MAX-PLANCK-INSTITUT FÜR PHYSIK

COSMIC STRINGS AND DOMAIN WALLS OF THE QCD QUARK CONDENSATE WITH AND WITHOUT A HIDDEN AXION

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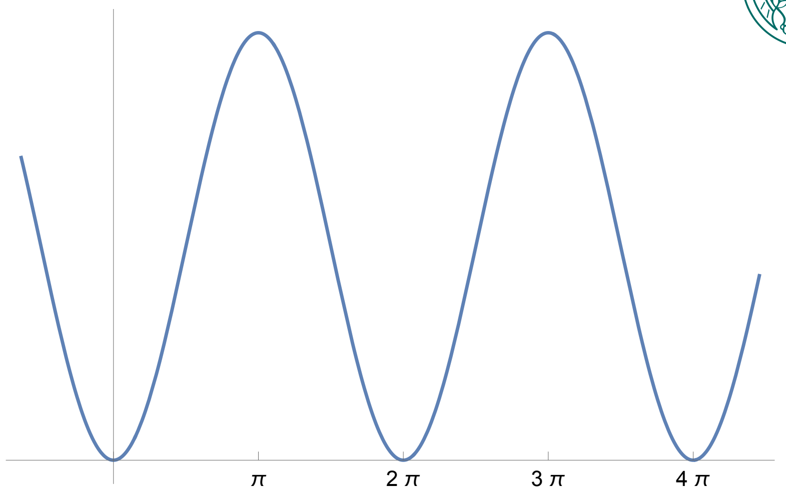
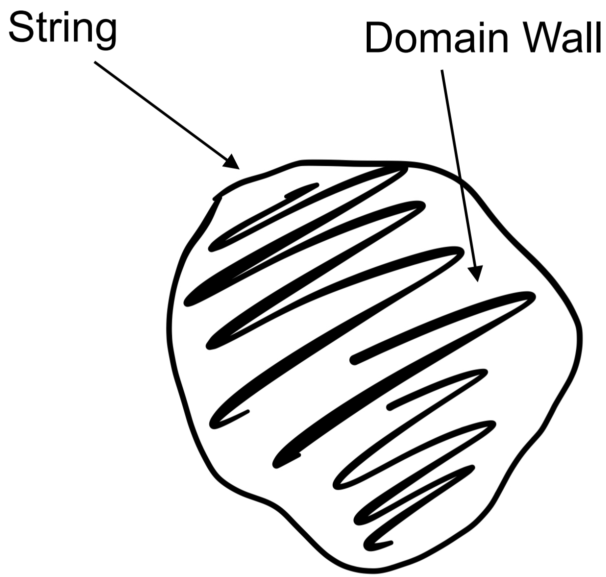
TOPOLOGICAL DEFECTS IN η' AND π^0



- Vacuum of QCD is 2π periodic in the phases of the quark condensate
- Spontaneous breaking of chiral symmetry

  $\Rightarrow$ Topological defects in η', π^0
- 2π-domain walls bounded by strings
- Closed bubbles, membrane bounded by string, …
- Unstable, can punch hole through DW

