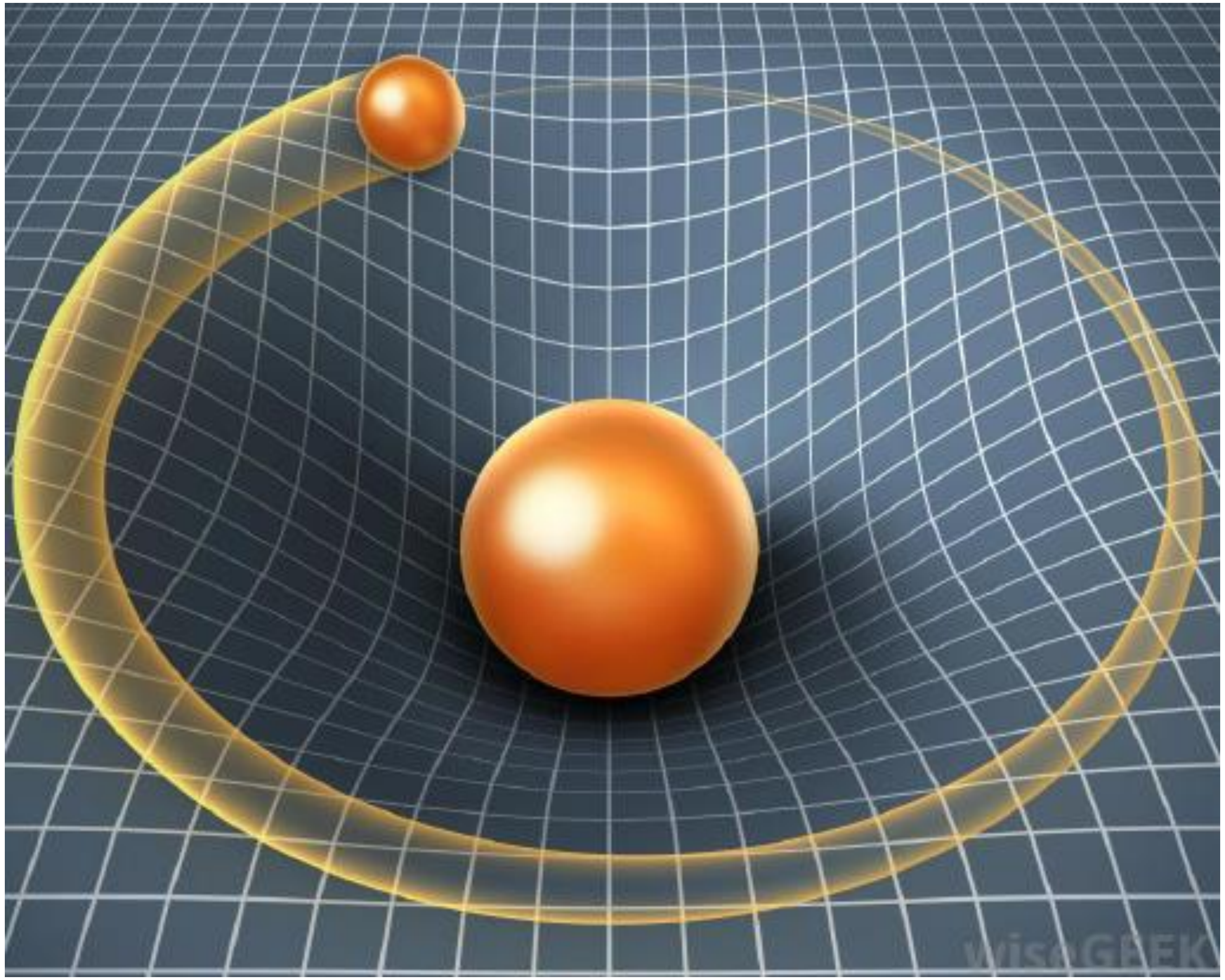
- the Ricci curvature tensor

$$\mathbb{R}_{\mu\nu}({}^g\Gamma) = \partial_\lambda{}^g\Gamma^\lambda_{\mu\nu} - \partial_\nu{}^g\Gamma^\lambda_{\lambda\mu} + {}^g\Gamma^\rho_{\rho\lambda}{}^g\Gamma^\lambda_{\mu\nu} - {}^g\Gamma^\rho_{\nu\lambda}{}^g\Gamma^\lambda_{\rho\mu}$$

# General Relativity

Einstein field equations:

$$\boxed{R_{\mu\nu}({}^{g}\Gamma) - \frac{1}{2} g^{\alpha\beta} R_{\alpha\beta}({}^{g}\Gamma) g_{\mu\nu} = T_{\mu\nu}}$$




- The Einstein-Hilbert action is known not to lead to the Einstein field equations.

# Palatini formalism is the remedy:

$$\underbrace{S\left[g,\Gamma\right]} = \int d^4x\,\sqrt{-g}\left\{\frac{M_{Pl}^2}{2} g^{\mu\nu}\mathbb{R}_{\mu\nu}(\Gamma) + \mathcal{L}\left({}^g\Gamma, \Psi_m\right)\right\}$$

affine Ricci curvature tensor

affine connection
(symmetric tensor)

$$\Gamma^\lambda_{\mu\nu} = \Gamma^\lambda_{\nu\mu}$$

two independent dynamical variables:

- metric tensor
- affine connection

M. Tsamparlis, M. Math. Phys. 19, 555 (1978)

# Palatini gravity

$$\frac{\partial S[g,\Gamma]}{\partial \Gamma^{\lambda}_{\mu\nu}} = 0 \longrightarrow {}^{\Gamma}\nabla_{\lambda} g_{\mu\nu} = 0 \longrightarrow \Gamma^{\lambda}_{\mu\nu} = {}^{g}\Gamma^{\lambda}_{\mu\nu}$$

$$\frac{\partial S[g,\Gamma]}{\partial g^{\mu\nu}} = 0 \longrightarrow \mathbb{R}_{\mu\nu}(\Gamma) - \frac{1}{2} g^{\alpha\beta}\mathbb{R}_{\alpha\beta}(\Gamma) g_{\mu\nu} = T_{\mu\nu}$$

- Einstein field equations in GR arise dynamically:

$$\boxed{R_{\mu\nu}({}^{g}\Gamma) - \frac{1}{2} g^{\alpha\beta} R_{\alpha\beta}({}^{g}\Gamma) g_{\mu\nu} = T_{\mu\nu}}$$

✓ The Palatini action is the right framework for getting the Einstein field equations.


J. York, Phys. Rev. Lett. 28 (1972) 1082 G.

G. Gibbons & S. Hawking, Phys. Rev. D15 (1977)

# Extended Palatini gravity

$$S_{EP}\left[g,\Gamma\right]=\int d^4x\,\sqrt{-g}\left\{\frac{M_{Pl}^2}{2}g^{\mu\nu}\mathbb{R}_{\mu\nu}(\Gamma)-\frac{\xi}{4}\overline{\mathbb{R}}_{\mu\nu}(\Gamma)\overline{\mathbb{R}}^{\mu\nu}(\Gamma)+\mathcal{L}\left({}^g\Gamma,\Psi_m\right)\right\}$$

- antisymmetric part of the affine Ricci curvature tensor → $\boxed{\overline{\mathbb{R}}_{\mu\nu}(\Gamma)=\partial_\mu\Gamma^\lambda_{\lambda\nu}-\partial_\nu\Gamma^\lambda_{\lambda\mu}}$

- antisymmetric affine Ricci acts like field strength tensor of an Abelian vector field → $\boxed{V_\mu=\Gamma^\lambda_{\lambda\mu}}$ and $\boxed{\overline{\mathbb{R}}_{\mu\nu}=\partial_\mu V_\nu-\partial_\nu V_\mu}$

➢ $V_\mu$ is the source of geometric Z' and can be related to non-metricity for a symmetric affine connection $\Gamma^\lambda_{\mu\nu}=\Gamma^\lambda_{\nu\mu}$


V. Vitagliano et al., Phys. Rev. D 82 (2010) 084007

D. Demir, BP, JCAP 04 (2020) 51

# Extended Palatini gravity with Matter

$$S_{EPm}\left[g,\Gamma\right] = \int d^4x\,\sqrt{-g}\left\{\frac{M_{Pl}^2}{2}g^{\mu\nu}\mathbb{R}_{\mu\nu}(\Gamma) - \frac{\xi}{4}\overline{\mathbb{R}}_{\mu\nu}(\Gamma)\overline{\mathbb{R}}^{\mu\nu}(\Gamma) + \mathcal{L}\left(\Gamma,\Psi_m\right)\right\}$$

matter sector involves affine connection

$\Gamma^{\lambda}_{\mu\nu}$ (not ${}^{g}\Gamma^{\lambda}_{\mu\nu}$ )

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D. Demir, BP, Eur. Phys. J. C 82 (2022) 996

- affine connection appears in the fermion kinetic term:

$${}^{\Gamma}\nabla_{\mu}\psi = (\nabla_{\mu} + \frac{1}{2}Q_{\mu})\psi$$

L. Fatibene et al., gr-qc/9608003 (1996)
M.Adak, T. Dereli, L. Ryder,
Int. J. Mod. Phys. D 12 (2003) 145

## Extended Palatini gravity with Matter

- In general affine connection is decomposed as

$$\Gamma^{\lambda}_{\mu\nu} = {}^{g}\Gamma^{\lambda}_{\mu\nu} + \Delta^{\lambda}_{\mu\nu}$$

symmetric tensor field

$$\Delta^{\lambda}_{\mu\nu} = \Delta^{\lambda}_{\nu\mu}$$

- which leads to affine Ricci curvatures:

$$\begin{aligned} \mathbb{R}_{\mu\nu}(\Gamma) &= \mathbb{R}_{\mu\nu}({}^{g}\Gamma) + \nabla_{\lambda}\Delta^{\lambda}_{\mu\nu} - \nabla_{\nu}\Delta^{\lambda}_{\lambda\mu} + \Delta^{\rho}_{\rho\lambda}\Delta^{\lambda}_{\mu\nu} - \Delta^{\rho}_{\nu\lambda}\Delta^{\lambda}_{\rho\mu} \\ \overline{\mathbb{R}}_{\mu\nu}(\Gamma) &= \partial_{\mu}\Delta^{\lambda}_{\lambda\nu} - \partial_{\nu}\Delta^{\lambda}_{\lambda\mu} \end{aligned}$$

