Direct detection of sub-GeV dark matter: a theoretical search for optimal detector materials

(work in progress)

Michał Iglicki

in colalboration with R. Catena

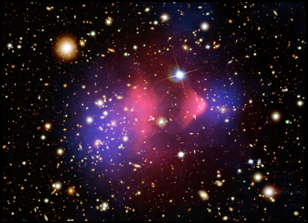
CHALMERS
UNIVERSITY OF TECHNOLOGY

Swedish Research Council

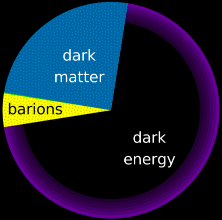
PASCOS 2025
Durham, 22 July 2025

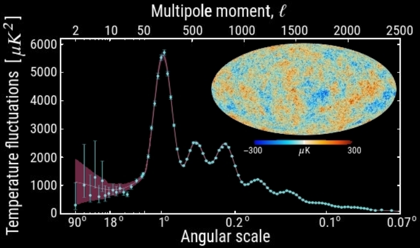
KUNGL. VETENSKAPS-AKADEMIEN
THE ROYAL SWEDISH ACADEMY OF SCIENCES

Dark matter

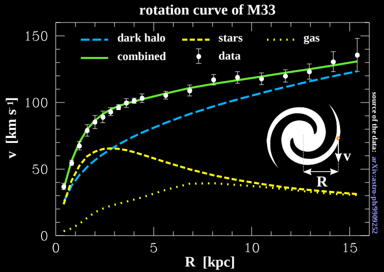


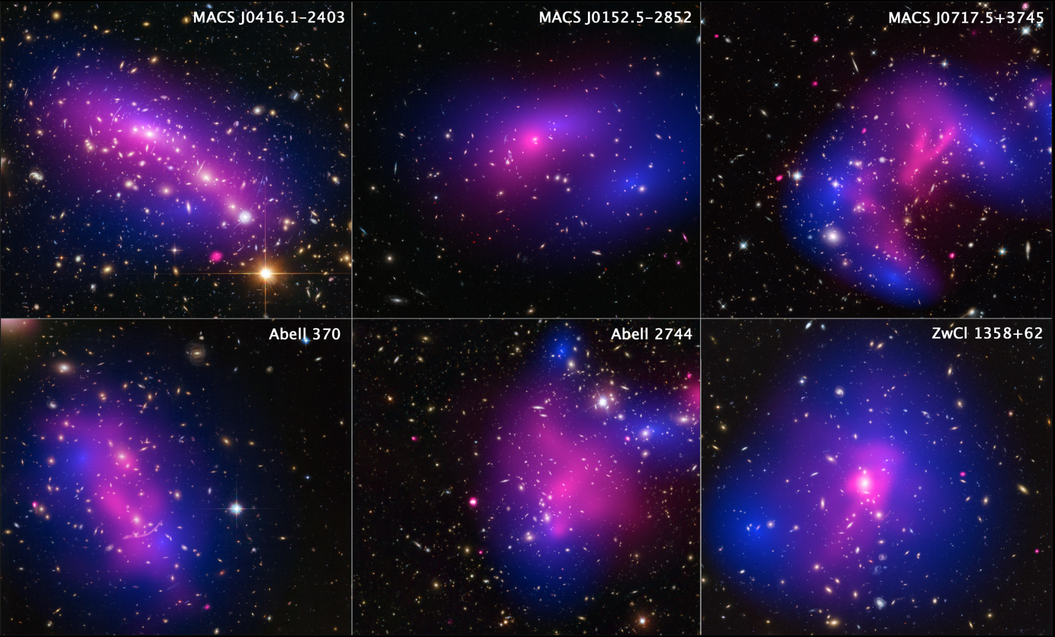
https://apod.nasa.gov/apod





http://sci.esa.int/planck
https://wiki.cosmos.esa.int/planck-legacy-archive





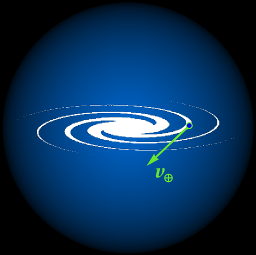
https://hubblesite.org

Local distribution of DM

Baxter et al., 2105.00599

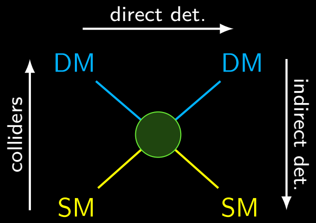
$$n_{\text{DM}} \simeq \frac{1\ \text{GeV}}{m_{\text{DM}}} \times 0.3\ \text{cm}^{-3}$$

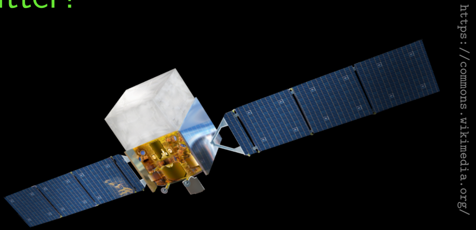
$$\rho(\boldsymbol{v}) = \mathcal{N}\ \exp\left[-\frac{(\boldsymbol{v}+\boldsymbol{v}_\oplus)^2}{v_0^2}\right]\theta(v_{\text{esc}} - |\boldsymbol{v}+\boldsymbol{v}_\oplus|)$$

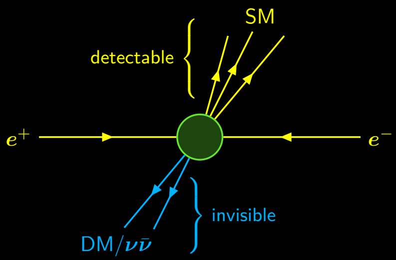


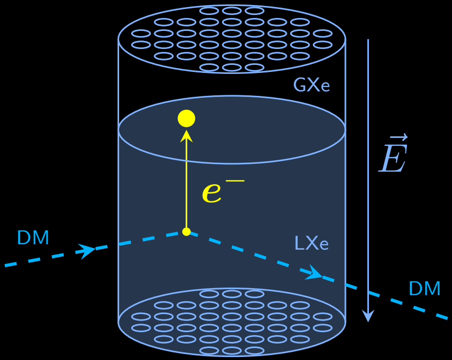
$$v_{\text{esc}} = 544\,\text{km/s}$$
$$v_\oplus = 250.5\,\text{km/s}$$
$$v_0 = 238\,\text{km/s}$$

How to detect particle dark matter?