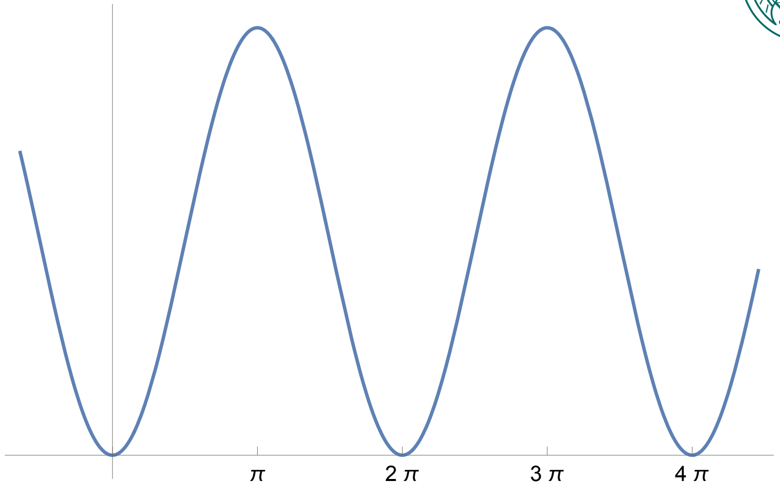


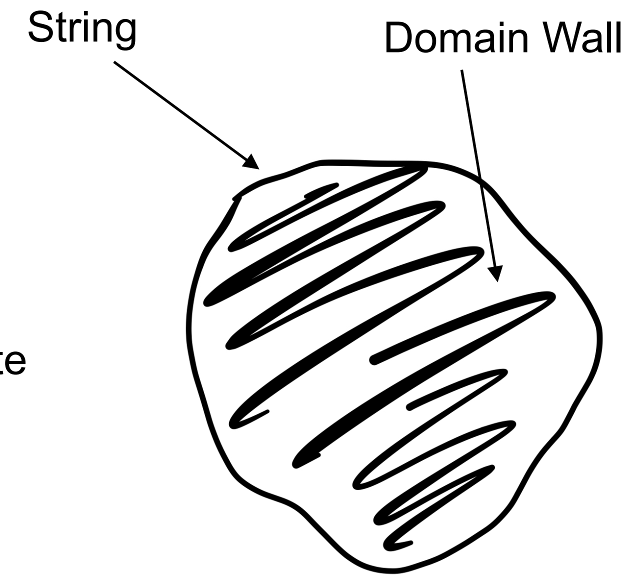
TOPOLOGICAL DEFECTS IN η' AND π^0 WITH HIDDEN AXION

- $\mathscr{L} = -\frac{1}{4}G^a_{\mu\nu}G^{\mu\nu,a} + i\overline{\psi}D_\mu\gamma^\mu\psi - \overline{\psi}_L M_q \psi_R + \theta\frac{g^2}{32\pi^2}G^a_{\mu\nu}\tilde{G}^{\mu\nu,a}$
- CP violation: $\overline{\theta} = \theta + \arg(\det M_q)$
- Strong CP puzzle: Why do we live in sector with tiny or zero $\overline{\theta}$?

  ⇒ Solution: Axion

- Axion strings and walls are accompanied by windings in quark condensate





PECCEI QUINN '77 VS MASSLESS QUARK SOLUTION

Anomalous $U(1)_{PQ}$

Heavy quark $\Psi \rightarrow e^{\mathrm{i}\frac{1}{2}\alpha\gamma_5}\Psi$

PQ field $\langle\Phi\rangle = f_\phi$

Axion: $\theta_\phi = \dfrac{\phi}{\sqrt{2}f_\phi}$ [Weinberg, Wilczek '78]

Instanton induced potential: ['t Hooft '76]

$$V(\theta_\phi) = -\Lambda^4 \cos(\theta_\phi(x) - \bar{\theta})$$

Vacuum angle $\theta_{\text{eff}}(x) = \theta_\phi(x) - \bar{\theta}$ becomes dynamical

Anomalous $U(1)_A$

Massless quark $\psi \rightarrow e^{-\mathrm{i}\frac{1}{2}\alpha\gamma_5}\psi$

Quark condensate $\langle\bar{\psi}\psi\rangle = \Lambda^3$

Axion: $\theta_\eta = \dfrac{\eta'}{\sqrt{2}f_\eta}$

Instanton induced potential: ['t Hooft '76]

$$V(\theta_\eta) = -\Lambda^4 \cos(\theta_\eta(x) - \bar{\theta})$$

Vacuum angle $\theta_{\text{eff}}(x) = \theta_\eta(x) - \bar{\theta}$ becomes dynamical

$\Rightarrow$ η' **is the axion** [Dvali '05]

η' AS POOR-QUALITY AXION

$m_\psi = 0$: η' is the axion

$m_\psi \neq 0$ explicitly breaks $U(1)_A$

$\Rightarrow$ Effective vacuum angle $\theta_{\text{eff}} = \frac{m_\psi}{\Lambda}\bar{\theta}$