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# Extended Palatini gravity with Matter

- Action after decomposition:

$$S\left[g,\Delta,\Psi_m\right] = \int d^4x\,\sqrt{-g}\Big\{\frac{M_{Pl}^2}{2}g^{\mu\nu}\mathbb{R}_{\mu\nu}({}^g\Gamma)\boxed{-\frac{1}{4}\xi g^{\mu\alpha}g^{\nu\beta}\left(\partial_\mu\Delta^\lambda_{\lambda\nu}-\partial_\nu\Delta^\lambda_{\lambda\mu}\right)\left(\partial_\alpha\Delta^\rho_{\rho\beta}-\partial_\beta\Delta^\rho_{\rho\alpha}\right)}$$
$$+\boxed{\frac{M_{Pl}^2}{2}g^{\mu\nu}\left(\Delta^\rho_{\rho\lambda}\Delta^\lambda_{\mu\nu}-\Delta^\rho_{\nu\lambda}\Delta^\lambda_{\rho\mu}\right)}+\mathcal{L}\left(g,{}^g\Gamma,\Delta,\Psi_m\right)\Big\}$$

Quadratic term involves all components of $\Delta^\lambda_{\mu\nu}$

Kinetic term involves only the vector field $\Delta^\lambda_{\lambda\nu}$


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[D. Demir, BP, Eur. Phys. J. C 82 (2022) 996]

# Extended Palatini gravity with Matter

- One can express $\Delta^{\lambda}_{\mu\nu}$ in terms of nonmetricity tensors

$$\Delta^{\lambda}_{\mu\nu} = \frac{1}{2} g^{\lambda\rho} (Q_{\mu\nu\rho} + Q_{\nu\mu\rho} - Q_{\rho\mu\nu})$$

nonmetricity tensor

$$Q_{\lambda\mu\nu} = -{}^{\Gamma}\nabla_{\lambda} g_{\mu\nu}$$

- For an action of a consistent vector field theory, tensor field enjoys the form

$$\Delta^{\lambda}_{\mu\nu} = -3Q^{\lambda} g_{\mu\nu} + Q_{\nu}\delta^{\lambda}_{\mu} + Q_{\mu}\delta^{\lambda}_{\nu}$$


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# Geometric-Z' with Matter:

$$S[g, Y, \Psi_m] = \int d^4x \sqrt{-g} \Big\{ \underbrace{\frac{M_{Pl}^2}{2} R({}^g\Gamma)}_{\text{general relativity}} \underbrace{- \frac{1}{4} Y_{\mu\nu} Y^{\mu\nu} - \frac{1}{2} M_Y^2 Y_\mu Y^\mu + g_Y \overline{f} \gamma^\mu f Y_\mu}_{\text{vector field theory}} + \overline{\mathcal{L}}(g, {}^g\Gamma, \Psi_m) \Big\}$$

matter sector without $Y_\mu$

canonical geometric Z' : $\boxed{Y_\mu \equiv 2\sqrt{\xi} Q_\mu}$

- Geometric Z' mass: $\boxed{M_Y^2 = \frac{3}{2\xi} M_{Pl}^2}$
- Geometric Z' couples to fermions universally: $\boxed{g_Y = \frac{1}{4\sqrt{\xi}}}$
- Quarks and leptons (not the Higgs and gauge bosons) couple to the geometrical Z' directly, universally and in an Abelian gauge field fashion.


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# Geometric-Z’ Field -> Geometric-Z’ Boson:

- In the flat metric limit $g_{\mu\nu} \rightarrow \eta_{\mu\nu}$, geometric-Z’ can be quantized:

$$\hat{Y}_\mu(x) = \sum_{\lambda=0}^{3} \int \frac{d^3\vec{p}}{(2\pi)^{3/2}} \frac{1}{\sqrt{2\omega(\vec{p})}} \left\{ \hat{a}(\vec{p},\lambda)\epsilon^\mu(\vec{p},\lambda)e^{-ip\cdot x} + \hat{a}^\dagger(\vec{p},\lambda)\epsilon^{\mu\star}(\vec{p},\lambda)e^{ip\cdot x} \right\}$$

- with the commutation relation: $\left[\hat{a}(\vec{p},\lambda), \hat{a}^\dagger(\vec{p}',\lambda')\right] = i\delta^3\left(\vec{p}-\vec{p}'\right)\delta_{\lambda\lambda'}$

- and the polarization sum: $\sum_{\lambda=1}^{3} \epsilon^\mu(\vec{p},\lambda)\epsilon^{\nu\star}(\vec{p},\lambda) = \eta^{\mu\nu} - \frac{p^\mu p^\nu}{M_Y^2}$

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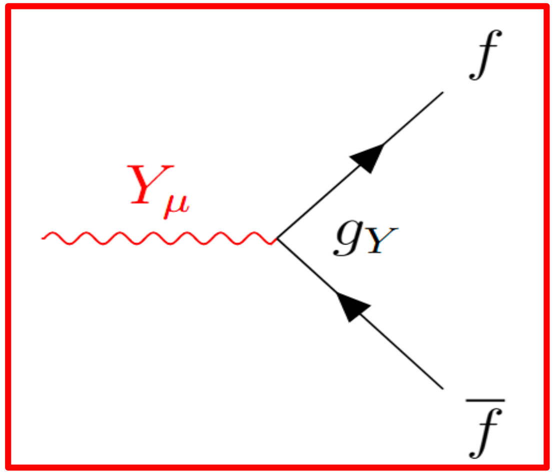
# Geometric Dark Matter

✓ First thing to check is the geometric Z' lifetime, hence we find its decay rate




$$\Gamma\left(Y \to f\overline{f}\right) = \frac{N_c^f}{8\pi}\left(\frac{3}{2\xi}\right)^{\frac{3}{2}}\left(1+\frac{4\xi m_f^2}{3M_{Pl}^2}\right)\left(1-\frac{8\xi m_f^2}{3M_{Pl}^2}\right)^{\frac{1}{2}} M_{Pl}$$

✓ For the summation over fermions, the lifetime of the $Y_\mu$ becomes

$$\tau_Y = \frac{1}{\Gamma_{\rm tot}} = \frac{4\pi}{5}\left(\frac{2}{3}\right)^{3/2}\frac{\xi^{3/2}}{M_{\rm Pl}}$$

➢ $\tau_Y > t_U = 13.8 \times 10^9$ years if $\xi > 1.1 \times 10^{40}$

➢ Decay rate stays physical if $\xi < 1 \times 10^{41}$

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# Geometric Dark Matter

- Bounds on the quadratic term parameter lead to the allowed mass range of $Y_\mu$ :

$$\boxed{9.4 \text{ MeV} < M_Y < 28.4 \text{ MeV}}$$

- and its lifetime ranges from $\boxed{4.4 \times 10^{17} \; s \text{ to } 1.2 \times 10^{19} \; s}$

⇒ $Y_\mu$ exists today to contribute to galactic dynamics

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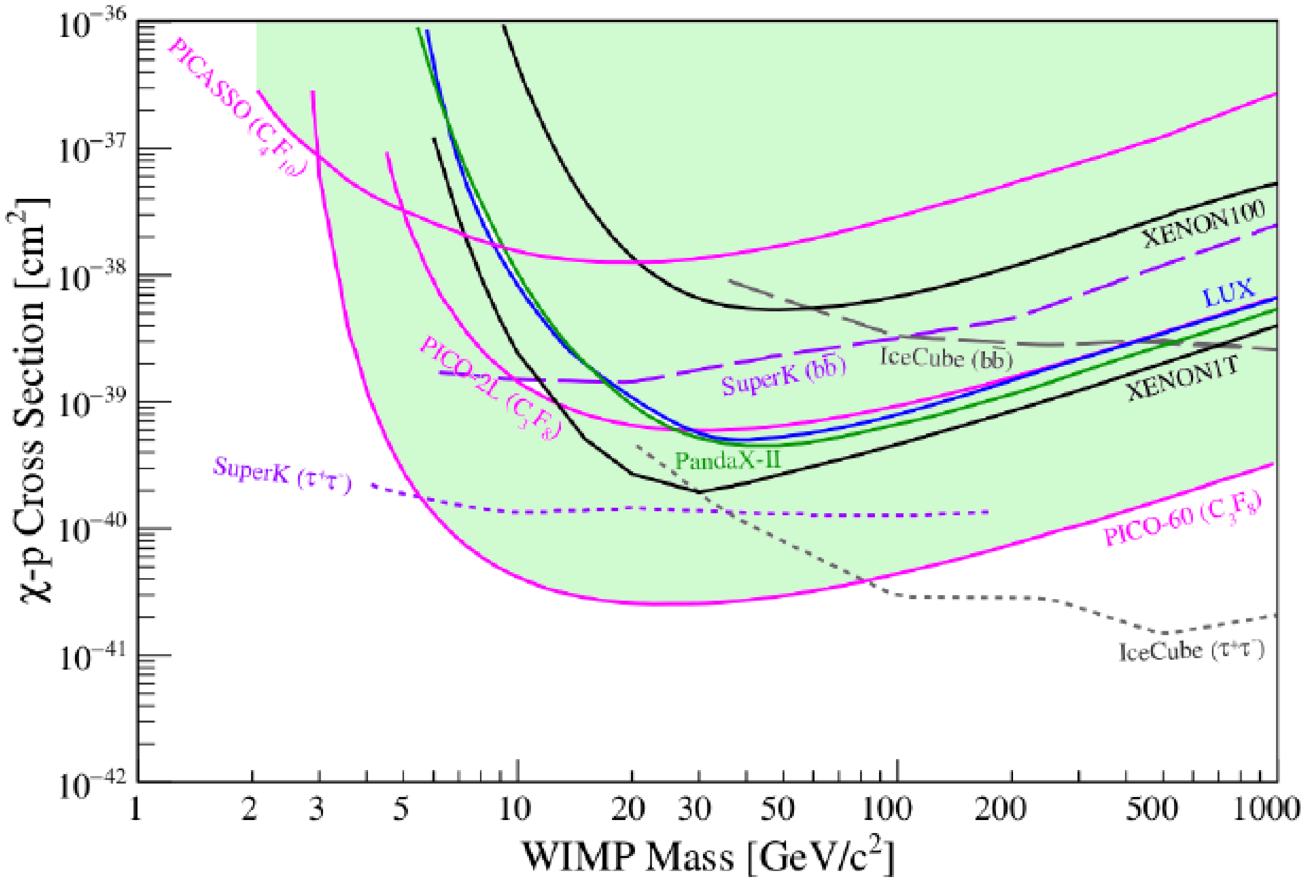
# Geometric-Z' Boson