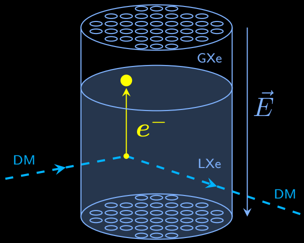






Direct detection experiments

- LUX-ZEPLIN, PandaX-4T, XENONnT, SuperCDMS, ...

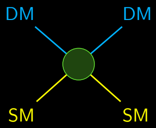


- WIMP paradigm: GeV+ range of masses
  - no success so far $\Rightarrow$ **sub-GeV** DM?
- nuclear vs. electronic recoil of **non-relativistic** DM

$$\Delta E_{\text{SM}} \leq \frac{4\,\mu}{(1+\mu)^2}\, E_{\text{DM}}^{\text{in}} \qquad \leftarrow \text{ maximized for } \mu \equiv m_{\text{SM}}/m_{\text{DM}} = 1$$

$\Rightarrow$ m_{SM} should be as close to m_{DM} as possible

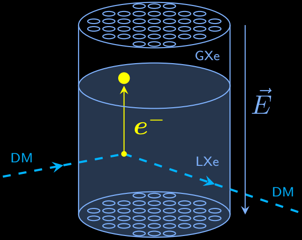
$\Rightarrow$ **electrons** preferable for **light DM**



- what material to use?

Direct detection experiments

- LUX-ZEPLIN, PandaX-4T, XENONnT, SuperCDMS, ...

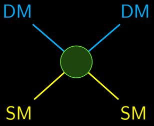


- WIMP paradigm: GeV+ range of masses
  - no success so far $\Rightarrow$ **sub-GeV** DM?
- nuclear vs. electronic recoil of **non-relativistic** DM

$$\Delta E_{\mathrm{SM}} \leq \frac{4\,\mu}{(1+\mu)^2}\, E_{\mathrm{DM}}^{\mathrm{in}} \qquad \leftarrow \text{ maximized for } \mu \equiv m_{\mathrm{SM}}/m_{\mathrm{DM}} = 1$$

$\Rightarrow$ m_{SM} should be as close to m_{DM} as possible

$\Rightarrow$ **electrons** preferable for **light DM**



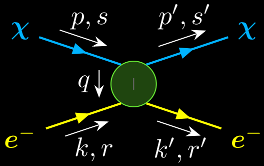
- what material to use?

Effective non-relativistic theory for spin-$1/2$ DM

Catena et al., 2105.02233

$$\boldsymbol{v}^{\perp} \equiv \frac{\boldsymbol{p}+\boldsymbol{p}'}{2\,m_\chi} - \frac{\boldsymbol{k}+\boldsymbol{k}'}{2\,m_e}$$

$$\boldsymbol{q}\cdot\boldsymbol{v}^{\perp} \xrightarrow{\text{en. cons.}} 0$$



non-relativistic limit
Lorentz (Galilean) invariance } $\Rightarrow$ $\mathcal{M} = \sum_i c_i\,\mathcal{O}_i$

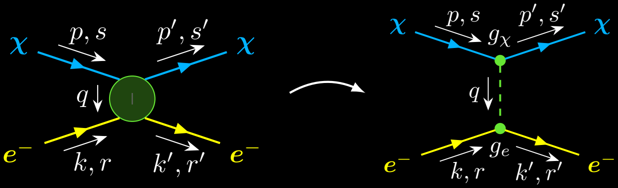
14 simple operators in the leading order

Effective non-relativistic theory for spin-1/2 DM

Catena et al., 2105.02233

$$\boldsymbol{v}^\perp \equiv \frac{\boldsymbol{p}+\boldsymbol{p}'}{2\,m_\chi} - \frac{\boldsymbol{k}+\boldsymbol{k}'}{2\,m_e}$$

$$\boldsymbol{q}\cdot\boldsymbol{v}^\perp \xrightarrow{\text{en. cons.}} 0$$



non-relativistic limit
Lorentz (Galilean) invariance } $\Rightarrow$ $\mathcal{M} = \sum_i c_i\,\mathcal{O}_i$

14 simple operators
in the leading order

- example: scalar coupling

$$\mathcal{M} \simeq \underbrace{-i\,\frac{g_\chi\,g_e}{q^2+M^2}\,4\,m_\chi m_e}_{c_1}\;\underbrace{\delta^{ss'}\delta^{rr'}}_{\mathcal{O}_1}$$

- other examples:

$$\mathcal{O}_4^{rr'ss'} = \frac{\boldsymbol{\sigma}^{rr'}}{2}\cdot\frac{\boldsymbol{\sigma}^{ss'}}{2}\,, \qquad \mathcal{O}_{15}^{rr'ss'} = \left[\left(\frac{\boldsymbol{\sigma}^{rr'}}{2}\times\frac{\boldsymbol{q}}{m_e}\right)\cdot\boldsymbol{v}^\perp\right]\left(\frac{\boldsymbol{\sigma}^{ss'}}{2}\cdot\frac{\boldsymbol{q}}{m_e}\right)$$

