# Constraining Non-Perturbative Parameters in Inclusive $\Lambda_b$ Decays

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Heavy Quark Expansion (HQE) – for studying weak inclusive decays of heavy hadrons (decays, lifetimes, $V_{ub}$- extraction...)

- HQE – systematic expansion in $1/m_Q$ and $\alpha_s$; for $\Lambda_b$ up to dimension-six:

$$\Gamma(\Lambda_b) = \Gamma_0 \left[ C_3 + C_\pi \frac{\hat{\mu}_\pi^2}{m_Q^2} + C_D \frac{\hat{\rho}_D^3}{m_Q^3} + \dots + 16\pi^2 \left( \sum_{i,q} C_{6,i}^q \frac{\langle \mathcal{O}_i^q \rangle}{m_Q^3} + \dots \right) \right]$$

- Wilson coefficients $C_i$ multiply non-perturbative matrix elements (MEs):

$$\hat{\mu}_\pi^2 = -\frac{1}{2M_{\Lambda_b}} \langle \Lambda_b | \bar{b}_v (iD_\mu)(iD^\mu) b_v | \Lambda_b \rangle \qquad \text{kinetic operator}$$

$$\hat{\rho}_D^3 = \frac{1}{2M_{\Lambda_b}} \langle \Lambda_b | \bar{b}_v (iD_\mu)(iv \cdot D)(iD^\mu) b_v | \Lambda_b \rangle \qquad \text{Darwin operator}$$

$$\langle O_i \rangle = \frac{1}{2M_{\Lambda_b}} \langle \Lambda_b | (\bar{b}_v \Gamma_A q)(\bar{q} \Gamma_B b_v) | \Lambda_b \rangle \qquad \text{four} - \text{quark operator (shematic)}$$

- **accurate predictions require good knowledge of MEs** ($C_D$ appears to be unexpectedly large)

- experimental data for $\Lambda_b$ decays (unlike for $B$-mesons) – insufficient to determine hadronic parameters through fits
- by applying the equation of motion for gluon fields, $\hat{\rho}_D^3$ is related to four-quark MEs:

$$2M_{\Lambda_b}\,\hat{\rho}_D^3 = g_s^2 \sum_{q=u,d,s} \left\langle \Lambda_b \middle| -\tfrac{1}{8}\,\mathcal{O}_1^q + \tfrac{1}{24}\,\tilde{\mathcal{O}}_1^q + \tfrac{1}{4}\,\mathcal{O}_2^q - \tfrac{1}{12}\,\tilde{\mathcal{O}}_2^q \middle| \Lambda_b \right\rangle + \mathcal{O}(1/m_b)$$

- four-quark MEs for $\Lambda_b$ carry large uncertainties – estimates within nonrelativistic constituent quark model (NRCQM) from heavy-baryon spectroscopy see e.g. [Gratrex, Lenz, BM, Nisandzic, Piscopo, Rusov, 2301.07698].
- effect of Darwin term $\hat{\rho}_D^3$ ($\sim$ large $C_D$) could be easily comparable to that of spectator four-quark contributions in $\Gamma(\Lambda_b)$ - same $1/m_b^3$ suppression

## Plan:

- use lattice QCD results for exclusive form factors in $\Lambda_b \to \Lambda_c^{1/2^+}$, $\Lambda_c^{1/2^-}$, and $\Lambda_c^{3/2^-}$

  [Detmold, Lehner, Meinel, 1503.01421; Meinel, Rendon, 2107.13140, 2103.08775]

  to constrain $\hat{\mu}_\pi^2$ and $\hat{\rho}_D^3$ via sum rules near zero recoil in inclusive semileptonic decays $\Lambda_b \to X_c e\bar{\nu}$.

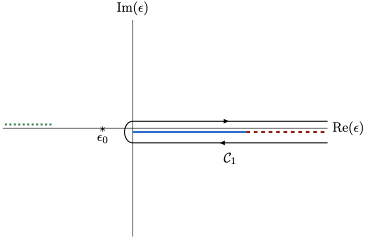
SMALL VELOCITY SUM RULES (SVSR): [Bigi, Shifman, Uraltsev, Vainshtein, 9405410]