- $Y_\mu$ couples only to fermions and coupling is universal
- Lighter the $Y_\mu$ smaller its couplings to fermions
- Lighter the $Y_\mu$ slower its decay into fermion/anti-fermion pairs

# Direct search experiments

- Direct search experiments put stringent upper limits on the cross section for scattering of dark matter off the SM particles.
- Present status of a WIMP-proton cross section:




Dark matter comes in various types but detection cross-section is getting smaller and smaller at each new experiment:

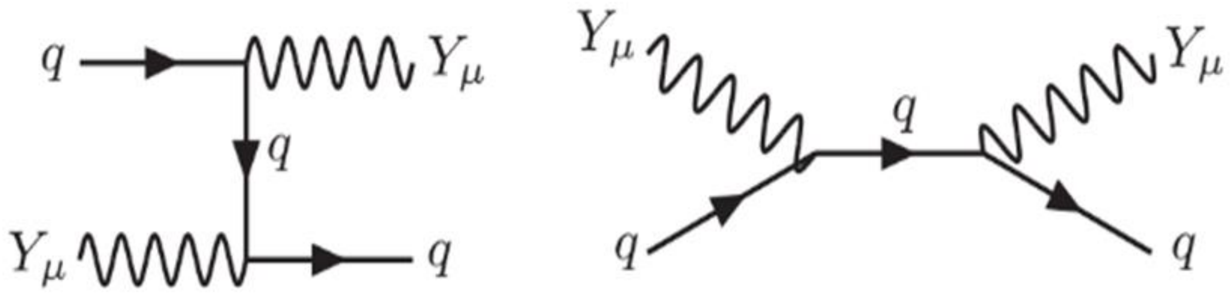
$$\boxed{\sigma_p^{SD} \sim \mathcal{O}(10^{-41})\ \text{cm}^2}$$

M. Schumann, J. Phys. G46 (2019) 10

# Geometric Dark Matter

- It is required to compute the scattering rate of the geometric Z' to check its detection in direct searches

- Relevant diagrams:




$Y_\mu q \to Y_\mu q$ scattering

$$\mathcal{M} = -i\frac{9}{4\xi}\bar{u}(k')\left(\gamma^\nu \frac{\not{k} - \not{p}' + m_q}{(k-p')^2 - m_q^2}\gamma^\mu + \gamma^\mu \frac{\not{k} + \not{p} + m_q}{(k+p)^2 - m_q^2}\gamma^\nu\right) u(k)\epsilon^*_\mu(p')\epsilon_\nu(p)$$

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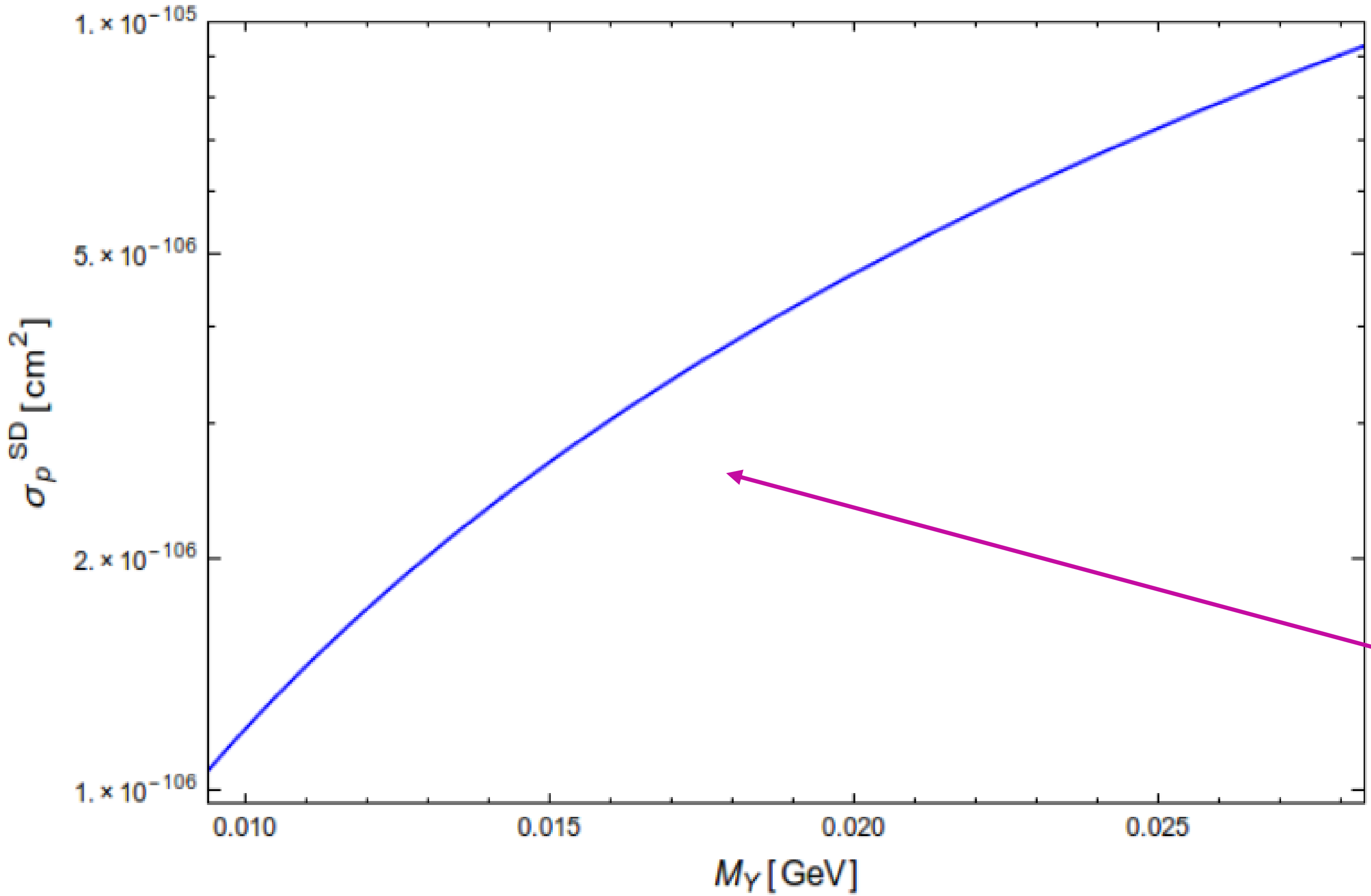
# Geometric Dark Matter

- $Y_\mu$ - proton cross section:




- ➢ Geometric Z’ is undetectable within today’s experimental precision limits
- ➢ Cross section is too small to be measurable by any of the current experiments.

$$\boxed{\mathcal{O}\left(10^{-106}\right)\ \ \mathrm{cm}^2}$$

This result might explain the current dark matter conundrum.

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# Geometric Dark Matter

We have set forth

- a new dark matter candidate, which seems to agree with all the existing bounds

- a genuinely geometrical field provided by the metric-affine gravity

- It explains the current conundrum by its exceedingly small scattering cross section from nucleon

- It can be difficult to detect it with today's technology but future experiments might reach the required accuracy

# Geometric Dark Matter

- We propose a fundamentally different vector dark matter candidate from all the other vector dark matter candidates in the literature

- We show that due to its geometrical origin, the geometric vector dark matter does not couple to scalars and gauge bosons. It couples only to fermions

- There is no need to impose any symmetry to prevent the gauge kinetic interaction in the vector portal

- Its feebly interacting nature is all that is needed for its longevity

- Geometric dark matter $Y_\mu$ is the truly minimal model of dark matter

# Extended Metric-Palatini Gravity (EMPG) augmented *with metrical curvature*

The EMPG framework is defined by the action

$$S\left[g, \varGamma, \varPsi_m\right] = \int d^4x \sqrt{-g}\left\{ \underbrace{\frac{M^2}{2} g^{\mu\nu} R_{\mu\nu}\left({}^{g}\varGamma\right)}_{\text{metrical}} + \underbrace{\frac{\overline{M}^2}{2} g^{\mu\nu} \mathbb{R}_{\mu\nu}\left(\varGamma\right)}_{\text{affine}} - \frac{\xi}{4} \overline{\mathbb{R}}_{\mu\nu}\left(\varGamma\right) \overline{\mathbb{R}}^{\mu\nu}\left(\varGamma\right) + \mathcal{L}\left(g, \varGamma, \varPsi_m\right) \right\}$$

- Einstein-Hilbert term will only partially be generated by the affine curvature

[D. Demir, BP, Eur. Phys. J. C 82 (2022) 996]

# Einstein-geometric Proca theory (EGP)

$$S[g,Y,\Psi_m] = \int d^4x\sqrt{-g}\left\{\frac{M_{Pl}^2}{2}R({}^g\Gamma) - \frac{1}{4}Y_{\mu\nu}Y^{\mu\nu} - \frac{1}{2}M_Y^2 Y_\mu Y^\mu + g_Y \bar{f}\gamma^\mu f Y_\mu + \mathcal{L}_{rest}(g, {}^g\Gamma, \Psi_m)\right\}$$

metrical curvature scalar

normalized canonical vector field

$$\boxed{Y_\mu \equiv 2\sqrt{\xi}Q_\mu}$$

- Planck scale is composed of the two masses:

$$M_{Pl}^2 = M^2 + \overline{M}^2$$

- Geometric-Z' mass:

$$M_Y^2 = \frac{3\overline{M}^2}{2\xi}$$

- Geometric-Z' - fermion coupling:

$$g_Y = \frac{1}{4\sqrt{\xi}}$$

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# Spherically-Symmetric Static EGP, with Perfect Fluid

$$S[g,Y] = \int d^4x \sqrt{-g} \left\{ \frac{1}{2\kappa} R({}^{g}\Gamma) - \frac{1}{4} Y_{\mu\nu} Y^{\mu\nu} - \frac{1}{2} M_Y^2 Y_\mu Y^\mu + g_Y Y_\mu J^\mu + \mathcal{L}_{rest} \right\}$$

- Einstein field equations:

$$R_{\mu\nu}({}^{g}\Gamma) - \frac{1}{2} R(g) g_{\mu\nu} = \kappa (T^{Y}_{\mu\nu} + T^{rest}_{\mu\nu})$$