Linear response theory

Catena & Spaldin, 2402.06817

- another decomposition:

$$\mathcal{M} = \sum_i c_i \, \mathcal{O}_i$$

$$= \sum_a \underbrace{F_a^{ss'}(\boldsymbol{q}, \boldsymbol{v}_\chi)}_{\text{DM model}} \times \underbrace{J_a^{rr'}(\boldsymbol{v}_e^\perp)}_{\text{electronic part}} \,, \qquad \boldsymbol{v}_\chi \equiv \frac{\boldsymbol{p}}{m_\chi} \,, \qquad \boldsymbol{v}_e^\perp \equiv \frac{\boldsymbol{k} + \boldsymbol{k}'}{2 m_e}$$

- electronic operators:

$$J_{n_0}^{rr'} \equiv \delta^{rr'} \,, \quad J_{n_A}^{rr'} \equiv \boldsymbol{v}_e^\perp \cdot \sigma^{rr'} \,,$$

$$\boldsymbol{J}_{\boldsymbol{j}_5}^{rr'} \equiv \boldsymbol{\sigma}^{rr'} \,, \quad \boldsymbol{J}_{\boldsymbol{j}_M}^{rr'} \equiv \boldsymbol{v}_e^\perp \, \delta^{rr'} \,, \quad \boldsymbol{J}_{\boldsymbol{j}_E}^{rr'} \equiv -i \, \boldsymbol{v}_e^\perp \times \boldsymbol{\sigma}^{rr'}$$

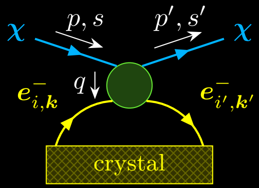
- example: scalar coupling

$$\mathcal{M} \simeq \overbrace{\underbrace{-i \frac{g_\chi \, g_e}{q^2 + M^2} 4 \, m_\chi m_e}_{c_1} \, \delta^{ss'}}^{F_0} \; \underbrace{\overbrace{\delta^{rr'}}^{J_0}}_{\mathcal{O}_1}$$

Int. rate for bounded electrons & generalized susceptibilities

Catena et al., 1912.08204

- electronic states ≠ momentum eigenstates



$$\mathcal{M} = \sum_a \underbrace{F_a^{ss'}(\boldsymbol{q}, \boldsymbol{v}_\chi)}_{\text{DM model}} \times \underbrace{J_a^{rr'}(\boldsymbol{v}_e^\perp)}_{\text{electronic part}}$$

$$\Big\downarrow \left| \begin{array}{l}\text{integral over} \\ e^- \text{ wave fun.}\end{array} \right|^2$$

$$|\widetilde{\mathcal{M}}_{i\boldsymbol{k}\to i'\boldsymbol{k}'}|^2 \simeq \sum_{ab} \underbrace{\mathcal{F}_{ab}(\boldsymbol{q}, \boldsymbol{v}_\chi)}_{\text{DM model}} \times \overbrace{\Big[\; |f_{i\boldsymbol{k}\to i'\boldsymbol{k}'}(\boldsymbol{q})|^2 \, \mathcal{J}_{ab}(\boldsymbol{q}) + f_{i\boldsymbol{k}\to i'\boldsymbol{k}'}(\boldsymbol{q}) \, \boldsymbol{f}^*_{i\boldsymbol{k}\to i'\boldsymbol{k}'}(\boldsymbol{q}) \cdot \boldsymbol{\mathcal{J}}_{ab}(\boldsymbol{q}) + \text{c.c.} + \boldsymbol{f}_{i\boldsymbol{k}\to i'\boldsymbol{k}'}(\boldsymbol{q})^* \cdot \hat{\mathcal{J}}_{ab}(\boldsymbol{q}) \cdot \boldsymbol{f}_{i\boldsymbol{k}\to i'\boldsymbol{k}'}(\boldsymbol{q}) \Big]}^{\text{material response}}$$

where

$$f_{i\boldsymbol{k}\to i'\boldsymbol{k}'}(\boldsymbol{q}) \equiv \int \frac{d^3 l}{(2\pi)^3} \, \psi_{i\boldsymbol{k}}(\boldsymbol{l}) \, \psi^*_{i'\boldsymbol{k}'}(\boldsymbol{l}+\boldsymbol{q}) \qquad \boldsymbol{f}_{i\boldsymbol{k}\to i'\boldsymbol{k}'}(\boldsymbol{q}) \equiv \int \frac{d^3 l}{(2\pi)^3} \, \psi_{i\boldsymbol{k}}(\boldsymbol{l}) \, \psi^*_{i'\boldsymbol{k}'}(\boldsymbol{l}+\boldsymbol{q}) \, \frac{\boldsymbol{l}}{m_e}$$

$$\mathcal{F}_{ab}(\boldsymbol{q}, \boldsymbol{v}) \equiv \frac{1}{2} \sum_{\chi \text{ sp.}} F_a^*(\boldsymbol{q}, \boldsymbol{v}) \, F_b(\boldsymbol{q}, \boldsymbol{v}) \,, \qquad \mathcal{J}_{ab}(\boldsymbol{q}) \equiv \frac{1}{2} \sum_{e^- \text{ sp.}} J_a^{0*}(\boldsymbol{q}) \, J_b^0(\boldsymbol{q}) \,,$$

$$\boldsymbol{\mathcal{J}}_{ab}(\boldsymbol{q}) \equiv \frac{1}{2} \sum_{e^- \text{ sp.}} \boldsymbol{J}_a^{1*}(\boldsymbol{q}) \, J_b^0(\boldsymbol{q}) \,, \qquad \hat{\mathcal{J}}_{ab}(\boldsymbol{q}) \equiv \frac{1}{2} \sum_{e^- \text{ sp.}} \boldsymbol{J}_a^{1*}(\boldsymbol{q}) \, \boldsymbol{J}_b^1(\boldsymbol{q})$$

no dot product here!