**relating the inclusive decay $\Lambda_b \to X_c e \bar{\nu}_e$ to a sum of exclusive channels**

starting point: time-ordered product of weak $J = \bar{c}\,\Gamma b$ currents:

$$T = \frac{i}{4m_{\Lambda_b}} a^{\alpha\beta} \int d^4x\, e^{-iq\cdot x} \langle \Lambda_b(v,s)|\, T\{J_\alpha^\dagger(x) J_\beta(0)\}\, |\Lambda_b(v,s)\rangle$$

- moments of $T$ may be compared making use of
  - an OPE on the inclusive side $\sim$ Im $h_{\alpha\beta}$
  - inserting a complete set of states of $\Lambda_c$ states on the exclusive side
- the dispersion relation connects both sides at the semileptonic (SL) cut $(0,\, m_{\Lambda_b} - E_{\Lambda_c})$




$\epsilon = E_{X_c} - E_{\Lambda_c}$: excitation energy along SL cut

- we compute $n^{th}$-moment of various Dirac structures $\Gamma_i = \{V, A\}$ and for different spatial or temporal components $a^{\alpha\beta}$:

$$
\begin{aligned}
\mathcal{I}_a^{\Gamma_1\Gamma_2(n)} &= \frac{1}{\pi}\int_0^{\Delta} d\epsilon\, \epsilon^n\, a^{\alpha\beta}\, \mathrm{Im}\, h_{\alpha\beta}^{\Gamma_1\Gamma_2\,\text{(OPE)}} \\
&= \frac{1}{8m_{\Lambda_b}} \sum_{X_c,s,s'} \frac{(E_{X_c}-E_{\Lambda_c})^n}{E_{X_c}} a^{\alpha\beta} \langle \Lambda_b(s)|J_\alpha^{\dagger\,\Gamma_1}|X_c(s')\rangle\langle X_c(s')|J_\beta^{\Gamma_2}|\Lambda_b(s)\rangle + \ldots
\end{aligned}
$$

- the sum is over $s_l^{\pi_l} = 1^-$ radial excitations with excitation energies up to a scale $\Delta$
- we'll stay at and close to zero recoil ($|\vec{q}| = 0$) and explicitly include only the states $\Lambda_c^{1/2^+}$, $\Lambda_c^{1/2^-}$, and $\Lambda_c^{3/2^-}$ into the hadronic sum and take $\Delta = 0.75$ GeV.
- positivity of omitted hadronic contributions implies $\mathcal{I}_{ii}^{\Gamma\Gamma\,(n),\,\text{OPE}} > \mathcal{I}_{ii}^{\Gamma\Gamma\,(n),\,\text{had}}$

For zeroth moments (with $\Gamma = A, V$) at zero recoil:

$$\mathcal{I}_{kk}^{\Gamma\Gamma\,(0),\,\text{OPE}}\Big|_{\text{0-recoil}} > \mathcal{I}_{kk}^{\Gamma\Gamma\,(0),\,\text{had}}\Big|_{\text{0-recoil}} \qquad \mathcal{I}_{00}^{\Gamma\Gamma\,(0),\,\text{OPE}}\Big|_{\text{0-recoil}} > \mathcal{I}_{00}^{\Gamma\Gamma\,(0),\,\text{had}}\Big|_{\text{0-recoil}}$$

OPE-side expressions:

$$\mathcal{I}_{kk}^{VV\,(0),\,\text{OPE}}\Big|_{\text{0-recoil}} = \frac{\hat{\mu}_\pi^2}{4}\left(\frac{3}{m_c^2} + \frac{3}{m_b^2} - \frac{2}{m_b m_c}\right) - \frac{\hat{\rho}_D^3}{4}\left(\frac{3}{m_c^3} - \frac{3}{m_b^3} - \frac{1}{m_b m_c^2} + \frac{1}{m_b^2 m_c}\right)$$

$$\mathcal{I}_{00}^{VV\,(0),\,\text{OPE}}\Big|_{\text{0-recoil}} = 1 + O(\alpha_s) - \frac{\hat{\mu}_\pi^2}{4}\left(\frac{1}{m_c} - \frac{1}{m_b}\right)^2 - \frac{\hat{\rho}_D^3}{4}\left(\frac{1}{m_c} - \frac{1}{m_b}\right)^2\left(\frac{1}{m_c} + \frac{1}{m_b}\right)$$