- Motion equation for the geometric field:

$$\nabla_\mu Y^{\mu\nu} - M_Y^2 Y^\nu = -g_Y J^\nu$$

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# Spherically-Symmetric Static EGP, with Perfect Fluid

- Metric ansatz:

$$\boxed{g_{\mu\nu} = \mathrm{diag}\left(-f^2(r), \frac{g^2(r)}{f^2(r)}, r^2, r^2\sin^2\theta\right)}$$

geometry
(black hole spacetime)

- Geometric field can be taken as a purely time-like field

$$Y_\mu = \frac{u(r)}{r}\delta^0_\mu$$

- Energy-momentum tensor can be taken to be a perfect fluid

$$\boxed{T^{rest}_{\mu\nu} = (\rho_M + p_M)v_\mu v_\nu + p_M g_{\mu\nu}}$$

matter
(dust)

- Source of the geometric field

$$J_\mu = \rho_C v_\mu$$

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# Spherically-Symmetric Static EGP, with Perfect Fluid

- Einstein field equations lead to coupled nonlinear ordinary differential equations; and equation of motion of the geometric field reads:

$$
\boxed{
\begin{aligned}
2\left(\hat{r}\frac{df^2}{d\hat{r}} + f^2 - g^2\right) &= \hat{M}_Y^2\frac{g^2}{f^2}u^2 - \left(\frac{du}{d\hat{r}} - \frac{u}{\hat{r}}\right)^2 + 2\hat{p}_M\hat{r}^2g^2 + \frac{2g_Y\hat{\rho}_C\hat{r}g^2}{f}u, \\
\hat{r}\frac{dg^2}{d\hat{r}} &= \hat{M}_Y^2\frac{g^4}{f^4}u^2 + \frac{\hat{r}^2g^4}{f^2}(\hat{\rho}_M + \hat{p}_M) + \frac{2g_Y\hat{\rho}_C\hat{r}g^4}{f^3}u, \\
\frac{d^2u}{d\hat{r}^2} &= \hat{M}_Y^2\frac{g^2}{f^2}u + \frac{1}{2g^2}\frac{dg^2}{d\hat{r}}\left(\frac{du}{d\hat{r}} - \frac{u}{\hat{r}}\right) + g_Y\frac{\hat{\rho}_C\hat{r}}{f}
\end{aligned}
}
$$

where we carry the equations into gravitational units by

$$\hat{r} := \kappa^{-1/2}r,\ \hat{M}_Y^2 := \kappa M_Y^2,\ \hat{p}_M := \kappa^2 p_M,\ \hat{\rho}_M := \kappa^2\rho_M,$$
$$\hat{\rho}_C := \kappa^{3/2}\rho_C$$

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# Dusty Black Hole Solutions

- We take for the dust:

$$\hat{p}_M = 0,$$
$$\hat{\rho}_M = \frac{\hat{M}_D}{\hat{r}^3},$$
$$\hat{\rho}_C = \frac{Q_D}{\hat{r}^3}$$

geometric charge

to find the asymptotic solutions for a large distance $\hat{r} \to \infty$

$$f_\infty^2 \to c_0^2 \left( \underbrace{\underbrace{1 - \frac{2\hat{M}_B}{\hat{r}_\infty}}_{\text{Sch}} + \frac{Q_\infty^2}{2c_0^2 \hat{r}_\infty^2}}_{\text{RN}} - \underbrace{g_Y \frac{Q_\infty Q_D}{\hat{r}_\infty^2}}_{} \right)$$

due to coupling of $Y_\mu$ to the geometrical charge density of the dust distribution

$$u_\infty \to Q_\infty e^{-\hat{M}_Y \hat{r}_\infty} - \frac{g_Y Q_D}{2} \left( e^{\hat{M}_Y \hat{r}_\infty} \mathrm{Ei}(-\hat{M}_Y \hat{r}_\infty) + e^{-\hat{M}_Y \hat{r}_\infty} \mathrm{Ei}(\hat{M}_Y \hat{r}_\infty) \right)$$

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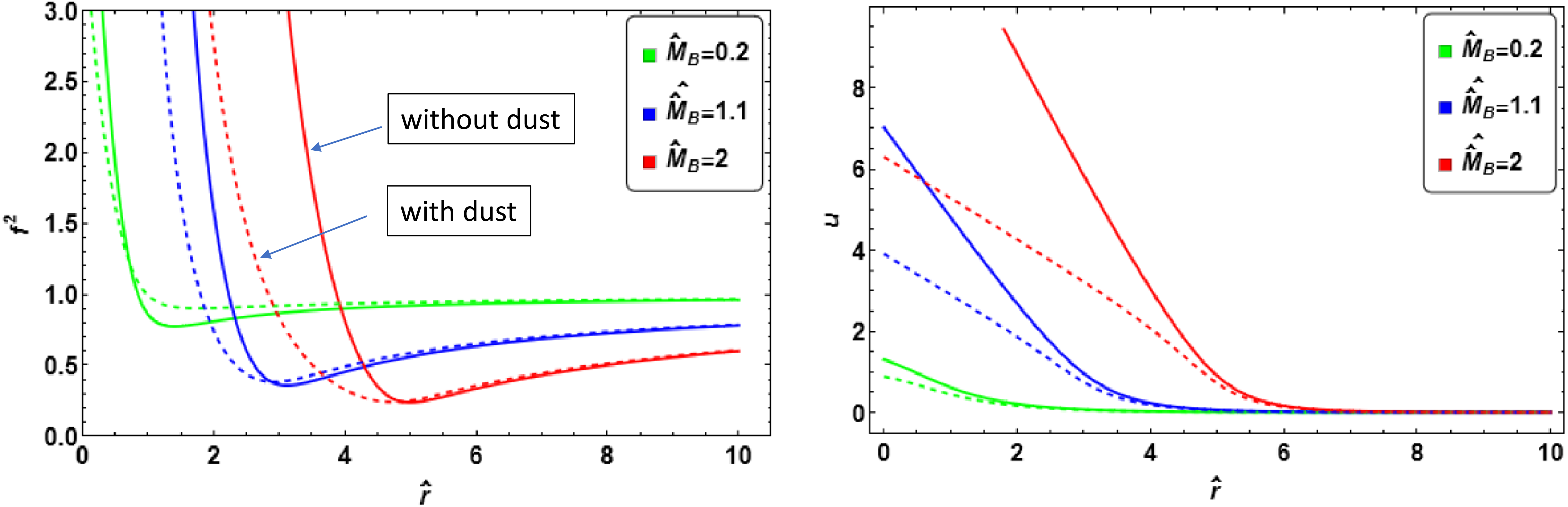
# Dusty Black Hole Solutions

## Geometrically - Neutral Dust $(Q_D = 0)$




- Effects of geometrically - neutral dust get pronounced at low $\hat{r}$ and high $\widehat{M}_B$ values
- Black hole solutions in the EGP system develop a horizon neither with dust nor without dust.

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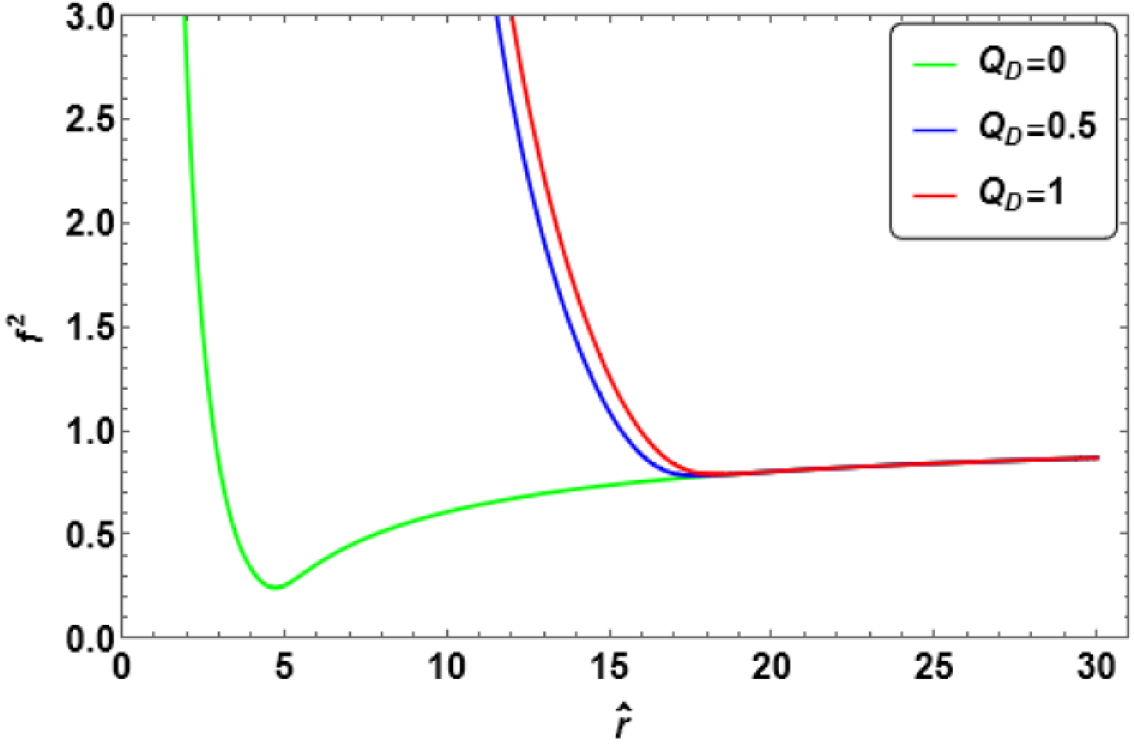
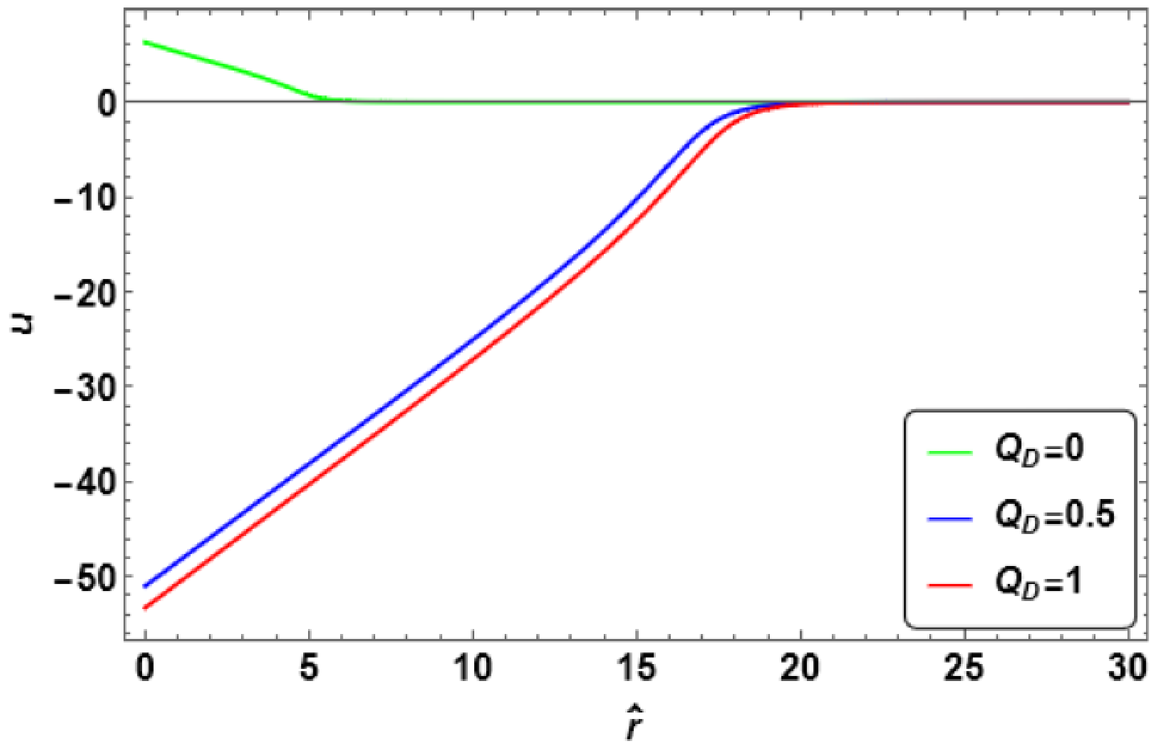
# Dusty Black Hole Solutions