Crystal response functions

Catena et al., 2105.02233

- interaction rate per dark particle

$$\Gamma(\boldsymbol{v}_\chi) \sim \int \frac{d^3q}{(2\pi)^3} \overbrace{\sum_{ii'} \int \frac{d^3k}{(2\pi)^3} \frac{d^3k'}{(2\pi)^3}}^{\text{sum over electronic states}} \underbrace{|\widetilde{\mathcal{M}}_{ik\to i'k'}|^2 \, \delta(\text{cons.})}$$

- total interaction rate

$$\Gamma = \frac{-i}{16 m_e^2 m_\chi^2} \int \frac{d^3q}{(2\pi)^3} d^3v \, \boldsymbol{\rho}(\boldsymbol{v}_\chi) \overbrace{\sum_{ab} \mathcal{F}_{ab}(\boldsymbol{q}, \boldsymbol{v}_\chi) \, c^i_{ab}(\boldsymbol{q}, \omega_{v,q}) \, W_i(\omega, q)}$$

where

$$W_i(\omega, q) = (4\pi)^2 \, \omega \, V_{\text{cell}} \sum_{ii'} \int \frac{d^3k}{(2\pi)^3} \frac{d^3k'}{(2\pi)^3} \delta(\text{cons.}) \times \begin{cases} |f_{ik\to i'k'}|^2 \\ \Re\left(\frac{\boldsymbol{q}}{m_e} \cdot \boldsymbol{f}^*_{ik\to i'k'} f_{ik\to i'k'}\right) \\ |\boldsymbol{f}_{ik\to i'k'}|^2 \\ \left|\frac{\boldsymbol{q}}{m_e} \cdot \boldsymbol{f}_{ik\to i'k'}\right|^2 \end{cases}$$

Parametrization of the crystal response functions

- Mermin parametrization of the dielectric function
  Mermin, DOI:10.1103/PhysRevB.1.2362
  Vos & Grande, DOI:10.1016/j.cpc.2025.109657 ← nice review and num. implementation

$$\varepsilon_M(\omega, q\,;\omega_p, \gamma) \equiv 1 + \frac{\left(1 + i\frac{\gamma}{\omega}\right)\varepsilon_L(\omega + i\gamma, q\,;\omega_p)}{1 + i\frac{\gamma}{\omega}\frac{\varepsilon_L(\omega+i\gamma, q\,;\omega_p)}{\varepsilon_L(0\,;\omega_p)}}$$

where $$\varepsilon_L(\omega, q\,;\omega_p) = \frac{3}{2}\frac{m_e^2}{k_F^2}\frac{\omega_p^2}{q^2}\left(1 + \frac{g(z+u) + g(z-u)}{4z}\right)$$

$$g(x) = (1 - x^2)\ln\frac{x+1}{x-1}\,, \quad u = \frac{m_e}{k_F}\frac{\omega}{q}\,, \quad z = \frac{1}{2}\frac{q}{k_F}\,, \quad k_F = \left(\frac{3\pi}{4\alpha}m_e\omega_p^2\right)^{1/3}$$

- dielectric function ε vs. W_1

$$W_1(\omega, q) = \frac{\omega V_c}{\pi^2}U(q)^{-1}\,\Im\left[\frac{\varepsilon(\omega, q) - 1}{1 + G(q)(\varepsilon(\omega, q) - 1)}\right], \quad U(q) = \frac{4\pi\alpha}{q^2}\,, \quad 0 < G(q) < \frac{1}{2}$$

- hence, we heuristically approximate:

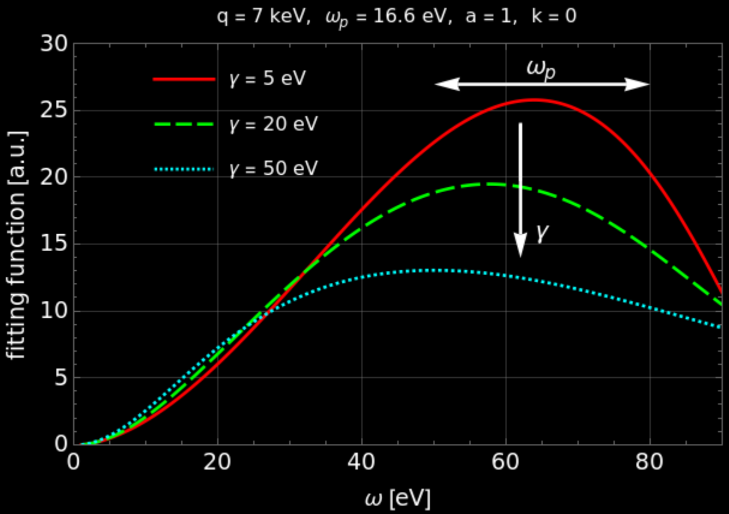
$$W_i(\omega, q) \approx a_i\left(\frac{q}{m_e}\right)^{k_i}\frac{\omega V_c}{\pi^2}U(q)^{-1}\,\Im\,\frac{\varepsilon_M(\omega, q\,;\omega_{pi}, \gamma_i) - 1}{1 + G(q)(\varepsilon_M(\omega, q\,;\omega_{pi}, \gamma_i) - 1)}$$

for some parameters a_i, k_i, ω_{pi}, γ_i

Shape of the fitting function

$$W_i(\omega, q) \approx a_i \left(\frac{q}{m_e}\right)^{k_i} \frac{\omega V_c}{\pi^2} U(q)^{-1} \Im \frac{\varepsilon_M(\omega, q\,;\,\omega_{pi}, \gamma_i) - 1}{1 + G(q)(\varepsilon_M(\omega, q\,;\,\omega_{pi}, \gamma_i) - 1)}$$