$\Rightarrow$ η' is a poor-quality axion [Dvali '05]

$\Rightarrow$ need PQ axion

COUPLED SYSTEM OF η' AND AXION

Massless quark ψ

- Combination of $U(1)_{PQ}$ and previous $U(1)_A$:
- $U(1)_V$: $\Psi \to e^{\mathrm{i}\frac{1}{2}\alpha\gamma_5}\Psi$, $\psi \to e^{-\mathrm{i}\frac{1}{2}\alpha\gamma_5}\psi$ (anomaly free)
- $U(1)_A$: $\Psi \to e^{\mathrm{i}\frac{1}{2}\alpha\gamma_5}\Psi$, $\psi \to e^{\mathrm{i}\frac{1}{2}\alpha\gamma_5}\psi$ (anomalous)
- Potential: $V(\theta_\phi, \theta_\eta) = -\Lambda^4 \cos(\theta_\phi + \theta_\eta - \bar{\theta})$
- Massive $\dfrac{a_\eta}{\sqrt{2}\tilde{f}} = \theta_\phi + \theta_\eta \quad \Rightarrow \quad a_\eta \simeq \eta'$ is the axion
- Massless $a_\phi \simeq \phi$

COUPLED SYSTEM OF η' AND AXION

Massive quark ψ

- Combination of $U(1)_{PQ}$ and previous $U(1)_A$:
- $U(1)_V$: $\Psi \to e^{\mathrm{i}\frac{1}{2}\alpha\gamma_5}\Psi$, $\psi \to e^{-\mathrm{i}\frac{1}{2}\alpha\gamma_5}\psi$ (anomaly free, explicitly broken by m_ψ)
- $U(1)_A$: $\Psi \to e^{\mathrm{i}\frac{1}{2}\alpha\gamma_5}\Psi$, $\psi \to e^{\mathrm{i}\frac{1}{2}\alpha\gamma_5}\psi$ (anomalous)
- Potential: $V(\theta_\phi, \theta_\eta) = -\Lambda^4 \cos(\theta_\phi + \theta_\eta - \bar{\theta}) - m_\psi \Lambda^3 \cos(\theta_\eta)$
- Massive $\dfrac{a_\eta}{\sqrt{2}\tilde{f}} = \theta_\phi + \theta_\eta$
- **Massive $a_\phi \simeq \phi$** is the **axion**

STRINGS AND WALLS OF η' AND AXION

Massless quark ψ

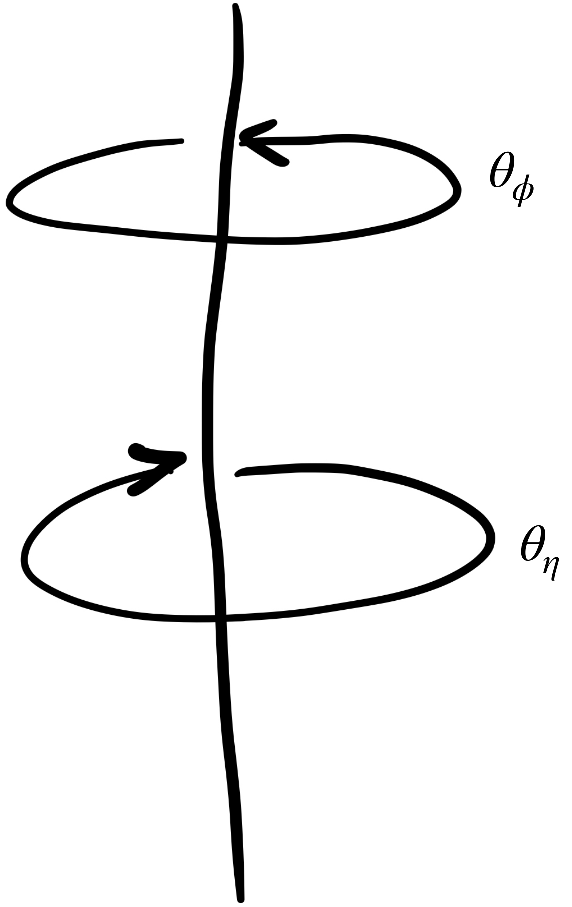
$U(1)_V$ (anomaly free) and $U(1)_A$ (anomalous)