## Geometrically - Charged Dust $(Q_D \neq 0)$




- Particular solution gets abruptly shifted from the homogeneous solution by the presence of the geometrically-charged dust distribution
- Charged dust causes push $f^2$ away from the zero-axis and therefore diminishes possibility of developing a horizon

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# Extended Metric-Palatini Gravity in AdS background

- EMPG action:

$$S[g,\Gamma] = \int d^4x\sqrt{-g}\left\{\frac{M^2}{2}R\left(g\right) + \frac{\overline{M}^2}{2}\mathbb{R}\left(g,\Gamma\right) + \xi\overline{\mathbb{R}}_{\mu\nu}\left(\Gamma\right)\overline{\mathbb{R}}^{\mu\nu}\left(\Gamma\right) - V_0 + \mathcal{L}_m({}^g\Gamma,\psi)\right\}$$

reduces to GR plus a massive vector field theory with matter

$$S[g,Y] = \int d^4x\sqrt{-g}\left\{\frac{1}{16\pi G_N}R(g) - V_0 - \frac{1}{4}Y_{\mu\nu}Y^{\mu\nu} - \frac{1}{2}M_Y^2 Y_\mu Y^\mu + \mathcal{L}_m({}^g\Gamma,\psi)\right\}$$

Newton's constant

$$\boxed{G_N = \frac{1}{8\pi(M^2+\overline{M}^2)}}$$

geometric field mass

$$\boxed{M_Y^2 = \frac{3\overline{M}^2}{2\xi}}$$

canonical geometric field

$$\boxed{Y_\mu = 2\sqrt{\xi}Q_\mu}$$

E. Ghorani, BP, F. Atamurotov, J.Rayimbaev, A. Abduljabarov, D. Demir, EPJC 318 (2023)

# Static Black Hole Solutions of the Einstein-Geometric Proca  (EGP) Model

- Einstein field equations:

$$R_{\mu\nu} - \Lambda g_{\mu\nu} - \hat{Y}_{\alpha\mu}\hat{Y}^{\alpha}{}_{\nu} + \frac{1}{4}\hat{Y}_{\alpha\beta}\hat{Y}^{\alpha\beta}g_{\mu\nu} - M_Y^2\hat{Y}_\mu\hat{Y}_\nu = 0$$

$$\hat{Y}_\mu \equiv \sqrt{\kappa}Y_\mu$$

$$\Lambda = \kappa V_0$$

$$\hat{M}_Y^2 := \kappa M_Y^2$$

- Proca equation:

$$\nabla_\mu \hat{Y}^{\mu\nu} - M_Y^2 \hat{Y}^\nu = 0$$

with geometric Proca field as a purely time-like field

$$\boxed{\hat{Y}_\mu = \hat{\phi}(r)\delta_\mu^0}$$

- Metric ansatz:

$$g_{\mu\nu} = \mathrm{diag}\left(-h(r), \frac{1}{f(r)}, r^2, r^2\sin^2\theta\right)$$

# Static Black Hole Solutions of the Einstein-Geometric Proca (EGP) Model

- Field equations:

$$\hat{\phi}^2 = \frac{1}{\hat{M}_Y^2 \hat{r}}(fh' - f'h)$$

$$1 - f - \frac{\hat{r}(hf)'}{2h} - \Lambda \hat{r}^2 - \frac{f\hat{r}^2}{2h}\hat{\phi}'^2 = 0$$

$$\frac{\sqrt{hf}}{\hat{r}^2}\left(\hat{r}^2\sqrt{\frac{f}{h}}\hat{\phi}'\right)' - \hat{M}_Y^2\hat{\phi} = 0$$

# Static Black Hole Solutions of the Einstein-Geometric Proca (EGP) Model

- Proca field:

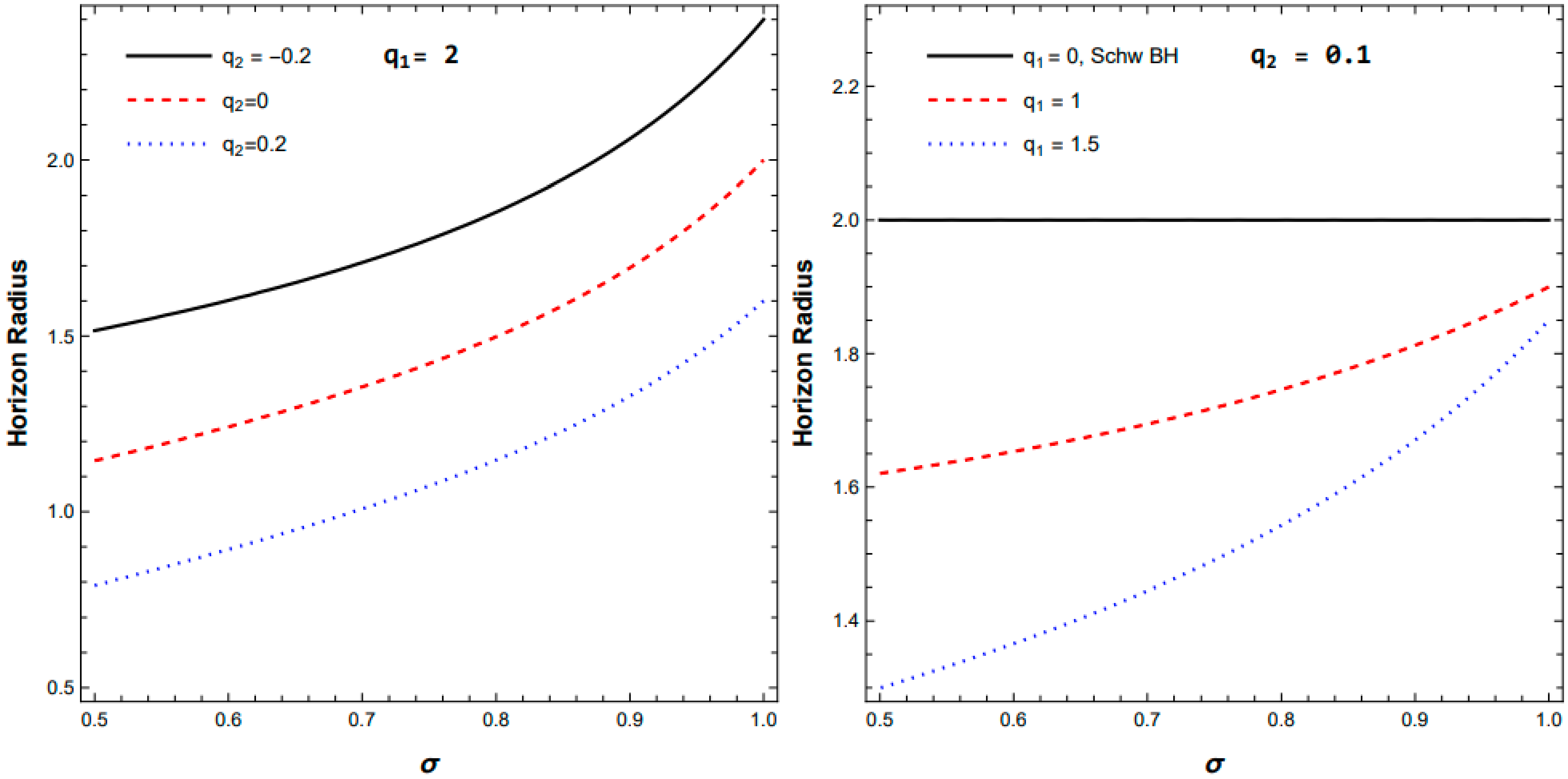
$$\boxed{\hat{\phi}(\hat{r}) = \frac{q_1}{\hat{r}^{\frac{1-\sigma}{2}}} + \frac{q_2}{\hat{r}^{\frac{1+\sigma}{2}}}}$$