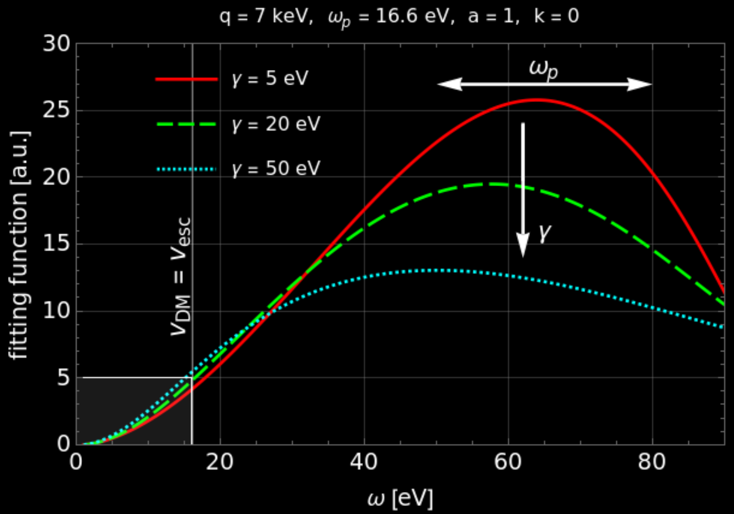
for some a_i, k_i, ω_{pi}, γ_i



Shape of the fitting function

$$W_i(\omega, q) \approx a_i \left(\frac{q}{m_e}\right)^{k_i} \frac{\omega V_c}{\pi^2} U(q)^{-1} \Im \frac{\varepsilon_M(\omega, q; \omega_{pi}, \gamma_i) - 1}{1 + G(q)(\varepsilon_M(\omega, q; \omega_{pi}, \gamma_i) - 1)}$$

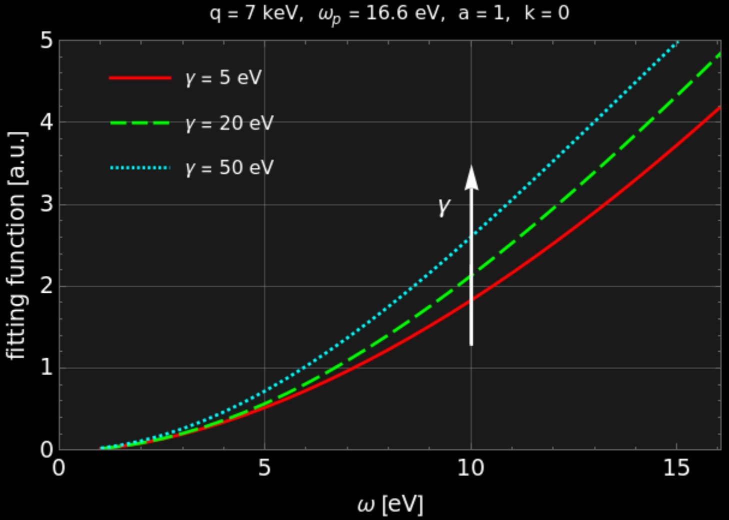
for some a_i, k_i, ω_{pi}, γ_i



Shape of the fitting function

$$W_i(\omega,q) \approx a_i \left(\frac{q}{m_e}\right)^{k_i} \frac{\omega V_c}{\pi^2} U(q)^{-1} \Im \frac{\varepsilon_M(\omega,q\,;\,\omega_{pi},\gamma_i)-1}{1+G(q)(\varepsilon_M(\omega,q\,;\,\omega_{pi},\gamma_i)-1)}$$

for some a_i, k_i, ω_{pi}, γ_i



Procedure

1. Fit the Mermin-inspired parametrization of the material's W_i's
   numerical data from codes darkELF (Knapen et al.) and DarkART (Emken)
2. Check the quality of the fit by calculating the interaction rate
3. Find values of ω_{pi}, γ_i maximizing the interaction rate
4. Compare the maximizing values to the fits

- effective models of DM-e^- interactions:
  - anapole: $\mathcal{L}_{\text{int}} = \frac{g}{2\,\Lambda^2}\,\bar{\chi}\gamma^\mu\gamma_5\chi\,\partial^\nu F_{\mu\nu}$
  - electric dipole: $\mathcal{L}_{\text{int}} = \frac{g}{\Lambda}\,i\,\bar{\chi}\sigma^{\mu\nu}\gamma_5\chi\,F_{\mu\nu}$
  - magnetic dipole: $\mathcal{L}_{\text{int}} = \frac{g}{\Lambda}\,\bar{\chi}\sigma^{\mu\nu}\chi\,F_{\mu\nu}$
- materials: Si, Ge

Results: fitted parameters

$$W_i(\omega, q) \approx a_i \left(\frac{q}{m_e}\right)^{k_i} \frac{\omega V_c}{\pi^2} U(q)^{-1} \Im\left[\frac{\varepsilon_M(\omega, q\,;\omega_{pi}, \gamma_i) - 1}{1 + G(q)(\varepsilon_M(\omega, q\,;\omega_{pi}, \gamma_i) - 1)}\right]$$