$$\mathcal{I}_{kk}^{AA\,(0),\,\text{OPE}}\Big|_{\text{0-recoil}} = 3\left[1 + O(\alpha_s) - \frac{\hat{\mu}_\pi^2}{4}\left(\frac{1}{m_c^2} + \frac{1}{m_b^2} + \frac{2}{3m_b m_c}\right) - \frac{\hat{\rho}_D^3}{4}\left(\frac{1}{m_c^3} + \frac{1}{3m_b m_c^2} + \frac{1}{m_b^3} + \frac{1}{3m_b^2 m_c}\right)\right]$$

$$\mathcal{I}_{00}^{AA\,(0),\,\text{OPE}}\Big|_{\text{0-recoil}} = \frac{\hat{\mu}_\pi^2}{4}\left(\frac{1}{m_b} + \frac{1}{m_c}\right)^2 - \frac{\hat{\rho}_D^3}{4}\left(\frac{1}{m_c^2} - \frac{1}{m_b^2}\right)\left(\frac{1}{m_c} + \frac{1}{m_b}\right)$$

## Hadronic-side expressions:

- Hadronic contributions to the right-hand side — $\mathcal{I}_{\alpha\beta}^{\Gamma\Gamma\,(n),\,\text{had}} \sim |\text{form factors}|^2$
- form factors near zero recoil for $\Lambda_b \to \Lambda_c^{1/2^+}$, $\Lambda_c^{1/2^-}$, and $\Lambda_c^{3/2^-}$ — from lattice QCD

[Detmold, Lehner, Meinel, 1503.01421; Meinel, Rendon, 2107.13140, 2103.08775]:

$$f_i = F^{f_i} + (w-1)A^{f_i}$$

non-vanishing contributions:

$w = v \cdot v'$ and $(w-1) \simeq \vec{q}^{\,2}/(2M_{\Lambda_c}^2)$

$$\mathcal{I}_{kk}^{VV\,(0),\,\text{had}}\Big|_{\text{0-recoil}} = 3\left|f_+^{(1/2^-)}(w=1)\right|^2 + 2\left|f_{\perp'}^{(3/2^-)}(w=1)\right|^2 = 0.094(8)\,,$$

$$\mathcal{I}_{00}^{VV\,(0),\,\text{had}}\Big|_{\text{0-recoil}} = \left|f_0^{(1/2^+)}(w=1)\right|^2 = 0.97(3).$$

$$\mathcal{I}_{kk}^{AA\,(0),\,\text{had}}\Big|_{\text{0-recoil}} = 3\left|g_+^{(1/2^+)}(w=1)\right|^2 = 2.45(6)\,,$$

$$\mathcal{I}_{00}^{AA\,(0),\,\text{had}}\Big|_{\text{0-recoil}} = \left|g_0^{(1/2^-)}(w=1)\right|^2 = 0.048(4)\,.$$

## Higher moments

- For higher moments we use combinations of zeroth, second, and third moments:

$$Z_n = \frac{\mathcal{I}_{kk}^{AA(3)}}{E_n - E_0} - \mathcal{I}_{kk}^{AA(2)}, \qquad Y_n = \frac{\mathcal{I}_{kk}^{AA(2)}}{(E_n - E_0)^2} - \mathcal{I}_{kk}^{AA(0)},$$

with $n = 0$ for the ground state $\Lambda_c$, $n = 1$ for $\Lambda_c^{1/2^-}$, and $n = 2$ for $\Lambda_c^{3/2^-}$.

## The combination $Z_n(Y_n)$ subtracts the contribution of the $n$th excited state

- some moments vanish at zero recoil, other give loose constraints $(Y_n)$;

  the most relevant constraint comes from $Z_2$ as

$$\left.\frac{dZ_2^{\mathrm{OPE}}}{d|\vec{q}|^2}\right|_{|\vec{q}|^2=0} > \frac{d}{d|\vec{q}|^2}\left[(E_{\Lambda_c^{1/2^-}} - E_{\Lambda_c})^2\left(\frac{E_{\Lambda_c^{1/2^-}} - E_{\Lambda_c}}{E_{\Lambda_c^{3/2^-}} - E_{\Lambda_c}} - 1\right)\frac{\mathcal{F}^{AA}_{kk,\Lambda_c^{1/2^-}}}{8m_{\Lambda_b}\, E_{\Lambda_c^{1/2^-}}}\right]_{|\vec{q}|^2=0}.$$