$$\sigma = \sqrt{1 + 4\hat{M}_Y^2 l^2}$$

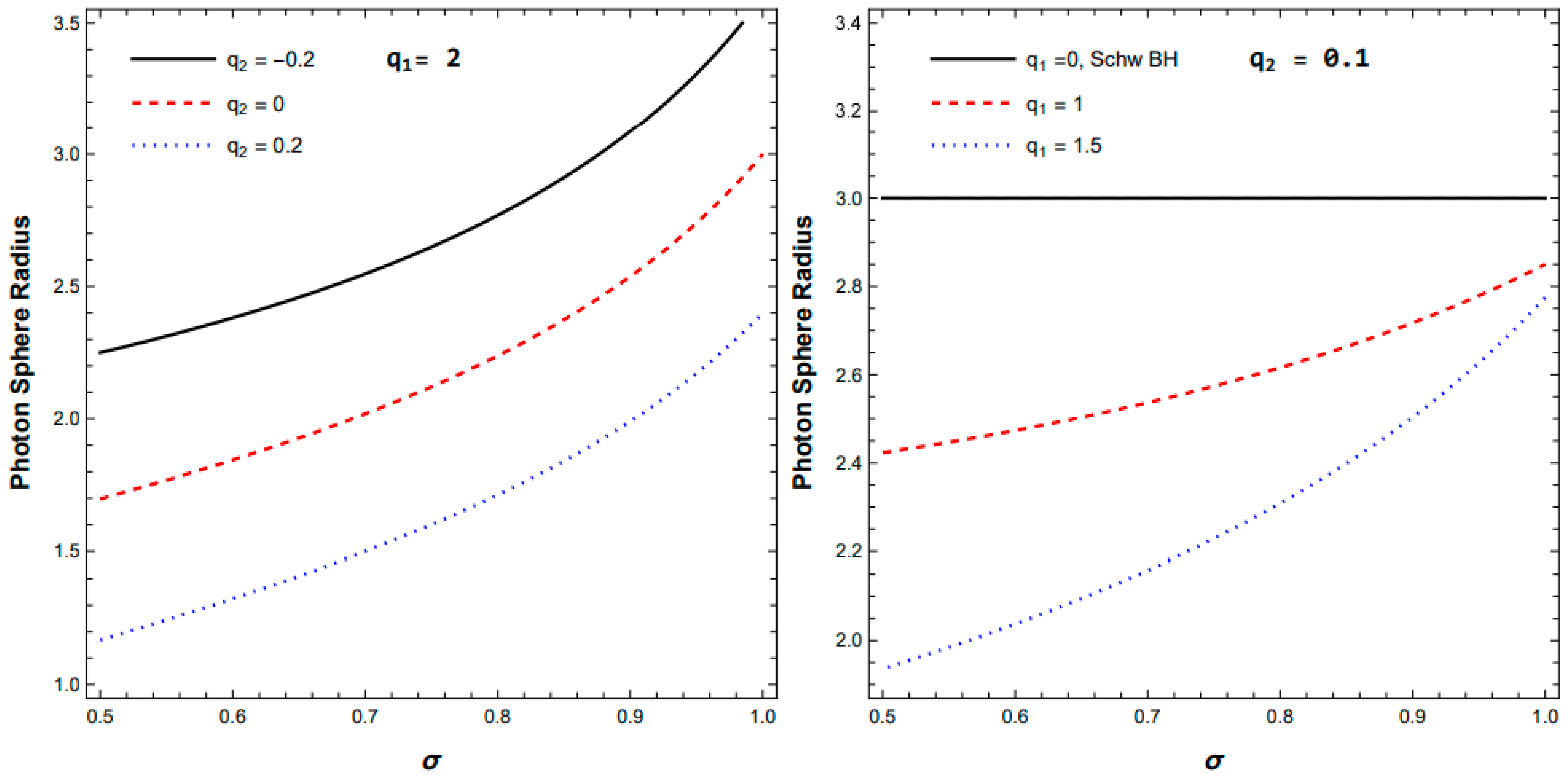
- **Black hole solutions**: Proca field backreaction on the solutions

$$g_{\mu\nu} = \mathrm{diag}\left(-h(r), \frac{1}{f(r)}, r^2, r^2 \sin^2\theta\right)$$

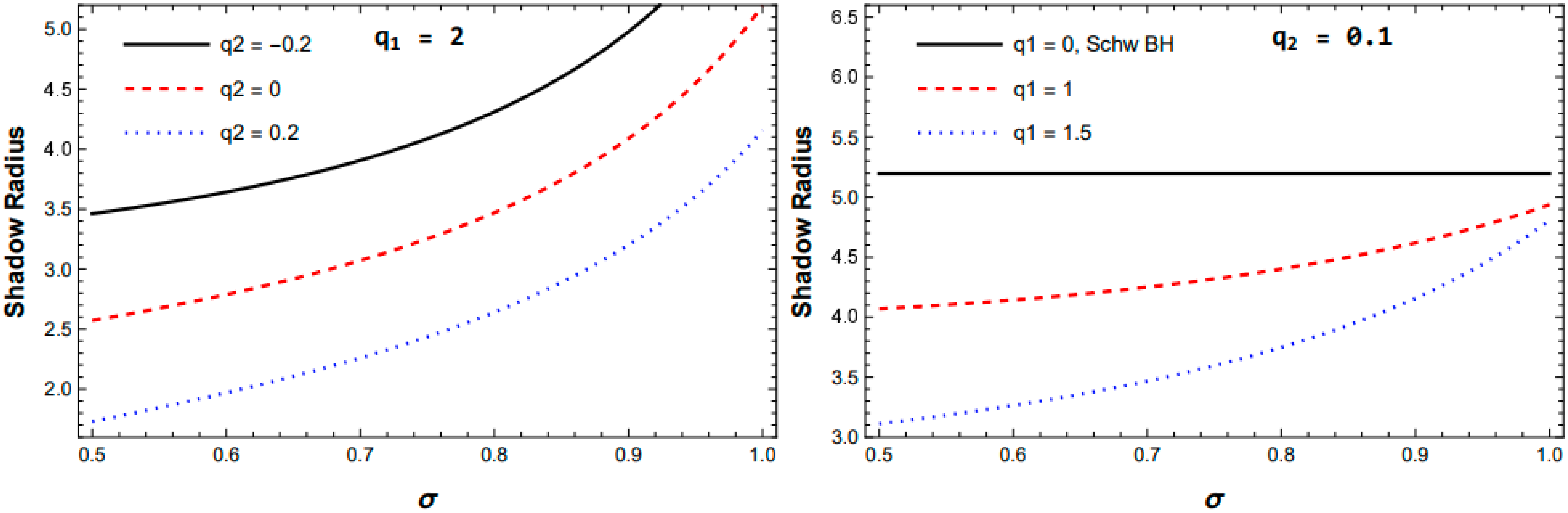
$$\boxed{\begin{aligned} f(\hat{r}) &= 1 + \frac{(1-\sigma)q_1^2}{4\hat{r}^{1-\sigma}} + \frac{-12 + q_1 q_2\left(7 + 6(\gamma - 1)\sigma - \sigma^2\right)}{6\hat{r}} \\ h(\hat{r}) &= 1 + \frac{(1-\sigma)q_1^2}{(3-\sigma)\hat{r}^{1-\sigma}} + \frac{q_1 q_2\left(\gamma\sigma + \frac{1}{3}(1-\sigma)(4+\sigma)\right) - 2}{\hat{r}} \end{aligned}}$$



q2 = −0.2
q1= 2
q2=0
q2=0.2
Horizon Radius
σ
q1 = 0, Schw BH
q2 = 0.1
q1 = 1
q1 = 1.5
Horizon Radius
σ



q2 = −0.2
q1= 2
q2 = 0
q2 = 0.2
Photon Sphere Radius
σ
q1 =0, Schw BH
q2 = 0.1
q1 = 1
q1 = 1.5
Photon Sphere Radius
σ


Shadow Radius
q2 = −0.2
q2 = 0
q2 = 0.2
$q_1 = 2$
σ
Shadow Radius
q1 = 0, Schw BH
q1 = 1
q1 = 1.5
$q_2 = 0.1$
σ

# Charged Black Hole Solutions in Symmergent gravity

Symmergent gravity is described by the action

$$S[g] = \frac{1}{16\pi G} \int d^4x \sqrt{-g} \left( f(R) - 2\Lambda - \frac{1}{2} \hat{F}_{\mu\nu} \hat{F}^{\mu\nu} \right)$$

$$\boxed{f(R) = R + \beta R^2}$$

$$\beta = -\pi G c_{\mathrm{O}}$$

Cosmological constant

$$\Lambda = 8\pi G V_{\mathrm{O}}$$

Electromagnetic field strength tensor

$$\hat{F}_{\mu\nu} = \partial_\mu \hat{A}_\nu - \partial_\nu \hat{A}_\mu$$

$$\hat{A}_\mu = A_\mu / \sqrt{8\pi G}$$

total number of bosonic and fermionic degrees of freedom

$$c_{\mathrm{O}} = \frac{n_{\mathrm{B}} - n_{\mathrm{F}}}{128\pi^2}$$

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# Charged Black Hole Solutions in Symmergent gravity

Einstein field equations

$$E_{\mu\nu} \equiv R_{\mu\nu}F(R) - \frac{1}{2}g_{\mu\nu}f(R) + g_{\mu\nu}\Lambda + (g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu)F(R) - \hat{T}_{\mu\nu} = 0$$

Maxwell field equations

$$\partial_\mu(\sqrt{-g}\hat{F}^{\mu\nu}) = 0$$

For a static spherically-symmetric solution, we propose the metric

$$ds^2 = -h(r)dt^2 + \frac{1}{h(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

and the electromagnetic scalar potential

$$\hat{A}_0 = \hat{q}(r)$$

which leads to the Ricci curvature scalar

$$R = -h'' - \frac{4}{r}h' - \frac{2}{r^2}(h-1)$$

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## Field equations in the Symmergent gravity

$$
\begin{aligned}
E_0{}^0 = {} & \Lambda + \frac{h'(r)}{r} + \frac{h(r)}{r^2} - \frac{1}{r^2} + \frac{1}{2}\hat{q}'(r)^2 \\
& + \beta\left(-2h''''(r)h(r) - h'''(r)h'(r) + \frac{1}{2}h''(r)^2 - \frac{12h'''(r)h(r)}{r} - \frac{2h'(r)h''(r)}{r} - \frac{4h(r)h''(r)}{r^2}\right. \\
& \left. + \frac{2h'(r)^2}{r^2} + \frac{8h(r)h'(r)}{r^3} - \frac{10h(r)^2}{r^4} + \frac{12h(r)}{r^4} - \frac{2}{r^4}\right),
\end{aligned}
$$

$$
\begin{aligned}
E_1{}^1 = {} & \Lambda + \frac{h'(r)}{r} + \frac{h(r)}{r^2} - \frac{1}{r^2} + \frac{1}{2}\hat{q}'(r)^2 \\
& + \beta\left(-h'''(r)h'(r) + \frac{1}{2}h''(r)^2 - \frac{4h'''(r)h(r)}{r} - \frac{2h'(r)h''(r)}{r} - \frac{16h(r)h''(r)}{r^2} + \frac{2h'(r)^2}{r^2}\right. \\
& \left. + \frac{8h(r)h'(r)}{r^3} + \frac{14h(r)^2}{r^4} - \frac{12h(r)}{r^4} - \frac{2}{r^4}\right),
\end{aligned}
$$

$$
\begin{aligned}
E_2{}^2 = {} & \Lambda + \frac{h'(r)}{r} + \frac{h''(r)}{2} - \frac{1}{2}\hat{q}'(r)^2 \\
& + \beta\left(-2h''''(r)h(r) - 2h'''(r)h'(r) - \frac{1}{2}h''(r)^2 - \frac{10h'''(r)h(r)}{r} - \frac{10h'(r)h''(r)}{r} + \frac{4h'(r)^2}{r^2}\right. \\
& \left. + \frac{4h(r)h''(r)}{r^2} + \frac{16h(r)h'(r)}{r^3} - \frac{12h'(r)}{r^3} - \frac{14h(r)^2}{r^4} + \frac{12h(r)}{r^4} + \frac{2}{r^4}\right),
\end{aligned}
$$