material: Ge

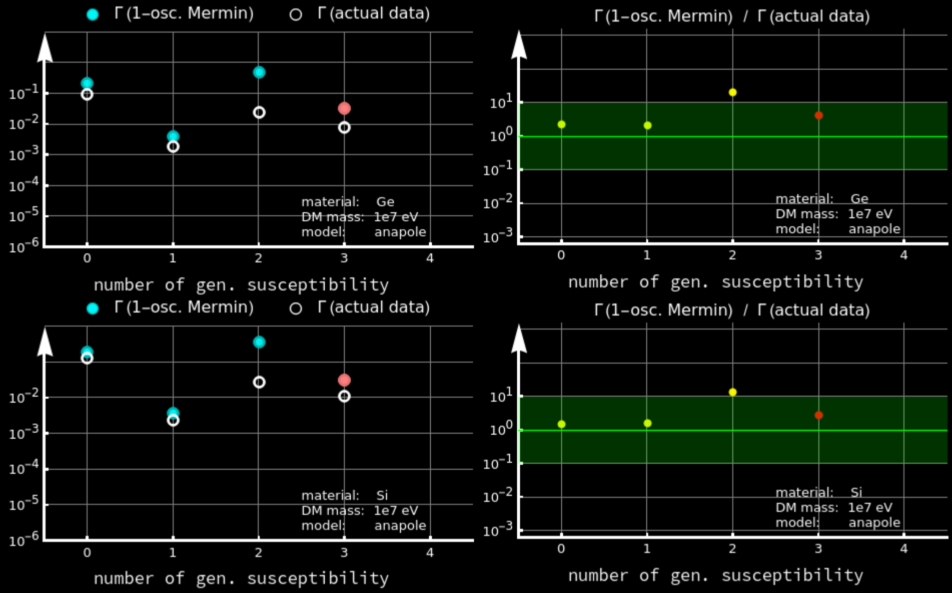
i	k	a	ω_p	γ
1	0	1	16	24.2
2	1	3.0×10^{-3}	15.7	3.2
3	−1	2.9×10^{-7}	70.8	139.4
4	1	1.5×10^{-7}	78.3	150.

material: Si

i	k	a	ω_p	γ
1	0	1	16.6	15.4
2	1	2.7×10^{-3}	17.2	3.4
3	−1	3.8×10^{-7}	28.3	46.9
4	1	8.3×10^{-8}	40.3	57.6