$V = -\Lambda^4 \cos(\theta_\phi + \theta_\eta)$

Lowest energy topological defects:

$\Rightarrow$ String of $U(1)_V$: $\theta_\phi = -\theta_\eta = n\varphi$

$\Rightarrow$ String-wall of combination of $U(1)_V \times U(1)_A$: $\theta_\eta = n\varphi$



STRINGS AND WALLS OF η' AND AXION

Massive quark ψ

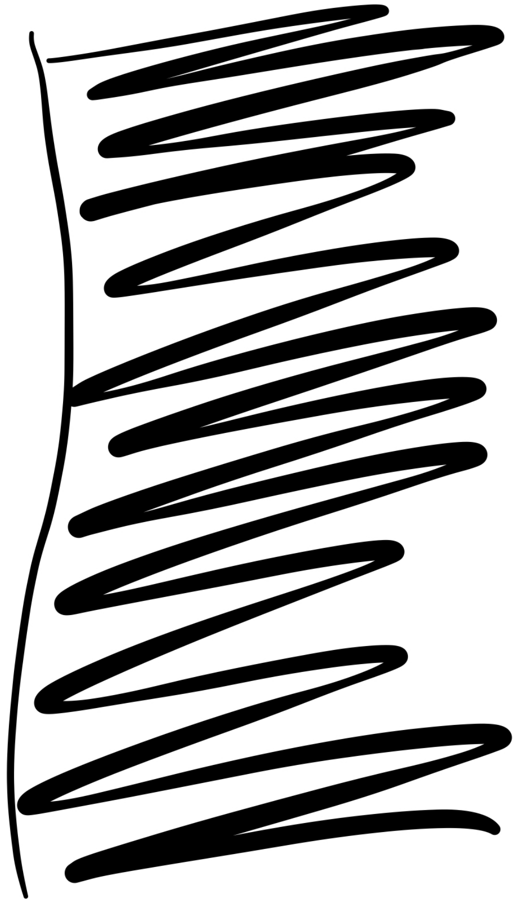
$U(1)_V$ (anomaly free) and $U(1)_A$ (anomalous)

$V = -\Lambda^4 \cos(\theta_\phi + \theta_\eta) - m_\psi \Lambda^3 \cos(\theta_\eta)$

Lowest energy topological defects:

$\Rightarrow$ **String-wall** of $U(1)_V$: $\theta_\phi = -\theta_\eta = n\varphi$

$\Rightarrow$ String-wall of combination of $U(1)_V \times U(1)_A$: $\theta_\eta = n\varphi$



DFSZ TYPE AXION MODEL

PQ field Φ, two Higgs doublets H_u and H_d (with phases χ_u and χ_d), two light quarks u and d

$m_u = 0$: anomaly free $U(1)_V$

Lowest energy axion string: $\theta_\phi = -\chi_u = n\varphi$ combination of $U(1)_V$ and $U(1)_Z$

$\Rightarrow$ Axion strings carry non-integer Z-flux (semiglobal) [Dvali, Senjanovic ’93],

can have winding in π^0

$m_u \neq 0$: String-wall system

IMPLICATIONS FOR

- Anomaly inflow & Superconductivity [Callan, Harvey ’85, Witten ’85]:

  winding of phase of quark condensate around string changes its anomaly content

- Cosmology: string-wall systems of phases of QCD condensate can dominate early cosmology if $\Lambda \gg f_\phi$ in early Universe [Dvali ’95]

- QCD phase transition: produce string-wall system of phases of QCD quark condensate, their collapse produces gravitational waves

- Heavy Ion colliders: production of string-wall systems of QCD condensate?