$r^2\hat{q}''(r) + 2r\hat{q}'(r) = 0$ with $\hat{q}(r) = \dfrac{Q}{r}$

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# Charged Black Hole Solutions in Symmergent gravity

The metric function solution takes the form

$$h(r) \ = \ 1 - \frac{2MG}{r} + \frac{1}{(1+8\beta\Lambda)} \frac{Q^2}{2r^2} - \frac{\Lambda r^2}{3}$$

Using Symmergent gravity parameters

$$\boxed{h(r) = 1 - \frac{2MG}{r} + \frac{Q^2}{2\hat{\alpha} r^2} - \frac{(1-\hat{\alpha})}{24\pi G c_{\rm O}} r^2}$$

$$\hat{Q}^2 = \frac{Q^2}{2\hat{\alpha}}$$

$$\hat{\Lambda} = \frac{(1-\hat{\alpha})}{8\pi G c_{\rm O}}$$

RN – AdS/dS black hole

$\hat{\alpha} > 1$ corresponds to fermion dominance and AdS space

$\hat{\alpha} < 1$ corresponds to boson dominance and dS space

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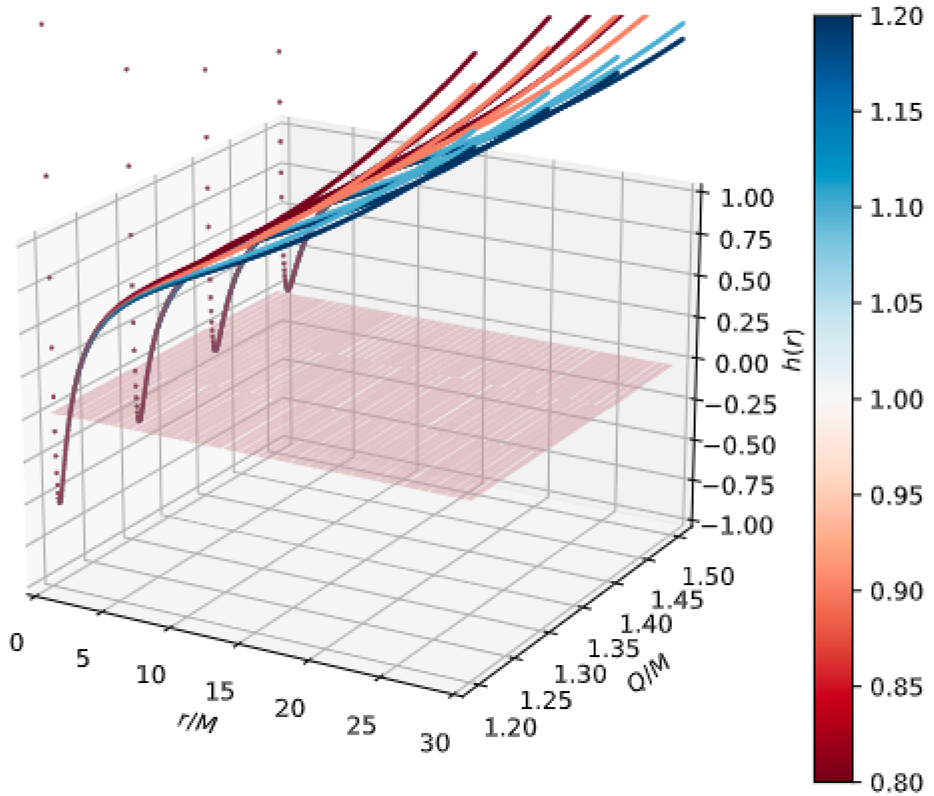
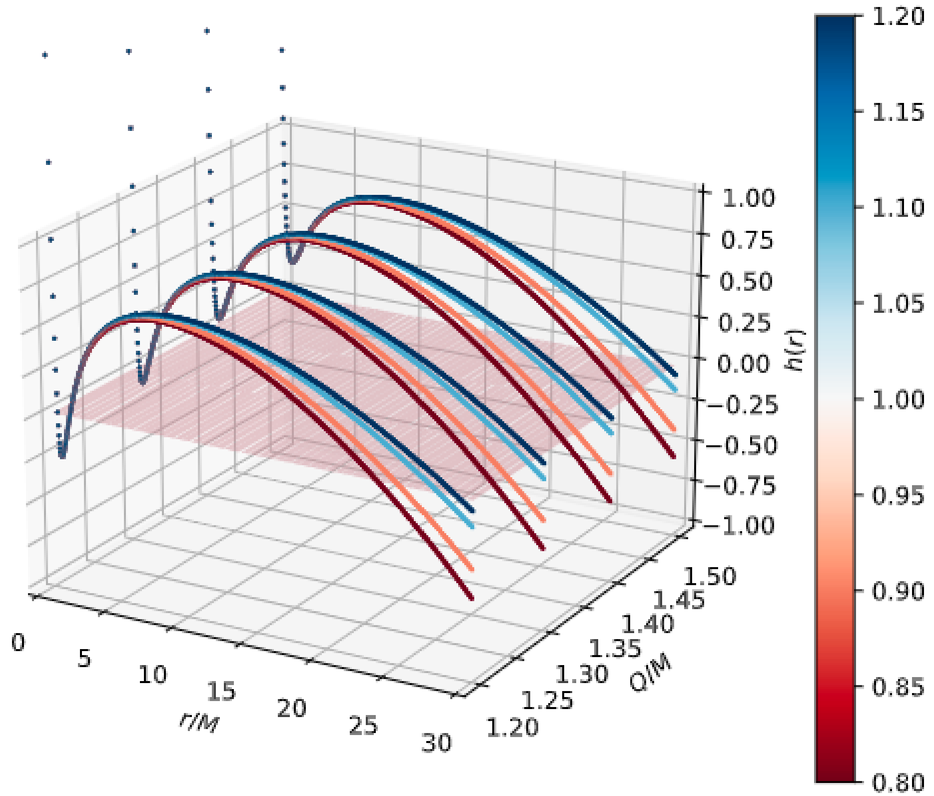
# Charged Black Hole Solutions in Symmergent gravity

## Metric functions

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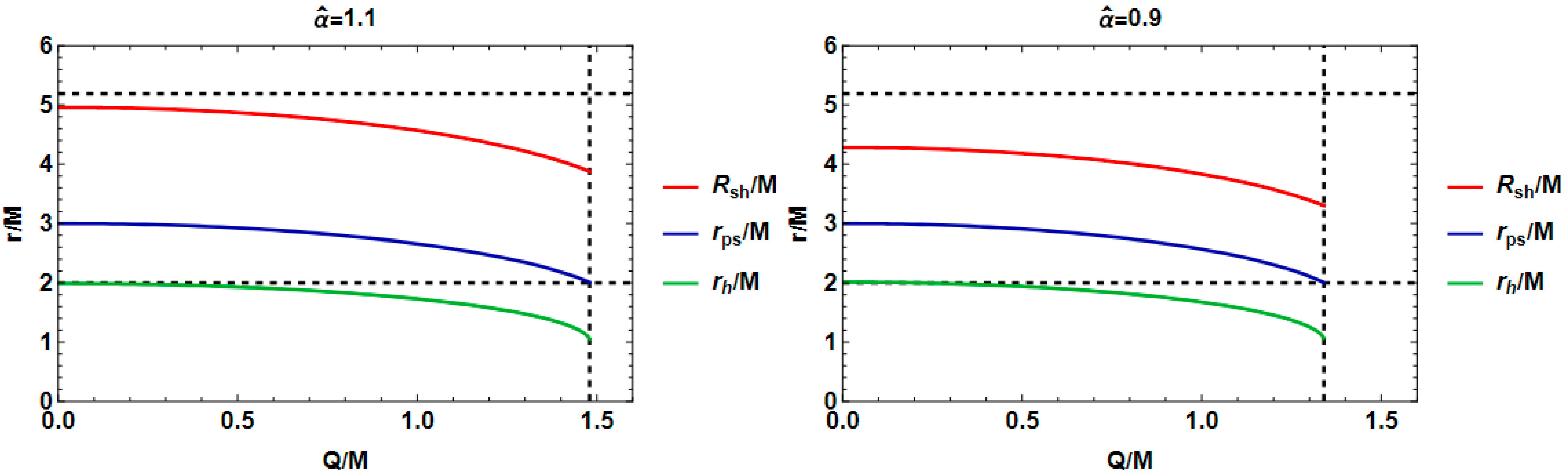
# Charged Black Hole Solutions in Symmergent gravity