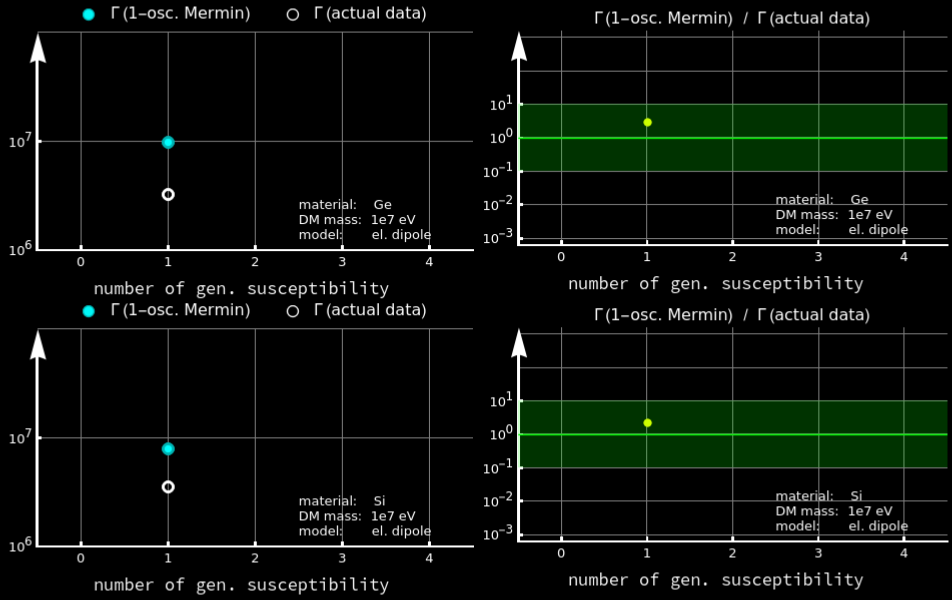
Results: fit quality

anapole model



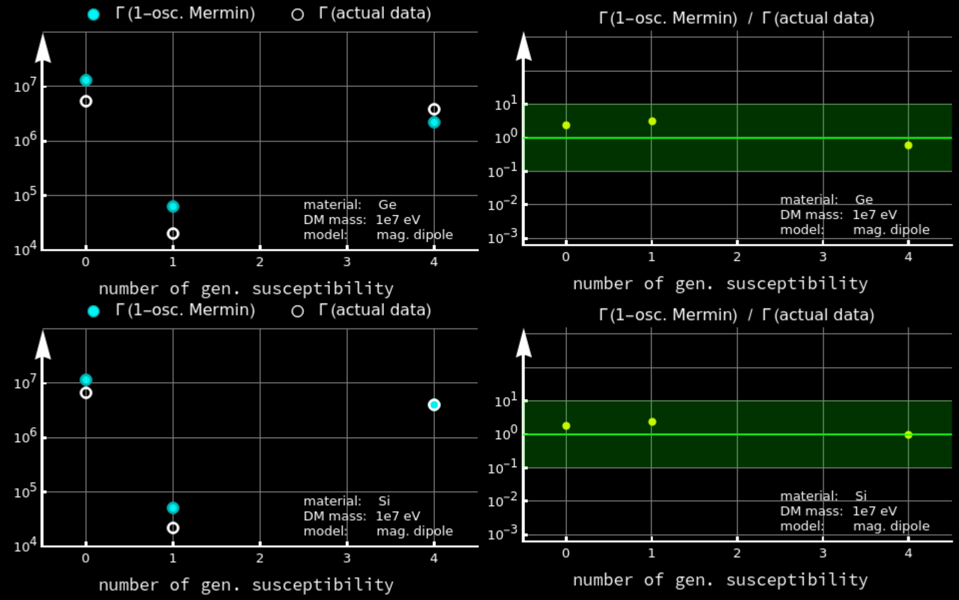
Results: fit quality

electric dipole model



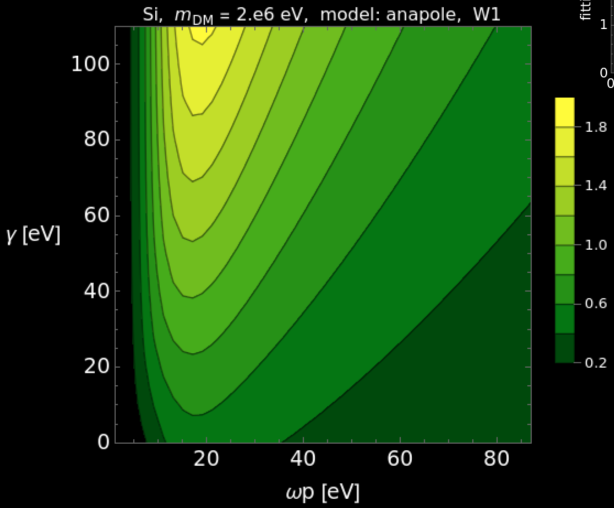
Results: fit quality

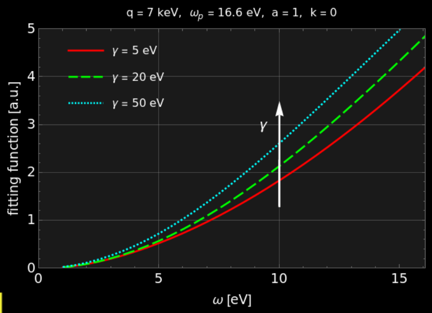
magnetic dipole model



Results: optimal parameters

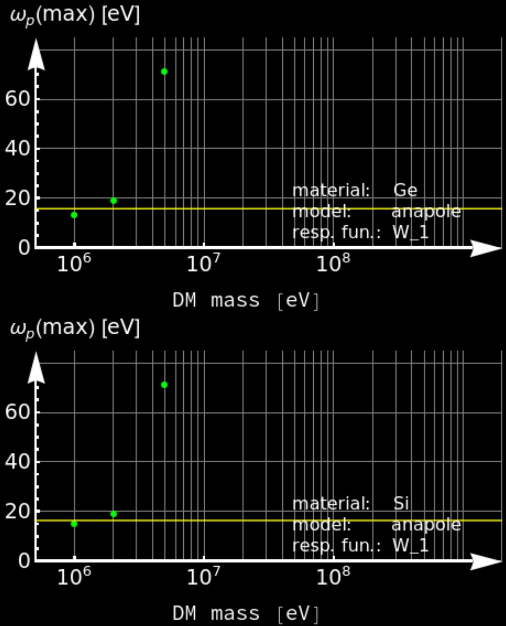
example: Si, $m_{\mathrm{DM}} = 2$ MeV, anapole DM model, W1)