SUMMARY

- Topological defects in phases of the QCD quark condensate
- Axion string-wall systems accompanied by windings in QCD quark condensate

  ⇒ Affects physical properties of axion defects
- Relevant for QCD phase transition, early Universe, Cosmology

BACKUP SLIDES

DFSZ TYPE AXION MODEL

Effective potential for $m_u = 0$:

$$V = -\mu f_\phi v_u v_d \cos(\theta_\phi + \chi_u + \chi_d) - m_d \Lambda_d^3 \cos(\chi_d + \theta_d) - m_d \Lambda_u^3 \cos(\theta_u - \chi_d) - \Lambda^4 \cos(\theta_u + \theta_d)$$

Comes from $\mu \Phi H_u H_d$, Yukawa of d, Yuk of d + 'tHooft det, 'tHooft det

$$U(1)_V: \theta_\phi \to \theta_\phi - 2\alpha, \;\; \chi_u \to \chi_u + \alpha, \;\; \chi_d \to \chi_d + \alpha, \;\; \theta_u \to \theta_u + \alpha, \;\; \theta_d \to \theta_d - \alpha$$

Lowest energy string: $n_\phi = 1, \;\; n_{\chi_u} = -1$

Z-flux: $\oint dx_\mu Z^\mu = \frac{1}{g_z} \frac{v_u^2 n_{\chi_u} - v_d^2 n_{\chi_d}}{v_u^2 + v_d^2}$

INTERNAL STRUCTURE OF QCD WALLS

- Is the phase of the quark condensate well-defined throughout the wall?
- The absolute value of the quark condensate has to be non-zero throughout the wall
- θ_{eff} changes by 2π through wall, so corrections beyond dilute instant gas approximation may be important for η' domain walls
- Closest cousins: domain walls in $\mathcal{N} = 1$ SQCD [Dvali, Shifman '97]
- Exact solution in large N: gaugino condensate is non-zero throughout wall and phase is well-defined everywhere [Dvali, Kakushadze '99]

ANOMALY INFLOW

- Fermions with Yukawa couplings to string field (Φ) deposit 0-mode on string [Jackiw, Rossi '81]
- Zero modes of charged fermions carry superconducting current along string [Witten '85]
- Anomalous global symmetry forming string $\Rightarrow$ 1+1 dimensional theory is anomalous
- Anomaly of 1+1 dimensional theory is canceled by anomaly inflow from bulk [Callan, Harvey '85]
- Important astrophysical implications (electromagnetic radiation [Ostriker, Thompson, Witten '86], String-magnetic field interactions [Agrawal, Hook, Huang, Marques-Tavares '21])

- Axion string with one heavy Ψ and one light quark ψ is anomaly free
- Zero modes of Ψ and ψ have opposite chirality

COSMOLOGY

- QCD gauge coupling can strongly depend on the inflaton Σ due to effective operators $W\left(\frac{\Sigma}{M}\right)\mathrm{Tr}G_{\mu\nu}G^{\mu\nu}$
- possible early epoch hierarchy $\left.\frac{f_\phi}{\Lambda}\right|_{\mathrm{early}} \ll 1$ [Dvali '95]
- string-wall systems of phases of QCD condensate can dominate early cosmology
- cosmic strings (PQ or QCD condensate) immediately accompanied by domain walls

PION STRING-WALL SYSTEMS

Phases of light quarks u and d: θ_u and θ_d

Global string in π^0-direction: $n_{\theta_u} = -n_{\theta_d} = 1$

Quark masses ⇒ String-walls

If $v \ll f_u, f_d$ at the moment of quark condensation (early strong QCD):

Z eats up π^0

$n_{\theta_u} = -n_{\theta_d} = -n_\chi = 1$

Integer Z-flux, similar to semilocal string [Achucarro, Vachaspati '00]

HEAVY ION COLLISIONS

- formation of solitons during the collision of a small number of high energy quanta is exponentially suppressed [Brown ’92, Voloshin ’92, Argyres, Kleiss, Papadopoulos ’93, …]
- If final state soliton has maximal microstate degeneracy, it’s possible to overcome suppression
- For our case, requires further investigation