## Shadow radius, photon sphere radius, horizon radius

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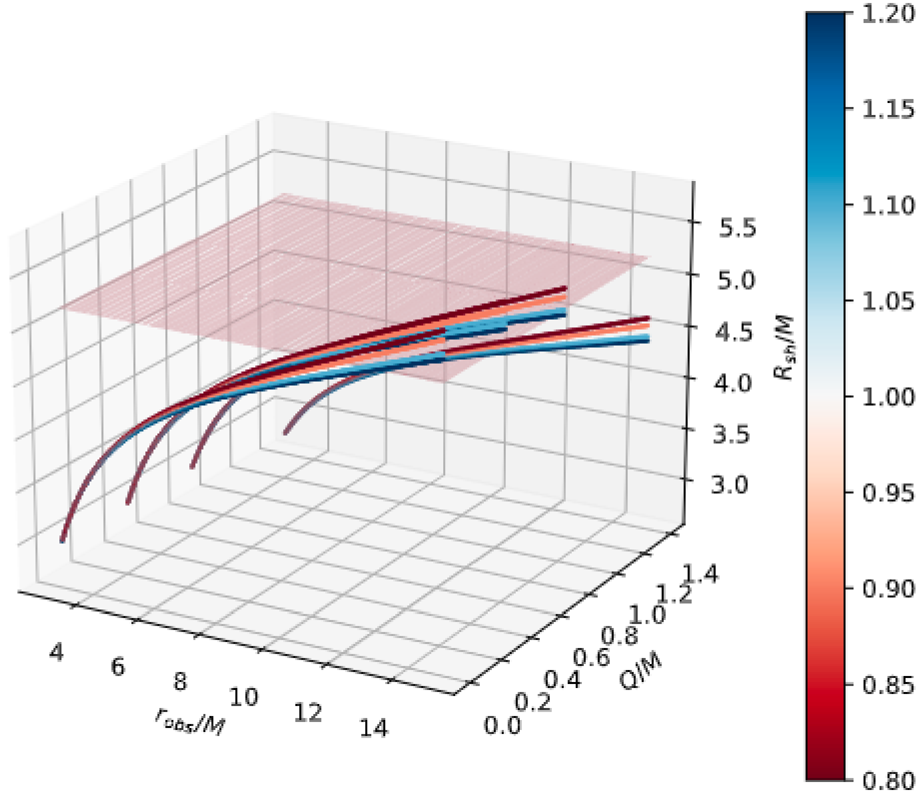
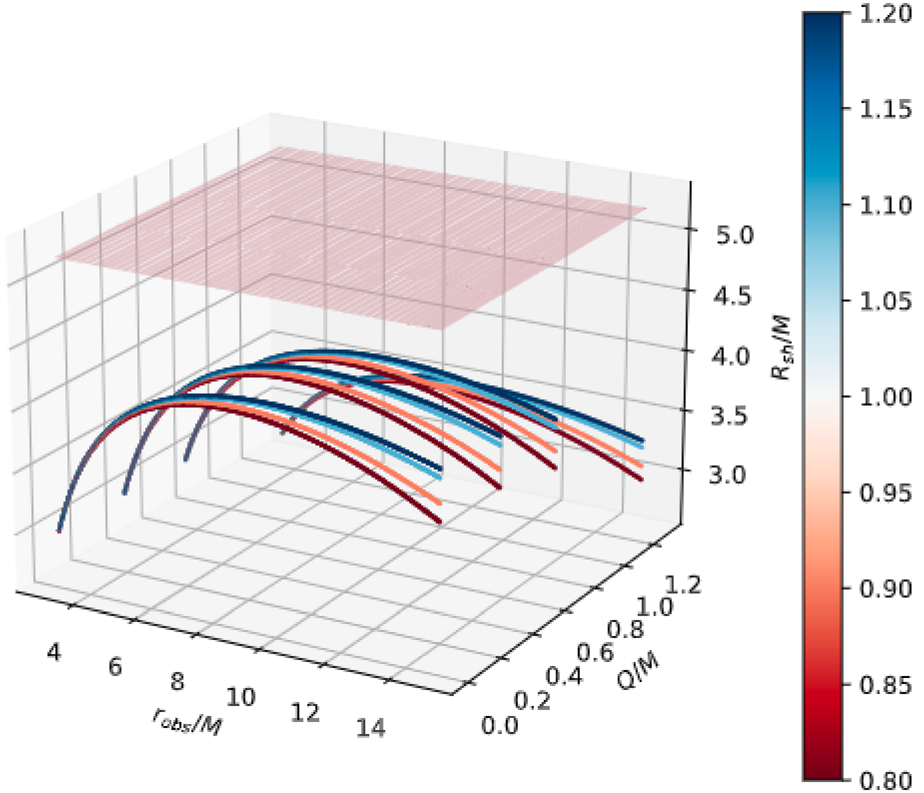
# Charged Black Hole Solutions in Symmergent gravity

## Shadow radius

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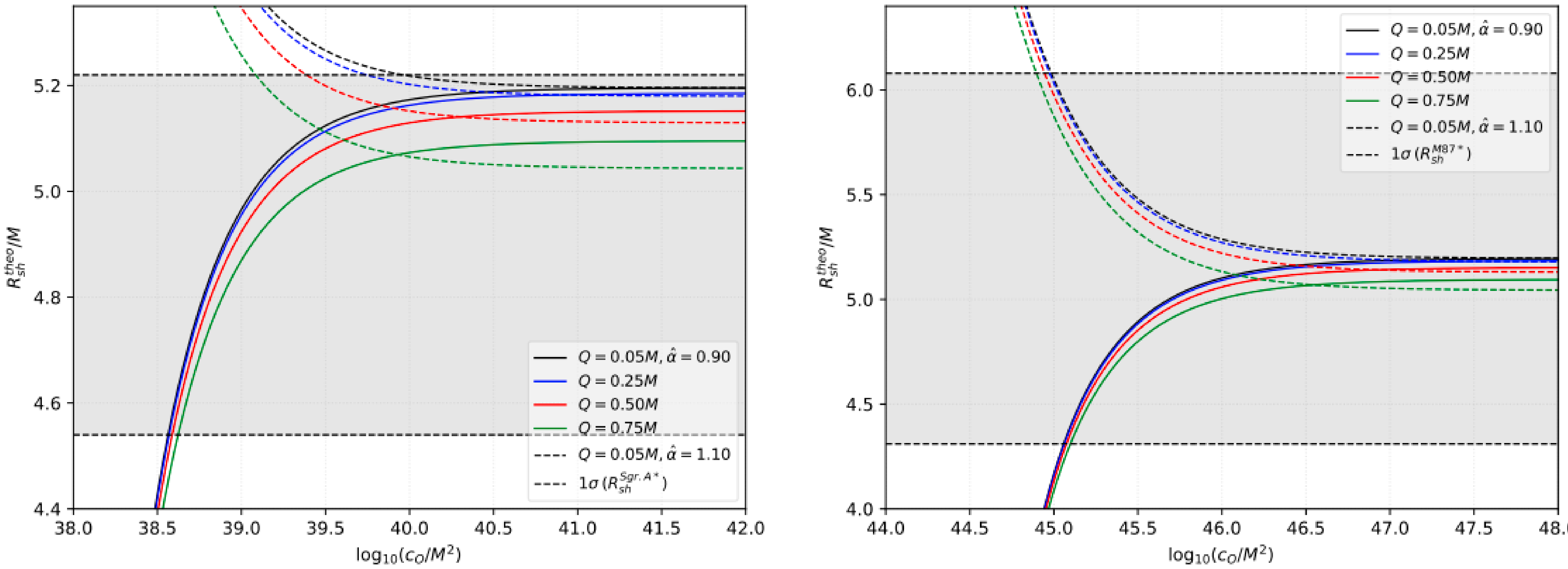

$Q = 0.05M, \hat{\alpha} = 0.90$
$Q = 0.25M$
$Q = 0.50M$
$Q = 0.75M$
$Q = 0.05M, \hat{\alpha} = 1.10$
$1\sigma\,(R_{sh}^{Sgr.A^*})$
$R_{sh}^{theo}/M$
$\log_{10}(c_O/M^2)$
$1\sigma\,(R_{sh}^{M87^*})$

- Metric-Palatini gravity theory reduces to the GR plus a geometrical massive vector theory, which we call the Einstein-Geometric Proca (EGP) theory
- EGP model differs from similar models in the literature by its explicit involvement of the direct coupling between the geometric field and the fermions in the theory
- We propose a fundamentally different vector dark matter candidate from all the other vector dark matter candidates in the literature, mainly due to its geometrical origin
- Our candidate particle, a genuinely geometrical field provided by the extended Palatini gravity, is a viable dark matter candidate, and explains the current conundrum by its exceedingly small scattering cross section from nucleon
- Static spherically symmetric solutions in the EGP system develop no horizon and show the impossibility of black hole type solutions in the presence of dust with or without geometrical charge

- Formation of black holes is possible in the EGP theory in the AdS background. Photon motion and shadow of the EGP AdS black hole are analyzed
- Metric-Palatini action in the Symmergent gravity reduces to the metrical gravity theory
- We report on exact charged black hole solutions in Symmergent gravity with Maxwell field
- Our results provide new insights into the behavior of charged black holes in the context of Symmergent gravity

# Recent works: *intersection of particle physics and gravitation*

- Verbin, Y. , **P. B.**, Övgün, A., Huang, H. (2025), "New black hole solutions of second and first order formulations of nonlinear electrodynamics", *Physical Review D,* 111 (2025) 8, 084061
- Ghorani, E.; Matri, S., Rayimbaev, J., **P. B.**, Atamurotov, F.; Abdujabbarov, A.; Demir, D. (2024), "Constraints on metric-Palatini gravity from QPO Data", *European Physical Journal C,* 84 (2024) 10, 1022
- **P. B.**; Pantig, R. C.; Övgün, A. and Demir, D., (2024), "Asymptotically-Flat Black Hole Solutions in Symmergent Gravity", *Fortschritte der Physik,* 72 (2024) 4, 2300138
- **P. B.**; Pantig, R. C.; Övgün, A. and Demir, D., (2023)," Constraints on charged Symmergent black hole from shadow and lensing ", *Classical and Quantum Gravity*, 40 195003
- Ghorani, E.; **P. B.**; Atamurotov, F.; Rayimbaev, J.; Abdujabbarov, A.; Demir, D., (2023), "Probing Geometric Proca with Black Hole Shadow and Photon Motion", *European Physical Journal C,* 83, 318 (2023)
- Demir, D. and **P. B.** (2022), "Geometric Proca with matter in metric-Palatini gravity", *European Physical Journal C*, 82 (2022) 11, 996
- **P. B.** (2021), "A Family-nonuniversal U (1)' Model for Excited Beryllium Decays", *Chinese Journal of Physics*, 71 (2021) 506-517
- Demir, D. and **P. B.** (2020), "Geometric Dark Matter", *Journal of Cosmology and Astroparticle Physics*, 04 (2020) 051

Thank you..