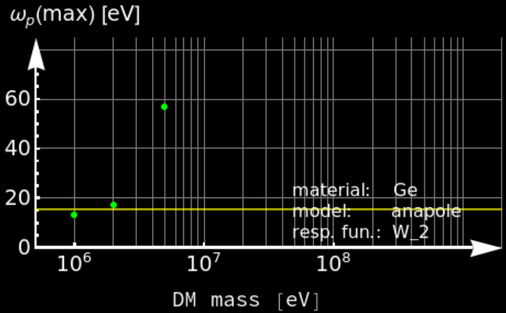
Results: optimal parameters

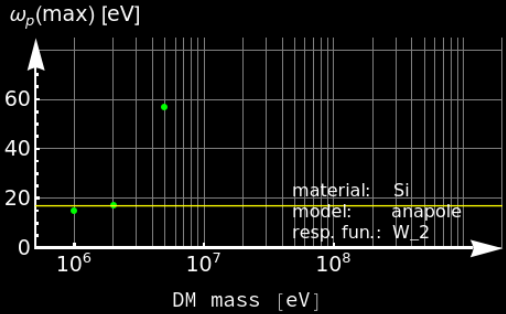
anapole model, W_1



Results: optimal parameters

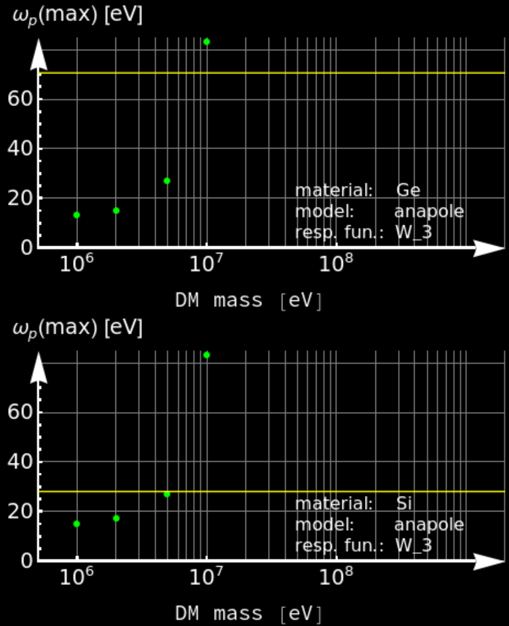
anapole model, W_2





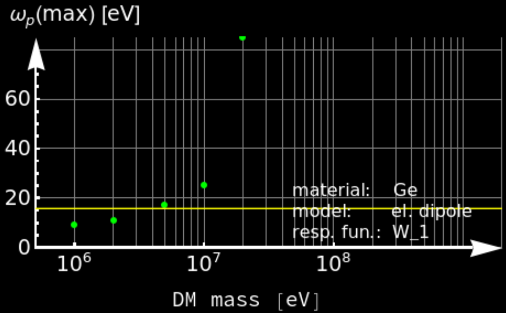
Results: optimal parameters

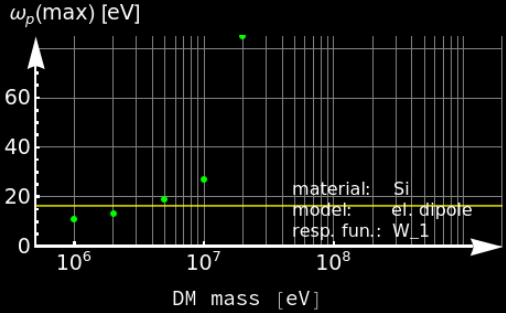
anapole model, W_3



Results: optimal parameters

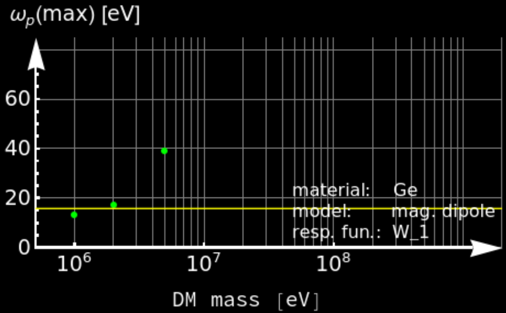
electric dipole model, W_1

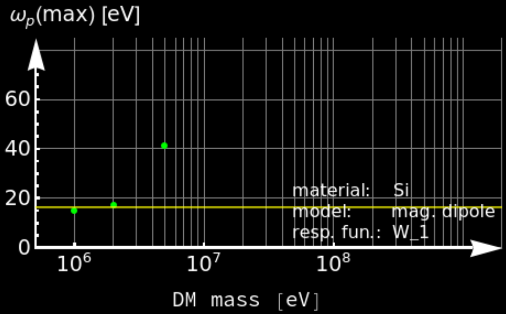




Results: optimal parameters

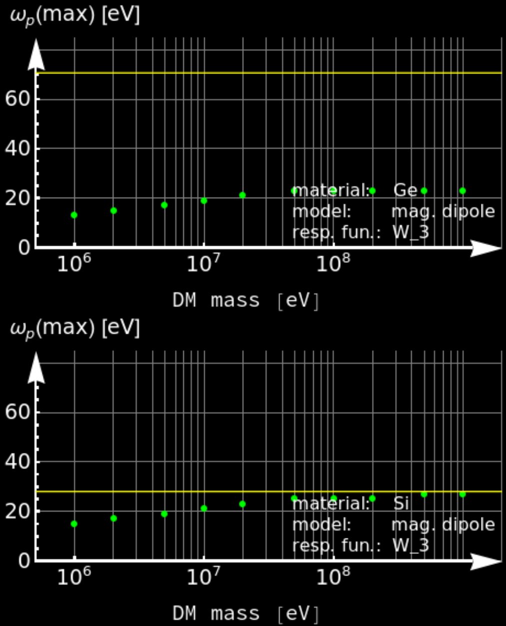
magnetic dipole model, W_1





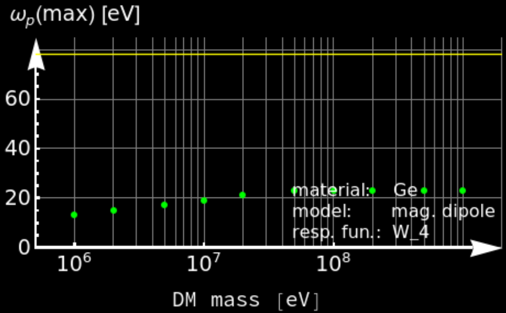
Results: optimal parameters

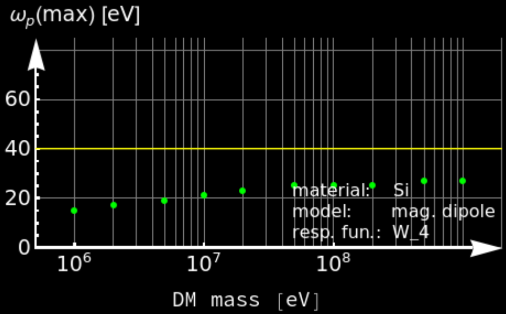
magnetic dipole model, W_3



Results: optimal parameters

magnetic dipole model, W_4





Summary

- **effective approach** to non-relativistic DM-e^- interactions
  - small set of operators in the leading order
- **linear response theory**

$$[\text{interaction rate}] = \int [\text{DM model}] \times [\text{material response of the detector}]$$

- material response $\rightarrow$ crystal response functions $\boldsymbol{W}_i(\omega, q)$
- $\boldsymbol{W}_i$'s approximated using **Mermin parametrization**
- preliminary results:
  - 1-osc. approximation of W_i's works **quite well**
  - important role of parameter γ
  - fitted $\boldsymbol{\omega}_p$ often **close to optimal** for $1\,\text{MeV} \lesssim \boldsymbol{m}_{\text{DM}} \lesssim 10\,\text{MeV}$

WORK IN PROGRESS...

Summary

thank you!

- **effective approach** to non-relativistic DM-e^- interactions
  - small set of operators in the leading order
- **linear response theory**

$$[\text{interaction rate}] = \int [\text{DM model}] \times [\text{material response of the detector}]$$

- material response → crystal response functions $\boldsymbol{W}_i(\omega, q)$
- $\boldsymbol{W}_i$'s approximated using **Mermin parametrization**
- preliminary results:
  - 1-osc. approximation of W_i's works **quite well**
  - important role of parameter γ
  - fitted $\boldsymbol{\omega}_p$ often **close to optimal** for $1\,\text{MeV} \lesssim \boldsymbol{m}_{\text{DM}} \lesssim 10\,\text{MeV}$

WORK IN PROGRESS...