Relevant OPE expressions up to $\mathcal{O}[\vec{q}^2(\Lambda^3_{\mathsf{QCD}}, \alpha_s)]$:

$$\mathcal{I}_{kk}^{AA(0)\,\mathsf{OPE}} = \mathcal{I}_{kk}^{AA(0)\,\mathsf{OPE}}\Big|_{\vec{q}=0} + \frac{|\vec{q}|^2}{2m_c^2}\left[-\frac{3}{2} - \frac{8\alpha_s(\mu_s)}{9\pi}\left(5 - 3\log\frac{4\Delta^2}{\mu_s^2}\right)\right.$$
$$\left. + \mu_\pi^2\left(\frac{15}{4m_c^2} + \frac{3}{4m_b^2} + \frac{5}{6m_b m_c}\right) + \rho_D^3\left(\frac{11}{4m_c^3} + \frac{3}{4m_b m_c^2} + \frac{5}{12m_b^2 m_c} + \frac{3}{4m_b^3}\right)\right],$$

$$\mathcal{I}_{kk}^{AA(2)\,\mathsf{OPE}} = \frac{|\vec{q}|^2}{2m_c^2}\left[2\mu_\pi^2 + \frac{8\alpha_s}{3\pi}\Delta^2 - \rho_D^3\left(\frac{1}{m_c} + \frac{1}{3m_b}\right)\right],$$
$$\mathcal{I}_{kk}^{AA(3)} = \frac{|\vec{q}|^2}{2m_c^2}\left(2\rho_D^3 + \frac{16\alpha_s}{9\pi}\Delta^3\right).$$

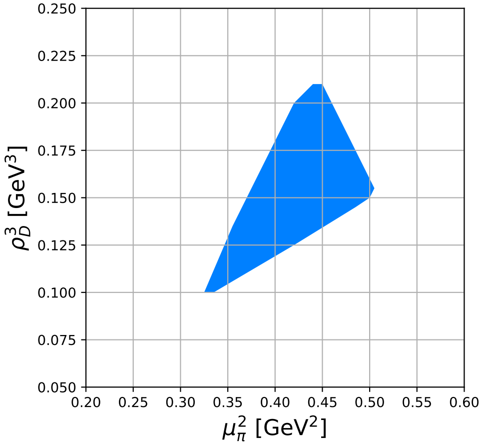
Adopt the kinetic scheme [Bigi, Shifman, Vainshtain, 9704245] , redefine heavy-quark masses and matrix elements accordingly [Fael, Schönwald, Steinhauser, 2011.11655] . Set $\Delta = \mu = 0.75\,\mathrm{GeV}$.

- Constrain $\hat{\mu}_\pi^2$ and $\hat{\rho}_D^3$ via sum-rule inequalities, using lattice inputs with uncertainties
- For each point $(\hat{\mu}_\pi^2, \hat{\rho}_D^3)$, evaluate the joint probability that the lattice values of moments $\mathbf{M}^{\rm had} \equiv \left( \frac{dY_3}{d|\vec{q}|^2}\Big|_{|\vec{q}|^2=0}, \frac{dZ_3^{\rm had}}{d|\vec{q}|^2}\Big|_{|\vec{q}|^2=0}, \mathcal{I}_{kk}^{VV\,(0),\rm had}, \mathcal{I}_{kk}^{AA\,(0),\rm had}, \mathcal{I}_{00}^{VV\,(0),\rm had}, \mathcal{I}_{00}^{AA\,(0),\rm had} \right)$ lie below the corresponding OPE values $\mathbf{V}(\hat{\mu}_\pi^2, \hat{\rho}_D^3)$:

$$\mathrm{CDF}(\hat{\mu}_\pi^2, \hat{\rho}_D^3) = \int_{-\infty}^{\mathbf{V}(\hat{\mu}_\pi^2, \hat{\rho}_D^3)} p\big(\mathbf{M}^{\rm had}, \mathcal{C}^{\rm had}\big)\, d^n \mathbf{M}^{\rm had}$$

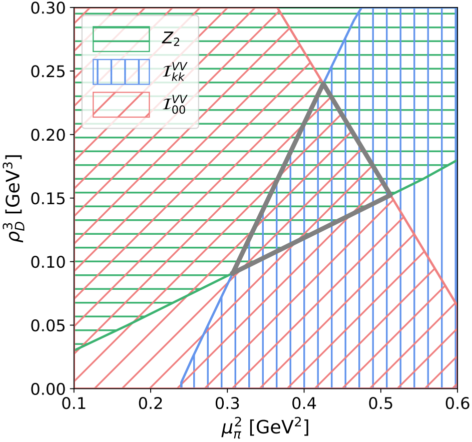
- We define the allowed region as the set of the points for which the probability exceeds a chosen threshold $P_{\rm th}$ =50%.

similar analysis in [Mannel, van Dyk, 1506.08780] - checking for the saturation in the two zeroth moments in inclusive $B$-meson decays




The value of $\hat{\rho}_D^3$ from the low-scale estimate $\hat{\rho}^3_{D\,\mathrm{kin}} \sim 0.07\,$GeV$^3$ based on the NRCQM is slightly below the SVSR region.

The SVSR values of $\mu_\pi^2$ are consistent with the extraction of the parameter from spectroscopy $\mu_\pi^2(\Lambda_b) = 0.43(4)$ GeV$^2$ as $(\overline{M}_D - M_{\Lambda_c}) - (\overline{M}_B - M_{\Lambda_b}) \simeq \left(\frac{1}{2m_c} - \frac{1}{2m_b}\right)(\mu_\pi^2(B) - \mu_\pi^2(\Lambda_b)$




Illustrations of the regions where the OPE expressions exceed the central lattice values **for the three most constraining observables**

- Applied zero-recoil sum rules to constrain the nonperturbative parameters $\hat{\mu}_\pi^2$ and $\hat{\rho}_D^3$ in inclusive $\Lambda_b$ decays.
- Extracted the allowed region in $(\hat{\mu}_\pi^2, \hat{\rho}_D^3)$ by matching OPE moments to hadronic moments computed from lattice QCD form factors for $\Lambda_b \to \Lambda_c^{(*)}$ transitions.
- Found the allowed region lies in the same ballpark as low-scale NRCQM estimates $(\hat{\rho}_D^3)$ and heavy hadron mass expansions $(\hat{\mu}_\pi^2)$
- the Darwin term enters the decay width with a negative Wilson coefficient - its larger value favoured by our SVSR region would reduce the total decay width of $\Lambda_b$ and thus increase the recently predicted ratio

  $\tau(\Lambda_b)/\tau(B_d) = 0.955(14)$ [Gratrex, Lenz, BM, Nisandzic, Piscopo, Rusov, 2301.07698]

  shifting it even closer to the measured value.
- Future lattice input and further exploration of excited state contributions may refine these constraints.

## Near saturation

- For $(\hat{\mu}_\pi^2, \hat{\rho}_D^3)$ values consistent with other constraints, the OPE predictions for $\mathcal{I}_{00}^{VV}$ and $\mathcal{I}_{kk}^{AA}$ do not significantly exceed the central lattice values — indicating near saturation by the ground-state $\Lambda_c$.
- This is consistent with the findings of [Mannel, van Dyk, 1506.08780] who used kinetic term estimates from spectroscopy and Darwin term values from inclusive $B$ decays to check for the saturation in the two zeroth moments
- We adopt the threshold $P_{\mathrm{th}} = 50\%$, primarily driven by $\mathcal{I}_{00}^{VV,\text{had}}$
- At present, near-saturation does not indicate a discrepancy—we lack information on higher-state contributions.
- Nonetheless, independent lattice determinations of the $\Lambda_b \to \Lambda_c$ form factors would be valuable.

## $\mu_\pi^2(\Lambda_b)$ from spectroscopy

- The kinetic term can also be estimated using HQE for heavy hadron masses:

$$(\overline{M}_D - M_{\Lambda_c}) - (\overline{M}_B - M_{\Lambda_b}) = \left(\frac{1}{2m_c} - \frac{1}{2m_b}\right)\left(\mu_\pi^2(B) - \mu_\pi^2(\Lambda_b)\right) + \mathcal{O}(1/m_c^2)$$

- Assumptions:
  - $\mu_\pi^2(B) = \mu_\pi^2(D)$, $\mu_\pi^2(\Lambda_b) = \mu_\pi^2(\Lambda_c)$,
  - residual mass differences $\bar{\Lambda}_{B_q} - \bar{\Lambda}_{\mathcal{B}}$ are the same in the $b$ and $c$ sectors
- Using $\mu_\pi^2(B)$ from the fits [Bordone, Capdevila, Gambino, 2107.00604] gives

$$\mu_\pi^2(\Lambda_b) = 0.50(6)\,\text{GeV}^2$$

which lies on the edge of our sum-rule region but remains consistent within the error bar